Pregunta 2

Si $f(x, y) = \ln y$, entonces $\nabla f(x, y) = \frac{1}{y}$

$f(x,y) = \ln y$

$f(x,y) = \quad \nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

$\frac{\partial f}{\partial x} = 0$ $\quad \ln y$ no depende de x

$\frac{\partial f}{\partial y} = \frac{1}{y}$

$\nabla f(x,y) = \left(0, \frac{1}{y}\right)$

Si (2,1) es un punto crítico de f y $f_{xx}(2,1)f_{yy}(2,1) < [f(2,1)]^2$ entonces f tiene un punto silla en (2, 1).

(2,1) PUNTO CRITICO f

$f_{xx}(2,1)\, f_{yy}(2,1) < [f(2,1)]^2$

PUNTO CRITICO = $\nabla f(x,y) = 0$ o INDEFINIDO

$f(2,1) = \ln 1 = 0$

$\underbrace{f_{xx}(2,1) \wedge f_{yy}(2,1)}_{\text{NEGATIVO}} < 0^2 = 0$

en el PUNTO (2,1)

f Tiene PUNTO sillo en (2,1)

Pregunta 4

Determine los valores máximos y mínimos absolutos de $f(x, y) = 2x^3 + 4y^4$ sobre el conjunto $D = \{(x, y) | x^2 + y^2 \leq 1\}$

$$f_x = \frac{\partial}{\partial x}(2x^3 + 4y^4) = 6x^2, \quad f_y = \frac{\partial}{\partial y}(2x^3 + 4y^4) = 16y^3$$

$$6x^2 = 0 \Rightarrow 0, \quad 16y^3 = 0 \Rightarrow 0$$

$$f(0,0) = 2 \cdot 0^3 + 4 \cdot 0^4 = 0$$

$$D = x^2 + y^2 = 1 \qquad x = \cos(\theta), \quad y = \sin(\theta)$$

$$f(\cos(\theta), \sin(\theta)) = 2\cos^3(\theta) + 4\sin^4(\theta) \qquad \theta \in [0, 2\pi]$$

$$f(\cos(\theta), \sin(\theta))$$

$$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

$$f(1,0) = 2, \quad f(0,1) = 4, \quad f(-1,0) = -2, \quad f(0,-1) = 4$$

MAXIMO ABSOLUTO DE f EN D es 4 y el MINIMO -2

Pregunta 5

Calcule los valores máximo y mínimo locales, y punto o puntos sillas de la función $f(x,y) = (x-y)(1-xy)$

$f(x,y) = (x-y)(1-xy)$

$f_x = \frac{\partial}{\partial x}\left[(x-y)(1-xy)\right] = (1-xy)-(x-y)y = 1-2xy+y^2$

$f_y = \frac{\partial}{\partial y}\left[(x-y)(1-xy)\right] = -(1-xy)-(x-y)x = 1-2xy+x^2$

$1-2xy+y^2=0$

$1-2xy+x^2=0$

$y^2-x^2=0 \Rightarrow y = \pm x$

$1-2x^2+x^2=0 \Rightarrow 1-x^2=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1,\ y=\pm 1$

$1-2x^2(-x)+x^2=0 \Rightarrow 1+x^2=0$ no tiene soluciones reales

$f_{xx} = \frac{\partial}{\partial x}(1-2xy+y^2) = -2y$

$f_{yy} = \frac{\partial}{\partial y}(-1+2xy-x^2) = 2x$

$f_{xy} = f_{yx} = (1-2xy+y^2) = -2x+2y$

$H = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} -2y & -2x+2y \\ -2x+2y & 2x \end{vmatrix}$

$H(1,1) \wedge (-1,-1)$

$H \begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} = -4$

Pregunta 6

Mediante la regla de la cadena encuentre $\frac{\partial z}{\partial s}$ y $\frac{\partial z}{\partial t}$, donde $z = \tan(u/v)$, $u = 2s + 3t$, $v = 3s - 2t$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial s}$$

$$\frac{\partial z}{\partial T} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial T} + \frac{\partial z}{\partial v}\frac{\partial u}{\partial T}$$

$$\frac{\partial u}{\partial s} = 2, \quad \frac{\partial u}{\partial T} = 3, \quad \frac{\partial v}{\partial s} = 3, \quad \frac{\partial v}{\partial T} = -2$$

$$\frac{\partial z}{\partial u} = \frac{1}{v}\sec^2\left(\frac{u}{v}\right), \qquad \frac{\partial z}{\partial v} = \frac{u}{v^2}\sec^2\left(\frac{u}{v}\right)$$

$$\frac{\partial z}{\partial s} = \frac{1}{v}\sec^2\left(\frac{u}{v}\right)\cdot 2 + \left(-\frac{u}{v^2}\sec^2\left(\frac{u}{v}\right)\right)\cdot 3$$

$$\frac{\partial z}{\partial T} = \frac{1}{v}\sec^2\left(\frac{u}{v}\right)\cdot 3 + \left(-\frac{u}{v^2}\sec^2\left(\frac{u}{v}\right)\right)\cdot(-2)$$