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An almost necessary and sufficient condition for robust stability of closed-loop systems with disturbance observer[☆]

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ABSTRACT

The disturbance observer (DOB)-based controller has been widely employed in industrial applications due to its powerful ability to reject disturbances and compensate plant uncertainties. In spite of various successful applications, no necessary and sufficient condition for robust stability of the closed loop systems with the DOB has been reported in the literature. In this paper, we present an almost necessary and sufficient condition for robust stability when the Q-filter has a sufficiently small time constant. The proposed condition indicates that robust stabilization can be achieved against *arbitrarily large* (but bounded) uncertain parameters, provided that an outer-loop controller stabilizes the *nominal* system, and uncertain plant is of minimum phase.

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1. Introduction

Due to a growing demand for high accuracy performance, the development of an advanced control system has attracted considerable attention. Several attempts have been made to achieve high precision control in the presence of various disturbances and plant uncertainties. For example, robust output regulation via internal model principle (Francis & Wonham, 1976; Huang, 2004; Isidori, Marconi & Serrani, 2003), adaptive robust control (ARC) (Yao, Al-Majed & Tomizuka, 1997), model-based disturbance attenuation (MBDA) (Choi, Choi, & Lim, 1999), and disturbance observer (DOB) have been proposed in the literature. In particular, the DOB-based controller has been widely employed in industrial applications due to simple structure as well as a powerful ability to reject disturbances and compensate plant uncertainties (Back & Shim, 2008; Kempf & Kobayashi, 1999; Lee & Tomizuka, 1996; Ohishi, Ohnishi & Miyachi, 1983; Radke & Gao, 2006; Shim & Joo, 2007; White, Tomizuka & Smith, 2000; Yang, Choi & Chung, 2005).

The structure of the DOB-based controller is shown in Fig. 1, where $P(s)$ and $P_n(s)$ represent the single-input–single-output real plant and its nominal model, respectively, and $Q(s)$ (called as the *Q-filter*) is a stable low-pass filter with the unity dc gain. The signals d and n represent the input disturbance and the measurement noise, respectively. From Fig. 1, the plant output y can be expressed as

$$y(s) = T_{yr}(s)r(s) + T_{yd}(s)\mathrm{d}(s) - T_{yn}(s)n(s), \tag{1}$$

where $T_{yr} = \frac{P_nPC}{P_n(1+PC)+Q(P-P_n)}$, $T_{yd} = \frac{P_nP(1-Q)}{P_n(1+PC)+Q(P-P_n)}$ and $T_{yn} = \frac{P(Q+P_nC)}{P_n(1+PC)+Q(P-P_n)}$. In the low frequency range for which $Q(j\omega) \approx 1$, it follows that $T_{yr} \approx \frac{P_nC}{1+P_nC}$ and $T_{yd} \approx 0$. Thus, below the cutoff frequency of $Q(j\omega)$, (1) is reduced to $y(j\omega) \approx \frac{P_nC}{1+P_nC}(j\omega)r(j\omega)$ since $n(j\omega) \approx 0$ at low frequencies. This implies that, *assuming that all the transfer functions are stable*, the real uncertain closed loop system with the DOB behaves as if it were the nominal closed loop system in the absence of disturbance. In other words, the DOB can be combined with any (pre-existing) outer-loop controller $C(s)$ in order to enhance the robustness and disturbance rejection performance.

One of the important issues of the DOB is how to design the low-pass filter $Q(s)$ in order that the closed loop system is internally stable for all uncertain plants $P(s)$. The plant uncertainty under consideration is summarized as follows.

Assumption 1. Let the set $\mathcal{P}$ of transfer functions be

$$\mathcal{P} = \left\{ P(s) = \frac{\beta_{n-r}s^{n-r} + \beta_{n-r-1}s^{n-r-1} + \cdots + \beta_0}{\alpha_n s^n + \alpha^{n-1}s^{n-1} + \cdots + \alpha_0} \right.$$
$$\left. : \alpha_i \in [\underline{\alpha}_i, \bar{\alpha}_i],\ \beta_i \in [\underline{\beta}_i, \bar{\beta}_i] \right\}$$

