No.: Date:

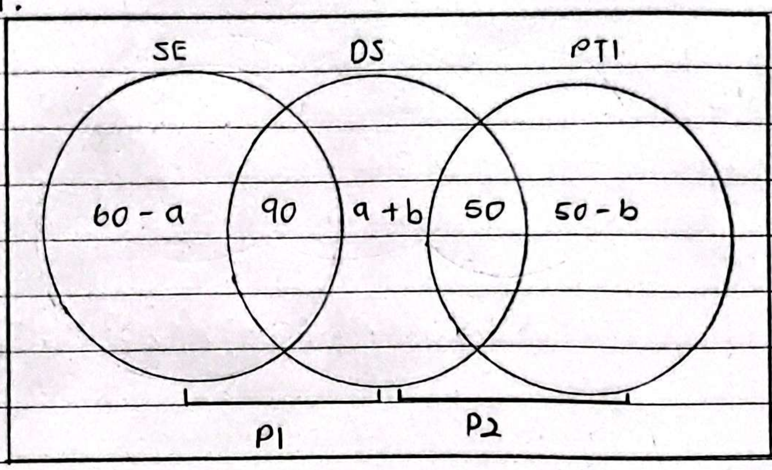
Assignment 1 (Discrete structure)

Group members :

1. Aw Xu Yuen A24CS0052
2. Fong Jin Xuan A24CS0074
3. Lai Shi Ni A24CS0298
4. Zahin Irdina binti Mohd Zabidy A24CS0216

Question 1

1 (a) i.



a = number of students in program 1 taking only discrete structure (DS)

b = number of students in program 2 taking only discrete structure (DS)

ii. $|DS| = 90 + a + b + 50$

$= 140 + a + b$

iii. $|(SE \cap DS) \cup (PTI \cap DS)| = 90 + 50$

$= 140$

iv. Number of students took less than 2 subjects $= (60 - a) + a + b + (50 - b)$

$= 60 + 50$

$= 110$

No.: Date:

1 (b) N = { Natural Number }

A = { 3, 5, 7, 9, 11, 13, 15, 17, 19 }

B = { 2, 3, 5, 7, 11, 13, 17, 19 }

C = { 5 }

i. $|A| = 9$

$|B| = 8$

$|C| = 1$

ii. $|A| = 9$

$|P(A)| = 2^9$

$= 512$

Number of proper subsets of A = 512 - 1

= 511

iii. $C \times B = \{ (5,2), (5,3), (5,5), (5,7), (5,11), (5,13), (5,17), (5,19) \}$

Question 2

2. (a) i. $m \wedge n$: You play table tennis and you miss the midterm test.

ii. $\neg(m \vee n) \vee o = \neg m \wedge \neg n \vee o$ (De Morgan's Law)

: You are not playing table tennis and not miss the midterm test or you pass the subject.

(b) Truth table

a	b	¬a	(¬a ∨ b)	(a → b)
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore \ (a \rightarrow b) \equiv (\neg a \vee b)$

No.: Date:

2. (C) i. $x \rightarrow y$

ii. $(\neg x \vee \neg z) \rightarrow \neg y$

$= \neg(x \wedge z) \rightarrow \neg y$

iii. $y \leftrightarrow (x \wedge z)$

Question 3

3. (a) $P(n)$ = n divides 15. Domain of discourse = $Z+ \times Z+$

i. $P(5)$ = 5 divides 15.

$15/5 = 3$ (no remainder)

∴ This statement is true.

ii. $\forall n P(n)$ = For all values of n, $P(n)$ is true.

Let $P(2) = 15/2$

$= 7.5$ (have remainder)

∴ This statement is false because 2 cannot divides 15.

iii. $\exists n \neg P(n)$ = For some values of n, $P(n)$ is not true.

Let $P(2) = 15/2$

$= 7.5$ (have remainder)

∴ This statement is true because 2 cannot divides 15.

(b) $P(m,n) : m \geq n$, domain of discourse = $Z+ \times Z+$

$\exists m \exists n P(m,n)$ = For some values of m and n, $P(m,n)$ is true.

Let $m=2$, $n=2$, $P(m,n) = m \geq n$

Let $m=9$, $n=5$, $P(m,n) = m \geq n$

Let $m=1$, $n=3$, $P(m,n) \neq m \geq n$

∴ When $m=2$ and $n=2$ or $m=9$ and $n=5$, $P(m,n)$ are true. However, when $m=1$ and $n=3$, $P(m,n)$ is false. Thus, this statement is true.

$\neg(\exists m \exists n P(m,n)) = \neg \forall m \forall n P(m,n)$

The negation of the proposition is none of the value of m and n for $P(m,n)$ is true.

