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H_∞ optimal design of robust observer against disturbances

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This paper considers the robust observer design problem for linear dynamic systems subject to the interference of external disturbances. For such systems, the state estimate from the conventional Luenberger is normally biased with respect to the true system state. To remedy this situation, this paper proposes a new structure for robust observers. With this new structure, the robust observer design problem is skillfully transformed into the well-known disturbance rejection control problem. The H_∞ optimal control design technique can then be applied to shape the proposed robust observer in the frequency domain. The proposed robust observer is a joint state and disturbance observer, which simultaneously estimates both the system state and unknown disturbances, and can be applied to non-minimum-phase systems.

Keywords: disturbance observer; robust observer; H_∞ control; frequency shaping design

1. Introduction

In many control applications, the controlled system is subject to the interference of external disturbances. For such systems, the state estimate provided by the conventional Luenberger is usually biased with respect to the true system state. Using a biased state estimate in the control law results in deviations of system performance from its expected standard. Hence, robust observer design problem, in which the goal is to reduce state estimation bias due to disturbances, has become an important issue in control area.

The first approach to the robust observer design is to independently estimate the disturbance acting on the system (Santamarina & Fratta, 2005; Trujillo & Busby, 1997) and then design an observer that is decoupled from the disturbance. In control engineering, disturbance observer (DOB) is the most popular approach to *on-line* disturbance estimation. The DOB approach (Nakao, Ohnishi, & Miyachi, 1987; Ohnishi, Shibata, & Murakami, 1996) refers to using direct system inverse to estimate the input disturbance. To avoid a non-proper system inverse, a low-pass Q-filter is cascaded with the system inverse, where the Q-filter is chosen to achieve accurate disturbance estimation and to avoid amplification of measurement noise. Different design considerations and design skills have been proposed in the literature (Choi, Yang, Chung, Kim, & Suh, 2003; Kim & Chung, 2003; Komada, Machii, & Hori, 2000; Shim & Jo, 2009; Tesfaye, Lee, & Tomizuka, 2000; Umeno, Kaneko, & Hori, 1993). Robust control based on DOB has been proved to be very effective especially in motion control applications. Since the original DOB design requires system inverse, its application is limited to minimum-phase systems (systems with stable zeros) only. To deal with DOB design for non-minimum-phase systems, Bajcinca and Bunte (2005) propose inverting only the invertible (stable-zero) part of the system. In Son, Shim, Jo, and Kim (2007), a parallel compensator is added to the non-minimum-phase system so that the parallel connection becomes minimum phase and invertible. However, for this approach to be exact, the parallel compensator must have almost zero gain in the frequency range of disturbance. An improved version of this approach is proposed in Shim, Jo, and Son (2008). The DOB in Chen, Zhai, and Fukuda (2004) is based on polynomial approximation to find approximate inverse of the non-minimum-phase system. There have been some works (El-Shaer & Tomizuka, 2013; Wang & Tomizuka, 2004) studying the H_∞ design of DOB. However, their designs are still limited to minimum-phase systems.

Another approach for robust observer design is the joint state and disturbance observer (Radke & Gao, 2006), where an observer is constructed to simultaneously estimate both the unknown state variables and the unknown disturbance. This approach originates in the study of unknown input observer; see, for example, the designs by Wang and Daley (1996), Gao (2006) and Han (2009). If knowledge of a disturbance model is known *a priori*, Luenberger type state space DOB can be constructed (Hostetter & Meditch, 1973; Johnson, 1971, 1975; Profeta, Vogt, & Mickle, 1990). When no disturbance model is available, the design in Corless and Tu (1998), Xiong and Saif (2003) is applicable. However, as explained in Duarte-Mermoud and La Rosa (2003), most unknown input observer designs restrict the transfer function from the unknown disturbance to the

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system output be relative degree one and minimum phase. The proportional integral DOB in Chang (2006) can apply to non-minimum-phase systems but only if the disturbance does not vary too much between two sampling instants.

In this paper, a previously proposed joint state and disturbance observer design (Hostetter & Meditch, 1973; Johnson, 1971, 1975) will first be reviewed. Since this previous observer design requires information of a disturbance model, its application is limited. This paper proposes a frequency domain redesign of this observer based on the finding that the structure of this previous observer is analogous to that of an H_∞ feedback control system. By taking advantage of this analogy, this paper proposes a new frequency shaping design for the robust observer. The new frequency shaping design waives the disturbance model constraint and can apply to a much larger class of disturbances than the original design. It takes into account both the estimation tracking ability and the measurement noise sensitivity. The new design can be easily carried out using standard H_∞ control design tools. Another advantage of the proposed robust observer is that it applies to both minimum-phase and non-minimum-phase systems, while most previous DOB design and unknown input observer design require that the system be minimum phase. It is mentioned that this paper deals with estimation of external disturbances, excluding the estimation of uncertainties such as parametric uncertainties and unmodelled dynamics, as is done in the works (Ohnishi et al., 1996; Han 2009).

