

An Introduction to Multilevel Modeling with Longitudinal Data

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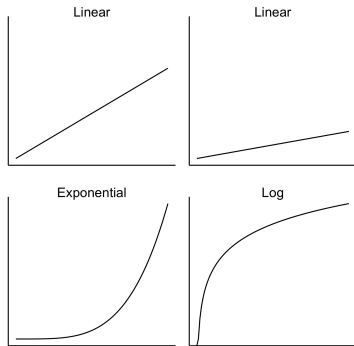
Section 1

The Scientific Study of Change

Two Questions

- ① How does the outcome change over time?
 - This question asks us how we can characterize (or describe) each person's pattern of change over time.
 - Example:
 - How might a person's depression change as a result of receiving therapy for a year?
 - How might a child's height change over time?

Two Questions



Two Questions

- ② Can we predict differences in these changes?
 - This question asks us to consider the relationship between predictors and the pattern of change.
 - Example:
 - Which variables predict individual differences in the changes in depression (while receiving therapy)?
 - Which variables predict differences in the changes in child height?

Two Questions

- MLM can be conceptualized as a pair of statistical models: one for each question.
- In the Level-1 model, we examine within-individual change over time and describe each person's individual growth trajectory.
- In the Level-2 model, we examine how these patterns of change differ among participants and which variables explain these differences.

Characteristics of Data

- Longitudinal research questions ask about change over time.
- The research context dictates how we measure time.
- As psychologists, we may consider:
 - age
 - grade
 - time from onset of symptoms
 - number of therapy sessions
 - number of exposures to a stimulus.

Characteristics of Data

- When deciding on a metric of time, we must consider practical and theoretical issues.
 - It may not be realistic to expect the patient to know the exact onset of depressive symptoms.
 - If your research question asks about how working memory span changes during adulthood, you may measure age in years, as opposed to months or days.

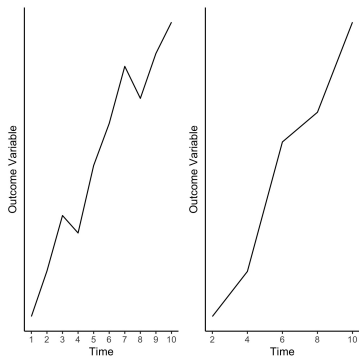
Characteristics of Data

- Can you give examples of specific longitudinal questions in your sub-field?
- How do people measure time in this context?

Waves of Measurement

- Each measurement is called a *wave* of data. What is the optimal number of waves?
- This will depend on the exact research question. We must have three or more waves to study change, but there is no upper limit.
- In repeated measures research, we can easily imagine having 20 or 30 waves (e.g., a sustained attention task may have 30 blocks of 15 unique stimuli for a total of 450 trials). In most longitudinal research, we have fewer waves of data.

Waves of Measurement



Long and Wide Data

- In the wide-format, each row represents everything we know about a single participant. This is the typical way of representing data.

Participant	Time 1	Time 2	CV
1	20	30	12
2	25	28	14
3	19	33	12

- This format is ideal for obtaining descriptive statistics and lends itself to thinking about between-person questions.

Long and Wide Data

- In the long-format, each row represents a single observation. This may seem counter-intuitive at first glance because we repeat much information.

Participant	Time	Outcome	CV
1	Time 1	20	12
1	Time 2	30	12
2	Time 1	33	14
2	Time 2	30	14
3	Time 1	19	12
3	Time 2	33	12

- This format lends itself to thinking about changes over time.

Long and Wide Data

- Most MLM software requires your data to be in a long-format, though every statistical software package makes it easy to switch between formats.
- It is often useful to get descriptive statistics using the wide format, then conduct your modeling in the long format.

Statistical Modeling

- We said that our goal is to model change over time. What is a statistical model?
- A statistical model is a set of assumptions that allow us to make inferences about the data generating process. The model allows us to represent the way we think variables relate.
- The goal of modeling is to use sample data to draw inferences about a population or the value of a parameter.
- Linear regression is probably the most commonly used statistical model.

