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A Formal Set Of Algorithms For Project Scheduling With Critical Path Scheduling/Material Requirements Planning

Nicholas J. Aquilano*
Dwight E. Smith**

ABSTRACT

A hybrid model combining the critical path method (CPM) with material requirements planning (MRP) has been suggested (Aquilano) as a more robust method for scheduling projects and resources. The primary advantage of this technique is that resource acquisition lead times as well as inventory records are integrated into the process of computing the project schedule. This paper presents a set of formal CPM/MRP algorithms that may be used to compute the early and late start schedules as well as the critical sequence. A number of modifications have been incorporated into the CPM/MRP technique to improve the viability of CPM/MRP as a tool for application to actual project scheduling problems. A simple example project is used to demonstrate the CPM/MRP model.

The CPM/MRP technique is designed to overcome a basic shortcoming of previously suggested project scheduling methodologies. CPM was initially designed to schedule projects subject to technological constraints only. Later, additional techniques were introduced to consider constraints upon various aspects of resource availability (Davis). None of the suggested techniques attempted to integrate resource acquisition lead time with the generation of requirements for resources. Obviously such a technique would require the integration of inventory records into the scheduling technique.

The combination of CPM and MRP provides a possible vehicle for overcoming this drawback in CPM. Both CPM and MRP are linear models that generate schedules based upon precedence relationships. An integrated approach is useful since activities could be scheduled subject to information about the inventory position. An activity may be scheduled as soon as all resources are on hand. It is only delayed by those resources which must be acquired and activities which preceed it in the project network.

CPM/MRP also shows promise as an aid to constrained resource scheduling since computations regarding resource availability are an integrated part of the technique. The effect of resource allocation decisions is immediately evident in the MRP-type time phased records.

Results of the tests run on short projects of up to 300 activities and resources have shown that the program does work satisfactorily. Execution time for a 300 item network tested was approximately ten seconds on a CYBER 175.

INTRODUCTION

The critical path method (CPM) has been used for over twenty years as a technique for scheduling projects in a variety of industries. Various extensions to the basic model have improved the realism of the technique by considering the effects of resource constraints. None of these techniques have integrated resource acquisition (lead) times and inventory records into the scheduling technique. A model that combines the critical path method with material requirements planning (MRP) has been suggested [1] as a possible vehicle for overcoming this shortcoming of previous models. This paper will expand this idea and suggest a generalized set of CPM/MRP scheduling algorithms that may be used to build an integrated project scheduling technique that includes all of the primary aspects of the problem.

The first section of this paper will contain a definition and discussion of the project scheduling problem and a short review of previous work of interest in CPM and MRP. In the second section, the algorithms will be outlined with the aid of a short example project. The third section will discuss computational experience with a CPM/MRP computer program, followed by a fourth

*University of Arizona.
**Miami University.

section containing conclusions and some possible areas for future research.

THE PROJECT SCHEDULING PROBLEM AND PREVIOUS WORK IN THE AREA

CPM was initially designed to schedule projects subject to technological constraints only. The method was subject to criticism because there was an implicit assumption that resources required by project activities were available in unlimited quantities. Other criticisms, which are not of particular interest here, concerned the assumptions of deterministic activity times, fixed technological relationships, and certain performance of all activities. Although there are interrelationships between these factors and resource scheduling, this paper will treat only resource scheduling.

An excellent survey on constrained resource scheduling by Davis [2] notes that there are three basic appraoches to the resource scheduling problem:

1. The time-cost trade-off problem
2. The resource leveling problem
3. The constrained resource scheduling problem.

In each case, one of the objectives of the solution technique is to schedule the quantities of resources used at each point in time. A technique has not been suggested in any of the three areas that integrates resource acquisition lead time with the generation of requirements for resources. Obviously such an approach would require the integration of inventory records into the scheduling technique.

One promising approach to the problem that did consider resource lead times was a technique outlined by Smith [4]. In a rather sketchy account, he outlines a method for shop scheduling named *POWER* (PERT Oriented Workshop Scheduling and Evaluation Routine), which deals with lead time requirements for materials. However, while Smith lists a bill of materials and inventory records as components of the scheduling method, he does not explain how they are integrated into the scheduling of orders in the shop.

Inventory records can be included in project scheduling by combining the concepts of material requirements planning [3] with the basic critical path method. Both CPM and MRP are linear models. Both models generate schedules based upon precedence relationships; MRP schedules the requirements for physical, storable goods, based on current and planned inventory levels, while CPM schedules nonstorable items, such as activities, facilities, equipment, and labor. Although attempts have been made to modify CPM or MRP models to contain features of each, a totally integrated attempt has not taken place.

The complete problem of project scheduling is one of scheduling projects subject to technological constraints and resource availability. This requires that inventory records be maintained, since a resource doesn't need to be ordered if it is on hand. A set of inventory records aids in the solution of the resource allocation problem since an activity may be scheduled earlier than originally proposed if all resources are on hand, rather than on order and subject to lead time restrictions. These advantages and others will become evident as the CPM/MRP technique is described.

