THE UNIVERSITY *of* EDINBURGH **informatics**

Probabilistic Modelling and Reasoning
Tutorial 2 — Notes

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These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

Factor representation — Define factors ϕ_i that do not necessarily correspond to probability distributions, this requires the use of a normalising constant $Z = \sum_{x,y,z} \phi_A(x,z)\phi_B(y,z)$.

$$x \perp\!\!\!\perp y \mid z \Leftrightarrow p(x,y,z) = a(x,z)b(y,z) \tag{1}$$

$$p(x,y,z) = \frac{1}{Z}\phi_A(x,z)\phi_B(y,z) \tag{2}$$

Gibbs distribution — A class of pdfs/pmfs that factorise into factors of sets of variables.

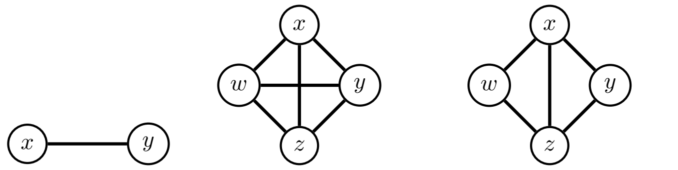
$$p(x_1,\ldots,x_d) = \frac{1}{Z}\prod_c \phi_c(\mathcal{X}_c) \qquad \mathcal{X}_c \subseteq \{x_1,\ldots,x_d\} \tag{3}$$

Energy based model — A model where energy functions are used in place of factors, this is useful as we can work with sums of energies which are in log-space.

$$p(x_1,\ldots,x_d) = \frac{1}{Z}\exp\left[-\sum_c E_c(\mathcal{X}_c)\right] \qquad E_c(\mathcal{X}_c) = -\log\left(\phi_c(\mathcal{X}_c)\right) \tag{4}$$

Undirected graphical model — All variables x_i are associated with one node, each set of variables $\mathcal{X}_c$ for a factor ϕ_c are maximally connected with edges.

$\phi(x,y)$ $\qquad$ $\phi(w,x,y,z)$ $\qquad$ $\phi_1(w,x,y)\phi_2(x,y,z)$



Separation in undirected graphical models — Conditioning on a set of variables Z removes the nodes corresponding to those variables from the graph. Two sets of variables X and Y are independent if there is no path between any variable $x \in X$ and $y \in Y$.

Local Markov property — For any node x in an undirected graphical model, if we condition on it's neighbours $ne(x)$, it will be separated from all other nodes $X \setminus (x \cup \mathrm{ne}(x))$.

$$x \perp\!\!\!\perp X \setminus (x \cup \mathrm{ne}(x)) \mid \mathrm{ne}(x) \quad \forall x \in X \tag{5}$$

Pairwise Markov property — Any non-neighbouring nodes x_i and x_j in an undirected graphical model are separated if we condition on all other nodes in the graph $X \setminus (x_i, x_j)$.

$$x_i \perp\!\!\!\perp x_j \mid X \setminus (x_i, x_j) \quad \forall x_i, x_j \in X \quad \textbf{s.t. } x_i \notin ne(x_j) \tag{6}$$