Comparison Between Distributed Observer And Adaptive Distributed Observer

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Abstract— ***This paper presents a comparison between distributed and adaptive distributed observer-based methods for the cooperative output regulation problem of linear multi agent systems. For the system involving the distributed observer, it is assumed that all the followers have access to system matrix from the leader system. The adaptive distributed observer method however assumes that the followers have no information about the leader signal and leader matrix the followers will estimate the leader matrix and signal adaptively. The analysis is carried out by applying both techniques on linear model of servo motors.***

Keywords—Distributed observer, Adaptive Distributed observer, Multi Agent System, Cooperative output regulation.

I. Introduction

In control theory, network multi-agent systems (MAS) have attracted the attention of the researchers in the past few decades. MAS is an efficient way to describe the dynamic agents that exchange information through communication. Recently researchers are now focusing on cooperative control of network MAS in a number of practical applications [1], [2], [3]. The MAS system has a potential advantage over a single agent in terms of increasing effectiveness in quality and time. MAS have been able to achieve tasks that a single robot cannot execute. They are flexible in task execution, exceptionally robust, easily maintainable, highly adaptive, sustainable and low in cost. There are number of phenomena's in nature which exhibits the cooperative behavior: such as flocking, shoaling and swarm. In all these systems every agent follows the leader in a very systematic way. Similarly, in robotics we also use cooperative output regulation to accomplish the offered objective.

This paper discusses two different techniques to achieve cooperative output regulation for MAS by considering two different estimator based approaches namely distributed and adaptive observers. For distributed observer the design needs to become distributed in the sense that each system can only make use of the information of its own and its neighbors [6], [7]. In order to overcome this issue, the distributed observer approach was suggested in [8] for handling the cooperative regulation. Distributed observer has been additionally extended for many multiple leaders in [9] to resolve the control issue for linear MAS methods. The other method is Adaptive distributed observer method that provides an estimation of exogenous system generator to each follower. This research focuses on a comparison of the above mentioned techniques.

This paper is distributed as follows. Section II describes the distributed observer approach while the Section III mentions the adaptive distributed observer. This is followed by application of both techniques on a cluster of servo motors followed by conclusion.

II. Distributed Observer

In the output regulation of N heterogeneous linear MAS, it is anticipated that at different nodes the common reference signal r is tracked and disturbance d is going to be rejected. The dynamics of the nodes are given by the equations:

$$\begin{aligned} \dot{x} &= A_i x_i + B_i u_i + E_i w \\ y_{m_i} &= C_{m_i} x_i + D_{m_i} u_i + F_{m_i} w \\ e_i &= C_i x_i + D_i u_i + F_i w \end{aligned} \qquad (1)$$

Where $x_i \in \mathbb{R}^{n_i}$ is the vector state, u_i is the i^{th} control input, y_i is the output, e_i is the output error, and $A_i, B_i, C_{m_i}, D_{m_i}, F_{m_i}, C_i, D_i, F_i$ are the matrices of appropriate dimension. $w \in \mathbb{R}^q$ is the reference and disturbance signal which is created by following differential conditions separately

$$\dot{d} = A d_d, d(0) = d_0, \dot{r} = A r_r, r(0) = r_0$$

Here $r \in \mathbb{R}^g$ and $d \in \mathbb{R}^h$. For node both the input reference r and the disturbance d are lumped to form the exogenous signal which is donated as $\omega = [r\ d]$. So, from the exosystem.

$$\dot{w} = Sw \qquad (2)$$

Where $w \in \mathbb{R}^q$ is the leader system representing for the reference contribution being followed or probably the external disturbance being rejected and $S \in \mathbb{R}^{q \times q}$ is actually a constant matrix where appropriate matrix dimensions are

$$S = \begin{bmatrix} A_r & 0 \\ 0 & A_d \end{bmatrix} \qquad (3)$$

Here A_r is the reference signal and A_d is disturbance and Eq. 2 is known as the exosystem or some time also referred as leader system, whereas the Eq. 1 defines the agent's dynamics or also referred as the follower's system. The distributed observer is fit for delivering the approximation from the leader to each and every follower so the distributed control law could

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be incorporated for this problem. The distributed observer of states $\eta_i \in \mathbb{R}^q$ is described as follow:

$$\dot{\eta}_i(t) = S\eta_i(t) + \gamma\left(\sum_{i=1}^{n} a_{ij}(t)\,(\eta_j - \eta_i) + a_{io}(t)(\omega - \eta_i)\right) \quad (4)$$

