```python
import pandas as pd
import matplotlib.pyplot as plt
from tqdm import tqdm

import torch
from torch import nn
import torchvision
import torchvision.transforms as transforms

import numpy as np
import random
np.random.seed(0)
torch.manual_seed(0)
random.seed(0)
```

Automatic differentiation with pytorch

$$y=f(x)=5x^2+1$$

$$\frac{\partial y}{\partial x}=10x$$

$$\frac{\partial y}{\partial x}\text{¿}_{x=1}=10$$

```python
x = torch.tensor(1.0, requires_grad=True)
```

```python
y = 5 * x ** 2 + 1
```

```python
print("Before calling `backward`, x.grad: ", x.grad)
```

```
Before calling `backward`, x.grad:  None
```

```python
y.backward()
```

```python
print("After calling `backward`, x.grad: ", x.grad)
```

```
After calling `backward`, x.grad:  tensor(10.)
```

Another example

$$y=f(x1,x2)=5*x_1^2+x_2+1$$

$$\frac{\partial y}{\partial x_1}=10x_1$$

$$\frac{\partial y}{\partial x_2}=1$$

$$\frac{\partial y}{\partial x_1} \text{¿}_{x_1=1, x_2=10} = 10$$

$$\frac{\partial y}{\partial x_2} \text{¿}_{x_1=1, x_2=10} = 1$$

```python
# y = 5*x**2 + 1

x1 = torch.tensor(1.0, requires_grad=True)
x2 = torch.tensor(10.0, requires_grad=True)

y = 5 * x1**2 + 1 + x2

print("Before calling `backward`, (x1.grad, x2.grad): ", (x1.grad,
x2.grad))
```

```
Before calling `backward`, (x1.grad, x2.grad):  (None, None)
```

```python
y.backward()

print("After calling `backward`, (x1.grad, x2.grad): ", (x1.grad,
x2.grad))
```

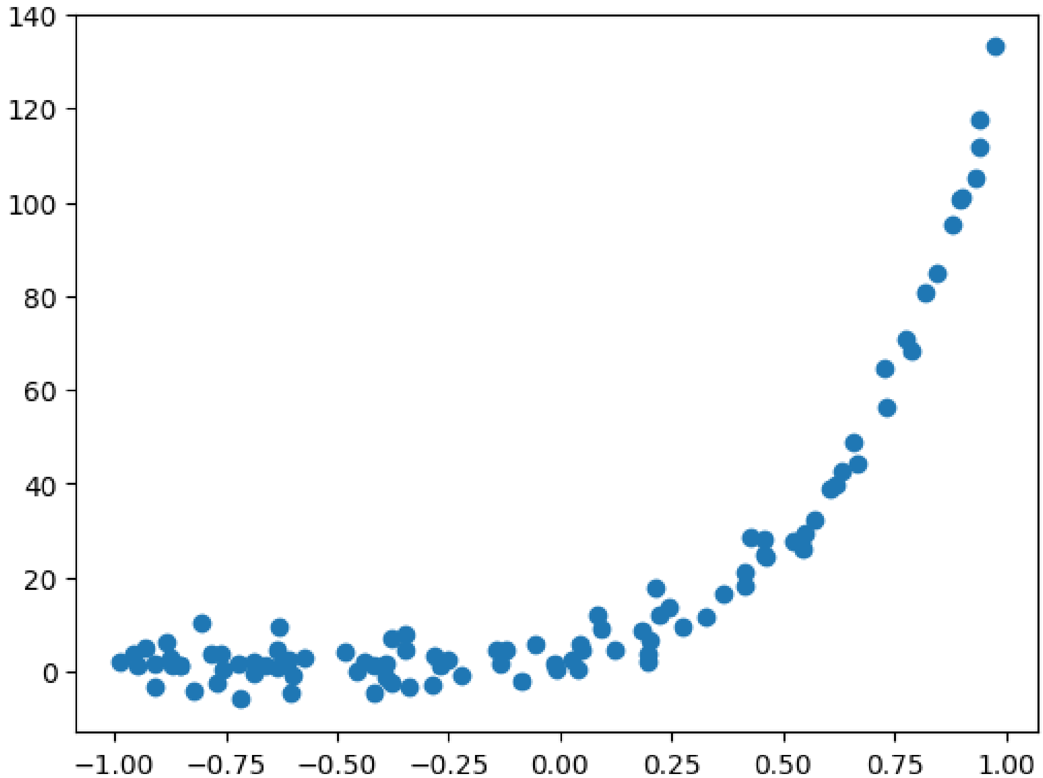
```
After calling `backward`, (x1.grad, x2.grad):  (tensor(10.),
tensor(1.))
```

Working with synthetic data

Data

```python
df = pd.read_csv("data.csv")

plt.scatter(df["x"], df["y"])
```

```
<matplotlib.collections.PathCollection at 0x7d9b56c475d0>
```



Model

```
a = torch.randn(1, requires_grad=True)
b = torch.randn(1, requires_grad=True)

X = torch.tensor(df["x"])[:, None].float()
Y = torch.tensor(df["y"])[:, None].float()


model = lambda x: a + b * x
weights = [a, b]
```

Loss function

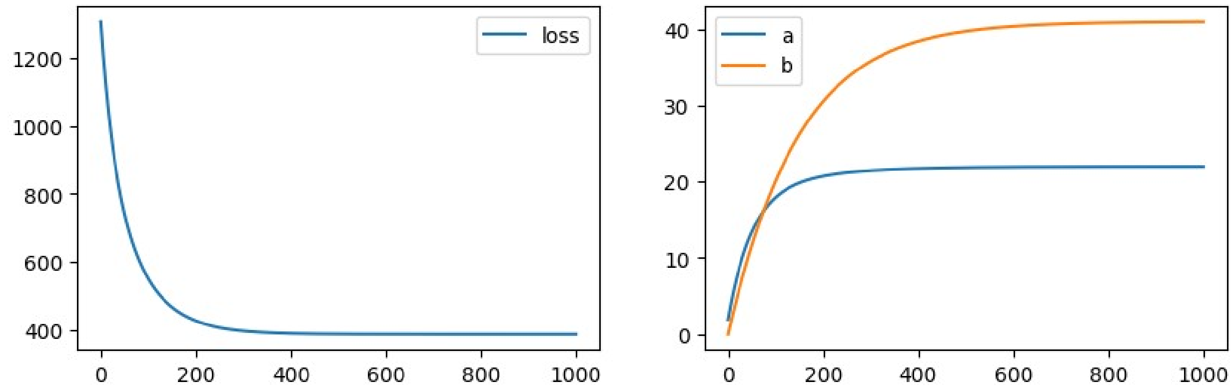
```
loss_fn = lambda y_pred, y_gt: ((y_pred - y_gt) ** 2).mean()
```

