Advanced Macroeconomics II

Lecture 2

Investment: Frictionless and Convex Adjustment Costs

Isaac Baley

UPF & Barcelona GSE

January 11, 2016

Motivation

- Investment increases productive capacity of the economy
  $\Rightarrow$ key to determine standards of living in the *long-run*.
- Investment is highly volatile
  $\Rightarrow$ key to understand *short run (business cycle)* fluctuations.
- Investment depends on real interest rates
  $\Rightarrow$ key to understand impact of *monetary policy*
- Investment is a channel through which many fiscal instruments act
  $\Rightarrow$ key to understand impact of *fiscal policy*
- Some facts...

Facts about investment

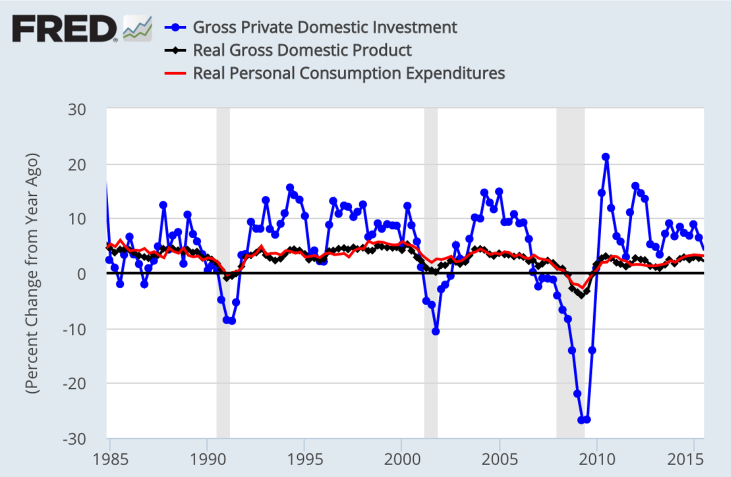
- **Fact 1: Aggregate investment is relatively volatile.**
- **Fact 2: High correlation of aggregate investment with output.**
  - HP-detrended, quarterly time series from US during 1954 –1991.

Variable	St Dev (%)	Correlation with GNP
GNP	1.72	1
Consumption (non-durables)	0.86	0.77
Investment (gross private domestic)	8.24	0.91
Hours Worked	1.59	0.86

Cooley and Prescott (1996)

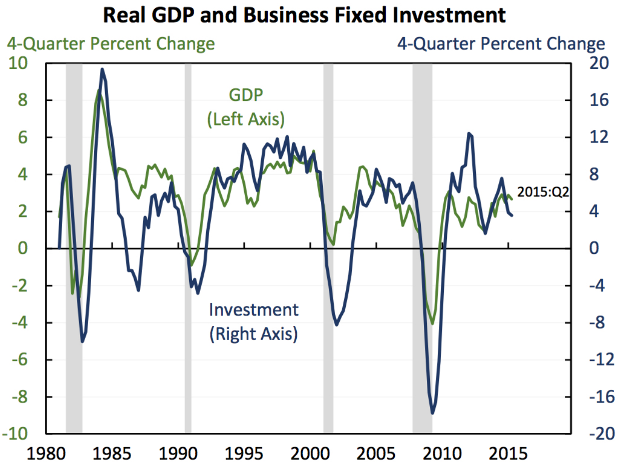
Facts about investment

- **Fact 1: Aggregate investment is relatively volatile.**



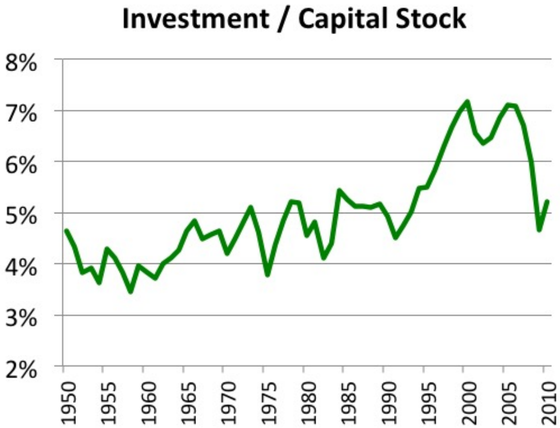
Facts about investment

- **Fact 2: High correlation of aggregate investment with output.**
  - Investment gradually increases and decreases along the business cycle.



