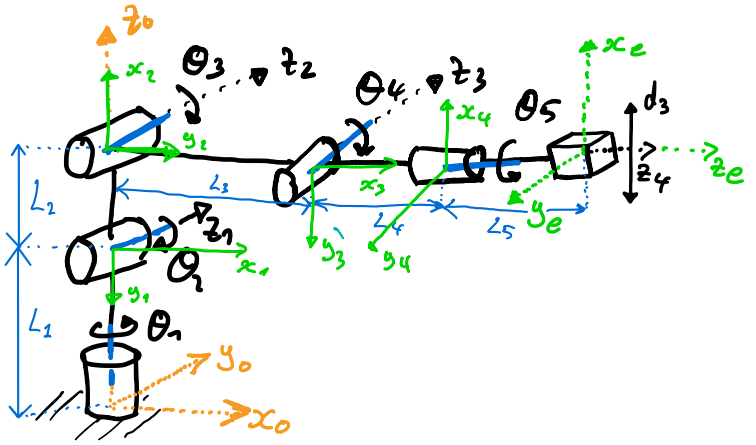
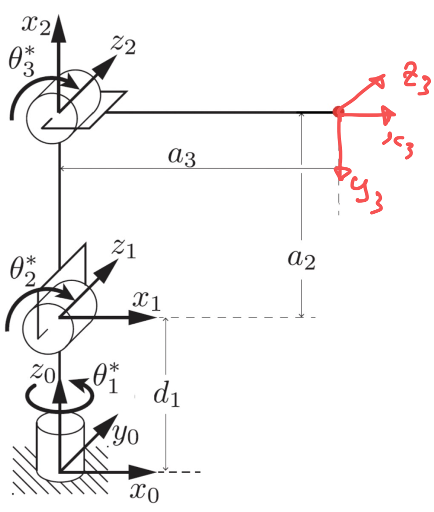
1 Pre-lab Tasks



Link	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	L_1	θ_1
2	L_2	0	0	$-\frac{\pi}{2}+\theta_2$
3	L_3	0	0	$\frac{\pi}{2}+\theta_3$
4	0	$-\pi/2$	0	$-\frac{\pi}{2}+\theta_4$
5	0	0	L_5	θ_5

$$H_4^3 = \begin{bmatrix} S\theta_4 & 0 & C\theta & L_4 C\theta \\ -C\theta_4 & 0 & S\theta & L_4 S\theta \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1)



$$\underline{T}_n^0 = \underline{A}_0^1\left(a=0, \alpha=-\frac{\pi}{2}, d_i=d_1, \theta=\theta_1^*\right)$$

$$\cdot\, \underline{A}_1^2\left(a=a_2, \alpha=0, d=0, \theta=\frac{\pi}{2}+\theta_2^*\right)$$

$$\cdot\, \underline{A}_2^3\left(a=a_3, \alpha=0, d=0, \theta=-\frac{\pi}{2}+\theta_3^*\right)$$

s.t. $\underline{q}_n^0 = \underline{T}_n^0 \cdot \underline{q}_0^0$

where

$$\underline{A}_0^1\left(a=0, \alpha=-\frac{\pi}{2}, d_i=d_1, \theta=\theta_1^*\right) =$$

$$\begin{bmatrix} C\theta_1^* & 0 & -S\theta_1^* & 0 \\ S\theta_1^* & 0 & C\theta_1^* & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{A}_1^2\left(a=a_2, \alpha=0, d=0, \theta=\frac{\pi}{2}+\theta_2^*\right) =$$

$$\begin{bmatrix} C_{\frac{\pi}{2}+\theta_2^*} & -S_{\frac{\pi}{2}+\theta_2^*} & 0 & a_2 C_{\frac{\pi}{2}+\theta_2^*} \\ S_{\frac{\pi}{2}+\theta_2^*} & C_{\frac{\pi}{2}+\theta_2^*} & 0 & a_2 S_{\frac{\pi}{2}+\theta_2^*} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{A}_2^3\left(a=a_3, \alpha=0, d=0, \theta=-\frac{\pi}{2}+\theta_3^*\right) =$$

$$\begin{bmatrix} C_{\frac{\pi}{2}+\theta_3^*} & -S_{\frac{\pi}{2}+\theta_3^*} & 0 & a_3 C_{\frac{\pi}{2}+\theta_3^*} \\ S_{\frac{\pi}{2}+\theta_3^*} & C_{\frac{\pi}{2}+\theta_3^*} & 0 & d_3 S_{\frac{\pi}{2}+\theta_3^*} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

