Robust Speed Control of DC Servomotors Using Modern Two Degrees-of-Freedom Controller Design

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Abstract—**We propose a robust speed control system of dc servomotors based on the parametrization of two degrees-of-freedom controllers. This servosystem can revolutionarily improve the characteristics of the closed loop systems, i.e., the disturbance torque suppression performance and the robustness to system parameter variations, without changing the command input response. We will show the excellent control performances we obtained during laboratory experiments by using a microprocessor-based controller.**

I. Introduction

RECENTLY, robust servosystems with excellent load disturbance suppression performance and their applications to motion control have been widely studied [1]–[4]. From a general control theory point of view, some of these servosystems have structures of two-degrees-of-freedom controller. Two-degrees-of-freedom control systems have the striking feature that one can design the command input response and the closed-loop characteristics independently. Although this fact has been known for over 35 years [5], [6], until now, there has been little work done on its practical application to actual control systems.

The classes of the stabilizing controllers are parametrized by the approach based on the matrix fraction descriptions [7], [8]. By this approach, we can solve various control problems, e.g., stabilizing problem [7], dead-beat problem [9], in a general way using only algebraic calculations, and the class of servocontrollers with two degrees of freedom has been clarified [10].

In this paper, we proposed the generalized speed control system design technique of dc servomotors based on the parametrization of two-degrees-of-freedom controllers. The servosystem in this paper includes a free parameter within the set of strictly proper and stable rational functions. We will show that the closed-loop characteristics can be improved by the appropriate choice of the free parameter without changing the command input response. We employ the design method of a Butterworth filter to determine the parameter.

The servosystem designed here is implemented using a microprocessor-based controller, and its excellent control performance was confirmed by laboratory experiments.

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II. Parameterization of Two-Degrees-of-Freedom Controllers

In this section, we review the design theory of controllers with two degrees of freedom. First, we define our notation. Let $R_-(s)$ denote the proper and stable rational functions, and $Rs_-(s)$ denote the strictly proper and stable rational functions. Then consider the control system configuration in Fig. 1.

For simplicity, it is assumed that the plant is a single-input single-output (SISO) system and free of hidden modes. Its transfer function $P(s)$ is strictly proper. We define the coprime fraction description of $P(s)$ as follows:

$$P(s) = ND^{-1} \tag{1}$$

where $N, D \in R_-(s)$. Thus, there exists $X, Y \in R_-(s)$ such that $XD + YN = 1$. Let the controller be described as

$$u = C_1(s)r - C_2(s)(y + \xi) \tag{2}$$

and let

$$C(s) = [C_1, -C_2] = D_c^{-1}[K, -N_c] \tag{3}$$

be a coprime fraction description of the controller $C(s)$, where D_c, $(K, -N_c) \in R_-(s)$. The system is internally stable if and only if D_c and N_c satisfy the equation

$$D_cD + N_cN = 1. \tag{4}$$

Consequently, the set of all stabilizing proper controllers $C(s)$ is parameterized as

$$C(s) = (X - RN)^{-1}[K, -(Y + RD)] \tag{5}$$

where K and R are arbitrary rational functions in $R_-(s)$. Expression (5) is called the Youla parametrization [8]. By simple analysis, it follows that the transfer functions $G_{ry}(s)$ from r to y and $G_{dy}(s)$ from d to y are given by

$$G_{ry}(s) = NK \tag{6}$$

and

$$G_{dy}(s) = (X - NR)N. \tag{7}$$

This shows that $G_{ry}(s)$ and $G_{dy}(s)$ can be independently designed through the choice of two arbitrary parameters $K, R \in R_-(s)$.

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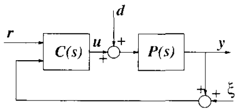


Fig. 1. General configuration of two degrees of freedom control system. r: command input, u: control input, y: controlled output, d: disturbance, and ξ: sensor noise.

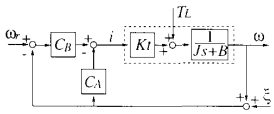


Fig. 2. Servo system design based on the two degrees of freedom con troller.

III. Design of Robust Servosystems with Two Degrees of Freedom

A. Parameterization

We now present the robust speed control systems of dc motors using the format explained above. We assume that the current of the dc motor is well controlled. Then, the transfer function of a dc motor can be given by

$$P(s) = \frac{K_t}{Js + B} \tag{8}$$

where J, B, and K_t are the total inertia, the friction constant, and the torque constant, respectively. Since $P(s) \in R_-(s)$, one of the simplest descriptions of (1) is that $N = P(s)$ and $D = 1$, which yields $X = 1$ and $Y = 0$ as the solutions of (4).