Optimizer

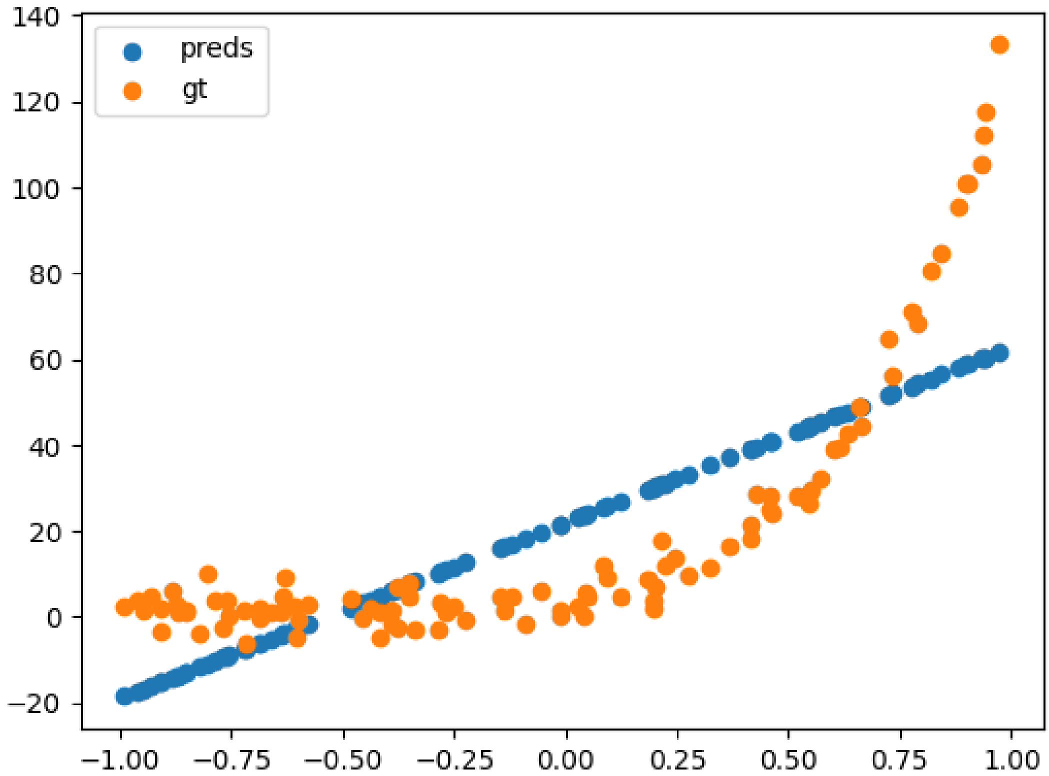
```
optimizer = torch.optim.SGD(weights, lr=1e-2)
```

```
def train_one_epoch(model, optimizer, X, Y, BS=100):
    indexes = np.random.permutation(len(X))
    losses = []
    a_values = []
    b_values = []
    for i, batch_start in enumerate(range(0, len(X), BS)):
        optimizer.zero_grad()

        x = X[indexes[batch_start:batch_start+BS]]
        y = Y[indexes[batch_start:batch_start+BS]]

        y_preds = model(x)

        loss = loss_fn(y_preds, y)
        loss.backward()

        optimizer.step()

        losses.append(loss.item())
        a_values.append(a.item())
        b_values.append(b.item())
        # print(f"step={i}, loss={loss.item():0.03f},
ws={[round(x.item(), 2) for x in weights]}")
    return losses, a_values, b_values

losses = []
a_values = []
b_values = []

pbar = tqdm(range(1000))

for ep in pbar:
    epoch_loss, epoch_a, epoch_b = train_one_epoch(model, optimizer,
X, Y)
    losses.extend(epoch_loss)
    a_values.extend(epoch_a)
    b_values.extend(epoch_b)
    if (ep + 1) % 100 == 0:
        pbar.set_description(f"loss={losses[-1]:0.03f}, a={a_values[-
1]:0.03f}, b={b_values[-1]:0.03f}")
plt.figure(figsize=(10, 3))
plt.subplot(1, 2, 1)
plt.plot(losses, label="loss")
plt.legend()
plt.subplot(1, 2, 2)
plt.plot(a_values, label="a")
plt.plot(b_values, label="b")
plt.legend()
```

```
loss=386.797, a=21.939, b=40.924: 100%|██████████| 1000/1000
[00:00<00:00, 1801.47it/s]

<matplotlib.legend.Legend at 0x7d9b54a83a10>
```




```
with torch.no_grad():
    y_preds = model(X)
    plt.scatter(X, y_preds, label="preds")
    plt.scatter(X, Y, label="gt")

plt.legend()

<matplotlib.legend.Legend at 0x7d9b54b0b2d0>
```

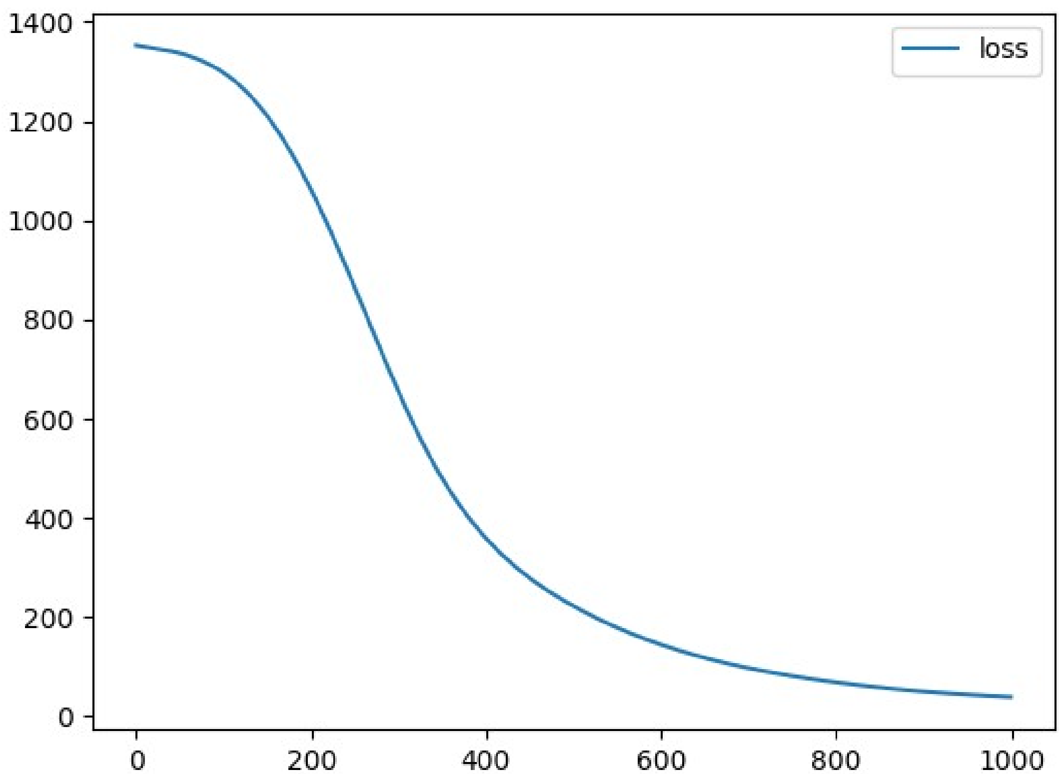



```
class Model(nn.Module):
    def __init__(self, input_shape, output_shape, hidden_act="relu",
n_layers=3, hidden_dim=128):
        super().__init__()
        self.layers = nn.ModuleList(
            [
                nn.Linear(input_shape if i == 0 else hidden_dim,
hidden_dim)
                for i in range(n_layers)
            ]
        )
        if hidden_act == "relu":
            self.hidden_act = nn.functional.relu
        else:
            raise ValueError

        self.output_layer = nn.Linear(hidden_dim, output_shape)

    def forward(self, x):
        for layer in self.layers:
            x = layer(x)
            x = self.hidden_act(x)
        output = self.output_layer(x)
        return output
```

```python
model = Model(input_shape=1, output_shape=1)

optimizer = torch.optim.Adam(model.parameters(), lr=1e-4)

losses = []
a_values = []
b_values = []

pbar = tqdm(range(1000))
for ep in pbar:
    epoch_loss, epoch_a, epoch_b = train_one_epoch(model, optimizer,
X[:, None], Y[:, None])
    losses.extend(epoch_loss)
    a_values.extend(epoch_a)
    b_values.extend(epoch_b)
    if (ep + 1) % 100 == 0:
        pbar.set_description(f"loss={losses[-1]:0.03f}")
plt.plot(losses, label="loss")
plt.legend()
```

```
loss=39.172: 100%|██████████| 1000/1000 [00:03<00:00, 332.57it/s]
```

```
<matplotlib.legend.Legend at 0x7d9b541b2d50>
```