The CPM/MRP Model

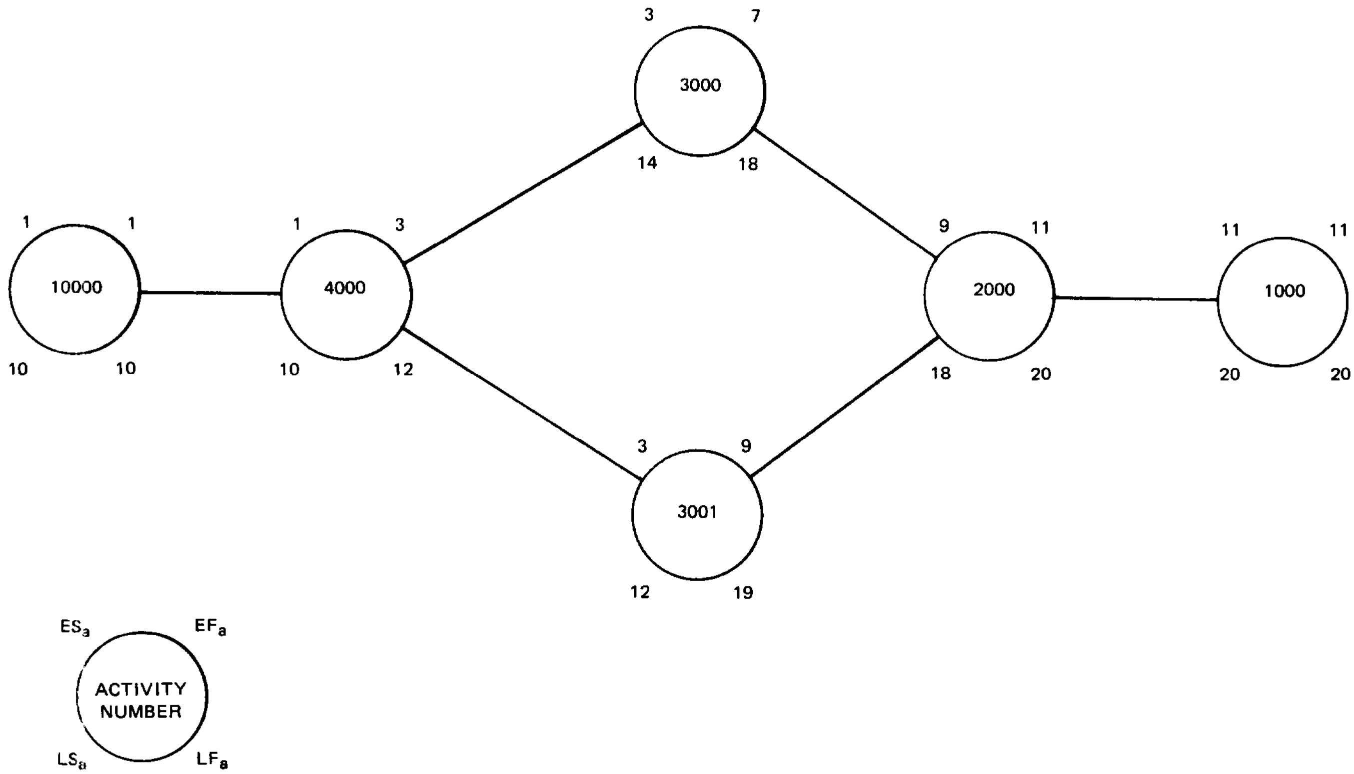
The CPM/MRP model will be explained in this section with the aid of an example project, a simple patio bench. The CPM/MRP solution will be compared to the simple CPM schedule. The example project will be scheduled with the assumption that unlimited amounts of each resource are available.

The project network used for the CPM solution (Figure 1) lists all of the project activities. The early and late times listed on the diagram were derived using the common CPM forward and backward pass algorithms. Note that time measurement begins with the first day, which is consistent with the shop calendar format used in MRP. Activities are started and finished at the beginning of the indicated periods.

If the project is not due until day twenty, then there is no critical path, since the project may be completed in ten days. If the project were indeed due on day eleven, then activities 1000, 2000, 3001, 4000, and 10000 would be critical, while activity 3001 would have two days of free slack.

The project can also be listed in the form of an MRP-type indented bill of materials (Figure 2). This bill includes activities, labor requirements, and facility and equipment needs as well as the materials normally listed in the MRP bill. Again, while bills of labor have been used in MRP, they were not in the integrated form used here. Activities and resources are numbered in descending order from the top of the project structure tree such that a successor (child) always has a higher number than its predecessor (parent). For instance, activity 3000 in Figure 1 is a parent, while resource 9000 is a child. The lead time required to order each resource and the performance time for

FIGURE 1
Activity-On-Node Diagram And Schedule Computed With Critical Path Method



each activity are listed along with inventory records in the inventory record file (Figure 3).

