

Assignment 3

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Question 1 Discrete Probability Theory

1. (a) All people = 2015 = 25

$$P(41) = \frac{{}^5C_4}{{}^{25}C_4} = \frac{1}{2530} = 3.9526 \times 10^{-4}$$

$$(b) P(2T2P) = \frac{{}^{20}C_2 \cdot {}^5C_2}{{}^{25}C_4} = \frac{38}{253} = 0.1502$$

2. N: Kamal buy a new car

C: Kamal live on campus

$$P(N \cap C) = 0.37$$

$$P(C) = 0.73$$

$$P(N|C) = \frac{P(N \cap C)}{P(C)}$$

$$= \frac{0.37}{0.73}$$

$$= \frac{37}{73}$$

$$= 0.5068$$

3. (a) E: Sum of numbers on dice is even

$$E = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), \\ (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$$

(b) N = Sum of numbers is 9

$$N = \{(3,6), (4,5), (5,4), (6,3)\}$$

$$P(N) = \frac{4}{6 \times 6} = \frac{1}{9} = 0.1111$$

(c) A = Sum of numbers is 7

$$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

B = Sum of numbers is 8

$$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$A \cup B = \{(1,6), (2,5), (2,6), (3,4), (3,5), (4,3), (4,4), (5,2), (5,3), (6,1), (6,2)\}$$

$$P(A \cup B) = \frac{6}{36} + \frac{5}{36}$$

$$= \frac{11}{36}$$

$$= 0.3056$$

Question 2 Bayes' Theorem

1. (a) $P(A) = 0.55$

$$P(D) = 0.10$$

$$P(N) = 0.35$$

(b) $P(B|A) = 0.01$

$$P(B|D) = 0.03$$

$$P(B|N) = 0.03$$

$$P(B) = P(B|A)P(A) + P(B|D)P(D) + P(B|N)P(N)$$

$$= 0.01(0.55) + 0.03(0.10) + 0.03(0.35)$$

$$= 0.019$$

2. (a) R : RT-PCR test Positive

I : Active Covid 19 Infection

$$P(I) = 0.15$$

$$P(R|I) = 0.95$$

$$P(R|I') = 0.02$$

$$P(I') = 1 - 0.15$$

$$= 0.85$$

$$P(I|R) = \frac{P(R|I)P(I)}{P(R|I)P(I) + P(R|I')P(I')}$$

$$= \frac{(0.95)(0.15)}{(0.95)(0.15) + (0.02)(0.85)}$$

$$= \frac{142.5}{319}$$

$$= 0.8934$$

$$= 0.8934$$

(b) $P(I'|R) = \frac{P(R|I')P(I')}{P(R|I')P(I') + P(R|I)P(I)}$

$$= \frac{(0.02)(0.85)}{(0.02)(0.85) + (0.95)(0.15)}$$

$$= \frac{34}{319}$$

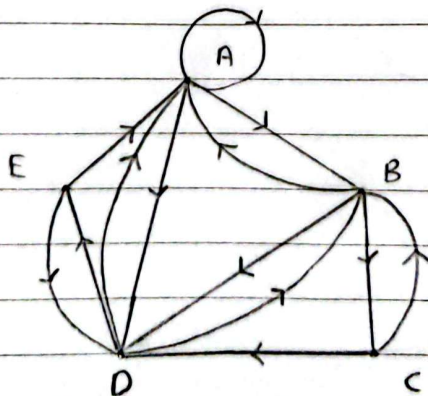
$$= 0.1066$$

$$= 0.1066$$

$$= 0.1066$$

Question 3 Graph Definition and Notation

1. (a)



$$(b) \deg(A) = 7$$

$$\deg(B) = 6$$

$$\deg(C) = 3$$

$$\deg(D) = 7$$

$$\deg(E) = 3$$

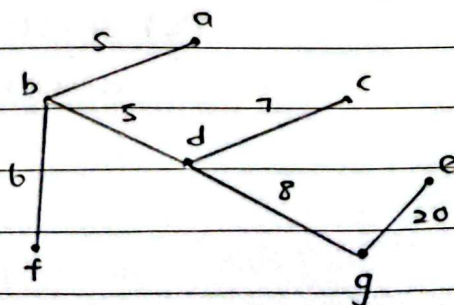
2. (a) Vertex : A point or dot where two or more lines, curves, rays, edges, or line segments meet.
- (b) Edge : A line that connect between the nodes (vertices) of the network in graph.
- (c) Loop : An edge incident on a single vertex.
- (d) Parallel edges : Two or more distinct edges with the same set of end points.
- (e) Degree of vertex : The number of edges incident with vertices.

Question 4 Representation of graphs

1. (a) disjoint vertex set 1 : $\{v_1, v_2, v_5\}$
disjoint vertex set 2 : $\{v_3, v_4\}$

- (b) simple graph, connected graph, undirected graph

2.



$$\begin{aligned} \text{Cost} &= 5 + 6 + 5 + 7 + 8 + 20 \\ &= 51 \end{aligned}$$

Question 5 Isomorphism of Graph

Properties	G_1	G_2
Number of vertices	7	7
Number of edges	7	7
Number of degrees	All vertices have 2 degree	All vertices have 2 degree

Both G_1 and G_2 have 7 vertices and 7 edges.

Both are connected and simple graph.

Both have 7 vertices with 2 degree.

$f: G_1 \rightarrow G_2$ can be defined.

$f(a)=1, f(b)=6, f(c)=4, f(d)=2, f(e)=7, f(f)=5, f(g)=3$

Adjacency matrix:

		a	b	c	d	e	f	g			1	6	4	2	7	5	3
	a	0	1	0	0	0	0	1		1	0	1	0	0	0	0	1
$A_{G_1} =$	b	1	0	1	0	0	0	0		6	1	0	1	0	0	0	0
	c	0	1	0	1	0	0	0		4	0	1	0	1	0	0	0
	d	0	0	1	0	1	0	0		2	0	0	1	0	1	0	0
	e	0	0	0	1	0	1	0		7	0	0	0	1	0	1	0
	f	0	0	0	0	1	0	1		5	0	0	0	0	1	0	1
	g	1	0	0	0	0	1	0		3	0	0	0	1	0	1	0

$$\therefore A_{G_1} = A_{G_2}$$

Hence, both G_1 and G_2 are isomorphic.

Properties	G_1	G_2
Number of vertices	6	6
Number of edges	9	9
Number of degrees	All vertices have 3 degree	All vertices have 3 degree

Both G_1 and G_2 have 6 vertices and 9 edges.

Both are connected and simple graph.

Both have 6 vertices with 3 degree.

However, $f: G_1 \rightarrow G_2$ cannot be defined.

Hence, G_1 and G_2 are not isomorphic.

No.:

Date:

(b) Properties	G_1	G_2
Number of vertices	6	6
Number of edges	11	11
Number of degrees	2 vertices with 3 degree, 4 vertices with 4 degree	2 vertices with 3 degree, 4 vertices with 4 degree

Both G_1 and G_2 have 6 vertices and 11 edges.

Both are simple and connected graph.

Both have 2 vertices with 3 degree and 4 vertices with 4 degree.

However, function $f: G_1 \rightarrow G_2$ cannot be defined.

Hence, G_1 and G_2 are not isomorphic.

3.

