Muestre que la función $f(x,y) = \frac{x(\cos y - 1)}{x^3+y^3}$ no tiene límite cuando (x,y) tiende a $(0,0)$

Tray 1 $y = 0$

Tray 2 $y = x$

Tray 1 $y = 0$

$$f(x,0) = \frac{x(\cos 0 - 1)}{x^3 + 0^3} = \frac{x(1-1)}{x^3} = \frac{0}{x^3} = 0$$

Tray 2 $y = x$

$$f(x,x) = \frac{x(\cos x - 1)}{x^3 + x^3} = \frac{x(\cos x - 1)}{2x^3}$$

$$f(x,x) = \frac{\cos x - 1}{2x^2}$$

$$f(x,x) \approx \frac{1 - \frac{x^2}{2} - 1}{2x^2} = \frac{-\frac{x^2}{2}}{2x^2} = -\frac{1}{4}$$

Tray $y = x$, $f(x,y) \to -\frac{1}{4}$. $x \to 0$

$\lim f(x,y)$: $(x,y) \to (0,0)$ No existe

Muestre que la función $f(x,y)=\frac{2x^2y}{x^4+y^2}$ no tiene límite cuando (x,y) tiende a $(0,0)$

TRAY 1: $x=0$

TRAY 2: $y=0$

$x=0$

$$f(0,y)=\frac{2\cdot 0^2\ y}{0^4+y^2}=\frac{0}{y^2}=0$$

$f(x,y)\to 0$ CUANDO $y\to 0$

TRAY 2 $y=0$

$$f(x,0)\ \frac{2x^2\ 0}{x^4+0^2}=\frac{0}{x^4}=0$$

$f(x,y)=0$ CUANDO $x\to 0$

TRAY 3 $y=x^2$

$$f(x,x^2)\ \frac{2x^2\ x^2}{x^4\ \ (x^2)^2}=\frac{2x^4}{2x^4}=1$$

$y=x^2$, $f(x,y)\to 1$ CUANDO $x\to 0$

$\lim f(x,y)$ CUANDO $(x,y)\to(0,0)$ NO EXISTE

Evalúe el límite de $f(x,y) = \frac{x^2-2xy+y^2}{x-y}$ cuando (x,y) tiende a $(1,1)$

$$x^2 - 2xy + y^2 = (x-y)^2$$

$$f(x,y) = \frac{(x-y)^{\cancel{2}}}{\cancel{x-y}} = x - y$$

$$\lim_{(x,y)\to(1,1)} (x-y) = 1 - 1 = 0$$ FUNCION INDEFINIDA

Evalúe el límite de $f(x,y) = \frac{y^3}{5x^4+y^2}$ cuando (x,y) tiende a $(0,0)$

$x = 0$

$$f(0,y) = \frac{y^3}{5 \cdot 0^4 + y^2} = \frac{y^3}{y^2} = y$$

$$\lim_{y \to 0} y = 0$$

$y = 0$

$$f(x,0) = \frac{0^3}{5x^4+0^2} = 0$$

$$\lim_{(x \to 0)} 0 = 0$$

$x = x^2$

$$f(x, x^2) = \frac{(x^2)^3}{5x^4 + (x^2)^2} = \frac{x^6}{5x^4 + x^4} = \frac{x^6}{6x^4} = \frac{x^2}{6}$$

$$\lim_{x \to 0} \frac{x^2}{6} = 0$$

$\lim f(x,y) = \frac{y^3}{5x^4+y^2}$ CUANDO $(x,y) \to (0,0)$ NO ESTA DEFINIDO

Encuentre el mayor conjunto donde la función $f(x, y, z) = \sqrt{xy} \tan z$ es continua

$\sqrt{xy}$ si es continua siempre que $xy \geq 0$

$xy \geq 0 \longrightarrow x \wedge y \geq 0$

$xy \geq 0 \longrightarrow x \wedge y \leq 0$

tan z = c mientras que $z \neq \frac{\pi}{2} + K\pi$ para cualquier K entero

$f(x, y, z)$ es continua si $xy \geq 0 \wedge z \neq \frac{\pi}{2} + K\pi$ para K

$f(x,y,z) = \{(x,y,z) \in \mathbb{R}^3 : xy \geq 0, z \neq \frac{\pi}{2} + K\pi, K \in \mathbb{Z}$