It is possible to list this project in the form of the product structure tree used in MRP (Figure 4). Note that due to the occurrences of multiple paths through most project networks, the project structure tree may list some activities more then once. As a consequence, resource inputs to these activities would be duplicated if left unchanged. To prevent this duplication, an activity is only permitted to occur once; at the time when it is first needed in the schedule. Later requirements are ignored.

THE SCHEDULING ALGORITHMS

Both early and late start scheduling algorithms will be described to facilitate the description of these algorithms. Let:

a = an activity
r = a resource input of labor, material, or facilities and equipment
D = project delivery date.

For activity a:
ES_a = Early start time for a
LS_a = Late start time for a
EF_a = Early finish time for a
LF_a = Late finish time for a
PT_a = Performance time for a

For resource r:
EO_r = Earliest time at which the resource should be ordered
LO_r = Latest time at which the resource could be ordered
EA_r = Earliest time at which the resource should be acquired
LA_r = Latest time at which the resource could be acquired
LT_r = Lead time on an order for this resource

This notation is similar to the accepted CPM notation, [6] the nature of these times still requires some explanation. All activity start times, as noted previously, are subject to both the completion of previous activities and the acquisition

FIGURE 2
Bill Of Materials For The Example Project

Assembly (Parent)	Part (Child)	Quantity	Description
1000			Complete project
	2000	1	Assemble bench
2000			Assemble bench
	3000	1	Assemble back
	3001	1	Assemble seat
	9001	1	¼ lb., 10¢ nails
	9002	2	Carpenter hours
3000			Assemble back
	4000	1	Collect tools
	9000	2	2 × 4's
	9001	1	¼ lb., 10¢ nails
	9002	4	Carpenter hours
	9003	2	Handsaw hours
3001			Assemble seat
	4000	1	Collect tools
	9000	4	2 × 4's
	9001	1	¼ lb., 10¢ nails
	9002	12	Carpenter hours
	9003	4	Handsaw hours
	9004	2	Wrought iron legs
4000			Collect tools
	10000	1	Start project
9000			2 × 4's
	10000	1	Start project
9001			¼ lb., 10¢ nails
	10000	1	Start project
9002			Carpenter hours
	10000	1	Start project
9003			Handsaw hours
	10000	1	Start project
9004			Wrought iron legs
	10000	1	Start project
10000			Start project

FIGURE 3
Lead Time, Quantity On Hand, And Planned Receipts For The Activities And Resources In The Example Project

Type	Number	On Hand	Performance Time	Lead Time	On Order
Activity	1000	0	0		0
Activity	2000	0	2		0
Activity	3000	0	4		0
Activity	3001	0	6		0
Activity	4000	0	2		0
Material	9000	0		6	0
Material	9001	0		3	0
Labor	9002	0		2	0
Facility & Equipment	9003	0		5	0
Material	9004	0		8	0
Activity	10000	0	0		0

of the necessary resources. Early and late resource order times should be scheduled such that the resource acquisition times coincide with the start times of activities. As in CPM, times in the late schedule are the latest times at which activities and resource acquisitions should take place if the project is to be completed in the shortest time.

