

Project I - Explanation
COMP 472 NN 2214
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Experiment #1 (Greater Challenge)

Start State: (5 1 4 7 B 6 3 8 2)

5	1	4
7		6
3	8	2

Goal State: (1 2 3 8 B 4 7 6 5)

1	2	3
8		4
7	6	5

	Best First Search		A* Search	
Heuristics	Nodes Explored	Solution Path Length	Nodes Explored	Solution Path Length
Hamming Distance	1,037	52	4,030	20
Manhattan Distance	237	38	139	20
Permutation Inverse	721	76	310	20
Nilsson's Sequence (Inadmissible)	96	46	377	30

Search Type	Nodes Explored	Solution Path Length
Depth First Search (DFS)	Recursion limit reached	Recursion limit reached
Breadth First Search (BFS)	48,169	20

Experiment #2 (Challenge)

Start State: (2 8 3 1 6 4 7 B 5)

2	8	3
1	6	4
7		5

Goal State: (1 2 3 8 B 4 7 6 5)

1	2	3
8		4
7	6	5

	Best First Search		A* Search	
Heuristics	Nodes Explored	Solution Path Length	Nodes Explored	Solution Path Length
Hamming Distance	7	5	7	5
Manhattan Distance	6	5	6	5
Permutation Inverse	6	5	7	5
Nilsson's Sequence (Inadmissible)	6	5	6	5

Search Type	Nodes Explored	Solution Path Length
Depth First Search (DFS)	181,417	31
Breadth First Search (BFS)	46	5

Experiment #3

Start State: (3 8 6 4 5 B 1 7 2)

3	8	6
4	5	
1	7	2

Goal State: (1 2 3 8 B 4 7 6 5)

1	2	3
8		4
7	6	5

	Best First Search		A* Search	
Heuristics	Nodes Explored	Solution Path Length	Nodes Explored	Solution Path Length
Hamming Distance	133	21	5,501	21
Manhattan Distance	156	21	274	21
Permutation Inverse	398	31	896	21
Nilsson's Sequence (Inadmissible)	34	29	36	21

Search Type	Nodes Explored	Solution Path Length
Depth First Search (DFS)	Recursion Limit Reached	Recursion Limit Reached
Breadth First Search (BFS)	60,366	21

Experiment #4

Start State: (1 5 7 6 2 3 8 B 4)

1	5	7
6	2	3
8		4

Goal State: (1 2 3 8 B 4 7 6 5)

1	2	3
8		4
7	6	5

	Best First Search		A* Search	
Heuristics	Nodes Explored	Solution Path Length	Nodes Explored	Solution Path Length
Hamming Distance	305	35	2,516	19
Manhattan Distance	256	49	252	19
Permutation Inverse	398	33	367	19
Nilsson's Sequence (Inadmissible)	72	37	334	31

Search Type	Nodes Explored	Solution Path Length
Depth First Search (DFS)	431	419
Breadth First Search (BFS)	38,577	19

Experiment #5

Start State: (3 8 4 7 5 1 6 2 B)

3	8	4
7	5	1
6	2	

Goal State: (1 2 3 8 B 4 7 6 5)

1	2	3
8		4
7	6	5

Heuristics	Best First Search		A* Search	
	Nodes Explored	Solution Path Length	Nodes Explored	Solution Path Length
Hamming Distance	458	36	540	16
Manhattan Distance	22	16	35	16
Permutation Inverse	266	32	113	16
Nilsson's Sequence (Inadmissible)	79	32	190	24

Search Type	Nodes Explored	Solution Path Length
Depth First Search (DFS)	Recursion Limit Reached	Recursion Limit Reached
Breadth First Search (BFS)	9,847	16

Overall Results

	Best First Search		A* Search	
Heuristics	Nodes Explored	Solution Path	Nodes Explored	Solution Path
Hamming Distance	N_1	$\leq N_1$	M_1	K ($K \leq M_1$)
Manhattan Distance	N_2	$\leq N_2$	M_2	K ($K \leq M_2$)
Permutation Inverse	N_3	$\leq N_3$	M_3	K ($K \leq M_3$)
Nilsson's Sequence (Inadmissible)	N_4	$\leq N_4$	M_4	K^* ($K^* \leq M_4$) ($K^* \geq K$)

After running 5 experiments we can compile the table above with a certain degree of confidence. The experiments run are consistent with prior theoretical knowledge of search algorithms. Mainly in identifying a few keypoints.

- As long as the heuristic chosen is admissible – the A* algorithm will always return the most optimal path from the start node to the goal node.
- Depending on the heuristic, a different amount of nodes can be explored.
- Best first search does not guarantee the most optimal path, but given a good heuristic best first search may give the most optimal path.
- Best first search tends to explore fewer nodes before reaching a solution
- Inadmissible heuristics overestimate the shortest path in the case of an A* algorithm. In doing so an inadmissible heuristic using an A* algorithm does not always return the most optimal path from the start node to the goal node.
- The number of nodes explored are generally smaller in best first searches compared to A* searches. This is because an A* algorithm is in search for the most optimal solution so it must explore more possible nodes and in turn paths.

Inadmissible Heuristic (Explanation)
Nilsson's Heuristic for 8-puzzle

$$\text{score} = \text{distances} + 3 * (\text{center} + 2 * \text{successors})$$

Center

If the center tile of the 8 puzzle state does not match the center tile of the goal state then center = 1, else center = 0.

Distances

Sum of Manhattan Distances of the tiles in the 8 puzzle state.

Successors

Traverse state in clockwise order and compare to clockwise traversal of goal state.
Suppose goal state is:

A	B	C
H		D
G	F	E

Suppose current state is:

	A	C
H	B	D
G	F	E

In clockwise arrangement the current state and goal state are given as

Current state: _ACDEFGH

Goal state: ABCDEFGH

the [tile, tile clockwise to it] pairs in the current state are:

[A,C], [C,D], [D,E], [E,F], [F,G], [G,H], [H,_]

([,A] is not considered)

the "goal" pairs are:

[A,B], [B,C], [C,D], [D,E], [E,F], [F,G], [G,H], [H,A]

There are 2 pairs that do not match the goal pairs:

[A,C], [H,_]

Hence successors = 2

$$\text{Score} = 2 + 3 * (1 + 2 * (2)) = 17$$

Example

Suppose goal state is:

A	B	C
H		D
G	F	E

Suppose current state is:

A	B	C
	H	D
G	F	E

$h^*(n) = 1$, since tile H needs to move to the left by 1 in order to reach the goal state.

Since the center tile in the current state and goal state do not match, center = 1

With the score formula

$$\text{score} = \text{distances} + 3 * (\text{center} + 2 * \text{successors})$$

center will be multiplied by 3.

$3 * 1 > 1$. Therefore the heuristic value obtained is an overestimate and therefore the Nilsson heuristic is an inadmissible heuristic.