For any arbitrary constant γ. If the node I can access the exosystem then $a_i0(t) > 0$ else $a_i0(t) = 0$. The dependency of η_j *and* η_i on each other can be understood from Eq. 4. The distributed observer in the literature is also referred as a dynamic compensator. The dynamic measurement output feedback control can be summarized as

$$\begin{aligned} u_i(t) &= K_{1i}\xi_i(t) + K_{2i}\eta_i(t) \\ \dot{\eta}_i(t) &= S\eta_i + \gamma\left(\sum_{i=1}^{n} a_{ij}(t)\,(\eta_j - \eta_i) + a_{io}(t)(\omega - \eta_i)\right) \\ \dot{\xi}(t) &= A_i\xi_i(t) + B_iu_i(t) + E_i\eta_i(t) + L_i(C_{m_i}\xi_i(t) \\ &\quad + D_{m_i}u_i(t) + F_{m_i}\eta_i(t) - y_{m_i}(t)) \end{aligned} \quad (5)$$

Where the controller gains dynamics are $K_{1i} \in \mathbb{R}^{\mathrm{mi}\times\mathrm{ni}}$ *and* K_{2i} is $\mathbb{R}^{\mathrm{mi}\times\mathrm{q}}$. The observer gain dynamics are L_i that is $\mathbb{R}^{\mathrm{mi}\times\mathrm{pi}}$. The follower system defined in Eq. 1, the exosystem in Eq. 2 and the output measurement feedback control law in Eq. 4 is put in a closed loop form as follow:

$$\begin{aligned} x &= A_{\sigma(t)}x(t) + B_{\sigma(t)}w(t), x(t) = x_0 \\ e(t) &= C_{\sigma(t)}x(t) + D_{\sigma(t)}w(t) \end{aligned} \quad (6)$$

In Eq. 6 the closed loop matrices $A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}$ *and* $D_{\sigma(t)}$ are explained in theorem 1. Now for homogenous MAS the cooperative output regulation is described as follow:

Definition 1: for a given MAS (Eq. 1), exosystem (Eq. 2) and a corresponding diagraph $\bar{\mathcal{G}}_{\sigma(t)}$, the output measurement feedback control law (Eq. 5) can be found if the following properties are satisfied.

1. For $w = 0$, the closed loop system x_c origin should be exponentially stable.
2. For the given $x_i(0), w(0)$ and $\cdots\cdots$ $\eta_i(0)$ with initial conditions $\lim_{t\to\infty} e_i(t) = 0$ and $i = 1, \dots, N$.

For review the classical linear output regulation and switched system regulation problem related assumptions are as follow:

A1: All the S matrix eigenvalues should not have a negative real part.

A2: For $i = 1, \dots, N$, *the* (A_i, B_i) pair should be stabilizable.

A3: F or $i = 1, \dots, N$, *the* (C_{m_i}, A_i) pair should be detectable.

A4: For the linear matrix equation with X_i *and* U_i as the solutions $i = 1, \dots, N$.

$$\begin{aligned} X_iS &= A_iX_i + B_iU_i + E_i \\ 0 &= C_iX_i + D_iU_i + F_i \end{aligned} \quad (7)$$

A5: For a subsequent i_k with $t_{iK+1} - t_{ik} < v$ *for* $v > 0$ such that the in the union graph $\cup_{j=i_k}^{i_{k+1}} \bar{\mathcal{G}}_{(t)}$ of the exosystem can contact all the available nodes.

Remark 1: For standard the only **A1** to **A4** assumptions are being assumed and being used in [10], [11]. In results the follower's nodes are of higher order and the assumption **A5** is specifically used for the digraph with fixed and switching network topologies. An assumption Like **A5** is being discussed in [11].

Lemma 1: Suppose the unforced portion of closed loop linear system of Eq. 6.

$$\dot{x}_c = A_{c,\sigma(t)}x_c \quad (8)$$

Have an exponentially stable origin [11]. Then for all positive time the constant matrix X_c satisfy the following equations

$$\begin{aligned} XS &= A_{\sigma(t)}X + B_{\sigma(t)} \\ 0 &= C_{\sigma(t)}X + D_{\sigma(t)} \end{aligned} \quad (9)$$

Such that the error equation becomes

$$\lim_{t\to\infty} e(t) = 0$$

Remark 2: Y. Su in [10] discussed **Lemma 1** for homogenous agents.

For $\Delta_{\sigma(t)} = (a_{io}, \dots, a_{no})$ such that $a > 0$ only then portioned of $\bar{\mathcal{L}}_{\sigma(t)}$ can be

$$\bar{\mathcal{L}}_{\sigma(t)} = \left(\begin{array}{c|c} \sum_{j=1} N\, a_{oj(t)} & -[a_{01}(t), \dots, a_{0N}(t)] \\ \hline -\Delta_{\sigma(t)}1_N & H_{\sigma(t)} \end{array}\right) \quad (10)$$

Here $a_{0j} > 0\ \forall\, j = 1, \dots, N$.