2. Conventional joint state and disturbance observer

Consider an multi-input multi-output linear time-invariant system subject to unknown disturbance excitation,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Gd(t), \\ y(t) &= Cx(t) + n(t), \end{aligned} \tag{1}$$

where $x \in R^n$ is the unknown system state, $y \in R^p$ the measured system output, $u \in R^m$ the control input, $d \in R^p$ an unknown disturbance and $n \in R^p$ the measurement noise. The input–output representation of the system is given by

$$\begin{aligned} y = G(s)u + P(s)d + n, \quad G(s) = C(sI - A)^{-1}B, \\ P(s) = C(sI - A)^{-1}G. \end{aligned} \tag{2}$$

Assumption A1: *The pair* (A, C) *is observable and* (A, G) *controllable. Hence, there is no pole-zero cancellation in the transfer function* $P(s)$.

Assumption A2: *The number of output* y *should be no less than that of disturbance d. In this paper, they are assumed to be equal. Hence,* $P(s)$ *is a square matrix transfer function.*

The problem of robust observer design is to obtain an *on-line* close estimate of the system state x from output measurement y and control input u. No preview of the output y is necessary. In this section, one will review a joint state and disturbance observer design (Hostetter & Meditch, 1973; Johnson, 1971, 1975). This design can simultaneously estimate the system state x and the unknown disturbance d from the signals u and y if a disturbance model, as stated in Assumption A3, is available. Note that this disturbance model assumption will be waived in the new robust observer design in the next section.

Assumption A3: *The disturbance d is generated from an autonomous dynamic system of order r,*

$$\dot{z}(t) = Fz(t), \quad d(t) = Hz(t), \tag{3}$$

where $z \in R^r$ *is the unknown disturbance state, and system matrices* $F \in R^{r\times r}$ *and* $H \in R^{p\times r}$ *are known a priori. Examples of such disturbance d include polynomial time functions with unknown coefficients and sinusoidal time functions with known frequencies.*

One can combine the disturbance model (3) with the system model (1) to form an expanded system,

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} &= \begin{bmatrix} A & GH \\ 0 & F \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + n(t). \end{aligned} \tag{4}$$

If this expanded system is observable, a Luenberger observer can be constructed for the expanded system to estimate the state x, z, and hence the disturbance d,

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} &= \begin{bmatrix} A & GH \\ 0 & F \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y - C\hat{x}), \\ \hat{d}(t) &= \begin{bmatrix} 0 & H \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{z}(t) \end{bmatrix}. \end{aligned} \tag{5}$$

where the upper observer feedback gain L_1 and the lower observer feedback gain L_2 are chosen based on the eigenvalue assignment design or the Kalman filter design to ensure convergence of the state estimation error, and hence the disturbance estimation error. Such a design provides a simple and accurate solution to the disturbance estimation problem and the robust observer problem, nevertheless, it applies to very limited classes of disturbance functions such as the polynomial time functions or sinusoidal time functions. Hence, in the next section, one will propose a new robust observer that operates without Assumption A3 on the disturbance.

3. Redesign of the robust observer

The previous section reviews a joint state and disturbance observer design that can handle the disturbance estimation problem for a small class of disturbances satisfying Assumption A3. To enlarge the application domain of this robust observer, the same observer will be redesigned in the frequency domain without using Assumption A3. In this section, one assumes Assumptions A1 and A2, but not Assumption A3.

The robust observer proposed in this section has a structure very much similar to that in Equation (5), except that a feedthrough term $J(y - C\hat{x})$ is added,

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & GN \\ 0 & M \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y - C\hat{x}), \qquad (6)$$
$$\hat{d}(t) = \begin{bmatrix} 0 & N \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{z}(t) \end{bmatrix} + J(y - C\hat{x}),$$

where $M \in R^{r\times r}$, $N \in R^{p\times r}$. The newly included feedthrough term is to ensure that the structure of the robust observer considered is general enough to cover all the design results proposed below.

The design parameters of the proposed robust observer (6) include matrices L_1, L_2, M, N, J. This new robust observer will be redesigned in the frequency domain to determine these design parameters. The frequency domain design will rely on closed-loop transfer functions of the estimation error dynamics, which will be derived below.