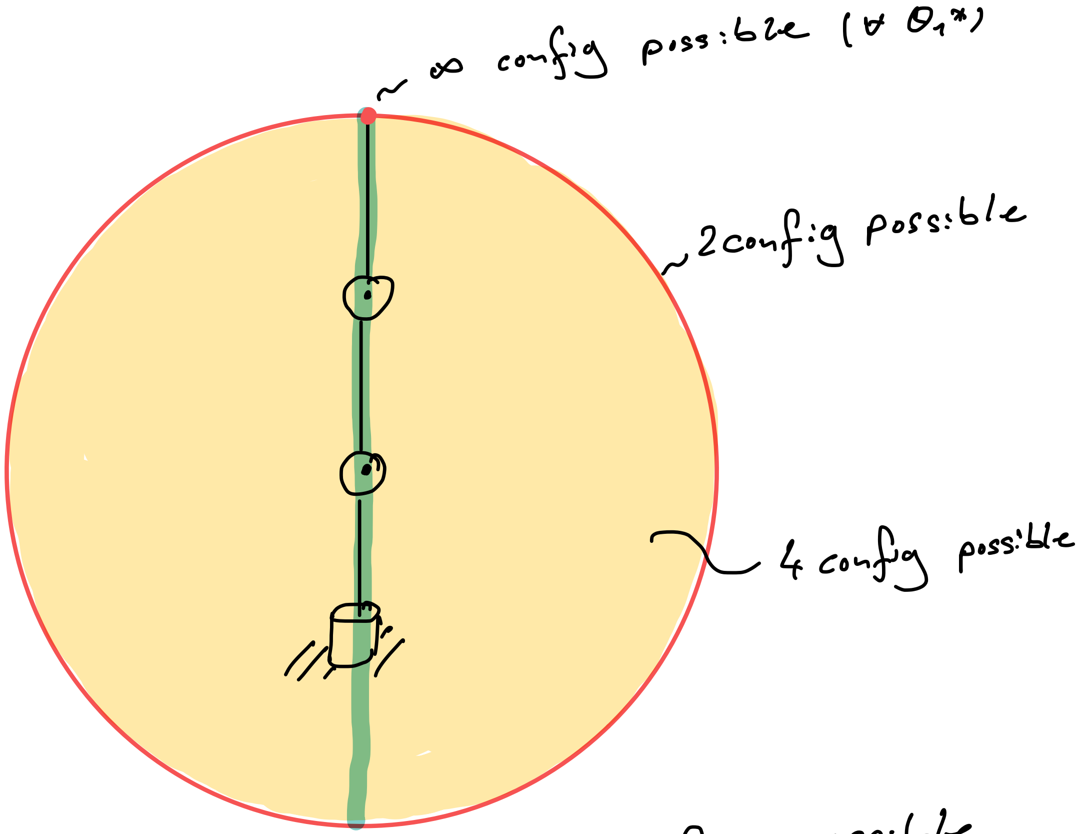
$$\underline{q}_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{T}_n^0 = \begin{pmatrix} c_1 c_2 c_3 - c_1 s_2 s_3 & -c_1 c_3 s_2 - c_1 c_2 s_3 & -s_1 & -a_3 c_1 s_2 s_3 + a_2 c_1 c_2 + a_3 c_1 c_3 c_2 \\ c_2 c_3 s_1 - s_1 s_2 s_3 & -c_3 s_1 s_2 - c_2 s_1 s_3 & c_1 & a_2 c_2 s_1 + a_3 c_2 c_3 s_1 - a_3 s_2 s_3 s_1 \\ -c_3 s_2 - c_2 s_3 & s_2 s_3 - c_2 c_3 & 0 & -a_3 c_3 s_2 - a_3 c_2 s_3 - a_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where: $S/C_1 = S/C_{\theta_1^*}$, $S/C_2 = S/C_{\frac{\pi}{2}+\theta_2^*}$, $S/C_3 = S/C_{\frac{\pi}{2}+\theta_3^*}$

