Multivariate Regression

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Controls

- Last time we talked about multivariate regression and controlling for variables
- If our X of interest is related to the error term, we have endogeniety, and the mean of the sampling variation of $\hat{\beta}_1$ won't be β_1 (i.e. it's biased)
- If we can figure out all the *other reasons* why X and Y might be related, the back doors. And we can close those back doors to identify our effect by controlling for variables along those back door paths
- We draw a causal diagram and list the paths
- If we can control for one thing on each path, then that washes out the variation from those paths, and the only remaining reason X and Y are related is that X causes Y - causal identification!

Today

- Today we're going to talk more about controlling
- Including when you *don't* want to control for things
- We'll also talk about how to actually perform multivariate regression
- As well as do some things that go along with it (those F-statistics will finally be useful!)

Bad Controls

- Last time, we were careful to draw a causal diagram, list out the paths, and control for something that would close each path we don't want to include
- Couldn't we just skip this all and control for everything we can think of?
- No! There are a few reasons why adding controls when you shouldn't can make your estimate worse:
  - Washing out too much variation
  - Post-treatment bias
  - Collider bias

Washing out Too Much Variation

- When we control for a variable Z, we wash out all the variation in X and Y associated with that variable
- Which means that we can also think of $\hat{\beta}_1$ from that regression as saying "*within the same value of* Z, a one-unit change in X raises Y by $\hat{\beta}_1$"
- So we have to think carefully about whether that statement actually makes sense!
- (this is very similar to the concept of "collinearity")

Washing out Too Much Variation

- For example, let's say we want to know the effect of being in a business school IBA on $Earnings$
- However, we also know that your college major is strongly related to whether you're in IBA, and causes your $Earnings$
- But if we control for $Major$, we're saying "within the same major, being in the business school vs. not has a $\hat{\beta}_1$ effect on earnings"
- What does that even mean? You're comparing econ majors in vs. out of the business school... but who are the econ majors not in the business school? And who are the English majors *in* the business school to compare against the English majors not in IBA?
- Controlling for major would make the regression impossible to interpret. Plus, it would provide an estimate based entirely on the few people it can find who are English majors in IBA or Econ majors out of IBA - is that representative?

Washing out Too Much Variation

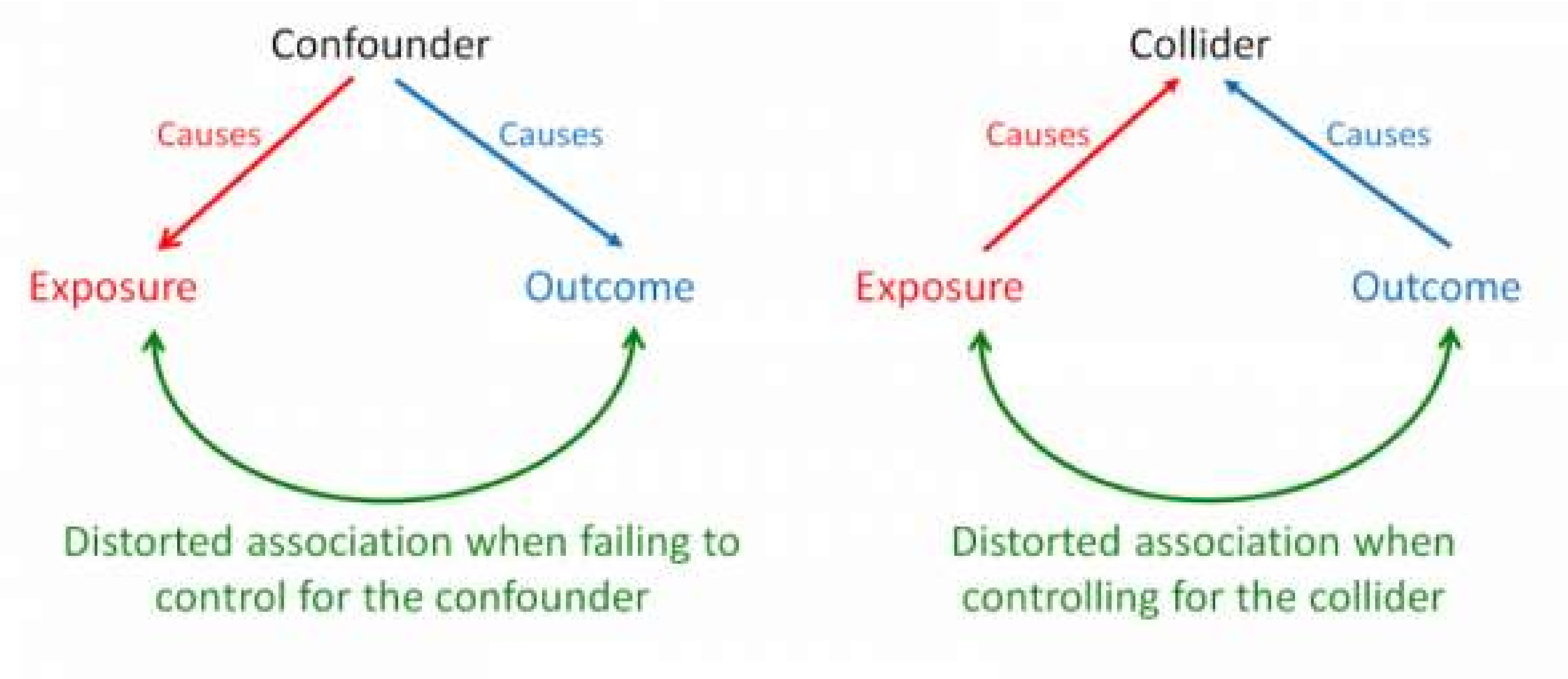
- When adding a control, it's always useful to think about *who you're comparing* and if that variation really exists in the data
- In thinking about the effect of being in IBA, we really want to compare people *between* in-IBA and out-of-IBA majors
- Controlling for $Major$ is asking OLS to compare people *within* majors
- So it doesn't really make sense to control for major. We want that effect in there!