Statistical Modeling

ID	Gender	Peer	SE 6	SE 7	SE 8	SE 9
1	female	5	13	11	8	8
2	female	5	16	15	15	14
3	male	3	16	15	11	8
4	female	6	12	9	7	6
5	male	2	12	NA	7	6

Note: Peer = Peer Support; SE = Self-Esteem

Descriptive Statistics

Our sample consisted of 163 (54%) females and 137 males.

Variable	M	SD	Range
Peer Support	4.53	1.13	0.92 - 7.72
Self-Esteem (6 th grade)	15.36	3.26	4 - 25
Self-Esteem (7 th grade)	14.16	3.42	2 - 25
Self-Esteem (8 th grade)	12.79	3.67	1 - 25
Self-Esteem (9 th grade)	11.87	3.88	1 - 26

Statistical Modeling

- We call predictors time invariant when they do not differ across observations. Both of our predictors are time invariant.
- We included missing data in our dataset. This was done to show the flexibility of MLM.
- We assume our data is missing at random.

OLS Regression

- We will start with a linear regression to illustrate some principles of statistical modeling.
- In this example, we will regress self-esteem on gender and peer support at the first time point.
- We note that traditional linear regression cannot effectively model change over time.

OLS Regression

- Gender is dummy coded with female serving as the reference group. This means that females are coded 0 and males are coded 1.
- Peer support is mean centered, so, for example, scores greater than 0 indicate above average peer support.
- Centering quantitative predictors can be helpful in linear regression. However, the practice of centering the time variable is essential in MLM.
- We also include an interaction between gender and peer support.
- Some participants are missing data on at least one variable in the regression, so none of their data can be used in this model. We will not have this limitation in MLM.

OLS Regression

Based on our assumptions, our statistical model is given by:

$$Y_i = \beta_0 + \beta_1 GENDER_i + \beta_2 PEER_i + \beta_3 (GENDER_i \times PEER_i) + \epsilon_i,$$

where $\epsilon_i \stackrel{iid}{\sim} \text{Normal}(0, \sigma^2)$.

Interpretation of Coefficients

$$Y_i = \beta_0 + \beta_1 GENDER_i + \beta_2 PEER_i + \beta_3 (GENDER_i \times PEER_i) + \epsilon_i,$$

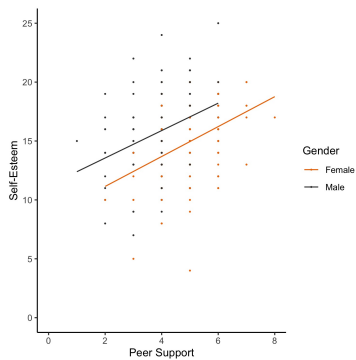
where:

- β_0 is the average self-esteem for female students with average peer-support
- β_1 is the difference in average self-esteem between male and female students
- β_2 is the change in average slope given a one unit change in peer support
- β_3 is the difference in average slope between male and female students
- ϵ_{1i} is individual i 's deviation between the predicted and observed values

OLS Regression

Fixed Effect	Coefficient	<i>S.E.</i>
$\beta_0 = \text{Intercept}$	14.36	0.27
$\beta_1 = \text{Slope for } GENDER$	2.14	0.40
$\beta_2 = \text{Slope for } PEER$	1.27	0.24
$\beta_3 = \text{Slope for } (GENDER \times PEER)$	-0.11	0.34
Random Effect	Variance Component	
$\sigma^2 = \text{var}(\epsilon_i)$	8.86	
Deviance	2365.03	

Visualizing OLS Regression



Predicted Value

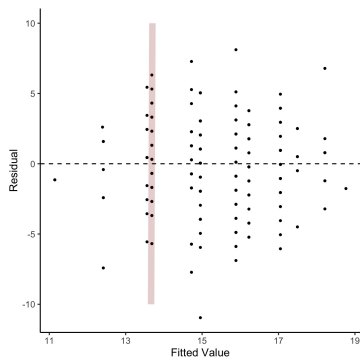
$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 GENDER_i + \hat{\beta}_2 PEER_i + \hat{\beta}_3 (GENDER_i \times PEER_i)$$

$$\begin{aligned}\hat{Y}_1 &= 14.36 + 2.14(0) + 1.27(0.47) - 0.11(0 \times 0.47) \\ &= 14.36 + 1.27(0.47) \\ &= 14.96\end{aligned}$$

The residual is given by

$$14.96 - 13 = 1.96.$$

OLS Regression



Substantive Interpretation

- We do not show the associated p -values for each effect. A coefficient estimate larger than two standard errors would indicate a p -value less than .05.
- Females with average peer support have a predicted self-esteem of 14.36.
- The results reveal that female students, on average, have lower self-esteem than male students (at least during 6th grade). Specifically, male students average 2.14 units higher self-esteem.
- Students with higher peer support tend to have a higher self-esteem, which is expected. A one-unit change in peer support is associated with a 1.27 unit change in self-esteem.