Facts about investment

- **Fact 3: Investment to Capital Ratio is relatively stable**



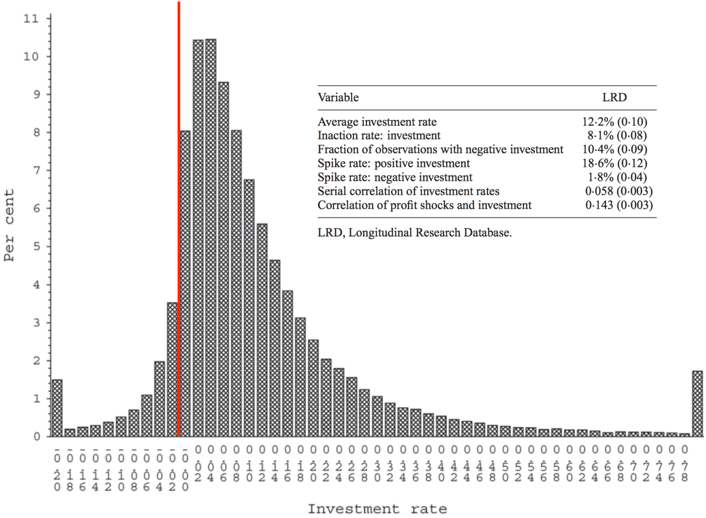
- Law of motion: $K_{t+1} = (1-\delta)K_t + I_t$
- In steady state, $K_{t+1} = K_t \implies \frac{I_t}{K_t} = \delta$.

Facts about investment

- **Fact 4: Investment at the plant level is "lumpy".**
- Lumpiness: Spikes (infrequent and large changes) and Inaction
- Doms and Dunne (1998), Cooper and Haltiwanger (2005)
  - Longitudinal Research Database, 7000 US plants during 1972-1988.
  - 18% of plants report investment rates of 20% (relative to capital).
  - 50% of plants experience a 1 year capital adjustment of $\geq$ 37%.
  - 80% of plants in a given year change their net capital stock $<$10%.
- Becker et al (2006)
  - Between 9 and 28% of plants have exactly zero investment in a year.

Facts about investment

- **Fact 4: Investment at the plant level is "lumpy".**



Variable	LRD
Average investment rate	12·2% (0·10)
Inaction rate: investment	8·1% (0·08)
Fraction of observations with negative investment	10·4% (0·09)
Spike rate: positive investment	18·6% (0·12)
Spike rate: negative investment	1·8% (0·04)
Serial correlation of investment rates	0·058 (0·003)
Correlation of profit shocks and investment	0·143 (0·003)

LRD, Longitudinal Research Database.

Source: Cooper and Haltiwanger (2005), *On the Nature of Capital Adjustment Costs*, REStud.

Roadmap

1. **Frictionless investment**
   - Rental model
   - Ownership model and user cost of capital
   - Some empirical evidence and a quick fix
2. Convex adjustment costs
   - Tobin's q theory
   - Quadratic adjustment cost (microfounds q theory)
   - Empirical Evidence

Frictionless investment

- Start from model **without adjustment costs**.
- First, we assume that firms **rent** capital every period from households.
  - Static problem.
  - No cost to change the level of capital they rent.
  - Same as in the Solow and Ramsey models.
- Second, we assume that the representative firm **owns** capital.
  - Dynamic problem: depreciation and future production.
  - No costs to change their investment (capital they purchase).
- Note: Without adjustment costs, who owns the capital does not matter.
- In both models: $\underbrace{\text{marginal benefit}}_{\text{marginal productivity of capital}} = \underbrace{\text{marginal cost}}_{\text{rental rate or user cost}}$

Frictionless investment: Rental model

- A firm **rents** capital K_t every period to produce.
- Suppose we can write profits, after optimizing over other inputs, as $\Pi(K_t, x_t)$, where x_t are other inputs' costs.
- Let r_K the rental cost of a unit of capital, then the firm solves:

$$\max_{K_t} \quad \Pi(K_t, x_t) - r_K K_t$$

- The first order condition (FOC) for the demand of capital is:

$$\Pi_K(K_t, x_t) = r_K \tag{1}$$

- If profit function exhibits decreasing returns to capital and the usual Inada conditions, then LHS is decreasing in K and RHS is constant $\Rightarrow$ unique K^* that solves (1).

Frictionless investment: Ownership model (1)

- A (risk neutral) representative firm **owns** capital K_t, rents labour L_t, produces output Y_t and maximizes profits Π_t.
- Let V_0 be the expected discounted value of profits for the firm.

$$V_0 = \max_{\{K_t, L_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{R} \right)^t \Pi_t \right] \quad (2)$$

where period profits and investment are given by

$$\begin{aligned} \Pi_t &= Y_t - p_t I_t - w L_t \\ I_t &= K_t - (1-\delta) K_{t-1} \end{aligned}$$

  - $p_t =$ price of the capital goods.
  - $w =$ wage (constant, partial eq.)
  - $R = 1 + r =$ gross risk free rate, assumed constant (partial eq.).
  - $\delta =$ depreciation rate.

