1. Para $F^{\to}(x,y,z) = 2xy^3z^4 i + 3x^2y^2z^4 j + 4x^2y^3z^3 k$, encuentre una función f tal que $\nabla f = F$ y utilícela para evaluar $\int F \bullet dr$ a lo largo de la curva $C : x = t, y = t^2, z = t^3, 0 \le t \le 2$

$$\frac{\partial f}{\partial x} = 2xy^2z^4$$

$$\frac{\partial f}{\partial y} = 3x^2y^2z^4$$

$$\frac{\partial f}{\partial z} = 4x^2y^3z^3$$

$$\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$$

$$\frac{\partial}{\partial y} = (2xy^3z^4) = (6xy^2z^4)$$

$$\frac{\partial}{\partial x} = (3x^2y^2z^4) = (6xy^2z^4)$$

$$f(x,y,z) = \int 2xy^3z^4\,dx = x^2y^3z^4 + g_y(y,z)$$

$$f_2 = 3x^2y^2z^4 +$$

$$3x^2y^2z^4 + gy(y,z) = 3x^2y^2z^4$$

$gy(y,z) = 0$ implica que $g(y,z) = H(z)$

$$f_3 = \frac{\partial}{\partial z}(x^3y^3z^4 + H(z)) = 4x^2y^3z^4 + H'(z)$$

$$f_3 = 4x^2y^3z^3 \longrightarrow 4x^2y^3z^3 + H'(z) = 4x^2y^3z^3$$

$$H'(z) = 0 = H(z) = C$$

$$f(x,y,z) = x^2y^3z^4 + C$$

$$\int_C F\cdot dr = f(2,4,8) - f(0,0,0)$$

$$f(2,4,8) = (2^2)(4^3)(8^4) = 4 \cdot 64 \cdot 4096 = 1048576$$

$$f(0,0,0) = 0$$

$$\int_C F\cdot dr = 1048576$$

2. Trace el campo vectorial $F(x, y) = -xi + 2yj$ dibujando un diagrama

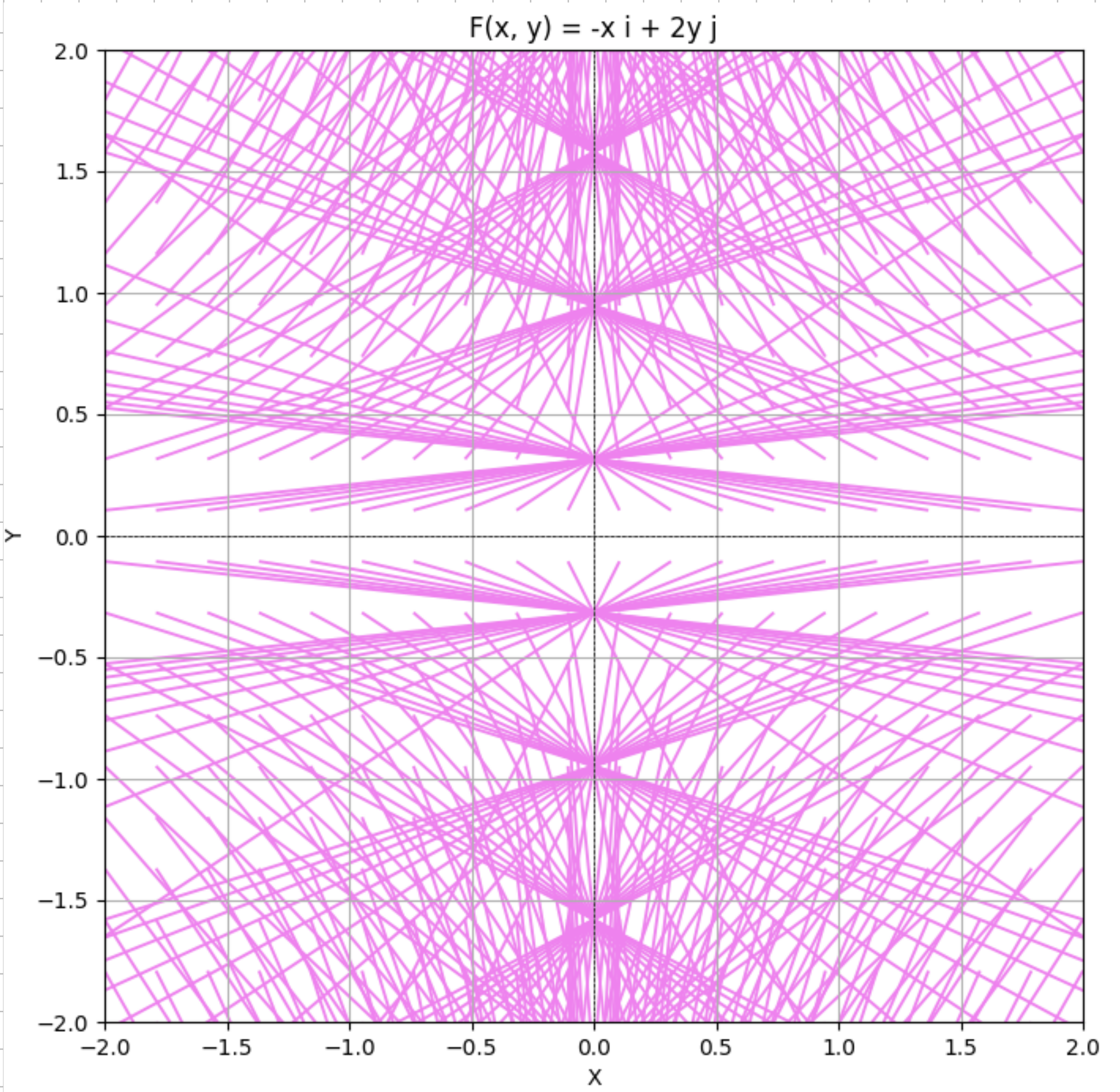
$(0, 0)$, $f(0, 0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$(1, 1)$, $f(1, 1) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$(2, 1)$, $f(2, 1) = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$(-1, 1)$, $f(-1, 1) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$(0, 2)$ $f(0, 2) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$



