15. Corporate decision-making under uncertainty: review and future research directions

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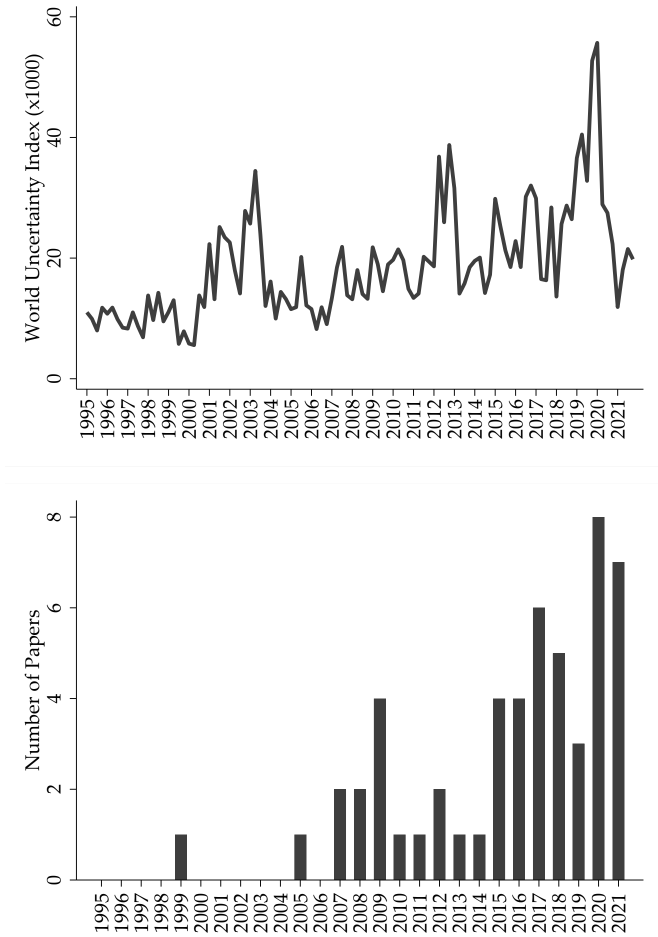
1 INTRODUCTION

Uncertainty in corporate decision-making arises from a variety of factors, ranging from prices of production inputs to market demand conditions, from technological progress to credit access, and from trade and tax regulation to disruptions in the geo-political environment. This subject has long fascinated financial economists. The predominant approach used in early theoretical models involves casting the firm's investment problem as the choice to exercise an option whose value may increase or decrease with uncertainty (see Dixit and Pindyck (1994) for a textbook discussion). These real-options models have shaped how applied researchers think about corporate decision-making under uncertainty. They are tractable and help formulate important questions regarding whether and how managers respond to uncertainty as they set firms' real and financial policies, including investment and disinvestment in physical and human capital, innovation adoption, liquidity management, financing choices, payout policy, and trade in the market for corporate control. Uncertainty-laden events appear to be occurring with increasing frequency around the world (as shown in Panel A of Figure 15.1). The rise of uncertainty as a matter of concern has been accompanied by a growing volume of empirical corporate finance research on how corporate policies respond to uncertainty (Figure 5.1, Panel B).[1] Yet, significant issues remain unresolved, and vast swathes of the corporate decision space remain unexplored. In this review, we strive to strike a balance between two objectives. First, we inform readers of important themes that have emerged in the literature on corporate decision-making under uncertainty. Second, we highlight challenges and promising directions for future work.

Regarding the first objective, we begin by presenting a real-options-based conceptual framework of corporate decision-making under uncertainty. Our framework provides micro-economic foundations for much of the empirical work that we discuss, shedding light on a number of challenges and limitations faced by researchers in the field. Our applied model encompasses a variety of corporate decisions (e.g., investment in capital and labor, disinvestment, and R&D) while simplifying and making accessible to empirical researchers key insights from complex theories. We derive five testable propositions. The first two predictions reflect the classical intuition of Bernanke (1983)—that, when investment and disinvestment are costly to reverse, firms face a greater incentive to "wait and see" before committing to either of those decisions. The next two predictions consider the modulating effects of reversibility costs. They show that wait-and-see dynamics are more pronounced as those costs increase. The final prediction relates to firms' investment in "growth-options-type" projects. In particular, we show how increased uncertainty may stimulate investment in R&D when such projects are costly to delay and are undertaken in stages. We conclude our theoretical discussion by raising an important issue that



Note: Panel A plots the GDP-weighted World Uncertainty Index developed by Ahir et al. (2022). Panel B plots the number of empirical corporate finance papers studying the impact of uncertainty on the relevant corporate decisions we cover in this review appearing in top economics journals, finance journals, and the NBER working paper series.

Figure 15.1 Trends in world uncertainty index and empirical corporate finance research on uncertainty

confronts the empirical literature: the notion that inferences on the effects of uncertainty (often proxied by changes in the second moment of a distribution of interest) may be confounded by the economic environment (changes in the distribution's first moment). Our model demonstrates how some corporate outcomes (like investment) are particularly prone to this problem, while others (like disinvestment and R&D) can actually be used to tease out whether a given corporate response is likely due to changing first or second moments of the distribution of interest.

The next section of our review delves into the measurement of uncertainty. The issue of measurement is first order in this context, given that business uncertainty is inherently latent, multidimensional, and time-varying. On that front, the empirical literature has made progress in the measurement of economic policy uncertainty, with two common classes of approach emerging. The first is an event-based approach in which researchers exploit recurrent features of the political process, like elections, as well as "one-off occurrences"—such as wars, terrorist attacks, and major geo-political events (e.g., the Brexit referendum)—to capture discrete changes in aggregate uncertainty affecting identifiable sets of firms. The second method, pioneered by Baker et al. (2016), relies on indices constructed by applying textual analysis techniques to the corpus of newspaper articles. While macroeconomic policy uncertainty is undoubtedly important for corporate decision-making, we highlight limitations in both approaches, and more broadly in the excessive focus on policy uncertainty that has emerged in empirical work. We conjecture that this has arisen from the "easy-to-use templates" that advances in the measurement of policy uncertainty have afforded researchers. One by-product of this phenomenon is that the literature has paid relatively little attention to more fundamental sources of *firm-specific* business uncertainty. This is likely due to the difficulty in finding appropriate measures and research designs in which causal inferences can be obtained. Fortunately, a recent stream of research attempts to tackle these issues head on. Our discussion sheds light on how information contained in financial markets (particularly derivatives markets) has been used by researchers to measure granular variation in business-specific uncertainty and identify plausible controls for confounding factors in the economic environment. We also discuss the development of new tools researchers can use to gauge business uncertainty, ranging from machine learning (ML) techniques, sources of big and unconventional data, and survey instruments.

We proceed with a thematic overview of empirical work across a range of corporate variables. Some outcomes exhibit an established consensus across papers. For example, it is generally accepted that uncertainty causes firms to cut fixed capital spending. Evidence on other outcomes is more mixed, nonetheless, with the literature showing considerable disagreement on, for example, how firms' innovation activity and payout policy respond to uncertainty. Concurrently, many corporate decisions remain thoroughly under-studied, including the role of uncertainty in shaping disinvestment, human capital investment, firm creation, and financial distress and bankruptcy. We propose various promising avenues for future work.

Before we conclude our introductory remarks, we must highlight what we do not cover in this review. Given our corporate focus, we largely omit any discussion of the vast literature on the aggregate economic effects of uncertainty shocks. For excellent reviews of work in this field, we refer readers to Bloom (2009 and 2014). Furthermore, we are unable to discuss in depth the literature in asset pricing and macro-finance on the impact of volatility on financial market prices and risk premia. Readers interested in this topic should refer to studies such as Brennan and Xia (2001), Bekaert et al. (2009), Bali et al. (2017), and Brogaard et al. (2020). We also avoid a full-on discussion of the effects of uncertainty on banks' decision-making, given the different considerations of risk and regulatory environment that banks operate under. A selection of studies addressing this topic includes Bordo et al. (2016), Gissler et al. (2016), Altavilla et al. (2019), and Wu and Suardi (2021).

Finally, it is important to address a definitional issue. For the purposes of this review, we will not draw a clear line of distinction between the concepts of "uncertainty" and "risk." We do this in order to adhere to (and be conversant with) a literature that predominantly treats the concepts as interchangeable (see Bloom, 2009 and 2014 for a related discussion). Ever since Knight (1921) there has been considerable debate over what constitutes "risk," in the sense of knowable probabilistic distributions over outcomes, and "uncertainty," where such parameters are unquantifiable.[2] It seems natural that empirical measures seeking to "quantify" uncertainty have gravitated towards a notion that is closer to risk, since it is easier to operationalize. We revisit this discussion later on in our analysis, identifying it prominently among our suggested directions for future empirical research.

2 CONCEPTUAL FRAMEWORK

In this section, we present a simple yet unified theoretical framework that establishes the micro-economic underpinnings of the impact of uncertainty on various types of corporate decisions. The framework has two salient features. First, following Campello et al. (2022 and 2023), it represents changes in uncertainty using the statistical concept of mean-preserving spread (MPS). Specifically, an uncertainty-increasing MPS to the distribution of an uncertain outcome is obtained by adding a zero-mean, non-degenerate source of randomness to its distribution. This approach is flexible and general as it does not require one to pre-specify the exact distribution governing the uncertain outcome.[3] Second, the framework incorporates the notion that corporate decisions are costly to reverse. This (partial) irreversibility is key in that it results in firms facing a tradeoff in the face of increased uncertainty between the benefits of committing early to a course of action and those of "waiting and seeing" (Bernanke (1983)).

2.1 Setup

Consider the investment decision of a firm operating for three periods, $t = 0,1$, and 2.[4] The firm's decision problem involves choosing whether and when to invest in two types of projects: traditional fixed investments ("capital" or "labor") and growth option-type investments ("R&D"). The firm faces a continuum of potential capital investment projects indexed by n, which lies on the interval $[0, N]$. It also has access to a continuum of potential *R&D* projects, indexed by m, on the interval $[0, M]$. The firm is endowed with existing capital investment projects that it had invested in prior to $t = 0$, which it may choose to disinvest (sell). The continuum of existing projects is indexed by w, on the interval $[0, W]$. For simplicity, we assume that there is no time discounting.

2.1.1 Investment income

Let the firm's cash flows from investing in a fixed capital project, n, at $t = 1,2$, be $v_t^{(n)} > 0$, an independently and identically distributed (IID) random variable of the form:

$$v_t^{(n)} = v_t. \tag{15.1}$$

Likewise, let the cash flows from investing in an R&D project, m, at $t = 1,2, u_t^{(m)} > 0$, be:

$$u_t^{(m)} = u_t. \tag{15.2}$$

Finally, let the firm's cash flows from disinvesting an existing project from its capital endowment, w, at $t = 1{,}2$, be $s_t^{(w)} > 0$, such that:

$$s_t^{(w)} = s_t. \tag{15.3}$$

In this setting, $v_t > 0$ represents the time-varying demand for output generated by the firm's fixed assets, u_t is the demand for output generated by the firm's R&D assets, and s_t is the proceeds from disinvestment. The firm's fixed asset investment project's cash flow, v_t, is distributed as $v_t \sim P(\overline{v}_t, p)$, where the mean of v_t is equal to $\overline{v}_t$, the variance is equal to $\sigma^2(p)$, and p is an index of the mean-preserving spread. Specifically, $p' > p \Longrightarrow P(\cdot\ ,p')$ is an MPS of $P(\cdot\ ,p)$ and $\int v_t dP(\cdot\ ,p) = \overline{v}_t \forall p$. The firm's cash flow from each R&D project can be characterized as an MPS with distribution $u_t \sim Q(\overline{u}_t, q)$ and mean $\overline{u}_t$, with variance $\omega^2(q)$. Finally, the cash flows from disinvesting each existing project can be characterized as an MPS with distribution $s_t \sim R(\overline{s}_t, r)$ and mean $\overline{s}_t$, with variance $\psi^2(r)$.

2.1.2 Investment costs

In order to undertake investment project n, the firm must incur a sunk cost of capital, denoted by $F_K(\kappa, n) = \kappa n$, and a sunk cost of labor, denoted by $F_L(\lambda, n) = \lambda n$, both of which increase linearly in n.[5] The parameters $\kappa > 0$ and $\lambda > 0$ capture the degree of irreversibility of the respective investments in capital and labor, as these components (which also scale linearly) cannot be recovered if the investment decisions are reversed. If the firm chooses to invest in a traditional project n, it can either invest at $t = 0$ or $t = 1$. If it invests in n at $t = 0$, it incurs the sunk costs $\lambda n + \kappa n$ at $t = 1$ and earns the revenues $v_1 + v_2$. If it chooses not to invest at $t = 0$, waiting instead to invest at $t = 1$, it incurs the fixed costs $\lambda n + \kappa n$ at $t = 2$, earning only the revenue v_2. In our framework, a negative effect of uncertainty on fixed asset investment arises due to the joint presence of the option to delay and costly reversibility.

Innovative, growth-option-type projects, m, differ from fixed asset investment projects, n, in two critical ways. First, the option to invest in these projects is only available at $t = 0$. That is, the firm has only one chance to decide whether to invest. If it declines, these projects cease to become available at any future period ($t = 1$ or 2). This assumption plausibly matches several features of the innovation process such as the "race to patent" a novel piece of intellectual property or bring a new invention to the market before other rival firms ("winner takes all"). Second, R&D investments can be staged, meaning the firm can observe interim signals before deciding to proceed with the project. In order to pursue m R&D projects, the firm incurs an upfront cost of m^α, which is convex in the size of its R&D portfolio (i.e., $\alpha > 1$). This parametrization reflects the scaling of the costs of monitoring these initial "seed" investments.[6] To pursue an R&D project further, the firm must pay a development cost d_t for each period in which the project continues to exist. That is, in order to earn $t = 1$ revenue u_1, it must pay d_1; similarly, in order to earn $t = 2$ revenue u_2, it must pay d_2. However, at $t = 1$, the firm observes the first-stage cash flow, u_1, which serves as a signal of the second-stage revenue of the project.[7] Based on this signal, the firm has the choice to either abandon the project at the end of $t = 1$ or re-invest and take it to completion at $t = 2$. In the former case, it does not receive any revenue from the project at $t = 2$; that is, $u_2 = 0$. On the other hand, if, at the end of $t = 1$, it chooses to keep the project alive, the process recurs and it must pay the second-period development cost, d_2, to receive u_2. This, too, resembles a typical feature of *R&D* projects (e.g., pharmaceutical drug trials and corporate venture capital investments) in which decisions are staged and the firm does not need to pre-commit to follow through on all stages at once. Critically, the joint absence of (1) the option to delay and (2) nearly irreversible, fixed costs in our framework generates a positive effect of uncertainty on R&D investment.[8]

Finally, the firm can choose at time $t = 0$ or $t = 1$ to disinvest any of its existing endowment of fixed asset projects, w. If the firm sells a project at time t, it must pay a scrapping cost δw (e.g., sunk costs of lost synergies and environmental costs), but receives the cash flow from disinvestment of s_t. Otherwise, for each period t that the project remains alive, the firm earns a known x_t (for example, rent accruing from a real-estate holding). The process of disinvesting a project is nearly irreversible; and, as with fixed asset investment, this irreversibility induces a negative effect of uncertainty on disinvestment in our framework.[9]

2.2 Model Analysis and Results

In what follows, we present analysis and results corresponding to the firm's canonical real-options investment problem. We also model the firm's decision to disinvest, and derive results on the cross-sectional implications of the role played by irreversibility costs. Finally, we solve for the firm's optimal R&D project selection decision.[10]

2.2.1 Capital and labor investment decisions

In order to solve the firm's fixed investment problem, we must first consider its decision at $t = 1$, and then iterate backwards. If the firm had initiated any projects at $t = 0$, it obtains the second-period cash flow v_2 per project. Among those projects that the firm left uninvested at $t = 0$, the firm can choose to invest in any of them at $t = 1$ and earn $v_2 - (\kappa + \lambda)n$ per project. Alternatively, it can discard any uninvested projects and earn 0. The firm will optimally discard a given project, $\tilde{n}$, when its expected revenue is less than the associated costs of investment and hiring. The firm ceases to operate at the end of $t = 2$, and any project that is not undertaken at either $t = 0$ or $t = 1$ has a value of 0 by the end of $t = 2$. The firm's investment decision at $t = 1$ will therefore be guided by value in the second period that is generated by project $\tilde{n}$. The value function, π_2, can be characterized as:

$$\pi_2(\tilde{n}) = \begin{cases} v_2 & \text{(Early Investment)} \\ v_2 - (\kappa + \lambda)\tilde{n} \quad \text{if } v_2 > (\kappa + \lambda)\tilde{n} & \text{(Delayed Investment)} \\ 0 \quad \text{if } v_2 \leq (\kappa + \lambda)\tilde{n} & \text{(No Investment)} \end{cases} \tag{15.4}$$

Next, we consider the firm's decision at $t = 0$. The optimal total investment level at $t = 0$ can be expressed in terms of n^*, the breakeven project. The firm will invest in all projects in the range $[0, n^*)$, and not invest in projects in the range $[n^*, N]$, instead waiting until $t = 1$ to decide whether to undertake any of those projects. The firm's expected profit from investing in project $\tilde{n}$ at $t = 0$ is $v_1 + \mathbb{E}[v_2] - (\kappa + \lambda)\tilde{n}$. Its expected profit from not investing in $\tilde{n}$ at $t = 0$, and choosing instead to wait until $t = 1$ to decide, is $\mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)\tilde{n}, 0\right)\right]$. The firm invests in project $\tilde{n}$ at $t = 0$ if:

$$\underbrace{v_1 + \mathbb{E}[v_2]}_{\text{Expected Revenue}} \geq \underbrace{(\kappa + \lambda)\tilde{n}}_{\text{Cost of Investment}} + \underbrace{\mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)\tilde{n}, 0\right)\right]}_{\text{Value of Waiting}} \tag{15.5}$$

The breakeven condition for determining the optimal investment level n^* at $t = 0$ is:

$$v_1 + \mathbb{E}[v_2] = (\kappa + \lambda)n^* + \mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)n^*, 0\right)\right] \tag{15.6}$$

In Lemma 1, we prove the existence of the optimal $t = 0$ investment level, n^*.