Exercise

You are tasked with training the following models on the given dataset.

1. $y = a + b * x $
2. $y=a+b*x+c*x^2$
3. $y=ae^{bx}$
4. a MLP. If you decide to use the `Model` class above, you must change one or more of the number of layers, the number of neurons or the activation function.

Deliverables

- A plot with the learning curves depicting 1k epochs for each model.
- A single plot containing the predictions of the trained models and the ground truth data.
- A brief write-up explaining how the choice of the model (hypothesis function) influences the final outcome, along with suggestions for improving the results.

1- y=a+b∗x

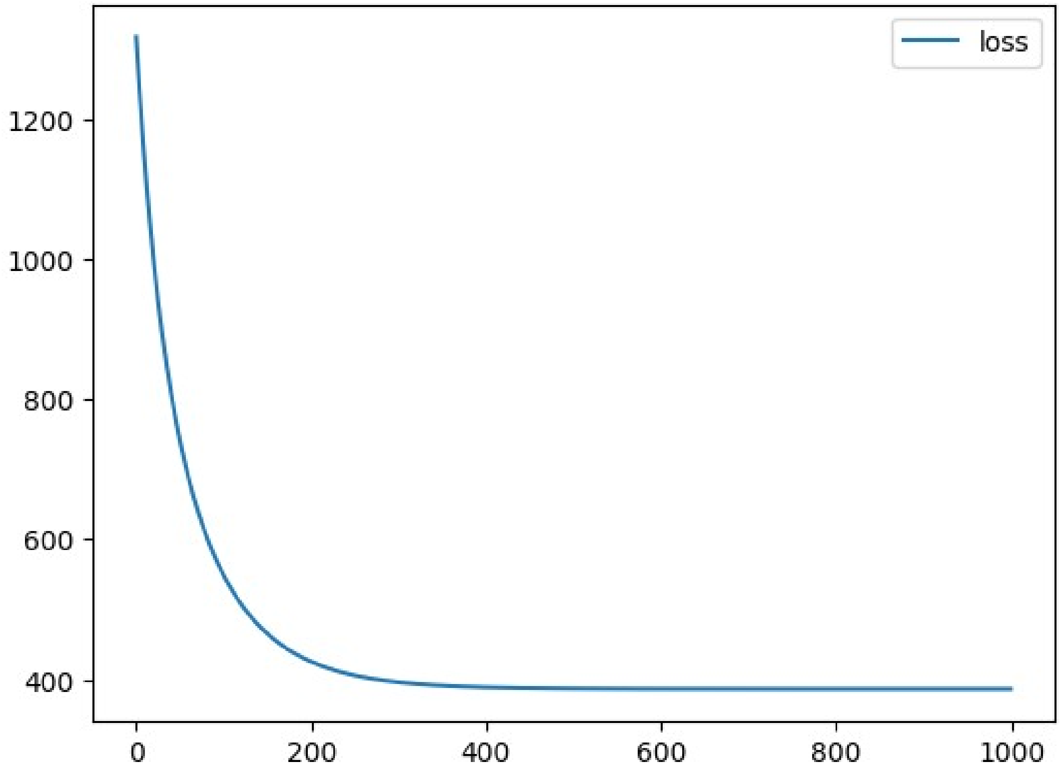
```
a = torch.randn(1, requires_grad=True)
b = torch.randn(1, requires_grad=True)

X = torch.tensor(df["x"])[:, None].float()
Y = torch.tensor(df["y"])[:, None].float()

model = lambda x: a + b * x
weights = [a, b]

loss_function = lambda y_pred , y_actual : ((y_pred - y_actual) **
2).mean()

optimizer = torch.optim.SGD(weights , lr=0.001)

def train_one_epoch(model , optimizer , X ,Y , batch_size =100):
  index = np.random.permutation(len(X))
  x_shuffled = X[index]
  y_shuffled = Y[index]

  losses = []
  a_values = []
  b_values = []


  for i in range(0 , len(X) , batch_size):
    optimizer.zero_grad()
    x = x_shuffled[i : i +batch_size]
    y = y_shuffled[i : i +batch_size]
```

```
    y_pred = model(x)
    loss = loss_function(y_pred , y)
    loss.backward()
    optimizer.step()

    losses.append(loss.item())
    a_values.append(a.item())
    b_values.append(b.item())

  return losses , a_values , b_values

losses =[]
a_values = []
b_values = []

pbar = tqdm(range(100000))

for ep in pbar:
  epoch_loss , epoch_a , epoch_b  = train_one_epoch(model ,
optimizer , X ,Y , batch_size =100)
  losses.extend(epoch_loss)
  a_values.extend(epoch_a)
  b_values.extend(epoch_b)

  if (ep + 1) % 100 == 0:
     pbar.set_description(f"loss={losses[-1]:0.03f}, a={a_values[-
1]:0.03f}, b={b_values[-1]:0.03f}")
```

```
loss=386.796, a=21.942, b=40.961: 100%|██████████| 100000/100000
[01:02<00:00, 1596.05it/s]
```

```
plt.plot(losses, label="loss")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b5414bf10>
```

