Disturbance Observers for Rigid Mechanical Systems: Equivalence, Stability, and Design

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Mechanical (direct-drive) systems designed for high-speed and high-accuracy applications require control systems that eliminate the influence of disturbances like cogging forces and friction. One way to achieve additional disturbance rejection is to extend the usual (P(I)D) controller with a disturbance observer. There are two distinct ways to design, represent, and implement a disturbance observer, but in this paper it is shown that the one is a generalization of the other. A general systematic design procedure for disturbance observers that incorporates stability requirements is given. Furthermore, it is shown that a disturbance observer can be transformed into a classical feedback structure, enabling numerous well-known tools to be used for the design and analysis of disturbance observers. Using this feedback interpretation of disturbance observers, it will be shown that a disturbance observer based robot tracking controller can be constructed that is equivalent to a passivity based controller. By this equivalence not only stability proofs of the disturbance observer based controller are obtained, but it also provides more transparent controller parameter selection rules for the passivity based controller.
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1 Introduction

After the introduction of the disturbance observer as an Unknown Input (state space) observer [1], it has been used and analyzed for linear systems by several researchers [2,3]. The key idea of a disturbance observer is to augment the plant with a fictitious autonomous dynamical system that generates the disturbances. An observer is then constructed to estimate the states of both the plant and the disturbance generator. By using the reconstructed disturbance state for feedback, disturbance rejection is accomplished. As the feedback signal is only an estimate of the actual disturbance acting on the system, the disturbance observer does not *control* the plant, i.e., a normal feedback controller is still needed to achieve performance. The benefit of the disturbance observer is that it adds disturbance rejection to the nominal feedback controller without effecting the performance. This separation property enables two independent design stages for the overall controller design: one for performance and one for disturbance rejection.

Several years after the introduction of the disturbance observer concept, the two-degree-of-freedom (2-dof) controller was described [4–8]. In the 2-dof controller configuration, 1 dof is used to design the command input response, while the other is designed to obtain disturbance rejection. In this paper, it is shown that the disturbance rejection properties of the 2-dof controller is equivalent to the previously known disturbance observer of Johnson [1], for a linear plant and for some specific design choices. By virtue of the equivalence, we will denote the disturbance rejection part of the 2-dof structure again a disturbance observer. To distinguish the two design methodologies, we denote the original disturbance observer as introduced by Johnson an Unknown Input Disturbance Observer (UIDO), and the one resulting from the 2-dof controller structure as a Disturbance Estimating Filter (DEF). Note that, as a result of the equivalence, the terms UIDO and DEF only refer to the *design methodology* and/or the *representation*, and not to the disturbance observer itself.

As the 2-dof structure is more general than the original disturbance observer as introduced by Johnson, we will mainly focus on the 2-dof structure in this paper. However, because of their equivalence, some properties known for the UIDO can be transformed to the DEF, and vice versa. Furthermore, by closely examining the equivalence, a new structure for the design of the low-pass filter in the DEF design is derived, which enables the use of the disturbance observer for non-minimum-phase plants.

Although the disturbance observer was originally designed to obtain additional disturbance rejection for linear systems, some researchers also included the disturbance observer to make the controller more robust (e.g., [9]). The disturbance observer makes the real plant behave like the nominal plant model for a certain frequency range, thereby simplifying the design of the feedback and feedforward controllers. However, it has recently been shown that only a carefully designed disturbance observer enlarges the stability margin [3,10–12]. A systematic design procedure that incorporates these stability requirements, is presented in this paper (Section 2.4).

The stability properties of disturbance observer applied to nonlinear plants are not shown rigorously yet, except for rigid robot manipulators [8,13]. In these disturbance observer based robot controller designs, the robot is treated as a set of linear decoupled moving mass systems, plus disturbances acting on these separate linear systems representing the nonlinearities, coupling terms, and any other (unmodeled, but bounded) effects. In [8], it is shown that this type of robot controller results in the same error dynamics as with a specific Passivity-Based controller, thereby establishing stability of the error dynamics for the disturbance observer based approach to robot tracking control. By comparing the resulting error dynamics, however, no insight is gained in how the similarity is achieved. Besides, it only shows that the error dy-

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namics are equivalent. In contrast, this paper will show that the disturbance observer based controller and the passivity based controller themselves are equivalent. To accomplish this equivalence, it will be shown (Section 2.5) that the disturbance observer can be transformed into a classical feedback structure. This implies that basically any feedback controller design tool can be used to construct a disturbance observer. Furthermore, the resulting feedback representation enables a structural analysis of disturbance observers applied to nonlinear systems. This will be illustrated in Section 4, where a disturbance observer based robot tracking controller is designed, based on intuitive arguments. Using the feedback interpretation of the disturbance observer, it will be shown that the resulting controller is equivalent to the passivity based controller as used in [8] (Section 4.4).

The outline of the paper is as follows: First, the DEF disturbance observer structure is introduced and analyzed in Section 2. Based on some general and intuitive arguments together with a robustness analysis, this results in a systematic design procedure (Section 2.4). In Section 2.5, it will be shown that the disturbance observer can be restructured in a feedback form, which renders an interesting interpretation of disturbance observers for linear mechanical plants.

Then, in Section 3, it will be shown that for some specific design choices of the DEF design, the DEF and the UIDO are equivalent, implying that the UIDO structure is a subset of the DEF structure. Some consequences of the equivalence are given.

In the last section, we will focus on disturbance observers for mechanical plants specifically. Using the feedback interpretation of the DEF, an intuitively designed disturbance observer based robot tracking controller will be transformed into a Passivity based controller (Section 4.1). By this equivalence, stability of the disturbance observer based tracking controller can be guaranteed.

2 Disturbance Estimating Filter

The DEF disturbance observer structure that originated from the 2-dof control structure is depicted in Fig. 1 [4–6,8]. The disturbance observer is based on using the inverse nominal plant model $P_n^{-1}(s)$ to estimate u_0. By directly feeding back the difference between $\hat{u}_0$ and u, which is thus an estimate of the disturbance d, disturbance rejection is accomplished. However, direct feedback of $\hat{d}$ cannot be realized, as in general the inverse plant model is non-proper. Besides, direct feedback would also result in an algebraic loop. Therefore, the filter $Q(s)$ is introduced. The design of the disturbance observer is now reduced to the design of the filter $Q(s)$.

This section will continue with a derivation of some general properties of the DEF, resulting in specific requirements on $Q(s)$ (Section 2.1). The sensitivity and complementary sensitivity functions realized by the DEF are described in Section 2.2. Then, a specific structure for $Q(s)$ is selected in Section 2.3. The derived requirements on Q are summarized in a systematic design procedure (Section 2.4). Finally, an interesting feedback interpretation of disturbance observers is presented in Section 2.5.