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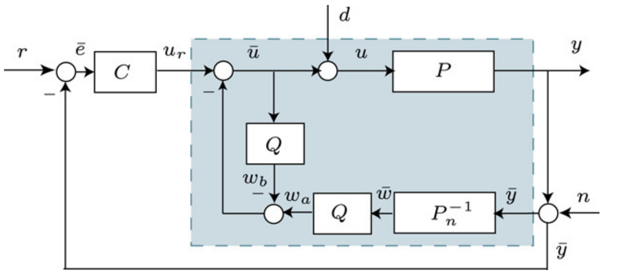

Fig. 1. Control system with disturbance observer (DOB). The shaded block represents the actual plant $P(s)$ augmented with the DOB. For convenience, the DOB and $C(s)$ are referred to as an inner-loop controller and an outer-loop controller, respectively.

where n and r are positive integers, and all $\underline{\alpha}_i$, $\bar{\alpha}_i$, $\underline{\beta}_i$, and $\bar{\beta}_i$ are known constants such that $0 \notin [\underline{\alpha}_n, \bar{\alpha}_n]$ and $0 \notin [\underline{\beta}_{n-r}, \bar{\beta}_{n-r}]$. We assume that both the real uncertain plant $P(s)$ and its nominal model $P_n(s)$ are contained in $\mathscr{P}$.

Usually the Q-filter $Q(s)$ has the form (Choi, Yang, Chung, Kim & Suh, 2003; Lee & Tomizuka, 1996; Umeno & Hori, 1993) of

$$Q(s) = \frac{c_k(\tau s)^k + c_{k-1}(\tau s)^{k-1} + \cdots + c_0}{(\tau s)^l + a_{l-1}(\tau s)^{l-1} + \cdots + a_1(\tau s) + a_0} \tag{2}$$

where $\tau > 0$ is the filter time constant, and k and l are some nonnegative integers with $k \le l - r$ so that the $Q(s)P_n^{-1}(s)$ block in Fig. 1 becomes proper and implementable. It is also assumed that $a_0 = c_0$ so that it has the unity dc gain, *i.e.*, $Q(0) = 1$. In order to enjoy the virtue of the DOB (*i.e.*, rejecting disturbances and compensating plant uncertainties) over a wide frequency range, τ is usually chosen as small as possible so that the bandwidth for which $Q(j\omega) \approx 1$ is increased (Choi et al., 1999, 2003; Yao et al., 1997). Motivated by this fact, we present a new condition for robust internal stability of the closed loop system when τ is chosen sufficiently small. While there have been some research works on the stability of the DOB (Choi et al., 2003; Kim & Chung, 2003; Schrijver & Dijk, 2002; Yao et al., 1997), most of them have presented somewhat conservative sufficient conditions. In contrast, as will be discussed in Remark 1, the proposed condition is almost necessary and sufficient for robust stability. Moreover, the proposed condition presents a clear relationship between the Q-filter and the robust stabilization, which yields a considerably efficient way to design $Q(s)$ for robust stability. (Of course, the ability of disturbance rejection and uncertainty compensation is still preserved since τ is chosen sufficiently small.)

Notation. Let $D(s)$ be a polynomial with real coefficients expressed as $D(s) = d_n s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0$. The polynomial $D(s)$ is said to be of *degree n* if $d_n \neq 0$, which will be denoted by $\deg(D) = n$. For a rational transfer function $G(s) = N(s)/D(s)$ (it is assumed that $N(s)$ and $D(s)$ are coprime polynomials), the degree and the relative degree of $G(s)$ are defined as $\deg(D)$ and $\deg(D) - \deg(N)$, respectively. A stable transfer function implies that its denominator is a Hurwitz polynomial. LHP (RHP, respectively) stands for the open left (right, respectively) half plane.