- Think: Why does the firm discount with R? How would the discount change if firm was owned by a household?

Frictionless investment: Ownership model (2)

- The **production function** F transforms inputs (K_t, L_t) into output Y_t:

$$Y_t = F(K_t, L_t)$$
$$where: \quad F_K > 0, \quad F_L > 0, \quad F_{KK} < 0, \quad F_{LL} < 0, \quad F_{LK} > 0$$

  *Note 1: Capital is productive in the same period it is purchased. It can be modelled also with a time to build, where $Y_t = F(K_{t-1}, L_t)$.

  *Note 2: In partial equilibrium, no need to impose constant returns to scale.

- We restate the problem net of the flexible factors (only labor in this case):

$$L_t^*(K_t, w, p_t) = \arg\max F(K_t, L_t^*) - p_t I_t - wL_t^*$$

  and substitute into the value function:

$$V_0 = \max_{\{K_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t \mathbb{E}_0 \left[F(K_t, L_t^*) - p_t I_t - wL_t^* \right] \tag{3}$$

Frictionless investment: Ownership model (3)

- Substituting the expression for investment I_t, the problem becomes:

$$V_0 = \max_{\{K_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t \mathbb{E}_0 \left\{ F\left(K_t, L_t^*\right) - p_t \overbrace{\left[K_t - (1-\delta) K_{t-1}\right]}^{I_t} - wL_t^* \right\} \tag{4}$$

- **Optimality:** First order condition for capital at a generic time t is:

$$F_k\left(K_t, L_t^*\right) - p_t + \frac{1}{R}(1-\delta)\,\mathbb{E}_t\left[p_{t+1}\right] = 0$$

- Rearrange to express as:

$$p_t = F_k\left(K_t, L_t^*\right) + \frac{1}{R}(1-\delta)\,\mathbb{E}_t\left[p_{t+1}\right] \tag{5}$$

  - Pay p_t today (units of output)
  - Produce today F_k (units of output)
  - Future resale value of undepreciated capital $(1-\delta)\,\mathbb{E}_t\left[p_{t+1}\right]$

Frictionless investment: Ownership model (4)

- Iterate on (5), use the law of iterated expectations and a transversality condition to express the **price of capital as discounted marginal product**:

$$p_t = \sum_{j=0}^{\infty} \left(\frac{1-\delta}{R} \right)^j \mathbb{E} \left[F_k \left(K_{t+j}, L^*_{t+j} \right) \right]$$

- Comparing FOC to that of rental model, we define the **user cost of capital**:

$$UK_t \equiv p_t - \frac{1}{R} (1-\delta) \mathbb{E}_t \left[p_{t+1} \right]$$

- UK_t is an estimate of the rental rate of capital r_K.
- UK_t increases with riskless rate R, depreciation δ, and current price p_t, and decreases with future price of capital goods $\mathbb{E}[p_{t+1}]$.

Frictionless investment: Ownership model (5)

- Particular case: Cobb-Douglas production with productivity θ_t:

$$Y_t = \theta_t K_t^{\alpha} L_t^{\beta} \tag{6}$$

Then the FOC reads:

$$\alpha \theta_t K_t^{\alpha - 1} L_t^{\beta} = UK_t$$

- Solve for K_t, we get the "desired" level of capital without adustment costs:

$$K_t^* = (L_t)^{\frac{\beta}{1-\alpha}} \left(\frac{\alpha \theta_t}{UK_t} \right)^{\frac{1}{1-\alpha}} \tag{7}$$

- If labour $\overline{L}$ is fixed, K_t^* follows closely θ_t and UK_t.

$$K_t^* = \left(\overline{L}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{\alpha \theta_t}{UK_t} \right)^{\frac{1}{1-\alpha}}$$

Frictionless investment: Consequences

- The UK model determines the stock of capital:

$$K_t^* = \left(\overline{L}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{\alpha\theta_t}{UK_t}\right)^{\frac{1}{1-\alpha}}$$

- Fluctuation in "desired" capital K_t^* are matched by equal fluctuations in observed capital K_t.
- Empirical studies: Not rejected as a long run relation.
- But it does not explain short-run fluctuations:
  - K_t^* does not depend on past or expected future levels of capital.
  - K_t^* is a jump variable $\Rightarrow$ Investment adjusts immediately.
  - Excessive volatility of I_t against data (Cooper & Haltinwanger, 2000).
- Need for something that **slows down adjustment of capital stock**.