THE LATE START SCHEDULE

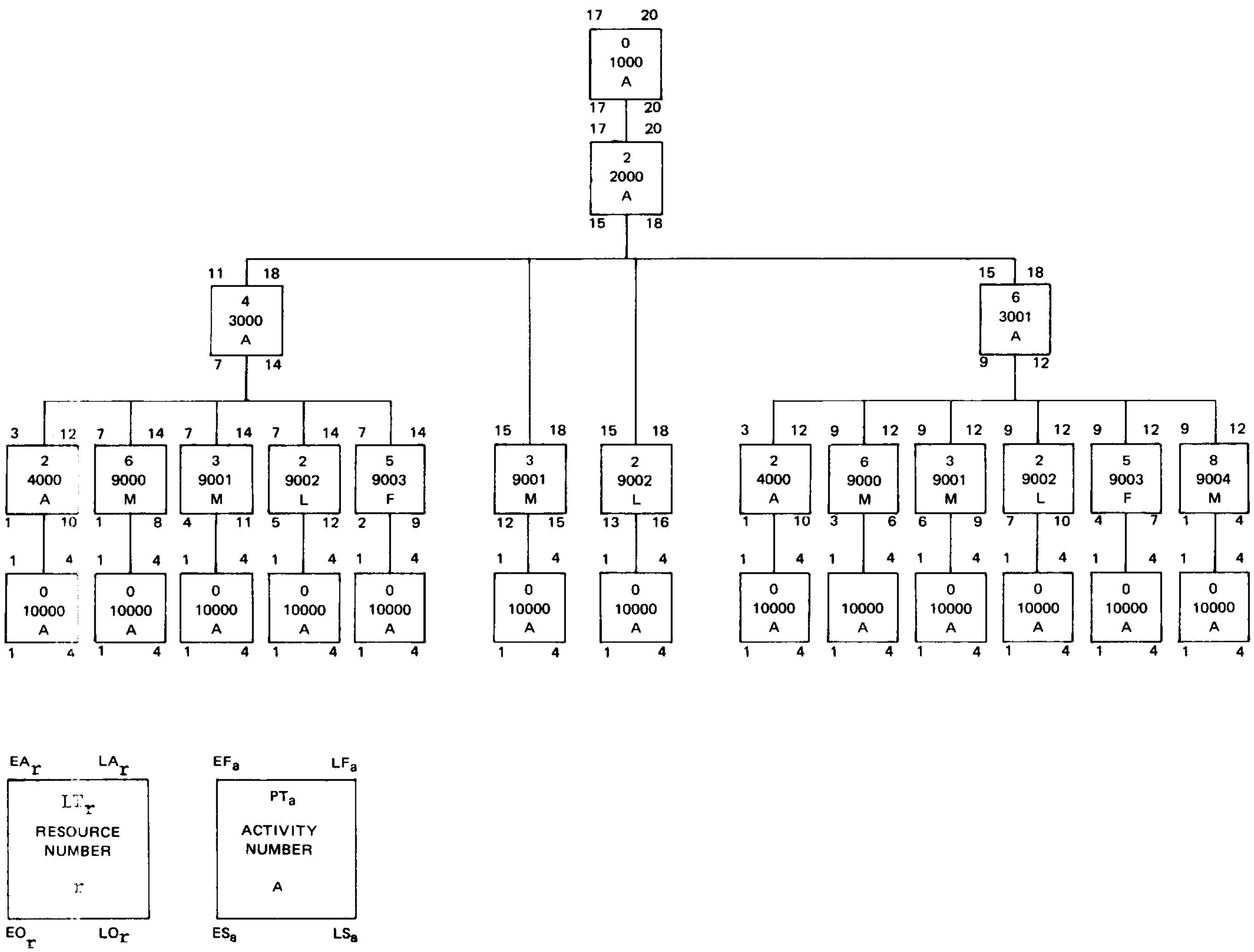
As in MRP, the CPM/MRP technique branches out through the project structure tree (or more properly "explodes" the project bill of materials) so as to find the late start schedule. All material requirement plans are late start schedules, since they schedule materials to arrive when they are needed in the due date completion schedule, and not before. In the CPM/MRP program discussed in this paper, the project due date is established and the project is exploded to find the late start schedule.

The late start scheduling algorithm will be listed first, followed by the calculations necessary to compute resource requirements:

1. Schedule the last activity in the project. Set $LF_a = D$ and $LS_a = LF_a - PT_a$.
2. In ascending order of activity or resource number, compute the late times for each activity or resource within the bill. Schedule each item depending upon whether it is:
   a) A unique occurrence of an activity; that is, it only occurs once in the tree.
   $LF_{a(\text{child})} = LS_{a(\text{parent})}$; or, $LF_{a(\text{child})} = LO_{r(\text{parent})}$
   $LS_{a(\text{child})} = LF_{a(\text{child})} - PT_{a(\text{child})}$
   b) An activity with multiple occurrences in the project structure tree.
   $LF_{a(\text{child})} = \underset{a,r}{\text{MIN}}\ [LS_{a(\text{parents})}, LO_{r(\text{parents})}]$
   $LS_{a(\text{child})} = LF_{a(\text{child})} - PT_{a(\text{child})}$
   c) Any resource. Since explosion proceeds downwards, a resource required by an activity with multiple occurrences will be scheduled only after the activity is scheduled.
   $LA_r = LS_{a(\text{parent})}$
   $LO_r = LA_r - LT_r$
3. Continue scheduling by returning to the beginning of Step 2 until all activities are scheduled.

The late schedule listed on the example project structure tree was found using this algorithm (Figure 4).

FIGURE 4
Project Structure Tree With Early And Late Schedules Using CPM/MRP Technique



1. Set $LF_{1000} = D = 20$ and $LS_{1000} = LF_{1000} - PT_{1000} = 20 - 0 = 20$

2. Step 2 at each level of the tree:

   Level 1

   $LF_{2000} = LS_{1000} = 20$ and $LS_{2000} = LF_{2000} - PT_{2000} = 20 - 2 = 18$

   Level 2

   $LF_{3000} = LS_{2000} = 18$ and $LS_{3000} = LF_{3000} - PT_{3000} = 18 - 4 = 14$

   $LF_{3001} = LS_{2000}$ 18 and $LS_{3001} = LF_{3001} - PT_{3001} = 18 - 6 = 12$

   Level 3

   a) Activity 4000 occurs twice in the tree, so its times are based on the minimum LS of activities 3000 and 3001.