In Fig. 2, we illustrate the actual configuration of the proposed servosystem. The reason why we selected this configuration among the various kinds of two-degrees-of-freedom controllers is that this configuration is very similar to that of conventional "one-degree-of-freedom" controllers. The only—but important—difference is the existence of $C_A(s)$ in our proposed configuration. The relation between $C_A(s)$, $C_B(s)$ (Fig. 2) and $C_1(s)$, $C_2(s)$ (see (2)) is given by

$$C_1 = C_B \qquad C_2 = C_A + C_B. \tag{9}$$

These relations imply that $R = K + R'$ where $R' \in R_-(s)$. Therefore, the class of stabilizing controllers in Fig. 2 is parametrized as

$$C_B = \left(1 - G_{ry} - R'N\right)^{-1} K \tag{10a}$$

$$C_A = \left(1 - G_{ry} - R'N\right)^{-1} R'. \tag{10b}$$

Next, we should determine free parameters R' and K for our control specifications as follows:

Specification 1: The motor speed must follow the stepwise command inputs ω_r asymptotically without any steady state error. This can be performed by conventional PI controllers given by

$$C(s) = K_1 \frac{T_1 s + 1}{T_1 s}. \tag{11}$$

The command input response $G_{ry}(s)$ then takes the form

$$G_{ry}(s) = \frac{T_1 s + 1}{T_1 s + 1 + T_1 s(Js + B)/(K_1 K_t)} = \frac{T_1 s + 1}{\phi(s)}. \tag{12}$$

Using (6), K, which satisfies specification 1, can be given by

$$K = \left(\frac{K_t}{Js + B} + \frac{1}{K_1}\frac{T_1 s}{T_1 s + 1}\right)^{-1}. \tag{13}$$

Specification 2: Speed fluctuation caused by the step load disturbances must be asymptotically removed. This specification can be satisfied by designing R' in (10b) in the following manner. Let a coprime description of the stepwise load disturbance be

$$T_L = \frac{T_{L0}}{s} = \left(\frac{s}{a(s)}\right)^{-1}\left(\frac{T_{L0}}{a(s)}\right) \tag{14}$$

where $a(s)$ is an arbitrary stable polynomial of the first degree and T_{L0} is a constant real number. Using (14), we can derive the speed fluctuation for this disturbance as

$$\omega_d = \left(1 - Gry - \frac{Kt}{Js + B} R'\right)\left(\frac{s}{a(s)}\right)^{-1} \cdot \frac{1}{Js + B}\frac{T_{L0}}{a(s)}. \tag{15}$$

For the purpose of the asymptotic regulation, ω_d must be a rational function in $R_-(s)$. Thus, the parameter $R' \in R_-(s)$ should satisfy the following condition:

$$\left(1 - G_{ry} - \frac{K_t}{Js + B} R'\right)\left(\frac{s}{a(s)}\right)^{-1} = W \in R_-(s). \tag{16}$$

Equation (16) yields (17).

$$W \frac{s}{a(s)} + \frac{K_t}{Js + B} R' = \frac{T_1(Js + B)^2 s}{K_1 K_t \phi(s)}. \tag{17}$$

We can find that the general solution R' of this equation as

$$R' \in \{R: R'/s \in Rs_-(s)\} \tag{18}$$

and R' can be represented without any lack of generality as follows:

$$R' = \frac{(Js + B)^2 s}{K_t \phi(s)} \frac{T_1}{K_1} Q(s) \tag{19}$$

where $Q(s)$ is an arbitrary rational function in $Rs_-(s)$.

As a result, our proposed servocontrollers are parametrized by

$$C_A(s) = \frac{Js + B}{K_t} Q(s)(1 - Q(s))^{-1} \tag{20a}$$

$$C_B(s) = K_1 \frac{T_1 s + 1}{T_1 s}(1 - Q(s))^{-1}. \tag{20b}$$

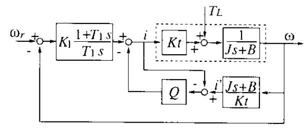


Fig. 3. Interpretative configuration of the proposed servosystem in Fig. 2.

