Cambie el orden de integración y evalúe la integral resultante

$$\int_0^1 \int_x^1 e^{x/y}\,dydx$$

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$x \leq y \leq 1$

$0 \leq x \leq 1$

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$0 \leq x \leq y$
$0 \leq y \leq 1$

$$\int_0^1 \int_0^y e^{x/y}\,dxdy$$

$$\int_0^1 ey - y\,dy$$

$$\int ey - y\,dy$$

$$\int ey\,dy - \int y\,dy$$

$$\frac{ey^2}{2} - \int y\,dy$$

$$\frac{ey^2}{2} - \frac{y^2}{2}$$

$$\frac{ey^2 - y^2}{2}\Big|_0^1 \quad \frac{e\ 1^2 - 1^2}{2} - \frac{e\ 0^2 - 0^2}{2}$$

$$\frac{e-1}{2}$$

El volumen de la esfera $x^2 + y^2 + z^2 = 1$ está dado por la integral

$$V = 8 \int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1 - x^2 - y^2}\, dx dy$$

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$x = r\cos(\theta)$ ^ $y = r\sin(\theta)$

θ = Angulo desde el eje x

$dxdy = r\, dr\, d\theta$

r varia de 0 a 1

θ varia de 0 a $\frac{\pi}{2}$ 1 cuadrante

$$V = 8 \int_0^{\frac{\pi}{2}} \int_0^1 \sqrt{1-r^2}\, r\, dr\, d\theta$$

$$\int_0^1 r\sqrt{1-r^2}\, dr$$

$u = 1 - r^2 \longrightarrow du = -2r\, dr$ ^ $r dr = -\frac{1}{2} du$

si $r = 0,\ u = 1;$

si $r = 1,\ u = 0;$

$$\int_1^0 \sqrt{u}\left(-\frac{1}{2}\right) du = \frac{1}{2}\int_0^1 \sqrt{u}\, du$$

$$\frac{1}{2}\int_0^1 u^{\frac{1}{2}} du = \frac{1}{2}\left[\frac{2}{3} u^{\frac{3}{2}}\right]_0^1 = \frac{1}{3}$$

$$V = 8\int_0^{\frac{\pi}{2}} \frac{1}{3} d\theta = \frac{8}{3}\left[\theta\right]_0^{\frac{\pi}{2}} = \frac{8}{3} \cdot \frac{\pi}{2} = \boxed{\frac{4\pi}{3}}$$

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$$\int_0^2 \int_{y^2}^4 ycosx^2 dxdy$$

$$\int_0^2 \int_{y^2}^4 y\cos x^2 \, dx\,dy$$

$$y^2 \leq x \leq 4$$

$$0 \leq y \leq 2$$

$$\sqrt{y^2} \leq \sqrt{x} \leq \sqrt{4}$$

$$0 \leq y \leq 2$$

$$y \leq \sqrt{x} \leq 2$$

$$0 \leq y \leq \sqrt{x} \leq 2$$

$$0 \leq \sqrt{x} \leq 2$$

$$0^2 \leq \sqrt{x}^2 \leq 2^2 = 0 \leq x \leq 4$$

$$\int_0^4 \int_0^{\sqrt{x}} y\, x\cos(x^2)\, dy\, dx$$

$$\int y\, x\cos(x^2)\, dy$$

$$\cos(x^2) \cdot x \int y\, dy$$

$$\cos(x^2) \cdot x\, \frac{y^2}{2}$$

$$\frac{\cos(x^2)\, y^2}{2} \Bigg|_0^{\sqrt{x}} \quad \frac{\cos(x^2)\cdot \sqrt{x}^2}{2} - \frac{\cos(x^2)\cdot 0^2}{2} = \frac{\cos(x^2)\cdot x}{2}$$

$$\int \frac{\cos(x^2)\cdot x}{2}\, dx$$

$$\frac{1}{2}\int \cos(x^2)\cdot x\, dx$$

$$T = x^2$$

$$\frac{1}{2}\int \frac{\cos(T)}{2}\, dT$$

$$\frac{1}{2}\cdot\frac{1}{2}\int \cos(T)\, dT = \frac{1}{4}\int \cos(T)\, dT = \frac{1}{4}\sin(T) = \frac{1}{4}\sin(x^2) = \frac{\sin(x^2)}{4}$$

$$\frac{\sin(x^2)}{4}\Bigg|_0^4 = \frac{\sin(4^2)}{4} - \frac{\sin(0^2)}{4} = \frac{\sin(16)}{4}$$