Sensitive Analysis

$\text{Max } Z = 3x_1 + 2x_2 + 5x_3$

Subject to

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 0x_2 + 2x_3 \leq 460$

$x_1 + 4x_2 + 0x_3 \leq 420$

Case I
R.H.S

Augmented form

$Z - 3x_1 - 2x_2 - 5x_3 + 0S_1 + 0S_2 + 0S_3 = 0$

$x_1 + 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 = 430$

$3x_1 + 0x_2 + 2x_3 + 0S_1 + S_2 + 0S_3 = 460$

$x_1 + 4x_2 + 0x_3 + 0S_1 + 0S_2 + S_3 = 420$

Table

B.V	x_1	x_2	x_3	S_1	S_2	S_3	sol	R_1	R_2	R_3	Ratio
Z	-3	-2	-5	0	0	0	0	0	0	0	
S_1	1	2	1	1	0	0	430	1	0	0	430
S_2	3	0	2	0	1	0	420	0	1	0	230
S_3	1	4	0	0	0	1	460	0	0	1	∞

similar values copy in R_1, R_2, R_3 column wise

$x_1 + 2x_2 + x_3 \leq 430 + R_1$

$3x_1 + 0x_2 + 2x_3 \leq 460 + R_2$

$x_1 + 4x_2 + 0x_3 \leq 420 + R_3$

Attemp all steps to reach at optimal Table. Here is optimal Table.

B.V	x_1	x_2	x_3	S_1	S_2	S_3	Sol	R_1	R_2	R_3
Z	4	0	0	1	2	0	1350	1	2	0
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100	$1/2$	$-1/4$	0
x_3	$3/2$	0	1	0	$1/2$	0	230	0	$1/2$	0
S_3	2	0	0	-2	1	1	20	-2	1	1

Here we had to find values of R_1, R_2 and R_3

$x_2 \Rightarrow 100 + \frac{1}{2}R_1 - \frac{1}{4}R_2 + 0R_3 \geq 0$ ———— ①

$x_3 \Rightarrow 230 + 0R_1 + \frac{1}{2}R_2 + 0R_3 \geq 0$ ———— ②

$S_3 \Rightarrow 20 - 2R_1 + R_2 + R_3 \geq 0$ ———— ③

Now find values of R_1 put R_2 and $R_3 = 0$

(i) $100 + \frac{1}{2}R_1 \geq 0$ $\qquad R_1 \geq -200$

(ii) $230 + 0R_1 \geq 0$

(iii) $20 - 2R_1 \geq 0$ $\qquad R_1 \leq 10$

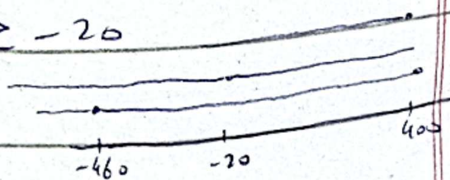
$\boxed{-200 \leq R_1 \leq 10}$

Now find R_2 put R_1 & $R_3 = 0$

(i) $100 - \frac{1}{4}R_2 \geq 0$ $\qquad R_2 \leq 400$

(ii) $230 + \frac{1}{2}R_2 \geq 0$ $\qquad R_2 \geq -460$

(iii) $20 + R_2 \geq 0$ $\qquad R_2 \geq -20$



$\boxed{-20 \leq R_2 \leq 400}$

Reduced cost ⇒ $C_B B^{-1}A - C$

shadow price ⇒ $C_B B^{-1}$

Duality/opt sol ⇒ $C_B B^{-1} b$

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Now find value of R_3 put R_1 & $R_2 = 0$

