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1. (a)

$\Omega \in \Sigma \wedge \Omega^c = \emptyset \implies \emptyset \in \Sigma$

(b)

$A_k \in \Sigma \implies A_k^c \in \Sigma \implies \bigcup_{k \in \mathbb{N}} A_k^c \in \Sigma \implies (\bigcup_{k \in \mathbb{N}} A_k^c)^c \in \Sigma \implies$

De Morgan's Law

$\implies \bigcap_{k \in N} A_k \in \Sigma$

2. a) $\{\emptyset, \Omega\}$, $\{\emptyset, \{a\}, \{b,c\}, \Omega\}$, $\{\emptyset, \{a\}, \{b\}, \{a,b\}, \{c\}, \{a,c\}, \{b,c\}, \Omega\}$

$\{\emptyset, \{b\}, \{a,c\}, \Omega\}$

$\{\emptyset, \{c\}, \{a,b\}, \Omega\}$

b) for σ-form $\Sigma = \{\emptyset, \{a\}, \{b,c\}, \Omega\}$

X: $a \to 1$, $b \to 2$, $c \to 2$

Y: $a \to 1$, $b \to 1$, $c \to 2$, because $\{a,b\} \notin \Sigma$ "$Y^{-1}(\{1\})$"

3. $\Omega = \{1,2,3,4,5\}$, $S = \{1,4\}$

$\Sigma = \{\emptyset, \{1,4\}, \{2,3,5\}, \{1,2,3,4,5\}\}$

4. Let F be CDF of X

$$F(x) = \begin{cases} 0 & \text{for } x \leq 2 \\ 0,2 & \text{for } 2 < x \leq 3 \\ 0,6 & \text{for } 3 < x \leq 4 \\ 0,7 & \text{for } 4 < x \leq 5 \\ 1 & \text{for } 5 < x \end{cases}$$

$$E(X) = 0,2 \cdot 2 + 0,4 \cdot 3 + 0,1 \cdot 4 + 0,3 \cdot 5 = 0,4 + 1,2 + 0,4 + 1,5 =$$
$$= 3,5$$

5.

x_i	2	3	5
p_i	0,2	0,5	0,3

6. $E(X) = \sum_{x_i \in \Omega} x_i \cdot p_i$, $\sum_{x_i \in \Omega} p_i = 1$

$$E(aX+b) = \sum_{x_i \in \Omega} (a x_i + b) p_i = \sum_{x_i \in \Omega} a x_i p_i + \sum_{x_i \in \Omega} b p_i = a \underbrace{\sum_{x_i \in \Omega} x_i p_i}_{E(X)} + b \underbrace{\sum_{x_i \in \Omega} p_i}_{1} =$$
$$= a E(X) + b$$

7. $\int_{\mathbb{R}} f(x)\, dx = 1$, $E(X) = \int_{\mathbb{R}} x f(x)\, dx$

$$E(aX+b) = \int_{\mathbb{R}} (ax+b) f(x)\, dx = a \int_{\mathbb{R}} x f(x)\, dx + b \int_{\mathbb{R}} f(x)\, dx =$$
$$= a E(X) + b$$

8. a) $$B(p,q) \cdot \frac{q}{p+q} = B(p,q) \cdot \frac{\frac{\Gamma(q+1)}{\Gamma(q)}}{\frac{\Gamma(p+q+1)}{\Gamma(p+q)}} \overset{\text{from } 9.}{=} \frac{\Gamma(p) \cdot \cancel{\Gamma(q)}}{\cancel{\Gamma(p+q)}} \cdot$$

$$\cdot \frac{\frac{\Gamma(q+1)}{\cancel{\Gamma(q)}}}{\frac{\Gamma(p+q+1)}{\cancel{\Gamma(p+q)}}} = \frac{\Gamma(q+1) \cdot \Gamma(p)}{\Gamma(p+q+1)} \overset{\text{from } 9}{=} B(p,q+1)$$

b) $$B(p,q+1) + B(p+1,q) = \int_0^1 t^{p-1}(1-t)^q + t^p (1-t)^{q-1}\, dt = \int_0^1 t^{p-1}(1-t)^{q-1}\, dt =$$
$$= B(p,q)$$