When the upper observer feedback gain $L_1 = GJ$, the state estimation error dynamics ($\tilde{x} = x - \hat{x}$) can be obtained from Equations (1) and (6),

$$\dot{\tilde{x}} = A\tilde{x} + G(d - \hat{d}), \qquad \hat{d} = C(s)(y - C\hat{x}), \qquad (7)$$

where $C(s) = N(sI - M)^{-1}L_2 + J$. From the first equation in Equation (7), one obtains $\tilde{x} = (sI - A)^{-1}G(d - \hat{d})$. Substituting this into the second equation of Equation (7) and using $P(s) = C(sI - A)^{-1}G$ lead to $\hat{d} = C(s)P(s)(d - \hat{d}) + C(s)n$. Rearranging the equation gives $\hat{d} = (I + C(s)P(s))^{-1}C(s)P(s)d + (I + C(s)P(s))^{-1}C(s)n$. Finally, the disturbance estimation error is obtained as $\tilde{d} = d - \hat{d} = (I + C(s)P(s))^{-1}d - (I + C(s)P(s))^{-1}C(s)n$.

In summary, consider the joint state and disturbance observer (6) for the system (1), which satisfies Assumptions A1 and A2 (not Assumption A3). If the upper observer feedback gain is chosen to be $L_1 = GJ$ with J given as in Equation (10), the disturbance estimation error $\tilde{d} = d - \hat{d}$ and the state estimation error $\tilde{x} = x - \hat{x}$ satisfy the closed-loop transfer function relationships,

$$\tilde{d} = S(s)d - T(s)n, \quad \text{and} \quad \tilde{x} = (sI - A)^{-1}G\tilde{d}, \qquad (8)$$

where $S(s)$ and $T(s)$ are given by

$$S(s) = (1 + C(s)P(s))^{-1},$$
$$T(s) = (1 + C(s)P(s))^{-1}C(s), \qquad (9)$$

with $P(s)$ the system transfer function in Equation (2), and the design filter $C(s)$ given by

$$C(s) = N(sI - M)^{-1}L_2 + J. \qquad (10)$$

It is noted that in deriving the transfer function relationship (8) above, one has used Equations (2) and (6), but not the disturbance model constraint (3). Hence, the relationship (8) holds for any disturbance d as long as its Laplace transform exists. The disturbance d is not constrained to be generated from the disturbance model (3) for the new design in this section. In the sequel, one will choose the design parameters M, N, L_1, L_2, and J in Equation (6) such that the transfer functions $S(s)$ and $T(s)$ in Equation (8) have desired frequency response characteristics. Since L_1 is set to be $L_1 = GJ$, all the other design parameters are summarised in the design filter transfer function matrix $C(s) = N(sI - M)^{-1}L_2 + J$. Finding the design parameters M, N, L_2, J is equivalent to finding the design filter transfer function matrix $C(s)$.

The two transfer functions $S(s) = (1 + C(s)P(s))^{-1}$ and $T(s) = (1 + C(s)P(s))^{-1}C(s)$ in Equation (8) are familiar to control engineers since they are closed-loop transfer functions in the disturbance rejection control problem in Figure 1. Figure 1 describes a system $P(s)$ subject to input disturbance d and measurement noise n. The control objective is to design the feedback controller $C(s)$ to achieve disturbance rejection. Ideally, the control input u achieves disturbance rejection if it cancels the disturbance; that is, $e \triangleq u + d \to 0$ in Figure 1. From the block diagram, e satisfies

$$e = S(s)d - T(s)n. \qquad (11)$$

To minimise the error e in the above equation, the controller $C(s)$ should be chosen, so that the sensitivity function $S(s) = (1 + C(s)P(s))^{-1}$ is small over the frequency range of d, and

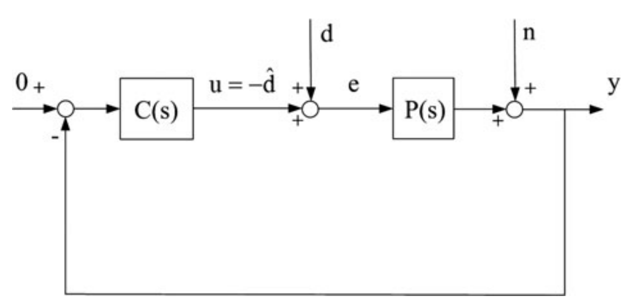

Figure 1. Disturbance rejection control problem.

the high frequency gain of $T(s) = (1 + C(s)P(s))^{-1}C(s)$ is suppressed over the high frequency range of n. Such frequency shaping design of $S(s)$ and $T(s)$ has been studied thoroughly in the control area. An efficient and systematic way to design $S(s)$ and $T(s)$ is the H_∞ control design (Zhou & Doyle, 1998), which ensures closed-loop stability as well as performance in the frequency domain.