(i) $100 + 0R_3 \geq 0 \rightarrow$

(ii) $230 + 0R_3 \geq 0 \rightarrow$

(iii) $20 + R_3 \geq 0$ $\quad R_3 \geq -20$

$$\boxed{-20 \leq R_3 \leq \infty}$$

Actual values for R.H.S $\quad$ eq 1 $\quad x_1 + 2x_2 + x_3 + S_1 = 430 + R_1$

For Minimum ⇒ $430 - 200 = 230$

For Max ⇒ $430 + 10 = 440$

$(230 - 440)$

"For equ 2" $\quad x_1 + 4x_2 + S_2 = 420 + R_2$

Min ⇒ $420 - 20 = 400$

Max ⇒ $420 + 400 = 820$

$(400 - 820)$

"For Equ 3" $\quad 3x_1 + 2x_3 + S_3 = 460 + R_3$

Min ⇒ $460 - 20 \rightarrow 440$

Max ⇒ $460 + \infty \rightarrow \infty$

$(440 - \infty)$

Dual price

$1350 + R_1 + 2R_2 + 0R_3$

⇒ A unit change in operation 1 $(R_1 = \pm 1 \text{ min})$ changes Z by 1\$

⇒ " " " " " $(R_2 = \pm 1 \text{ min})$ change Z by 2\$

⇒ " " " " $(R_3 = \pm 1 \text{ min})$ changed by 0\$

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Imp we can compute values of feasible solution when D_1, D_2, D_3 setting the feasible sol by computing values of x_2, x_3 and S_3

$$x_2 \Rightarrow 100 + \frac{1}{2}R_1 - \frac{1}{4}R_2 \geq 0$$

$$x_3 \Rightarrow 230 + \frac{1}{2}R_2 \geq 0$$

$$S_3 \Rightarrow 20 - 2R_1 + R_2 + R_3 \geq 0$$

Let Here

$\boxed{R_1 = -30}$ $\boxed{R_2 = -12}$ $\boxed{R_3 = 10}$

then

$\boxed{x_2 = 88}$ $\boxed{x_3 = 224}$ $\boxed{S_3 = 78}$

$$\therefore Z = 3x_1 + 2x_2 + 5x_3$$

Here $Z = 3(0) + 2(88) + 5(224) = \boxed{1296}$

other way

$$Z = 1350 + 1R_1 + 2R_2 + 0R_3$$

$$= 1350 + 1(-30) + 2(-12) + 0$$

$$= \boxed{1296}$$

Case II objective Function change

~~Z = (3+d1)~~

$$Z = 3x_1 + 2x_2 + 5x_3$$

$$Z = (3 + d_1)x_1 + (2 + d_2)x_2 + (5 + d_3)x_3$$

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 0x_2 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

Simplex solution

		D_1	D_2	D_3	0	0	0	
	B.V	x_1	x_2	x_3	s_1	s_2	s_3	sol
I	Z	4	0	0	1	2	0	1350
D_2	x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100
D_3	x_3	$3/2$	0	1	0	$1/2$	0	230
0	s_3	2	0	0	-2	1	1	20

$$(4\times1) - \left(\frac{1}{4}D_2\right) + \frac{3}{2}D_3 - D_1 \geq 0 \quad \text{—— (1)}$$

$$(1\times1) + \frac{1}{2}D_2 \geq 0 \quad \text{—— (2)}$$

$$2 - \frac{1}{4}D_2 + \frac{1}{2}D_3 \geq 0 \quad \text{—— (3)}$$

Ye Expression bta rhi hein k kitna change kr sktY hein. Agr hum D_1, D_2, D_3 ki value ko fy krein aur is mein put krein Agr vo satisfy krein to solution feasible ho jaa, warna nae.

our old Z

$$Z_{old} = 3x_1 + 2x_2 + 5x_3$$

New Z basically chang in Z

$$Z_{new} \Rightarrow 2x_1 + x_2 + 6x_3$$

$D_1 \Rightarrow 2 - 3 = -1$

$D_2 \Rightarrow 1 - 2 = -1$

$D_3 \Rightarrow 6 - 5 = 1$

put D_1, D_2 and D_3 in above equat.

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$$4 - \frac{1}{4}(-1) + \frac{3}{2}(1) - (-1) \geq 0$$

$$6.75 \geq 0 \quad \text{feasible} \checkmark$$

$$1 + \frac{1}{2}(-1) \geq 0$$

$$0.57 \geq 0 \quad \text{feasible} \checkmark$$

$$2 - \frac{1}{4}(-1) + \frac{1}{2}(1) \geq 0$$

$$2.757 \geq 0 \quad \text{feasible} \checkmark$$

Now find the solution (New optimal)

$$Z = (3+d_1)x_1 + (2+d_2)x_2 + (5+d_3)x_3$$

As $Z = 1350 + 100D_2 + 230D_3$ —— (VIP) from Table

$$Z = 1350 + 100(-1) + 230(1)$$

$$\boxed{Z = 1480}$$

Case III Chang in Constraint

$$Z \Rightarrow 3x_1 + 2x_2 + 5x_3$$

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 0x_2 + 2x_3 \leq 460$$

x_1 change

no

$$x_1 + 4x_2 + 0x_3 \leq 420$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 4 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix}$$