3. Calcule el trabajo realizado por $F(x,y) = x^2 i - xyj$ sobre una partícula que se mueve a lo largo de la trayectoria

$C : x = cos^3 t, y = sen^3 t$ desde (1,0) hasta (0,1)

$$w = \int_C f \, dr$$

$$\left. \begin{array}{l} x = \cos^3 T \\ y = sen^3 T \end{array} \right\} \quad 0 \leq t \leq \frac{\pi}{2}$$

$$dr = \frac{dx}{dT} \, dt \, i + \frac{dy}{dt} \, dt \, J$$

$$\frac{dx}{dT} = 3\cos^2 + (-sen\, T) = -3 \cos^2 T \quad sen\, T$$

$$\frac{dy}{dT} = 3 \, sen^2 T \, \cos T$$

$$dr = \left(-3 \cos^2 t \, \cos t\right) dT i + \left(3 \, sen^2 T \, \cos t\right) dT J$$

$$F(x,y) = \left(\cos^6 T\right) i - \left(\cos^3 T \; sen^3 T\right) J$$

$$F \, dr = \left(\cos^6 t\right)\left(-3 \cos^2 T \, sen\, t\right) + \left(-\cos^3 T + sen^3 t\right)\left(3 \, sen^2 + \cos t\right)$$

$$= -3 \cos^8 t \quad sen\, t - 3 \cos^4 T \quad sen^5 T$$

$$= -3 \cos^4 t \quad sen\, T \left(\cos^4 T + sen^4 T\right)$$

$$w = \int_1^0 3u^4\left(u^4 + (1-u^2)^2\right) du$$

$$u = \cos T$$

$$dU = -sen\, T \, dT$$

$$(1-u^2)^2 = 1 - 2u^2 + u^4$$

$$w \int_1^0 3u^4\left(u^4 + 1 - 2U + u^4\right) dU$$

$$w \int_1^0 3u^4\left(2u^4 - 2u^2 + 1\right) dU$$

$$w \int_1^0 \left(6u^8 - 6u^6 + 3u^4\right) du$$

$$\int_1^0 u^4 du = \frac{u^{n+1}}{n+1} \Big|_1^0 = \frac{0-1}{4+1} = \frac{1}{5}$$

$$w = 6\left(-\frac{1}{9}\right) - 6\left(-\frac{1}{7}\right) + 3\left(-\frac{1}{5}\right)$$

$$= -\frac{6}{9} + \frac{6}{7} - \frac{3}{5}$$

$$= -\frac{2}{3} + \frac{6}{7} - \frac{3}{5}$$

$$W = \frac{-70}{105} + \frac{90}{105} - \frac{63}{105} = \frac{-43}{105}$$

$MCD = 105$