Class worksheet: Alg2H
Geometric sequence and series
(book chapter 14)

Warm up:	$\frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{2}{8}, \frac{2}{4},$ ...

$\overset{+4}{1}, \overset{+3}{5}, \overset{+2}{8}, \overset{+1}{10}, \overset{+0}{11}, \overset{-1}{11}, \overset{-2}{10}, -8$ ...

$2, 4, 8, 16,$ ...

Definition: Geometric sequence

$3, 9, 27, 81$

$3, 1, \frac{1}{3}, \frac{1}{9},$ ...

common factor: $\underline{\underline{r}}$

$3, -1, \frac{1}{3}, -\frac{1}{9},$ ...

Recursive formula

$$\boxed{a_n = a_{n-1} \cdot r}$$

Explicit formula

$a_1 = 3$

$a_2 = a_1 \cdot \frac{1}{3} = a_1 \cdot r$

$a_3 = a_2 \cdot r = a_1 \cdot r^2$

$a_4 = \qquad = a_1 \cdot r^3$

$\vdots$

$$\boxed{a_n = \qquad = a_1 \cdot r^{n-1}}$$

Find the n'th element:

$$a_n = a_1 \cdot r^{(n-1)}$$

$3, -15, 75, \ldots$

$a_6 = ?$

$r = -5, \quad a_6 = a_1 \cdot r^{(6-1)} = 3 \cdot (-5)^{6-1} = 3(-3125) = \boxed{-9375}$

Given two elements, find the sequence:

$a_2 = 8, \quad a_7 = 256$

$a_7 = a_2 \cdot r^5 \quad \rightarrow \quad 32 = r^5 \quad \rightarrow \quad \boxed{r = 2}$

$\dfrac{a_7}{a_2} = \dfrac{256}{8} = 32$

$\boxed{a_1 = 1} \Big) \cdot 2$
$a_2 = 8 \Big) \cdot 2$
$a_3 = 16 \Big) \cdot 2$
$a_4 = 32$

Sum

$$S_n = a_1 + a_1 \cdot r + a_1 r^2 + \ldots + a_1 r^{n-1}$$

multiply by r:

$$r \cdot S_n = r a_1 + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^n$$

subtract:

$$S_n - r S_n = a_1 - a_1 r^n$$

$$S_n(1-r) = a_1(1-r^n) \quad \rightarrow \quad \boxed{S_n = \dfrac{a_1(1-r^n)}{1-r}}$$

$$\sum_{n=1}^{5} \left(\frac{1}{2}\right)^{n+1}$$

① Write down first 3 elements/terms, and last one.

$a_1 = \left(\frac{1}{2}\right)^{1+1} = \frac{1}{4}$

$a_2 = \left(\frac{1}{2}\right)^{2+1} = \frac{1}{8}$

$a_3 = \frac{1}{2}^{3+1} = \frac{1}{16}$

$\vdots$

$a_5 = \left(\frac{1}{2}\right)^{5+1} = \frac{1}{64}$

② Is it geometric? → fixed ratio, $\boxed{r = \frac{1}{2}}$

YES

③ Calculate sum:

$S_5 = \frac{1}{4} \cdot \frac{\left(1 - \frac{1}{2}^5\right)}{1 - \frac{1}{2}} = \frac{1}{4} \cdot \frac{\frac{31}{32}}{\frac{1}{2}} = \boxed{\frac{31}{64}}$