Post-Treatment Bias

Post-Treatment Bias

- We determined that we needed to control for $Location$ and $Background$ but not $Skills$. Why?
- Because $Skills$ is *part of the effect we're trying to pick up!*
- If $Preschool$ affects $Earnings$ *because it improves your $Skills$, then we'd count that as being a valid way that $Preschool$ affects $Earnings$
- $Skills$ is *post-treatment*, i.e. caused by treatment
- (note that all the arrows on the path $Preschool \rightarrow Skills \rightarrow Earnings$ point away from $Preschool$)

Post-Treatment Bias

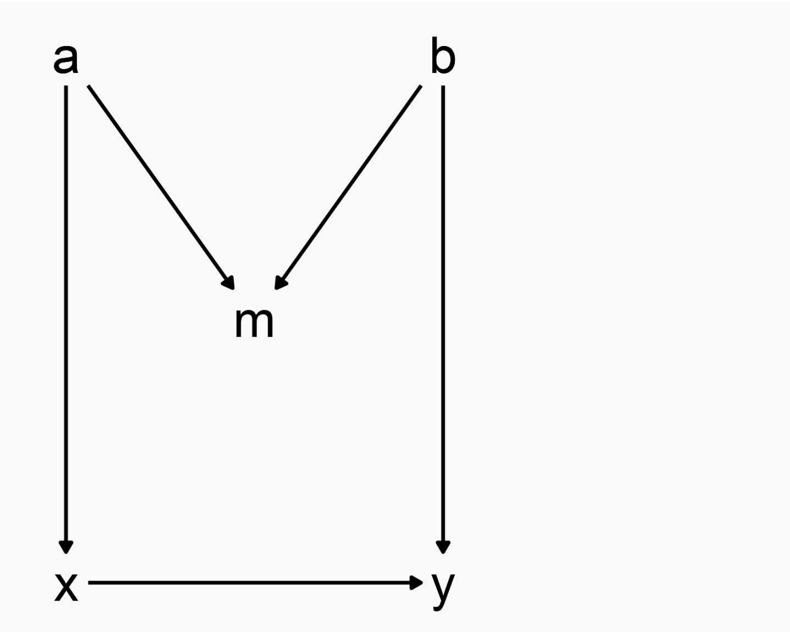

Confounder
Causes
Causes
Exposure
Outcome
Distorted association when failing to control for the confounder
Collider
Causes
Causes
Exposure
Outcome
Distorted association when controlling for the collider

Colliders

- One last reason to not control for something, and this one's a bit harder to wrap your head around
- On a causal path from X to Y, if there's a variable on that path where *both arrows on either side point at it*, that's a *collider variable* on that path
- Like this: `X <- W -> C <- Z -> Y`. The arrows "collide" at `C`
- If there's a collider on a path, *that path is automatically closed already*
- But if you control for the collider, *it opens back up!*
- You can go from identified to endogenous by *adding* a control!