Frictionless investment: A quick fix (1)

- Distinction between net investment I_t^n (after depreciation) and replacement investment $I_t^r = \delta K_{t-1}$

- The **flexible accelerator model**:

$$I_t^n = \beta \left(K_t^* - K_t \right), \qquad \beta < 1$$

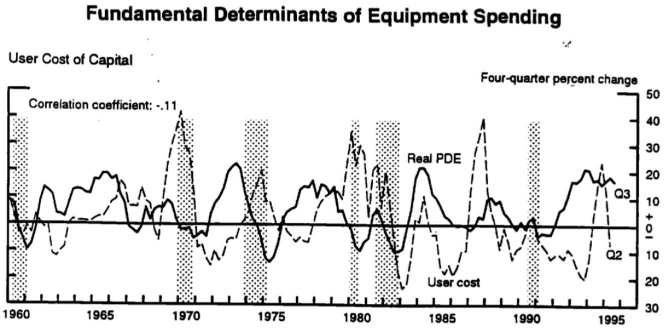
  Net investment closes the gap between desired and current capital stock.

- Delayed adjustment of K_t to K_t^* generates a negative correlation over time between UK_t and net investment rate.

- But such correlation is not strong in the data!

Frictionless investment: A quick fix (2)

- Low empirical correlation between user cost UK_t and investment rate (equipment spending).



- Better strategy: micro-founded model with convex adjustment costs.

Roadmap

1. **Frictionless investment**
    - Rental model
    - Ownership model and user cost of capital
    - Some empirical evidence and a quick fix
2. **Convex adjustment costs**
    - Tobin's q theory
    - Quadratic adjustment cost (microfounds q theory)
    - Empirical Evidence

Tobin's q

- Define the discounted return of **installed capital** at t, which assumes no further investments, as:

$$W(K_t) \equiv \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j F((1-\delta)^j K_t)$$

  where the argument of the production function $F(\cdot)$ assumes capital evolves as $K_{t+j} = (1-\delta)^j K_t$ since $I_{t+j} = 0 \ \ \forall j > 0$.

- For simplicity we do not write explicitly labor and productivity but they do affect firms production.
- Define **average return on installed capital** Q as follows:

$$Q_t \equiv \frac{W(K_t)/K_t}{p_t}$$

  - Note: we must divide by p_t to convert units, since the numerator is measured in terms of output and the denominator in terms of capital.

Tobin's q

- Define **marginal return on capital** q as follows:

$$q_t \equiv \frac{\mathbb{E}_t\left[\partial W(K_t)/\partial K_t\right]}{p_t} = \frac{\mathbb{E}_t\left[\sum_{j=0}^{\infty}\left(\frac{1-\delta}{R}\right)^j F_K((1-\delta)^j K_t)\right]}{p_t} \tag{8}$$

- Note the derivative F_K in the previous expression.
- In other words:

$$q = \frac{\text{Market value of installed capital}}{\text{Replacement cost of installed capital}}$$

- q **measures the perpetual return from one marginal unit of installed capital.**
- Tobin's q theory states that investment is a function of q and r:

$$I = I\left(q, r\right) \quad \text{with} \quad \frac{\partial I}{\partial q} > 0 \;\; \text{and} \quad \frac{\partial I}{\partial r} < 0$$

- Simple rule: Invest if $q > 1$.

Tobin's q and frictionless model

- Is $q > 1$ consistent with frictionless model (i.e. $F_k = UK_t$)? No!!
- From the definition of q we have that

$$q_t - 1 = \frac{\mathbb{E}_t \left[\partial W(K_t) / \partial K_t \right] - p_t}{p_t} \tag{9}$$

where $\mathbb{E}_t \left[\partial W(K_t) / \partial K_t \right] - p_t$ is the NPV of marginal profits net of the cost of investment p_t.