Verifique que $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$ para $f(x, y) = x^3 e^{-2y} + y^{-2} \cos x$

$$\frac{\partial f}{\partial x}$$

$$f(x, y) = x^3 e^{-2y} + y^{-2} \cos x$$

$$\frac{\partial^2 f}{\partial x} = 3x^2 e^{-2y} - y^{-2} \sin x$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial}{\partial x}\left(3x^2 e^{-2y} - y^{-2} \sin x\right) = -6x^2 e^{-2y} + 2y^{-3} \sin x$$

$$\frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial y} = -2x^3 e^{-2y} + 2y^{-3} \cos x$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial}{\partial x}\left(-2x^3 e^{-2y} + 2y^{-3} \cos x\right) = -6x^2 e^{-2y} - 2y^{-3} \sin x$$

$$\frac{\partial^2 f}{\partial y \partial x} \wedge \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial^2 f}{\partial y \partial x} = -6x^2 e^{-2y} + 2y^{-3} \sin x$$

$$\frac{\partial^2 f}{\partial x \partial y} = -6x^2 e^{-2y} + 2y^{-3} \sin x$$

$$\boxed{\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}}$$

Hallar $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial y \partial x}$ para $f(x,y) = x^2 y - 4x + 3seny$

$\frac{\partial f}{\partial x}$

$f(x,y) = x^2 y - 4x + 3 \sin y$

$\frac{\partial f}{\partial x} = 2xy - 4$

$\frac{\partial^2 f}{\partial x^2}$

$\frac{\partial^2 f}{\partial x^2} = 2y$

$\frac{\partial f}{\partial y}$

$\frac{\partial f}{\partial y} = x^2 + 3 \cos y$

$\frac{\partial^2 f}{\partial y}$

$\frac{\partial^2 f}{\partial y^2} = -3 \sin y$

$\frac{\partial^2 f}{\partial y \partial x}$

$\frac{\partial^2 f}{\partial y \partial x} = 2x$

$$\boxed{\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= 2y \\ \frac{\partial^2 f}{\partial y^2} &= -3 \sin y \\ \frac{\partial^2 f}{\partial y \partial x} &= 2x \end{aligned}}$$

En el análisis de algunos circuitos eléctricos se utiliza la fórmula $I = \frac{V}{\sqrt{R^2+L^2w^2}}$, donde I es la corriente, V la tensión o voltaje, R la resistencia, L la inductancia y w una constante positiva. Calcule e Interprete $\frac{\partial I}{\partial R}$ y $\frac{\partial I}{\partial L}$

$$I = \frac{V}{\sqrt{R^2+L^2w^2}}$$

$$u = R^2+L^2w^2 \qquad \sqrt{u} = (u)^{1/2} \quad \wedge \quad \frac{d}{dR}\sqrt{u} = \frac{1}{2}u^{-1/2}\cdot\frac{du}{dR}$$

$$u \quad R: \frac{du}{dR} = 2R$$

$$\frac{d}{dR}\sqrt{R^2+L^2w^2} = \frac{1}{2}(R^2+L^2w^2)^{-1/2}\cdot 2R = \frac{R}{\sqrt{R^2+L^2w^2}}$$

$$\frac{\partial I}{\partial R} = \frac{0\cdot\sqrt{R^2+L^2w^2} - V\cdot\frac{R}{\sqrt{R^2+L^2w^2}}}{R^2+L^2w^2} = -\frac{VR}{(R^2+L^2w^2)^{3/2}}$$

$$\frac{\partial I}{\partial R} \qquad \frac{\partial I}{\partial R} = \frac{VR}{(R^2L^2w^2)^{3/2}}$$

$$\frac{\partial I}{\partial L} \qquad L: \frac{\partial u}{\partial L} = 2Lw^2$$

$$\frac{d}{dL}\sqrt{R^2+L^2w^2} = \frac{1}{2}(R^2+L^2w^2)^{-1/2}\cdot 2Lw^2 = \frac{Lw^2}{\sqrt{R^2+L^2w^2}}$$

$$\frac{\partial I}{\partial L} = \frac{0\cdot\sqrt{R^2+L^2w^2} - V\cdot\frac{Lw^2}{\sqrt{R^2+L^2w^2}}}{R^2+L^2w^2} = -\frac{VLw^2}{(R^2+L^2w^2)^{3/2}}$$

$$\boxed{\frac{\partial I}{\partial L} = -\frac{VLw^2}{(R^2+L^2w^2)^{3/2}}}$$