The characteristics of the closed loop system are given by

$$S(s) = (1 - G_{ry}(s))(1 - Q(s)) \quad \text{sensitivity function} \tag{21a}$$

$$G_{dy}(s) = \frac{T_1 s}{K_t K_1 \phi(s)}(1 - Q(s)) \tag{21b}$$

transfer function from T_L to ω.

Note that the value of $Q(s)$ has no relation with that of $G_{ry}(s)$. This is shown more clearly in Fig. 3 which is the interpretative configuration of our two-degrees-of-freedom servosystem based on (20). This figure shows that our servosystem improves its closed loop characteristics by feeding back the difference between the control input i and estimated input i'. The parameter $Q(s)$ is the keypoint of our controller design.

B. Design of the Parameter Q(s)

The characteristics of a closed-loop system depends upon the choice of parameter $Q(s)$. To determine $Q(s)$, we employ a Butterworth filter design. $Q(s)$ is defined such that $1 - Q(s)$ has the frequency characteristics of a high pass filter. We name the servosystems by the order of its parameter $Q(s)$.

Type I:
$$Q(s) = \frac{1}{(s\tau) + 1} \tag{22a}$$

Type II:
$$Q(s) = \frac{1.41(s\tau) + 1}{(s\tau)^2 + 1.41(s\tau) + 1} \tag{22b}$$

Type III:
$$Q(s) = \frac{2(s\tau)^2 + 2(s\tau) + 1}{(s\tau)^3 + 2(s\tau)^2 + 2(s\tau) + 1} \tag{22c}$$

where τ determines the cutoff frequency of the parameter. For convenience, the conventional PI controller expressed by (12) is defined as Type 0 because it can be derived from (20) by setting $Q(s) = 0$.

IV. Advantages of Our Servosystems

A. Equivalence to the Disturbance Torque Observer-Based Servosystem

Consider the servosystem Type I in Fig. 4, which has the smallest order among our servosystems. It can be modified to the servosystem shown in Fig. 5 using the equation

$$\frac{1}{\tau s + 1}\,\frac{Js + 1}{K_t} = \frac{1}{K_t}\left(\frac{J}{\tau} - \frac{J/\tau - B}{\tau s + 1}\right). \tag{23}$$

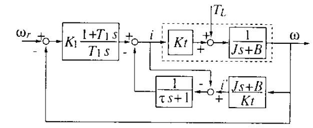


Fig. 4. Type I servo system.

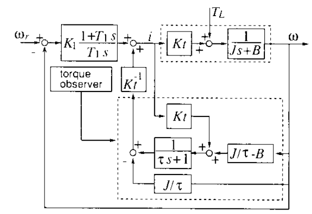


Fig. 5. Disturbance observer-based servo system.

We see that the disturbance torque observer developed in [1] appears in Fig. 5. In other words, Type I is equivalent to the torque observer based servosystem [2]. Furthermore, we can expect that Type II, Type III and so on will be superior to Type I. In that sense, the parametrization expressed in (20) gives us a more general method of configurating robust servosystems.

B. Robustness to the Motor Parameter Variation and Load Disturbances

Let the relation between the actual plant $P'(s)$ and the nominal plant $P(s)$, which is used for the controller design, be represented as

$$P(s) = (1 + \Delta P(s))P'(s),$$

where

$$\Delta P(s) \in R_-(s). \tag{24}$$

If we assume a variation of the motor, the parameters can be written as

$$\Delta P(s) = \frac{(J'/K_t' - J/K_t)s + (B'/K_t' - B/K_t)}{(Js + B)/K_t} \tag{25}$$

the characteristics of the command input response can be changed to

$$G_{ry}(s) = (1 + S(s)\Delta P(s))G_{ry}' \tag{26}$$

where $S(s)$ is the sensitivity function expressed in (21a). $G_{ry}(s)$ is the desired characteristic and $G_{ry}'(s)$ is the actual one. From the point of desensitization to the motor parameter variation, we have to reduce the norm of the sensitivity function $|S(j\omega)|$.