2.1 Analysis. The transfer functions realized by the (SISO) disturbance observer of Fig. 1 are [8]:

$$H_{u_a y}=\frac{y}{u_a}=\frac{PP_n}{Q(P-P_n)+P_n} \tag{1}$$

$$H_{dy}=\frac{y}{d}=\frac{PP_n(1-Q)}{Q(P-P_n)+P_n} \tag{2}$$

$$H_{\xi y}=\frac{y}{\xi}=\frac{PQ}{Q(P-P_n)+P_n} \tag{3}$$

(d is the disturbance input, ξ is measurement noise, and u_a is the new control input). Assume that the nominal model of the plant is correct (i.e., $P=P_n$). Then the transfer-functions simplify to:

$$H_{u_a y}=P \tag{4}$$

$$H_{dy}=P(1-Q) \tag{5}$$

$$H_{\xi y}=Q \tag{6}$$

Because $H_{u_a y}$ is equal to P, the feedback controller that is used in conjunction with the disturbance observer (see Fig. 2) does not notice the presence of the disturbance observer. So, the disturbance observer and the feedback controller can be designed independently (which is known as the separation property) as long as the nominal model is correct.

As mentioned, the design of the disturbance observer comes down to the design of the filter $Q(s)$. From an engineering point of view, some requirements on $Q(s)$ can be derived. The requirements are:

1. *Relative degree*—In order to enable practical implementation of the disturbance observer, $Q(s)$ should have a relative degree ρ_q larger than or equal to the relative degree ρ_p of the nominal plant model.
2. *Global shape*—As disturbances should be rejected as much as possible, from the transfer function H_{dy} (5), it can be concluded that $Q(s)$ should be unity. However, as a result of the requirement on the relative degree, this will in general only be possible for a particular frequency range. Besides, from the transfer function $H_{\xi y}$ (6), it is clear that $Q(s)$ should go to zero for high frequencies in order to reject measurement noise. As a result, $Q(s)$ should be a low-pass filter.
3. *Peaking*—From H_{dy} (5), is can be concluded that the maximum amplification of $1-Q(s)$ should be small for all frequencies. However, from $H_{\xi y}$ (6), it follows that also $Q(s)$ should be small. This is a conflicting requirement.

2.1.1 Model Misfits. Now suppose the filter $Q(s)$ has been designed as a low-pass filter and applies to all the requirements derived before. The properties of the DEF can then be determined for the case that the nominal plant model does not match the actual plant. First, consider high frequencies. For high frequencies $Q(s)\approx 0$, resulting in:

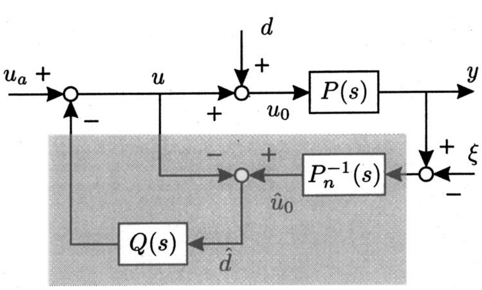

Fig. 1 Disturbance estimating filter

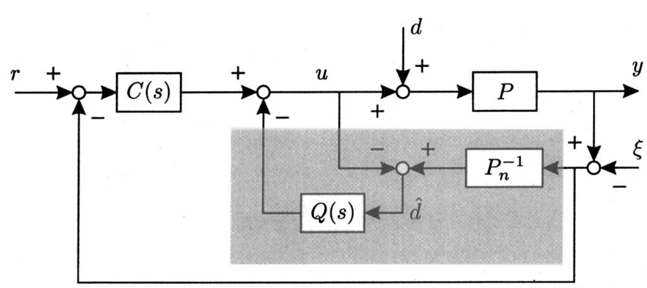

Fig. 2 Disturbance observer combined with a controller

$$H_{ry}=\frac{PP_n}{P_n}=P \tag{7}$$

$$H_{dy}=\frac{PP_n(1-Q)}{P_n}=P \tag{8}$$

$$H_{\xi y}=\frac{PQ}{P_n}=0 \tag{9}$$

Clearly, for high frequencies the system behaves like the plant without DEF. For low frequencies, $Q(s)\approx 1$, so:

$$H_{ry}=\frac{PP_n}{P}=P_n \tag{10}$$

$$H_{dy}=\frac{PP_n(1-Q)}{P}=0 \tag{11}$$

$$H_{\xi y}=\frac{-P}{P}=1 \tag{12}$$

It follows that for low frequencies, the resulting system behaves like the nominal plant. Robustness for parameter uncertainty is therefore guaranteed [5,6]. However, for this to be true, the DEF should result in robust stable dynamics. The robustness properties are further analyzed in the next section.

2.2 Sensitivity Analysis. To continue with a more structural analysis of the DEF, notice that the disturbance observer can be written in the form of Fig. 3. Assume the plant is SISO. From this figure, the sensitivity-functions $T=L/(1+L)$ and $S=1/(1+L)$ (where L is the loop gain) of the disturbance observer are easily derived as [3,13]:

$$T=\frac{QP}{Q(P-P_n)+P_n} \tag{13}$$

$$S=\frac{P_n(1-Q)}{Q(P-P_n)+P_n} \tag{14}$$

This results in $T=Q$ and $S=(1-Q)$ for $P=P_n$ which shows the importance of the filter $Q(s)$ in the design of the disturbance observer. Using Eqs. (13) and (14), the requirements derived before ($\|1-Q(s)\|_\infty$ and $\|Q(s)\|_\infty$ small) can simply be restated as requiring limited peaking of the sensitivity and complementary sensitivity function (Small Gain Theorem [14]) [15].

When a controller $C(s)$ is applied (see Fig. 2), the realized transfer functions are given by (assume $P=P_n$):

$$H_{ry}=\frac{PC}{1+PC}=T_c \tag{15}$$

$$H_{dy}=\frac{P(1-Q)}{1+PC}=PS_c(1-Q)=P\cdot S \tag{16}$$

$$H_{\xi y}=\frac{PC+Q}{1+PC}=T_c+\frac{Q}{1+PC}=T \tag{17}$$

where $S_c=1/(1+PC)$ and $T_c=PC/(1+PC)$ are the sensitivity and complementary sensitivity functions of the controlled system without DEF. From the transfer functions (16) and (17), it is clear that the peaking in $1-Q(s)$ and $Q(s)$ should not coincide with the peaking of the sensitivity (S_c) and complementary sensitivity functions (T_c) which would have been realized by the controller without the DEF, to limit the peaking of the overall sensitivity functions. As a result, the DEF and the controller $C(s)$ cannot be designed independently!

