SECI 1013 Discrete Structure Assignment 4

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Q1 Dijkstra's Shortest Path Algorithm

1. Dijkstra's algorithm efficiently finds the shortest path from a source verticle to all other verticles in a weight graph without negative. Let G as a weighted graph, V as verticles in G, u is verticles such that $u \in V$, a is starting verticle and $a \in V$.

   Let $S = \phi, N = V$

   Length of path: $L(a) = 0$, $L(u) = \infty$

   Starting from the starting verticle, a, the length of path to the adjacent verticles, L(w) are calculated.

   If $L(w) < L(u)$, $L(u) = L(w)$. Else, value of L(u) remains unchanged. When all adjacent verticles are choosed and checked, the verticle is moving from N to S, and the next verticle with smallest L(u) is chosen to repeat the procedure.

   The steps are repeated until the ending verticle, $z \in S$. Value of L(z) will be the shortest length from a to z.

2. Let $S = \phi$, $N = \{a, b, c, d, e, f, g, h, i, j, z\}$

   (a) a, f

Iteration	S	N	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	L(h)	L(i)	L(j)	L(z)
0	{}	{a,b,c,d,e,f,g,h,i,j,z}	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
1	{a}	{b,c,d,e,f,g,h,i,j,z}	0	3	∞	∞	5	∞	∞	4	∞	∞	∞
2	{a,b}	{c,d,e,f,g,h,i,j,z}	0	3	5	∞	5	∞	∞	4	∞	∞	∞
3	{a,b,h}	{c,d,e,f,g,h,i,j,z}	0	3	5	∞	5	9	∞	4	6	∞	∞
4	{a,b,h,e}	{c,d,f,g,i,j,z}	0	3	5	∞	5	9	∞	4	6	∞	∞
5	{a,b,h,e,c}	{d,f,g,i,j,z}	0	3	5	8	5	7	11	4	6	∞	∞
6	{a,b,h,e,c,i}	{d,f,g,j,z}	0	3	5	8	5	7	11	4	6	12	∞
7	{a,b,h,e,c,i,f}	{d,g,j,z}	0	3	5	8	5	7	11	4	6	12	∞

∴ Shortest path is $a \to b \to c \to f$, with the shortest length is 7.

$L(f) = 3+2+2$

$= 7$

(b) b, j

Iteration	S	N	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	L(h)	L(i)	L(j)	L(z)
0	{ }	{a,b,c,d,e,f,g,h,i,j,z}	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	∞
1	{b}	{a,c,d,e,f,g,h,i,j,z}	3	0	2	∞	5	7	∞	∞	∞	∞	∞
2	{b,c}	{a,d,e,f,g,h,i,j,z}	3	0	2	5	5	4	8	∞	∞	∞	∞
3	{b,c,a}	{d,e,f,g,h,i,j,z}	3	0	2	5	5	4	8	7	∞	∞	∞
4	{b,c,a,f}	{d,e,g,h,i,j,z}	3	0	2	5	5	4	8	7	8	7	∞
5	{b,c,a,f,d}	{e,g,h,i,j,z}	3	0	2	5	5	4	8	7	8	7	10
6	{b,c,a,f,d,e}	{g,h,i,j,z}	3	0	2	5	5	4	8	7	8	7	10
7	{b,c,a,f,d,e,j}	{g,h,i,z}	3	0	2	5	5	4	8	7	8	7	10

∴ Shortest path is b→c→f→j, with the shortest length is 7.

$L(j) = 3+2+2$
$= 7$

(c) a, g

Iteration	S	N	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	L(h)	L(i)	L(j)	L(z)
0	{ }	{a,b,c,d,e,f,g,h,i,j,z}	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
1	{a}	{b,c,d,e,f,g,h,i,j,z}	0	3	∞	∞	5	∞	∞	4	∞	∞	∞
2	{a,b}	{c,d,e,f,g,h,i,j,z}	0	3	5	∞	5	∞	∞	4	∞	∞	∞
3	{a,b,h}	{c,d,e,f,g,i,j,z}	0	3	5	∞	5	9	∞	4	6	∞	∞
4	{a,b,h,e}	{c,d,f,g,i,j,z}	0	3	5	∞	5	9	∞	4	6	∞	∞
5	{a,b,h,e,c}	{d,f,g,i,j,z}	0	3	5	8	5	7	11	4	6	∞	∞
6	{a,b,h,e,c,i}	{d,f,g,j,z}	0	3	5	8	5	7	11	4	6	12	∞
7	{a,b,h,e,c,i,f}	{d,g,j,z}	0	3	5	8	5	7	11	4	6	10	∞
8	{a,b,h,e,c,i,f,d}	{g,j,z}	0	3	5	8	5	7	11	4	6	10	10
9	{a,b,h,e,c,i,f,d,j}	{g,z}	0	3	5	8	5	7	11	4	6	10	10
10	{a,b,h,e,c,i,f,d,j,g}	{z}	0	3	5	8	5	7	11	4	6	10	10

∴ shortest path is a→b→c→f→g, with the shortest length is 11.

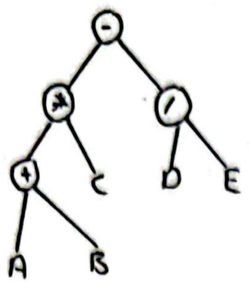
$L(g) = 3+2+6$
$= 11$

Q2 Trees

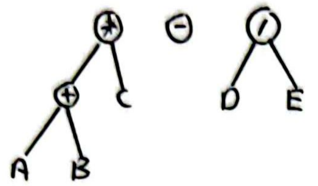
1. a is balanced because a is a rooted full 1-ary tree and all leaves are at level 2 and level 1 only.