   $LF_{4000} = \text{MIN}\ (LS_{3000}, LS_{3001}) = 12,$
   $LS_{4000} = LF_{4000} - LT_{4000} = 12 - 2 = 10$

   b) For purposes of brevity, resource times for only activity 3001 will be listed:

   $LA_{9000} = LA_{9001} = LA_{9002} = LA_{9003} = LA_{9004} = LS_{3001} = 12$
   $LO_{9000} = LA_{9000} - LT_{9000} = 12 - 6 = 6$
   $LO_{9001} = LA_{9001} - LT_{9001} = 12 - 3 = 9$
   $LO_{9002} = LA_{9002} - LT_{9002} = 12 - 2 = 10$
   $LO_{9003} = LA_{9003} - LT_{9003} = 12 - 5 = 7$
   $LO_{9004} = LA_{9004} - LT_{9004} = 12 - 8 = 4$

   Level 4

   $\text{MIN}(LS_{a(\text{parents})}, LO_{r(\text{parents})}) = 4$. Therefore set $LF_{10000} = 4$ and $LS_{10000} = LF_{10000} - PT_{10000} = 4 - 0 = 4$

FIGURE 5
The CPM/MRP Late Start Schedule

LATE START SCHEDULE

ACTIVITY COMPLETE PROJECT

1000 LT = 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
ORDER RELEASE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

ACTIVITY ASSEMBLE BENCH

2000 LT = 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
ORDER RELEASE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

ACTIVITY ASSEMBLE BACK

3000 LT = 4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1
ORDER RELEASE	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

ACTIVITY ASSEMBLE SEAT

3001 LT = 6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1
ORDER RELEASE	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

ACTIVITY COLLECT TOOLS

4000 LT = 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−2	−2	−2	−2	−2	−2	−2
ORDER RELEASE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

MATERIAL START PROJECT

9000 LT = 6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	4	0	2	0	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	−6	−6	−6	−6	−6	−6	−6
ORDER RELEASE	0	0	0	0	0	4	0	2	0	0	0	0	0	0	0	0	0	0	0	0

MATERIAL 10 P. NAILS

9001 LT = 3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−2	−2	−2	−2	−3	−3	−3
ORDER RELEASE	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0

LABOR CARPENTER HOURS

9002 LT = 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	2.0	3.0	3.0	3.0	3.0	1.0	1.0	0.0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	−12	−12	−16	−16	−16	−16	−18	−18	−18
ORDER RELEASE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	2.0	3.0	3.0	3.0	3.0	1.0	1.0	0.0	0.0	0.0

FAC & EQ HANDSAW HOURS

9003 LT = 5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	1.2	1.2	1.2	1.2	0.0	0.0	0.0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	−6	−6	−6	−6	−6	−6	−6
ORDER RELEASE	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	1.2	1.2	1.2	1.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

MATERIAL WROUGHT IRON LEGS

9004 LT = 8	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	−2	−2	−2	−2	−2	−2
ORDER RELEASE	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

ACTIVITY START PROJECT

10000 LT = 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	2	0	4	4	2	3	13	1	4	0	0	1	2	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	−2	−2	−6	−10	−12	−15	−28	−29	−33	−33	−33	−34	−36	−36	−36	−36	−36
ORDER RELEASE	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

All activities and resources have been scheduled. The computation of net requirements for resources is essentially performed the same way as in MRP. If lot-by-lot order quantities are used, orders for storable resources in each period are found using the slightly modified version of the common MRP equations [3, pp. 76–77]. If it is assumed that actual receipts equals planned receipts, then:

$$ORD_{im} = TREQ_{jm} - TREC_{jm} - ONH_{om}$$
$$TREC_{jm} = TREC_{j-1,m} + REC_{jm}$$
$$ONH_{jm} = ONH_{j-1,m} + TREC_{jm} - TREQ_{jm}$$

where

m = Storable resource (material) number m
ORD_{im} = Quantity of m to be ordered in period i
$TREQ_{jm}$ = Total requirements for m as of period j
$TREC_{jm}$ = Total receipts of m as of period j
REC_{jm} = Planned receipts of m in period j
ONH_{jm} = Total of m on hand in period j
OHN_{om} = Initial on hand
$i = j - LT_m$

In its original form, CPM/MRP treated all resources as storable, noncapital resources. Any amount of a resource remaining at the end of a period could be stored for use in a future period. In practice, most projects require both storable and non-storable resources such as labor that may not be stored for future use. This means that while one order may be generated for the entire quantity of a resource used by an activity, the resource will probably arrive in increments over the course of the activity. Therefore, the equations used to generate requirements for non-storable resources are slightly different than those listed for storable resources:

$$UNR_{jn} = REC_{jn} - REQ_{jn}$$
$$ORD_{in} = REQ_{in} - REC_{jn}$$

UNR_{jn} = Amount of non-storable resource n unused at the end of period j
REQ_{jn} = Requirements for resource n in period j
n = Non-storable resource, either labor input, or facility and equipment

Thus, the quantity of a resource on hand at the end of a day is not saved for use in the following day. Both sets of formulas may be demonstrated by referrring to storable resource 9000, and non-storable resource 9002. These equations can be used along with the previously discussed late start schedule to find an MRP-style late start schedule. (Figure 5) The negative numbers in the on hand row follow the common procedure used by many firms. When a planned order is actually released, this amount is added to the on hand [3, Chapter 4]. Activity 10000 is the starting node of the entire project. Therefore, the first elements (those without predecessors) show requirements for 10000 which means "start the project."

