

# Modeling and Analysis of Model Reference Adaptive Control by Using MIT and Modified MIT Rule for Speed Control of DC Motor

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**Abstract**— Normal feedback controllers may not perform well, because of the variations in process or Plant due to nonlinear actuators, changes in environmental conditions. The design of a controller for speed control of DC Motor with Model Reference Adaptive Control scheme using the MIT rule for adaptive mechanism is presented in this paper. The controller gives reasonable results, but to the changes in the amplitude of reference signal it is very sensitive. It is shown from the simulation work carried out in this paper that adaptive system becomes oscillatory if the value of adaptation gain or the amplitude of reference signal is sufficiently large. This paper also deals with the use of MIT rule along with the normalized algorithm to handle the variations in the reference signal, and this adaptation law is referred as modified MIT rule. The Modeling of MRAC is shown by means of simulation on MATLAB.

**Index Terms**— Model Reference Adaptive control, MIT Rule, Modified MIT Rule.

## I. INTRODUCTION

The device which controls or regulates performance of any other plant or process is called as control system. An adaptive control strategy is used to design advanced control systems which gives better performance and accuracy [1]. For industrial applications there is a need of better performance control system which has created great research efforts for the application in adaptive control. Adaptive control is a modern class of control technique, though there is a long and vibrant history for research in adaptive control. First it was encouraged in 1950 for the problem of designing of aircraft which operating at wide range of altitudes and speeds for autopilots. From many adaptive control methods, in this paper model reference adaptive control approach based on MIT rule has explained [2].

In adaptive control methods, now a day's most effective method is Model Reference Adaptive control method. This Method uses the model either indirectly or directly. In this the model output should be followed by the plant output. at Massachusetts Institute of Technology, USA

Pioneer research work was carried out which focused on flight control for MRAC. H. P. Whitaker, P. V. Osburn and A. Kezer are the Contributors for initial work during 1960s. In the Designing of MRAS new developments were presented by them. It is considered as the initial work of MRAC. Firstly MIT rule was suggested and Then by P. C. Parks, D. D. Donalson, C. T. Leonde etc. Lyapunov design of MRAC and model reference parameter tracking techniques were presented and proposed to carry this work further. [3].

## II. MODEL REFERENCE ADAPTIVE CONTROLLER

The controller consists of two loops. Inner loop is feedback loop. It is composed of the plant and the controller. The outer loop is adjustment mechanism loop. It adjusts the controller parameters in such a way that the error which is difference between process output  $y$  is and model output  $y_m$  is reduced to zero

As shown in Figure.1 MRAC consist of four parts,

Plant: it is assumed that plant have known structure, even though the parameters are not known

Reference Model: To give an idyllic response of the adaptive control system to the reference input this reference model is used. The choice of the reference model has to satisfy two requirements.

1. It must satisfy the specifications which are used for performance in the control tasks
2. The reference model should reach the ideally behaviour for the adaptive control system.

Controller: It is generally standardised by a number of adjustable parameters. That implies there exist different sets of controller parameter values for which the desired control task is achievable. Usually in terms of the adjustable parameters the control law is linear. Here only one control parameter  $\theta$  is taken. The value of  $\theta$  dependent on adaptation gain.

Parameter Adjustment block: parameters in the control law are adjusted here. Adaptation law finds the parameters in such a way that the response of the plant will follow the reference

model. It is designed to achieve the stability of the control system as well as for reduction of tracking error to zero

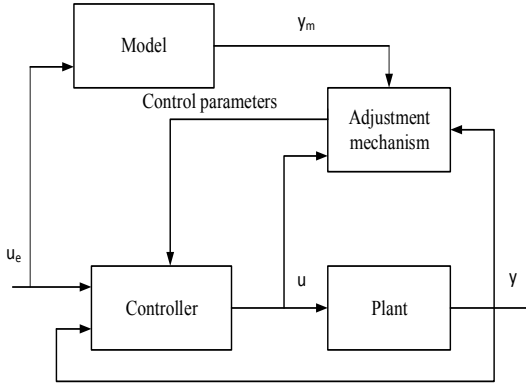


Fig.1.Block diagram for MRAC

As shown in Figure.1

$$e(t) = y(t) - y_m(t) \dots \dots \dots (1)$$

### III. MRAC WITH MIT RULE

(Massachusetts Institute of Technology (MIT) has developed the MIT Rule and used to design the autopilot system for aircrafts in 1960. MIT rule can be used to design the controller for any system. In this rule, a cost function is defined as,[1]

$$J(\theta) = \frac{e^2}{2} \dots \dots \dots (2)$$

Where  $e$  indicates the error between the plant and the model output.