Colliders

- So here, `x <- a -> m <- b -> y` is pre-blocked because of `m`, no problem. `a` and `b` are unrelated, so no back door issue!
- Control for `m` and now `a` and `b` are related, back door path open.

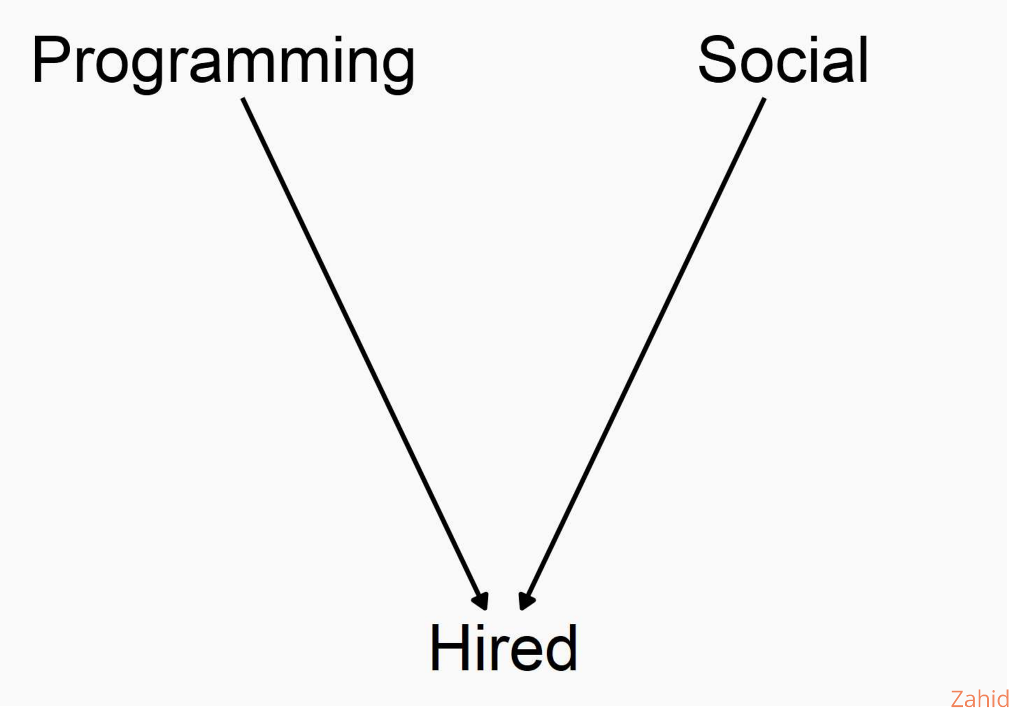


Example

- You want to know if programming skills reduce your social skills
- So you go to a tech company and test all their employees on programming and social skills
- Let’s imagine that the *truth* is that programming skills and social skills are unrelated
- But you find a negative relationship! What gives?