Question 4

4. $P(x)$: $x < a < y$

domain of discourse : $x < y$ with x and y are real numbers, a is rational number

$\exists a P(x) : \exists a (x < a < y)$

Proving method: Contrapositive

Suppose there is no rational number a satisfy $x < a < y$.
This means the interval (x, y) contain no rational number a.
Thus, $a \leq x$ or $a \geq y$.
However, when $x = 1$, $y = 2$, the interval is $(1, 2)$.
There has rational number lies between interval $(1,2)$ which is $1/2$.
So, rational number a exist and satisfying $x < a < y$.

$\therefore$ The statement of if x and y are real numbers with $x < y$, there exist a rational number a satisfying $x < a < y$ is true.

Question 5

5. (a) $(m,n) \in R$, $A = \{1,2,3,4,5\}$, R: 2 divides $m-n$

$R = \{ (1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5) \}$

$R^{-1} = \{ (1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5) \}$

(b) $(m,n) \in R$, $A = \{1, 2, 3, 4, 5\}$, R: $m + n <= 4$

$R = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (3,1) \}$

$R^{-1} = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (3,1) \}$

(c)

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \qquad \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

R in question (a) R in question (b)

$\therefore$ Both relations, R in question (a) and question (b) are symmetric.

No.: Date:

5. (d) i. $(x,y) \in R$ if $xy = 1$

$R = \{(1,1)\}$

$\therefore$ The relation is reflexive, symmetric, not antisymmetric, transitive and not partial order.

ii. $(x,y) \in R$ if $x = y^2$

$R = \{(1,1), (4,2), (9,3), (16,4), \ldots \ldots\}$

$\therefore$ The relation is not reflexive, not symmetric, antisymmetric, not transitive and not partial order.

iii. $(x,y) \in R$ if $x = y$

$R = \{(1,1), (2,2), (3,3), (4,4), \ldots\ldots\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \otimes \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ The relation is reflexive, symmetric, antisymmetric, transitive and partial order.

扫描全能王 创建

No.: Date:

Question 6

6. (a) i. $f(n) = n+1$

Let $f(n_1) = f(n_2)$

$n_1 + 1 = n_2 + 1$

$n_1 = n_2$

The function is one - to one

$f(n) = n+1$

Let $m = n+1$

$n = m-1$

$f^{-1}(n) = n-1$

Each element in $f^{-1}(n)$ has at least one relation with n, Thus, the function is onto function.

$\therefore$ The function is both one-to-one and onto.

ii. $f(n) = |n|$

Let $n_1 = 1$, $n_2 = -1$

$f(n_1) = |1| = 1$

$f(n_2) = |-1| = 1$

The function is not one-to-one because it is many-to-one relation.

$f(n) = |n|$

If n positive, let $n_1 = 2$, $f(n_1) = |2| = 2$

If n negative, let $n_2 = -2$, $f(n_2) = |-2| = 2$

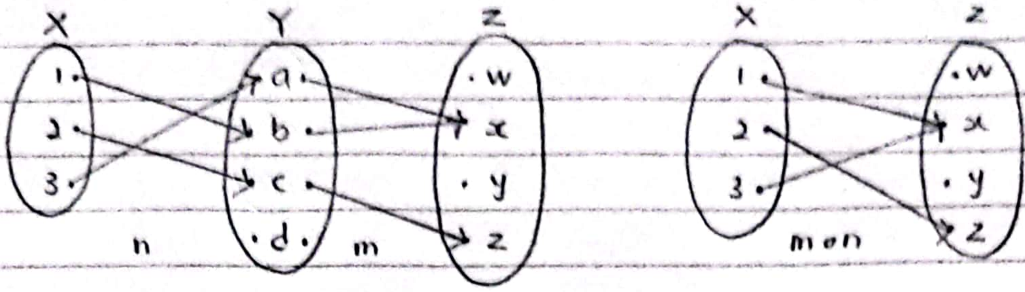
Since all positive and negative input will get the positive output, so the function is not onto.

$\therefore$ The function is not one-to-one and not onto.

6\. (b) $n = \{(1,b),(2,c),(3,a)\}$

$m = \{(a,x),(b,x),(c,z)\}$

$(m \circ n)(x) = \{(1,x),(2,z),(3,x)\}$



(c) $g = \{(1,a),(2,c),(3,c)\}$

$X = \{1,2,3\}$, $Y = \{a,b,c,d\}$

$S = \{1\}$, $T = \{1,3\}$, $U = \{a\}$, $V = \{a,c\}$

$g(S) = \{a\}$

$g(T) = \{a,c\}$

$g^{-1}(U) = \{1\}$

$g^{-1}(V) = \{1,2,3\}$