$\theta$  -Adjustable parameter and it is adjusted in such a way so that the cost function is reduced to zero.

$\theta$  is placed in the way of the negative gradient of  $J$ .

$$\frac{d(\theta)}{d(t)} = \gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \dots \dots \dots (3)$$

Where,  $\frac{\partial e}{\partial \theta}$  is called as the sensitivity derivative of the system. It shows how the error is changing with the change in parameter  $\theta$ . And Eq. (3) shows the change in the parameter  $\theta$  with change in time with that the cost function  $J(\theta)$  will be decreased to zero. Here  $\gamma$  is known as adaptation gain of the controller.

Here it is assumed that the process is linear with transfer function  $K_p G(s)$ ,

$K_p$  -unknown parameter

$G(s)$  -second order transfer function. Now design a controller such that our process could track the reference model.

$$Y_m = K_m G(s) U_c(s) \text{ where } K_m \text{ is a known parameter}$$

Defining a control law

$$U = \theta U_c \dots \dots \dots (4)$$

Then from Eq (1)

$$e(t) = y(t) - y_m(t)$$

We get

$$E(s) = K_p G(s) U(s) - K_m G(s) U_c(s) \dots \dots \dots (5)$$

From Eq.(4) we get

$$E(s) = K_p G(s) \theta U_c(s) - K_m G(s) U_c(s) \dots \dots \dots (6)$$

Now by taking partial derivative

$$\frac{\partial E(s)}{\partial \theta} = K_p G(s) U_c(s) = \frac{K_p}{K_m} Y_m(s) \dots \dots \dots (7)$$

From Eq (3) and (7)

$$\frac{d(\theta)}{dt} = -\gamma e \frac{K_p}{K_m} Y_m(s) = -\gamma' e Y_m \dots \dots \dots (8)$$

Eq.(8) will give us the law for adjusting the parameter  $\theta$

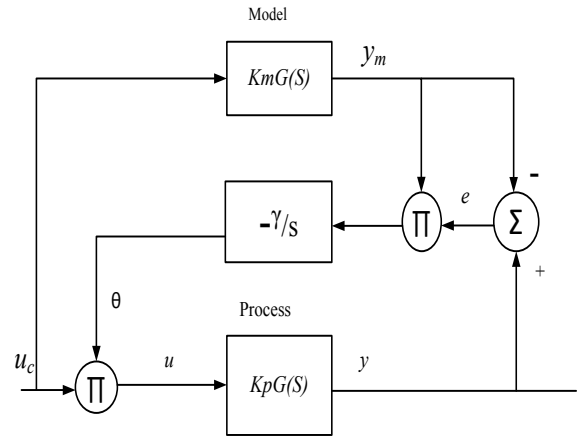


Fig.2 MIT rule for adjusting feed forward gain

### IV MODIFIED MIT RULE

The designed controller using MIT rule gives adequate results but is very susceptible to the changes in the amplitude of the reference input. For high values of reference input, system may become unbalanced. Hence to overcome this problem, Normalized algorithm is used with MIT rule to develop the control law. In order to protect against dependence on the signal amplitude Normalization can be used.

.Normalized algorithm modifies the adaptation law in the following manner,

$$\frac{d\theta}{dt} = \frac{-\gamma e \varphi}{\alpha + \varphi \varphi} \dots \dots \dots (9)$$

Where,  $\varphi = \frac{\partial e}{\partial \theta}$  and  $\alpha (\alpha > 0)$  is used to remove the difficulty of zero division when  $\varphi$  is small.

From Eq.(7)

$$\varphi = \frac{\partial e}{\partial \theta} = \frac{K_p}{K_m} Y_m \dots \dots \dots (10)$$

$$\frac{d\theta}{dt} = -\gamma\varphi e \dots\dots\dots (11)$$


$$J \frac{d^2 \theta}{dt^2} + f_0 \frac{d\theta}{dt} = K_T i_a \dots\dots\dots (17)$$