2. Main result

2.1. Internal stability of the closed loop system with the DOB

With the configuration of Fig. 1, nine transfer functions from $[r, d, n]^{\mathrm{T}}$ to $[\bar{e}, u, \bar{y}]^{\mathrm{T}}$ are given by

$$\frac{1}{\Delta(s)}\begin{bmatrix} Q(P-P_n)+P_n, & (Q-1)PP_n, & (Q-1)P_n \\ CP_n, & (1-Q)P_n, & -Q-CP_n \\ CPP_n, & (1-Q)PP_n, & (1-Q)P_n \end{bmatrix}$$

where $\Delta(s) = (1 + PC)P_n + Q(P - P_n)$. If the above nine transfer functions are stable, then the closed loop system is said to be *internally stable*. Now let us write P, P_n, C, and Q as ratios of coprime polynomials; that is, $P(s) = \frac{N(s)}{D(s)}$, $P_n(s) = \frac{N_n(s)}{D_n(s)}$, $C(s) = \frac{N_c(s)}{D_c(s)}$, and $Q(s) = \frac{N_Q(s)}{D_Q(s)}$. In fact, in order to express the explicit dependency of τ, $N_Q(s;\tau)$ and $D_Q(s;\tau)$ will be used instead of $N_Q(s)$ and $D_Q(s)$, respectively; *i.e.*, $N_Q(s;\tau) = c_k(\tau s)^k + c_{k-1}(\tau s)^{k-1} + \cdots + c_0$ and $D_Q(s;\tau) = (\tau s)^l + a_{l-1}(\tau s)^{l-1} + \cdots + a_1(\tau s) + a_0$. Then, it can be shown in a similar way to (Doyle, Francis & Tannenbaum, 1992, p. 37, Theorem 1) that, for given $\tau > 0$, the closed loop system is internally stable if and only if the *characteristic* polynomial

$$\delta(s;\tau) := (DD_c + NN_c)N_n D_Q + N_Q D_c(ND_n - N_n D)$$

is Hurwitz.

In order to present our main result, we first establish the following preliminary lemma.

Lemma 1. *Let $p(s)$ and $q_j(s)$, $j = 1, \ldots, k$, be polynomials of complex variable s. Define $R(s;\tau) := p(s) + \tau q_1(s) + \tau^2 q_2(s) + \cdots + \tau^k q_k(s)$. Assume that $\deg(p) = n$ and let s_i^*, $i = 1, \ldots, n$, be the roots of $R(s;0) = 0$. Then, for a sufficiently small $\tau > 0$, there exist n roots of $R(s;\tau) = 0$, say $s_i(\tau)$, $i = 1, \ldots, n$, such that $\lim_{\tau\to 0} s_i(\tau) = s_i^*$ (even if $R(s;\tau)$ may have more than n roots for $\tau > 0$).*

Proof. Let $\epsilon > 0$ be given. Then, there exists a positive constant $r \le \epsilon$ such that, for each $i = 1, \ldots, n$, $p(s)$ has no root on $B(s_i^*, r) := \{s : |s - s_i^*| \le r\}$, except at s_i^*. Since $q_j(s), j = 1, \ldots, k$, are continuous on $B(s_i^*, r)$, there exists a positive constant M such that $|q_j(s)| \le M$ for all $j = 1, \ldots, k$ and $s \in B(s_i^*, r)$. Now, let $\alpha := \min_{1\le i\le n}\left[\min_{|s-s_i^*|=r}|p(s)|\right]$ and choose a positive constant $\bar{\tau} < \min\left(1, \frac{\alpha}{Mk}\right)$. Then, for all $0 \le \tau \le \bar{\tau}$ and all $i = 1, \ldots, n$,

$$\begin{aligned} &|\tau q_1(s) + \tau^2 q_2(s) + \cdots + \tau^k q_k(s)| \\ &\quad \le \tau|q_1(s)| + \tau^2|q_2(s)| + \cdots + \tau^k|q_k(s)| \\ &\quad < \frac{\alpha}{Mk}M + \frac{\alpha}{Mk}M + \cdots + \frac{\alpha}{Mk}M = \alpha \le |p(s)| \end{aligned}$$

on $\{s : |s - s_i^*| = r\}$. Applying Rouché's Theorem in Appendix with $f(s) = p(s)$ and $g(s) = \tau q_1(s) + \tau^2 q_2(s) + \cdots + \tau^k q_k(s)$, we conclude that, for $0 \le \tau \le \bar{\tau}$, $p(s)$ and $R(s;\tau)$ have the same number of roots inside the contour $|s - s_i^*| = r \le \epsilon$. Since ϵ is arbitrary, the claim is proved. □

