Question #5

Transform

$\lambda \equiv$ rate of housefire calls (per month)

$n \equiv$ total calls in a one year period

$t \equiv$ time interval (months)

assume:

- part(b): $\lambda = 171$ calls/month — Poisson Distribution
- $n = 2050$ calls
- $t = 12$ months

(A) part a: $\frac{2050 \text{ calls}}{12 \text{ months}}$ = estimated rate of $\boxed{170.8\overline{3} \text{ calls/month}}$

(B) part b: $\lambda = 171$; use a Poisson Distribution

$t = 12 \times 171 = 2052 = n$

$$\frac{n}{12 \cdot 171} \pm \frac{2\sqrt{n}}{12 \cdot 171} = 1$$

Upper:

Lower:

(C)

$\neq 171$

$$\frac{2050}{12\lambda} \pm \frac{2\sqrt{2050}}{12\lambda} = 1$$

upper: 178.37

Lower: 128

Question 6

Transform

$$\Pr\{N_t = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

(A) **part(a):** $X \equiv$ calls in time interval t with rate λ

$X \sim \text{Pois}(\lambda t)$

$$P(X = x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$$

$$E(X) = \sum_x x \cdot p_x \rightarrow \sum_x x \frac{e^{-\lambda t}(\lambda t)^x}{x!} \rightarrow (\lambda t) e^{-\lambda t} \sum_{x=1} \frac{(\lambda t)^x (\lambda t)^{-1}}{(x-1)!}$$

$$\rightarrow \lambda t e^{-\lambda t} \sum_{x=1} \frac{(\lambda t)^{x-1}}{(x-1)!} \rightarrow u = x-1 \rightarrow \lambda t e^{-\lambda t} \sum_{u=0} \frac{(\lambda t)^u}{u!} \rightarrow e^{\lambda t}$$

$$\lambda t \cancel{e^{-\lambda t}} \cdot \cancel{e^{\lambda t}} \qquad \boxed{E(X) = \lambda t}$$

$$\star\, V(X) = E(X^2) - (E(X))^2 \rightarrow -(\lambda t)^2$$

$$x^2 \cdot p(x) \rightarrow \sum_x x^2 \frac{e^{-\lambda t}(\lambda t)^x}{x!} \rightarrow e^{-\lambda t} \sum_x \frac{x\, \lambda t^x}{(x-1)!}$$

(B) **part b:** mean $= 171 \pm 18$

possible values $= [153, 189]$

(See R for Code) Varies As much as $18 = \underline{0.843}$

Ⓒ <u>part c:</u> (See R-Code)

To account for .953, λ could vary between <u>145</u> and <u>197</u>.

Ⓓ <u>part d:</u> — Estimation would be better for a larger Dataset (ie a year) where an exact method may be better for short-term.

Problem 11

Transform

Assumption: - Detected + Acquired, once the plane will get fired o[n]

Low	high
fire 20x	fire 5x3 times

- full 5/1 minute interval to be fired on.
- weapons have ability to fire at numerous targets
- P_{detect} and $P_{acquire}$ are independent.

$$P(A \cap D) = P(A/D) = P(D) = P(A) \cdot P(D)$$

$$P(A/D) = P(A)$$

Ⓐ a) low $0.90 \times 0.80 = \underline{0.72} \times 16 = 11.5 = E(X)$ $\qquad 4.5 + 4.02 = \boxed{8.52}$ AC

11.5 AC $\qquad 1 - 0.35 = 0.65$

$X \sim bin(n=20, p=0.1) \quad P(X=0) = \binom{20}{0}(0.05)^0(0.95)^{20} = 0.35$

low = 8.52 Aircraft survive

high: $0.75 \times 0.95 = \underline{0.7125} \times 16 = 11.4 = E(X)$ $\qquad 4.6 + 0 = \boxed{4.6 \text{ AC}}$

11.4 AC

$X \sim bin(n=15, p=0.1) \quad P(X=0) = \binom{15}{0}(0.70)^0(0.30)^{15} = (1.4 \times 10^{-8}) \times 11.4$

high = 4.6 Aircraft survive

Ⓑ b) low: 8.52

$$P(X=0) = \binom{8.52}{0} = (0.7)(0.3)^{8.52} = 0.000066 \rightarrow \text{chance Aircraft d[...]}$$

$$1 - P(X=0) = \boxed{0.99}$$

high: 4.6

$$P(X=0)\binom{4.6}{0} = (0.7)^0(0.3)^{4.6} = 0.00393 \rightarrow \text{chance AC fail}$$

$$1 - P(X=0) = \boxed{.996}$$

Low: 99% Success RATE

high: 99.6% Success Rate

Ⓒ c)

I apologize sir; this is how far I got before Running out of time.