Example

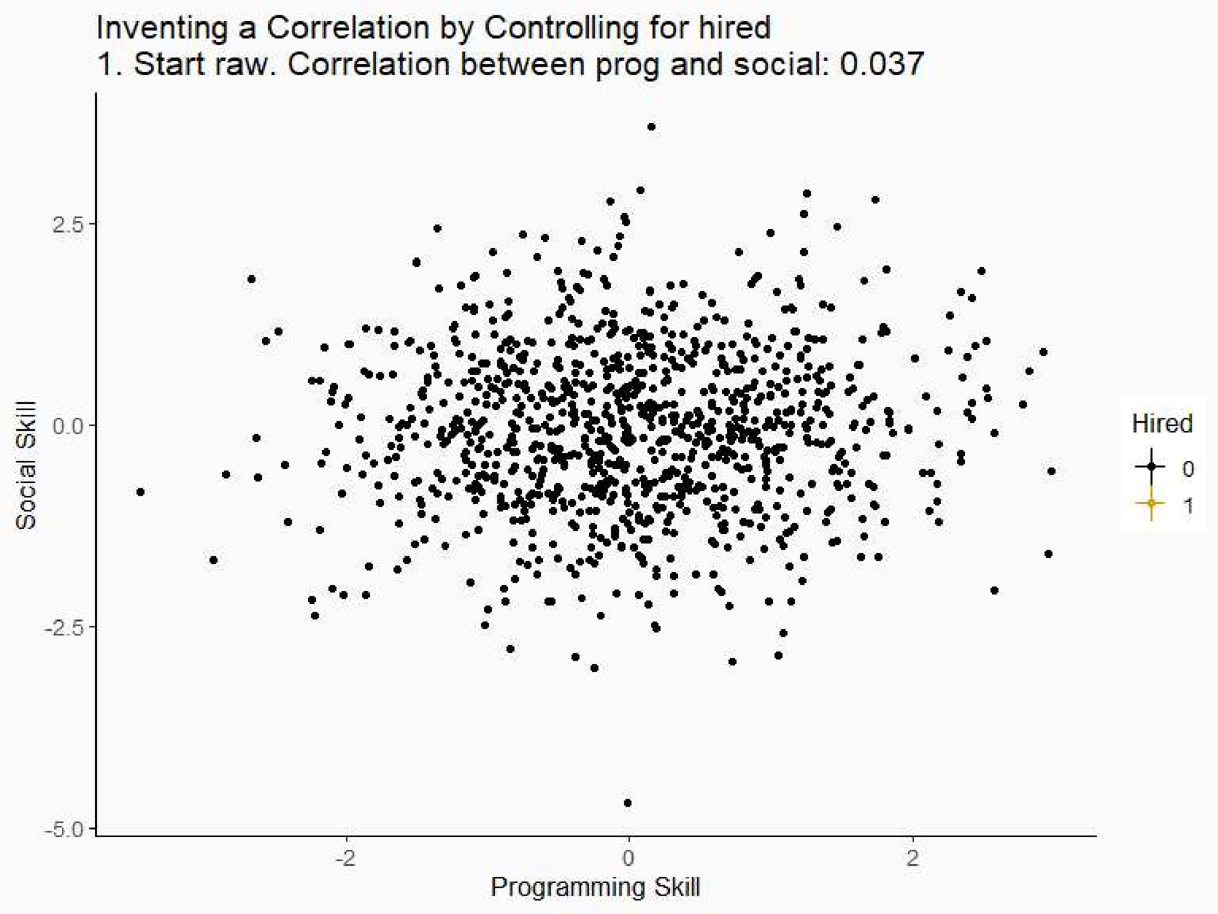
- Oops! By surveying only the tech company, you controlled for “works in a tech company”
- To do that, you need programming skills, social skills, or both! It’s a collider!



