

Theme: Regular or context free grammar

Laboratory tasks:

1. For the formal grammar $G=(V_N, V_T, P, S)$ need be obtained five strings, that belong to the language $L(G)$, that is generated by this grammar. The length of strings must be no lesser than the number of characters from the alphabet V_N+2 .
2. For each string build the noninverted (derivation) tree and derivation table.
3. Convert regular grammar to Finite Automaton (FA).
4. Determine the grammar type by the Chomsky classification.

Theoretical notions

A **grammar** is an ordered quadruple $G = (V_N, V_T, P, S)$, where:

- V_N is the alphabet of **variables** (or **nonterminal symbols**),

- V_T is the alphabet of **terminal symbols**.

- $P \subseteq (V_N \cup V_T)^* V_N (V_N \cup V_T)^* \times (V_N \cup V_T)^*$, P - set of *productions*, which are rules for replacing (or rewriting) nonterminal symbols (on the left side of the production) in a string with other nonterminal or terminal symbols (on the right side of the production).

- $S \in V_N$, S - **start symbol**, which is a special nonterminal symbol that appears in the initial string generated by the grammar.

If $(\alpha, \beta) \in P$, then this production is denoted: $\alpha \rightarrow \beta$ and this mean that α is replaced by the β .

A grammar represents a way of specifying a language. Let is given the grammar $G = (V_N, V_T, P, S)$. On the set $(V_N \cup V_T)^*$ is defined the binary relations:

\Rightarrow (direct derivation);

$\overset{k}{\Rightarrow}$ (k derivation);

$\overset{+}{\Rightarrow}$ (+ derivation);

$\overset{*}{\Rightarrow}$ (* derivation).

The language generated by the grammar $G = (V_N, V_T, P, S)$ is the set

$$L(G) = \left\{ w \mid w \in V_T^*, S \overset{*}{\Rightarrow} w \right\}.$$

So $L(G)$ contains all words over the alphabet V_T which can be derived from the start symbol S using the productions from P .

Let $G = (V_N, V_T, P, S)$ be a grammar. If $S \overset{*}{\Rightarrow} x$ and $x \in (V_N \cup V_T)^*$ then x is a **sentential form**.

A sentential form that not contains nonterminal symbols is called **phrase**.

Two grammars G_1 and G_2 are **equivalent**, and this is denoted if they generate the same language, so $L(G_1) = L(G_2)$.

By the Chomsky the grammar is classified in 4 types:

1. A grammar G is of type 0 (**phrase-structure grammar**) if there are no restrictions on productions.

2. A grammar G is of type 1 (**context-sensitive grammar**) if all of its productions are of the form $\alpha_1 A \alpha_2 \rightarrow \alpha_1 \beta \alpha_2$, where $\alpha_1, \beta, \alpha_2 \in (V_N \cup V_T)^*$, $A \in V_N$.

The **left-context**- and **right-context**-sensitive grammars are defined by restricting the rules to just the form $\alpha A \rightarrow \alpha \gamma$ and to just $A \beta \rightarrow \gamma \beta$, respectively, where $\alpha, \beta, \gamma \in (V_N \cup V_T)^*$, $A \in V_N$.

We have Type 1 \subseteq Type 0.

3. A grammar G is of type 2 (**context-free grammar**) if all of its productions are of the form $A \rightarrow \beta$, where $A \in V_N$, $\beta \in (V_N \cup V_T)^*$.

We have Type 2 \subseteq Type 1

4. A grammar G is of type 3 (**regular grammar**) if its productions are of the form

1. $A \rightarrow aB$;

2. $A \rightarrow b$,

where $a, b \in V_T$ and $A, B \in V_N$.

If a grammar G is of type i then language $L(G)$ is also of type i .

We have Type 3 \subseteq Type 2.

In this way we have Type 3 \subset Type 2 \subset Type 1 \subset Type 0.

Chomsky demonstrated that:

- a) Exist languages Type 0 that are not Type 1.
- b) Exist languages Type 1 that are not Type 2.
- c) Exist languages Type 2 that are not Type 3.

Variant 1.