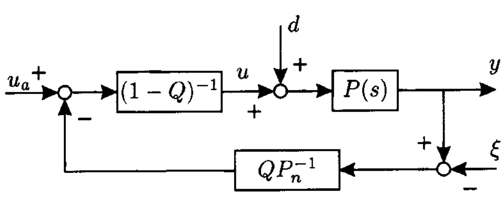

Fig. 3 Equivalent form of the disturbance observer

2.3 Low-Pass Filter *Q*. Despite the fact that $Q(s)$ should adhere to some basic requirements as derived before, there is still a fair amount of freedom to select $Q(s)$. This freedom can be used to include a specific disturbance model in the disturbance observer design. To illustrate this, suppose the disturbances to be rejected can be described by $1/s^{n_i}$. The Internal Model Principle [16] then prescribes that the controller should also include a factor $1/s^{n_i}$, i.e., it should include n_i integrating actions. For a specific choice of $Q(s)$, the disturbance observer indeed implements n_i-integrating actions. To achieve this, $Q(s)$ can be selected according to the following structure [5,8,13]:

$$Q(s)=\frac{1+\sum_{m=1}^{n_q-\rho_q}f_m s^m}{1+\sum_{m=1}^{n_q}f_m s^m} \tag{18}$$

where n_q is the order of $Q(s)$ and ρ_q is the relative degree of $Q(s)$. As a result of this specific choice for $Q(s)$, $1/(1-Q)$ (see Fig. 3) can be written as:

$$\frac{1}{1-Q(s)}=\frac{1+\sum_{m=1}^{n_q}a_m s^m}{\sum_{m=n_q-\rho_q+1}^{n_q}a_m s^m}=\frac{1+\sum_{m=1}^{n_q}a_m s^m}{s^{n_i}(a_{n_i}+\sum_{m=1}^{\rho_q-1}a_{n_i+m}s^m)} \tag{19}$$

where $n_i=n_q-\rho_q+1$. It follows that the disturbance observer implements $n_q-\rho_q+1$ pure integrators. As a result of the selected structure for $Q(s)$, only two of the three parameters n_q, ρ_q and n_i can be selected freely.

Although this structure (Eq. (18)) is presented by many researchers [5,8,13], in here we present a new structure. This new structure has the benefit that it enables the application of the DEF to non-minimum-phase plants. Assume that the nominal, linear, stable, SISO plant model is proper (relative degree $\rho_p \geqslant 0$):

$$P_n(s)=\frac{N(s)}{D(s)}=\frac{N_u(s)N_s(s)}{D(s)} \tag{20}$$

where n_p is the order of $P_n(s)$ and n_n the order of the polynomial $N(s)$. The relative order of the plant is $\rho_p=n_p-n_n\geqslant 0$. The numerator polynomial is divided into two sections N_u (of order n_u) and N_s. The only requirement on this separation is that the polynomial N_s is stable. The new structure for the $Q(s)$ filter is now given by:

$$Q(s)=\frac{1+\sum_{m=1}^{n_i-1}f_m s^m+\sum_{m=1}^{n_u}g_m s^{n_i+m-1}}{1+\sum_{m=1}^{n_q}f_m s^m} \tag{21}$$

In this new structure, the coefficients $g_1,\ldots,g_{n_n}$ are selected such that the numerator polynomial of Q can be written as $N_u(s)\cdot(r_{n_i-1}s^{n_i-1}+\ldots+r_1 s+r_0)$ for some $r_0,\ldots,r_{n_i-1}$.

It is straightforward to verify that this new structure complies with all requirements derived so far, including the requirement that $1/(1-Q)$ should include a model of the disturbance generator. By using this new structure, the disturbance observer structure of Fig. 3 is implementable even for non-minimum-phase plants, as the unstable poles of P_n^{-1} are cancelled by the right-half-plane zeros of $Q(s)$.

Note that the order of the disturbance model n_i realized by the DEF is now given by $n_i=n_q-\rho_q-n_u+1$. Note furthermore that for minimum-phase plants, N_u can be selected equal to 1 which renders the new structure (Eq. (21)) equal to the old structure (Eq. (18)).

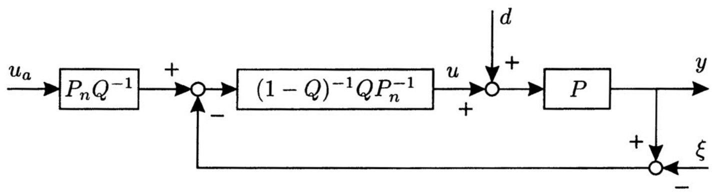

Fig. 4 Feedback interpretation of the disturbance observer

2.4 Design Procedure. In this section, an overview is given of choices to be made to design a disturbance observer. The consequences of every step are discussed.

1. Determine the nominal plant. It is stated in many papers that only $Q(s)$ is a design entity. However, from (10), it follows that for low frequencies the plant is transformed into the nominal plant, and it might therefore be interesting to select P_n deliberately different from P. From (10), one could conclude that P_n should be selected as simple as possible, to facilitate the feedback controller design. However, for $P \neq P_n$, stability and performance might be in danger. After selecting $P_n = N/D$, the numerator polynomial $N(s)$ has to be divided into two parts N_u and N_s (see Eq. (20)). The only requirement on this separation is that the polynomial N_s is stable. To minimize the order of the final $Q(s)$ filter, the order n_u of N_u should be minimal (i.e., if the plant has no right half plane zeros, N_u should be selected as a static gain, resulting in $n_u = 0$).
2. Select the relative degree $\rho_q \geqslant \rho_p$ of the filter $Q(s)$. For $\rho_q = \rho_p + q$, there is $q \times 20$ dB/decade high-frequency roll-off in the transfer function from measurement noise ξ to the control action u. More roll-off might be needed for robustness. However, excessive roll-off will result in peaking of the sensitivity and complementary sensitivity functions. Furthermore, the additional roll-off is only effective when the feedback controller includes the same amount of roll-off (see Eq. (17)).
3. Determine the desired number of integrating actions n_i. For example the desired shape of the sensitivity function (14) might be a selection criterion. More integrating actions improve the disturbance rejection for low frequencies, but results also in more peaking of the sensitivity and complementary sensitivity functions. The order of $Q(s)$ can now be determined:
$$n_q = n_i + \rho_q + n_u - 1 \tag{22}$$
4. Select the cut-off frequency of $Q(s)$. This will depend on, among others, the amount of measurement noise and disturbances present in the system, the amount of model uncertainty, and the desired bandwidth of the overall system when a controller $C(s)$ is applied, see Fig. 2. As already stated in Section 2.2, the cut-off frequency should be selected such that the peaks in $Q(s)$ and $1 - Q(s)$ do not coincide with the peaks of the sensitivity (S_c) and complementary sensitivity functions (T_c) which would have been realized by the controller without the DEF. From this step onwards, the design of the DEF requires knowledge of the controller $C(s)$.
5. The next step is to determine the actual filter $Q(s)$, using the structure of Eq. (18). Several authors [5,6,8] suggest a Butterworth or binomial model to be used to select the pole locations (i.e., de coefficients f_m, $m \in [1, \ldots, n_q]$). The location of the zeros then follows from (21). Afterwards, it should be checked that $\|Q(s)\|_\infty$ and $\|1 - Q(s)\|_\infty$ are small. Further research has to be done to derive methods to select $Q(s)$ according to the structure (21) while minimizing $\|Q(s)\|_\infty$ and $\|1 - Q(s)\|_\infty$.
6. The final step is checking the robustness of the system, for example by employing the small-gain-theorem. If the system is not robust, the robustness can be increased by reconsidering the design choices made. Once robust stability of the DEF is achieved, robust stability of the DEF including the feedback controller $C(s)$ should be checked (see Eq. (17)) for the definition of the complementary sensitivity function of the closed-loop system).