While the late start schedule is computed, a set of records are constructed that list where each child is used in a parent assembly. A set of peg records are generated simultaneously with the late start schedule that are similar to the peg records in MRP. This set of records "pegs" the requirement for a child to each of its parents, and lists the time of the requirement for the child placed by each of its parents in the late start schedule, as well as the earliest requirement (Figure 6). Note that this information is useful for resource allocation purposes, because it lists not only the activities that a delayed activity or resource will effect immediately, but those that it will effect as it is delayed to greater degrees. The time information stored in the peg records is used to find the early start schedule.

THE EARLY START SCHEDULE

The early start times in CPM/MRP are equivalent to those in CPM in that activities are scheduled to begin as early as possible, subject to technological and precedence constraints. Resource orders are timed so that resources arrive as they are required. They arrive as late as possible so that holding costs are not incurred. The algorithm used is:

1. Set start and finish for first activity in project.
   Set $ES_a = 1$ and $EF_a = ES_a + PT_a$
2. Consider each successive activity in descending order of part number. Using the peg records:
   a) Find all children (resources and activities) of the activity currently being scheduled.
   b) Set $ES_{a\,(\text{parent})} = \underset{a,r}{\text{MAX}}\,[EF_{a(\text{children})}, EA_{r(\text{children})}]$
   (Set EA_r tentatively equal to $EF_{a(\text{parent})} + LT_r$ where a is the first activity in the project.)
   c) Set $EF_{a(\text{parent})} = ES_{a(\text{parent})} + PT_{a(\text{parent})}$.
3. Schedule each succeeding lower numbered activity until all have been scheduled.
4. For all resources,
   $EA_{r(\text{child})} = ES_{a(\text{parent})}$, and $EO_r = EA_r - LT_r$
5. Continue until all resources have been scheduled.

The early schedule times for the example project would be found in the following fashion:

FIGURE 6
The Peg Record File

Child	Parent	Earliest REQ	This REQ	QTY
2000	1000	18	18	1
3000	2000	14	14	1
3001	2000	12	12	1
9001	2000	9	15	1
9002	2000	10	16	2
4000	3000	10	12	1
9000	3000	6	8	2
9001	3000	9	11	1
9002	3000	10	12	4
9003	3000	7	9	2
4000	3001	10	10	1
9000	3001	6	6	4
9001	3001	9	9	1
9002	3001	10	10	12
9003	3001	7	7	4
9004	3001	4	4	2
10000	4000	4	10	1
10000	9000	4	6	1
10000	9001	4	9	1
10000	9002	4	10	1
10000	9003	4	7	1
10000	9004	4	4	1

1. The initial activity in the project is activity 10000.
   $ES_{10000} = 1$, and $EF_{10000} = ES_{10000} + LT_{10000} = 1 + 0 = 1$

2 & 3. Consider each succeeding lower numbered activity:

Activity 4000
Child: Activity 10000 $EF_{10000} = 1$
$ES_{4000} = EF_{10000} = 1$
$EF_{4000} = ES_{4000} + PT_{4000} = 1 + 2 = 3$

Activity 3000
Children: Activity 4000 $EF_{4000} = 3$
Resource 9000 $EA_{9000} = 1 + 6 = 7$ (tentative)
Resource 9001 $EA_{9001} = 1 + 3 = 4$ (tentative)
Resource 9002 $EA_{9002} = 1 + 2 = 3$ (tentative)
Resource 9003 $EA_{9003} = 1 + 5 = 6$ (tentative)

$\underset{a,r}{\text{MAX}}\ [EF_a, EA_r]$

Then, $ES_{3000} = 7$, and $EF_{3000} = ES_{3000} + PT_{3000} = 7 + 4 = 11$

Activity 3001
Children: Activity 4000 $EF_{4000} = 3$
Resource 9000 $EA_{9000} = 1 + 6 = 7$ (tentative)
Resource 9001 $EA_{9001} = 1 + 3 = 4$ (tentative)
Resource 9002 $EA_{9002} = 1 + 2 + 3$ (tentative)
Resource 9003 $EA_{9003} = 1 + 5 = 6$ (tentative)
Resource 9004 $EA_{9004} = 1 + 8 = 9$ (tentative)

$\underset{a,r}{\text{MAX}}\ [EF_a, EA_r]$

Then, $ES_{3001} = 9$, and $EF_{3001} = ES_{3001} + PT_{3001} = 9 + 6 = 15$

Activity 2000
Children: Activity 3000 $EF_{3000} = 11$
Activity 3001 $EF_{3001} = 15$
Resource 9001 $EA_{9001} = 4$ (tentative)
Resource 9002 $EA_{9002} = 3$ (tentative)

