These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

Inverse transform sampling — Given we have a cdf $F_x(\alpha)$ which is invertible, we can generate samples $\bar{x}$ from our distribution $p_x(x)$ using uniform samples $\bar{y} \sim \mathcal{U}(0,1)$,

$$F_x(\alpha) = \mathbb{P}(x \leq \alpha) = \int_{-\infty}^{\alpha} p_x(y) \mathrm{d}y \tag{1}$$

Using the inverse cdf $F_x^{-1}(y)$, a sample $\bar{x} \sim p_x(x)$ can be generated using

$$\bar{x} = F_x^{-1}(\bar{y}) \qquad\qquad \bar{y} \sim \mathcal{U}(0,1) \tag{2}$$

Gibbs sampling — Given a multivariate pdf $p(\mathbf{x})$ and an initial state $\mathbf{x}^1 = (x_1^1, \ldots, x_d^1)$, we obtain multivariate samples $\mathbf{x}^k$ by sampling from a univariate distribution $p(x_i \mid \mathbf{x}_{\setminus i})$, and updating individual variables many times.

$$\mathbf{x}^2 = (x_1^1, \ldots, x_{i-1}^1, x_i^2, x_{i+1}^1, \ldots, x_d^1) \qquad\qquad i \sim \{0, \ldots, d\} \tag{3}$$

$$\ldots$$

$$\mathbf{x}^n = (x_1^{n-1}, \ldots, x_{j-1}^{n-1}, x_j^n, x_{j+1}^{n-1}, \ldots, x_d^{n-1}) \qquad\qquad j \sim \{0, \ldots, d\} \tag{4}$$

In the multidimensional space of $\mathbf{x}$, the iterative Gibbs sampling process will appear as a path in orthogonal axes, generally moving towards the densest region of the univariate distribution $p(x_i \mid \mathbf{x}_{\setminus i})$ corresponding to axis of movement x_i.

Gibbs sampling often exhibits a warm-up period, where the samples $\mathbf{x}^k$ are not representative of any modes of the distribution $p(\mathbf{x})$. This occurs when the initial state $\mathbf{x}^1$ is not close to a mode of the pdf. For multi-modal distributions Gibbs sampling may fail to sample from one or more modes, especially if the modes do not overlap when projected onto any of axes.