2.5 Feedback Interpretation. In the previous sections, the disturbance observer was described for a general linear plant P. We now confine ourselves to linear *mechanical* plants, to simplify the analysis and to illustrate the disturbance observer design on a more practical level. Finally, we will show that for specific plant and disturbance model combinations (in this case rigid mechanical plants and $1/s^k$ disturbance models), the disturbance observer reduces to a simple feedback controller.

Consider a rigid moving mass system (input is force, output is the velocity), for which the linear nominal plant model is given as:

$$P_n(s) = \frac{1}{ms} \tag{23}$$

For most mechanical applications, at least one integrating action is required to reduce the influence of friction. This integrating action will be implemented by a disturbance observer. Using the analysis of the first part of this section, it results that the $1/(1-Q)$ block of the disturbance observer in Fig. 3 should contain a factor $1/s$. The most simple Q filter that accomplishes this, is [5,8,13]:

$$Q(s) = \frac{1}{s\tau + 1} \tag{24}$$

where τ is the only design parameter. When we now calculate the block QP_n^{-1} in the feedback path of Fig. 3:

$$QP_n^{-1} = \frac{ms}{s\tau + 1} \tag{25}$$

we notice that this block contains the factor s in the numerator, which will be cancelled by the integrator of the $1/(1-Q)$ block when the loop-gain is computed. As these two factors will not be effective in the loop (as they cancel each other), it is perhaps more convenient to restructure Fig. 3 as is Fig. 4 (this is only possible if Q was designed with the same relative degree as P_n, which is actually the case in this development). For the system defined so far, the resulting feedback controller block of Fig. 4 evaluates to:

$$(1 - Q(s))^{-1} Q(s) P_n^{-1}(s) = \frac{\tau s + 1}{\tau s} \frac{ms}{\tau s + 1} = \frac{m}{\tau} \tag{26}$$

Notice that this is simply a static gain. In conclusion, in this specific case, we could have designed the disturbance observer by simply designing a proportional controller. Only the pre-filter $P_n Q^{-1}$ remains to be designed. The function of this pre-filter becomes clear after computing the transfer function of the internal loop of the disturbance observer (see Fig. 4), assuming that $P = P_n$:

$$H=\frac{\frac{PQ}{P_n(1-Q)}}{1+\frac{PQ}{P_n(1-Q)}}=\frac{Q}{1-Q+Q}=Q \tag{27}$$

Note from this equation that the pre-filter block P_nQ^{-1} cancels the closed loop dynamics of Eq. (27), and replaces it by the nominal plant model. It is tempting to omit this pre-filter, and consider the closed loop dynamics of Eq. (27) as the new plant for which a controller (see Fig. 2) should be designed to achieve the desired overall performance (e.g., tracking of a reference signal). However, that would also cancel the desired disturbance rejection, as the loop no longer contains the disturbance generator dynamics (Internal Model Principle [16]).

Remark: A similar result can be obtained if not the velocity of the mass but its position was taken as output, combined with a second order disturbance observer, i.e.:

$$P_n=\frac{1}{ms^2} \tag{28}$$

$$Q=\frac{2\tau s+1}{(\tau s)^3+2(\tau s)^2+2\tau s+1} \tag{29}$$

$$(1-Q)^{-1}QP_n^{-1}=m\frac{2\tau s+1}{\tau^3 s+2\tau^2} \tag{30}$$

The resulting feedback controller is in that case a lead-lag compensator.

3 Equivalence

This section will show that the DEF disturbance observer structure as described in the previous section, is a generalization of the Unknown Input Disturbance Observer (UIDO). The section is organized as follows. First, the UIDO structure as introduced by Johnson [1], is described (Section 3.1). Then the UIDO structure for a linear SISO plant is transformed from a state-space description to a set of transfer-functions (Section 3.2). This step is performed to enable comparison of the UIDO with the DEF (Section 3.3). Finally, in the last section (Section 3.4), some properties known for the UIDO will be applied on the DEF design and vise versa.

3.1 Unknown Input Disturbance Observer. The disturbance observer structure of Fig. 5, referred to as an Unknown Input Disturbance Observer (UIDO) [2,3,17], originated from an Unknown Input Observer [18]. For the construction of the disturbance observer, it is assumed that the disturbance signal $d(t)$ can be thought of as being generated by a fictitious autonomous dynamic system of order k:

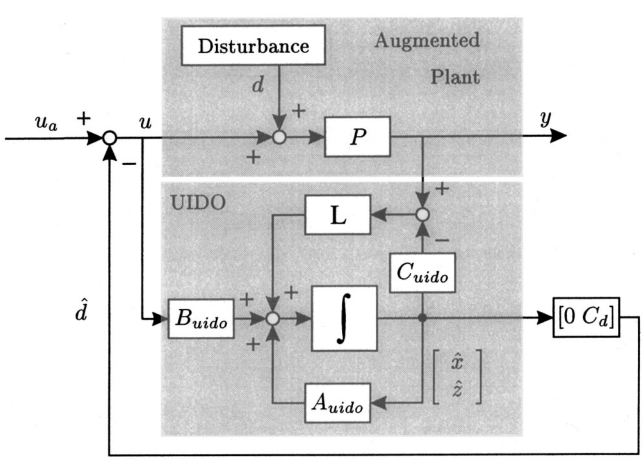

Fig. 5 Unknown input disturbance observer (UIDO)

$$\dot{z}=A_dz \tag{31}$$

$$d=C_dz \tag{32}$$

As an example, assume the disturbances are piece-wise constant in time. The Laplace transform of $d(t)$ is then given by $d(s)=1/s$, resulting in $A_d=0$ and $C_d=1$. This fictitious disturbance generator can be included in the plant, resulting in the *augmented plant*. Where the original plant $P(s)$ has two inputs (one known input $u(t)$, and one unknown disturbance input $d(t)$), the augmented plant has only one input $u(t)$. And under some mild conditions [3], an observer can be constructed which estimates not only the states of the plant, but also the states z of the fictitious disturbance generator. Using this estimated $\hat{z}$, an estimate of the disturbance can be constructed and used for feedback.