Lemma 1. *The optimal capital investment level n^* at $t = 0$ is given by Eq. (15.6) for sufficiently large N.*

The breakeven condition in Eq. (15.6) implies that the firm invests in all projects at $t = 0$ up to project n^*, for which the benefits are expected to exceed the costs. The embedded optionality in the firm's investment decision is key in generating a negative relation between uncertainty and investment. An increase in uncertainty in the distribution of v_t reduces the breakeven project level n^*, and correspondingly shrinks the set of projects the firm invests in at $t = 0$, namely the interval $[0, n^*)$. We establish this result in Proposition 1.

Proposition 1. *Increased uncertainty leads to less investment at $t = 0$. For $p' > p$, namely when $P(\cdot, p')$ is obtained by a mean-preserving spread of $P(\cdot, p), n^*(p') < n^*(p)$. That is, $\frac{dn^*}{dp} < 0$.*

2.2.2 Disinvestment decisions

In solving a firm's disinvestment problem, we must, likewise, consider its decision at $t = 1$. If the firm had disinvested any of its endowed projects at $t = 0$, then it earns 0 for those projects. Among projects that were not disinvested at $t = 0$ (i.e., remain alive a $t = 1$), the firm can choose to sell any of them at $t = 1$ and receive cash flows of $s_2 + x_2 - \delta w$ per project. Alternatively, it can choose not to sell and receive x_2 per project. As in the case of the investment decision, the firm's disinvestment policy is guided by the cash flows at $t = 2$ generated by project $\tilde{w}$. These cash flows can be characterized as:

$$\pi_2(\tilde{w}) = \begin{cases} 0 & & \text{(Early Disinvestment)} \\ s_2 + x_2 - \delta\tilde{w} & \text{if } s_2 > \delta\tilde{w} & \text{(Delayed Disinvestment)} \\ x_2 & \text{if } s_2 \leq \delta\tilde{w} & \text{(No Disinvestment)} \end{cases} \tag{15.7}$$

Next, we consider the firm's disinvestment decision at $t = 0$. The optimal level of disinvestment at $t = 0$ can be expressed in terms of w^*, the breakeven project. The firm will optimally disinvest (sell) all projects in the range $[0, w^*)$, and not disinvest (choose to retain) any projects in the range $[w^*, W]$ instead of waiting until $t = 1$ to decide whether or not to disinvest. The firm's cash flows from disinvesting project $\tilde{w}$ at $t = 0$ is $s_1 + x_1 - \delta\tilde{w}$. Its expected cash flows from not disinvesting project $\tilde{w}$ at $t = 0$, and choosing instead to wait till $t = 1$ to decide, is $x_1 + \mathbb{E}\left[\max\left(s_2 + x_2 - \delta\widetilde{w}, x_{i2}\right)\right]$. Simplifying these two expressions, the firm disinvests project $\widetilde{w}$ at $t = 0$ if:

$$s_1 - \delta\widetilde{w} \geq x_2 + \mathbb{E}\left[\max\left(s_2 - \delta\widetilde{w}, 0\right)\right] \tag{15.8}$$

The breakeven condition for determining the optimal disinvestment level w^* at $t = 0$ is:

$$s_1 - \delta w^* = x_2 + \mathbb{E}\left[\max\left(s_2 - \delta w^*, 0\right)\right] \tag{15.9}$$

In Lemma 2, we prove the existence of the optimal $t = 0$ investment level, w^*.

Lemma 2. *The optimal disinvestment level w^* at $t = 0$ is given by Eq. (15.9) for sufficiently large W.*

The breakeven condition in Eq. (15.9) implies that at $t = 0$ the firm sells all projects up to project w^*, as the benefits of doing so, s_1, are expected to exceed the costs. Costs are made of two components: (1) the cost of selling the project, δw; and (2) the option value of waiting to choose whether to disinvest. The embedded optionality in the firm's disinvestment decision is key in generating a negative relation between uncertainty and disinvestment, as is the case with investment. As before, while the addition of a zero-mean spread does not change the left-hand side of Eq. (15.8), it increases the right-hand side of that inequality given the firm's option to forgo disinvestment in high-income states. An increase in uncertainty reduces the breakeven project level w^*, and correspondingly shrinks the set of projects the firm disinvests at $t = 0$, namely the interval $[0, w^*)$. We establish this result in Proposition 2.

Proposition 2. *Increased uncertainty leads to less disinvestment at $t = 0$. For $r' > r$, namely when $R(\cdot, r')$ is obtained by a mean-preserving spread of $R(\cdot, r)$, $w^*(r') < w^*(r)$. That is, $\frac{dw^*}{dr} < 0$.*

Taken together, the results of Propositions 1 and 2 imply that, by increasing the value of the option to wait, greater uncertainty leads to decreases in both investment and disinvestment. In other words, a firm's "inaction zone" is increased in the face of heightened uncertainty.[11]

It is worth noting that an alternative class of models predicts that uncertainty increases firm investment when firm profits are convex in input costs or output prices, dubbed the "Oi–Hartman–Abel" effect (see Oi (1961), Hartman (1972), Abel (1983), and Pindyck (1993)). Caballero (1991) notes that the sign of the investment–uncertainty relationship depends on fundamental assumptions about competition and returns to scale. That analysis shows that models assuming perfect competition and constant returns to scale generally predict a positive investment–uncertainty relationship, while models predicting a negative relationship between investment and uncertainty tend to require (often, implicitly) assumptions of imperfect competition or decreasing returns to scale. Given this theoretical debate, empirical work on the role of competition in modulating the effects of uncertainty—not only on investment but also on various other important corporate decisions—is rather limited, and this issue merits further study going forward.[12]

2.2.3 The effect of input irreversibility

We now address the role played by the costs of reversibility of capital and labor, by way of two propositions.

Proposition 3. *An increase in the degree of irreversibility of capital leads to less investment for higher levels of uncertainty in the first period; i.e.,* $\frac{dn^*}{d\kappa} < 0$.

Proposition 4. *An increase in the degree of irreversibility of labor leads to less investment for higher levels of uncertainty in the first period; i.e.,* $\frac{dn^*}{d\kappa} < 0$.

Combining the last two propositions with Proposition 1, we have that, for an increase in uncertainty in the MPS sense (i.e., $p' > p$) and for greater degree of input irreversibility ($\kappa' > \kappa$ and $\lambda' > \lambda$), the following conditions hold with respect to investment:

$$\begin{aligned} n^*(p,\kappa,\lambda) > n^*(p',\kappa,\lambda) > n^*(p',\kappa',\lambda) \\ n^*(p,\kappa,\lambda) > n^*(p',\kappa,\lambda) > n^*(p',\kappa,\lambda') \end{aligned} \tag{15.10}$$

The above conditions state that an increase in uncertainty reduces the set of projects the firm is willing to invest in at $t = 0$, electing to wait until uncertainty is partially resolved at $t = 1$ before deciding whether to invest. Notably, when the firm faces higher reversibility costs, it invests even less at $t = 0$. Differently put, an increase in uncertainty reduces investment in the first period, and the effect is modulated by the degree to which reversing their capital and labor investment decisions incurs costs.

2.2.4 R&D investment decisions

Consider the firm's decision at $t = 0$, when it may opt to invest in a portfolio of R&D projects. The cash flows from a R&D portfolio of size $\overline{m}$ can be expressed as:

$$\pi_2(\overline{m}) = \begin{cases} 0 & & \text{(No Investment)} \\ u_1 - d_1 + u_2 - d_2 & \text{if } \mathbb{E}[u_2 \mid u_1 = u] > d_2 & \text{(Invest Both Stages)} \\ u_1 - d_1 & \text{if } \mathbb{E}[u_2 \mid u_1 = u] \leq d_2 & \text{(Invest First Stage Only)} \end{cases} \tag{15.11}$$

The firm will invest in a portfolio of R&D projects of size m at $t = 0$ to maximize the following:

$$\max_m m \times \mathbb{E}[\max(u_2 - d_2, 0)] + m u_1 - m d_1 - m^\alpha \tag{15.12}$$

The first-order condition of Eq. (15.12) implicitly defines the optimal R&D portfolio size, m^*, by the following:

$$\mathbb{E}[\max(u_2 - d_2, 0)] + u_1 = d_1 + \alpha m^{*\alpha-1} \tag{15.13}$$

As the first-stage cash flow, u_1, is a noisy but unbiased signal of the second-stage cash flow, u_2, and assuming that these variables are normally distributed, we can simplify Eq. (15.13) as:[13]

$$\mathbb{E}\left[u_1 - d_2 \mid u_1 > d_2\right] + u_1 = d_1 + \alpha m^{*\alpha-1},$$

$$\overline{u_1} + \frac{\sqrt{\omega^2(q) + \sigma_x^2}}{1 - \Phi\left(\frac{d_2 - \overline{u_1}}{\sqrt{\omega^2(q) + \sigma_x^2}}\right)}\left[\phi\left(\frac{d_2 - \overline{u_1}}{\sqrt{\omega^2(q) + \sigma_x^2}}\right)\right] + u_1 = d_1 + d_2 + \alpha m^{*\alpha-1} \qquad (15.14)$$

In Lemma 3, we prove the existence of the optimal R&D investment level m^*.

Lemma 3. *The optimal R&D investment level m^* at $t = 0$ is given by Eq. (15.14) for $\alpha > 1$.*

The breakeven condition in Eq. (15.14) implies that the firm invests in a portfolio of R&D projects up to the point at which benefits are expected to exceed costs. Since at $t = 1$ the firm can (upon observing a signal of the projects' future revenues) choose to abandon any project, and, if so, avoid paying the second-stage development cost, the decision to invest in the R&D projects at $t = 0$ is equivalent to the decision to buy a portfolio of call options expiring at $t = 1$. At the breakeven R&D investment level, m^*, the price of the option equals its value. Increased uncertainty in the distribution of u_t increases the value of this option, thereby increasing the breakeven project threshold m^*, expanding the set of R&D projects the firm undertakes. This argument is formalized in Proposition 5.

Proposition 5. *Increased uncertainty leads to greater R & D investment at $t = 0$. For $q' > q$, namely when $Q(\cdot\ ,q')$ is obtained by a mean-preserving spread of $Q(\ \cdot\ ,q), m^*(q') > m^*(q)$. That is, $\frac{dm^*}{dq} > 0$.*

Proposition 5 states that an increase in uncertainty increases the set of R&D-type projects that the firm is willing to undertake—given that the potential upside has increased and the downside is capped by the ability to abandon a project upon observing an interim signal as to its future profitability—and avoid paying part of the investment costs. This is *in contrast* to capital investment, whose initial costs, once paid, are largely sunk. The positive uncertainty–R&D relation is obtained in our framework due to the presence of "abandonment options," which increase in value under higher uncertainty.

We conclude the discussion of our theoretical framework by briefly discussing the distinction between the impact of changes in "expectations" about future cash flows (first moments) and changes in "uncertainty" about future cash flows (second moments) on the various corporate decisions we consider. Under our framework, a fall in first-moment expectations and a rise in second-moment expectations produce similar predictions of a decline in current-period investment in capital and labor. As we discuss subsequently, this poses a critical challenge for empirical work as, in a variety of contexts, uncertainty shocks have been shown to be counter-cyclical, at least in the aggregate (see, e.g., Bloom (2014) and Kozeniauskas et al. (2018)). Accordingly, findings of decreased investment in response to uncertainty shocks may instead be attributed to a failure to properly control for declining first-moment expectations.

While we subsequently discuss several attempts made by the empirical literature to separate the two moments, we stress that our conceptual framework offers an *alternative*, *complementary* approach to deal with this challenge. Specifically, declining first-moment expectations provide contrasting predictions for *other* corporate decisions like disinvestment and R&D, relative to the predictions for these outcomes under a positive uncertainty shock. As such, studying a variety of corporate decisions with differential predictions allows researchers to empirically distinguish between positive second-moment shocks and negative first-moment shocks.[14] Campello et al. (2022 and 2023), for instance, show that investment declines in response to uncertainty shocks are accompanied by concurrent increases in R&D expenditures and decreases in disinvestment rates. The totality of these findings is consistent with the theoretical effects of increased second-moment expectations and *not* declining expectations in first moments. Such an approach is particularly useful given the difficulties in obtaining distinct measures of first- and second-moment expectations that we discuss next.

3 MEASURING UNCERTAINTY IN CORPORATE FINANCE

Any examination of the role of uncertainty on corporate decisions necessitates a relevant measure of uncertainty. In this section, we review various approaches adopted by prior studies in confronting this challenge.

We begin by considering a particular dimension of uncertainty that is undoubtedly relevant to corporate decision-makers—namely, policy uncertainty. We highlight two common approaches employed in the literature to measure policy uncertainty. The first relies on identifying events that lead to discrete and appreciable changes in policy uncertainty. The second comes out of a growing literature that applies textual-based measures of policy uncertainty.

Next, we discuss promising approaches to measuring uncertainty premised on the notion that financial market prices contain information on corporate decision-relevant uncertainty. In our discussion, we focus on the distinction between *business-level* uncertainty facing a firm and *macro-level* uncertainty. We argue that market-based metrics are particularly suitable for capturing corporate-relevant forms of uncertainty. We also highlight econometric approaches to measuring uncertainty, and conclude by revisiting the issue of first versus second moments.

3.1 Policy Uncertainty

Perhaps the largest subset of empirical work on corporate decision-making under uncertainty focuses on firm responses to changes in, specifically, policy uncertainty. These studies can be broadly divided into two sets. A first set of papers, which we label "event-based studies," relies on institutional features of the political system to identify discrete changes in policy-making. For a causal interpretation, it is essential that these changes in the policy environment impact some subset of exposed firms while leaving a "control" set of firms unaffected. Naturally, such settings may be limited, and concerns arise around generalizability. An alternative approach, therefore, relies on broad-based measures of policy uncertainty derived from the textual content of newspaper articles, building on the pioneering work of Baker et al. (2016). Several papers, which we label "index-based studies," exploit variation in various versions of economic policy uncertainty (EPU) indices developed in the spirit of Baker et al. to examine how corporate decision-making responds to uncertainty.

3.1.1 Event-based studies

Beginning with Julio and Yook (2012), a number of papers in corporate finance have exploited elections as policy uncertainty-inducing events.[15] The underlying logic is straightforward: elections are a turnover point for a nation's (or sub-national unit's) governing regime. Such changes may be accompanied by significant shifts in legislative priorities, regulatory initiatives, and judicial orientation, all of which influence the economic environment in which firms operate.

Julio and Yook (2012) study one particular set of elections—national elections—arguing that the potential for change in national governments around such elections represents a source of considerable uncertainty for firms in election years relative to non-election years. Using a sample of 248 national elections in 48 countries held between 1980 and 2005, they show that firms in election countries reduce their investment in election years relative to non-election years. Addressing the endogeneity concern that the ruling regime may strategically time elections and that election outcomes may be predictable, Julio and Yook show that results obtain in the subset of countries with fixed election timings and during elections that were, ex-post, shown to be closely contested. Consistent with the "wait-and-see" incentives being temporary, they show that firms (partially) return their investment to baseline levels post-election.