2.2. An almost necessary and sufficient condition for robust internal stability

Now, we present a necessary and sufficient condition for robust internal stability of the closed loop system in Fig. 1. (The closed loop system is said to be *robustly internally stable* if it is internally stable for all $P(s) \in \mathscr{P}$.) For convenience, let

$$m = \deg(DD_c N_n). \tag{3}$$

Then, since all transfer functions C, P, P_n, and Q are proper, P and P_n have the same relative degree r, and the relative degree of Q is not less than r, it follows that the highest power of s in $\delta(s;\tau)$ with $\tau > 0$ is $\deg(DD_cN_nD_Q) = m + l$. Thus, there exist $m + l$ roots of the characteristic equation $\delta(s;\tau) = 0$.

Lemma 2. *Let*

$$p_s(s) := N(s)(D_c(s)D_n(s) + N_c(s)N_n(s)),$$

$$p_f(s) := D_Q(s;1) + \left(\lim_{s\to\infty}\frac{P(s)}{P_n(s)} - 1\right)N_Q(s;1),$$

and $s_1^*, \ldots, s_m^*$ *and* $s_{m+1}^*, \ldots, s_{m+l}^*$ *be the roots of* $p_s(s) = 0$ *and* $p_f(s) = 0$*, respectively. Then,* $m+l$ *roots of* $\delta(s;\tau) = 0$*, say* $s_i(\tau)$*,* $i = 1, \ldots, m+l$*, have the property that*

$$\lim_{\tau\to 0} s_i(\tau) = s_i^*, \quad i = 1, \ldots, m,$$
$$\lim_{\tau\to 0} \tau s_i(\tau) = s_i^*, \quad i = m+1, \ldots, m+l.$$

Proof. Since $D_Q(s;0) = N_Q(s;0) = a_0 \neq 0$, we obtain

$$\delta(s;0) = a_0 N(s)(D_c(s)D_n(s) + N_c(s)N_n(s)) = a_0 p_s(s).$$

Therefore, the first claim directly follows from Lemma 1.
Now let

$$\bar{\delta}(s;\tau) := \tau^m \delta\left(\frac{s}{\tau};\tau\right) = \gamma_1(s;\tau) D_Q\left(\frac{s}{\tau};\tau\right) + N_Q\left(\frac{s}{\tau};\tau\right)\gamma_2(s;\tau)$$

where $\gamma_1(s;\tau) = \tau^m(DD_cN_n(s/\tau) + NN_cN_n(s/\tau))$ and $\gamma_2(s;\tau) = \tau^m(D_cND_n(s/\tau) - D_cN_nD(s/\tau))$. Since $m = \deg(DD_cN_n) = \deg(D_cND_n) > \deg(NN_cN_n)$, it follows that $\lim_{\tau\to 0}\gamma_1(s;\tau) = \lim_{\tau\to 0}\tau^m DD_cN_n(s/\tau) = \bar{\gamma}_1 s^m$ and $\lim_{\tau\to 0}\gamma_2(s;\tau) = \bar{\gamma}_2 s^m$ for all s with some constants $\bar{\gamma}_1 \neq 0$ and $\bar{\gamma}_2$. (Recalling the paragraph after (3), the above limits are well-defined.) On the other hand, because $D_Q(s/\tau;\tau) = D_Q(s;1)$ and $N_Q(s/\tau;\tau) = N_Q(s;1)$, we obtain $\bar{\delta}(s;0) = \bar{\gamma}_1 s^m\left[D_Q(s;1) + \frac{\bar{\gamma}_2}{\bar{\gamma}_1}N_Q(s;1)\right]$. Since

$$\frac{\bar{\gamma}_2}{\bar{\gamma}_1} = \frac{\lim_{\tau\to 0}\tau^m(D_cND_n(s/\tau) - D_cN_nD(s/\tau))}{\lim_{\tau\to 0}\tau^m DD_cN_n(s/\tau)} = \lim_{s\to\infty}\frac{N(s)D_n(s)}{N_n(s)D(s)} - 1,$$

it follows that

$$\bar{\delta}(s;0) = \bar{\gamma}_1 s^m p_f(s),$$

which implies that $\bar{\delta}(s;0) = 0$ has m roots at the origin and l roots at $s_{m+1}^*, \ldots, s_{m+l}^*$. Again, applying Lemma 1 with $R(s;\tau) = \bar{\delta}(s;\tau)$, we see that there exist l roots of $\bar{\delta}(s;\tau) = 0$, say $\bar{s}_i(\tau)$, $i = m+1, \ldots, m+l$, such that $\lim_{\tau\to 0}\bar{s}_i(\tau) = s_i^*$. Since $\bar{s}_i(\tau)/\tau$ are roots of $\delta(s;\tau) = 0$, the second claim is proved. □

Based on Lemma 2, the following theorem presents a new condition for robust internal stability.