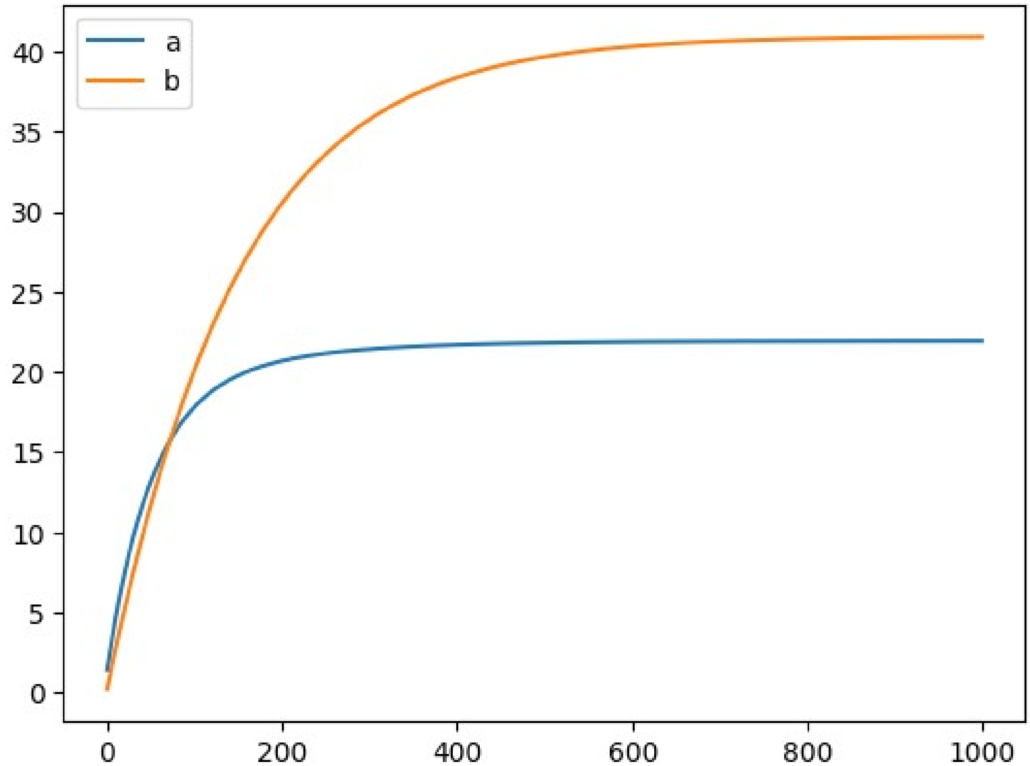



```
losses[-1]
```

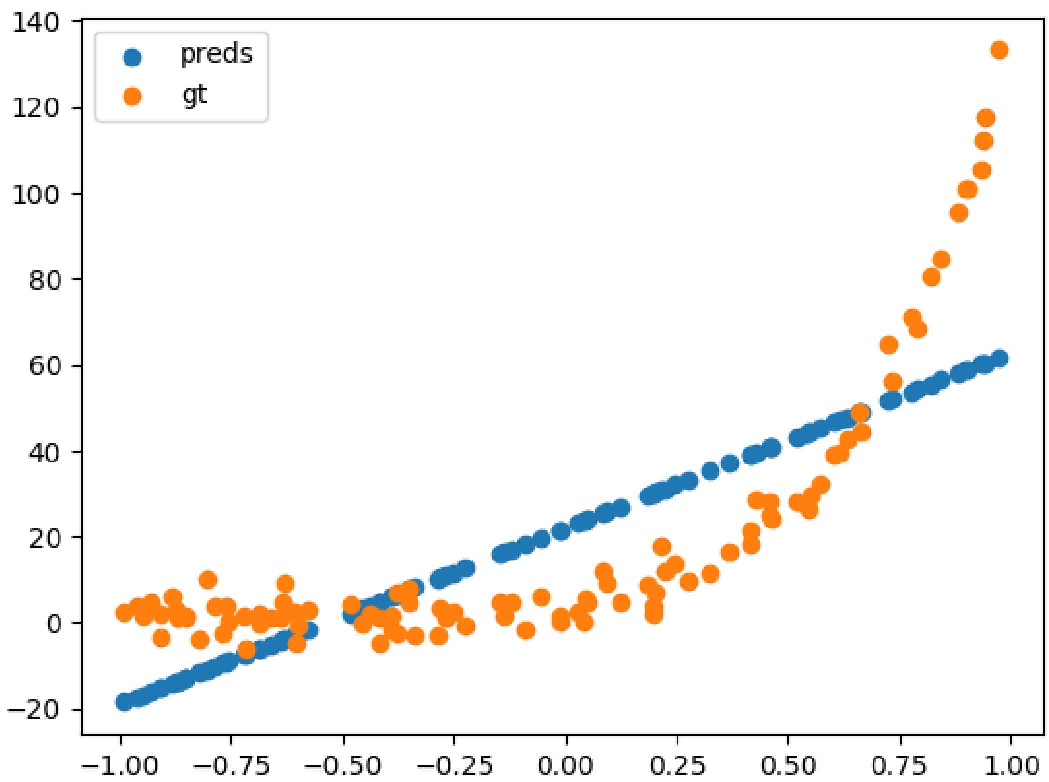
```
386.79644775390625
```

```
plt.plot(a_values, label="a")
plt.plot(b_values, label="b")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b52d57710>
```




```python
with torch.no_grad():
    y_preds = model(X)
    plt.scatter(X, y_preds, label="preds")
    plt.scatter(X, Y, label="gt")

plt.legend()

<matplotlib.legend.Legend at 0x7d9b52dce350>
```



2- y=a+b∗x+c∗x2

```
a = torch.randn(1 , requires_grad = True)
b = torch.randn(1 , requires_grad = True)
c = torch.randn(1 , requires_grad = True)

X = torch.tensor(df["x"])[:, None].float()
Y = torch.tensor(df["y"])[:, None].float()

model = lambda x : a + (b * x) + c * ( x ** 2)
weights = [a , b , c]

loss_function = lambda y_pred , y_actual : ((y_pred - y_actual) **
2).mean()

optimizer = torch.optim.SGD(weights , lr=0.001)

def train_one_epoch(model , optimizer , X ,Y , batch_size =100):
  index = np.random.permutation(len(X))
  x_shuffled = X[index]
  y_shuffled = Y[index]

  losses = []
  a_values = []
  b_values = []
```

```
  c_values = []

  for i in range(0 , len(X) , batch_size):
    optimizer.zero_grad()
    x = x_shuffled[i : i +batch_size]
    y = y_shuffled[i : i +batch_size]

    y_pred = model(x)
    loss = loss_function(y_pred , y)
    loss.backward()
    optimizer.step()

    losses.append(loss.item())
    a_values.append(a.item())
    b_values.append(b.item())
    c_values.append(c.item())

  return losses , a_values , b_values , c_values

losses =[]
a_values = []
b_values = []
c_values = []

pbar = tqdm(range(100000))

for ep in pbar:
  epoch_loss , epoch_a , epoch_b , epoch_c = train_one_epoch(model , optimizer , X ,Y , batch_size =100)
  losses.extend(epoch_loss)
  a_values.extend(epoch_a)
  b_values.extend(epoch_b)
  c_values.extend(epoch_c)

  if (ep + 1) % 100 == 0:
    pbar.set_description(f"loss={losses[-1]:0.03f}, a={a_values[-1]:0.03f}, b={b_values[-1]:0.03f}")
```