Complementing this cross-country evidence, Jens (2017) shows similar dynamics of reduced investment around U.S. gubernatorial elections. The identification strategy adopted by the author is useful in mitigating certain concerns that national-level political-business cycles may be driving investment responses, rather than the policy uncertainty engendered by the elections themselves. U.S. gubernatorial elections are predictable in timing, yet staggered across states, allowing for a comparison of investment rates of firms headquartered in different states, only a subset of which are experiencing elections at any given point in the national business cycle. The reliance on the ex-post closeness of elections to proxy for uncertainty is also discussed, with Jens arguing that poor economic conditions may be both driving closely contested election results and declining investment. To address this concern, the author proposes using term limits that naturally create more contested elections (as there is no incumbent towards whom the incumbency advantage may favor) for reasons unrelated to economic conditions or business cycle timing.

While using elections to identify increases in policy uncertainty has a number of advantages, several caveats remain. First, election time cycles are largely known, and both ex-ante and ex-post measures of election outcome predictability are likely to be affected by economic conditions that are also relevant to firms' decision-making. Second, while closely contested elections (ex-post) or term-limited elections (ex-ante) are likely to be more uncertainty-inducing, without a measure of policy uncertainty itself, it is unclear whether any individual election event actually did significantly increase policy uncertainty from the perspective of corporate decision-makers. Subsequent studies have made progress on both fronts.

In light of the likely anticipation effects of election timing and predictability of outcomes, several papers have focused on policy uncertainty shocks generated by non-election events. Campello et al. (2022) track U.S. firm responses to an entirely non-routine and unpredictable political event in a foreign country—the 2016 Brexit referendum in the United Kingdom.[16] On a similar note, Kim and Kung (2017) use the outbreak of the first Gulf War in 1990 and the 9/11 terrorist attacks in 2001 as shocks to uncertainty, comparing the investment responses of firms with varying degrees of asset redeployability (a proxy for investment irreversibility). Baker et al. (2023) utilize natural disasters as shocks to uncertainty, finding marked negative responses to disasters on output and growth consistent with a real-options effect.

The event-based approach, pioneered by the studies mentioned above, has been applied to studying uncertainty's effects on a number of corporate decision variables. A non-exhaustive list includes: corporate investment (An et al. (2016), Handley et al. (2020), Guceri and Albinowski (2021), and Jacob et al. (2022)); initial public offerings/IPOs (Çolak et al. (2017)); innovation (Bhattacharya et al. (2017), Atanassov et al. (2019), and Cong and Howell (2021)); cash savings (Acharya and Steffen (2020)); payouts (Buchanan et al. (2017)); and mergers and acquisitions (M&As) (Chen et al. (2023) and Cao et al. (2019)).

Resolving the second challenge faced by event-based studies—that is, assessing whether a given policy uncertainty-inducing event actually induces policy uncertainty—is considerably tougher. Doing so requires researchers to have access to a metric for policy uncertainty whose levels can be compared around any such events. We discuss several such metrics below.

3.1.2 Index-based studies

Developing a time-consistent, continuous measure of policy uncertainty is a formidable task. This is so especially because of the complex, multidimensional nature of uncertainty, as well as the difficulty in identifying ready-to-use proxies that are unconfounded with other aspects of the economy (e.g., volatility of macroeconomic aggregates). In their seminal work, Baker et al. (2016) turn to a novel data source in order to construct a measure of political uncertainty: the text of newspaper articles. Specifically, to capture policy uncertainty they create the economic policy uncertainty (EPU) index, which counts the frequency of articles in ten leading U.S. newspapers containing three sets of related words: (1) "economic" or "economy"; "uncertain" or "uncertainty"; and one or more of "Congress," "deficit," "Federal Reserve," "legislation," "regulation," or "White House." Those authors construct the time-series of the occurrence of these words, and show that their EPU index spikes around relevant events like the Gulf Wars, 9/11, debt ceiling negotiations in 2011, and so on.

Baker et al.'s (2016) EPU index spawned a large body of work creating similar textual-based indices of uncertainty and assessing the impact of changes in uncertainty on the decisions of economic agents. The variants of the EPU index that have emerged range from national uncertainty indices (Arbatli Saxegaard et al. (2022), Davis et al. (2019), and Huang and Luk (2020)) to sub-national indices (Baker et al. (2022)), policy-realm specific indices (Caldara et al. (2020), Husted et al. (2020), and Gavriilidis (2021)), and global indices (Davis (2016) and Ahir et al. (2022)).

As soon as textual-based uncertainty metrics emerged, empirical work proliferated applying these metrics to a gamut of corporate decision variables. While we will review a selection of these papers in more detail in the subsequent section, it is worth noting the sheer range of corporate outcomes that have been explored in relation to EPU-type indices. The set of outcomes scholars have argued to be influenced by variation in textual uncertainty indices includes: corporate investment (among others, Kang et al. (2014), Wang et al. (2014), Gulen and Ion (2016), Kim and Kung (2017), Drobetz et al. (2018), and Ilyas et al. (2021)); M&As (Nguyen and Phan (2017), Bonaimé et al. (2018), Borthwick et al. (2020), Sha et al. (2020), Gregoriou et al. (2021), Kumar et al. (2021), and Paudyal et al. (2021)); cash (Phan et al. (2019), Duong et al. (2020), and Goodell et al. (2021)); payouts (Pirgaip and Dinçergök (2019) and Attig et al. (2021)); innovation (Xu (2020), Cui et al. (2021), and Luis et al. (2022)); diversification (Hoang et al. (2021)); managerial turnover (Frye and Pham (2020)); stock issuance (Chan et al. (2021)); trade credit (D'Mello and Toscano (2020) and Jory et al. (2020)); credit spreads (Ashraf and Shen (2019) and Kaviani et al. (2020)); and disclosures (Jiang et al. (2022)).

Baker et al.'s (2016) EPU index, and the set of textual-based uncertainty indices it spawned, pushed forward the empirical measurement of uncertainty. The index-based approach has a distinct advantage over event-based studies in allowing researchers to benchmark the effects of uncertainty on corporate decisions across various time periods and countries. The accompanying flood of papers in the literature is a testament to the flexibility and generalizability of these EPU-type measures.

Yet this approach, too, has notable disadvantages. While the prevalence of EPU-type measures of uncertainty has simplified the research agenda, the reduction of uncertainty into a single index based on the count of a restricted set of keywords in newspaper articles will necessarily lead to a loss of context about what is driving changes in uncertainty. This is particularly important as different types of uncertainty-inducing events—all of which will similarly manifest as a spike in an EPU-type index—are likely to affect firms in heterogeneous ways.[17] As such, average correlations of corporate decisions with fluctuations in uncertainty indices may be uninformative without a fuller consideration of the context in which each fluctuation occurs. These concerns are aggravated when these EPU indices are mechanically applied to contexts where it is unclear that the source of uncertainty most relevant to managers is *policy* uncertainty. In some sense, the availability of these indices absolves researchers from making difficult choices about (1) *what* sort of uncertainty matters most to corporate decision-makers of (2) *which* firms and (3) for *which* corporate decisions. Doing so, we will argue, requires researchers to look to financial markets for more appropriate measures of *business-relevant* uncertainty.

3.2 Business-Level Uncertainty

Uncertainty about economic policy represents only one—albeit important—source of uncertainty that factors into corporate managers' decision-making calculus. Firms face multiple sources of uncertainty as they form expectations about the costs and availability of inputs into their production processes (capital, labor, technology) as well as output quantities and prices across the various markets in which they operate. Variation in these aspects is likely affected not only by macro-factors (including policy uncertainty) but also business-specific factors that are relevant to individual firms, product supply chains, and sector groups. Finding a sufficient statistic that summarizes the uncertainty facing managers across this multitude of dimensions is a difficult task.

3.2.1 Market prices

One potential solution is found by turning to financial markets. The information contained in capital market prices captures all aspects of a firm's future prospects that investors deem relevant. Notably, a large literature has shown that managers are particularly attuned to—and learn from—financial market prices in all crucial corporate decisions including investment, capital raising, mergers and acquisitions, and payouts.[18] A natural candidate measure for business uncertainty is, thus, the volatility of a firm's equity prices: these prices aggregate information from individual investors about firms' future stream of cash flows and are highly relevant to corporate decision makers. Stock price volatility, to a first approximation, may reasonably capture volatility in the various input and output factors that feed into an individual firm's earnings. Researchers have also applied econometric techniques to decompose the variation in returns volatility into components driven by firm-idiosyncratic factors, industry volatility, and aggregate, market-wide volatility.

In line with this approach, various studies utilize realized equity returns (or, analogously, cash flows) volatility to gauge the impact of uncertainty on corporate decisions and outcomes. These include: Bhagwat et al. (2016), who study the effects of equity volatility on M&A activity; Izhakian and Yermack (2017), who relate volatility to executive compensation; Chay and Suh (2009), who look at payout policy responses; Riddick and Whited (2009), who investigate the determinants of corporate savings; Im et al. (2020), who study effects on capital structure; Bushman et al. (2010), who look into CEO turnover; and Kraft et al. (2018), who investigate firm valuation effects.

A critical shortcoming of relying on realized volatility of equity prices (or realized cash flows) is that these are ex-post metrics, reflecting one particular realization of outcomes based on the observed state of the world.[19] Uncertainty, on the other hand, is a fundamentally ex-ante object, related to managers' expectations on the range of outcomes that may occur across all possible states of the world.[20] While under rational expectations realizations will (on average) coincide with expectations, this is a fairly restrictive assumption to impose on managers' expectations formation process. Instead, several recent papers turn to derivatives markets—most commonly option markets—to obtain ex-ante measures of uncertainty. In particular, option-implied volatility is a forward-looking measure of investors' expectations over the distribution of potential returns over a future period corresponding to the options' maturity. As such, option-implied volatility can be seen as a measure of *firm-specific* uncertainty. An example of an early adopter of this approach is Stein and Stone (2013), who use equity option-implied volatility to proxy for uncertainty surrounding firms' business conditions. Those authors show that such uncertainty depresses investment and hiring, while promoting R&D activities.

Other studies have taken a different approach, electing to model the expectation formation process itself. Such an approach requires one to specify and estimate a particular forecasting model and apply it to ex-post equity returns volatility to generate ex-ante predictions. In this context, Leahy and Whited (1996) apply a more general vector autoregression (VAR) model under the assumption that volatility follows a stationary stochastic process with a finite-lag structure to obtain forecast (forward-looking) volatility estimates. Gao et al. (2017) obtain forward-looking measures of latent uncertainty by modeling stochastic equity volatility as a generalized autoregressive conditional heteroskedasticity (GARCH) process.

Approaches relying on firm equity or option prices face an important shortcoming as they may confound the fact that uncertainty over individual inputs into (and outputs of) a firm's production process may differentially affect managers' decisions, depending on the relationship between cash flows and those factors. Fortunately, the increased "financialization" of markets observed in recent years provides a potential resolution. Nowadays, many of a firm's inputs and outputs have corresponding derivative securities traded on global financial markets. These prices provide unique insights into market participants' expectations on the evolution of specific sources of uncertainty. Recent papers have leveraged uncertainty measures derived from derivatives markets to test how managers respond to heightened uncertainty in their key input and output markets. One example is Campello et al. (2023), who focus on the shipping sector. The authors derive measures of uncertainty (and first-moment changes) by combining information from shipping firms' equity option-implied volatility and the prices of traded shipping derivative contracts corresponding to the carriage of specific goods on specific shipping routes, known as forward freight agreements. Kellogg (2014), likewise, obtains first- and second-moment proxies for oil-drilling firms using oil price futures, showing that such ex-ante measures produce a more theoretically consistent relationship with investment compared to ex-post, backward-looking realized volatility measures and model forecasts (e.g., GARCH predictions) based on them. Doshi et al. (2018) utilize a similar approach to explain how uncertainty affects oil-producing firms' corporate risk management strategies.

Capital markets data are also useful in that some of the same market prices that can inform us about changes in firms' second moments also embed information on expectations of changes in first moments. As we have discussed previously, studies on the impacts of uncertainty are often plagued by the problem that inferences may be capturing concurrent changes in first moments. Researchers largely recognize the issue, and several papers use derivatives prices as linear controls for changes in first moments in their regression specifications. This approach is limited as first-moment changes are likely to display higher-order correlations with the underlying data generation process that influences the object of interest, namely changes in second moments. Advances have been made in recent work on that front. Notably, Alfaro et al. (2023) design an identification strategy that relies on the differential exposure of firms to various input and output markets to construct firm-level instruments of first- and second-moment changes based on varying exposures to oil prices and exchange rates. This strategy relies on the plausible notion that volatility in input and output factors can hurt, not affect, or benefit individual firms based on the role those inputs and outputs play in each firm's production process—a nuance that is often lost in studies that examine *average* corporate responses to macro-level uncertainty metrics (like the EPU index). Empirically separating the effects of changes in the two moments on corporate decisions remains an open and active field of study.

A caveat applies to studies that use derivatives market information to capture changes in uncertainty: such studies tend to be industry-specific. This is naturally attributable to the fact that production processes (and their corresponding inputs and outputs) vary at the industry level, and the availability of comprehensive derivatives information on inputs and outputs is limited to a subset of industries for whom these factors are relatively homogeneous and trade on financial markets. For instance, neither the key inputs (human capital) nor outputs (software and internet products) for some of the largest technology firms are standardized or tradable in derivatives markets. As such, the generalizability of results outside samples consisting of firms in commodity-dependent industries with financially tradable outputs is limited.

There are further limitations of derivatives-based implied volatility measures worth noting. First, it is questionable the extent to which the model assumptions that generate implied volatility measures actually hold in reality.[21] Second, derivatives markets are often illiquid, and, as such, care should be taken when interpreting volatility and other distributional metrics derived from prices formed in such markets.

With FinTech opening new horizons for financialization (e.g., by expanding the menu of traded securities), we expect the market-based approach to measuring uncertainty to continue growing in its appeal. The rising attention towards climate concerns, for instance, points to weather derivatives as a source of information for quantifying climate-related uncertainty. On this front, Lin et al. (2021) gauge the impact of product price uncertainty facing electricity-producing firms on their liquidity management decisions by instrumenting the volatility of traded electricity futures prices with granular data on weather volatility. Relatedly, Ilhan et al. (2021) show that "carbon tail risk" is priced into options markets, particularly for carbon-intense firms that are most exposed to climate-related uncertainty.

3.2.2 Big data and artificial intelligence (AI)

In the age of big data, alternative sources of market information may prove useful in the quest to derive firm-specific uncertainty measures. Analogous to the Baker et al. (2016) economy-wide EPU index, firms' own textual disclosures contain a wealth of information that researchers have exploited to create firm-level measures of business uncertainty. A notable contribution on this front is Hassan et al. (2019), who develop firm-level metrics of political risk by applying textual-analysis methods to gauge discussion on related topics in firms' quarterly earnings conference calls. In a similar vein, Friberg and Seiler (2017) and Handley and Li (2020) develop metrics of firm business uncertainty through textual analysis of firms' 10-K filings, while Caldara et al. (2020) construct a measure of trade policy uncertainty at the firm level based on a test of firms' conference calls.

In what is somewhat of a hybrid between the market-based approach of extracting uncertainty measures and the textual-based EPU index approach, Manela and Moreira (2017) construct a news-based uncertainty counterpart to the market-based VIX index (NVIX) by analyzing articles that appear on the front page of the *Wall Street Journal*. Baker et al. (2021) apply the textual-based approach to the novel, large-scale, social-media data from Twitter to construct a variation of Baker et al.'s EPU index. It is worth noting that measuring uncertainty through simple word counts faces a challenge, in that it is unclear whether such measures are able to capture modulations in the level of uncertainty that a firm is facing. For instance, counting the times the word "risk" or "uncertainty" is cited in a firm disclosure is devoid of any "sentiment" about how serious or directly concerning to the firm a situation is.