Example

```
survey <- tibble(prog=rnorm(1000),social=rnorm(1000)) %>%
  mutate(hired = (prog + social > .25))
basic <- feols(prog~social, data = survey)
hiredonly <- feols(prog~social, data = survey %>% filter(hired))
withcontrol <- feols(prog ~ social + hired, data = survey)
etable(basic, hiredonly, withcontrol)
##                            basic            hiredonly          withcontrol
## Dependent Var.:             prog                 prog                 prog
##
## (Intercept)      0.0193 (0.0331)   1.018*** (0.0444) -0.7485*** (0.0354)
## social           0.0383 (0.0326) -0.4900*** (0.0433) -0.4245*** (0.0285)
## hiredTRUE                                               1.727*** (0.0583)
## _______________ ________________ ____________________ ____________________
## S.E. type                    IID                  IID                  IID
## Observations               1,000                  432                1,000
## R2                       0.00138              0.22937              0.46898
## Adj. R2                  0.00038              0.22757              0.46792
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Graphically



Concept Checks

In each case, we're controlling for something we shouldn't. Is this a case of washing out too much variation, post-treatment bias, or collider bias?

- Effect of a wife's eye color on her husband's eye color, controlling for the eye color of their biological child
- Effect of religious denomination on how often you attend church services, controlling for the specific church someone attends
- Effect of a new error-reducing accounting system on profits, controlling for the number of accounting errors
- Effect of a merger on market prices, controlling for the level of market concentration
- Effect of a state's intellectual property law on how many things an inventor invents, controlling for their hometown

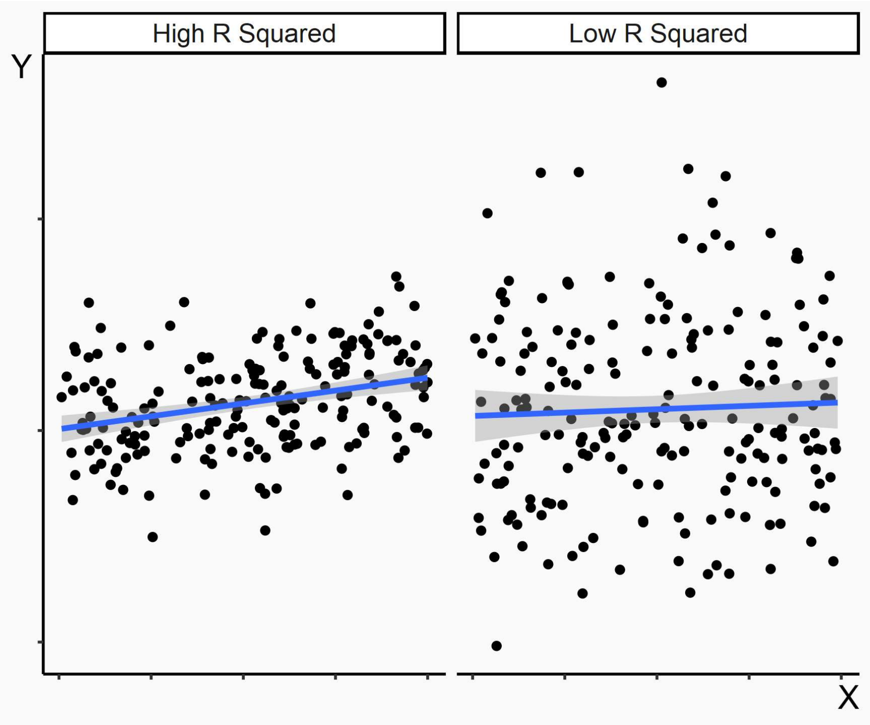
Goodness of Fit

- Let's switch gears a bit and talk about some of the statistical aspects of multivariate regression
- One is the *goodness of fit*. That is, OLS does as good a job as possible of using X and controls to explain Y, but how good a job does it do?
- If the residuals are really big, then there's a lot of noise in Y we're not explaining!
- If they're small, then most of what's going on in Y is accounted for

R squared

R squared

- In both, true effect is the same, no endogeneity. Only difference is how much *other, non -* X *- based* variation there is in Y



R squared

- Those same two regressions in a table (note the SEs are different too! Concept check: why is that?)

```
##                                high               low
## Dependent Var.:                   Y                 Y
##
## (Intercept)       0.1344 (0.1260)  -0.0961 (0.3199)
## X               1.871*** (0.2132) 2.570*** (0.5564)
## _______________ _________________ _________________
## S.E. type                     IID               IID
## Observations                1,000             1,000
## R2                        0.07165           0.02093
## Adj. R2                   0.07072           0.01995
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

F tests

- The R^2 reveals what was actually going on with those F-tests we did before
- An F-test of a regression sees if a regression *predicts more accurately* than a more restricted regression where some of the coefficients are forced to 0 (or to some other value)
- In other words, it might take the R^2 of each of these two models and calculate something from them that has a F distribution (remember, F distribution is the ratio of squared sums of normals!) to test if $\hat{\beta}_2$ and $\hat{\beta}_3$ are both zero at the same time
- The top model *IBA* have a higher R^2, but is it higher *than you'd expect by random chance?*

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 A + \varepsilon$$

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

F tests

- By the way, when we did this before with a single variable we were comparing:

$$Y = \beta_0 + \beta_1 X \varepsilon$$

to

$$Y = \beta_0 + \varepsilon$$

- It's also common to see an F-statistic at the bottom of a column for a regression table. This is, by convention, testing the full model in that column against the constant-only model. This pretty much always rejects the null and is mostly useless.