In our proposed servosystems, this point can be achieved directly and successfully. Now, we can describe the command input response approximately as

$$G_{ry} = \frac{1}{1 + (J/K_1 K_t)s} = \frac{1}{1 + \tau s} \tag{27}$$

with assumptions $B/(K_1 K_t) \ll 1$ and $\tau_r = J/(K_1 K_t) \ll T_1$, which are usually satisfied. The frequency characteristic of

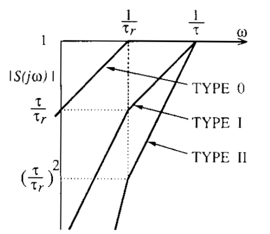


Fig. 6. Frequency characteristics of the sensitivity functions.

the sensitivity function $|S(j\omega)|^2$ is given by

$$|S(j\omega)|^2 = \frac{(\tau_r\omega)^2}{1+(\tau_r\omega)^2}\frac{(\tau\omega)^{2n}}{1+(\tau\omega)^{2n}} \tag{28}$$

where n is the order of the parameter $Q(s)$ in (22).

Fig. 6 illustrates the frequency characteristics of $S(s)$. We can see, from this figure, that the robustness improved as n increases. The load disturbance suppression performance expressed in (21b) also improved.

C. Influence of Sensor Noise

In this section, we investigate the influence of sensor noise ξ. The transfer function from ξ to ω is given by

$$T(s) = 1 - S(s) \tag{29}$$

which is called a complementary sensitivity function. Noise usually has larger components at higher frequencies. Thus $|T(j\omega)|$ should be small in high-frequency regions. In our servosystems, $T(s)$ can be described as

$$T(s) = Q(s) \tag{30}$$

with the assumption $\tau_r \gg \tau$. Fig. 7 shows its frequency characteristic for $T(s)$. Type I and Type II servosystems suppress the speed fluctuation caused by the noise ξ in frequencies over $1/\tau$ with -20 dB/decade. In other words, in our selection of $Q(s)$ by (22a)-(22c), an increase in the order of $Q(s)$ contributes only to the improvement of robustness not to noise reduction. In order to obtain exactly the same falloff characteristic using any type of servosystem we should adjust τ in (22). Type 0 servosystems are quite insensitive to sensor noise but have poor disturbance suppression performance.

V. Experimental Results

We will show some experimental results on our servocontrollers. The experiment was performed on Type 0, Type I, and Type II servosystems that were designed to have the same command input responses. All control algorithms are written in C language and implemented by using a microprocessor as illustrated in Fig. 8. The constants of the tested dc servomotor and the controller parameters are specified in Tables I and II, respectively.

A. Disturbance Suppression Performance

Fig. 9(a), (b), and (c) shows the speed fluctuations caused by rectangular load disturbance torque T_L with the amplitude

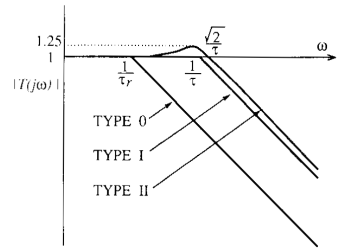


Fig. 7. Frequency characteristics of the complementary sensitivity functions.

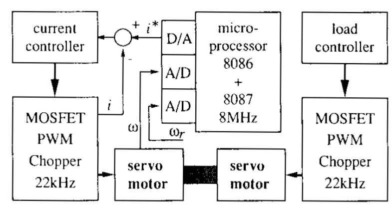


Fig. 8. Setup of the experimental system.

TABLE I
PARAMETERS OF THE TESTED MACHINE

Rated output	500 W
Rated current	6.5 A
Rated speed	1500 [r/min]
R	7.5 Ω
L	5.0 mH
K_t, K_e	0.809 Nm/A, Vs/rad
J	0.006 kgm^2
B	0.005 Nms

TABLE II
PARAMETERS OF THE CONTROLLERS

K_1	T_1	K_t	J	B
0.4	0.4	0.81	0.006	0.005

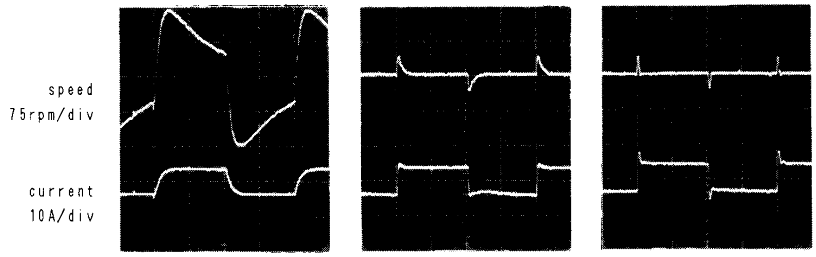
Sampling Time (ms)

Type 0	Type I	Type II
0.8	1.3	1.4

of $+/-$ 4 [Nm] when $\omega_r = 0$. Fig. 10 depicts the frequency responses for the sinusoidal load disturbances that have the same amplitude. We can see that our Type I and Type II servosystems suppress the load disturbances quite effectively, whereas the Type 0 servosystem is very much influenced by the disturbance.