3.1.1 Construction of the UIDO. As indicated, the UIDO design method starts with the construction of the augmented plant. Assume that the nominal, linear, stable, SISO plant model is strictly proper (relative degree $\rho_p=n_p-n_n>0$) and given by:

$$P_n(s)=\frac{N(s)}{D(s)}=\frac{b_{n_n}s^{n_n}+b_{n_n-1}s^{n_n-1}+\ldots+b_0}{s^{n_p}+a_{n_p-1}s^{n_p-1}+\ldots+a_0} \tag{33}$$

As the plant model is strictly proper, the plant model can be written in observer canonical form [19]:

$$\dot{x}=Ax+Bu \tag{34}$$

$$y=Cx \tag{35}$$

with

$$A=\begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_{n_p-1} \end{bmatrix}_{[n_p\times n_p]} \tag{36}$$

$$B=\begin{bmatrix} b_0 \\ \vdots \\ b_{n_p-1} \end{bmatrix}_{[n_p\times 1]} \tag{37}$$

$$C=[0\ \cdots\ 1]_{[1\times n_p]} \tag{38}$$

Write the fictitious disturbance generator model (of order k) also in observer canonical form (see (31) and (32)):

$$A_d=\begin{bmatrix} 0 & 0 & \cdots & 0 & -q_0 \\ 1 & 0 & \cdots & 0 & -q_1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -q_{k-1} \end{bmatrix}_{[k\times k]} \tag{39}$$

$$C_d=[0\ \cdots\ 0\ \ 1]_{[1\times k]} \tag{40}$$

Combining the plant with the disturbance generator, the augmented plant (Fig. 5) is constructed, resulting in the state-space matrices:

$$A_{uido}=\begin{bmatrix} A & BC_d \\ 0 & A_d \end{bmatrix} \tag{41}$$

$$B_{uido}=\begin{bmatrix} B \\ 0 \end{bmatrix} \tag{42}$$

$$C_{uido}=[C\ \ 0] \tag{43}$$

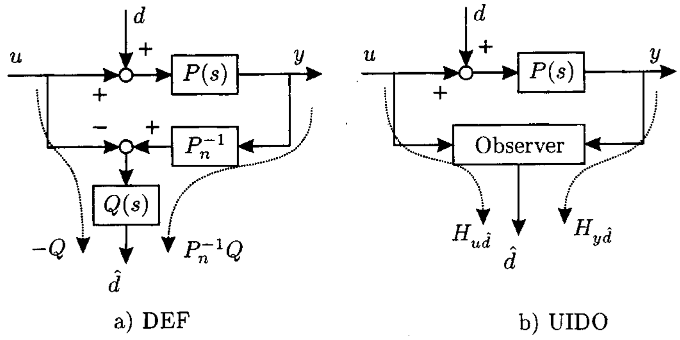

Fig. 6 Analogy of DEF and UIDO

Hostetter [18] and Benschop [20] proved that, if the pair $\{A,C\}$ and the pair $\{A_d, C_d\}$ are both observable, and no eigenvalues of A_d coincide with a zero of the plant (no pole-zero cancellation), then also the augmented plant is observable. An observer which estimates both the state x of the original plant and the state z of the disturbance generator can then be constructed. From the estimated state $\hat{z}$, an estimate of the disturbance $\hat{d}$ is available and can be used to compensate for the input disturbance (see Fig. 5).

Comparing the UIDO with the DEF, the UIDO has apparently some advantages:

1. The UIDO method can incorporate disturbances other than $1/s^k$ transparently.
2. Apart from estimating the disturbance states, the UIDO also estimates the system states. Instead of using a feedback controller to stabilize and control the plant, state-feedback is now applicable without an auxiliary observer.

3.2 Transfer functions of the UIDO. The UIDO structure is normally written in a state-space representation. This state-space representation will be transformed to a set of transfer functions to enable comparison of the UIDO method with the DEF method. Only SISO plants are considered. The transfer functions of interest, realized by the UIDO, are illustrated in Fig. 6*b* and are given by:

$$H_{y\hat{d}} = K_{uido}(sI - A_{uido} + LC_{uido})^{-1}Ly \tag{44}$$

$$H_{u\hat{d}} = K_{uido}(sI - A_{uido} + LC_{uido})^{-1}B_{uido}y \tag{45}$$

where

$$K_{uido} = [0 \quad C_d] \tag{46}$$

$$L = \begin{bmatrix} L_p \\ L_k \end{bmatrix} \tag{47}$$

$$L_p = [l_1 \ \cdots \ l_{n_p}]^T \tag{48}$$

$$L_k = [l_{n_p+1} \ \cdots \ l_{n_p+k}]^T \tag{49}$$

The poles of both transfer functions (44) and (45) are given by:

$$\det(sI - A_{uido} + LC_{uido}) = 0 \tag{50}$$

which, by construction of L, are all stable and specified for example by a binomial or Butterworth model, or resulted from a LQG design. Determining the poles, which is straightforward but laborious, gives:

$$\det(sI - A_{uido} + LC_{uido}) = \det\left[\begin{array}{ccccc|cccc} s & 0 & \cdots & 0 & l_1 + a_0 & 0 & \cdots & 0 & -b_0 \\ -1 & s & \cdots & 0 & l_2 + a_1 & 0 & \cdots & 0 & -b_1 \\ 0 & -1 & \cdots & 0 & l_3 + a_2 & 0 & \cdots & 0 & -b_2 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & s + l_{n_p} + a_{n_p-1} & 0 & \cdots & 0 & -b_{n_p-1} \\ \hline 0 & 0 & \cdots & 0 & l_{n_p+1} & s & \cdots & 0 & q_0 \\ 0 & 0 & \cdots & 0 & l_{n_p+2} & -1 & \cdots & 0 & q_1 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & l_{n_p+k} & 0 & \cdots & -1 & s + q_{k-1} \end{array}\right]$$
$$= \det\begin{bmatrix} \det(sI - A + L_pC) & -N(s) \\ l_{n_p+k}s^{k-1} + \cdots + l_{n_p+1} & \det(sI - A_d) \end{bmatrix} = \det(sI - A + L_pC)\cdot\det(sI - A_d) + N(s)\cdot\Xi(s) \tag{51}$$

where $\Xi(s)=l_{n_p+k}\det(sI-\Phi)$ and:

$$\Phi=\begin{bmatrix} 0 & \cdots & 0 & -\frac{l_{n_p+1}}{l_{n_p+k}} \\ 1 & \cdots & 0 & -\frac{l_{n_p+2}}{l_{n_p+k}} \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & 1 & -\frac{l_{n_p+k-1}}{l_{n_p+k}} \end{bmatrix}_{[(k-1)\times(k-1)]} \tag{52}$$

Notice that $\Xi(s)$ is a function of L_k only, and has a degree of $(k-1)$. It also shows that although L_p places the roots of $\det(sI-A+L_pC)$, the combination of both L_p and L_k determines the overall closed loop poles of the disturbance observer.