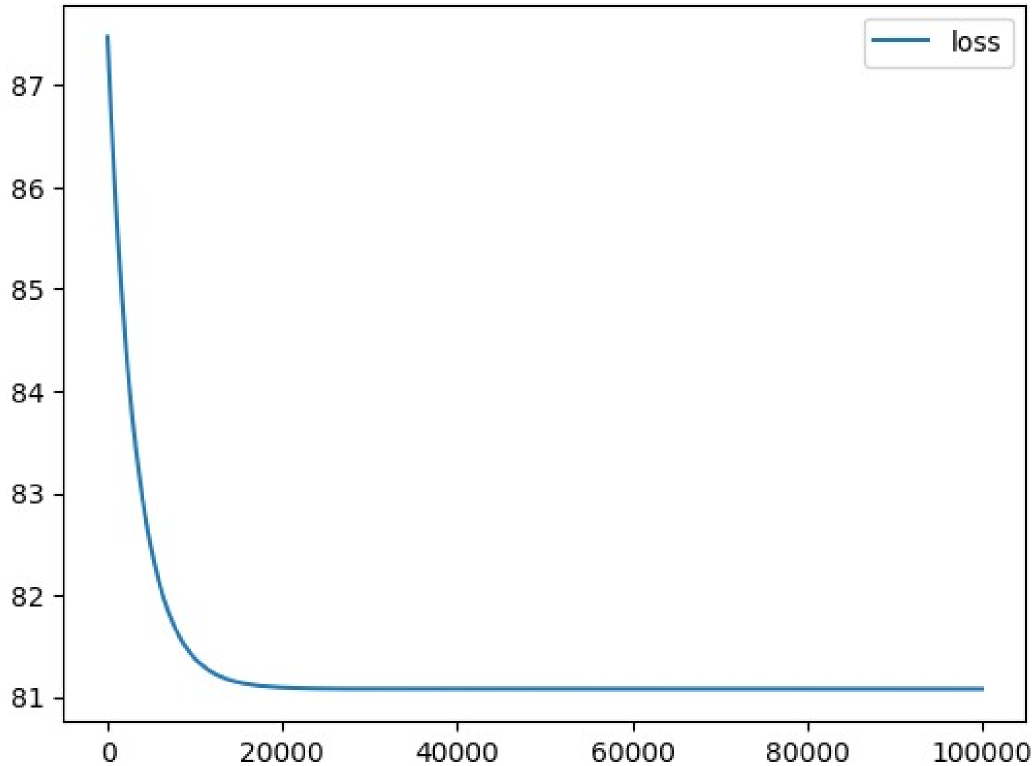
```
loss=81.080, a=1.155, b=44.111: 100%|██████████| 100000/100000
[01:09<00:00, 1444.35it/s]
```

```
losses[-1]
```

```
81.07974243164062
```

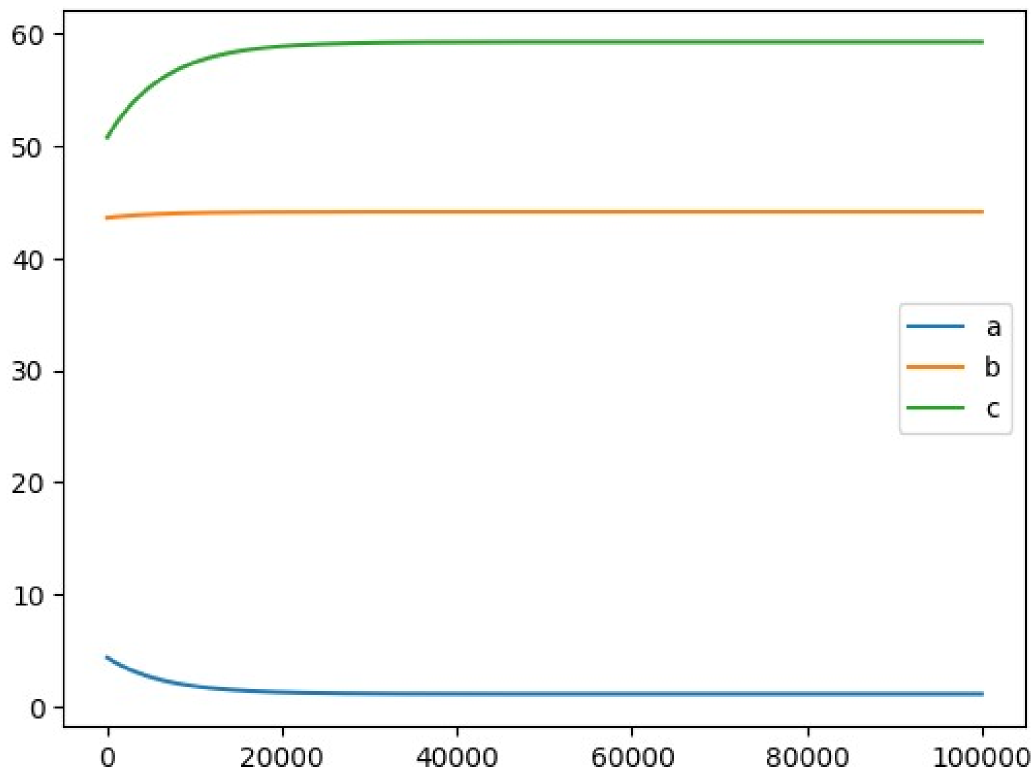
```
plt.plot(losses, label="loss")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b4d706390>
```