$$C = [3 \quad 2 \quad 5]$$

$$C_B = [0 \quad 0 \quad 0]$$

Z	$C_B B^{-1} A - C$	$C_B B^{-1}$	$C_B B^{-1} b$
	$B^{-1}A$	B^{-1}	$B^{-1}b$

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Case IV Agr Poora Constraint New ayye

$$2x_1 + x_2 + x_3 \le 550$$

prev sol

$\boxed{\begin{array}{l} x_2 = 100 \\ x_3 = 230 \\ S_3 = 20 \end{array}}$

isko dekhein k ye prev pr effect krta yaa nae.

$$2(0) + 100 + 230 \le 550$$

$330 \le 550$ ✓ satisfy so previous sol is optimal hi rhy gaa.

Imp

if $2x_1 + x_2 + 2x_3 \le 550$

$$2(0) + 100 + 2(230) \le 550$$

$$560 \le 550$$

Ab ye satisfy nae kr rha, Tu previous mein Adjustment krni ho gi.

So

$$2x_1 + x_2 + 2x_2 + S_4 = 550$$

Case V

Agr hr Constraint mein Ek New variable introduce krwa dein Tu?

Dual problem

$Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$

$x_1 + 2x_2 + x_3 + S_1 = 430 \longrightarrow y_1$

$3x_1 + 0x_2 + 2x_3 + S_2 = 460 \longrightarrow y_2$

$x_1 + 4x_2 + 0x_3 + S_3 = 420 \longrightarrow y_3$

objective

$\min Z = 430y_1 + 460y_2 + 420y_3$

Imp
1. max sy min mean $\geq$ consts
2. min sy max TU $\leq$ Constraint

$y_1 + 3y_2 + y_3 \geq 3$

$2y_1 + 0y_2 + 4y_3 \geq 2$

$y_1 + 2y_2 + 0y_3 \geq 5$

$\boxed{y_i \geq 0}$

Optimal Table

Reduce cost — shadow variable

B.V	x_1	x_2	x_3	S_1	S_2	S_3	Sol
Z	4	0	0	1	2	0	1350
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100
x_3	$3/2$	0	1	0	$1/2$	0	230
S_3	2	0	0	-2	1	1	20

Constraint coefficient — Inverse of primal prob

Understand b? in Constraint

Here $b = \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix}$

So $B^{-1}b$

Finding

I I can find $B^{-1}b$ = R-H-S of optimal Table

$B^{-1}b$ = R.H.S of sol

$$\begin{array}{ccc} S_1 & S_2 & S_3 \end{array}$$
$$\begin{bmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix} \Rightarrow \begin{bmatrix} 100 \\ 230 \\ 20 \end{bmatrix}$$

BV

II shadow price

Take Basic variable cofficient from objective function and multiply with B^{-1} iteration zero.

$C_B = [0 \quad 0 \quad 0]$

$$B^{-1}A = \begin{bmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 4 & 0 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} -1/4 & 1 & 0 \\ 3/2 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} \end{array}$$

$$X_B \Rightarrow \begin{bmatrix} x_2 \\ x_3 \\ S_3 \end{bmatrix} \Rightarrow \overset{C_B}{[2 \quad 5 \quad 0]}$$

$$C_B B^{-1} \Rightarrow [2 \quad 5 \quad 0] \begin{bmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{bmatrix} \Rightarrow [1 \quad 2 \quad 0]$$

III Reduced Cost Find

From dual problem

Find cofficient of x_1, x_2, x_3

$y_1 + 3y_2 + y_3 \geq 3$

$2y_2 + 4y_3 \geq 2$

Signature $y_1 + 2y_2 \geq 5$ No.

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$x_1 \Rightarrow y_1 + 3y_2 + y_3 - 3$ —— ①

$x_2 \Rightarrow 2y_2 + y_3 - 2$ —— ②

$x_3 \Rightarrow y_1 + 2y_2 - 5$ —— ③

now put the values of y_1, y_2, y_3

As $C_B B^{-1} = \begin{array}{ccc} [1 & 2 & 0] \\ y_1 & y_2 & y_3 \end{array}$

$x_1 = 1 + 3(2) + 0 - 3 = 4$

$x_2 = 4 + 0 - 2 \Rightarrow 2$

$x_3 \Rightarrow 1 + 2(2) - 5 \Rightarrow 0$