1. Calcule la siguiente integral $\int_0^{\pi} \int_0^1 \int_0^{\sqrt{1-y^2}} y sen x dz dy dx$

$$\int_0^{\pi} \int_0^1 \int_0^{\sqrt{1-y^2}} y \, sen x \, dz dy dx$$

LIMITES DE INTEGRACION PARA z, 0 A $\sqrt{1-y^2}$

PARA y, 0 A 1

PARA x, 0 A π

$$\int_0^{\sqrt{1-y^2}} dz = \sqrt{1-y^2}$$

$$\int_0^{\pi} \int_0^1 y \, sen \, x \sqrt{1-y^2} \, dy dx$$

$$\int_0^1 y \sqrt{1-y^2} \, dy$$

$u = 1 - y^2$

$du = -2y dy$

$dy = \frac{du}{2y}$

$$\int_1^0 y\sqrt{u}\left(-\frac{du}{2y}\right) = \int_0^1 \frac{\sqrt{u}}{2} du$$

$$\frac{1}{2} \int_0^1 u^{\frac{1}{2}} du = \frac{1}{2}\left[\frac{2}{3} u^{3/2}\right]\Bigg|_0^1 = \frac{1}{3}$$

$$\int_0^{\pi} sen \, x \, dx$$

$$[-\cos x]\Big|_0^{\pi} = -\cos(\pi) + \cos(0) = 2$$

$$\frac{1}{3} \cdot 2 = \boxed{\frac{2}{3}}$$

2. Calcule el valor de la siguiente integral , $\int\int\int_E x dV$ donde E está acotada por el paraboloide $x = 4y^2 + 4z^2$ y el plano $x = 4$

$E \quad x = 4y^2 + 4z^2 \quad x = 4 \quad x$ varia desde $4y^2 + 4z^2$ hasta 4

$x = 4y^2 + 4z^2 = 4r^2$

x varia de $4r^2$ hasta 4

r varia de 0 a 1 $\quad (4r^2 \leq 4 \rightarrow r^2 \leq 1)$

θ varia de 0 a 2π

$$\iiint_E x\,dV = \int_0^{2\pi}\int_0^1\int_{4r^2}^4 x r\,dx\,dr\,d\theta$$

$$\int_{4r^2}^4 x\,dx = \left[\frac{x^2}{2}\right]\Bigg|_{4r^2}^4 = \frac{16}{2} - \frac{(4r^2)^2}{2} = 8 - 8r^4$$

$$\int_0^{2\pi}\int_0^1 (8 - 8r^4) r\,dr\,d\theta = \int_0^{2\pi}\left(\int_0^1 8r\,dr - \int_0^1 8r^5\,dr\right)d\theta$$

$$= \int_0^{2\pi}\left[4r^2 - \frac{8r^6}{6}\right]\Bigg|_0^1 d\theta = \int_0^{2\pi}\left(4 - \frac{4}{3}\right)d\theta$$

$$= \int_0^{2\pi}\frac{8}{3}\,d\theta = \frac{8}{3}\times 2\pi = \boxed{\frac{16\pi}{3}}$$

3. Encuentre el volumen del sólido que está dentro del cilindro $x^2 + y^2 = 1$ y la esfera $x^2 + y^2 + z^2 = 4$.

$x^2 + y^2 = 1$ radio 1 se extiende indefinidamente en z

$x^2 + y^2 + z^2 = 4$

$x = r\cos(\theta)$,

$y = r\sin(\theta)$

$z = z$

$r = 1$

$r^2 + z^2 = 4$

θ varía de 0 a 2π

r varía de 0 a 1

z $\quad z^2 = 4 - r^2 \quad z = \pm\sqrt{4 - r^2}$ z varía $-\sqrt{4-r^2}$ a $\sqrt{4-r^2}$

$r\, dr\, d\theta\, dz$

$$V = \int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r}}^{\sqrt{4-r}} r\, dz\, dr\, d\theta$$

z es directo $2\sqrt{4-r^2}$ al integrar de $-\sqrt{4-r^2}$ a $\sqrt{4-r^2}$

$$V = \int_0^{2\pi}\int_0^1 2r\sqrt{4-r^2}\, dr\, d\theta$$

$$V = 2\pi\int_0^1 2r\sqrt{4-r^2}\, dr$$

$$V = 2\pi\int_4^3 \sqrt{u}\,\frac{-du}{2} = 2\pi\left[-\frac{2}{3}u^{\frac{3}{2}}\right]\Bigg|_4^3 = \frac{16\pi}{3}$$

$u = 4$

$du = -2r\, dr$

4. Halle el volumen del sólido que está dentro de la esfera $x^2 + y^2 + z^2 = 4$, por encima del plano xy y por abajo del cono $z = \sqrt{x^2 + y^2}$

$x^2 + y^2 + z^2 = 4$

$z = \sqrt{x^2 + y^2}$

$x = r\cos\theta$,

$y = r\sin\theta$

$z = z$

La de esfera $r^2 + z^2 = 4$

La del cono $z = r$

Angulo θ varía de 0 a 2π

Radio r varía 0 hasta la intersección entre el cono y la esfera $r^2 + r^2 = 4$

$r = \sqrt{2}$

z varía de r hasta $\sqrt{4-r^2}$

$$V = \int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

$$\int_r^{\sqrt{4-r^2}} r\,dz = r\left[\sqrt{4-r^2} - r\right]$$

$$\int_0^{\sqrt{2}} r\left[\sqrt{4-r^2} - r\right] dr$$

$$\int_0^{2\pi} \frac{-4\sqrt{2}+8}{3}\,d\theta$$

$$V = \frac{2\pi\left(4\sqrt{2}+8\right)}{3}$$

5. La integral $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi}\rho^2 sen\phi d\rho d\phi d\theta$, representa el volumen que yace arriba del cono $z=\sqrt{x^2+y^2}$ y debajo de la esfera $x^2+y^2+z^2=z$

lim

θ varia de 0 a 2π

ϕ varia de 0 a $\frac{\pi}{4}$

ρ varia de 0 a $\cos\phi$

cono $\quad z=\sqrt{x^2+y^2}$

esfera $\quad x^2+y^2+z^2=z = x^2+y^2+z^2-z=0$

$x^2+y^2+\left(z-\frac{1}{2}\right)^2=\frac{1}{4}$ centro en $\left(0,0,\frac{1}{2}\right)$ y $r=\frac{1}{2}$

$x=\rho\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

$z=\cos\phi$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\cos\phi}\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$\int_0^{\cos\phi}\rho^2 d\rho=\frac{1}{3}\rho^3\Big|_0^{\cos\phi}=\frac{1}{3}(\cos\phi)^3$$

$$\int_0^{\frac{\pi}{4}}\frac{1}{3}(\cos\phi)^3\sin\phi\, d\phi$$

$u = \cos\phi\, du$
$du=-\sin\phi\, d\phi$
$u=\cos(0)=1$ a $u=\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

$$\int_1^{\frac{\sqrt{2}}{2}}\frac{1}{3}u^3(-du)=\frac{1}{3}\int_1^{\frac{\sqrt{2}}{2}}u^3\, du=\frac{1}{12}\left(-\frac{3}{4}\right)=-\frac{1}{16}$$

$$\int_0^{2\pi}-\frac{1}{16}\, d\theta=-\frac{1}{16}\cdot 2\pi=-\frac{\pi}{8}$$