Intro to Q theory of investment

FIN 971: Corporate Finance

Professor Oliver Levine

Predicting firm investment

- One of the major questions in finance is trying to understand firm invest behavior
- Tobin (1969), Abel (1979), Hayashi (1982)

Take a recursive formulation of the firm's problem:

$$\begin{aligned} V(A,K) &= \max_{K'} D + \beta \mathbb{E}_A[V(A',K')] \\ &\text{s.t.} \\ D &= \Pi(A,K) - I - \Phi(I,K) \\ I &= K' - (1-\delta)K \end{aligned}$$

FOC: marginal cost of investment = marginal q :

$$1 + \Phi_1(K',K) = \beta \mathbb{E}_A[V_2(A',K')] \equiv q$$

Characterizing optimal investment

FOC for the general firm's problem can be written:

$$q_t = 1 + \Phi_I(I_t, K_t)$$

This should characterize investment. How can we test it?

- One option: solve for I_t

$$I_t = G(q_t - 1, K_t)$$

where $G = \Phi_I^{-1}$

In principle, we can estimate this investment equation

But we need marginal q, which isn't directly observable …

Additional restrictions

Impose additional restrictions that $\Pi()$ and $\Phi()$ are homogeneous degree 1.

- [Homog. deg k $\Rightarrow f(ax_1, ax_2) = a^k f(x_1, x_2)$]
- In this case, it can be shown (Hayashi, 1982) that ($Q \equiv$ Tobin's Q)

$$q_t = Q_t = \frac{V_t}{K_t}$$

and

$$\Phi_I(I_t, K_t) = \Phi_I\left(\frac{I_t}{K_t}\right)$$

this means

$$\frac{I}{K} = G(Q - 1)$$

Common special case

As a special case, suppose

$$\Phi(I,K) = \frac{b}{2}\left(\frac{I}{K} - \delta\right)^2 K$$

$$\begin{aligned} \frac{I_t}{K_t} &= \delta + \frac{1}{b}(Q_t - 1) \\ &= \beta_0 + \beta_1 Q_t \end{aligned}$$

1. Here Tobin's Q is a sufficient statistic for investment
   - Q completely summarizes the firm's investment decision

2. Investment is positively related to Q
   - Firm invests more than rate δ when $Q_t > 1$
   - Firm wants to invest when marginal value of an additional unit of capital is high

"Q 'controls' for investment opportunities" (high Q $\Rightarrow$ high investment opportunities)

Testing Q theory

$$\frac{I_t}{K_t} = \beta_0 + \beta_1 Q_t$$

where $\beta_1 = 1/b$.

- Q doesn't appear to be a sufficient stat for investment
- Original Hayashi (1982) work: $\beta_1 = .04$ and $R^2 = 0.46$ for aggregate data
- Panel data estimates typically lower, esp for R^2

Implied adjustment cost

A low coefficient estimate for β_1 implies a very large estimated marginal adjustment cost parameter b. Hayashi's estimate suggests $b \simeq 25$.

Implies total cost in terms of lost dividends of additional unit of investment given by

$$\text{Direct cost} + \text{Marginal Adj Cost} = 1 + \Phi_I = 1 + b\left(\frac{I}{K} - \delta\right)$$

If $I = 2\delta K$ and $\delta = 15\%$, then

$$\Phi_I = b\delta = 25 \times 0.15 = 3.75$$

Appears high for a direct purchase cost of 1.

Andrei, Moyen, and Mann (2017)

- Explanatory power has improved significantly since the mid 1990s