Theorem 3. *Under Assumption 1, there exists a constant* $\tau^* > 0$ *such that, for all* $0 < \tau \le \tau^*$*, the closed loop system is robustly internally stable if the following three conditions hold;*

(1) $P_nC/(1+P_nC)$ *is stable,*
(2) $P(s)$ *is of minimum phase for all* $P(s) \in \mathcal{P}$*,*
(3) $p_f(s)$ *is Hurwitz for all* $P(s) \in \mathcal{P}$*.*

On the contrary, there is $\tau^* > 0$ *such that, for all* $0 < \tau \le \tau^*$*, the closed loop system is not robustly internally stable if at least one of the conditions (1)–(3) is violated in the sense that* $P_nC/(1+P_nC)$ *has some poles in the RHP, or some zeros of* $P(s)$ *or some roots of* $p_f(s) = 0$ *are located in the RHP for some* $P(s) \in \mathcal{P}$*.*

Proof. Since the denominator of $P_nC/(1+P_nC)$ is $(D_cD_n+N_cN_n)(s)$ and the numerator of $P(s)$ is $N(s)$, the conditions (1) and (2) are equivalent to that $p_s(s)$ is Hurwitz for all $P(s) \in \mathcal{P}$. Thus, the proof follows from Lemma 2. □

Remark 1. Theorem 3 is not able to determine robust internal stability when some poles of $P_nC/(1+P_nC)$, or some zeros of $P(s)$, or some roots of $p_f(s) = 0$ are located on the imaginary axis in the complex plane, but the remaining poles, zeros, and roots are located in the LHP.[1] If we exclude such situations, the conditions (1)–(3) are not only sufficient but also necessary for robust internal stability. In this sense, we call them as an almost necessary and sufficient condition for robust stability.

Theorem 3 enlightens many important issues for the DOB applications. First, the condition (1) implies that the unity feedback system consisting of $P_n(s)$ and $C(s)$ is stable. It is quite a natural one since the primary goal of the outer-loop controller $C(s)$ is the stabilization of the *nominal* model $P_n(s)$. On the other hand, since the restriction on $C(s)$ is imposed by the condition (1) only, it is unnecessary to take plant uncertainties and disturbances into consideration when designing $C(s)$. Secondly, the condition (2) requires the uncertain plant to be of minimum phase. Even though it is rather restrictive, Theorem 3 indicates that it cannot be removed in most DOB approaches such as (Choi et al., 1999, 2003; Yao et al., 1997), where a small τ was used. Finally, the condition (3), which is our new finding, has some important implications for robust stability. One is that it automatically holds if plant uncertainty is small in the sense that $\lim_{s\to\infty}\frac{P(s)}{P_n(s)} \approx 1$. Indeed, since $\lim_{s\to\infty}\frac{P(s)}{P_n(s)} \approx 1$ implies that $p_f(s) \approx D_Q(s;1)$ and $D_Q(s;1)$ is Hurwitz (recalling that $Q(s)$ is a stable filter), $p_f(s)$ is likely to be Hurwitz. On the other hand, the condition (3) suggests a guideline to design $Q(s)$ for robust stabilization against arbitrarily large but bounded uncertainties. This will be discussed further in the next subsection.