Moving beyond simple "word count" approaches, Davis et al. (2021) apply supervised machine learning to extract firm exposures to Covid-19 risk, while Azqueta-Gavaldón (2017) and Azqueta-Gavaldón et al. (2023) refine the Baker et al. (2016) approach using unsupervised ML. While novel and informative, one concern that applies to uncertainty measures based on firms' disclosures (both written and spoken) is the notion of strategic disclosure—firms are known to manage the content and tone of their public communications (see, e.g., Hollander et al. (2010), Bushee et al. (2011), and Matsumoto et al. (2011)). Such strategic considerations may plausibly extend to managers' discussions of uncertainty, raising concerns of potential selection and reverse causality biases. For instance, managers may wish to create the impression of greater uncertainty following poor performance; or, equivalently, may wish to reduce public perceptions of uncertainty when their private information suggests heightened uncertainty.

3.2.3 Hybrid approaches

A key insight that emerges from our discussion of approaches to measuring uncertainty so far is the inherent tradeoff between measurement precision and generalizability of results. Papers that successfully navigate this tradeoff tend to adopt hybrid approaches, combining and validating measures and settings that leverage the advantages of the various approaches while ameliorating their limitations.

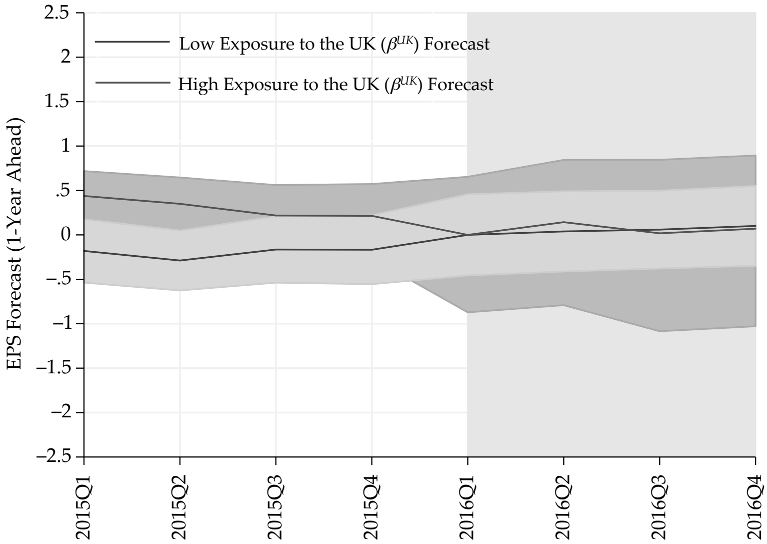
Campello et al. (2022), for example, use what is essentially an event-based design by centering their study on the unexpected outcome of the Brexit vote. An important aspect of the authors' hybrid approach is their use of both stock-return volatility betas and textual analysis of 10-K filings to gauge U.S. firms' exposures to Brexit. The authors validate that the Brexit vote was indeed associated with a spike in uncertainty for exposed firms by marshaling evidence from a number of metrics, including textual-based measures—the corresponding UK EPU index showed the largest spike in its history till that date following the vote outcome—and market-based proxies. Specifically, the authors use analyst forecast data to show, at the micro-firm level, that expectations of earnings uncertainty (forecast dispersion) for exposed firms increased markedly around the time of the Brexit vote, relative to control firms, while mean consensus forecasts (a first-moment proxy) remained stable for both exposed and control firms. Their results are reproduced in Figure 15.2.[22]

Turning to options data, Campello et al. (2022) use time-variation in the term structure of implied volatility for the UK market index to identify the exact period over which market participants incorporate uncertainty embedded by Brexit, as manifested in their options' trading activity, and to put ex-ante bounds on the duration of heightened uncertainty during which corporate decisions were hypothesized, and shown, to be affected. Their results are shown in Figure 15.3.

The lower (dotted) curve in Figure 15.3 serves as a benchmark since expectations at the time were uncontaminated by Brexit. Responses to official news about the exact referendum date suggest that market participants were quick to incorporate uncertainty embedded by Brexit in their trading activity—before the actual outcome of the vote. In particular, options trading taking place on February 22, 2016 (solid curve)—the first trading day following Prime Minister David Cameron's announcement of the Brexit vote date—were priced to reflect a significant drop in market volatility for the period leading up to the vote date (on June 23), only to show a spike in volatility right after the vote.

Campello et al.'s (2022) analysis also touches upon an important issue surrounding the resolution of uncertainty (see also Bloom (2009)). In this context, they show that expectations around Brexit-induced uncertainty were relatively short-lived: on June 24, 2016 (dashed curve), the first trading day following the vote, market uncertainty seemed unusually high in the short run, but declined to more normal levels over a longer horizon.

Kim and Kung (2017), too, adopt a hybrid approach, showing complementary results based on various measures of uncertainty, including market-based (VIX), text-based (EPU index), and event-based (the Gulf War of 1991 and the 9/11 terrorist attacks of 2001). Favara et al. (2021) also combine a number of approaches to gauge changes in uncertainty, including the EPU index, firms' cash flow volatility, and election events.



Note: This figure shows how analyst Earnings Per Share (EPS) forecasts behaved around Brexit's key dates. Confidence intervals are calculated as ± 1.5 standard deviations from the mean forecast. Each line represents a group of firms, sorted by their exposure to the UK economy as measured by β^{UK} (or the sensitivity of a firm's equity return volatility to the volatility of the U.K. FTSE100 Index). The shaded area marks the beginning of Brexit-related events with the announcement of the date of the EU referendum by then Prime Minister David Cameron (2016:Q1). Both series are normalized to take the value of 0 in 2016:Q1.

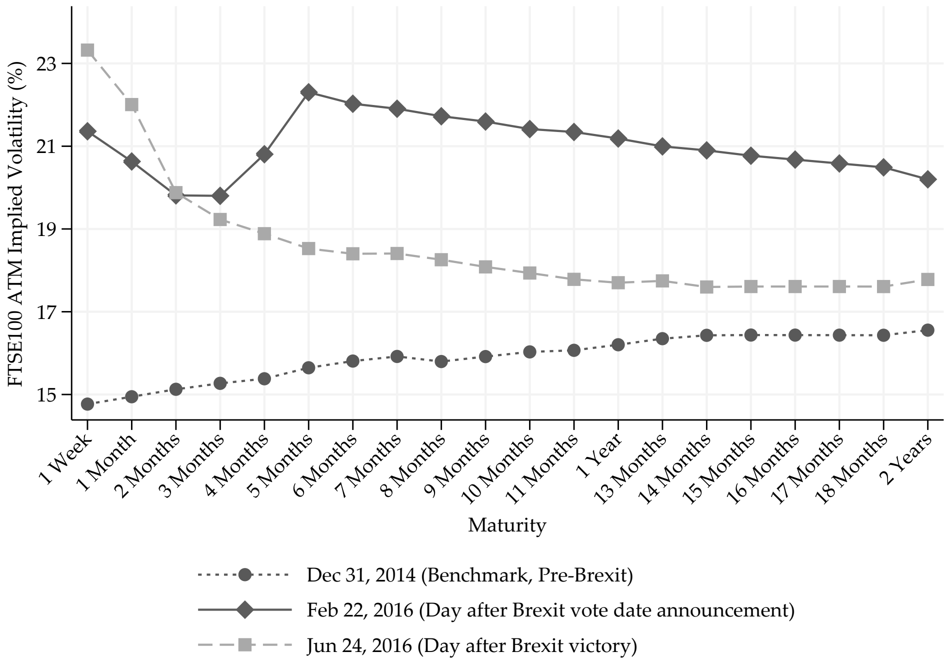
Source: Reproduced from Campello et al. (2022).

Figure 15.2 Analysts' earnings per share forecasts around Brexit

3.2.4 Other approaches

Another approach to gauge uncertainty that has emerged in the economics literature is to directly survey the relevant economic agents about their expectations. This has not found widespread use in corporate finance due to the costs and difficulties of systematically and repeatedly surveying the senior managers of U.S. public firms. A notable exception is a set of studies that draws on data from a regular survey of the Chief Financial Officers (CFOs) of large U.S. corporations to measure managerial expectations (see, e.g., Campello et al. (2010), Ben-David et al. (2013), Giambona et al. (2017), Barry et al. (2022), and Graham (2022)).[23]

We wrap up our overview of approaches used in the measurement of uncertainty by highlighting advances in the econometrics literature on this issue, which have, as yet, found limited application in the study of corporate responses. In their aptly titled paper ("Measuring uncertainty"), Jurado et al. (2015) obtain econometric estimates of macroeconomic uncertainty by extracting an un-forecastable common component across a large number of economic indicators.[24] A fruitful avenue for future research on corporate decisions would involve applying novel machine learning and artificial intelligence techniques to extract the common unpredictable component of the various uncertainty indicators outlined in this section.



Note: This figure shows the term structure of the FTSE100 Index at three different dates, constructed from average Black–Scholes implied volatilities derived from quoted prices of at-the-money options on the index. The values plotted reflect the market's expectation of the volatility of the index over various maturities considered.

Source: Reproduced from Campello et al. (2022),

Figure 15.3 Term structure of FTSE100 implied volatility around Brexit

3.3 Open Issues in the Measurement of Uncertainty

While much progress has been made on the front of measurement of uncertainty in the context of empirical corporate finance research, we identify several open issues that require further examination.

3.3.1 Duration and resolution of uncertainty

It is important to carefully contemplate the horizon over which uncertainty is being considered. In order to capture decision-relevant uncertainty, the duration over which uncertainty is measured has to reasonably approximate the planning horizon of managers. A failure to do so may confound inferences in a serious way, given that theory predicts that firms are likely to respond differently and in different ways across different decision margins—when uncertainty is expected to be short-lived versus persistent. In the case of investment, for instance, temporary uncertainty shocks may induce inter-temporal reallocation of investment (see, e.g., Bloom (2009) and Stokey (2016)), while long-lived shocks may trigger a variety of other mitigating responses, including operational or financial hedging and other forms of risk management. Theoretical models provide some guidance on this front, and it will be useful for empirical work to translate and test the different sets of implications that arise from varying uncertainty horizons (see Gorbenko and Strebulaev (2010) and Decamps et al. (2017)). Grenadier and Malenko (2010) highlight a Bayesian mechanism by which managers "learn" about past shocks over longer horizons, and their response to this "option to learn" may lead to a further temporal tradeoff in the timing of investment decisions. Econometric methods of decomposing uncertainty shocks, such as those proposed by Gryglewicz et al. (2022) and Schlag et al. (2021), may prove particularly useful in this regard.

It is also useful to consider the link between the related concepts of "duration of uncertainty" and "resolution of uncertainty." Temporary uncertainty shocks may be resolved over short time horizons with potentially short-lived firm responses. On the other hand, long-lasting periods of heightened uncertainty may, in effect, be resolved through firms' own long-term actions, rather than through the external circumstances that typically resolve "short-term" uncertainty shocks. On this front, Graham (2022) presents survey evidence that CFOs' planning horizons have declined over time. Investigating this phenomenon in relation to uncertainty may provide insights into how firms manage the resolution of longer-lasting uncertainty.[25] In sum, empirically distinguishing between these two forms of "resolutions" of uncertainty in corporate settings might serve as a fruitful avenue for further exploration.

3.3.2 Sources of uncertainty

Another issue we have briefly alluded to in the previous sections is the need to carefully consider the underlying *source* of uncertainty. It is evident from theory that uncertainty about different aspects of a business will impact various types of decisions in very different ways. Yet, most often, the uncertainty that is empirically identified under the approaches we have discussed above is either highly aggregated (at the macro-level) or, at most, measured at the level of a firm's bottom-line cash flows. Parsing out the various underlying sources from firm-level measures will require new techniques and novel data sources; for example, on firms' individual capital and labor input investment decisions, technology adoption, and supply chain linkages.

3.3.3 Types of uncertainty

A final open issue centers on the *type* of uncertainty. Corporate managers face a particularly difficult decision in responding to unanticipated uncertainty shocks—shocks that are inherently more difficult to forecast and plan for. A theoretically significant but empirically elusive form of uncertainty is "Knightian" uncertainty, or uncertainty over "unknown unknowns" (as opposed to "known" and quantifiable unknowns). To the extent that firms are persistently surprised by increasingly volatile shocks to their expectations, this is likely to indicate a more persistent and pervasive type of uncertainty over the underlying form and parameters of the distribution itself. This, in turn, has far-ranging implications, some of which have been explored in individual laboratory experimental settings where Knightian uncertainty—often referred to as "ambiguity"—can be plausibly manipulated.[26] Lab experiments, however, are likely to have limited value in informing us about how real-world corporate managers (with real stakes at play, and in command of complex organizations) respond to Knightian uncertainty. We note that much of the theory on decision-making under Knightian-type uncertainty centers on individual investors or entrepreneurs, who may be thought of as being highly averse to ambiguity (see, e.g., Schröder (2011), Malenko and Tsoy (2020), and Hansen (2022)). It is unclear the extent to which such preferences aggregate up in a manner that would affect corporate decision-making in the larger organizations that form the focus of our review.

Creative means of eliciting empirical variation in managerial expectations of Knightian uncertainty are needed in order to open up this area of study to corporate finance researchers. Some potential directions may be available in the asset pricing literature (see, e.g., Epstein and Schneider (2010), Guidolin and Rinaldi (2013), Brenner and Izhakian (2018), and Augustin and Izhakian (2020)). A smaller, but growing set of corporate finance theory-based papers has addressed concepts like ambiguity and model uncertainty (Nishimura and Ozaki (2007), Miao and Wang (2011), and Dicks and Fulghieri (2022)).

4 THE LITERATURE ON UNCERTAINTY AND CORPORATE DECISIONS

4.1 Investment, Disinvestment, and Asset Allocation and Composition

In light of the large theoretical literature on the uncertainty–investment relationship, it is natural that the bulk of empirical studies on how uncertainty affects corporate decisions focuses on its impact on investment. Among the earliest works to directly address this question with firm-level data is Leahy and Whited (1996), who motivate their study by noting the theoretical debate referenced in Section 2.2 on the sign of the uncertainty–investment relationship. Using forecasts of volatility from a VAR model fitted on realized equity returns volatility data as their proxy of firm uncertainty, the authors show that the sign of the relationship between firm investment rates (measured by reported capital expenditures) and uncertainty is negative. Based on the results from a host of panel regression specifications, they conclude that the empirical evidence on uncertainty–investment dynamics provides support for the irreversibility-based theoretical view.

Indeed, an entire body of empirical work has emerged in recent years aimed at elucidating the nuances of the uncertainty–investment relationship. These studies draw simultaneously from the development of improved measures of uncertainty and the popularization of causal identification techniques in empirical corporate finance. On the latter front, Julio and Yook (2012) use the staggered occurrence of national elections as a natural experiment in a difference-in-differences testing setup to assess the impact of uncertainty on firm investment. This study is notable in that it is among the earliest to acknowledge the potential source of endogeneity arising from the fact that events that increase uncertainty facing firms are likely to also affect their prospects directly. Thus, comparing firms across countries—some of which are experiencing elections (with varying degrees of outcome predictability)—allows Julio and Yook to net out the effects of common global trends affecting firms' investment rates. They also uncover a channel particular to political uncertainty—the role of government spending in the economy and political connections to firms—in accentuating the negative uncertainty–investment relationship. Jens (2017) provides confirmatory evidence among U.S.-domiciled firms around gubernatorial elections, a context in which it is easier to control for election closeness.

Jens (2017) is also notable in testing a related prediction of real-options models on uncertainty and investment by showing that firms' disinvestment rates—proxied by the reported sale of property, plant, and equipment—decline during heightened uncertainty. On the front of corporate disinvestment responses to uncertainty, Carvalho (2018) uses plant-level data from Brazil to show that input-price uncertainty (measured by exchange rate volatility exposures of importing firms) leads firms to reduce the rate at which they close manufacturing plants.

Taking an alternative approach, Gulen and Ion (2016) rely on the Baker et al. (2016) EPU index to assess the impact of changes in political uncertainty on corporate investment. A notable contribution of that study is that Gulen and Ion test an important modulating force in the uncertainty–investment relationship, namely investment irreversibility. They do so relying on a number of proxies, the most compelling of which was introduced by Kim and Kung (2017), whose measure of the costs of irreversibility is based on the idea that assets that are more redeployable are more liquid. Thus, when faced with increased uncertainty, a firm with more redeployable assets is less likely to cut investment more aggressively as their invested assets can be reallocated to other firms (e.g., through trade in secondary asset markets) if conditions turn out to be less favorable than anticipated. Firms whose assets have limited alternate uses, on the other hand, are likely to be more cautious in their investment decisions when faced with greater uncertainty. Kim and Kung operationalize the notion of asset redeployability by building a measure of the fraction of a firm's operations that are in an industry where used assets make up a greater share of the overall asset base. They build this measure using data on the value of used versus new assets from the Bureau of Economic Analysis (see also Almeida and Campello (2007)).

