Observations on strategies for Goofspiel

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***Abstract*—Goofspiel is a zero-sum two player card game in which all information is known by both players. Many strategies exist that leverage random, deterministic, and learning approaches to play, however, no strategy dominates all others. It has been suggested that a hybrid strategy combining two or more of these approaches may provide better results than any of these alone. In this note, we review the strengths and weaknesses of each traditional strategy and make a cursory evaluation of a hybrid 'Good' strategy.**

I. Introduction

Goofspiel, also known as the Game of Pure Strategy, in its classical version is a zero-sum two player card game. The basic play of Goofspiel is simple - the four suits are divided from a standard deck of 52 cards. One suit is discarded, each player is given a suit, and the final suit is shuffled and placed face down between the two players. This is the value deck. The cards from the value deck are revealed one at a time as the "value card" ($A = 1, 2, \ldots, K = 13$) at times referred to as the upcard. After the value card is revealed, each player selects a card from their hand to bid and the two players' cards are revealed simultaneously. The winning player receives the number of points shown on the value card and all three cards are discarded. If both players select the same bid card (a tie), no points are awarded and all three cards are discarded. After all thirteen value cards have been exposed, the player with the most points wins the round by the corresponding point margin. In repeated Goofspiel, the process repeats for an agreed upon number of rounds and the player that wins the most points over all the rounds played is the overall winner by the corresponding point margin. For a more complete description of the game, the interested reader is encouraged to review [3].

Basic play of Goofspiel is very straightforward. All information is known - each player can observe which cards remain in their hand and can observe which cards have been played, thereby allowing deduction of which cards remain in their opponent's hand and in the value deck. The cards could, in essence, be left exposed in full view of both players from the start without any impact on the game's strategies. As players attempt to decide which card to play, however, the complexity of the game rises quickly. For the duration of this note we will briefly explore a few general strategies for Goofspiel, highlight the benefits and shortcomings of each, and suggest a direction for future research.

II. Play Schemes and Strategies

A few basic strategies for Goofspiel exist, each with their own benefits and shortcomings. These strategies may be random, deterministic, learning, or some combination of each. Generally speaking, we find that random play is easily defeated by deterministic strategies, deterministic strategies are easily defeated by learning strategies, and learning strategies, while not dominated, are typically disrupted by random play. Across these three approaches it seems no one strategy consistently dominates. For this reason, it is proposed that a hybrid approach using two or more of these techniques seems a reasonable avenue to success.

A. Random Play

Random play in Goofspiel is just what it sounds like - for each hand, an unused card is selected randomly from the cards remaining in the player's hand with no regard for the value card, the cards remaining in play, or the score. If random play is observed over many rounds, we expect to see each bid card wagered against each value card approximately the same number of times. When pitting two opponents both using random play against one another, we find that each player will win approximately the same number of games.

B. Deterministic Strategies

When using deterministic strategies, players use a set of static rules that determine what bid card to wager with respect to the upturned value card. Two common deterministic strategies are the matching strategy (MS) and the upcard+n strategy. Using MS, the player simply plays a card that matches the exposed value card. Independently this strategy makes sense - winning an ace has very little value and thus few resources should be wagered. Likewise, winning a king is of great value so the increase risk of playing in kind is justified.

Previous research concerning stochastic games has shown that deterministic strategies can often be easily defeated [1]. For example, when confronted with MS, one's opponent might counter by using an upcard+1 strategy to win every hand except one. It follows that Player I may anticipate Player II's upcard+1 play and bid upcard+2 in response. This, of course, may be anticipated by Player II who may respond by playing upcard+3, and so on. As the bid values continue to increase, one player may eventually decide it would be foolish to wager a card that is significantly higher than the value card and "throw away" the hand by playing a low card. If their opponent can anticipate this play, they may seize this opportunity to win a high value card with a low wager. This cycle of one-upmanship could continue indefinitely, with the strategy eventually devolving into what is essentially random play. As this scenario illustrates deterministic strategies are fundamentally flawed and not suitable for play against an intelligent opponent.