- From (8) we have that:

$$\begin{aligned}
\mathbb{E}_t \left[\frac{\partial W(K_t)}{\partial K_t} \right] - p_t &= \mathbb{E}_t \left[\sum_{j=0}^{\infty} \left(\frac{1-\delta}{R} \right)^j F_K((1-\delta)^j K_t) \right] - p_t \\
&= F_K(K_t) - p_t + \frac{1-\delta}{R} \mathbb{E}_t \left[F_K(K_{t+1}) \right] \\
&+ \left(\frac{1-\delta}{R} \right)^2 \mathbb{E}_t \left[F_K(K_{t+2}) \right] + \dots
\end{aligned}$$

Tobin's q and frictionless model

- Adding and subtracting prices, we recover user cost UK_t at every period:

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{\partial W(K_t)}{\partial K_t}\right] - p_t &= F_K(K_t) - \underbrace{p_t + \frac{1-\delta}{R}\mathbb{E}_t[p_{t+1}]}_{UK_t} \\
&+ \frac{1-\delta}{R}\mathbb{E}_t\left[F_K(K_{t+1}) - \underbrace{p_{t+1} + \frac{1-\delta}{R}\mathbb{E}_t[p_{t+2}]}_{UK_{t+1}}\right] \\
&+ \left(\frac{1-\delta}{R}\right)^2 \mathbb{E}_t\left[F_K(K_{t+2}) - \underbrace{p_{t+2} + \frac{1-\delta}{R}\mathbb{E}_t[p_{t+3}]}_{UK_{t+2}}\right] + \ldots
\end{aligned}
$$

- Define the net marginal profits at time t, denoted π_t, as:

$$\pi_t \equiv F_K(K_t) - p_t + \frac{1}{R}(1-\delta)\mathbb{E}_t[p_{t+1}] = F_K(K_t) - UK_t$$

and rewrite as:

$$\mathbb{E}_t\left[\frac{\partial W(K_t)}{\partial K_t}\right] - p_t = \sum_{j=0}^{\infty}\left(\frac{1-\delta}{R}\right)^j \mathbb{E}_t[\pi_{t+j}] \tag{10}$$

Tobin's q and frictionless model (cont...)

- Summarizing, the expression for q under the frictionless model reads:

$$q_t - 1 = \frac{\mathbb{E}_t\left[\frac{\partial W(K_t)}{\partial K_t}\right] - p_t}{p_t} = \frac{\sum_{j=0}^{\infty}\left(\frac{1-\delta}{R}\right)^j \mathbb{E}_t\left[F_{K_{t+j}} - UK_{t+j}\right]}{p_t} = \frac{\sum_{j=0}^{\infty}\left(\frac{1-\delta}{R}\right)^j \mathbb{E}_t\left[\pi_{t+j}\right]}{p_t} \tag{11}$$

- But the profit maximizing condition of the **frictionless model** implies that:

$$F_{K_{t+j}} = UK_{t+j} \text{ for any j} \geq 0 \qquad \Longrightarrow \qquad \pi_{t+j} = 0 \text{ for any j} \geq 0$$

- It follows that without adjustment costs $q_t = 1$ always.

- Why? Since investment is frictionless, any situation with $q_t > 1$ triggers an immediate increase in investment until $q_t = 1$.

Reviving Tobin's q

- Tobin's q theory sounds appealing, and intuitively right.
- Empirically we observe a positive (even though weak) correlation between $\frac{I_t}{K_t}$ and average Q_t, both at the aggregate and at the firm level.
- How can we modify the model to make it compatible with the $q - theory$?
- Adjustment costs!
  - They reduce the volatility of investment, and are consistent with the realistic feature that the cost of installing capital adds up to the cost of purchasing it.

Convex adjustment costs (1): Idea

- Convex adjustment costs: small investments can be easily integrated in the current structure of the firm, while big investments create larger disruptions.
- Idea: adjustment costs generate positive relation between q and I.
  - Suppose that $q = 1$ and suddenly future net revenues are expected to increase.
  - $\mathbb{E}_t \left[\frac{\partial W(K_t)}{\partial K_t} \right]$ increases (one marginal unit of capital installed today generates more return in the future).
  - Investment goes up, but spread over time to minimize adjustment costs.
  - Since investments goes up a little today, $\mathbb{E}_t \left[\frac{\partial W(K_t)}{\partial K_t} \right]$ falls only little, and q is still larger than 1.
  - Therefore, q and I become positively related.