Consistent with real-options theories of irreversible investment, both Gulen and Ion (2016) and Kim and Kung (2017) empirically show that the negative uncertainty–investment relationship is more pronounced for firms with less redeployable assets. A number of the aforementioned studies additionally investigate the duration of the investment response to uncertainty shocks, finding evidence that the investment cuts are transitory (following a "drop-and-rebound" pattern) and confirming the theoretical and macro-empirical insights of Bloom (2009).

Campello et al. (2023) make a number of advancements in the study of uncertainty and firms' capital allocation decisions. The authors conduct their investigation in the context of the shipping industry, an industry in which: (1) firms' investment and disinvestment decisions in discrete assets (ships) are well documented; (2) ex-ante measures of secondary asset market liquidity are readily available; and (3) derivatives markets provide suitable proxies for the evolution of first- and second-moment expectations. Campello et al. show that shipping firms curtail investment and disinvestment of ships when uncertainty is high. Notably, they show that sunk costs of investment and disinvestment modulate firm responses to uncertainty, leading firms to cut back on new ship orders (decisions involving fixed costs of customization) and demolitions (decisions involving sunk costs of lost synergies and regulatory exposures due to adverse environmental impacts). Further, the authors show that liquidity conditions in secondary ship markets accentuate shipping firms' responses, with investment and disinvestment rates falling the most (least) in the least (most) liquid shipping subsectors.

Finally, and new to the literature, Campello et al. (2023) assess the effects of uncertainty on the overall composition of shipping firms' asset base (ship fleets). They document that firms concentrate their fleets into narrow market segments, and that the investment (disinvestment) declines are confined to the most (least) productive ships, worsening the composition of fleets. As such, their results point to a slowdown of "creative destruction" dynamics when uncertainty is high, as firms seem to be frozen in place instead of renewing their fleets by acquiring assets embodying new and productive technologies while disposing of their older and less productive assets.

While the literature on domestic investment effects of uncertainty is quite large, a growing number of studies have emerged focusing on cross-border effects. Using a similar empirical setup as their prior work on domestic investment, Julio and Yook (2016) show that U.S. firms' foreign direct investment (FDI) flows decline during election periods in recipient countries. Sarkar (2019) studies the investment behavior of multinational mining companies in response to EPU shocks in various countries in which they operate, finding evidence of reallocation of investment from higher to lower uncertainty markets. Campello et al. (2022) investigate the cross-border transmission of uncertainty. They show that U.K.-exposed American firms cut their *domestic* (U.S.-based) investment—in both capital and labor—and disinvestment in response to uncertainty induced by the unexpected result of the Brexit referendum. Studying the same event, Hassan et al. (2020) find consistent cross-border responses to Brexit-induced uncertainty in a large sample of global firms.

4.2 Innovation

In our theoretical framework, we posit that the impact of uncertainty on firm innovation may be positive, to the extent that such projects display a number of distinguishing characteristics with respect to typical capital and labor investments. These include increased costs of delaying projects and the ability to stage (and partially recover) investments. The empirical evidence, however, has been mixed. Stein and Stone (2013), Atanassov et al. (2019), and Campello et al. (2022) find that firms increase their R&D spending in the face of heightened uncertainty, consistent with the "growth options" nature of R&D. Relatedly, Kankanhalli (2021) shows that investment in early-stage startups is accelerated under heightened uncertainty.

However, another set of studies reports contrasting effects. Bhattacharya et al. (2017) use patent-based proxies for firm innovation to show that heightened uncertainty around national elections depresses firm patenting. Focusing on smaller, entrepreneurial firms in Italy, Caggese (2012) shows that such firms cut back on their investment in innovative activities when uncertainty (measured by unexplained volatility in profits) is high. Cong and Howell (2021), likewise, show that uncertainty in the IPO process leads Chinese firms to cut back on their innovation activity.

Several factors may account for the different findings of the two sets of papers. First, the source of uncertainty is likely to matter: aggregate economic uncertainty, policy-induced uncertainty, business uncertainty, and technological uncertainty may all have different impacts on firms' incentives to bring forward versus delay their innovation activities.[27] Second, managers' innovation efforts, or the inputs into the R&D process, are distinct from firm innovation outputs, such as patents. It remains an open question as to whether increased innovation efforts (i.e., R&D expenses) during periods of higher uncertainty translate into more, and higher-quality, innovation outputs (i.e., highly cited patents). Firms may, for instance, adopt different innovation strategies during different uncertainty regimes, resulting in fundamental changes in the R&D input–output mapping. Third, firms may respond differently at various points in their life cycle as growth options occupy a lesser fraction of firm value relative to assets-in-place as a firm ages. Lastly, R&D spending-based measures may co-mingle investing activities that are more speculative with those that are likely more traditional (e.g., building a physical facility to house a lab), and therefore more subject to irreversibility-led uncertainty responses. Teasing out these empirical nuances presents a promising direction for future work.

4.3 IPOs and M&As

Compared to the volume of work on firm investment responses to uncertainty, the literature has paid relatively little attention to its impact on other major corporate decisions, including decisions to go public and to transact in the market for corporate control. Work in this space has, perhaps, been limited by the lack of clear theoretical guidance. Regarding the going public decision, Çolak et al. (2017) identify uncertainty shocks using U.S. gubernatorial elections and show that IPO activity is lower in the run-up to elections, which they attribute to higher costs resulting from lower IPO first-day offer prices when uncertainty is high.

On the front of merger activity, Bhagwat et al. (2016) find a negative uncertainty–M&A activity relationship, driven exclusively by public targets. The authors argue that firms face greater incentives to delay their acquisitions of such targets during periods of heightened uncertainty due to greater risks related to pricing the deal and executing the transaction. Nguyen and Phan (2017) show consistent evidence, reporting that deals announced during higher uncertainty periods take longer to complete yet exhibit greater wealth creation for shareholders, suggesting increased prudence in deal selection. Bonaimé et al. (2018) show that higher policy uncertainty is associated with a lower propensity to engage in M&A deals. Their evidence supports an irreversibility channel. In particular, deals that are easier to reverse, involving targets with more redeployable assets, are less affected by uncertainty. Kumar et al. (2021) exploit a change in U.S. trade policy towards China to identify a discrete reduction in trade policy uncertainty, finding that this resolution of uncertainty boosts M&A propensity. Chen et al. (2023) show how acquirers mitigate the effects of uncertainty by restructuring deals to avoid uncertain periods around elections, pursuing targets in non-election states, decreasing deal size, and substituting cash for equity financing. Turning to cross-border acquisitions, Cao et al. (2019) show that increases in uncertainty around national elections depress inbound cross-border deals while increasing the propensity of domestic firms to seek outbound acquisition targets.

In all, corporate decisions like IPOs and M&As appear to be significantly affected by uncertainty; however, our understanding of the underlying economic mechanisms remains limited. Firm- and industry-specific settings in which researchers have explored the nuances of the uncertainty–investment relationship may prove useful in this regard. Applying those contexts to the study of IPOs and M&As (and other corporate transactions) will allow researchers to mitigate the confounding effects of first- and second-moment changes on these decisions.

4.4 Liquidity Management and Hedging

Corporate liquidity management and hedging, while outside the scope of our theoretical framework, represent an important class of corporate decision variables that are likely affected by uncertainty. Studies on liquidity policies under uncertainty are usually motivated by theories of precautionary saving, finding that cash balances tend to be higher when firms face more uncertain environments (see, e.g., Riddick and Whited (2009) and Duong et al. (2020)). Consistent with a precautionary motive, Gao et al. (2017) find that firms exposed to greater systematic uncertainty (estimated via a GARCH model applied to equity volatility data) increase their cash holdings. The authors hypothesize two motives for this cash accumulation: first, that increased uncertainty reduces the predictability of firms' future cash needs; and, second, that increased uncertainty increases fluctuations in firms' costs of external financing. Both channels predict an increase in cash holdings when systematic uncertainty spikes, with the first channel suggesting such increases should be predominantly observed among firms with more volatile cash flows, and the latter channel predicting more pronounced effects among firms with a more cyclical cost of capital. Both channels find support in the data. These insights build upon fundamental studies of the determinants of firms' liquidity policies, which show that firms with greater systematic risk exposures prefer cash over credit lines to meet their liquidity needs (Acharya et al. (2013)).

In a similar vein, using the onset of the Covid-19 pandemic as a shock to uncertainty, Acharya and Steffen (2020) show that firms at risk of being "fallen angels" (namely, facing a potential credit rating down-grade to non-investment grade status) accumulated sizeable cash reserves, even at the expense of pulling back on their investment outlays. Recent work by Segal and Shaliastovich (2021) provides a novel perspective, showing that firms' precautionary savings response to uncertainty may result in a decrease in the utilization—therefore, depreciation—of their existing physical capital stock.

Unlike corporate cash management policies, which fulfill a number of firm objectives (see Almeida et al. (2014)), firms' risk management policies are explicitly designed with a view to navigating uncertainty. Most notably, a number of papers have examined whether and to what extent managers choose to hedge their firms' exposures to various sources of uncertainty, and the resulting consequences on firms' real responses. Early work on this subject provides theoretical foundations characterizing firms' optimal financial hedging policies (see, e.g., Telser (1955), Stulz (1984), and Smith and Stulz (1985)).

On the empirical front, Campello et al. (2011) articulate and test the idea that interest rate and foreign currency exchange hedging increases firm value by reducing financial distress risk. The authors show that hedging lowers the cost of borrowed funds and allows for borrowing contracts with few covenants, with the implication that a hedged firm invests more as a result. These findings raise the interesting possibility that uncertainty may not depress investment if measures of hedging are put in place. This notion finds empirical support in subsequent studies (Pérez-González and Yun (2013) and Doshi et al. (2018)). Corporate diversification (both across business lines and geography) may also offer a strategy that managers can use to mitigate uncertainty's impacts. While several studies have examined the real and valuation effects of corporate diversification, they have not placed that discussion in the domain of decision-making under uncertainty. Future work could be undertaken with the aim of explicitly examining this connection.

4.5 Investor Payout and Managerial Compensation

Theoretical priors on how managers' payout policy decisions should respond to uncertainty are less than clear-cut. On one hand, payout policy is, at least in theory, a residual decision that should be unaffected by uncertainty surrounding firms' cash flows. Yet, prior literature shows that managers often depart from the residual view of payout policy, especially in the presence of tax frictions, liquidity concerns, and financing constraints. Such considerations raise the possibility that uncertainty in managers' cash flow expectations feed into their decisions on how much (and in what form) to pay out corporate earnings. Empirical work suggests an important role for tax policy uncertainty. In particular, firms display a lower likelihood of initiating regular dividends (and a higher likelihood of paying special dividends) during periods of lower tax policy uncertainty (Buchanan et al. (2017)). This is rationalized by the fact that firms take advantage of periods of "tax certainty" to make tax-favorable payouts to investors while avoiding the need to make long-term, sticky commitments.

Turning to cash flow uncertainty, Chay and Suh (2009) find that firms reduce both the intensive and extensive margins of their dividend payouts when uncertainty is high, attributing this decline to tightening financing constraints and the stickiness of dividend payout commitments. Conversely, Attig et al. (2021) find that firms increase their dividend payouts during periods of heightened policy uncertainty (measured by the EPU index) as a way of mitigating agency problems. In their study of corporate responses to uncertainty during the 2008–2009 crisis, Bliss et al. (2015) show that firms curtailed their payouts as a means of retaining precautionary cash balances. In all, much remains to be understood about how firms manage the interactions between their investment, liquidity, and payout decisions in the face of heightened uncertainty.

The extent to which corporate decision-making responds to uncertainty is also a function of managers' incentives. In the presence of agency frictions, these incentives are shaped by firms' managerial compensation arrangements—specifically, whether and to what degree managers' contracts expose them to the various sources of uncertainty facing the firm. A handful of studies investigate the relationship between managers' compensation, their incentives, and firm-level responses to uncertainty. Chakraborty et al. (1999) show that CEOs exposed to a high degree of compensation uncertainty tend to invest less than those with less volatile earnings. Glover and Levine (2015) take this a step further, demonstrating both theoretically and empirically that managers' incentive-based compensation contract structure influences the sign and magnitude of their firms' investment response to uncertainty shocks. Notably, they show that managers with a higher equity exposure in their compensation contracts respond more sharply in terms of cutting investment when uncertainty spikes. Their conclusions support those from an earlier study by Panousi and Papanikolaou (2012), who show that more risk-averse managers—those that own a larger share of their firms—respond more strongly in cutting investment in response to firm-specific uncertainty shocks. Recent work by Ion and Yin (2021) examines the relationship between economy-wide policy uncertainty and firm-level uncertainty. Those authors find that aspects of CEO incentives, including the "delta" and "vega" of their compensation contracts, play a critical conditioning role.

4.6 Financing Frictions

In the final section of our review, we briefly discuss the role of uncertainty in shaping corporate decisions under financing constraints. We devote relatively less attention to this stream of the literature as it lies outside the scope of the motivating real-options-based framework we discuss in Section 2. Yet, it is reasonable to expect that uncertainty affects a wide range of corporate decisions through its impact on the cost and availability of credit for firms.

In their seminal work, Gilchrist et al. (2014) develop a general equilibrium framework in which idiosyncratic uncertainty shocks affect investment through two channels: (1) the real-options-based "wait-and-see" effect; and (2) increases in credit spreads. The latter channel results in debt holders curtailing the supply of credit available to firms, which, in turn, cut back on investment. This effect arises as, under a canonical framework, debt holders require additional risk compensation when firm uncertainty rises. In those models, the return profile of a creditor mimics that of the seller of a put option on the firm's assets—a position whose value declines in uncertainty (cf. Merton (1974)). Confirming this logic, Kaviani et al. (2020) examine micro-level data on corporate bond yields, and find that high levels of policy uncertainty are associated with heightened credit spreads. On the bank side, several studies document a tightening of credit supply and increased pricing during high uncertainty periods (see, e.g., Alessandri and Bottero (2020) and Berger et al. (2022)).

The effects of uncertainty that operate through the amplification of financing frictions are expected to be more pronounced for firms that are ex-ante credit constrained. Consistent with this argument, Favara et al. (2021) show that access to debt markets mitigates the negative effects of uncertainty on corporate financial decisions. The authors exploit a quasi-natural experiment brought about by the staggered introduction of anti-recharacterization laws that eased credit availability for firms in certain states in the U.S. Comparing the responses of exposed and control firms to uncertainty shocks, those authors find that exposed firms engaged in a significantly lower degree of precautionary savings behavior (increasing cash while cutting payouts, leverage, and investment in intangibles). In similar fashion, Alfaro et al. (2023) find that firms facing greater financing frictions display an amplified response to uncertainty shocks, hoarding cash and reducing their investments in capital and labor beyond the "wait-and-see" effects observed among unconstrained firms.

5 CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

Empirical work on corporate decision-making under uncertainty has grown rapidly in recent years. A first-order challenge facing any such study is the need to define and measure decision-relevant uncertainty. An important branch of the literature focuses on the effects of policy uncertainty. Two main approaches have been adopted by studies in this branch. The first relies on quasi-natural experiments to generate plausibly exogenous variation in policy uncertainty that affects an identifiable subset of firms (examples range from scheduled national elections to one-off events such as outbreaks of war, terror attacks, and pandemics). The second measurement approach relies on policy uncertainty indices, which researchers have developed by applying textual analysis methods to newspaper articles.

Moving beyond policy uncertainty, another branch of the literature strives to gauge information about other sources of uncertainty that affect managerial decisions. Researchers in this branch have often leveraged information contained in financial market prices to measure firm-specific uncertainty. Sophisticated implementations of this approach involve isolating the particular inputs and outputs of a firm's production process and developing specific metrics of uncertainty from the prices of financial instruments, primarily derivatives, corresponding to those factors. Recent trends towards big data and alternative data will allow researchers to take this approach further. We are optimistic that future studies will be able to broaden the scope of uncertainty sources considered, encompassing hitherto unstudied factors for which traded financial instruments may not exist.

We expect that researchers will continue to devise innovative approaches to tackle the fundamental source of endogeneity plaguing most, if not all, studies of firm responses to uncertainty—the fact that changes in uncertainty are accompanied by concurrent, confounding changes in the economic environment (first-moment expectations). The predominant approach in existing studies is to saturate empirical specifications with first-moment controls. While reasonable to a first approximation, this approach fails to account for higher-order relationships between the first and second moments of the process of interest (e.g., firm cash flows).[28] Notably, isolating the two moments is critical to our understanding of heterogeneous responses to second-moment shocks that occur in the presence of negative versus positive first-moment shocks—managers may respond differently to "good uncertainty" and "bad uncertainty" (Segal et al. (2015)). There may be significant value in adapting macroeconometric approaches designed to isolate the common unpredictable component of aggregate uncertainty indicators to micro-firm-level uncertainty metrics.