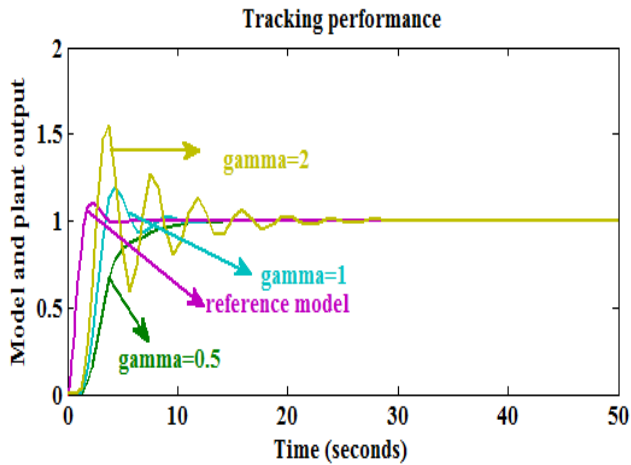


Fig.4.Tracking performance with MIT Rule for different Adaptive gains

It is clear from above result that the response of the system is fast with maximum peak overshoots for large values of adaption gain and is slow with minimum peak overshoot for smaller value of adaption gain. But beyond certain limit the performance of the system becomes very poor

Case2: study of tracking performance for different adaptive gain for Modified MIT rule

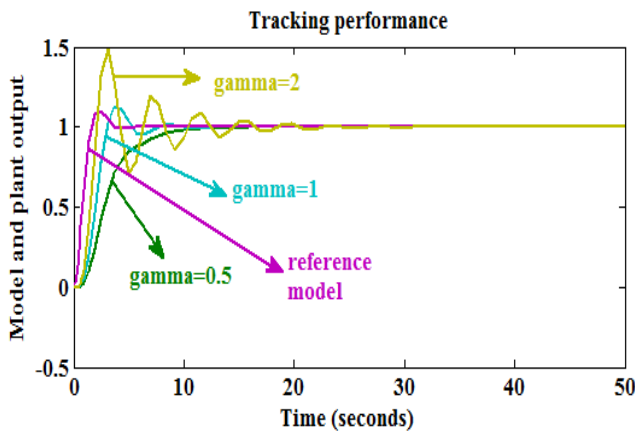


Fig.5 Tracking performance of Modified MIT Rule for different Adaptive gains

From the above obtained result the response of the system improves with the increase in adaption gain but beyond certain limit the performance of the system becomes very poor

Table.2 comparison of MIT Rule and Modified MIT Rule

Adaptive gain	MIT Rule		Modified MIT Rule	
	Maximum peak overshoot (%M <sub>p</sub> )	Settling time(t <sub>s</sub> ) (sec)	Maximum peak overshoot (%M <sub>p</sub> )	Settling time(t <sub>s</sub> ) (sec)
1	0	9	0	8
2	20	7	12	6
3	50	17	14	13

From the above table it is observed that in the Modified MIT rule the peak overshoot and settling time are less compared to MIT rule