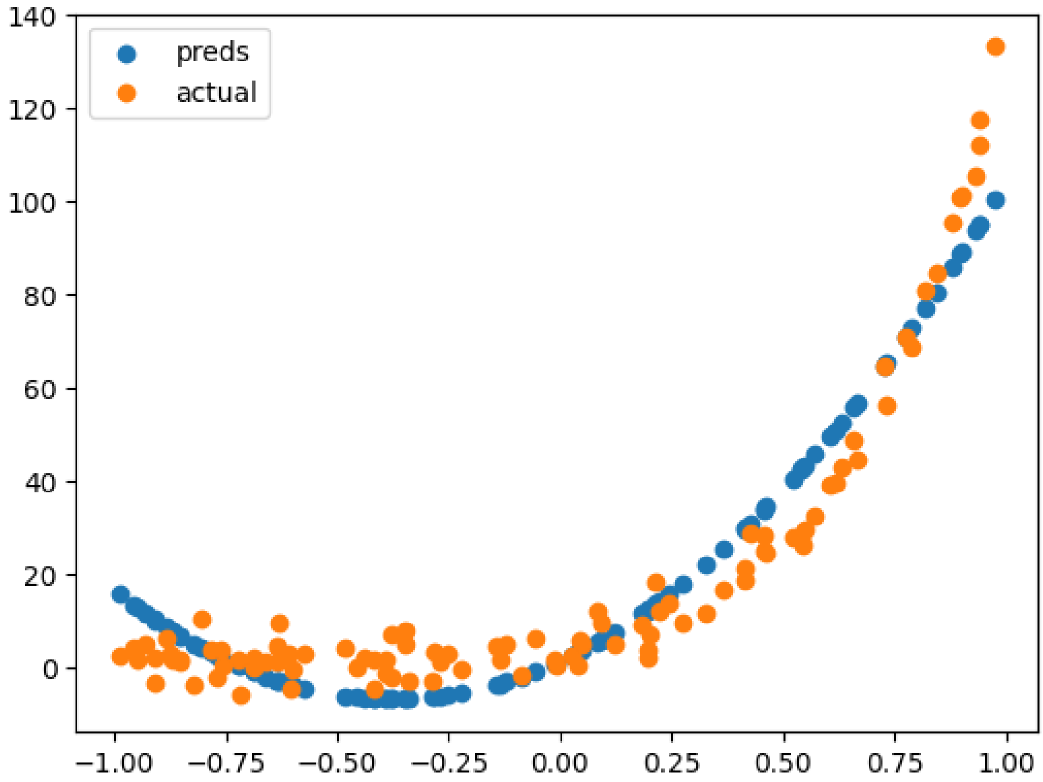



```
plt.plot(a_values, label="a")
plt.plot(b_values, label="b")
plt.plot(c_values, label="c")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b46ceba10>
```




```
with torch.no_grad():
    y_preds = model(X)
    plt.scatter(X, y_preds, label="preds")
    plt.scatter(X, Y, label="actual")

plt.legend()

<matplotlib.legend.Legend at 0x7d9b4752e350>
```



3- y=aebx

```
from math import exp
model = lambda x : a * (torch.exp(b * x))

optimizer = torch.optim.SGD(weights , lr=1e-6)

losses =[]
a_values = []
b_values = []

pbar = tqdm(range(1000000))

for ep in pbar:
  epoch_loss , epoch_a , epoch_b  = train_one_epoch(model ,
optimizer , X ,Y , batch_size =100)
  losses.extend(epoch_loss)
  a_values.extend(epoch_a)
  b_values.extend(epoch_b)
  c_values.extend(epoch_c)

  if (ep + 1) % 100 == 0:
     pbar.set_description(f"loss={losses[-1]:0.03f}, a={a_values[-
1]:0.03f}, b={b_values[-1]:0.03f}")
```

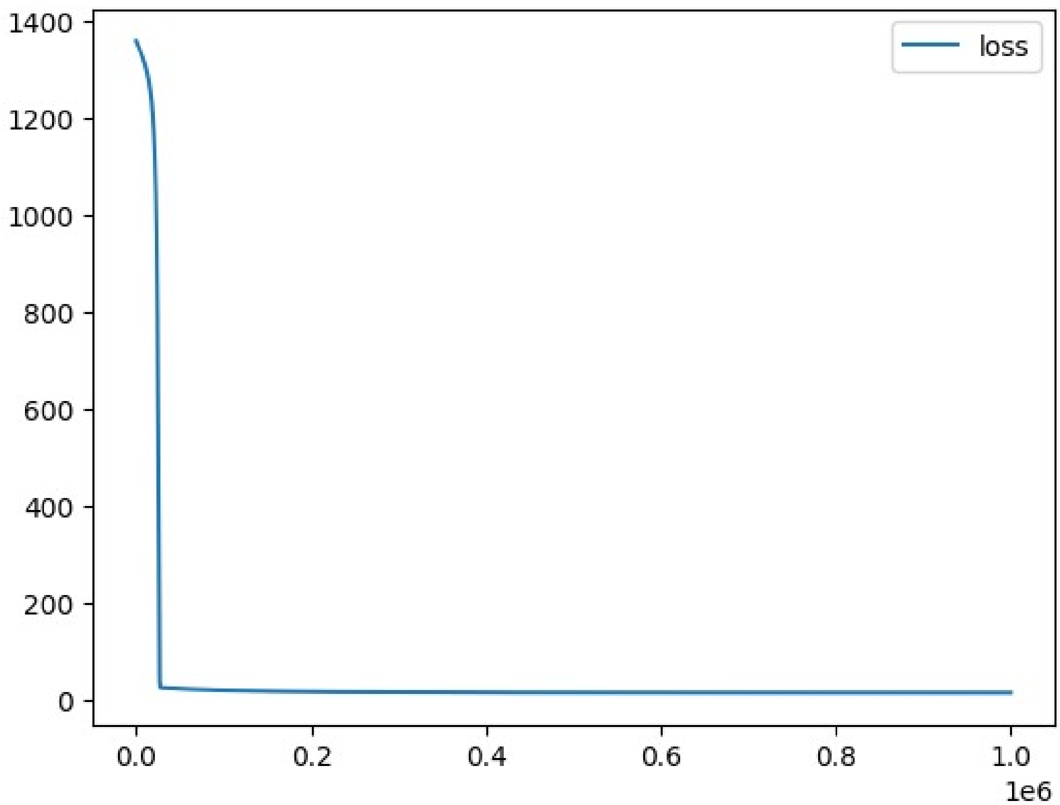
```
loss=13.176, a=4.933, b=3.360: 100%|██████████| 1000000/1000000
[12:01<00:00, 1385.68it/s]
```

```python
losses[-1]
```

```
13.176125526428223
```

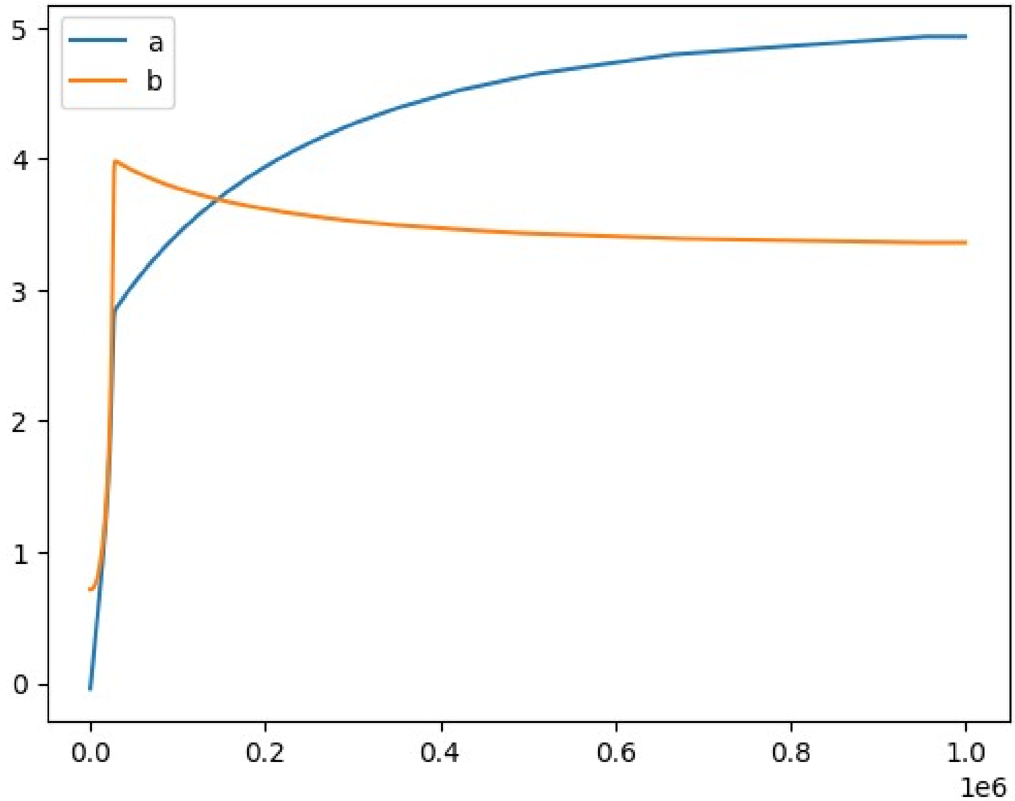
```python
plt.plot(losses, label="loss")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b47250110>
```

