Name:_____ Date:_____

Class worksheet: Alg2H

Geometric: Infinite sum

(book chapter 14, Page 630)

Warm up:

Geometric sequence:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$.

Geometric series:

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = ?$

$\frac{1}{2} \to \frac{1}{2}$

$\frac{1}{2} + \frac{1}{4} \to \frac{3}{4}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \to \frac{7}{8}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$ ...

Zeno's paradoxes (You can find on Wikipedia: Zeno's Paradoxes.

Achilles and the tortoise

Arrow Paradox.

Sum of infinite sequence:

Geometric sequence: r, a_1

$$S_\infty = \frac{1}{1-r} \cdot a_1 \qquad \text{only if } |r|<1$$

Convergence:

$$S_n = a_1 \cdot \frac{1-r^n}{1-r} \qquad S_{n\to\infty} = a_1 \cdot \frac{1-0}{1-r}$$

Examples:

$$4 - 1 + \frac{1}{4} - \frac{1}{16} + \frac{1}{64} \mp \dots$$

$r = -\frac{1}{4}$ $\quad |r|<1$ ✓ $\Rightarrow S_\infty = \frac{1}{1-(-\frac{1}{4})} \cdot 4 = \frac{4}{\frac{5}{4}} = \boxed{\frac{16}{5}}$

$$\sum_{i=0}^{\infty} \left(\frac{+1}{3}\right)^i = 1 + \frac{1}{3} + \frac{1}{9} + \dots$$

$r = \frac{1}{3} < 1$ ✓

$$= 1 \cdot \frac{1}{1-\frac{1}{3}} = \boxed{\frac{3}{2}}$$

Example:

$$2 + \frac{1}{2} + \frac{1}{2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{2\cdot 3\cdot 4\cdot 5} + \frac{1}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \dots$$

Is this geometric? No

Does it converge? Yes → what value? 2. TIP.. In Calc we will learn more convergence.