In order to retrieve the transfer-functions realized by the UIDO, the zeros of (44) and (45) have to be determined also. The zeros of (45) are given by [19]:

$$\det\begin{bmatrix} sI-A_{uido}+LC_{uido} & -B_{uido} \\ K_{uido} & 0 \end{bmatrix}=0 \tag{53}$$

Writing out the left-hand side of this equation results in:

$$\begin{aligned}\det\begin{bmatrix} sI-A_{uido}+LC_{uido} & -B_{uido} \\ K_{uido} & 0 \end{bmatrix} &= -\det\begin{bmatrix} \det(sI-A+L_pC) & -N(s) \\ l_{n_p+k}s^{k-1}+\ldots+l_{n_p+1} & 0 \end{bmatrix} \\ &= -N(s)\cdot\Xi(s)\end{aligned} \tag{54}$$

Equivalently, the zeros of (44) are given by:

$$\begin{aligned}\det\begin{bmatrix} sI-A_{uido}+LC_{uido} & -L \\ K_{uido} & 0 \end{bmatrix} &= \{s^{n_p}+a_{n_p-1}s^{n_p-1}+\ldots+a_0\}\cdot l_{n_p+k}\det(sI-\Phi) \\ &= \det(sI-A)\cdot\Xi(s)\end{aligned} \tag{55}$$

Using the derived formulas for the poles and zeros, the transfer functions (44) and (45) can be rewritten as:

$$H_{y\hat{d}}=\frac{\det(sI-A)\cdot\Xi(s)}{\det(sI-A_{uido}+LC_{uido})} \tag{56}$$

$$H_{u\hat{d}}=\frac{-N(s)\cdot\Xi(s)}{\det(sI-A_{uido}+LC_{uido})} \tag{57}$$

Now comparison of the UIDO with the DEF is possible, as both structures are represented in terms of transfer-functions.

3.3 Equivalence. To show the equivalence between the DEF and the UIDO, the derived transfer functions for the UIDO of the previous subsection are used. Suppose a UIDO has been designed. First it is shown that this disturbance observer can be represented (using the derived transfer functions) in a structure equivalent to a DEF. This equivalent structure has a filter $Q(s)$ that will be referred to as $Q_{UIDO}(s)$, to indicate that this filter resulted from the UIDO design.

As a DEF design is normally implicitly based on a disturbance model $1/s^{n_i}$, assume in the following that the UIDO is also designed for n_i-th order disturbances such that $\det(sI-A_d)=s^{n_i}$ (i.e., $k=n_i$).

Comparing Fig. 6*a* and *b*, it follows that for equivalence, $H_{u\hat{d}}$ should be $-Q$ and $H_{y\hat{d}}$ should equal QP_n^{-1}. It is easily derived, that if $H_{u\hat{d}}$ is equal to $-Q$, that then also $H_{y\hat{d}}$ equals QP_n^{-1}, viz.:

$$\begin{aligned}QP_n^{-1}=-H_{u\hat{d}}P_n^{-1} &= \frac{N(s)\cdot\Xi(s)}{\det(sI-A_{uido}+LC_{uido})}\cdot\frac{D(s)}{N(s)} \\ &= \frac{\det(sI-A)\cdot\Xi(s)}{\det(sI-A_{uido}+LC_{uido})}=H_{y\hat{d}}\end{aligned} \tag{58}$$

So, all that remains to show equivalence of the UIDO and the DEF, is proving that $-H_{u\hat{d}}$ complies to the structure of Q used in a DEF design (Eq. (21)). The transfer function $-H_{u\hat{d}}$ (57), which we will indicate by Q_{UIDO}, can be rewritten using (51):

$$-H_{u\hat{d}}=\frac{N(s)\cdot\Xi(s)}{\det(sI-A+L_pC)s^{n_i}+N(s)\cdot\Xi(s)}\triangleq Q_{UIDO} \tag{59}$$

We used the fact that the UIDO was designed for the disturbance model $1/s^{n_1}$. Note that the order of Q_{UIDO} is n_p+n_i.

We now design a Q filter for a DEF (indicated by Q_{DEF}), according to the structure of Eq. (21) and show that this filter is equivalent to Q_{UIDO} (Eq. (59)). We use the design parameters $n_q=n_p+n_i$ which is obviously needed to obtain equivalence. With this design choice, Q_{DEF} is given by:

$$\begin{aligned}Q_{DEF}(s) &= \frac{1+\sum_{m=1}^{n_i-1}f_ms^m+\sum_{m=1}^{n_u}g_ms^{k+m-1}}{1+\sum_{m=1}^{n_i-1}f_ms^m+(\sum_{m=1}^{n_u}g_ms^{k+m-1}-\sum_{m=1}^{n_u}g_ms^{k+m-1})+\sum_{m=k}^{n_p+n_i}f_ms^m} \\ &= \frac{N_u(s)\cdot(r_{n_i-1}s^{n_i-1}+\ldots+r_1s+r_0)}{\{\sum_{m=1}^{n_p}f_{n_i+m}s^m-\sum_{m=1}^{n_u}g_ms^{m-1}\}s^{n_i}+N_u(s)\cdot(r_{n_i-1}s^{n_i-1}+\ldots+r_1s+r_0)}\end{aligned} \tag{60}$$

To prove that Q_{UIDO} and Q_{DEF} are indeed equivalent, we should keep the following in mind:

1. In the DEF design we can always select $N_u=N$ and $N_s=1$. For this selection, the poles of the transfer function QP_n^{-1} are completely determined by the poles of Q. Furthermore, $n_u=n_n$.
2. $\Xi(s)$ is of order n_i-1, and is a function of the last component of L (i.e., L_k) only.
3. By proper selection of L, we can place the poles of the transfer functions (Eqs. (56) and (57)), i.e., the roots of $\det(sI-A+L_pC)s^{n_i}+N(s)\cdot\Xi(s)$ practically anywhere in the left-half plane.

Comparing Eq. (60) with Eq. (59), it follows that:

$$(r_{n_i-1}s^{n_i-1}+\ldots+r_1s+r_0)=\Xi(s)$$

$$\sum_{m=1}^{n_p} f_{n_i+m}s^m-\sum_{m=1}^{n_n} g_m s^{m-1}=\det(sI-A+L_pC) \tag{61}$$

$$\rho_{uido}=(n_p+n_i)-(n_n+n_i-1)=\rho_p+1$$

This proves that Q_{UIDO} complies with the proposed structure of Eq. (21). In conclusion, given any UIDO design for a disturbance model $1/s^k$, a DEF can always be constructed that results in exactly the same disturbance observer.