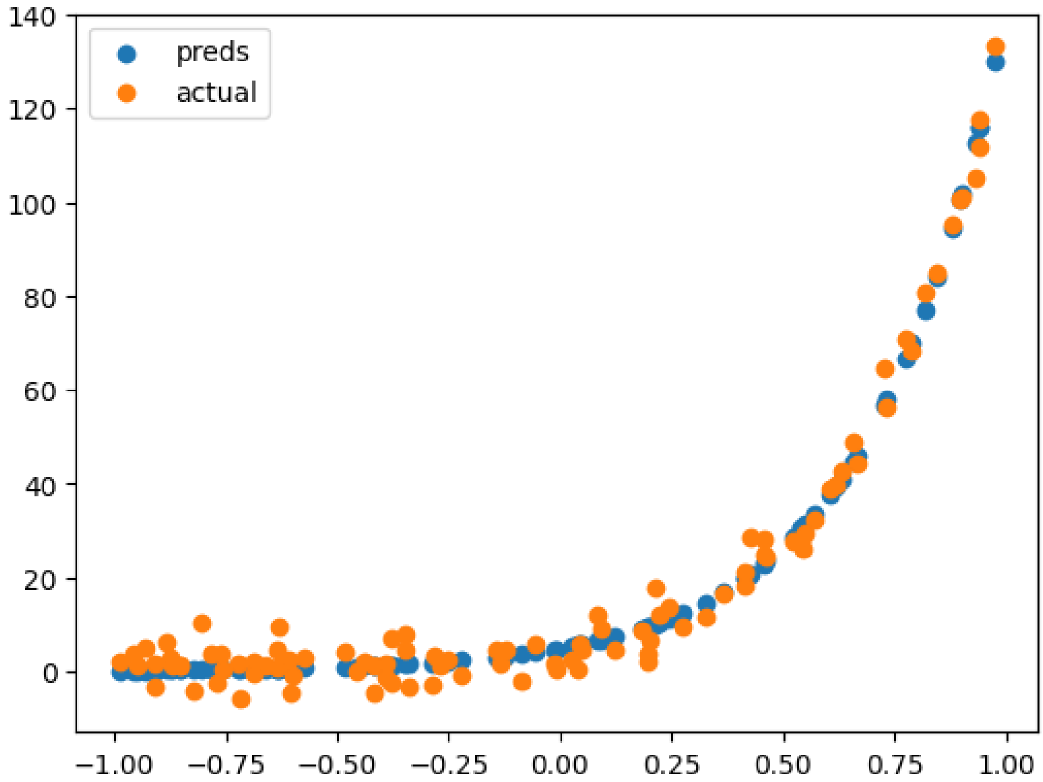



```python
plt.plot(a_values, label="a")
plt.plot(b_values, label="b")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b3c733a10>
```




```
with torch.no_grad():
    y_preds = model(X)
    plt.scatter(X, y_preds, label="preds")
    plt.scatter(X, Y, label="actual")

plt.legend()

<matplotlib.legend.Legend at 0x7d9b3c5dba10>
```



Conclusion

We can see that changing the hypothesis function can cleary change the performance of the model as at the beginning the first function underfit the model resulting in huge error. We improved that by trying to increase the degree of the function from first order to second order (polynomial regression) and we found that it fitted the model better. Eventually we use exponential function because we knew it can also fit the model and it improved the performance of the model leading to the least amount of losses

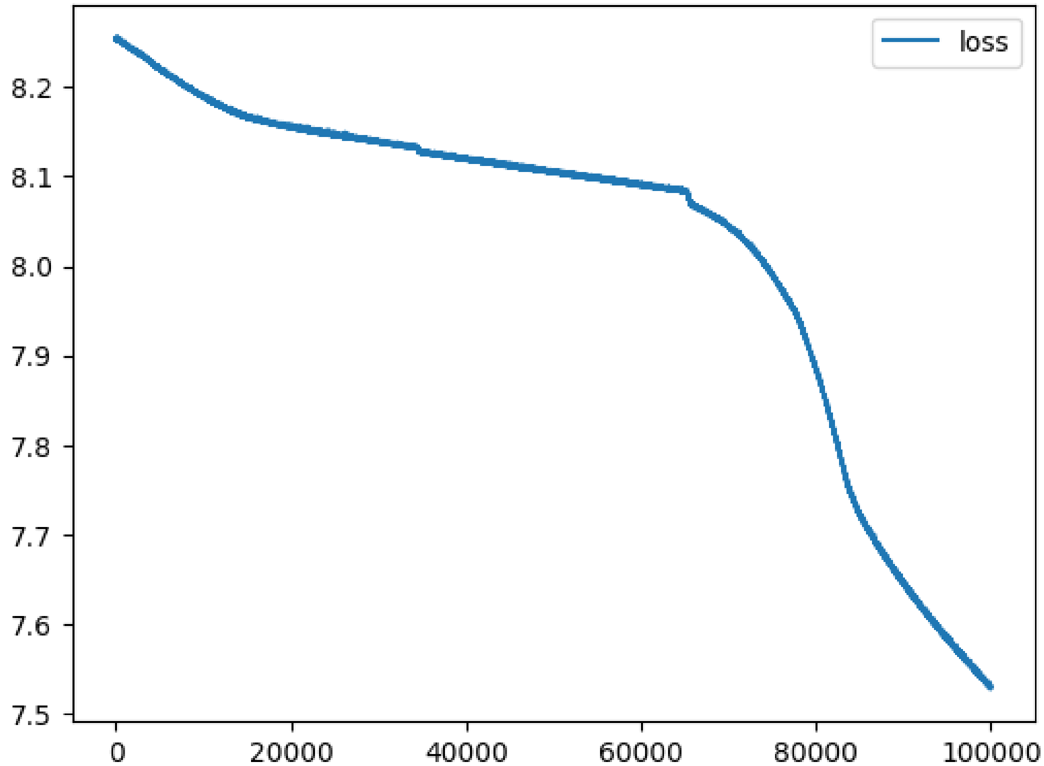
```
class NNModel(nn.Module):
  def __init__(self , input_shape , output_shape):
    super().__init__()
    self.fc1 = nn.Linear(input_shape , 64)
    self.fc2 = nn.Linear(64 , 32)
    self.fc3 = nn.Linear(32 , 16)
    self.out = nn.Linear(16 , output_shape)

    self.relu = nn.ReLU()
    self.tanh = nn.Tanh()

  def forward(self , x):
    x = self.tanh(self.fc1(x))
    x = self.relu(self.fc2(x))
```

```python
        x = self.relu(self.fc3(x))
        x = self.out(x)
        return x

model = NNModel(1 , 1)

optimizer = torch.optim.Adam(model.parameters() , lr=0.0001)

losses = []
a_values = []
b_values = []

pbar = tqdm(range(100000))
for ep in pbar:
    epoch_loss, epoch_a, epoch_b = train_one_epoch(model, optimizer, 
X[:, None], Y[:, None])
    losses.extend(epoch_loss)
    a_values.extend(epoch_a)
    b_values.extend(epoch_b)
    if (ep + 1) % 100 == 0:
        pbar.set_description(f"loss={losses[-1]:0.03f}")
```