Convex adjustment costs (2): Problem

- Formally we write the problem with convex adjustments costs as:

$$\max_{\{I_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t [y_t - p_t I_t - C(I_t)] \right\}$$

- It is equivalent to write the problem in terms of investment or capital.
- Adjustment costs $C(I_t)$:
  - Convex function of investment ($C_I > 0, C_{II} > 0$)
  - Measured in units of output.
  - We specialize to quadratic adjustment costs:

$$C(I_t) \equiv \frac{\gamma}{2} I_t^2, \qquad \gamma > 0$$

Convex adjustment costs (3): Quadratic costs

- For simplicity we omit labour (think of a per capita production function):

$$y_t = \theta_t K_t^{\alpha}$$

- Substitute production function and adjustment costs to obtain:

$$\max_{\{I_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{R} \right)^t \left[\theta_t K_t^{\alpha} - p_t I_t - \frac{\gamma}{2} I_t^2 \right] \right\}$$

- First order condition (with respect to K_t) is:

$$\alpha \theta_t K_t^{\alpha-1} - p_t - \gamma I_t + \frac{1}{R} \left[\mathbb{E}_t \left[p_{t+1} \right] (1-\delta) + \gamma (1-\delta) \mathbb{E}_t \left[I_{t+1} \right] \right] = 0$$

- Solving for I_t:

$$I_t = \frac{1}{\gamma} \left(F_{K_t} - UK_t \right) + \frac{1}{R} (1-\delta) \mathbb{E}_t \left[I_{t+1} \right] \qquad (12)$$

where:

$$UK_t = p_t - \frac{1-\delta}{R} \mathbb{E}_t \left[p_{t+1} \right] \quad \text{and} \quad F_{K_t} = \alpha \theta_t K_t^{\alpha-1}$$

Convex adjustment costs (4): Policy

- Iterating (12) forward:

$$I_t = \frac{1}{\gamma} \sum_{j=0}^{\infty} \left(\frac{1-\delta}{R} \right)^j \mathbb{E}_t \left[F_{K_{t+j}} - UK_{t+j} \right] \tag{13}$$

- Comparing this to the expression for q in (11) above, which read:

$$q_t - 1 = \frac{\sum_{j=0}^{\infty} \left(\frac{1-\delta}{R} \right)^j \mathbb{E}_t \left[F_{K_{t+j}} - UK_{t+j} \right]}{p_t}$$

  we obtain that:

$$I_t = \frac{1}{\gamma} \left(q_t - 1 \right) p_t \tag{14}$$

- Now investment is a positive function of q, as argued by Tobin.

Convex adjustment costs (5): Intuition

- Intuition for expression (14):

$$I_t = \frac{1}{\gamma}\left(q_t - 1\right) p_t$$

- Rewrite as:

$$p_t + \gamma I_t = q_t p_t$$

  - Since $C\left(I_t\right) = \frac{\gamma}{2} I_t^2$, then the marginal cost is $C'\left(I_t\right) = \gamma I_t$
  - From the definition of q_t, we have that: $q_t p_t = \mathbb{E}_t\left[\frac{\partial W(K_t)}{\partial K_t}\right]$

- Therefore, expression (14) implies that optimal investment satisfies that:

$$\underbrace{p_t + C'\left(I_t\right)}_{\text{Marginal cost of installing one unit of capital}} = \underbrace{\mathbb{E}_t\left[\frac{\partial W(K_t)}{\partial K_t}\right]}_{\text{Marginal profits expected from that unit of capital.}}$$

Convex adjustment costs (5): Transversality

- Consider the two equations derived before:

$$
\begin{aligned}
I_t &= \frac{1}{\gamma}\left(F_{K_t} - UK_t\right) + \frac{1}{R}\left(1-\delta\right)\mathbb{E}_t\left[I_{t+1}\right] \\
I_t &= \frac{1}{\gamma}\left(q_t - 1\right)p_t
\end{aligned}
$$

- Assume for simplicity that $p_t = 1$ and $\delta = 0$:

$$
\frac{1}{\gamma}\left(q_t - 1\right) = \frac{1}{\gamma}\left(F_{K_t} - 1 + \frac{1}{R}\right) + \frac{1}{R\gamma}\left(\mathbb{E}_t\left[q_{t+1}\right] - 1\right)
$$

- Simplifying:

$$
q_t = F_{K_t} + \frac{1}{R}\mathbb{E}_t\left[q_{t+1}\right]
$$

- Substituting recursively forward:

$$
q_t = \sum_{j=0}^{\infty} \frac{1}{R^j}\mathbb{E}_t\left[F_{K_{t+j}}\right] + \lim_{j\to\infty} \frac{1}{R^j}\mathbb{E}_t\left[q_{t+j+1}\right]
$$

Convex adjustment costs (5): Transversality

- Maximization requires the following **transversality condition**:

$$\lim_{j\to\infty} \frac{1}{R^j} \mathbb{E}_t \left(q_{t+j+1} \right) = 0$$

- This is not an exogenous constraint (e.g. "no Ponzi scheme" condition).
- It is a condition required from profit maximization.
    - If this condition is not satisfied, q grows too fast over time.
    - This cannot be compatible with profit maximization, it would be optimal to invest more.