Note that Equation (11) in the disturbance rejection control problem in Figure 1 is exactly the same as the disturbance estimation error equation (8) in the DOB problem. This reveals a parallelism between the input disturbance rejection control problem in Figure 1 and the robust observer problem treated in this section. Both problems address the shaping design of the same transfer functions $S(s)$ and $T(s)$. The objective of this section is to take advantage of this parallelism to create a new design methodology for the joint state and disturbance observer (6). The new robust observer design, which is described below, will be carried out in the frequency domain using the standard H_∞ control design tools.

3.1 *Design procedure step I*

One can incorporate the internal modelling principle (Chen, 1999) into the observer design. If it is known that part of the disturbance signal has a generating polynomial $E(s)$, then set an expanded system to be

$$\bar{P}(s) = \frac{1}{E(s)} \times P(s), \tag{12}$$

where $P(s)$ is the system transfer function in Equation (2). For example, if the disturbance d contains an unknown DC component and a linearly drifting term, one can set $E(s) = s^2$, and the internal modelling principle design will ensure that the proposed disturbance estimate $\hat{d}$ can accurately estimate this DC component and the linearly drifting term. One then sets the design filter $C(s)$ to be

$$C(s) = \bar{C}(s) \times \frac{1}{E(s)}, \tag{13}$$

where $\bar{C}(s)$ is to be found from design step II below.

3.2 *Design procedure step II*

According to Equation (8), $\tilde{d} = S(s)d - T(s)n$, where

$$\begin{aligned} S(s) &= (1 + C(s)P(s))^{-1} = (1 + \bar{C}(s)\frac{1}{E(s)}P(s))^{-1} \\ &= (1 + \bar{C}(s)\bar{P}(s))^{-1}, \end{aligned} \tag{14}$$

$$T(s) = (1 + \bar{C}(s)\bar{P}(s))^{-1}\bar{C}(s)\frac{1}{E(s)}. \tag{15}$$

The design task has two missions through the choice of the design filter $\bar{C}(s)$. The first mission is to achieve accurate disturbance estimation by shaping $S(s)$ small over the frequency range of d. The second mission is to suppress the high frequency gain of $T(s)$ so as to reduce the sensitivity of the observer with respect to measurement noise n. To achieve this, consider $S(s)$ and $T(s)$ in Equations (14) and (15), and perform an H_∞ frequency shaping design to find $\bar{C}(s)$ with mixed optimisation index,

$$\left\| \begin{bmatrix} W_1(s)S(s), \\ W_2(s)T(s) \end{bmatrix} \right\|_\infty, \tag{16}$$

where $W_1(s)$ is a stable weighting function aiming to lower the gain of $S(s)$ over the frequency range of d, and $W_2(s)$ is a stable weighting function aiming to suppress the high frequency gain of $T(s)$. In case that the level of measurement noise is low, one can perform an H_∞ frequency shaping design to find $\bar{C}(s)$ with a different optimisation index,

$$\| W_1(s)S(s) \|_\infty. \tag{17}$$

The above design procedures generates a $\bar{C}(s)$ that stabilises the closed-loop transfer function $(1 + C(s)P(s))^{-1} = (1 + \bar{C}(s)\bar{P}(s))^{-1}$. If the dimension of the generated $\bar{C}(s)$ is too high, one may perform model reduction (Zhou & Doyle, 1998) to obtain a reduced-order $\bar{C}(s)$.

3.3 *Design procedure step III*

Use Equation (13) to find the design filter $C(s)$ from $\bar{C}(s)$. The design parameters of the DOB (6), M, N, $L_1(= GJ)$, L_2 and J can then be determined from any state space realisation of the transfer function matrix $C(s) = N(sI - M)^{-1}L_2 + J$.