Lemma 2: A linear switched system under **A1** and **A5**. For a positive constant γ and a Hurwitz matrix the equation is exponentially stable [11].

Remark 3: For a switching network topology to be a fixed network topology the value of $\rho = 1$. A special case of $H \triangleq H_{(t)}$ *and* **A5** is as:

$$\Delta \triangleq \Delta_{\sigma(t)} \neq 0$$

A6 For the case of $\rho = 1$ to remove the Assumption **A1** dependency the **Lemma 2** is applied for a large positive value of γ.

A7 The graph $\bar{\mathcal{G}}$ contains a spreading over other networks with the hub 0 as the root.

Corollary 1: A linear system origin under assumption **A6** is Hurwitz for a positive large value of γ and a Hurwitz M matrix [11]. Under the Assumption **A2**, K_{1i} exist for $A_i+B_iK_{1i}$ to be Hurwitz for $i = 1, \dots, N$. Then for the equation K_{2i} will be

$$K_{2i} = U_i - K_{1i}X_i \quad (11)$$

If the matrices $A, B, C, D, E, F, X, U, K_{1i}$ *and* K_{2i} are block diagonal then the Eq. 7 and Eq. 11.

$$\begin{aligned} X(I_N \otimes S) &= AX + BK_1 + BK_2 + E \\ 0 &= CX + DK_1 + DK_2 + F \end{aligned} \quad (12)$$

Theorem 1: By assumptions **A1, A5** and **Definition 1** a linear heterogeneous MAS cooperative output regulation problem is solvable by Eq. 5 output feedback control law if and only if the closed loop Eq. 7 origin is exponentially stable, here K_{1i} is so that $(A_i + B_i K_{1i})$ is Routh Hurwitz and K_{2i} is expressed in Eq. 11. By Assumption **A3**, if the $A_i+L_i C_{mi}$ is Hurwitz then there exist L_i for $i = 1, \ldots, N$.

Remark 4: Make sure to select K_{1i} so that the $A_i+B_i K_{1i}$ eigenvalue is in the left plane and then K_{2i} is found by obtaining the solution of (X_i, U_i) by solving the linear matrix equation. Select $\mathcal{L}_i$ so that $A_i+L_{mi}C_i$ eigenvalue is left half plane.

III. Adaptive Distributed Observer

Distributed observer approach does not provide access to all the follower systems about the matrix S [4], [5], [9] and [13]. To overcome this issue, an adaptive distributed observer method is introduced that gives the observer estimates of the leader's signal and in addition estimates the leader's system matrix as well. For system (1) that was proposed in [4], to resolve the adaptive leader following consensus issue for Lagrange techniques was further extended to resolve the cooperative output regulation problem with regard to linear MAS in [5]. Therefore, the condition that each follower know the matrix S could be removed. So, from the adaptive distributed observer

$$\dot{S}_i = \mu_1 \textstyle\sum_{j=0}^{N} a_{ij}\left(S_j - S_i\right) \tag{13}$$

$$\dot{\eta}_i = S_i\eta_i + \mu_2 \textstyle\sum_{j=0}^{N} a_{ij}(\eta_j - \eta_i) \tag{14}$$

Where $S_0 = S, S_i \in \mathbb{R}^{q*q}$, $i = 1, \ldots, N$ and $\mu_1, \mu_2 > 0$. In Assumption A7 and with the assumptions that each one of the eigenvalues have simply no real parts which is semi simple also it is undirected. It is actually mentioned that, this observer incorporates a system for estimating the actual matrix, and it is known as the adaptive distributed observer. Through (13), it is actually observed that simply those followers that are the children of the leader knows the matrix S. For better understanding we will remove the assumption that the eigenvalues associated with matrix are often semi-simple with absolutely no real parts to make sure we may furthermore manage unbounded signals for example sinusoidal signal. Furthermore, we will even eliminate assumption that's it is undirected. In this context, consider the following lemmas,

Lemma 3. Let us consider the following system

$$\dot{x} = \epsilon F x + F_1(t)x + F_2(t) \tag{15}$$

Where $x \in \mathbb{R}^n, F \in \mathbb{R}^{n*n}$ is Routh Hurwitz, $\epsilon > 0, F_1(t) \in \mathbb{R}^{n*n}$ and $F_2(t) \in \mathbb{R}^n$ remains continuous and bounded for all $t \geq t_0$.

Lemma 4. Given the systems (2) let $\tilde{S}_i = S_i - S, \tilde{\eta}_i = \eta_i - w$ with initial conditions $\tilde{S}_i(0)$ *and* $\tilde{\eta}_i$, we have, (i) for any $\mu_1 > 0$, for i = 1,...,N, $\lim_{t\to\infty} \tilde{S}_i(t) = 0$ exponentially, and (ii) for suitably μ_1 *and* $\mu_2 > 0$, for $i = 1, \ldots, N, \lim_{t\to\infty} \tilde{\eta}_i(t) = 0$.