Substantive Interpretation

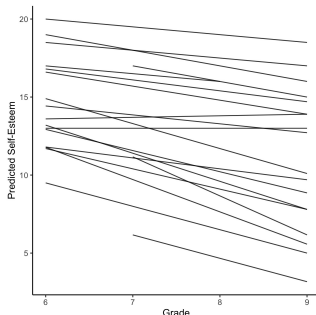
- The interaction between gender and peer support is negligible. The coefficient estimate is less than one standard error, which means that we have little evidence that the effect is different from zero in the population. Should we remove the effect from our model?
- The residual standard error ($\sqrt{8.86} = 2.98$) represents the average amount that self-esteem will deviate from the “true” regression line. Given that our estimate for females with an average self-esteem is 14.36, we can say that we expect any prediction to be off by $2.98 \div 14.36 = 20.75\%$.

Pooling

- We will show two naïve approaches modeling change using linear regression. However, as mentioned above, neither will be adequate.
- Both approaches can be seen as a response to the question: Can data from one person inform us about other people?
- Put differently, how much between-person variability should we expect?

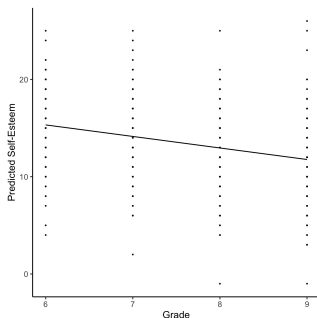
No Pooling

- In the No Pooling approach, each person is modeled separately.
- This can be problematic because it implicitly assumes between-person variation is infinite.
- Estimates will be especially unreliable when there are few observations for a given person.



Complete Pooling

- In the Complete Pooling approach, data are simply pooled and we compute aggregate estimates.
- This can be problematic because it implicitly assumes between-person variation is 0.
- Often our research goal is to understand individual variation. This approach assumes that there is no meaningful variation.



Pooling

- In most research contexts, we would intuitively think that each person contributes to our understanding of the underlying phenomena. However, we also appreciate that there will be individual differences.
- MLM involves a process of partial pooling. That is, we set a common parameter (called a *fixed effect*) and then allow each individual to deviate from the fixed effect (called a *random effect*).
- Partial pooling is especially useful when we have missing data because the model can use information from other participants to provide an estimate for the missing value. MLM does not drop a person if they have some missing data.

Section 2

Building an MLM with Longitudinal Data

With longitudinal data MLMs embody two types of research questions:

- ① Level-1 questions about *within-person change*
- ② Level-2 questions about *between-person differences in change*.

Examples:

- How does each child's neurological functioning change over time? (Level-1 question)
- Do children's trajectories of change vary by birth weight? (Level-2 question)

These questions correspond to two submodels that together constitute the MLM.

A Taxonomy of MLMs of Change

- We begin with the most basic model and then proceed to add to this base model in pursuit of greater flexibility.
- The 3 models we'll cover are the:
 - 1 Unconditional Means Model
 - 2 Unconditional Growth Model
 - 3 Conditional Growth Model
- The reasons behind the names of each model will become clear.

The Unconditional Means Model

- The **unconditional means model** doesn't attempt to track individual change over time.
- More precisely, it stipulates that each individual's trajectory (slope over time) is flat; making it an inappropriate specification for research questions regarding change over time.
- So why cover it?
- Its results (and those of the unconditional growth model) allow you to establish:
 - ① Whether there is systematic variation in your outcome that's worth exploring;
 - ② Where that variation resides (i.e., within or between individuals).