Remark: According to Equation (7), the estimation error dynamics of the proposed observer is plotted in Figure 2. Note that the above H_∞ design ensures that the closed-loop system in Figure 2 is stable (Zhou & Doyle, 1998); hence, the proposed robust observer is a stable observer in the sense that when there is no external disturbance $d = 0$, the state estimation error $\tilde{x}$ approaches zero exponentially. When there exists non-zero disturbance, the magnitudes of estimation error $\tilde{d}$ and $\tilde{x}$ depend on the frequency shaping result. When the transfer function $P(s)$ is minimum phase, one can shape the sensitivity $S(s)$ as desired by properly

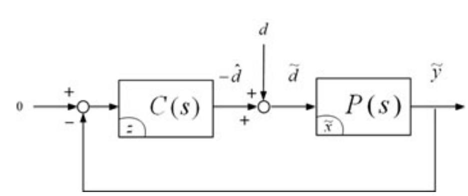

Figure 2. Estimation error dynamics of the new robust observer.

choosing the weighting function $W_1(s)$ in Equation (17). As a result, even if the disturbance has a very wide spectrum, the proposed robust observer can successfully estimate the system state and the disturbance accurately. However, when $P(s)$ is non-minimum phase, due to the constraint of Bode's integral theorem (Maciejowski, 1989), the achievable bandwidth of $S(s)$ is limited. The spectrum of trackable disturbances is then limited essentially by the position of unstable zeros of $P(s)$ (Maciejowski, 1989).

Note that most previous DOB designs or unknown input observer designs apply to minimum-phase systems only. Since the H_∞ frequency shaping design applies to both minimum-phase and non-minimum-phase system $P(s)$, the proposed robust observer design applies to both minimum-phase and non-minimum-phase systems.

Finally, it is mentioned that the proposed robust observer design applies to discrete-time systems as well. One can either use discrete-time H_∞ frequency shaping design tools, or use bilinear (or W) transformation (Astrom & Wittenmark, 1997) to transform the discrete transfer function $P(z)$ into a continuous transfer function, and then proceed with a continuous time frequency shaping design as introduced in this paper.

The following example demonstrates the effectiveness of the proposed robust observer.

Example 1: Consider a fifth-order non-minimum-phase control system (1) with

$$A = \begin{bmatrix} -15.9 & -76.4 & -192.2 & -180 & 20 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C^T = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -76 \\ -240 \end{bmatrix},$$

the unknown disturbance $d(t) = 10 + 5\cos(2\sqrt{t})$, the measurement noise $n(t)$ is a white stochastic noise with normal distribution, zero mean and standard deviation 0.2 and the initial condition $x(0) = [1, 1, 1, 1, 1]^T$. The transfer function $P(s) = C(sI - A)^{-1}G$ in Figure 2 is given by

$$P(s) = \frac{(s-10)(s+4)(s+6)}{(s-0.1)(s+2)(s+10)(s^2+4s+10)}.$$

Note also that $P(s)$ is of relative degree two, non-minimum phase (having one unstable zero at $s = 10$) and open-loop unstable (having one unstable pole at $s = 0.1$). The control input is simply $u(t) = 1$. Since $u(t)$ is not stabilising the system, the system state x explodes exponentially in the simulation due to the unstable pole at $s = 0.1$. The proposed DOB is found by performing a sensitivity minimisation in Equation (17) with the weighting function,

$$W_1(s) = \frac{s+1000}{(s+0.001)(s+10)}.$$

The resultant design filter $C(s)$ is given by

$$C(s) = \frac{b_7 s^7 + b_6 s^6 + b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^9 + a_8 s^8 + a_7 s^7 + a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0},$$

where $b_7 = -1.0623 \times 10^5$, $b_6 = -2.7720 \times 10^6$, $b_5 = -2.5543 \times 10^7$, $b_4 = -1.0649 \times 10^8$, $b_3 = -2.4352 \times 10^8$, $b_2 = -2.3448 \times 10^8$, $b_1 = -2.0016 \times 10^7$, $b_0 = 1.2863 \times 10^{-3}$, $a_8 = 72.3721$, $a_7 = 2.5189 \times 10^3$, $a_6 = 5.5047 \times 10^4$, $a_5 = 9.2286 \times 10^5$, $a_4 = 9.2912 \times 10^6$, $a_3 = 4.4307 \times 10^7$, $a_2 = 7.5858 \times 10^7$, $a_1 = 7.5199 \times 10^4$ and $a_0 = 0$. The design parameters M, N, L_2, J can be obtained from a state space realisation of $C(s)$.

The time response simulation is performed using a fourth-order Runge–Kutta's algorithm with fixed time step 1.5×10^{-4} second. Figure 3 shows the time response for the proposed robust observer, with the solid line representing the true disturbance d, and the dash line the disturbance estimate $\hat{d}$, which is able to track d accurately. Figure 4 shows the state variable x_4 and its estimate $\hat{x}_4$; it is seen that even though x_4 is exploding, its estimate $\hat{x}_4$ still converges to x_4. Figure 5 shows the exponential decaying state estimation error norm $\|x - \hat{x}\|$ versus time.

