Theme: Chomsky Normal Form (CNF)
Laboratory tasks:

1. Eliminate ε productions.
2. Eliminate any renaming.
3. Eliminate inaccessible symbols.
4. Eliminate the non productive symbols.
5. Obtain the Chomsky Normal Form.

Normal forms of the context-free languages
In the case of arbitrary grammars the normal form was defined as grammars with no terminals in the left-hand side of productions. The normal form in the case of the context-free languages will contains some restrictions on the right-hand sides of productions.
Chomsky Normal Form
A context-free grammar $G=(V_N, V_T, P, S)$ is in Chomsky normal form, if all productions have form $A\rightarrow a$ or $A\rightarrow BC$, where $A,B,C \in V_N$, $a\in V_T$.
To each ε -free context-free language can be associated an equivalent grammar is Chomsky normal form.

Example:
$G=(V_N, V_T, P, S)$ $V_N=\{S, A, B, C, D\}$ $V_T=\{a, b\}$
P={1. S→AC
2. S→bA
3. S→B
4. S→aA
5. A→ ε
6. A→aS
7. A→ABAb
8. B→a
9. B→AbSA
10. C→abC
11. D→AB}

1. Elimination of ε productions:
a) $N_\varepsilon = \varnothing$
b) for the production A→ ε $N_\varepsilon = \varnothing \cup \{A\}$
$N_\varepsilon = \{A\}$

P'={1. S→AC	11. **S→C**
2. S→bA	12. S→b
3**. S→B**	
4. S→aA	13. S→a
5. A→aS	
6. A→ABAb	14. A→BAb
	15. A→ABb
	16. A→Bb
7. B→a	
8. B→AbSA	17. B→bSA
	18. B→AbS
	19. B→bS
9. C→abC	
10. D→AB	20. **D→B** }

2. Elimination of renaming:

The production that has the form $X \to Y$, X and Y are nonterminal, is called renaming.

The renaming from P' are: $S \to B$, $S \to C$, $D \to B$

$R_S = \{S\},\ R_B = \{B\},\ R_C = \{C\},\ R_D = \{D\}$

for $S \to B$ $\quad R_B = R_B \cup R_S = \{B\} \cup \{S\} = \{B, S\}$

for $S \to C$ $\quad R_C = R_C \cup R_S = \{C\} \cup \{S\} = \{C, S\}$

for $D \to B$ $\quad R_B = R_B \cup R_D = \{B, S\} \cup \{D\} = \{B, S, D\}$

P''={ 1. S→AC
2. S→bA
3. S→aA
4. A→aS
5. A→ABAb
6. B→a
7. B→AbSA
8. C→abC
9. D→AB
10. S→b
11. S→a
12. A→BAb
13. A→ABb
14. A→Bb
15. B→bSA
16. B→AbS
17. B→bS
18. S→a
19. S→ AbSA
20. S→abC
21. D→a
22. S→ bSA
23. S→ AbS
24. S→bS
25. D→ bSA
26. D→ AbS
27. D→bS
28. D→ AbSA }

3. Elimination of nonproductive symbols.

$PROD(G) = \{A \mid A \in V_N, \exists A \Rightarrow v, v \in V_T\}$

$NEPROD(G) = V_N \setminus PROD(G)$

$V_N = \{S, A, B, C, D\}$

$PROD(G) = \{B, S, A, D\}$

$NEPROD(G) = \{S, A, B, C, D\} \setminus \{B, S, A, D\} = \{C\}$

P'''={ 1. S→a
2. S→bA
3. S→aA
4. A→aS
5. A→ABAb
6. B→a
7. B→AbSA
8. D→ bSA
9. D→AB
10. S→b
11. S→a
12. A→BAb
13. A→ABb
14. A→Bb
18. S→ AbSA
19. D→a
20. S→ bSA
21. S→ AbS
22. S→bS
23. D→ AbS
24. D→bS
25. D→ AbSA

15. $B \to bSA$
16. $B \to AbS$
17. $B \to bS$ }

4. Elimination of inaccesibile symbols:

Initial $ACCES(G) = \{S\}$

$ACCES(G) = \{x \mid \exists S \Rightarrow \alpha x \beta\}$

$INACCES(G) = (V_N \cup V_T) \setminus ACCES(G)$

$ACCES(G) = \{S, A, b, a, B\}$

$V_N = \{S, A, B, D\}$ $\quad V_T = \{a, b\}$

$INACCES(G) = \{S, A, B, D, a, b\} \setminus \{S, A, b, a, B\} = \{D\}$

P^{IV}= {

1. $S \to a$
2. $S \to bA$ — 18. $S \to AbSA$
3. $S \to aA$
4. $A \to aS$
5. $A \to ABAb$ — 19. $S \to bSA$
6. $B \to a$ — 20. $S \to AbS$
7. $B \to AbSA$ — 21. $S \to bS$
10. $S \to b$
11. $S \to a$
12. $A \to BAb$
13. $A \to ABb$
14. $A \to Bb$
15. $B \to bSA$
16. $B \to AbS$
17. $B \to bS$ }

5. The Chomsky Normal Form

A grammar in the Chomsky Normal Form is a grammar of rules that has a form $A \to BC$, $D \to i$, where $A,B,C,D \in V_N$ and $i \in V_T$

P^{V}= {

1. $S \to X_1A$ — 20. $X_1 \to b$
2. $S \to X_2A$ — 21. $X_2 \to a$
3. $A \to X_2S$
4. $A \to AY_1$ — 22. $Y_1 \to B Y_2$ — 23. $Y_2 \to AX_1$
5. $B \to a$
6. $B \to A Y_3$ — 24. $Y_3 \to X_1Y_4$ — 25. $Y_4 \to SA$
7. $S \to b$
8. $S \to a$
9. $A \to BY_2$
10. $A \to AY_5$ — 26. $Y_5 \to BX_1$
11. $A \to BX_1$
12. $B \to X_1Y_4$
13. $B \to AY_6$ — 27. $Y_6 \to X_1S$
14. $B \to X_1S$

17. $S \rightarrow X_1 Y_4$
18. $S \rightarrow A Y_6$
19. $S \rightarrow X_1 S$ }

$V_N=\{S, A, B, X_1, X_2, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6\}$