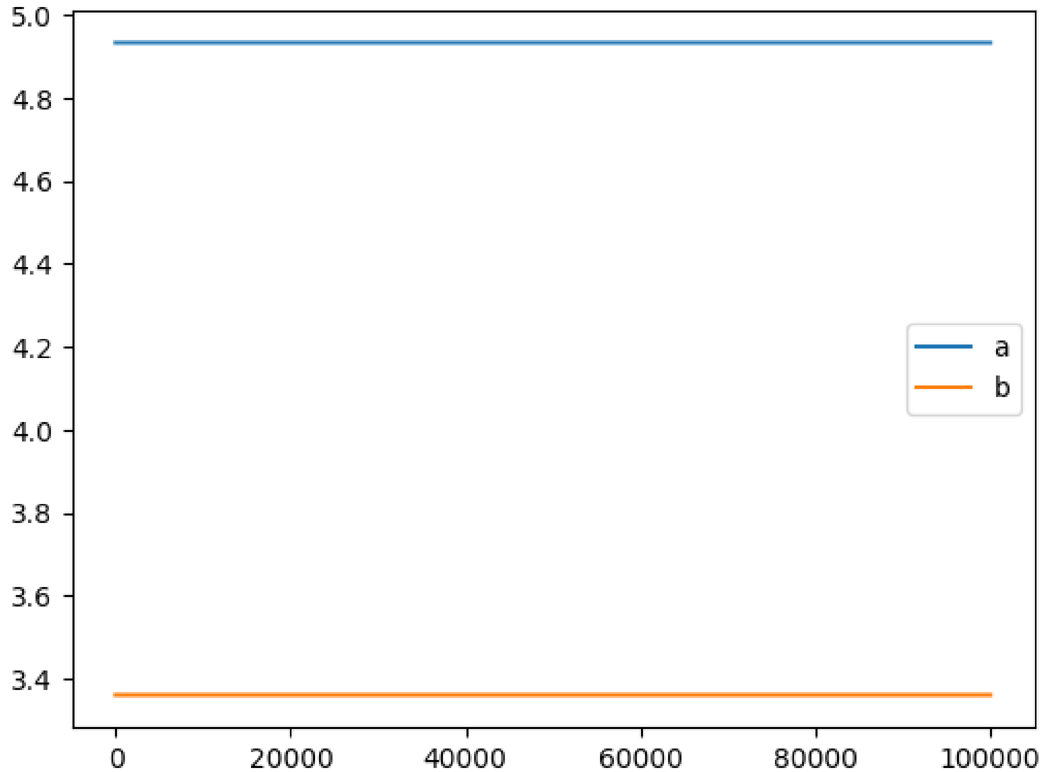
```
loss=7.529: 100%|██████████| 100000/100000 [03:44<00:00, 444.49it/s]
```

```python
plt.plot(losses, label="loss")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b3c6d9ad0>
```

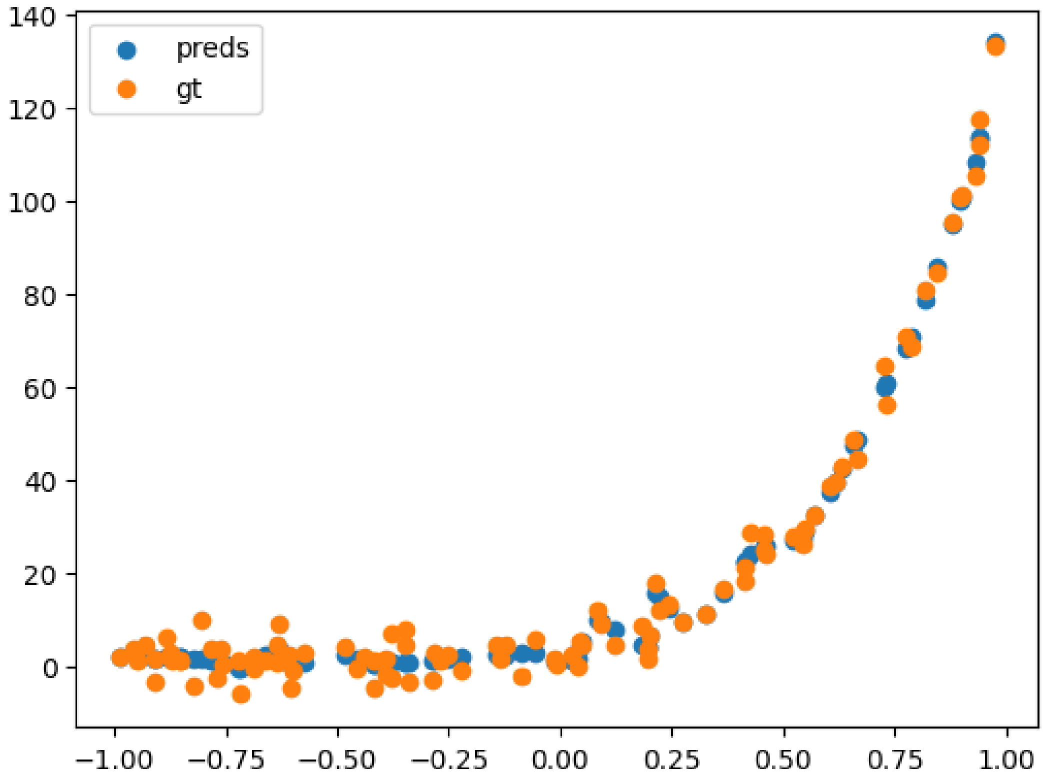



```
plt.plot(a_values, label="a")
plt.plot(b_values, label="b")
plt.legend()

<matplotlib.legend.Legend at 0x7d9b47277f10>
```




```
with torch.no_grad():
    y_preds = model(X)
    plt.scatter(X, y_preds, label="preds")
    plt.scatter(X, Y, label="gt")

plt.legend()
```

```
<matplotlib.legend.Legend at 0x7d9b473d7710>
```



preds
gt
140
120
100
80
60
40
20
0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00