Set 9 exercise 10

Wiktor Hamberger 308982

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We have to find probability that first significant digit of Z equals 1. From task 9. we know the density function of r.v. Z:

$$f_z(x) = \begin{cases} \frac{1}{2} & 0 \leq x \leq 1 \\ \frac{1}{2x^2} & 1 < x \end{cases}$$

We will need to integrals:

$$\int \frac{1}{2} dx = \frac{x}{2} + C$$

$$\int \frac{1}{2x^2} dx = -\frac{1}{2x} dx + C$$

Knowing that let's calculate:

$$\begin{gathered} P(\text{first significant digit of Z equals 1}) = \\ = P(1 \leq Z < 2) + P(10 \leq Z < 20) + P(100 \leq Z < 200) + \cdots + \\ + P(0,1 \leq Z < 0,2) + P(0,01 \leq Z < 0,02) + P(0,001 \leq Z < 0,002) + \cdots = \\ = \int_1^2 \frac{1}{2x^2} dx + \int_{10}^{20} \frac{1}{2x^2} dx + \int_{100}^{200} \frac{1}{2x^2} dx + \cdots + \\ + \int_{0,1}^{0,2} \frac{1}{2} dx + \int_{00,1}^{0,02} \frac{1}{2} dx + \int_{0,001}^{0,002} \frac{1}{2} dx + \cdots = \\ = \frac{1}{4} + \frac{1}{40} + \frac{1}{400} + \cdots + \frac{1}{20} + \frac{1}{200} + \frac{1}{2000} = \\ = \frac{1}{4} * \frac{1}{1 - \frac{1}{10}} + \frac{1}{20} * \frac{1}{1 - \frac{1}{10}} = \frac{1}{3} \end{gathered}$$