F tests

- Let's predict some professor salaries

```r
data(Salaries, package = 'carData')
unrestricted <- feols(salary ~ yrs.since.phd + yrs.service + sex, data = Salaries)
restricted <- feols(salary ~ yrs.since.phd, data = Salaries)

fitstat(unrestricted, 'r2')
## R2: 0.195102
fitstat(restricted, 'r2')
## R2: 0.175755
```

F tests

```
##                              unrestricted            restricted
## Dependent Var.:                    salary                salary
##
## (Intercept)     82,875.9*** (4,800.6) 91,718.7*** (2,765.8)
## yrs.since.phd      1,552.8*** (256.1)       985.3*** (107.4)
## yrs.service           -649.8* (254.0)
## sexMale            8,457.1. (4,656.1)
## _______________ ______________________ ______________________
## S.E. type                         IID                    IID
## Observations                      397                    397
## R2                            0.19510                0.17575
## Adj. R2                       0.18896                0.17367
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

F tests

- We basically just take a ratio based on R^2 values. On top: additional explanatory power scaled by the number of restrictions (on top), and on bottom the explanatory power left to go scaled by N minus the number of variables

```
r2_unres <- fitstat(unrestricted, 'r2')$r2 %>% unname()
r2_res <- fitstat(restricted, 'r2')$r2 %>% unname()

# Calculate by hand
((r2_unres - r2_res)/2) / ((1 - r2_unres)/(397 - 4))
## [1] 4.72

# Have the wald() function do it for us (note F is the same!)
wald(unrestricted, c('yrs.service','sexMale'))
## Wald test, H0: joint nullity of yrs.service and sexMale
##  stat = 4.72326, p-value = 0.009397, on 2 and 393 DoF, VCOV: IID.
```

F tests

- Why would we want to do this?
- We might want to see if a *set of variables* has explanatory power: for example, does adding a bunch of background variables improve prediction?
- A t-test asks if a single coefficient is 0. The F-test asks if a bunch of coefficients are 0 at the same time
- If those variables overlap a lot, then each individual one might be insignificant but the group could be important
- Also, we might want to know if two coefficients are not just nonzero, but *equal to each other*
- "Does this variable have a similarly-sized effect to this other variable?"

F tests

- F tests are broader, too - you can test other restrictions on coefficients, not just seeing if they're all 0
- Do `yrs.since.phd` and `yrs.service` have the same effect but of opposite signs (in other words, if you add them together do you get 0)? No!

```
library(multcomp)
# Generalized Linear Hypothesis Test
glht(unrestricted, 'yrs.since.phd + yrs.service = 0') %>% summary()
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Fit: feols(fml = salary ~ yrs.since.phd + yrs.service + sex, data = Salaries)
##
## Linear Hypotheses:
##                                  Estimate Std. Error t value Pr(>|t|)
## yrs.since.phd + yrs.service == 0      903        110    8.23  2.7e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

Concept Checks

- In a sentence, describe what R^2 measures
- Give two reasons why you shouldn't pick one model over another just based on its R^2 value
- Finish the sentence: "The F-statistic shows whether the difference between R^2 in an unrestricted model and a restricted model is bigger than…"
- What does it mean to say that two coefficients are "zero at the same time"?

Multivariate OLS in R

- Conveniently, adding more variables to an OLS model in R is just an issue of… literally adding them

```
model <- feols(salary ~ yrs.since.phd + yrs.service + sex, data = Salaries)
```

Multivariate OLS in R

What else might we want to know?

- How to get the R^2
- How to do an F test
- How to look at residuals and control for something by hand

Getting R squared

- Well, we can just see it at the bottom of the default `etable()` table, that works
- We can also pull it out using `fitstat()` which can also be used to get all sorts of other statistics

```
# Look at a summary of the statistic
fitstat(model, 'r2')
## R2: 0.195102
# Get the actual value out
fitstat(model, 'r2')$r2
##    r2
## 0.195
```

Doing an F test

- We already did one by hand! using the R^2 values
- We can also use `wald()` to test a set of coefficients being jointly 0
- Or `glht()` in **multcomp** for testing more complex equations, or things not tested against 0 (rearrange the equation to have a constant on one side)
- To account for heteroskedasticity, you can use `vcov = 'hetero'` in the `feols()` you feed either, or set `vcov` directly in `wald()` (but not `glht`)

```
wald(model, c('X1','X2'))
glht(model, 'X1 = 3')
glht(model, 'X1 - X2 = 0')
```

Predictions and Residuals

- We can get a vector of predictions from a regression object with `predict()` (or, with `feols()`, optionally `model$fitted.values`)
- And a vector of residuals with `resid()` or optionally with `feols()`, `model$residuals`
- This turns out to be handy often in applied work! For example, maybe we want to plot those predicted values or residuals!
- Although for what we've covered so far it's mostly just good for doing R^2 or controlling by hand.
- Which can be good to get a feel for how this all works. Or just to learn how to use `predict()` and `resid()`