$V_N = \{S, P, Q\}$,
 $V_T = \{a, b, c, d, e, f\}$,
 $P = \{$
 1. $S \rightarrow aP$
 2. $S \rightarrow bQ$
 3. $P \rightarrow bP$
 4. $P \rightarrow cP$
 5. $P \rightarrow dQ$
 6. $P \rightarrow e$
 7. $Q \rightarrow eQ$
 8. $Q \rightarrow fQ$
 9. $Q \rightarrow a \}$

Variant 2.

$V_N = \{S, R, L\}$,
 $V_T = \{a, b, c, d, e, f\}$,
 $P = \{$

1. $S \rightarrow aS$
2. $S \rightarrow bS$
3. $S \rightarrow cR$
4. $S \rightarrow dL$
5. $R \rightarrow dL$
6. $R \rightarrow e$
7. $L \rightarrow fL$
8. $L \rightarrow eL$
9. $L \rightarrow d \}$

Variant 3.

- $V_N = \{S, D, R\}$,
 $V_T = \{a, b, c, d, f\}$,
 $P = \{$
1. $S \rightarrow aS$
 2. $S \rightarrow bD$
 3. $S \rightarrow fR$
 4. $D \rightarrow cD$
 5. $D \rightarrow dR$
 6. $R \rightarrow bR$
 7. $R \rightarrow f$
 8. $D \rightarrow d \}$

Variant 4.

- $V_N = \{S, L, D\}$,
 $V_T = \{a, b, c, d, e, f, j\}$,
 $P = \{$
1. $S \rightarrow aS$
 2. $S \rightarrow bS$
 3. $S \rightarrow cD$
 4. $S \rightarrow dL$
 5. $S \rightarrow e$
 6. $L \rightarrow eL$
 7. $L \rightarrow fL$
 8. $L \rightarrow jD$
 9. $L \rightarrow e$
 10. $D \rightarrow eD$
 11. $D \rightarrow d\}$

Variant 5.

- $V_N = \{S, F, L\}$,
 $V_T = \{a, b, c, d\}$,
 $P = \{$
1. $S \rightarrow bS$
 2. $S \rightarrow aF$
 3. $S \rightarrow d$
 4. $F \rightarrow cF$
 5. $F \rightarrow dF$
 6. $F \rightarrow aL$
 7. $L \rightarrow aL$
 8. $L \rightarrow c$

9. $F \rightarrow b$ }

Variant 6.

$V_N = \{S, I, J, K\}$,

$V_T = \{a, b, c, e, n, f, m\}$,

$P = \{$

1. $S \rightarrow cI$

2. $I \rightarrow bJ$

3. $I \rightarrow fI$

4. $J \rightarrow nJ$

5. $J \rightarrow cS$

6. $I \rightarrow eK$

7. $K \rightarrow nK$

8. $I \rightarrow e$

9. $K \rightarrow m$ }

Variant 7.

$V_N = \{S, D, E, F, L\}$,

$V_T = \{a, b, c, d\}$,

$P = \{$

1. $S \rightarrow aD$

2. $D \rightarrow bE$

3. $E \rightarrow cF$

4. $F \rightarrow dD$

5. $E \rightarrow dL$

6. $L \rightarrow aL$

7. $L \rightarrow bL$

8. $L \rightarrow c$ }

Variant 8.

$V_N = \{S, D, E, J\}$,

$V_T = \{a, b, c, d, e\}$,

$P = \{$

1. $S \rightarrow aD$

2. $D \rightarrow dE$

3. $D \rightarrow bJ$

4. $J \rightarrow cS$

5. $E \rightarrow e$

6. $E \rightarrow aE$

7. $D \rightarrow aE$ }

Variant 9.

$V_N = \{S, B, D, Q\}$,

$V_T = \{a, b, c, d\}$,

$P = \{$

1. $S \rightarrow aB$

2. $S \rightarrow bB$

3. $B \rightarrow cD$

4. $D \rightarrow dQ$

5. $Q \rightarrow bB$

6. $D \rightarrow a$

7. $Q \rightarrow dQ$ }

Variant 10.