As a comparison, a simulation test is also performed for the DOB design using high-order sliding modes (HOSM) (Davila, Fridman, & Levant, 2007). Figure 6 shows the disturbance estimation results, where the solid line represents the unknown disturbance d, and the dash line the disturbance estimate $\hat{d}$. It is seen that the disturbance estimate

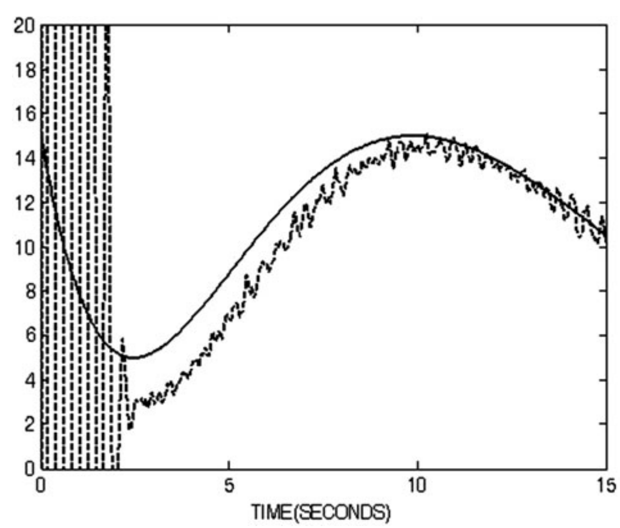

Figure 3. Disturbance d and disturbance estimate $\hat{d}$.

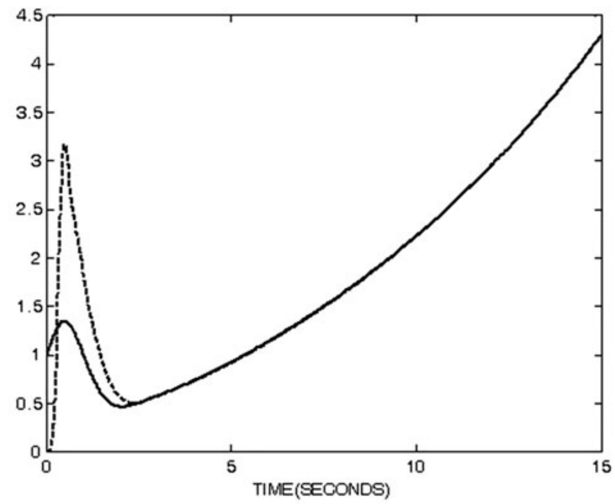

Figure 4. State x_4 and state estimate $\hat{x}_4$.

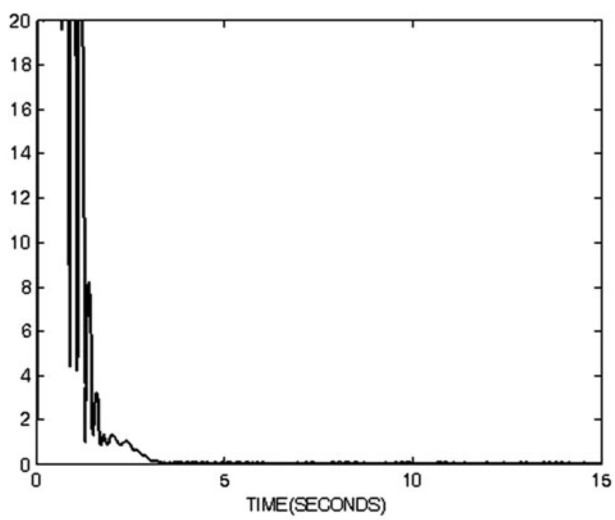

Figure 5. Norm of state estimation error $\|x(t) - \hat{x}(t)\|$.

fails to track the disturbance and explodes to infinity since their design cannot apply to non-minimum-phase systems.

The second example compares the performance of the proposed robust observer with that of the HOSM observer (Davila et al., 2007) for a minimum-phase system.

Example 2: Consider a fourth-order minimum-phase system (1) with

$$A = \begin{bmatrix} -5.9 & -17.4 & -18.2 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 1 \\ 10 \\ 24 \end{bmatrix}^T,$$