$\underset{a,r}{\text{MAX}}\ [EF_a, EA_r]$

Then, $ES_{2000} = 15$ and $EF_{2000} = ES_{2000} + PT_{2000} = 15 + 2 = 17$

Activity 1000
Children: Activity 2000 $EF_{2000} = 17$
$ES_{1000} = 17$, $EF_{1000} = ES_{1000} + PT_{1000} = 17 + 0 = 17$

4 & 5. Set the resource order times:

Parent:
3000, Child: 9000: $EA_m = ES_{3000} = 7$, $EO_m = 7 - LT_{9000} = 7 - 6 = 1$
9001: $EA_m = ES_{3000} = 7$, $EO_m = 7 - LT_{9001} = 7 - 3 = 4$
9002: $EA_l = ES_{3000} = 7$, $EO_l = 7 - LT_{9002} = 7 - 2 = 5$
9003: $EA_f = ES_{3000} = 7$, $EO_f = 7 - LT_{9003} = 7 - 5 = 2$

Parent:
3001, Child: 9000: $EA_m = ES_{3001} = 9$, $EO_m = 9 - LT_{9000} = 9 - 6 = 3$
9001: $EA_m = ES_{3001} = 9$, $EO_m = 9 - LT_{9001} = 9 - 3 = 6$
9002: $EA_l = ES_{3001} = 9$, $EO_l = 9 - LT_{9002} = 9 - 2 = 7$
9003: $EA_f = ES_{3001} = 9$, $EO_f = 9 - LT_{9003} = 9 - 5 = 4$
9004: $EA_m = ES_{3001} = 9$, $EO_m = 9 - LT_{9004} = 9 - 8 = 1$

Parent:
2000, Child: 9001: $EA_m = ES_{2000} = 15$, $EO_m = 15 - LT_{9001} = 15 - 3 = 12$
9002: $EA_l = ES_{2000} = 15$, $EO_l = 15 - LT_{9002} = 15 - 2 = 13$

FIGURE 7
The CPM/MRP Early Start Schedule

EARLY START SCHEDULE

ACTIVITY COMPLETE PROJECT

1000 LT = 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1
ORDER RELEASE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

ACTIVITY ASSEMBLE BENCH

2000 LT = 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	−2
ORDER RELEASE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

ACTIVITY ASSEMBLE BACK

3000 LT = 4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−2	−2	−2	−2	−2	−2
ORDER RELEASE	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

ACTIVITY ASSEMBLE SEAT

3001 LT = 6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	−2	−2	−2
ORDER RELEASE	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0

ACTIVITY COLLECT TOOLS

4000 LT = 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	−1	−1	−1	−1	−2	−2	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
ORDER RELEASE	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

MATERIAL START PROJECT

9000 LT = 6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	2	0	4	0	0	0	0	0	0	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	−2	−2	−6	−6	−6	−6	−6	−6	−6	−6	−6	−6	−6	−6
ORDER RELEASE	2	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

MATERIAL 10 P. NAILS

9001 LT = 3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	−1	−1	−2	−2	−2	−2	−2	−2	−3	−3	−3	−3	−3	−3
ORDER RELEASE	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0

LABOR CARPENTER HOURS

9002 LT = 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	3.0	3.0	2.0	2.0	2.0	2.0	1.0	1.0	0.0	0.0	0.0	0.0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	−4	−4	−16	−16	−16	−16	−16	−16	−18	−18	−18	−18	−18	−18
ORDER RELEASE	0.0	0.0	0.0	0.0	1.0	1.0	3.0	3.0	2.0	2.0	2.0	2.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0

FAC & EQ HANDSAW HOURS

9003 LT = 5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.5	1.2	1.2	0.7	0.7	0.7	0.7	0.0	0.0	0.0	0.0	0.0	0.0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	−2	−2	−6	−6	−6	−6	−6	−6	−6	−6	−6	−6	−6	−6
ORDER RELEASE	0.0	0.5	0.5	1.2	1.2	0.7	0.7	0.7	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

MATERIAL WROUGHT IRON LEGS

9004 LT = 8	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	0	0	0	0	0	0	0	0	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2
ORDER RELEASE	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

ACTIVITY START PROJECT

10000 LT = 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
RQMTS	6	2	4	5	4	1	12	0	0	0	0	1	2	0	0	0	0	0	0	0
PL'ED RECEIPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENDING INV 0	−6	−8	−12	−17	−21	−22	−34	−34	−34	−34	−34	−35	−37	−37	−37	−37	−37	−37	−37	−37
ORDER RELEASE	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The order quantities are again computed using the previously explained formulas and the results are stored in the MRP time phased schedule format (Figure 7).

A final comparison between this schedule and the original CPM schedule will show that while CPM estimated project duration is 10 days, the project duration would be 17 days if all resources must be ordered. Therefore, if the project is commenced after day 3, in day 9 as the CPM late start schedule suggests, the project would not be completed on time. This underestimation of project length may contribute to operational problems where CPM is used. Note also that quantities of activity slack are overesitmated when CPM is used (Figure 8). If only part of the resources are on hand, the duration of the project should be somewhere between 11 and 17 days.