Controlling by hand

```
etable(model)
##                                      model
## Dependent Var.:                     salary
##
## (Intercept)      82,875.9*** (4,800.6)
## yrs.since.phd      1,552.8*** (256.1)
## yrs.service           -649.8* (254.0)
## sexMale            8,457.1. (4,656.1)
## _______________ ______________________
## S.E. type                          IID
## Observations                       397
## R2                             0.19510
## Adj. R2                        0.18896
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

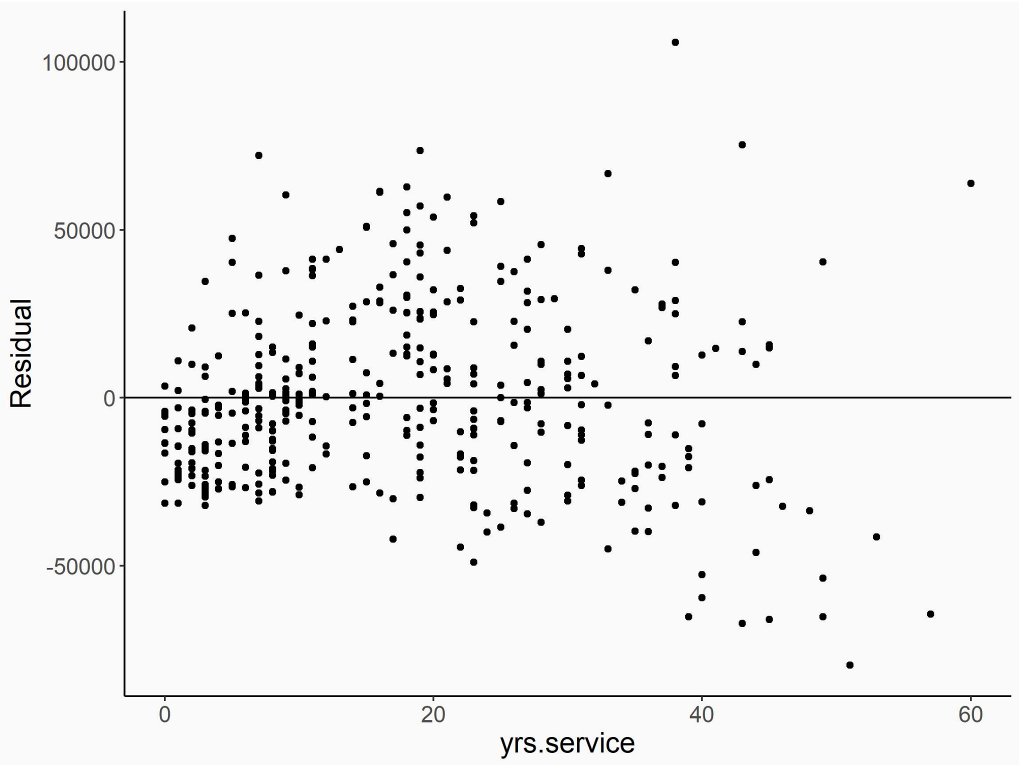
Controlling by hand

```
yrs_model <- feols(yrs.since.phd ~ yrs.service + sex, data = Salaries)
salary_model <- feols(salary ~ yrs.service + sex, data = Salaries)
my_data <- tibble(yrs_resid = resid(yrs_model),
                  salary_resid = resid(salary_model))

resid_model <- feols(salary_resid ~ yrs_resid, data = my_data)
etable(resid_model)
##                        resid_model
## Dependent Var.:       salary_resid
##
## (Intercept)     2.81e-10 (1,365.6)
## yrs_resid       1,552.8*** (255.5)
## _______________ __________________
## S.E. type                      IID
## Observations                   397
## R2                         0.08552
## Adj. R2                    0.08320
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Plotting Residuals

Plotting residuals from the full model against X is a good way to check for heteroskedasticity (or anything else super strange)



R squared by hand

```
fitstat(model, 'r2')$r2
##     r2
## 0.195


predicted_values <- predict(model)
cor(predicted_values,Salaries$salary)^2
## [1] 0.195
```

Swirl

Let’s do the Multivariate Regression Swirl!