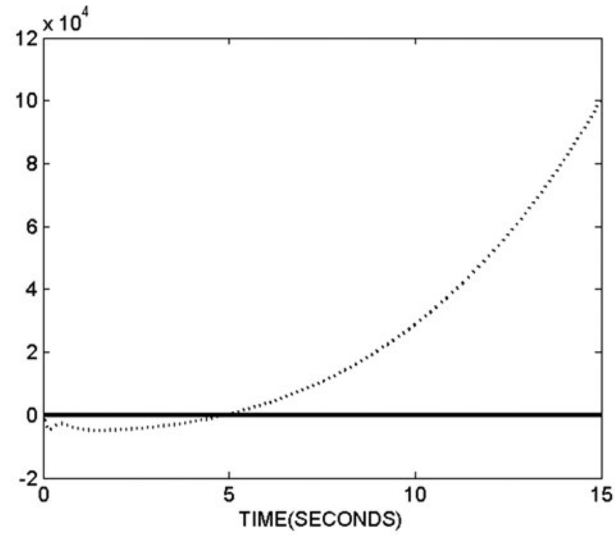

Figure 6. d and $\hat{d}$ of HOSM observer.

with the unknown disturbance $d(t) = 10 + 5\cos(2\sqrt{(t)})$, the measurement noise $n(t)$ is also a white stochastic noise with Gaussian distribution, zero mean and standard deviation 0.2 and the initial condition $x(0) = [1, 1, 1, 1]^T$. The transfer function $P(s) = C(sI - A)^{-1}G$ is

$$P(s) = \frac{(s+4)(s+6)}{(s-0.1)(s+2)(s^2+4s+10)}.$$

Note that this transfer function has only stable zeros, and hence is minimum phase. The HOSM observer (Davila et al., 2007) is, therefore, applicable. The control input is a simple step function $u(t) = 1$, and the proposed robust disturbance in this paper is found with the weighting functions,

$$W_1 = \left[\frac{s+1000}{(s+0.001)(s+10)}\right]^2,$$
$$W_2 = \frac{1.3\times10^5 s + 5.2\times10^6}{s + 2\times10^6},$$

in Equation (16). The resultant filter $C(s)$ is given by

$$C(s) = \frac{b_8 s^8 + b_7 s^7 + b_6 s^6 + b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s^1 + b_0}{s^9 + a_8 s^8 + a_7 s^7 + a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0},$$

where $b_8 = 534.7$, $b_7 = 1.069 \times 10^9$, $b_6 = 3.193 \times 10^{10}$, $b_5 = 3.654 \times 10^{11}$, $b_4 = 2.104 \times 10^{12}$, $b_3 = 6.817 \times 10^{12}$, $b_2 = 1.249 \times 10^{13}$, $b_1 = 1.009 \times 10^{13}$, $b_0 = 8.913 \times 10^{11}$, $a_8 = 2.131 \times 10^5$, $a_7 = 1.784 \times 10^7$, $a_6 = 5.545 \times 10^8$, $a_5 = 8.307 \times 10^9$, $a_4 = 6.4 \times 10^{10}$, $a_3 = 2.416 \times 10^{11}$, $a_2 = 3.496 \times 10^{11}$, $a_1 = 6.984 \times 10^8$ and $a_0 = 3.491 \times 10^5$. Figure 7 shows the state estimation error norm $\|x - \tilde{x}\|$ of the proposed robust observer,

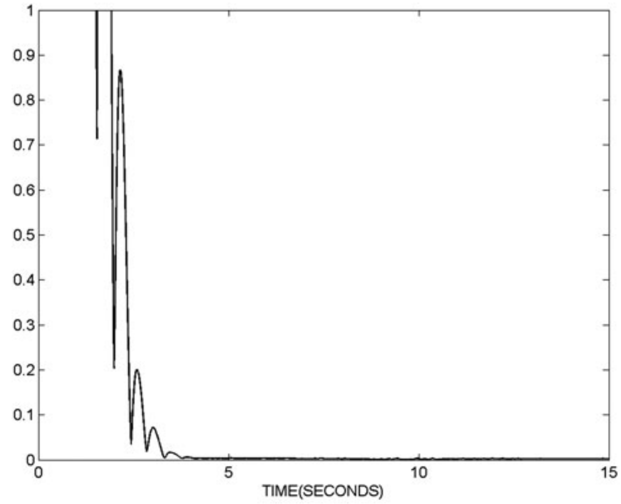

Figure 7. State estimation error norm of proposed observer.

and Figure 8 shows the unknown input $d(t)$ (solid line) and its estimate $\hat{d}(t)$ (dashed line). To make a comparison, one tests the HOSM observer (Davila et al., 2007) with design parameters $\alpha_1 = 1.1$, $\alpha_2 = 1.5$, $\alpha_3 = 2$, $M = 3$, the observer gain $L = [-18.39, 6.1, 1, 0]^T$ and the transformation matrix,

$$T = \begin{bmatrix} 0 & 1 & 16 & 60 \\ 1 & -6.1 & -161 & -530.4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Figure 9 shows the state estimation error norm $\|x - \tilde{x}\|$ of the HOSM observer, and Figure 10 depicts the unknown input $d(t)$ (solid line) and its reconstruction $\hat{d}(t)$ (dashed line).