3.4 Consequences of the Equivalence. In this subsection, the equivalence of the UIDO and the DEF will be used to link some well known properties and characterizations of the DEF to the UIDO and vice versa.

3.4.1 Relative Degree. Contrary to the DEF design method, where the relative degree ρ_q of the filter $Q(s)$ can be chosen freely (that is, under the constraint $\rho_q \geqslant \rho_p$), the UIDO always implements a filter $Q(s)$ with relative degree $\rho_q=\rho_p+1$. So the UIDO always implements at least 20 dB/decade additional high-frequency roll-off, which might be beneficial for robust stability. However, it might as well be superfluous, resulting in a higher order controller than needed. The fixed relative degree also means that a DEF cannot (in general) be rewritten in a UIDO structure.

3.4.2 Order of the Disturbance Observer. As indicated in the design procedure of Section 2.4, the order of N_u should be selected as small as possible to minimize the order of the resulting Q filter. However, in the UIDO design, there is no design freedom to select N_u, and N_u is implicitly always selected as $N_u=N$. This again shows the more general nature of the DEF. Furthermore, for plants with stable zeros, the UIDO will result in a higher order disturbance observer than the DEF would.

3.4.3 Sensitivity Functions. An important aspect involved in DEF designs are the (complementary) sensitivity functions (see Section 2.2). The shape of these function can be used to asses the robustness of the resulting controller. However, little literature is available that discusses robustness for the UIDO design method (see [3] for one of the exceptions). But because the UIDO can be written in terms of a DEF structure, it follows that the (complementary) sensitivity functions for the UIDO are given by $(1-Q_{UIDO})$ and Q_{UIDO}. This gives a systematic way to analyze the robustness of the UIDO, similar as in the DEF design.

3.4.4 Pole-zero Cancellation. One of the requirements of the UIDO method to succeed, is that the augmented plant model (i.e., plant and disturbance generator) is observable. To the best of our knowledge, similar requirements on pole-zero cancellation have never been reported for a DEF design. But because of the equivalence, this requirement can directly be translated to the DEF design.

3.4.5 Alternative Disturbance Models. When the disturbance to be rejected is not described by $1/s^k$, but more generally described by $1/(s^k+q_{k-1}s^{k-1}+\ldots q_0)$, the UIDO design method can simply be used to arrive at a disturbance observer. On the other hand, the DEF design method is less apparent. In fact, the filter $Q(s)$ has to be selected according to the more general (and, therefore, harder to determine) structure:

$$Q(s)=\frac{1+\Sigma_{m=1}^{n_q-\rho_q} g_m s^m}{1+\Sigma_{m=1}^{n_q} f_m s^m} \tag{62}$$

such that:

1. the numerator of $Q(s)$ contains the unstable transmission zeros $N(s)$ of the plant,
2. $1/(1-Q)$ includes the factor $1/(s^k+q_{k-1}s^{k-1}+\ldots q_0)$.

It is not trivial to select $Q(s)$ such that both requirement are satisfied. Though, as a result of the equivalence between the DEF and the UIDO, it is always possible to construct a $Q(s)$ that fulfills both requirements.

4 Disturbance Observer Based Robot Control

In this section, we consider the design of a tracking controller for rigid robot manipulators, based on a disturbance observer. All individual steps in this design are quite basic and intuitive. However, although the approach is intuitive and basic, it will result in the same controller as the one resulting from the more complex and abstract passivity based approach of Sadegh and Horowitz [21] and Slotine and Li [22].

The intuitive design procedure will go as follows: First, we will design a feedforward control signal to compensate most of the nonlinear robot dynamics, based on estimates of the robot dynamic properties (Section 4.1). Any discrepancies between this estimate and the real dynamics, including any (external and/or unmodeled) disturbances, will be cancelled by a linear disturbance observer (Section 4.2). The final step in the design is the design of the tracking controller (Section 4.3). As the plant is linearized by the disturbance observer and the nonlinear feedforward signal, the tracking controller can be designed for a linear nominal plant model.

Finally, in Section 4.4, it will be shown that the designed tracking controller is indeed the passivity based tracking controller of [21,22].

4.1 Robot Model. The dynamic equations of motion for a rigid robot with n degrees of freedom can be expressed as:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+d=u \tag{63}$$

where q is the $n\times 1$ vector of joint displacements, $M(q)$ the inertial matrix, $C(q,\dot{q})\dot{q}$ the vector of centripetal and Coriolis forces, $G(q)$ the vector of gravitation forces, d external (unmodelled) disturbances, and u the control input. Separating the linear and non-linear parts of Eq. (63) leads to:

$$\bar{M}\ddot{q}+u_d+d=u \tag{64}$$

$$u_d=(M(q)-\bar{M})\ddot{q}+C(q,\dot{q})\dot{q}+G(q) \tag{65}$$

The matrix $\bar{M}=\text{diag}(\bar{m}_1\ldots\bar{m}_n)$ is some constant, positive-definite, diagonal matrix representing the constant portion of the inertia matrix $M(q)$. The signal u_d is the collection of all nonlinear terms, and will be compensated for with a feedforward signal $\hat{u}_d$:

$$\hat{u}_d=(\hat{M}(q)-\bar{M})\ddot{q}_r+\hat{C}(q,\dot{q})\dot{q}_r+\hat{G}(q) \tag{66}$$

In this equation, q_r is some reference signal which will be defined later. A symbol with a hat $(\hat{\bullet})$ indicates an estimate. The difference between u_d and $\hat{u}_d$, and all other remaining disturbances d, will be compensated for by a disturbance observer, as described in the next subsection.

4.2 Disturbance Observer. As the feedforward signal $\hat{u}_d$ is based on estimates of the robot dynamic properties, the nonlinear and coupled robot dynamics will not be completely compensated for. We will therefore design a disturbance observer, which cancels not only the errors due to this model uncertainty, but also the disturbance signal d (see Fig. 7).