Specification

- We'll specify MLMs as two submodels: A level-1 model and a level-2 model.
- We write the unconditional means model as:

$$Y_{ti} = \pi_{0i} + \epsilon_{ti}$$

$$\pi_{0i} = \gamma_{00} + \zeta_{0i}.$$

- Parameter γ_{00} is the grand mean, while π_{0i} is a person-specific mean.
- Notice there are two error terms: a level-1 error ϵ_{ti} and level-2 error ζ_{0i} .
- Finally, notice time isn't included as a predictor, therefore the model predicts self esteem doesn't change over time.

Varying Intercepts

- The unconditional means model allows for people to have different intercepts through the inclusion of a person-specific mean π_{0i} .
- By substituting for π_{0i} in the level-1 submodel we obtain the **composite model**:

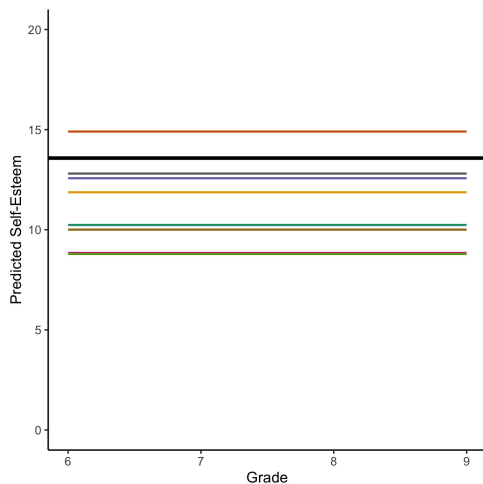
$$Y_{ti} = \gamma_{00} + \zeta_{0i} + \epsilon_{ti}.$$

- The primary reason we consider this rudimentary model is to estimate the variance components, $\text{var}(\epsilon_{ti})$ and $\text{var}(\zeta_{0i})$, which assess the amount of outcome variation at each level.

Unconditional Means Model

Fixed Effect	Coefficient	<i>S.E.</i>
$\gamma_{00} = \text{Intercept}$	13.58	0.20
Random Effect	Variance Component	
$\sigma_0^2 = \text{var}(\zeta_{0i})$	11.18	
$\sigma_\epsilon^2 = \text{var}(\epsilon_{ti})$	3.21	
Deviance	5100.04	

Unconditional Means Model



$$\hat{Y}_{ti} = 13.88 + \zeta_{0i} + \epsilon_{ti}$$

Fixed Intercept		13.58
Student	Random Intercept	
01	-3.34	
02	1.33	
03	-1.01	
04	-4.74	
05	-4.79	

An Unconditional Growth Model

- The unconditional means model assumes self-esteem does not change for students over time, although the initial level of self-esteem can differ across students.
- An unconditional growth model rectifies this lack of dynamics by allowing self esteem to change for each student over time.
- We'll now introduce the unconditional growth model and be more explicit about the assumptions made.

The Level-1 Submodel

The level-1 submodel is an individual *growth* model. It represents the change we expect each student to experience during the time period under study. We write it as:

$$Y_{ti} = \pi_{0i} + \pi_{1i} \text{GRADE}_{ti} + \epsilon_{ti},$$

where:

- π_{0i} is the intercept of individual i 's trajectory in the population
- π_{1i} is the slope of individual i 's trajectory in the population
- ϵ_{ti} is the level-1 error across time for individual i in the population.

Distributional Assumptions: Level-1 Error

- As a first approximation conventional regression assumptions of the level-1 error, ϵ_{ti} , are invoked.
- Namely, it's assumed that ϵ_{ti} are independently and identically distributed with mean 0, a constant variance across occasions and individuals.
- It's also standard to assume the ϵ_{ti} are distributed normally.
- Putting these assumptions together we can write:

$$\epsilon_{ti} \overset{iid}{\sim} \text{Normal}(0, \sigma_{\epsilon}^2),$$

where σ_{ϵ}^2 is the (constant) variance capturing the scatter of level-1 errors around each individual's trajectory.

Centering the Temporal Predictor

- It's essential to center the time predictor when working with longitudinal data.
- There are empirical and interpretative issues you'll need to consider when deciding how to center time, but a good starting point is to center time around the first wave of data.
- Doing so gives each intercept a straightforward interpretation: It's individual i 's *initial status*.
- If, as in our example, the first wave of data collected is 6th grade, then the level-1 submodel takes the form:

$$Y_{ti} = \pi_{0i} + \pi_{1i}(\text{GRADE}_{ti} - 6) + \epsilon_{ti},$$

where π_{0i} is now individual i 's self-esteem in the 6th grade.