Lagrange Multiplier Method

- We can also solve the model using the Lagrange multiplier (LM) method.
- Recall the investment equation:

$$K_t = I_t + (1 - \delta) K_{t-1}$$

- The Lagrange multiplier is the shadow value of capital:
  - What is the increase in firm value V when we relax constraint by 1 unit?
  - 1 unit of investment good has a value of p outside the firm and of pq inside the firm.
  - Therefore the LM associated to this constraint will measure exactly pq!

Lagrange Multiplier Method (cont...)

- Set the Lagrangian, using as multiplier $\lambda_t = q_t p_t$:

$$\max_{\{I_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{R} \right)^t \left[\theta_t K_t^{\alpha} - p_t I_t - \frac{\gamma}{2} I_t^2 + q_t p_t (I_t + (1-\delta) K_{t-1} - K_t) \right] \right\}$$

- FOC with respect to I_t :

$$-p_t - \gamma I_t + q_t p_t = 0$$

- And we recover back our expression for investment:

$$I_t = \frac{1}{\gamma} (q_t - 1) p_t$$

Implications of the q model

- Investment is a linear function of marginal q_t

$$I_t = \frac{1}{\gamma}\left(q_t - 1\right) p_t$$

  where q_t is a sufficient statistic: summarizes all relevant information about the future that is relevant for I_t

$$q_t = 1 + \frac{\sum_{j=0}^{\infty}\left[\frac{1}{R}\left(1-\delta\right)\right]^j \mathbb{E}_t\left[F_{K_{t+j}} - UK_{t+j}\right]}{p_t}$$

- Implications:
  - I_t is much smoother than $F_{K_t} - UK_t$
  - I_t reacts to $\mathbb{E}_t\left[F_{K_{t+j}} - UK_{t+j}\right]$ even if $F_{K_t} - UK_t$ does not change.

Tobin's q: Empirical evidence (1)

- Marginal q is the value of one marginal unit of capital in the firm divided by its purchasing price.
  - Problem: It is not observable!
- Average Q is the value of the whole firm divided by the replacement value of its assets.
  - Solution: Average Q can be estimated

$$\widehat{Q} = \frac{\text{Stock market value}}{\text{Book value}}$$

Tobin's q: Empirical Evidence (2)

- Hayashi (1982) provides a set of sufficient conditions for $q = Q$:

  1. The firm is price-taker (additional units of output do not imply selling at a lower price).
  2. Production technology and adjust. costs with constant returns to scale.

     $$\text{For example:} \quad y(K) = \theta K, \qquad C(i, K) = c(I/K)K$$

  3. Adjustment costs are convex in ratio I/K.

     $$\text{For example:} \qquad c\left(\frac{I}{K}\right) = \frac{\gamma}{2}\left(\frac{I}{K}\right)^2$$

Tobin's q: Empirical Evidence (3)

- Assume $Q = q$ holds. Model implies that:

$$\frac{I}{K} = \beta_0 + \beta_1 Q + \varepsilon, \qquad \varepsilon \sim \text{is an observational error}$$

- Obtain an empirical counterpart for Q, called $\widehat{Q}$, usually:

$$\widehat{Q} = \text{stock market value / book value}$$

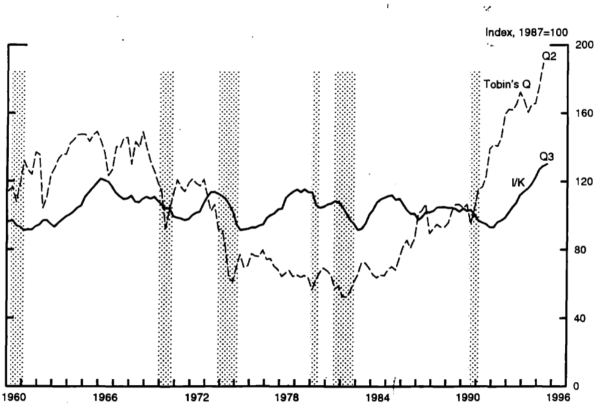
- Estimate with OLS the following regression using aggregate data

$$\left(\frac{I}{K}\right)_t = \beta_0 + \beta_1 \widehat{Q}_t + \widehat{\varepsilon}_t$$

Tobin's q: Empirical Evidence (4)