Case3: study of tracking performance for different input signals for MIT Rule

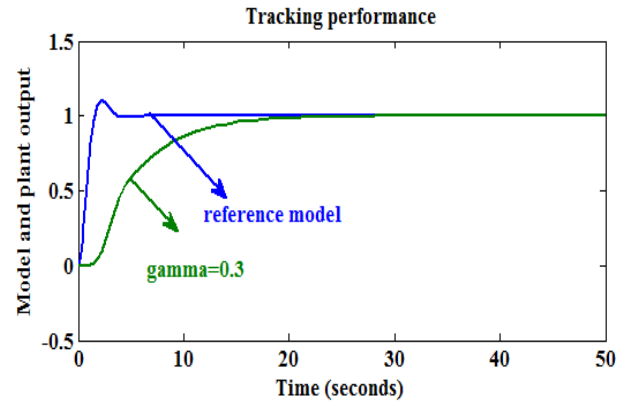


Fig.6 Tracking performance with MIT Rule for input signal  $U_c=1$

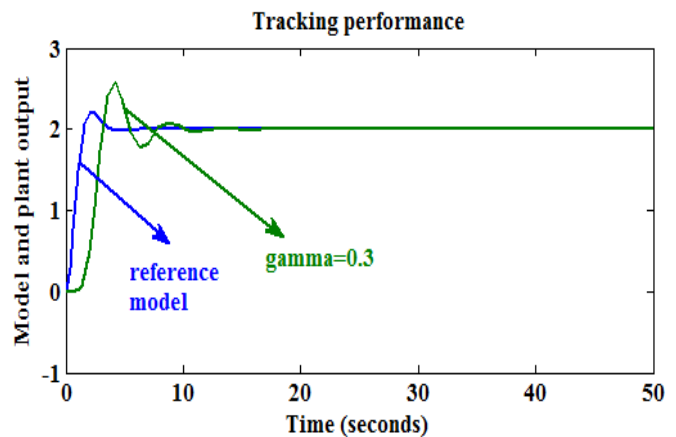


Fig.7.Tracking performance with MIT Rule for input signal  $U_c=2$

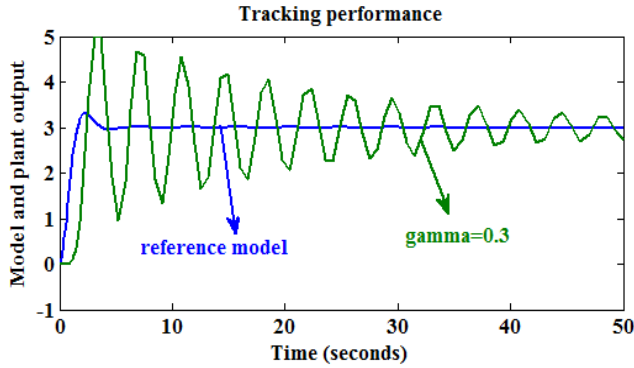


Fig.8 Tracking performance with MIT Rule for input signal  $U_C=3$

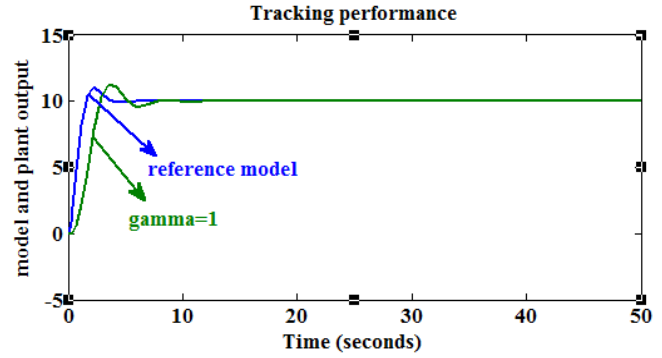


Fig.10. Tracking performance with Modified MIT Rule for input signal  $U_C=10$

Case4: study of tracking performance for different input signals for Modified MIT Rule.

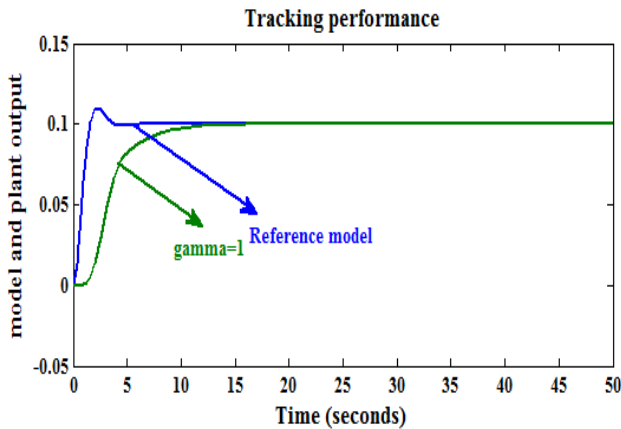


Fig.9 Tracking performance with Modified MIT Rule for input signal  $U_C=0.1$

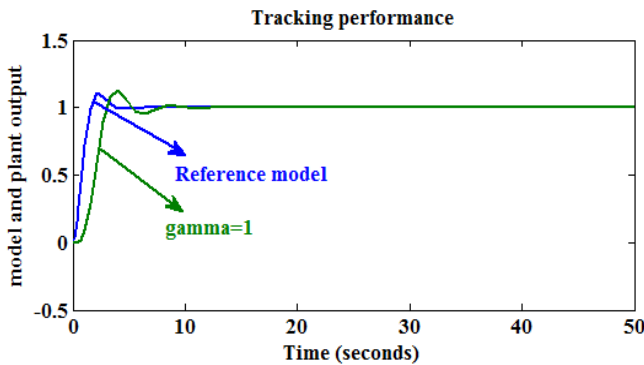


Fig.10. Tracking performance with Modified MIT Rule for input signal  $U_C=1$

Above results of case 3 and case 4 indicates that with the change in amplitude of the reference signal the controller designed by using MIT rule will be very sensitive and may become oscillatory for larger values of reference input where as Modified MIT rule makes the system almost insensitive to input amplitude changes.

## VII CONCLUSION

A thorough discussion on MRAC using MIT rule is done in this paper and the performance evaluation is carried out on SIMULINK.. The MIT rule is applied in many different cases. From above obtained results it is essential that selection of adaptive gain is very important. The Normalized algorithm,, is less sensitive for very large and very small amplitudes of reference input is observed. Therefore, it is shown in this paper that for acceptable values of adaptive gain, the MIT rule with normalization will make the plant to follow the model as precisely as possible.

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