C. Learning Strategies

Learning strategies observe play over time and attempt to deduce what an opponent's next play will be based on this information. There are a number of different approaches to learning which may take into account the cards remaining in each players hand, the cards remaining in the deck, and simple observation of what card the opponent has historically bid for the upturned value card. Since deterministic strategies use simple sets of rules to determine play and will bid the same values consistently, learning strategies can often identify these patterns and bid in such a way as to win each hand by playing one higher than their opponent.

When paired against random play, however, learning strategies perform less admirably. Since the random play strategy selects a bid card without respect to any information found in the game, over time each bid card will be played against each value card approximately the same number of times. Since there are no patterns to learn, a pure learning strategy will, in essence, learn randomness. Ultimately this results in game play similar to two random play strategies playing one another.

D. Hybrid Strategies

In light of the observation that there is no clear winner among random, deterministic and learning strategies, it has been proposed that a 'Good' strategy for Goofspiel will incorporate elements of each of these approaches [2]. While Chan and Craft do not provide details of their 'Good' strategy, elements of the approach they propose have been integrated into a strategy described in [5]. The strategy described in [5] utilizes a learning algorithm as well as deterministic play by placing value cards into three 'tiers' based on their contribution to the target (winning) score. When the learning algorithm cannot determine appropriate play, bid cards are pseudo randomly selected based on the tier of the value card. This strategy has been found to dominate both random and deterministic strategies and perform well against other learning and hybrid strategies as described in [4].

At the onset of play, the hybrid strategy uses semi-deterministic bounded random play. After a set number of rounds of play, the program has observed a sufficient amount of game play to attempt predicting future play. The learning algorithm looks for a card remaining in the opponent's hand that has been played a high percentage of the time against the upturned value card. Failing this, we look for a set of cards that have cumulatively been played a high percentage of the time, and attempt to play one higher than the highest card in this set. To keep from overbidding, if it is determined we must play a card that is significantly higher than the value card in order to win the hand, the learning algorithm's selection is discarded and we default to bounded random play.

In order to defend against learning algorithms employed by our opponent, once enough points have been obtained to win the round the rest of the hand is played randomly in an effort to corrupt their observations. While this irrational play may be detected and discarded by some learning algorithms, it may be an effective means of corrupting others.

It should be noted that it is impossible to win every hand in a round of Goofspiel. In a "perfect" game one would strive to win the 2-K value cards by bidding one higher than their opponent on each hand and lose the ace value card by bidding A when their opponent bids K. This would result in a final score of 90-1. Since such domination is highly unlikely, a reasonable goal is to win by the minimum number of points required to win the round. At the onset of the game this value is 46 points, however, during play this value may decrease as value cards are lost to ties. Therefore this target point and value tiers are recalculated at the start of each hand. While this classification is partially deterministic in nature, the tiers are generated dynamically at the start of each hand, thereby creating a new set of deterministic rules for each hand and making it more difficult for opponent learning algorithms to predict our play.

III. Discussion

Initial testing of the strategy described in [5] suggests this hybrid approach is highly effective against matching, semi-deterministic and random play strategies. The results of these tests (Table 1) are comparable to the results of the strategy alluded to in [2]. In the semi-deterministic strategy used here, the matching card is played 40% of the time, upcard + 1 is played 40% of the time, and a random card is played 20% of the time.

	vs. Matching	vs. Semi Det.	vs. Random
Wins	>99%	95%	85%

Table 1: Preliminary results

When playing against other 'Good' or hybrid strategies the performance of the strategy may vary considerably. Similar to the loop of deterministic dominating random, learning dominating deterministic, and random performing comparably to learning, we find that for a set of three 'Good' strategies it may be such that A >B >C >A. Identifying a single strategy that dominates all others is an intriguing and complex problem to pursue.

Games such as Goofspiel provide an entertaining venue for exploring complex decision problems related to real world scenarios of bidding on finite resources. Often we must work in concert with with other actors to bid on and determine distribution of finite resources and, just as in Goofspiel, purely deterministic strategies will often make our approach too predictable and open ourselves up to being taken advantage of. By analyzing past decisions of other actors in the situation and acting in a somewhat unpredictable way one may be able to ultimately claim more value.

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