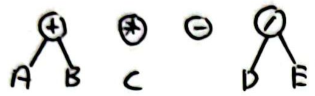
   b is balanced because b is a rooted full 1-ary tree and all leaves are at level 3 and level 2 only.

2.



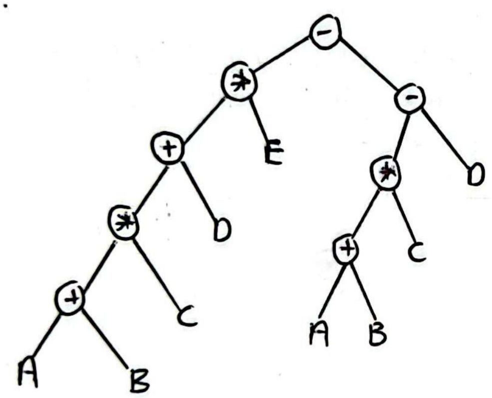
Inorder traversal = A, +, B, *, C, -, D, /, E





(A+B) * C - D / E

3.

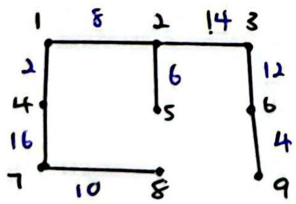


Prefix expression: - * + * + ABCDE - * + ABCD

Postfix expression: AB+C*D+E*AB+C*D--

((((A+B)*C)+D)*E)-(((A+B)*C)-D)

4.



Length of minimum spanning tree

= 8+14+12+4+6+2+16+10

= 72

Q3 : Finite state Machine

1. Let $M = \{S, I, q_0, f_s, F\}$

where $S = \{q_0, q_1, q_2, q_3, q_4\}$

$I = \{a, b\}$

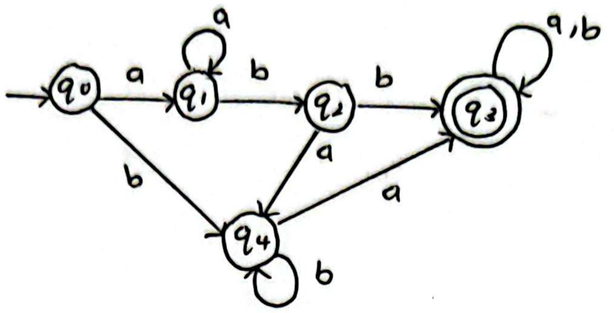
$F = \{q_3\}$

q_0 : initial state

Transition Table:

f_s	a	b
q_0	q_1	q_4
q_1	q_1	q_2
q_2	q_4	q_3
q_3	q_3	q_3
q_4	q_3	q_4

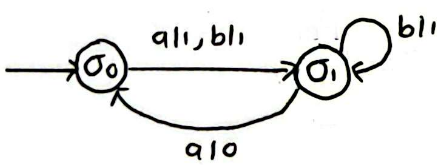
Transition Diagram:



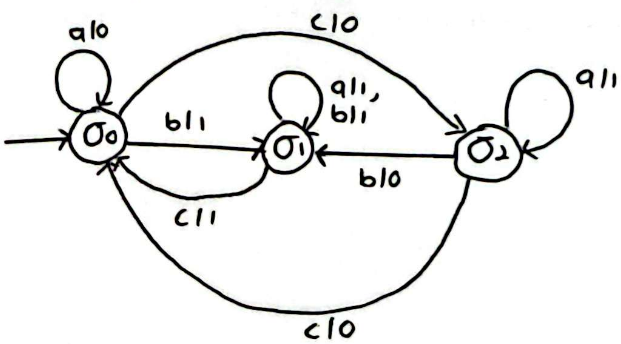
Q4 : Finite state Machine

1. Finite state machine is a mathematical model used to simulate sequential logic and control execution flow. Every state has an input and corresponding to the input, the state also has an output.
A finite state machine is written as $M = \{S, I, O, q_0, f_s, f_o\}$
where S is finite nonempty set of states, I is finite set of input alphabet, O is finite set of output alphabet, q_0 is initial state, the state transition function, $f_s : S \times I \to S$ and output function, $f_o : S \times I \to O$.

2. (a) $I = \{a, b\}$, $O = \{0, 1\}$, $S = \{\sigma_0, \sigma_1\}$



(b) $I = \{a, b, c\}$, $O = \{0, 1\}$, $S = \{\sigma_0, \sigma_1, \sigma_2\}$



3. $I = \{a, b\}$, $O = \{0, 1\}$, $S = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$, Initial state $= \sigma_0$

Transition Table:

	f_s		f_o	
S \ I	a	b	a	b
σ_0	σ_1	σ_2	0	0
σ_1	σ_0	σ_2	1	0
σ_2	σ_3	σ_0	0	1
σ_3	σ_1	σ_3	0	0

Output function:

$f_o(\sigma_0, a) = 0$, $f_o(\sigma_0, b) = 0$

$f_o(\sigma_1, a) = 1$, $f_o(\sigma_1, b) = 0$

$f_o(\sigma_2, a) = 0$, $f_o(\sigma_2, b) = 1$

$f_o(\sigma_3, a) = 0$, $f_o(\sigma_3, b) = 0$