The Level-2 Submodel

- The unconditional growth model, as suggested by its name, stipulates that each individual's trajectory can change over time.
- So now we include a second equation at level 2 that allows each individual to have their own slope.

$$\pi_{0i} = \gamma_{00} + \zeta_{0i}$$

$$\pi_{1i} = \gamma_{10} + \zeta_{1i}.$$

Varying Intercepts

The intercept of individual i 's trajectory can be written as:

$$\pi_{0i} = \gamma_{00} + \zeta_{0i},$$

where:

- γ_{00} is mean initial status of self-esteem
- ζ_{0i} represents individual i 's deviation from the mean initial status of self-esteem.

Varying Slopes

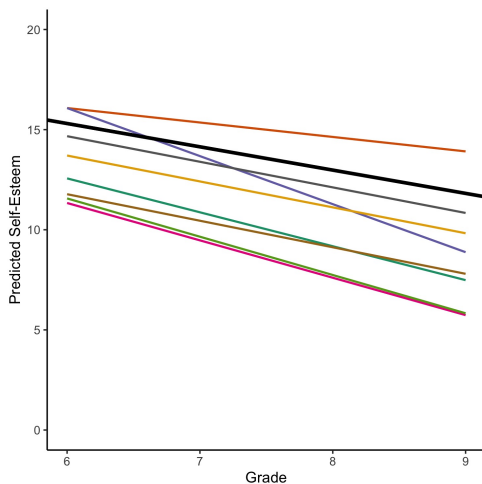
The slope of individual i 's trajectory can be written as:

$$\pi_{1i} = \gamma_{10} + \zeta_{1i}$$

where:

- γ_{10} is the mean rate of change in self-esteem over time.
- ζ_{1i} represents individual i 's deviation from the mean rate of change.

Unconditional Growth Model



$$\hat{Y}_{ti} = 15.31 - 1.16(\text{GRADE}_{ti} - 6) + \zeta_{0i} + \zeta_{1i}(\text{GRADE}_{ti} - 6) + \epsilon_{ti}$$

Fixed Intercept

15.31

Fixed Slope

-1.16

Student	Random Intercept	Random Slope
01	-2.74	-0.53
02	0.77	0.44
03	0.77	-1.24
04	-3.97	-0.70
05	-3.74	-0.75

Variance Components: Level-2 Variances

- Often we want to know how much variation exists between individual's at their initial status and growth trajectory. In other words, we often care about individual differences as a substantive research question.
- Level-2 errors represent deviations between the individual growth parameters and their respective averages.
- Their variances, σ_0^2 and σ_1^2 , summarize the variation in individual intercepts and slopes around these averages.
- For example, if σ_0^2 is large, then there's significant variation around self-esteem at grade 6.

Variance Components: Level-2 Covariance

- By positing a level-2 submodel we also allow for the association between initial status and rates of change.
- The covariance between ζ_{0i} and ζ_{1i} summarizes the association between individual intercepts and slopes.
- The covariance of ζ_{0i} and ζ_{1i} , denoted by σ_{01} , allows us to investigate whether initial status and rate of change are related.
- For example, if σ_{01} is positive and large, then students with higher than average self-esteem at grade 6 tend to experience larger increases in self-esteem over time.

Distributional Assumptions: Variance Components

- As before, we'll have to make distributional assumptions about the level-2 errors.
- The difference here is that there are *two* level-2 errors (ζ_{0i} and ζ_{1i}).
- The standard assumptions of the bivariate distribution of ζ_{0i} and ζ_{1i} can be compactly written as follows:

$$\begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim \text{Normal} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{bmatrix} \right).$$

The Composite Model

- As shown earlier the two submodels combine to form a composite model.
- Substituting for π_{0i} and π_{1i} the composite MLM of change becomes:

$$Y_{ti} = \underbrace{\gamma_{00} + \gamma_{10}(\text{GRADE}_{ti} - 6)}_{\text{Fixed}} + \underbrace{\zeta_{0i} + \zeta_{1i}(\text{GRADE}_{ti} - 6) + \epsilon_{ti}}_{\text{Random}}.$$