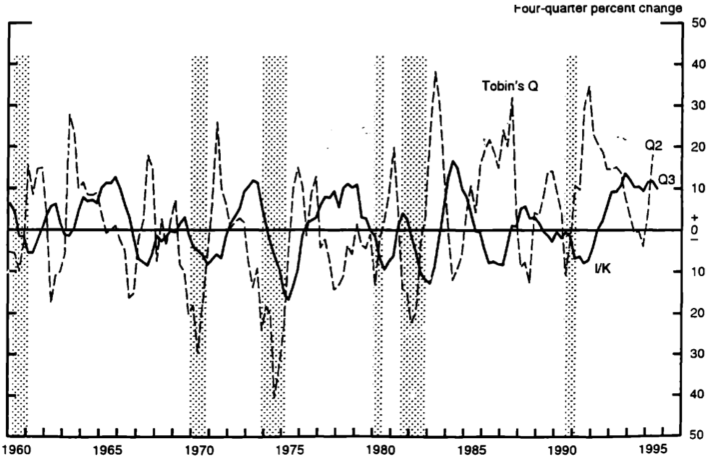
- Estimate $\left(\frac{I}{K}\right)_t = \beta_0 + \beta_1 \widehat{Q}_t + \widehat{\varepsilon}_t$ with yearly data.
- $\beta > 0$, but Q_t is not a sufficient statistic for $\frac{I}{K}$

Tobin's Q and the I/K Ratio



Tobin's q: Empirical Evidence (5)

- Even worse at a quarterly frequency.



Tobin's q: Empirical Evidence (6)

- More and more good firm level data available.
- Estimate a panel regression with firm (or plant) level data:

$$\left(\frac{I}{K}\right)_{it} = \beta_{i0} + \beta_1 \widehat{Q}_{it} + \beta_2 \left(\frac{X_{it}}{K_{it}}\right) + v_{it}$$

  - $\widehat{Q}_{it}$ is an empirical counterpart for Q_{it}
  - Individual fixed effects β_{i0}
  - Other firm characteristics X_{it} (should be insignificant)

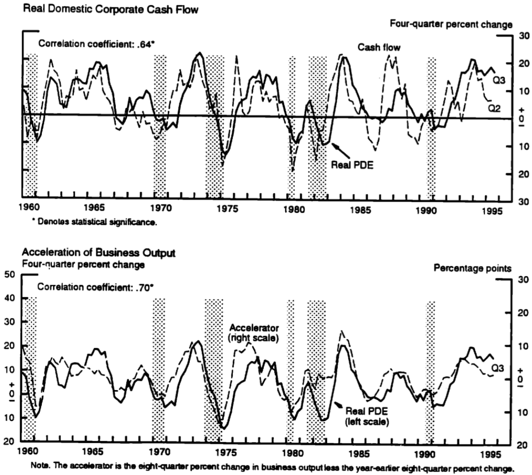
- Results:
  - β_1 is positive $\Rightarrow$ Evidence for q-theory
  - β_2 not zero $\Rightarrow$ Q_t is not a sufficient statistic for I/K
  - $\beta_2 > 0$ when X_{it} is cash flow $\Rightarrow$ evidence of financing constraints

Why is the q model rejected?

- Possible explanations for empirical rejection:
  1. $\widehat{Q}_t$ is a noisy measure of Q_t.
  2. Assumptions that make $Q_t = q_t$ do not hold
     - Imperfect competition (Cooper and Ejarque, 2001)
     - Decreasing returns to scale
  3. Adjustment costs not quadratic
     - Model misspecification
  4. Finance matters (borrowing constraints, capital market imperfections)
- The literature has explored these possibilities both at the aggregate and at the firm level and has found strong evidence for all the points above.
- Example, cash flow and acceleration of output predict investment very well.

Other factors beyond q?

- Cash flow and acceleration of business output explain much better investment (PDE: producers durable equipment)



Two main avenues

- The empirical failure of the Q model started two fields of research:
  1. Optimal Investment with financing constraints
  2. Optimal Investment with non convex adjustment costs
- Plenty of evidence has been found that both factors matter at *firm level*, but still unclear wether they matter at the *aggregate level*.
- Active debate about the Q model, especially in Finance, where cash flow, credit lines and other measures of liquidity affect investment beyond q
  - Abel and Eberly, ReStud (2011), Bolton, Chen, and Wang, JF (2011)
- In Macro, the debate is whether or not we should care about non-convex adjustment costs when we model aggregate investment.
  - Khan and Thomas, Econometrica (2008): we should not care.
  - Bachmann, Caballero and Engel, AEJ: Macro (2013): we should care.