$$\underline{P}_n^0 = \begin{bmatrix} -a_3 C\theta_1^* S_{\frac{\pi}{2}+\theta_2^*} \cdot S_{\frac{\pi}{2}+\theta_3^*} + a_2 C\theta_1^* C_{\frac{\pi}{2}+\theta_2^*} + a_3 C\theta_1^* C_{\frac{\pi}{2}+\theta_3^*} C_{\frac{\pi}{2}+\theta_2^*} \\ a_2 C_{\frac{\pi}{2}+\theta_2} S\theta_1^* + a_3 C_{\frac{\pi}{2}+\theta_2^*} C_{\frac{\pi}{2}+\theta_3^*} S\theta_1^* - a_3 S_{\frac{\pi}{2}+\theta_2^*} S_{\frac{\pi}{2}+\theta_3^*} S\theta_1^* \\ -a_3 C_{\frac{\pi}{2}+\theta_3^*} S_{\frac{\pi}{2}+\theta_2^*} + a_3 C_{\frac{\pi}{2}+\theta_2^*} \cdot S_{\frac{\pi}{2}+\theta_3^*} - a_2 S_{\frac{\pi}{2}+\theta_2^*} + d_1 \end{bmatrix}$$

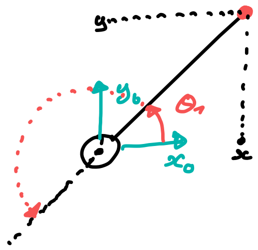
2)



Outsid red sphere : 0 config. possible

3)

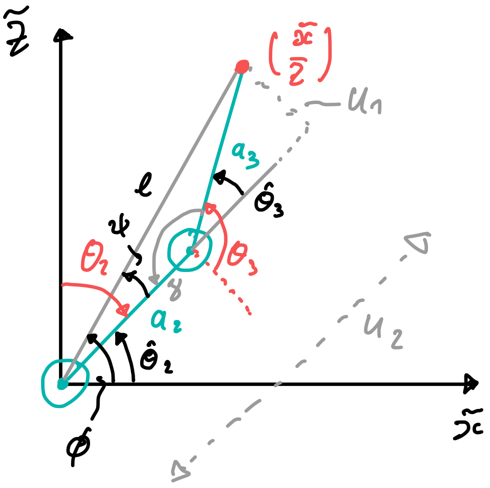
Top View :



$\Rightarrow$ $\boxed{\theta_1^* = \arctan\left(\frac{y}{x}\right)}$

Zero the system for side view for the sake of simplicity : $\tilde{z} = z - d_1$

$\tilde{x} = \sqrt{x^2 + y^2}$



$$\ell^2 = \tilde{x}^2 + \tilde{z}^2$$

Law of cosine :

$$\ell^2 = a_2^2 + a_3^2 - 2a_2 a_3 \underbrace{\overbrace{\cos(\pi - \theta_3)}^{\gamma}}_{= -\cos(\theta_3)} = \tilde{x}^2 + \tilde{z}^2$$

$$\Rightarrow \hat{\theta}_3 = \arccos\left[\frac{\tilde{x}^2 + \tilde{z}^2 - a_2^2 - a_3^2}{2 a_2 a_3}\right]$$

$$\boxed{\hat{\theta} = \arccos\left[\frac{(x^2+y^2) + (z-d_1)^2 - a_2^2 - a_3^2}{2 a_2 a_3}\right]}$$

$$\hat{\theta}_2 = \phi - \psi$$

$$\psi = \arctan\left(\frac{u_1}{u_2}\right)$$

$$u_1 = a_3 \cdot \sin(\hat{\theta}_3)$$

$$u_2 = a_2 + a_3 \cos(\theta_2)$$

$$\phi = \arctan\left(\frac{\tilde{z}}{\tilde{x}}\right)$$

$$\Rightarrow \boxed{\hat{\theta}_2 = \arctan\left(\frac{z-d_1}{\sqrt{x^2+y^2}}\right) - \arctan\left(\frac{a_3 \cdot \sin(\hat{\theta}_3)}{a_2 + a_3 \cos(\hat{\theta}_3)}\right)}$$