$V_N = \{S, B, L\},$

$V_T = \{a, b, c\},$

$P = \{$

1. $S \rightarrow aB$

2. $B \rightarrow bB$

3. $B \rightarrow cL$

4. $L \rightarrow cL$

5. $L \rightarrow aS$

6. $L \rightarrow b\}$

Variant 11.

$V_N = \{S, B, D\},$

$V_T = \{a, b, c\},$

$P = \{$

1. $S \rightarrow aB$

2. $S \rightarrow bB$

3. $B \rightarrow bD$

4. $D \rightarrow b$

5. $D \rightarrow aD$

6. $B \rightarrow cB$

7. $B \rightarrow aS\}$

Variant 12.

$V_N = \{S, F, D\},$

$V_T = \{a, b, c\},$

$P = \{$

1. $S \rightarrow aF$

2. $F \rightarrow bF$

3. $F \rightarrow cD$

4. $S \rightarrow bS$

5. $D \rightarrow cS$

6. $D \rightarrow a$

7. $F \rightarrow a\}$

Variant 13.

$V_N = \{S, B, D\},$

$V_T = \{a, b, c\},$

$P = \{$

1. $S \rightarrow aB$

2. $B \rightarrow aD$

3. $B \rightarrow bB$

4. $D \rightarrow aD$

5. $D \rightarrow bS$

6. $B \rightarrow cS$

7. $D \rightarrow c\}$

Variant 14.

$V_N = \{S, B, D\},$

$V_T = \{a, b, c, d\},$

- $P = \{$
1. $S \rightarrow aS$
 2. $S \rightarrow bB$
 3. $B \rightarrow cB$
 4. $B \rightarrow d$
 5. $B \rightarrow aD$
 6. $D \rightarrow aB$
 7. $D \rightarrow b \}$

Variant 15.

$V_N = \{S, A, B\}, V_T = \{a, b, c\},$

$P = \{$

1. $S \rightarrow aS$
2. $S \rightarrow bS$
3. $S \rightarrow cA$
4. $A \rightarrow aB$
5. $B \rightarrow aB$
6. $B \rightarrow bB$
7. $B \rightarrow c \}$

Variant 16.

$V_N = \{S, A, B\}, V_T = \{a, b, c, d\},$

$P = \{$

1. $S \rightarrow bS$
2. $S \rightarrow dA$
3. $A \rightarrow aA$
4. $A \rightarrow dB$
5. $B \rightarrow cB$
6. $A \rightarrow b$
7. $B \rightarrow a \}$

Variant 17.

$V_N = \{S, A, B, C\}, V_T = \{a, b, c, d\},$

$P = \{$

1. $S \rightarrow dA$
2. $A \rightarrow aB$
3. $B \rightarrow bC$
4. $C \rightarrow cB$
5. $A \rightarrow bA$
6. $B \rightarrow aB$
7. $B \rightarrow d \}$

Variant 18.

$V_N = \{S, A, B, C\}, V_T = \{a, b\},$

$P = \{$

1. $S \rightarrow aA$
2. $A \rightarrow bS$
3. $S \rightarrow aB$
4. $B \rightarrow aC$
5. $C \rightarrow a$
6. $C \rightarrow bS \}$

Variant 19.

$V_N = \{S, A, B, C\}$, $V_T = \{a, b\}$,

$P = \{$

1. $S \rightarrow aA$
2. $A \rightarrow bS$
3. $A \rightarrow aB$
4. $B \rightarrow bC$
5. $C \rightarrow aA$
6. $C \rightarrow b \}$

Variant 20.

$V_N = \{S, A, B, C\}$, $V_T = \{a, b, c, d\}$,

$P = \{$

1. $S \rightarrow dA$
2. $A \rightarrow d$
3. $A \rightarrow aB$
4. $B \rightarrow bC$
5. $C \rightarrow cA$
6. $C \rightarrow aS \}$

Variant 21.

$V_N = \{S, B, C, D\}$, $V_T = \{a, b, c\}$,

$P = \{$

1. $S \rightarrow aB$
2. $B \rightarrow bS$
3. $B \rightarrow aC$
4. $B \rightarrow b$
5. $C \rightarrow bD$
6. $D \rightarrow a$
7. $D \rightarrow bC$
8. $D \rightarrow cS \}$

Variant 22.