B. Robustness to Inertia Variation

Fig. 11(a), (b) and (c) shows the step command input responses for nominal inertia, and Fig. 12(a), (b), and (c) shows the step responses when inertia is increased to about three times of the nominal value. It is clear that Type 0 is very sensitive to an increase in inertia, however, we do not see notable changes for the proposed servosystems. In particular, Type II is the most robust among the tested servosystems.



(a) TYPE 0 (b) TYPE I (c) TYPE II

Fig. 9. Response to rectangular disturbance torque (τ = 3.0 ms, time: 0.1 s/div).

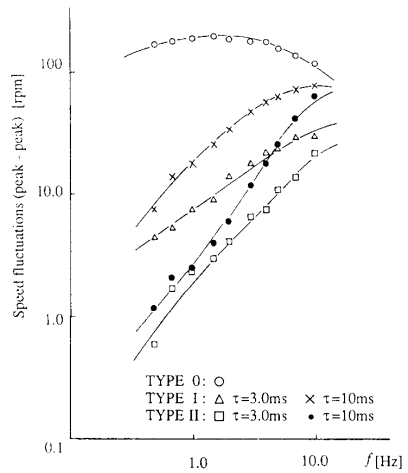
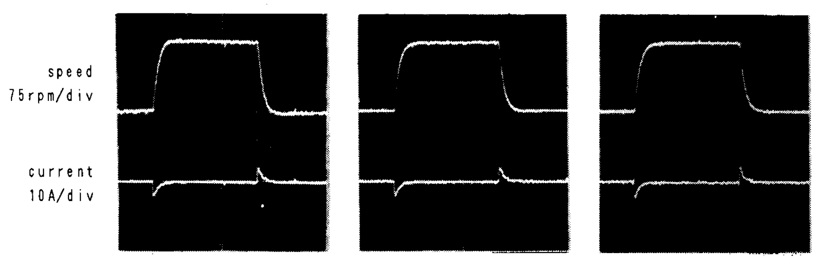
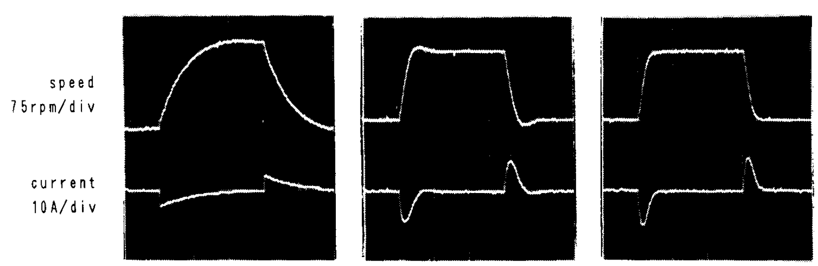


Fig. 10. Frequency responses to sinusoidal load disturbance.



(a) TYPE 0 (b) TYPE I (c) TYPE II

Fig. 11. Response to rectangular speed command input (τ = 3.0 ms, time: 0.2 s/div).



(a) TYPE 0 (b) TYPE I (c) TYPE II

Fig. 12. Step response to the speed command input when the load inertia moment is increased to three times its nominal value (τ = 3.0 ms, time: 0.2 s/div).

VI. Conclusions

We proposed the robust speed control system design method of a dc servomotor based on the parametrization of two-degrees-of-freedom controllers. The proposed servosystem has a simple configuration. We designed it by adding only a single control block to the conventional servocontroller. It has excellent closed loop characteristics, i.e., the disturbance suppression performance and the robustness to the machine's parameter variations. We implemented the servosystems designed here with a microprocessor and confirmed its excellent control performance by laboratory experiments.

We have much interest in designing a robust position servosystem, e.g., for robot manipulators. In this case, our parameterization developed here can not be applied directly because the controlled object is unstable. Insertion of an additional position controller consisting of a simple gain should be sufficiently effective for this if we design the position servosystem based on our proposed robust speed controller. However, we have already succeeded in the true augmentation of the parameterizaton applicable to unstable plants and realized a position servosystem without using any speed sensors [11]. We would like to report on this next time.

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