Fixed and Random Effects

It's standard to refer to certain coefficients as either “fixed” or “random effects.” This can be confusing because there is no agreed upon definition to identify which effects are fixed and which are random. Gelman and Hill (2007) found at least five conflicting definitions in the literature:

1. Fixed effects are constant across individuals, and random effects vary. For example, in a growth study, a model with random intercepts α_i and fixed slope β corresponds to parallel lines for different individuals i , or the model $y_{it} = \alpha_i + \beta t$. Kreft and De Leeuw (1998, p. 12) thus distinguish between fixed and random coefficients.
2. Effects are fixed if they are interesting in themselves or random if there is interest in the underlying population. Searle, Casella, and McCulloch (1992, section 1.4) explore this distinction in depth.
3. “When a sample exhausts the population, the corresponding variable is *fixed*; when the sample is a small (i.e., negligible) part of the population the corresponding variable is *random*” (Green and Tukey, 1960).
4. “If an effect is assumed to be a realized value of a random variable, it is called a random effect” (LaMotte, 1983).
5. Fixed effects are estimated using least squares (or, more generally, maximum likelihood) and random effects are estimated with shrinkage (“linear unbiased prediction” in the terminology of Robinson, 1991). This definition is standard in the multilevel modeling literature (see, for example, Snijders and Bosker, 1999, section 4.2) and in econometrics.

In a multilevel model, this definition implies that fixed effects β_j are estimated conditional on a group-level variance $\sigma_\beta = \infty$ and random effects β_j are estimated conditional on σ_β estimated from data.

Fixed and Random Effects

- The two γ parameters are referred to as **fixed effects**. They capture systematic interindividual differences in trajectories. In the present model they're simply mean initial status (γ_{00}) and mean rate of change (γ_{10}) across all students.
- The two ζ parameters are the model's **random effects**. Random effects allow each student's trajectory to be scattered around the average trajectory:
 - The initial status of all students scattered around mean initial status (γ_{00});
 - Each student's rate of change scattered around the mean rate of change (γ_{10}) of all students.

A Conditional Growth Model

- After investigating the change trajectories of students in the unconditional growth model, a reasonable next step would be to consider whether these trajectories differ systematically by other variables.
- We now analyze whether gender and peer support have significant impacts on these change trajectories.
- So: How should we introduce these variables into the model?

Adding Predictors

- In our conditional growth model we'll include predictors in the level-2 submodel that can predict individual differences in intercepts and slopes.

$$\pi_{0i} = \gamma_{00} + \gamma_{01}GENDER_i + \gamma_{02}PEER_i + \zeta_{0i}$$

$$\pi_{1i} = \gamma_{10} + \gamma_{11}GENDER_i + \gamma_{12}PEER_i + \zeta_{1i}.$$

Varying Intercepts

The intercept of individual i 's trajectory is now written as:

$$\pi_{0i} = \gamma_{00} + \gamma_{01}GENDER_i + \gamma_{02}PEER_i + \zeta_{0i},$$

where:

- γ_{00} is average initial status of self-esteem for female students with average peer support
- γ_{01} is the difference in average self-esteem between male and female students
- γ_{02} represents the mean change in self-esteem given a one unit change in peer support
- ζ_{0i} is individual i 's deviation **FINISH**

Varying Slopes

The slope of individual i 's trajectory is now written as:

$$\pi_{1i} = \gamma_{10} + \gamma_{11}GENDER_i + \gamma_{12}PEER_i + \zeta_{1i},$$

where:

- γ_{10} is average slope for female students with average peer support
- γ_{11} is the difference in average slope between male and female students
- γ_{12} represents the mean change in average slope given a one unit change in peer support
- ζ_{1i} is individual i 's deviation **FINISH**

Cross-Level Interactions

- Deriving the composite model with level-2 predictors is an algebraic mess because many terms are multiplied by the time predictor.
- The resulting equation will include two cross-level interactions:

$$(GRADE_{ti} - 6) \times GENDER_i \text{ and } (GRADE_{ti} - 6) \times PEER_i.$$