The control law must utilize the solution from the regulator conditions (7) to provide an appropriate feed forward control to complete the control goal. The arrangement from the controller conditions depends upon the system matrix in the leader matrix. Since the followers have no information about the system matrix so it will estimate the system matrix adaptively from eq. 13 and 14. The dynamic estimation output feedback control can be outlined as

$$\begin{aligned} u_i(t) &= K_{1i}x_i(t) + K_{2i}\eta_i(t) \\ \dot{\xi}(t) &= A_i\xi_i(t) + B_i u_i(t) + E_i\eta_i(t) + L_i(C_{m_i}\xi_i(t) \\ &\quad + D_{m_i}u_i(t) + F_{m_i}\eta_i(t) - y_{m_i}(t)) \end{aligned} \tag{16}$$

Where k_{1i} such that $A_i + B_i K_{1i}$ is Hurwitz and K_{2i} can be solved by regulator eq. 5.

IV. Example

Consider a servo motor defined as [15]:

$$J\ddot{\theta} = u(t) - d(t) \tag{17}$$

Where J is moment of inertia, u is input, θ represents position.

$$\ddot{\theta} = bu(t) + f(t) \tag{18}$$

Where $b = \frac{1}{J}, f(t) = -\frac{1}{J}d(t)$, Equation (18) can be expressed as

$$\dot{x} = Ax + B\left(bu + f(t)\right) \tag{19}$$

Where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0],$$

$$D = 0, \quad E_i = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 * i \end{bmatrix}, \quad F_i = [-1\,, 0],$$

$$L_i = col[-2, -5], \quad k_{1_i} = [-5, -6]$$

$$and\ i = 1, 2, \ldots \ldots, 6.$$

Let the leader system

$$S(w) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad w(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Whereas the communication graph is,

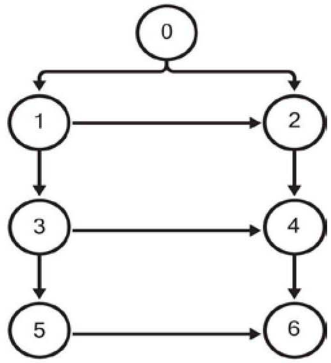

Fig 1: communication graph

V. RESULTS

For the above communication graph, we obtained the following laplacian matrix $\mathcal{L}$ *and* $\mathcal{H}$ matrix are:

$$\mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

$$\mathcal{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

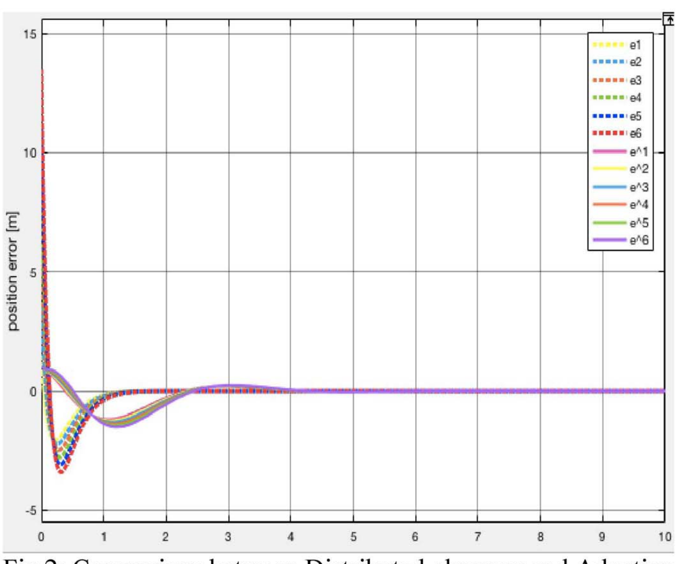

Fig 2: Comparison between Distributed observer and Adaptive distributed observer

Here e1, e2, e3, e4, e5, e6 are showing distributed observer tracking errors and e^1, e^2, e^3, e^4, e^5, e^6 are showing the adaptive distributed tracking errors.

VI. CONLUSION.

The cooperative output regulation problem of multi agent systems is discussed using two different observer based approaches namely distributed and adaptive distributed observers. By comparing both techniques, it was concluded that the distributed observer is converging faster than the adaptive distributed observer for the given scenario. In this way distributed observer method is better than adaptive distributed observer. For the case when the followers doesn't have any information regarding the system matrix from the leader program, adaptive observer may estimates the actual leader matrix adaptively, though it will not converge fast.

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