THE CRITICAL SEQUENCE

The critical sequence in CPM/MRP will be defined initially as the connected path through the project structure tree that connects those activities and resource orders with zero slack. This sequence can be found after the late start schedule and peg records have been computed, since those items that are members of the sequence will generate the earliest orders for each succeeding member of the sequence.

Note that it is assumed that there are initially no constraints on quantities of resources available. In cases where resources are constrained, the critical sequence in CPM/MRP would be an expanded version of the previous definition of critical sequence in constrained resource scheduling [5], since it would now include resource acquisition lead time.

To find the critical sequence:

1. Find the first activity in the project, and make this the first activity in the critical sequence.
2. Beginning with the first activity of the project, find the remaining members of the sequence:
   a) Consider the last activity or resource added to the sequence.
   b) Find all instances of this activity or resource where it occurs as a child in the peg records (Figure 6).
   c) Find the parent activity or resource that generated the earliest requirement for the child in b). Store this activity or resource as the next member of the sequence.
3. Continue until the last activity in the project has been added to the critical sequence.

This technique essentially branches through the network to find the longest connected path. The critical sequence for the example project (Figure 9) can be found using this technique.

FIGURE 8
Slack On Activities

Activity	Early Start	Late Start	Slack
1000	17	20	3
2000	15	18	3
3000	7	14	7
3001	9	12	3
4000	3	10	7
10000	1	4	3

COMPUTATIONAL EXPERIENCE WITH CPM/MRP

The original CPM/MRP program was run with projects ranging from 52 to 200 activities. Zastera [7] hypothesized that execution time seemed to rise in an exponential fashion as the number of activities and resources in a project increased. The algorithms suggested in this paper were incorporated into the program. The rerunning of these same projects then yielded an essentially *linear* relationship between the number of project activities and resources and computer execution time (Figure 10). Although it is obviously the case that execution time is affected by project structure, these results are nevertheless much more favorable to CPM/MRP than those found by Zastera.

CONCLUSIONS AND IMPLICATIONS

The CPM/MRP technique provides an integrated approach to project scheduling as it incorporates project activities and technological relationships with resource requirements and inventory records. A technique is currently being completed that combines the CPM/MRP model with a

FIGURE 9
The Critical Sequence For The Example Project

Type	Number
Activity	10000
Material	9004
Activity	3001
Activity	2000
Activity	1000

FIGURE 10
Execution Time Summary And Linear Regression Equations

Number of Activities & Resources	Execution Time (Sec.'s)	Print Time (Sec.'s)	Total Time (Sec.'s)	Y Exec	Y Total
52	.880	.736	1.616	.653	4.647
102	2.197	8.167	10.364	2.633	8.287
152	4.112	8.788	12.900	4.613	11.927
202	7.640	9.922	17.562	6.593	15.567
300	10.136	10.550	20.686	10.474	22.701

Regression Equations

Execution Time

$Y = -1.4064 + .0396X$ $\qquad r^2 = .9711$

Total Time

$Y = .8611 + .0728X$ $\qquad r^2 = .8958$

set of constrained resource heuristics. The total technique shows a great deal of promise as a method for both project planning and control.

Further work is necessary in the area of development of an efficient CPM/MRP computer algorithm. The current program does not require excessive execution time, but the core requirements are rather large. This can be solved by storing the resource information and requirements in the MRP-style format, while storing each activity in a much more compact format.

Finally, it would be useful to apply the method to empirical data. A multi-project example would be useful in testing the applicabilities of the CPM/MRP technique to real world problems. CPM/MRP provides the user with a technique for integrating resource acquisitions into the project schedule. It would also allow the user to determine the effects of changes in resource lead times on the project schedule, and the effects of changes in activity duration on resource order release dates.

REFERENCES

1. Aquilano, Nicholas J., "Project Scheduling With MRP," University of Arizona, College of Business and Public Administration, Working Paper, 1978.
2. Davis, Edward W., "Project Scheduling Under Resource Constraints-Historical Review and Categorization of Procedures," *AIEE Transactions*, December 1973, pp. 297–313.
3. Orlicky, Joseph, *Material Requirements Planning*, McGraw-Hill, New York, 1975.
4. Smith, O. C., "POWER—PERT Oriented Workshop Scheduling Evaluation Routine," *Proceedings of the Second International Congress on Product Planning by Network Analysis*, North Holland Publishing Company, Amsterdam, 1969.
5. Wiest, Jerome D., "A Heuristic Model for Scheduling Large Projects With Limited Resources," *Management Science*, Vol. 13, No. 6, February 1967, pp. B359–377.
6. Wiest, Jerome D. and Ferdinand K. Levy, *A Management Guide to PERT/CPM*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1977.
7. Zastera, Eugene P., "An Application of MRP to Engineering Management," Master's Thesis, University of Arizona, 1978.