- These interactions result from permitting individuals to have different change trajectories and positing that these trajectories are impacted by gender and peer support.
- **EXPLAIN: Read ALDA**

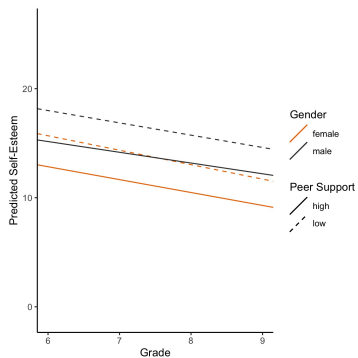
Reporting Results: Fixed Effects

Fixed Effects	Coefficient	SE
γ_{00} = Intercept	14.26	0.24
γ_{01} = Intercept (GENDER)	2.31	0.37
γ_{02} = Intercept (PEER)	1.23	0.16
γ_{10} = Slope for (GRADE)	-1.25	0.05
γ_{11} = Slope Differential (GENDER)	0.20	0.08
γ_{12} = Slope for PEER	-0.06	0.04

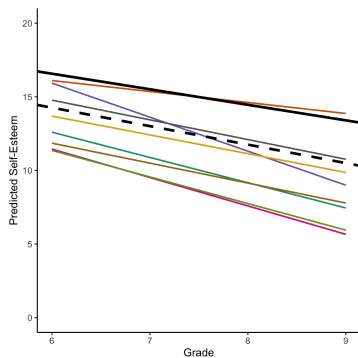
Reporting Results: Variance Components

Random Effects	Variance Component
Level 1:	
σ_{ϵ}^2	0.45
Level 2:	
$\sigma_0^2 = \text{var}(\zeta_{0i})$	8.40
$\sigma_1^2 = \text{var}(\zeta_{1i})$	0.29
$\sigma_{01} = \text{cov}(\zeta_{0i}, \zeta_{1i})$	0.18

Conditional Growth Model



Conditional Growth Model



Substantive Interpretation: Fixed Effects

- Students self-esteem gets lower from 6th grade to 9th grade. Holding all other effects constant, there is a drop of 1.3 units of self-esteem per year.
- On average, females have lower self-esteem than males. Specifically, males students average 2.3 units higher self-esteem.
- The cross-level interaction between grade and gender indicates that the gender difference gets slightly larger over time (by about 0.2 units per year).

Substantive Interpretation: Fixed Effects

- Peer support is a buffer against low self-esteem. A one unit increase in peer support is associated with a 1.2 unit increase in self-esteem.
- The cross-level interaction between grade and peer support is negligible.

Substantive Interpretation: Random Effects

- There is still a small correlation ($r = .18$) between the random intercept and random slope. This indicates that participants who have a higher than average initial status tend to have a larger (i.e., less negative) slope.
- We can compare the variance components from this model to the unconditional growth model.
 - Unconditional Growth Model: $\text{var}(\text{intercept})=10.33$; $\text{var}(\text{grade})=0.31$; $\text{var}(\text{Residual})=0.45$
 - Conditional Growth Model: $\text{var}(\text{intercept})=8.40$; $\text{var}(\text{grade})=0.29$; $\text{var}(\text{Residual})=0.45$
- This indicates that the predictors effectively reduced some of the variance in initial status, though there was a smaller reduction in the variability of random slopes.

What We Covered

- a review of multiple regression and a discussion of why these models are inadequate for studying change
- a discussion about the characteristics of longitudinal data
- an overview of the unconditional means, unconditional growth, and conditional growth models
- an illustrative analytic problem using longitudinal data

But wait, there's more!

- specification of nonlinear models or general linear models
- time varying predictors and unequal spacing between measurements
- troubleshooting when models don't converge
- implementation using specific statistical software

Resources: Books

- Singer, J. D., & Willet, J. B. (2003). *Applied Longitudinal Analysis: Modeling Change and Event Occurrence* (1st ed.). Oxford University Press.
- Snijders, T.A.B., & Bosker, R.J. (2012). *Multilevel analysis: An introduction to basic and advanced multilevel modeling* (2nd Ed.). London: Sage.
- Gelman, A., & Hill, J. (2006). *Data analysis using regression and multilevel/hierarchical models* Cambridge, England: Cambridge University Press.

Resources: Articles and Online Resources

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