$V_N = \{S, D, F\}$, $V_T = \{a, b, c, d\}$,

$P = \{$

1. $S \rightarrow aS$
2. $S \rightarrow bS$
3. $S \rightarrow cD$
4. $D \rightarrow dD$
5. $D \rightarrow bF$
6. $D \rightarrow a$
7. $F \rightarrow bS$
8. $F \rightarrow a \}$

Variant 23.

$V_N = \{S, B, C\}$, $V_T = \{a, b, c\}$,

$P = \{$

1. $S \rightarrow aB$
2. $B \rightarrow aC$
3. $C \rightarrow bB$

4. $C \rightarrow c$
5. $C \rightarrow aS$
6. $B \rightarrow bB \}$

Variant 24.

$V_N = \{S, A, C, D\}$, $V_T = \{a, b\}$,

$P = \{$

1. $S \rightarrow aA$
2. $A \rightarrow bS$
3. $A \rightarrow dD$
4. $D \rightarrow bC$
5. $C \rightarrow a$
6. $C \rightarrow bA$
7. $D \rightarrow aD \}$

Variant 25.

$V_N = \{S, A, B\}$, $V_T = \{a, b, c, d\}$,

$P = \{$

1. $S \rightarrow bS$
2. $S \rightarrow dA$
3. $A \rightarrow aA$
4. $A \rightarrow dB$
5. $B \rightarrow cB$
6. $A \rightarrow b$
7. $B \rightarrow a \}$

Variant 26.

$V_N = \{S, A, B, C\}$, $V_T = \{a, b, c, d\}$,

$P = \{$

1. $S \rightarrow dA$
2. $A \rightarrow aB$
3. $B \rightarrow bC$
4. $C \rightarrow cB$
5. $B \rightarrow d$
6. $C \rightarrow aA$
7. $A \rightarrow b \}$

Variant 27.

$V_N = \{S, A, B\}$, $V_T = \{a, b, c\}$,

$P = \{$

1. $S \rightarrow aA$
2. $A \rightarrow bS$
3. $S \rightarrow bB$
4. $A \rightarrow cA$
5. $A \rightarrow aB$
6. $B \rightarrow aB$
7. $B \rightarrow b \}$

Variant 28.

$V_N = \{S, A, B, C\}$, $V_T = \{a, b\}$,

$P = \{$

1. $S \rightarrow aA$

2. $A \rightarrow bS$
3. $A \rightarrow aB$
4. $B \rightarrow bC$
5. $C \rightarrow aA$
6. $B \rightarrow aB$
7. $C \rightarrow b \}$

Variant 29.

$V_N = \{S, A, B, C\}$, $V_T = \{a, b\}$,

$P = \{$

1. $S \rightarrow bA$
2. $A \rightarrow b$
3. $A \rightarrow aB$
4. $B \rightarrow bC$
5. $C \rightarrow cA$
6. $A \rightarrow bA$
7. $B \rightarrow aB \}$

Variant 30.

$V_N = \{S, B, C, D\}$, $V_T = \{a, b, c\}$,

$P = \{$

1. $S \rightarrow aB$
2. $B \rightarrow bS$
3. $B \rightarrow aC$
4. $B \rightarrow c$
5. $C \rightarrow bD$
6. $D \rightarrow c$
7. $D \rightarrow aC \}$

Variant 31.

$V_N = \{S, D, R\}$, $V_T = \{a, b, c\}$,

$P = \{$

1. $S \rightarrow aS$
2. $S \rightarrow bS$
3. $S \rightarrow cD$
4. $D \rightarrow bD$
5. $D \rightarrow cR$
6. $D \rightarrow a$
7. $R \rightarrow b \}$

Variant 32.

$V_N = \{S, D, R\}$, $V_T = \{a, b, c\}$,

$P = \{$

1. $S \rightarrow aS$
2. $S \rightarrow cD$
3. $D \rightarrow bR$
4. $R \rightarrow aR$
5. $R \rightarrow b$
6. $R \rightarrow cS \}$