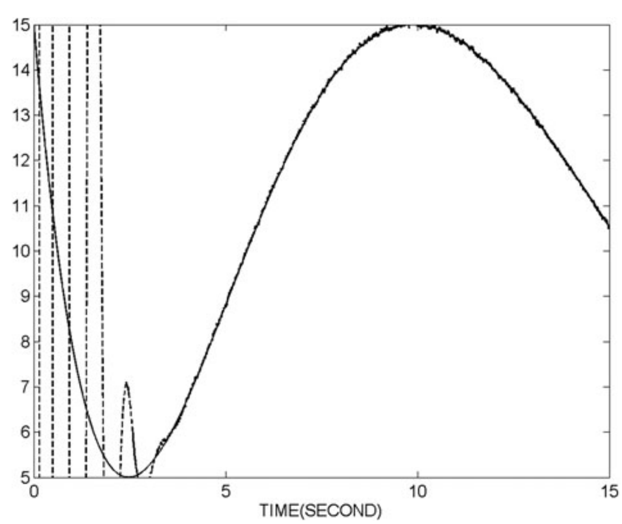

Figure 8. d and $\hat{d}$ of proposed observer.

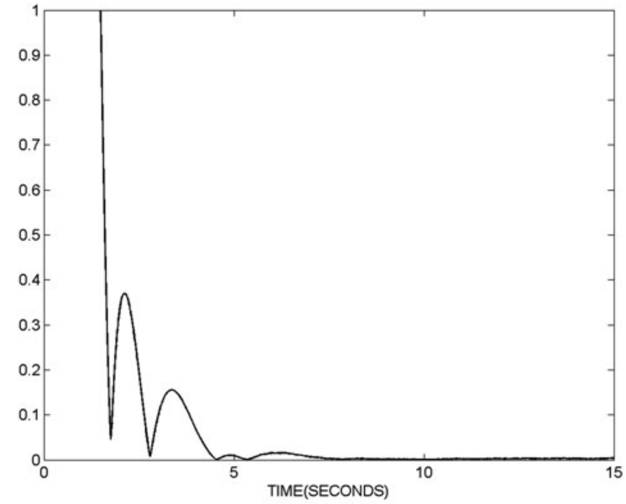

Figure 9. State estimation error norm of HOSM observer.

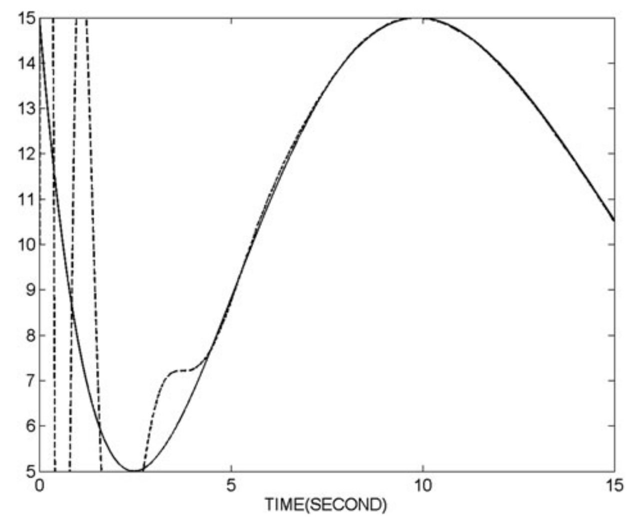

Figure 10. d and $\hat{d}$ of HOSM observer.

After comparison of the above figures, it is clear that the two observers, the robust observer proposed in this paper and the HOSM observer, have similar performance both in tracking ability and in noise sensitivity.

4. Conclusions

This paper proposes a new robust observer for linear dynamic systems subject to external disturbances. The proposed robust observer is a joint state and disturbance observer, since it estimates not only system state but also unknown disturbances acting on the system. The contribution of this paper is a new structure for robust observers. With this new structure, the robust observer design problem is skillfully transformed into a disturbance rejection control problem. The H_∞ frequency shaping control design can then be easily applied to shape the proposed robust observer in the frequency domain. Such a frequency shaping design has many advantages. First, the new design applies to estimation of the most general class of disturbances as long as the disturbance's Laplace transform exists. Second, the proposed robust observer design applies to minimum phase as well as non-minimum-phase systems. Third, the new design applies to both continuous-time systems and discrete-time systems. Finally, the new design takes into account not only the estimation tracking ability, but also sensitivity with respect to measurement noise.

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