Our work concludes with a discussion of several other gaps that we perceive in the growing literature on corporate decision-making under uncertainty. First, it is important to understand how firms respond to uncertainty in their human capital investment decisions. We speculate that existing research on this question is scant due to data limitations—worker-level data is generally only available via confidential, administrative datasets, which are difficult to access for most researchers. The recent availability of detailed job postings data that shed light on the nature of firms' demand for various types of labor may prove useful in this regard (see, e.g., Campello et al. (2023)). Relatedly, the nature of corporate investment itself has changed over time, reflecting a greater reliance on intangible capital such as knowledge capital embedded in employees (Corrado et al. (2022)). It may be fruitful to investigate whether and how firms' responses to uncertainty have adjusted to this fundamental economic transformation.

Second, competition plays a first-order role in shaping corporate responses to uncertainty. Grenadier (2002) and Décaire et al. (2020) provide theoretical and empirical evidence, respectively, of a significant direct effect of competition on firms' decisions to exercise real options. Recent work by Grigoris and Segal (2021) examines firms' responses to uncertainty shocks across their supply chains, showing that the impact of uncertainty shocks on investment depends on whether they occur upstream (negative) or downstream (positive) in the production network. As a logical next step, researchers could leverage new and detailed information on firms' product market positioning (e.g., Hoberg and Phillips (2016)) to empirically examine whether competitive interactions influence how firms react to uncertainty shocks.

Third, the interplay between financing frictions, real-options-based incentives, and uncertainty is complex and fascinating. Existing studies are only beginning to scratch the surface, and several aspects merit deeper investigation. For instance, an important question concerns whether uncertainty affects firms' propensity to file for bankruptcy protection. If so, further questions arise on what aspects of the bankruptcy regime and firms' pre-existing debt structure make distressed firms more susceptible to the impact of uncertainty (e.g., due to creditors' incentives to seek liquidation). Relatedly, we know very little about how the bankruptcy process itself, as well as the resolution of financial distress, is affected by uncertainty. Along these lines, researchers might study whether banks face increased incentives to keep firms alive as "zombies" when they are hit with uncertainty shocks. Taking this idea further, it would be important to see whether uncertainty-induced zombie firms distort capital allocation decisions of otherwise healthy firms operating in their industries—evidence of yet another channel through which financial frictions exacerbate the deleterious effects of uncertainty (see Aretz et al. (2023)).

Uncertainty is also likely to affect firms' capital structure policies, a central topic in corporate finance research. Existing work on this topic adopts a primarily macroeconomic focus, attempting to explain capital structure dynamics under counter-cyclical aggregate uncertainty shocks (see, e.g., Hackbarth et al. (2006), Christiano et al. (2014), Arellano et al. (2019), and Johnson (2022)). Yet, empirical studies of atomistic firm decisions in settings in which uncertainty shocks can be plausibly separated from first-moment shocks at the industry-specific or even idiosyncratic firm level are lacking. One would expect to see considerable firm heterogeneity as some firms benefit from uncertainty while others suffer—heterogeneity that is lost when studying firm responses to uncertainty measured at the level of macro-business cycles.

Our discussion of measurement also brings to the fore several directions in which future work may be oriented. Leveraging new data sources and machine learning methods is likely to be fruitful in advancing the text-based methods of measuring uncertainty, particularly on the front of gauging the amplitude and sentiment of a particular uncertainty episode as it relates to a firm. Researchers may also apply econometric methods from the macro literature designed to extract un-forecastable elements of firm-level uncertainty metrics or, equivalently, gauge long-term and short-term components of uncertainty.

Understanding differential sources of firm uncertainty is also paramount. New dimensions of uncertainty are becoming increasingly important for managers, yet remain understudied in the literature. A notable example is the rise of climate-related uncertainty. A small number of papers in finance has begun to emerge exploring various dimensions of climate risk—such as measurement (see, e.g., Barnett et al. (2020), Engle et al. (2020), Krueger et al. (2020), Ilhan et al. (2021), and Kruttli et al. (2023))—as well as implications for cost of capital (Chava (2014)), cash holdings (Dessaint and Matray (2017)), and investment (Jia and Li (2020)). This is certainly an area in which growing research interest is both expected and needed.

Lastly, and perhaps most speculatively, we urge empirical researchers to confront difficult questions that challenge the definition of uncertainty itself and how it arises.

Our framework and discussion so far rest on an implicit assumption that uncertainty is, in some sense, exogenous to the economic system in which firms operate. Yet, it is also apparent that some manifestations of uncertainty, particularly regulatory uncertainty, may be endogenous to the economic setting of interest as policy-makers respond to technological innovation expanding the frontiers of what is possible by imposing regulatory constraints, with firms, in turn, responding to these regulations (and uncertainty surrounding them). The domain of FinTech presents a particularly salient example of these dynamics in play. Empirical work in corporate finance ought to confront this phenomenon, perhaps taking inspiration from the literature in macroeconomics and banking on instability and indeterminacy under multiple equilibria (see, e.g., Cass and Shell (1983), Diamond and Dybvig (1983), and Gu et al. (2023)).

On the nature of uncertainty itself, anecdotal evidence suggests that managers are often confronted with uncertainty of the Knightian form—that is, uncertainty (or ambiguity) over outcomes for which neither the distribution nor the probabilities are known or readily estimated. As a first step in this direction, insights from theories on ambiguity (Ellsberg (1961) and Gilboa and Schmeidler (1989)) and model uncertainty (Hansen and Sargent (2001)) could be adapted and applied to the empirical corporate finance context. A small set of empirical studies have emerged looking into the implications of ambiguity on various corporate policies, including capital structure (Izhakian et al. (2022)), M&As (Herron and Izhakian (2020)), and cash holdings (Breuer et al. (2017)). We expect this to form a growing field of research in the coming years.

NOTES

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1. The finance papers studying the impact of uncertainty on the relevant corporate decisions we cover in this review appear in the top economics journals (American Economic Review, Quarterly Journal of Economics, Journal of Political Economy, Econometrica, Review of Economic Studies), finance journals (Journal of Finance, Review of Financial Studies, Journal of Financial Economics, Journal of Financial and Quantitative Analysis, Review of Finance, Management Science), and the National Bureau of Economic Research (NBER) working paper series.
2. Runde (1998) provides a thoughtful distillation of this debate.
3. See also Lee and Shin (2000) for a similar approach.
4. Our conceptual presentation in this section necessitates a number of simplifications, chief among which is the decision to represent a real-options framework in three discrete periods where some decision and payoff periods are collapsed. We note that this modeling is a rough representation of processes that realize at different points in the future, and not necessarily synchronously (as modeled).
5. Note that the continuous operating costs of both capital and labor inputs are embedded in the cash flows, v_t.
6. We assume convex costs for simplicity and to reflect the fact that the supply of promising ideas is finite (see Bloom et al. (2020)). Other plausible features of the R&D process may imply concave costs arising from specific assumptions about the correlation structure across projects and the presence of synergies across projects.

7. The revenues at the first and second stages have a common component and an idiosyncratic, IID component. As a result, the first-stage cash flow is a noisy but unbiased signal of the second-stage cash flow.
8. This is consistent with theories showing a positive effect of uncertainty on growth-options-type investment (see, e.g., Grossman and Shapiro (1986), Bar-Ilan and Strange (1996, 1998), and Kulatilaka and Perotti (1998)). Other theoretical work, including Dixit and Pindyck (1994), shows conditions under which R&D-type investments are curtailed in the face of increased uncertainty.
9. See also Guthrie (2011) and Hackbarth and Johnson (2015) for results on the role of uncertainty in shaping firm disinvestment.
10. Proofs of propositions and lemmas are in Appendix 15A.
11. See Bloom (2009) for dynamic equilibrium model generating similar implications via a "real options" effect.
12. Related theoretical work has posited other mechanisms for a positive link between investment and uncertainty. For instance, models that incorporate shareholders' risk-shifting incentives in the presence of debt also predict a positive uncertainty–investment relationship for financially distressed firms (see Eisdorfer (2008)). Boyle and Guthrie (2003) show that the possibility of future financing constraints arising may lead firms to accelerate investment in the face of uncertainty shocks. Lastly, models incorporating "time-to-build" often generate predictions consistent with a positive uncertainty–investment link (see, e.g., Aguerrevere (2003) and Marmer and Slade (2018)).
13. We assume, without loss of generality, normality of these distributions for analytical tractability. This allows us to write u_2 as u_1 plus an IID zero-mean normal noise with variance σ_x^2 and to provide a closed-form expression for the firm's breakeven R&D project in Eq. (15.14).
14. We note that, while our model assumes the cash flow processes underlying each corporate decision are independent, in reality these processes are likely to be correlated. This correlation matters insofar as an empirical researcher is likely to observe the response of the same set of firms across multiple corporate outcomes. As an example, the same firm is unlikely to unconditionally be engaging in significant investment and disinvestment at the same time. Therefore, an empirical finding that firms concurrently reduce both investment and disinvestment in response to second-moment shocks is informative about the real-options-based dynamics we discuss, given that these decisions are otherwise likely to be negatively correlated. While our framework does not allow for correlations across corporate decisions for simplicity, a richer model that incorporates such correlations would deliver predictions that will allow empirical researchers to more concretely distinguish between the first- and second-moment responses across the various corporate outcomes.
15. A handful of earlier works that utilized national elections as a setting to study the effects of uncertainty focused on either macroeconomic aggregates (Alesina et al. (1992)) or stock return volatility (Bernhard and Leblang (2006), Białkowski et al. (2008), and Boutchkova et al. (2012)).
16. Bloom et al. (2018, 2019) and Hassan et al. (2020) also employ Brexit as a shock to uncertainty.
17. Consider the very different nature of uncertainty induced by the 9/11 terror attacks, Brexit, and Covid-19.
18. See Bond et al. (2012) for a review of this literature.
19. Realized volatility of stock returns may also embed leverage effects (see Choi and Richardson (2016)), suggesting that researchers ought to control for leverage in tests employing returns volatility to capture uncertainty. Measurement error may further confound inferences, requiring econometric corrections to be applied (see, e.g., Li and Sun (2023)).
20. A related concern is that any measure of uncertainty that is derived from realized prices reflects both uncertainty and the expected firm response to that uncertainty.
21. The classical equity option pricing models, for instance, assume constant volatility (Black and Scholes (1973) and Merton (1973)). Implied volatility may also be influenced by the presence of jumps, risk premia, and, under incomplete markets, investor preferences (Jarrow and Wiggins (1989)).

22. Bloom et al. (2007), Diether et al. (2002), and Güntay and Hackbarth (2010), among others, consider analyst forecast dispersion as an indicator of firm-level uncertainty.
23. A handful of other studies have used this approach to gauge uncertainty in more limited contexts (see, e.g., Guiso and Parigi (1999), Bachmann et al. (2013), Bloom et al. (2018), Altig et al. (2022), and Kumar et al. (2022)). Relatedly, macroeconomic studies on the effects of uncertainty on output often use the dispersion of forecasts on economic aggregates made by professional economic forecasters as an indicator of uncertainty (see Jurado et al. (2015)).
24. In a related application, Bekaert et al. (2013) decompose the VIX index into a "risk-aversion" component and an "uncertainty" component using various econometric volatility forecasting models, and study their relation with monetary policy.
25. For example, uncertainty arising from climate change is likely to be long-lasting; however, firms may choose to learn and adapt early so as to minimize their exposure to future regulatory changes in emission standards.
26. A large literature on individual decision-making under such form of uncertainty has been inspired by, among others, the seminal work of Ellsberg (1961) and Gilboa and Schmeidler (1989).
27. The degree of competitive pressure that a firm faces from its technology peers is also likely to be key, yet this has been unaccounted for in existing work.
28. Easterwood et al. (2021), for instance, discuss the role that third moments, that is, skewness, may play.

REFERENCES

Abel, A., 1983. Optimal investment under uncertainty. American Economic Review 73, 228–233.
Acharya, V., Almeida, H., Campello, M., 2013. Aggregate risk and the choice between cash and lines of credit. Journal of Finance 68, 2059–2116.
Acharya, V., Steffen, S., 2020. The risk of being a fallen angel and the corporate dash for cash in the midst of COVID. Review of Corporate Finance Studies 9, 430–471.
Aguerrevere, F., 2003. Equilibrium investment strategies and output price behavior: A real-options approach. Review of Financial Studies 16, 1239–1272.
Ahir, H., Bloom, N., Furceri, D., 2022. The world uncertainty index. Tech. rep., National Bureau of Economic Research.
Alesina, A., Cohen, G., Roubini, N., 1992. Macroeconomic policy and elections in OECD democracies. Economics & Politics 4, 1–30.
Alessandri, P., Bottero, M., 2020. Bank lending in uncertain times. European Economic Review 128, 103503.
Alfaro, I., Bloom, N., Lin, X., 2023. The finance uncertainty multiplier. Journal of Political Economy, Forthcoming.
Almeida, H., Campello, M., 2007. Financial constraints, asset tangibility, and corporate investment. Review of Financial Studies 20, 1429–1460.
Almeida, H., Campello, M., Cunha, I., Weisbach, M., 2014. Corporate liquidity management: A conceptual framework and survey. Annual Review of Financial Economics 6, 135–162.
Altavilla, C., Carboni, G., Lenza, M., Uhlig, H., 2019. Interbank rate uncertainty and bank lending. Unpublished working paper. European Central Bank.
Altig, D., Barrero, J., Bloom, N., Davis, S., Meyer, B., Parker, N., 2022. Surveying business uncertainty. Journal of Econometrics 231, 282–303.
An, H., Chen, Y., Luo, D., Zhang, T., 2016. Political uncertainty and corporate investment: Evidence from China. Journal of Corporate Finance 36, 174–189.
Arbatli Saxegaard, E., Davis, S., Ito, A., Miake, N., 2022. Policy uncertainty in Japan. Journal of the Japanese and International Economies 64, 101192.
Arellano, C., Bai, Y., Kehoe, P., 2019. Financial frictions and fluctuations in volatility. Journal of Political Economy 127, 2049–2103.

Aretz, K., Campello, M., Kankanhalli, G., Schneider, K., 2023. Uncertainty creates zombie firms: Implications for competition dynamics and creative destruction. Unpublished working paper. Cornell University.
Ashraf, B., Shen, Y., 2019. Economic policy uncertainty and banks' loan pricing. Journal of Financial Stability 44, 100695.
Atanassov, J., Julio, B., Leng, T., 2019. The bright side of political uncertainty: The case of R&D. Unpublished working paper. University of Nebraska, Lincoln.
Attig, N., El Ghoul, S., Guedhami, O., Zheng, X., 2021. Dividends and economic policy uncertainty: International evidence. Journal of Corporate Finance 66, 101785.
Augustin, P., Izhakian, Y., 2020. Ambiguity, volatility, and credit risk. Review of Financial Studies 33, 1618–1672.
Azqueta-Gavaldón, A., 2017. Developing news-based economic policy uncertainty index with unsupervised machine learning. Economics Letters 158, 47–50.
Azqueta-Gavaldón, A., Hirschbühl, D., Onorante, L., Saiz, L., 2023. Sources of economic policy uncertainty in the Euro area. European Economic Review 152, 104373.
Bachmann, R., Elstner, S., Sims, E., 2013. Uncertainty and economic activity: Evidence from business survey data. American Economic Journal: Macroeconomics 5, 217–249.
Baker, S., Bloom, N., Davis, S., 2016. Measuring economic policy uncertainty. Quarterly Journal of Economics 131, 1593–1636.
Baker, S., Bloom, N., Davis, S., Renault, T., 2021. Twitter-derived measures of economic uncertainty. Unpublished working paper. Stanford University.
Baker, S., Bloom, N., Terry, S., 2023. Using disasters to estimate the impact of uncertainty. Review of Economic Studies, rdad036.
Baker, S., Davis, S., Levy, J., 2022. State-level economic policy uncertainty. Journal of Monetary Economics 132, 81–99.
Bali, T., Brown, S., Tang, Y., 2017. Is economic uncertainty priced in the cross-section of stock returns? Journal of Financial Economics 126, 471–489.
Bar-Ilan, A., Strange, W., 1996. Investment lags. American Economic Review 86, 610–622.
Bar-Ilan, A., Strange, W., 1998. A model of sequential investment. Journal of Economic Dynamics and Control 22, 437–463.
Barnett, M., Brock, W., Hansen, L., 2020. Pricing uncertainty induced by climate change. Review of Financial Studies 33, 1024–1066.
Barry, J., Campello, M., Graham, J., Ma, Y., 2022. Corporate flexibility in a time of crisis. Journal of Financial Economics 144, 780–806.
Bekaert, G., Engstrom, E., Xing, Y., 2009. Risk, uncertainty, and asset prices. Journal of Financial Economics 91, 59–82.
Bekaert, G., Hoerova, M., Lo Duca, M., 2013. Risk, uncertainty and monetary policy. Journal of Monetary Economics 60, 771–788.
Ben-David, I., Graham, J., Harvey, C., 2013. Managerial miscalibration. Quarterly Journal of Economics 128, 1547–1584.
Berger, A., Guedhami, O., Kim, H., Li, X., 2022. Economic policy uncertainty and bank liquidity hoarding. Journal of Financial Intermediation 49, 100893.
Bernanke, B., 1983. Irreversibility, uncertainty, and cyclical investment. Quarterly Journal of Economics 98, 85–106.
Bernhard, W., Leblang, D., 2006. Democratic processes and financial markets: Pricing politics. Cambridge University Press, Cambridge, UK.
Bhagwat, V., Dam, R., Harford, J., 2016. The real effects of uncertainty on merger activity. Review of Financial Studies 29, 3000–3034.
Bhattacharya, U., Hsu, P.-H., Tian, X., Xu, Y., 2017. What affects innovation more: Policy or policy uncertainty? Journal of Financial and Quantitative Analysis 52, 1869–1901.
Białkowski, J., Gottschalk, K., Wisniewski, T., 2008. Stock market volatility around national elections. Journal of Banking & Finance 32, 1941–1953.
Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–654.