The nominal (linear) plant model used for the disturbance observer design is given as $P_n(s)=1/\bar{M}s$. The Q filter for the disturbance observer is selected as proposed in section II-E, for each individual joint separately, i.e.:

$$Q(s)=\text{diag}\left(\frac{1}{s\tau_1+1},\ldots,\frac{1}{s\tau_n+1}\right) \tag{67}$$

Now if the disturbance observer performs optimally, that is $\hat{d}=d+(u_d-\hat{u}_d)$, the dynamics from the new input u_a (see Fig. 7)

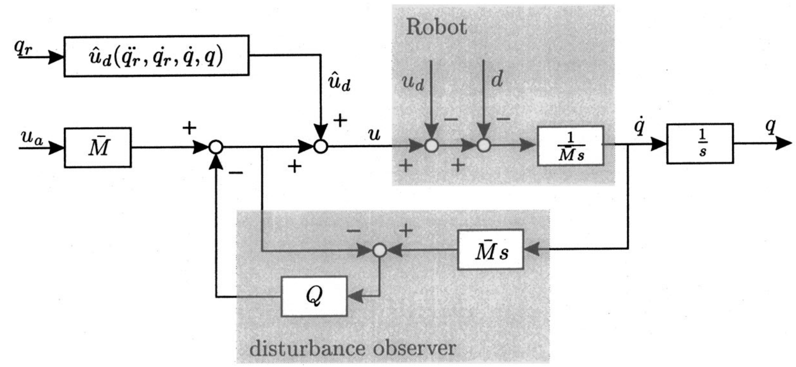

Fig. 7 Disturbance observer for a robot manipulator

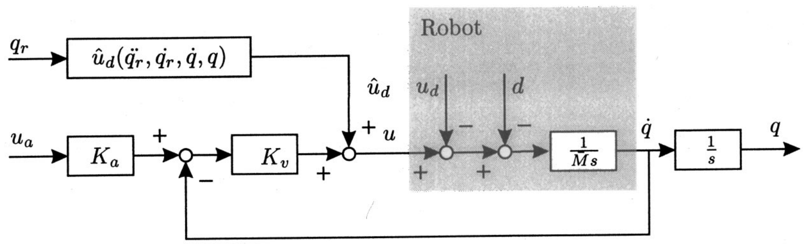

Fig. 8 Disturbance observer represented in feedback form

to the output of the robot manipulator is given by $\ddot{q}=u_a$. The new control input u_a can therefore be regarded the reference acceleration signal for the robot. With this interpretation of u_a, the signal q_r can be defined as $\ddot{q}_r=u_a$. Note that, in an intuitive way, some sort of feedback linearization has been performed. Note furthermore that all matrices involved in the disturbance observer design are diagonal (i.e., $\bar{M}$ and Q are designed diagonal), so that in the linearized system each individual joint can be treated separately.

To analyze the controller defined so far, the disturbance observer is represented in the feedback form (as derived in Section 2.5), resulting in the structure of Fig. 8. The transfer matrices K_a and K_v are given by:

$$K_a \triangleq \bar{M} \cdot P_n Q^{-1} = \operatorname{diag}\left(\frac{s\tau_1+1}{s} \ldots \frac{s\tau_n+1}{s}\right) \tag{68}$$

$$K_v \triangleq (1-Q(s))^{-1} Q(s) P_n^{-1}(s) = \operatorname{diag}\left(\frac{\bar{m}_1}{\tau_1} \ldots \frac{\bar{m}_n}{\tau_n}\right) \tag{69}$$

From these equations it is straightforward to compute the control action of the disturbance observer and the feedforward signal $\hat{u}_d$, using the fact that $K_a K_v = \bar{M} + K_v/s$ (from Eqs. (68) and (69)), and $\ddot{q}_r = u_a$:

$$\begin{aligned} u &= \hat{u}_d + K_v \cdot (K_a \ddot{q}_r - \dot{q}) \\ &= (\hat{M}(q) - \bar{M})\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) + \bar{M}\ddot{q}_r + K_v \dot{q}_r - K_v \dot{q} \\ &= \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) - K_v e_v \end{aligned} \tag{70}$$

where $e_v = \dot{q} - \dot{q}_r$. This result will be used later on in Section 4.4.

4.3 Tracking Controller. All that remains in the design of the disturbance observer based tracking controller, is the design of the tracking controller. As remarked before, the dynamics between the new control input $u_a \triangleq \ddot{q}_r$ and the output q is given by $\ddot{q} = u_a$. A basic tracking controller for this system is given by:

$$u_a = \ddot{q}_d - 2Z\Lambda\dot{e} - \Lambda^2 e \tag{71}$$

where $e = q - q_d$, $Z = \operatorname{diag}(\zeta_1 \ldots \zeta_n) > 0$, $\Lambda = \operatorname{diag}(\lambda_1 \ldots \lambda_n) > 0$ and q_d is the reference trajectory. This controller leads to the error dynamics:

$$(s^2 + 2Z\Lambda s + \Lambda^2)e = 0 \tag{72}$$

By proper selection of the bandwidth Λ and damping Z, these error-dynamics can be designed stable and as fast as necessary.

4.4 Overall Controller Dynamics. In the previous subsections, a disturbance observer based tracking controller has been designed, using fairly basic (linear) techniques. The only design parameters are τ_i for the disturbance observer (Eq. (67)), and the damping and bandwidth parameters ζ_i, λ_i of the tracking controller (Eq. (71)). The selection of these parameters is straightforward as the influence of each parameter on the overall performance is transparent. The overall stability of the resulting system has however not been addressed yet. For that purpose, checking the controller equations (Eqs. (70) and (71)), reveals that we have designed a passivity based controller equal to the one proposed by [8,22,23]. It is well proven that this passivity based controller is L_p input/output stable with respect to the pair (d, e_v) for all $p \in [1,\infty]$, and that the controller can be extended with a parameter update law to estimate the parameters in $\hat{M}(q)$, $\hat{C}(q,\dot{q})\dot{q}$ and $\hat{G}(q)$ without changing passivity properties [8]. As a result of the equivalence, this stability result also applies for the disturbance observer based tracking controller as designed in the previous subsections. Note that this stability result by itself is not new; it has already been reported by Bickel and Tomizuka [8]. However, we show that not only the resulting error dynamics are equivalent, but also that the controllers themselves are equivalent.

Remark: The stability result even holds (although performance will probably degrade) if the nonlinear feedforward signal $\hat{u}_d$ is selected zero, i.e., $\hat{M} = \bar{M}$, $\hat{C}(q,\dot{q}) = 0$ and $\hat{G}(q) = 0$. This selection of the robot property estimates implies that all nonlinear and/or coupling terms in the robot dynamics are compensated for by the disturbance observer. Remark furthermore that the specific structure of Eq. (71) is also not important for the stability results. The more general controller:

$$u_a = F_1^{-1}(s)e + F_2^{-1}(s)q_d \tag{73}$$

also yields stability, as long as $F_1(s)$ and $F_2(s)$ are strictly proper and stable [23].

5 Conclusions

In this paper, it is shown that the two commonly known distinct representations of disturbance observers are equivalent in that the one is a generalization of the other. A general systematic design procedure for disturbance observers is given. The design procedure incorporates robustness criteria for both the disturbance observer alone, and for the disturbance observer combined with a feedback controller. Finally, a disturbance observer based tracking controller for rigid robot manipulator is designed. By representing the disturbance observer as a feedback controller, the overall controller configuration appears to be equivalent to the passivity based controller as proposed by Sadegh and Horowitz, and Slotine and Li. As a result of that equivalence, we can state that, although the presented disturbance observer based controller is designed intuitively, input/ouput stability is guaranteed.

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