$\hat{\theta}_2$ & $\hat{\theta}_3$ do not match our settings. $\Rightarrow$ Transform

$$\boxed{\theta_3 = -\hat{\theta}_3 - \frac{\pi}{2} = -\arccos\left[\frac{(x^2+y^2) + (z-d_1)^2 - a_2^2 - a_3^2}{2 a_2 a_3}\right] - \frac{\pi}{2}}$$

$$\boxed{\theta_2 = \frac{\pi}{2} - \hat{\theta}_2 = \frac{\pi}{2} - \arctan\left(\frac{z-d_1}{\sqrt{x^2+y^2}}\right) + \arctan\left(\frac{a_3 \cdot \sin(-\hat{\theta}_3 - \frac{\pi}{2})}{a_2 + a_3 \cos(-\hat{\theta}_3 - \frac{\pi}{2})}\right)}$$

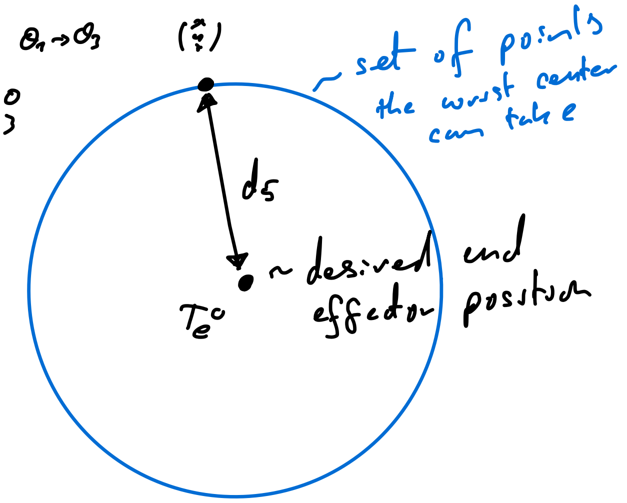
First I split the robot arm into two parts, s.t. I can work w/ 2D plains: Top view and side view. In the top view, I quickly solved for $\theta_1(x, y)$ using simple trigonometry. Next I treated the side view, but I zeroed the system to work in a simple, non θ_1 dependent frame: $\tilde{x}, \tilde{z}$. Here I realized the system was almost identical to the one treated in a lecture so again I transformed to system θ_2 & $\theta_3 \rightarrow \tilde{\theta}_2$ & $\tilde{\theta}_3$ to make it easier to solve, followed the steps in the lecture and finally reversed the transformations and found θ_2 & θ_3.

I took the geometric approach.

2.1

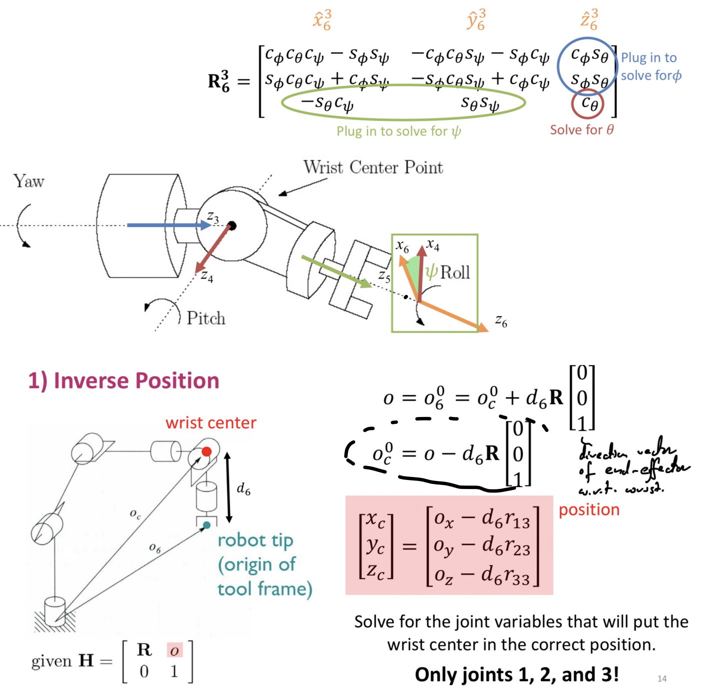
1) $\theta_1 \to \theta_3$ (?)