Bliss, B., Cheng, Y., Denis, D., 2015. Corporate payout, cash retention, and the supply of credit: Evidence from the 2008–2009 credit crisis. Journal of Financial Economics 115, 521–540.
Bloom, N., 2009. The impact of uncertainty shocks. Econometrica 77, 623–685.
Bloom, N., 2014. Fluctuations in uncertainty. Journal of Economic Perspectives 28, 153–176.
Bloom, N., Bond, S., Van Reenen, J., 2007. Uncertainty and investment dynamics. Review of Economic Studies 74, 391–415.
Bloom, N., Bunn, P., Chen, S., Mizen, P., Smietanka, P., Thwaites, G., 2019. The impact of Brexit on UK firms. Tech. rep., National Bureau of Economic Research.
Bloom, N., Bunn, P., Chen, S., Mizen, P., Smietanka, P., Thwaites, G., Young, G., 2018. Brexit and uncertainty: Insights from the Decision Maker Panel. Fiscal Studies 39, 555–580.
Bloom, N., Jones, C., Van Reenen, J., Webb, M., 2020. Are ideas getting harder to find? American Economic Review 110, 1104–1144.
Bonaimé, A., Gulen, H., Ion, M., 2018. Does policy uncertainty affect mergers and acquisitions? Journal of Financial Economics 129, 531–558.
Bond, P., Edmans, A., Goldstein, I., 2012. The real effects of financial markets. Annual Review of Financial Economics 4, 339–360.
Bordo, M., Duca, J., Koch, C., 2016. Economic policy uncertainty and the credit channel: Aggregate and bank level US evidence over several decades. Journal of Financial Stability 26, 90–106.
Borthwick, J., Ali, S., Pan, X., 2020. Does policy uncertainty influence mergers and acquisitions activities in China? A replication study. Pacific-Basin Finance Journal 62, 101381.
Boutchkova, M., Doshi, H., Durnev, A., Molchanov, A., 2012. Precarious politics and return volatility. Review of Financial Studies 25, 1111–1154.
Boyle, G., Guthrie, G., 2003. Investment, uncertainty, and liquidity. Journal of Finance 58, 2143–2166.
Brennan, M., Xia, Y., 2001. Stock price volatility and equity premium. Journal of Monetary Economics 47, 249–283.
Brenner, M., Izhakian, Y., 2018. Asset pricing and ambiguity: Empirical evidence. Journal of Financial Economics 130, 503–531.
Breuer, W., Rieger, M., Soypak, K., 2017. Corporate cash holdings and ambiguity aversion. Review of Finance 21, 1933–1974.
Brogaard, J., Dai, L., Ngo, P., Zhang, B., 2020. Global political uncertainty and asset prices. Review of Financial Studies 33, 1737–1780.
Buchanan, B., Cao, C., Liljeblom, E., Weihrich, S., 2017. Uncertainty and firm dividend policy: A natural experiment. Journal of Corporate Finance 42, 179–197.
Bushee, B., Jung, M., Miller, G., 2011. Conference presentations and the disclosure milieu. Journal of Accounting Research 49, 1163–1192.
Bushman, R., Dai, Z., Wang, X., 2010. Risk and CEO turnover. Journal of Financial Economics 96, 381–398.
Caballero, R., 1991. On the sign of the investment-uncertainty relationship. American Economic Review 81, 279–288.
Caggese, A., 2012. Entrepreneurial risk, investment, and innovation. Journal of Financial Economics 106, 287–307.
Caldara, D., Iacoviello, M., Molligo, P., Prestipino, A., Raffo, A., 2020. The economic effects of trade policy uncertainty. Journal of Monetary Economics 109, 38–59.
Campello, M., Cortes, G., d'Almeida, F., Kankanhalli, G., 2022. Exporting uncertainty: The impact of Brexit on corporate America. Journal of Financial and Quantitative Analysis 57, 3178–3222.
Campello, M., Graham, J., Harvey, C., 2010. The real effects of financial constraints: Evidence from a financial crisis. Journal of Financial Economics 97, 470–487.
Campello, M., Kankanhalli, G., Kim, H., 2023. Delayed creative destruction: How uncertainty shapes corporate assets. Journal of Financial Economics, Forthcoming.
Campello, M., Kankanhalli, G., Muthukrishnan, P., 2023. Corporate hiring under Covid-19: Financial constraints and the nature of new jobs. Journal of Financial and Quantitative Analysis, 1–45. doi:10.1017/S0022109023000522.
Campello, M., Lin, C., Ma, Y., Zou, H., 2011. The real and financial implications of corporate hedging. Journal of Finance 66, 1615–1647.

Cao, C., Li, X., Liu, G., 2019. Political uncertainty and cross-border acquisitions. Review of Finance 23, 439–470.
Carvalho, D., 2018. Uncertainty and exit decisions: Evidence from plant-level data. Unpublished working paper. Indiana University.
Cass, D., Shell, K., 1983. Do sunspots matter? Journal of Political Economy 91, 193–227.
Chakraborty, A., Kazarosian, M., Trahan, E., 1999. Uncertainty in executive compensation and capital investment: A panel study. Financial Management 28, 126–139.
Chan, Y.-C., Saffar, W., Wei, K., 2021. How economic policy uncertainty affects the cost of raising equity capital: Evidence from seasoned equity offerings. Journal of Financial Stability 53, 100841.
Chava, S., 2014. Environmental externalities and cost of capital. Management Science 60, 2223–2247.
Chay, J.-B., Suh, J., 2009. Payout policy and cash-flow uncertainty. Journal of Financial Economics 93, 88–107.
Chen, Z., Cihan, M., Jens, C., Page, T., 2023. Political uncertainty and firm investment: Project-level evidence from M&A activity. Journal of Financial and Quantitative Analysis 58, 71–103.
Choi, J., Richardson, M., 2016. The volatility of a firm's assets and the leverage effect. Journal of Financial Economics 121, 254–277.
Christiano, L., Motto, R., Rostagno, M., 2014. Risk shocks. American Economic Review 104, 27–65.
Çolak, G., Durnev, A., Qian, Y., 2017. Political uncertainty and IPO activity: Evidence from US gubernatorial elections. Journal of Financial and Quantitative Analysis 52, 2523–2564.
Cong, L., Howell, S., 2021. Policy uncertainty and innovation: Evidence from initial public offering interventions in China. Management Science 67, 7238–7261.
Corrado, C., Haskel, J., Jona-Lasinio, C., Iommi, M., 2022. Intangible capital and modern economies. Journal of Economic Perspectives 36, 3–28.
Cui, X., Wang, C., Liao, J., Fang, Z., Cheng, F., 2021. Economic policy uncertainty exposure and corporate innovation investment: Evidence from China. Pacific-Basin Finance Journal 67, 101533.
D'Mello, R., Toscano, F., 2020. Economic policy uncertainty and short-term financing: The case of trade credit. Journal of Corporate Finance 64, 101686.
Davis, S., 2016. An index of global economic policy uncertainty. Tech. rep., National Bureau of Economic Research.
Davis, S., Hansen, S., Seminario-Amez, C., 2021. Firm-level risk exposures and stock returns in the wake of COVID-19. Tech. rep., National Bureau of Economic Research.
Davis, S., Liu, D., Sheng, X., 2019. Economic policy uncertainty in China since 1949: The view from mainland newspapers. In: Fourth Annual IMF-Atlanta Fed Research Workshop on China's Economy Atlanta, vol. 19.
Décaire, P., Gilje, E., Taillard, J., 2020. Real option exercise: Empirical evidence. Review of Financial Studies 33, 3250–3306.
Decamps, J.-P., Gryglewicz, S., Morellec, E., Villeneuve, S., 2017. Corporate policies with permanent and transitory shocks. Review of Financial Studies 30, 162–210.
Dessaint, O., Matray, A., 2017. Do managers overreact to salient risks? Evidence from hurricane strikes. Journal of Financial Economics 126, 97–121.
Diamond, D., Dybvig, P., 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy 91, 401–419.
Dicks, D., Fulghieri, P., 2022. Uncertainty, disagreement, and allocation of control in firms. Unpublished working paper. Baylor University.
Diether, K., Malloy, C., Scherbina, A., 2002. Differences of opinion and the cross section of stock returns. Journal of Finance 57, 2113–2141.
Dixit, A., Pindyck, R., 1994. Investment under uncertainty. Princeton University Press, Princeton.
Doshi, H., Kumar, P., Yerramilli, V., 2018. Uncertainty, capital investment, and risk management. Management Science 64, 5769–5786.
Drobetz, W., El Ghoul, S., Guedhami, O., Janzen, M., 2018. Policy uncertainty, investment, and the cost of capital. Journal of Financial Stability 39, 28–45.
Duong, H., Nguyen, J., Nguyen, M., Rhee, S., 2020. Navigating through economic policy uncertainty: The role of corporate cash holdings. Journal of Corporate Finance 62, 101607.

Easterwood, J., Paye, B., Xie, Y., 2021. Firm uncertainty and corporate policies: The role of stock return skewness. Journal of Corporate Finance 69, 102032.
Eisdorfer, A., 2008. Empirical evidence of risk shifting in financially distressed firms. Journal of Finance 63, 609–637.
Ellsberg, D., 1961. Risk, ambiguity, and the Savage axioms. Quarterly Journal of Economics 75, 643–669.
Engle, R., Giglio, S., Kelly, B., Lee, H., Stroebel, J., 2020. Hedging climate change news. Review of Financial Studies 33, 1184–1216.
Epstein, L., Schneider, M., 2010. Ambiguity and asset markets. Annual Review of Financial Economics 2, 315–346.
Favara, G., Gao, J., Giannetti, M., 2021. Uncertainty, access to debt, and firm precautionary behavior. Journal of Financial Economics 141, 436–453.
Friberg, R., Seiler, T., 2017. Risk and ambiguity in 10-Ks: An examination of cash holding and derivatives use. Journal of Corporate Finance 45, 608–631.
Frye, M., Pham, D., 2020. Economic policy uncertainty and board monitoring: Evidence from CEO turnovers. Journal of Financial Research 43, 675–703.
Gao, J., Grinstein, Y., Wang, W., 2017. Cash holdings, precautionary motives, and systematic uncertainty. Unpublished working paper. Cornell University.
Gavriilidis, K., 2021. Measuring climate policy uncertainty. Unpublished working paper. University of Stirling.
Giambona, E., Graham, J., Harvey, C., 2017. The management of political risk. Journal of International Business Studies 48, 523–533.
Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. Journal of Mathematical Economics 18, 141–153.
Gilchrist, S., Sim, J., Zakrajšek, E., 2014. Uncertainty, financial frictions, and investment dynamics. Tech. rep., National Bureau of Economic Research.
Gissler, S., Oldfather, J., Ruffino, D., 2016. Lending on hold: Regulatory uncertainty and bank lending standards. Journal of Monetary Economics 81, 89–101.
Glover, B., Levine, O., 2015. Uncertainty, investment, and managerial incentives. Journal of Monetary Economics 69, 121–137.
Goodell, J., Goyal, A., Urquhart, A., 2021. Uncertainty of uncertainty and firm cash holdings. Journal of Financial Stability 56, 100922.
Gorbenko, A., Strebulaev, I., 2010. Temporary versus permanent shocks: Explaining corporate financial policies. Review of Financial Studies 23, 2591–2647.
Graham, J., 2022. Presidential address: Corporate finance and reality. Journal of Finance 77, 1975–2049.
Gregoriou, A., Nguyen, B., Nguyen, T., Le, H., Hudson, R., 2021. Economic policy uncertainty and cross-border mergers and acquisitions. International Review of Financial Analysis 78, 100926.
Grenadier, S., 2002. Option exercise games: An application to the equilibrium investment strategies of firms. Review of Financial Studies 15, 691–721.
Grenadier, S., Malenko, A., 2010. A Bayesian approach to real options: The case of distinguishing between temporary and permanent shocks. Journal of Finance 65, 1949–1986.
Grigoris, F., Segal, G., 2021. Investment under up-and downstream uncertainty. Unpublished working paper. Indiana University.
Grossman, G., Shapiro, C., 1986. Optimal dynamic R&D programs. RAND Journal of Economics 17, 581–593.
Gryglewicz, S., Mancini, L., Morellec, E., Schroth, E., Valta, P., 2022. Understanding cash flow risk. Review of Financial Studies 35, 3922–3972.
Gu, C., Monnet, C., Nosal, E., Wright, R., 2023. Diamond-Dybvig and beyond: On the instability of banking. European Economic Review 154, 104414.
Guceri, I., Albinowski, M., 2021. Investment responses to tax policy under uncertainty. Journal of Financial Economics 141, 1147–1170.
Guidolin, M., Rinaldi, F., 2013. Ambiguity in asset pricing and portfolio choice: A review of the literature. Theory and Decision 74, 183–217.
Guiso, L., Parigi, G., 1999. Investment and demand uncertainty. Quarterly Journal of Economics 114, 185–227.