2.3. An application to Robust stabilization against arbitrarily large uncertainties

In this subsection, we demonstrate that one can always choose $D_Q(s;1)$ and $N_Q(s;1)$ such that $p_f(s)$ becomes Hurwitz for all $P(s) \in \mathcal{P}$. Note that the size of variations in the uncertain parameters can be arbitrarily large. First, choose l such that $l \ge r$, and choose $a_{l-1}, \ldots, a_1$ so that $\rho(s) := s^{l-1} + a_{l-1}s^{l-2} + \cdots + a_2 s + a_1$ is Hurwitz. (If $l = 1$, let $\rho(s) = 1$.) Then, find a $\bar{k} > 0$ such that $s\rho(s) + k = s^l + a_{l-1}s^{l-1} + \cdots + a_1 s + k$ is Hurwitz for all $0 < k < \bar{k}$. Indeed, such a $\bar{k}$ always exists. For instance, by applying the root locus technique to the transfer function $\frac{1}{s\rho(s)}$, it is seen[2] that there exists a $\bar{k} > 0$ such that $\frac{k}{s\rho(s)+k} (= [\frac{k}{s\rho(s)}]/[1 + \frac{k}{s\rho(s)}])$ is stable for all $0 < k < \bar{k}$. Now, define $\lambda(P) := \lim_{s\to\infty}\frac{P(s)}{P_n(s)}$, $\lambda_m := \min_{P(s)\in\mathcal{P}}\lambda(P)$, and $\lambda_M := \max_{P(s)\in\mathcal{P}}\lambda(P)$. Then, because of Assumption 1, $\lambda_M \ge \lambda_m > 0$. Finally, choose a_0 such that $0 < a_0 < \bar{k}/\lambda_M$, and let $D_Q(s;1) = s^l + a_{l-1}s^{l-1} + \cdots + a_1 s + a_0$ and $N_Q(s;1) = a_0$. Then, $p_f(s)$ is Hurwitz for all $P(s) \in \mathcal{P}$, because $p_f(s) = s\rho(s) + \lambda(P)a_0$ and $0 < \lambda(P)a_0 < \bar{k}$.

For example, suppose that, for a certain $\mathcal{P}$, $0.5 \le \lambda(P) \le 10$ and $r = 3$. (Note that $\lambda(P)$ depends only on $\underline{\alpha}_n$, $\bar{\alpha}_n$, $\underline{\beta}_{n-r}$, and $\bar{\beta}_{n-r}$.) If we choose $\rho(s) = (s+5)^2$, then it is seen that $s\rho(s)+k$ is Hurwitz for all $0 < k < 250$. Thus, by choosing $D_Q(s;1) = s(s+5)^2 + a_0$ and $N_Q(s;1) = a_0$ with $0 < a_0 < 25$, $p_f(s)$ is guaranteed to be Hurwitz for all $P(s) \in \mathcal{P}$.

3. Conclusion

In this paper, an almost necessary and sufficient condition for robust stability of closed loop systems with the DOB has been proposed under the condition that the Q-filter has a sufficiently

[1] In this case, it is necessary to inspect higher order terms in $\delta(s;\tau)$ and $\bar{\delta}(s;\tau)$ that are not considered in the proof. However, since those terms are closely coupled to plant uncertainty, it is hard to derive a simple and useful condition.

[2] We recall that the root locus includes all points along the real axis to the left of an odd number of poles and zeros of the transfer function under consideration. Since $1/[s\rho(s)]$ has no zeros and it has all poles in the LHP except one at the origin, it follows that, as the loop gain k is increased from zero by a small amount, the root locus starting at the origin moves to the left, while the others remain in the LHP.

small time constant. The proposed condition has revealed the following facts, which have been somewhat appreciated but never been rigorously proved in the literature (to the best of authors' knowledge): (1) for the closed loop stability, uncertain plant should be of minimum phase,[3] (2) for any given $C(s)$ that stabilizes the nominal plant, robust stabilization can be achieved by an appropriate choice of $Q(s)$ (without changing $C(s)$). Moreover, it has been shown that robust stabilization can always be achieved against arbitrarily large but bounded uncertainties, provided that the nominal system is stabilized by $C(s)$ and the uncertain plant is of minimum phase.

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Appendix

Rouché's Theorem (Flanigan, 1983): Let f and g be analytic on and inside a simple closed curve C, with $|g(s)| < |f(s)|$ on C. Then, $f(s)$ and $f(s) + g(s)$ have the same number of roots inside C (counting multiplicity). $\diamond$

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[3] Some extension to non-minimum phase systems, however, is presented in Shim, Jo and Son (2008) by modifying the standard structure of the DOB and sacrificing the performance of disturbance rejection.