R_3^0



· 2D plain defined by orientation θ_1

2)



R_6^0, we want R_6^3, R_3^0

· Center $\boxed{o_c^0 = o - d_6 R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$

· $o_c^0 \to \theta_{1:3} \to R_3^0 \to R_6^3$

$$R = \cancel{R_{z,\phi}}^{\,I} R_{y,\theta} R_{z,\psi} \qquad s_\theta = \sin\theta,\ c_\theta$$

$$R = \cancel{\begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\,I} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3^6 = \begin{bmatrix} c_\theta c_\psi & -s_\psi c_\theta & s_\theta \\ s_\psi & c_\psi & 0 \\ -s_\theta c_\psi & -s_\theta s_\psi & c_\theta \end{bmatrix}$$

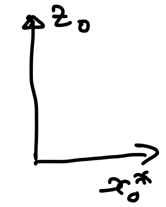
$$\theta = -\theta_4 \ ; \ \psi = \theta_5$$

$$\boxed{R_6^3 = \begin{bmatrix} c_{\theta_5} c_{-\theta_4} & -s_{\theta_5} c_{-\theta_4} & s_{-\theta_4} \\ s_{\theta_5} & c_{\theta_5} & 0 \\ -s_{-\theta_4} c_{\theta_5} & -s_{-\theta_4} s_{\theta_5} & c_{-\theta_4} \end{bmatrix}}$$

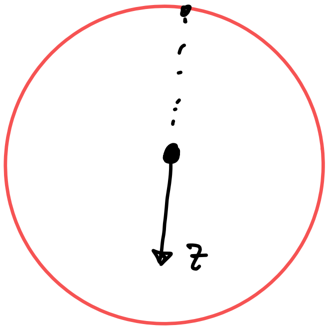
- $IK_{Prelab2}(O_c^0) \to \theta_{1:3}$
- $FK_{Lab1}(\theta_{1:3}) \to R_3^0$

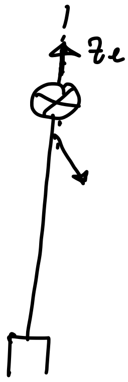
$\Rightarrow R_6^3 \Rightarrow \theta_4 \& 5$

3)



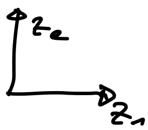
z_0 , x_0^*





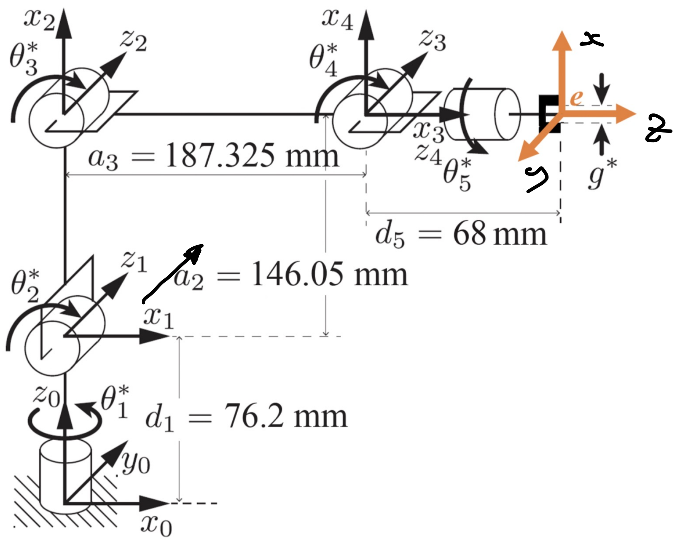
$\hat{z}_{R_0^1}$

$$\text{if } \left| \underline{z}_1 \cdot \begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \end{bmatrix} \right| > 0 \; : \; \text{error}$$



$: \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$: \begin{pmatrix} 0.5 \\ 0 \\ 0 \end{pmatrix}$



$$\underline{z}_e^* = \frac{(\hat{\underline{z}} - \hat{\underline{z}} \cdot \underline{z}_1)}{|(\hat{\underline{z}} - \hat{\underline{z}} \cdot \underline{z}_1)|}$$

$$\underline{y}_e^* = \frac{(\hat{\underline{y}} - \underline{y} \cdot \underline{x}_1)}{|(\hat{\underline{y}} - \underline{y} \cdot \underline{x}_1)|}$$

$$\underline{x}_e^* = \underline{y}^* \times \underline{z}^*$$