Gulen, H., Ion, M., 2016. Policy uncertainty and corporate investment. Review of Financial Studies 29, 523–564.
Güntay, L., Hackbarth, D., 2010. Corporate bond credit spreads and forecast dispersion. Journal of Banking & Finance 34, 2328–2345.
Guthrie, G., 2011. A note on operating leverage and expected rates of return. Finance Research Letters 8, 88–100.
Hackbarth, D., Johnson, T., 2015. Real options and risk dynamics. Review of Economic Studies 82, 1449–1482.
Hackbarth, D., Miao, J., Morellec, E., 2006. Capital structure, credit risk, and macroeconomic conditions. Journal of Financial Economics 82, 519–550.
Handley, K., Li, J., 2020. Measuring the effects of firm uncertainty on economic activity: New evidence from one million documents. Tech. rep., National Bureau of Economic Research.
Handley, K., Limão, N., Ludema, R., Yu, Z., 2020. Firm input choice under trade policy uncertainty. Tech. rep., National Bureau of Economic Research.
Hansen, L., Sargent, T., 2001. Robust control and model uncertainty. American Economic Review 91, 60–66.
Hansen, P., 2022. Valuation uncertainty and dynamic entrepreneurial finance. Unpublished working paper. Purdue University.
Hartman, R., 1972. The effects of price and cost uncertainty on investment. Journal of Economic Theory 5, 258–266.
Hassan, T., Hollander, S., Van Lent, L., Tahoun, A., 2019. Firm-level political risk: Measurement and effects. Quarterly Journal of Economics 134, 2135–2202.
Hassan, T., Hollander, S., Van Lent, L., Tahoun, A., 2020. The global impact of Brexit uncertainty. Tech. rep., National Bureau of Economic Research.
Herron, R., Izhakian, Y., 2020. Mergers and acquisitions: The role of ambiguity. Unpublished working paper. Northeastern University.
Hoang, K., Nguyen, C., Zhang, H., 2021. How does economic policy uncertainty affect corporate diversification? International Review of Economics & Finance 72, 254–269.
Hoberg, G., Phillips, G., 2016. Text-based network industries and endogenous product differentiation. Journal of Political Economy 124, 1423–1465.
Hollander, S., Pronk, M., Roelofsen, E., 2010. Does silence speak? An empirical analysis of disclosure choices during conference calls. Journal of Accounting Research 48, 531–563.
Huang, Y., Luk, P., 2020. Measuring economic policy uncertainty in China. China Economic Review 59, 101367.
Husted, L., Rogers, J., Sun, B., 2020. Monetary policy uncertainty. Journal of Monetary Economics 115, 20–36.
Ilhan, E., Sautner, Z., Vilkov, G., 2021. Carbon tail risk. Review of Financial Studies 34, 1540–1571.
Ilyas, M., Khan, A., Nadeem, M., Suleman, M., 2021. Economic policy uncertainty, oil price shocks and corporate investment: Evidence from the oil industry. Energy Economics 97, 105193.
Im, H., Kang, Y., Shon, J., 2020. How does uncertainty influence target capital structure? Journal of Corporate Finance 64, 101642.
Ion, M., Yin, D., 2021. Policy uncertainty, corporate risk-taking, and CEO incentives. Unpublished working paper. University of Arizona.
Izhakian, Y., Yermack, D., 2017. Risk, ambiguity, and the exercise of employee stock options. Journal of Financial Economics 124, 65–85.
Izhakian, Y., Yermack, D., Zender, J., 2022. Ambiguity and the tradeoff theory of capital structure. Management Science 68, 4090–4111.
Jacob, M., Wentland, K., Wentland, S., 2022. Real effects of tax uncertainty: Evidence from firm capital investments. Management Science 68, 4065–4089.
Jarrow, R., Wiggins, J., 1989. Option pricing and implicit volatilities. Journal of Economic Surveys 3, 59–81.
Jens, C., 2017. Political uncertainty and investment: Causal evidence from US gubernatorial elections. Journal of Financial Economics 124, 563–579.
Jia, J., Li, Z., 2020. Does external uncertainty matter in corporate sustainability performance? Journal of Corporate Finance 65, 101743.

Jiang, L., Pittman, J., Saffar, W., 2022. Policy uncertainty and textual disclosure. Accounting Horizons 36, 113–131.
Johnson, T., 2022. Economic uncertainty, aggregate debt, and the real effects of corporate finance. Critical Finance Review 11, 79–116.
Jory, S., Khieu, H., Ngo, T., Phan, H., 2020. The influence of economic policy uncertainty on corporate trade credit and firm value. Journal of Corporate Finance 64, 101671.
Julio, B., Yook, Y., 2012. Political uncertainty and corporate investment cycles. Journal of Finance 67, 45–83.
Julio, B., Yook, Y., 2016. Policy uncertainty, irreversibility, and cross-border flows of capital. Journal of International Economics 103, 13–26.
Jurado, K., Ludvigson, S., Ng, S., 2015. Measuring uncertainty. American Economic Review 105, 1177–1216.
Kang, W., Lee, K., Ratti, R., 2014. Economic policy uncertainty and firm-level investment. Journal of Macroeconomics 39, 42–53.
Kankanhalli, G., 2021. Betting on disruption: How uncertainty shapes the US startup ecosystem. Unpublished working paper. University of Pittsburgh.
Kaviani, M., Kryzanowski, L., Maleki, H., Savor, P., 2020. Policy uncertainty and corporate credit spreads. Journal of Financial Economics 138, 838–865.
Kellogg, R., 2014. The effect of uncertainty on investment: Evidence from Texas oil drilling. American Economic Review 104, 1698–1734.
Kim, H., Kung, H., 2017. The asset redeployability channel: How uncertainty affects corporate investment. Review of Financial Studies 30, 245–280.
Knight, F., 1921. Risk, uncertainty and profit. Houghton Mifflin, Boston.
Kozeniauskas, N., Orlik, A., Veldkamp, L., 2018. What are uncertainty shocks? Journal of Monetary Economics 100, 1–15.
Kraft, H., Schwartz, E., Weiss, F., 2018. Growth options and firm valuation. European Financial Management 24, 209–238.
Krueger, P., Sautner, Z., Starks, L., 2020. The importance of climate risks for institutional investors. Review of Financial Studies 33, 1067–1111.
Kruttli, M., Roth Tran, B., Watugala, S., 2023. Pricing Poseidon: Extreme weather uncertainty and firm return dynamics. Unpublished working paper. Indiana University.
Kulatilaka, N., Perotti, E., 1998. Strategic growth options. Management Science 44, 1021–1031.
Kumar, P., Tian, G., Yerramilli, V., 2021. Trade policy uncertainty and shareholder returns in mergers and acquisitions. Unpublished working paper. University of Houston.
Kumar, S., Gorodnichenko, Y., Coibion, O., 2022. The effect of macroeconomic uncertainty on firm decisions. Tech. rep., National Bureau of Economic Research.
Leahy, J., Whited, T., 1996. The effect of uncertainty on investment: Some stylized facts. Journal of Money, Credit and Banking 28, 64–83.
Lee, J., Shin, K., 2000. The role of a variable input in the relationship between investment and uncertainty. American Economic Review 90, 667–680.
Li, D., Sun, Y., 2023. The impact of uncertainty on investment: Empirical challenges and a new estimator. Journal of Financial and Quantitative Analysis, 1–32. doi:10.1017/S0022109023000169.
Lin, C., Schmid, T., Weisbach, M., 2021. Product price risk and liquidity management: Evidence from the electricity industry. Management Science 67, 2519–2540.
Luis, P., Perote, J., de la Fuente, G., 2022. The impact of economic policy uncertainty and monetary policy on R&D investment: An option pricing approach. Economics Letters 214, 110413.
Malenko, A., Tsoy, A., 2020. Asymmetric information and security design under Knightian uncertainty. Unpublished working paper. University of Michigan.
Manela, A., Moreira, A., 2017. News implied volatility and disaster concerns. Journal of Financial Economics 123, 137–162.
Marmer, V., Slade, M., 2018. Investment and uncertainty with time to build: Evidence from entry into US copper mining. Journal of Economic Dynamics and Control 95, 233–254.
Matsumoto, D., Pronk, M., Roelofsen, E., 2011. What makes conference calls useful? The information content of managers' presentations and analysts' discussion sessions. Accounting Review 86, 1383–1414.

Merton, R., 1973. Theory of rational option pricing. Bell Journal of Economics and Management Science 4, 141–183.
Merton, R., 1974. On the pricing of corporate debt: The risk structure of interest rates. Journal of Finance 29, 449–470.
Miao, J., Wang, N., 2011. Risk, uncertainty, and option exercise. Journal of Economic Dynamics and Control 35, 442–461.
Nguyen, N., Phan, H., 2017. Policy uncertainty and mergers and acquisitions. Journal of Financial and Quantitative Analysis 52, 613–644.
Nishimura, K., Ozaki, H., 2007. Irreversible investment and Knightian uncertainty. Journal of Economic Theory 136, 668–694.
Oi, W., 1961. The desirability of price instability under perfect competition. Econometrica 29, 58–64.
Panousi, V., Papanikolaou, D., 2012. Investment, idiosyncratic risk, and ownership. Journal of Finance 67, 1113–1148.
Paudyal, K., Thapa, C., Koirala, S., Aldhawyan, S., 2021. Economic policy uncertainty and cross-border mergers and acquisitions. Journal of Financial Stability 56, 100926.
Pérez-González, F., Yun, H., 2013. Risk management and firm value: Evidence from weather derivatives. Journal of Finance 68, 2143–2176.
Phan, H., Nguyen, N., Nguyen, H., Hegde, S., 2019. Policy uncertainty and firm cash holdings. Journal of Business Research 95, 71–82.
Pindyck, R., 1993. A note on competitive investment under uncertainty. American Economic Review 83, 273–277.
Pirgaip, B., Dinçergök, B., 2019. Share repurchases under uncertainty: US evidence. Finance Research Letters 30, 130–138.
Riddick, L., Whited, T., 2009. The corporate propensity to save. Journal of Finance 64, 1729–1766.
Runde, J., 1998. Clarifying Frank Knight's discussion of the meaning of risk and uncertainty. Cambridge Journal of Economics 22, 539–546.
Sarkar, A., 2019. Policy uncertainty, multinational firms, and reallocation. Unpublished working paper. National University of Singapore.
Schlag, C., Thimme, J., Weber, R., 2021. Implied volatility duration: A measure for the timing of uncertainty resolution. Journal of Financial Economics 140, 127–144.
Schröder, D., 2011. Investment under ambiguity with the best and worst in mind. Mathematics and Financial Economics 4, 107–133.
Segal, G., Shaliastovich, I., 2021. Uncertainty, risk, and capital growth. Unpublished working paper. University of North Carolina, Chapel Hill.
Segal, G., Shaliastovich, I., Yaron, A., 2015. Good and bad uncertainty: Macroeconomic and financial market implications. Journal of Financial Economics 117, 369–397.
Sha, Y., Kang, C., Wang, Z., 2020. Economic policy uncertainty and mergers and acquisitions: Evidence from China. Economic Modelling 89, 590–600.
Smith, C., Stulz, R., 1985. The determinants of firms' hedging policies. Journal of Financial and Quantitative Analysis 20, 391–405.
Stein, L., Stone, E., 2013. The effect of uncertainty on investment, hiring, and R&D: Causal evidence from equity options. Unpublished working paper. Babson College.
Stokey, N., 2016. Wait-and-see: Investment options under policy uncertainty. Review of Economic Dynamics 21, 246–265.
Stulz, R., 1984. Optimal hedging policies. Journal of Financial and Quantitative Analysis 19, 127–140.
Telser, L., 1955. Safety first and hedging. Review of Economic Studies 23, 1–16.
Wang, Y., Chen, C., Huang, Y., 2014. Economic policy uncertainty and corporate investment: Evidence from China. Pacific-Basin Finance Journal 26, 227–243.
Wu, W.-S., Suardi, S., 2021. Economic uncertainty and bank lending. Journal of Money, Credit and Banking 53, 2037–2069.
Xu, Z., 2020. Economic policy uncertainty, cost of capital, and corporate innovation. Journal of Banking & Finance 111, 105698.

APPENDIX 15A: PROOFS

A.1 Proof of Lemma 1

Proof. Let us define

$$H(n^*) = v_1 + \mathbb{E}[v_2] - (\kappa + \lambda)n^* - \mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)n^*, 0\right)\right].$$

To guarantee the existence of n^* as characterized by Eq. (6), it suffices to show that $H(n^*) = 0$ for some $n^* \in [0, N]$. Since $H(\cdot)$ is a sum of continuous functions, it is itself continuous. Since $v_1 > 0$ and $v_2 > 0$, it follows that:

$$H(0) = v_1 + \mathbb{E}[v_2] - \mathbb{E}[\max(v_2, 0)] = v_1 > 0.$$

Finally, for $N \to \infty$, we have that:

$$\begin{aligned}\lim_{N\to\infty} H(N) &= \lim_{N\to\infty}\left(v_1 + \mathbb{E}[v_2] - (\kappa + \lambda)N\right) + \lim_{N\to\infty}\left(\mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)N, 0\right)\right]\right) \\ &= -\infty \quad + 0 = -\infty.\end{aligned}$$

Thus, there must exist an $\bar{N} \in R$ such that, for $N > \bar{N}, H(\bar{N}) < 0$. Putting these conditions together with the continuity of $H(\cdot)$ over $[0, N]$, the Intermediate Value Theorem guarantees that there exists an $n^* \in [0, N]$ such that $H(n^*) = 0$.

A.2 Proof of Proposition 1

Proof. Let us define

$$H(n^*; p) = v_1 + \mathbb{E}[v_2] - (\kappa + \lambda)n^* - \mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)n^*, 0\right); p\right] = 0.$$

By the Implicit Function Theorem,

$$\frac{dn^*}{dp} = -\frac{\partial H/\partial n^*}{\partial H/\partial p}.$$

Considering first the derivative of H with respect to n^*, we have:

$$\begin{aligned}\frac{\partial H(n^*; p)}{\partial n^*} &= -(\kappa + \lambda) - \frac{\partial}{\partial n^*}\mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)n^*, 0\right); p\right] \\ &= -(\kappa + \lambda) - \mathbb{E}\left[\frac{\partial}{\partial n^*}\max\left(v_2 - (\kappa + \lambda)n^*, 0\right); p\right] \\ &= -(\kappa + \lambda) - \mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda), 0\right); p\right] \\ &< 0.\end{aligned}$$

Next, considering the derivative of H with respect to p, we have:

$$\frac{\partial H(n^*; p)}{\partial p} = -\frac{\partial}{\partial p}\mathbb{E}\left[\max\left(v_2 - (\kappa + \lambda)n^*, 0\right); p\right].$$

Because $P(\cdot\ ,p')$ is an MPS of $P(\ \cdot\ ,p)$, for any convex function $J(\ \cdot\ )$,

$$\begin{aligned}\mathbb{E}\left[J(v_2);p'\right] &= \int J(v_2)dP(v_2,p')\\ &\geq \int J(v_2)dP(v_2,p)\\ &= \mathbb{E}\left[J(v_2);p\right].\end{aligned}$$

Since $\max\left(v_2 - (\kappa+\lambda)n^*,0\right)$ is convex in v_2, it follows that:

$$\mathbb{E}\left[\max\left(v_2 - (\kappa+\lambda)n^*,0\right);p'\right] \geq \mathbb{E}\left[\max\left(v_2 - (\kappa+\lambda)n^*,0\right);p\right] \forall p' > p.$$

This implies

$$\frac{\partial}{\partial p}\mathbb{E}\left[\max\left(v_2 - (\kappa+\lambda)n^*,0\right);p\right] \geq 0.$$

Thus,

$$\begin{aligned}\frac{\partial H(n^*;p)}{\partial p} &= -\frac{\partial}{\partial p}\mathbb{E}\left[\max\left(v_2 - (\kappa+\lambda)n^*,0\right);p\right]\\ &\leq 0.\end{aligned}$$

Putting these conditions together, we have:

$$\frac{dn^*}{dp} = -\frac{\partial H/\partial n^*}{\partial H/\partial p} < 0.$$

A.3 Proof of Lemma 2 and Proposition 2

Proof. Symmetric to the case of investment.

A.4 Proof of Proposition 3

Proof. Let us define

$$H(n^*;\kappa) = v_1 + \mathbb{E}\left[v_2\right] - \left(\kappa+\lambda\right)n^* - \mathbb{E}\left[\max\left(v_2 - (\kappa+\lambda)n^*,0\right)\right] = 0.$$

By the Implicit Function Theorem,

$$\frac{dn^*}{d\kappa} = -\frac{\partial H/\partial n^*}{\partial H/\partial \kappa}.$$

Considering first the numerator, we know from Proposition 1 that:

$$\frac{\partial H}{\partial n^*} < 0.$$

Next, considering the denominator,

$$\begin{aligned}\frac{\partial H}{\partial \kappa} &= -n^* - \frac{\partial}{\partial \kappa}\mathbb{E}\left[\max\left(v_2 - (\kappa+\lambda)n^*, 0\right)\right] \\ &= -n^* - \mathbb{E}\left[\frac{\partial}{\partial \kappa}\max\left(v_2 - (\kappa+\lambda)n^*, 0\right)\right] \\ &= -n^* - \mathbb{E}\left[\max\left(v_2 - n^*, 0\right)\right] \\ &< 0.\end{aligned}$$

Putting these together, we have:

$$\frac{dn^*}{d\kappa} = -\frac{\partial H/\partial n^*}{\partial H/\partial \kappa} < 0.$$

A.5 Proof of Proposition 4

Proof. Symmetric to the case of capital.

A.6 Proof of Proposition 5

Proof. Rearranging Eq. (15.14), we get:

$$m^* = \frac{1}{\alpha}\left(\overline{u}_1 - d_1 - d_2 + u_1 + \frac{\sqrt{\omega^2(q)+\sigma_x^2}}{1-\Phi\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right)}\left[\phi\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right)\right]\right)^{\frac{1}{\alpha-1}}$$

It can be shown that $\frac{dm^*}{dq} > 0$ as:

$$\phi\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right)\left[1-\frac{\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right)\phi\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right)}{1-\Phi\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right)}\right] - \phi'\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right)\left(\frac{d_2-\overline{u}_1}{\sqrt{\omega^2(q)+\sigma_x^2}}\right) > 0.$$