Name:_____

Class/Home worksheet: Alg2H
Quadratic equation: Discriminant and solutions.

An equation of the type

$$ax^2 + bx + c = 0$$

where a,b, and c are constants, and $a \neq 0$, is called **standard form of the quadratic equation.**

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac$$

Discriminant:

$\Delta < 0$	$\Delta = 0$	$\Delta > 0$
Two complex-conjugate Solutions.	One real solution	Two real solutions
$x^2 + 5x + 8 = 0$	$x^2 - 6x + 9 = 0$	$x^2 - 5x + 6 = 0.$
$\Delta = 5^2 - 4 \cdot 1 \cdot 8 =$ $25 - 32 = -7$	$\Delta = (-6)^2 - 4 \cdot 1 \cdot 4 = 0$	$\Delta = (-5)^2 - 4 \cdot 1 \cdot 6 =$ $25 - 24 = 1$
Two Complex Solutions	$x_{1,2} = \frac{6 \pm \sqrt{0}}{2} = 3$	$x_{1,2} = \frac{5 \pm \sqrt{1}}{2} = \nearrow \genfrac{}{}{0pt}{}{3}{2}$
	$(x-3)(x-3)$ $= (x-3)^2$	$(x-3)(x-2)$

Properties of the solution	
$$ax^2 + bx + c = 0$$ $$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$	
Sum of solutions: $$x_1 + x_2 = \frac{-b}{a}$$	If $a = 1$ $$b = -(x_1 + x_2)$$
Product of solutions: $$x_1 \cdot x_2 = \frac{c}{a}$$	$$c = x_1 \cdot x_2$$

Explanation: $a(x - x_1)(x - x_2) = a(x^2 - x(x_1 + x_2) + x_1 \cdot x_2)$

$$= ax^2 - x \cdot a \cdot (x_1 + x_2) + a x_1 x_2$$

Example:
Find a quadratic equation for which the sum
of solutions is 3, and product is 2.

Answer: $x^2 - 3x + 2 = 0$.

Check: $x_{1,2} = \frac{+3 \pm \sqrt{9 - 4 \cdot 2}}{2} = \frac{3 \pm 1}{2} = \begin{matrix} 2 \\ 1 \end{matrix}$ ✓