Assignment 2 Discrete Structure

Group Members :

1. AW XU YUEN A24CS0052
2. FONG JIN XUAN A24CS074
3. LAI SHI NI A24CS0298
4. ZAHIN IRDINA BINTI MOHD ZABIDY A24CS0216

Question 1: Recurrence relation

1.(i) $a_n=6a_{n-1}-9a_{n-2}$, $a_0=1$, $a_1=6$

$a_2= 6a_1-9a_0$

$= 6(6)-9(1)$

$=27$

$a_3= 6a_2-9a_1$

$= 6(27)-9(6)$

$= 108$

$a_4= 6a_3-9a_2$

$= 6(108)-9(27)$

$= 405$

$a_5= 6a_4-9a_3$

$= 6(405)-9(108)$

$=1458$

∴ Recurrence relation: 1, 6, 27, 108, 405, 1458, …

(ii) $a_n= 6a_{n-1} – 11a_{n-2} + 6a_{n-3}$, $a_0=2$, $a_1=5$, $a_2=15$

$a_3= 6a_2-11a_1+6a_0$

$= 6(15)-11(5)+6(2)$

$=47$

$a_4= 6a_3-11a_2+6a_1$

$= 6(47)-11(15)+6(5)$

$=147$

$a_5= 6a_4-11a_3+6a_2$

$= 6(147)-11(47)+6(15)$

$=455$

$a_6= 6a_5-11a_4+6a_3$

$= 6(455)-11(147)+6(47)$

$=1395$

∴ Recurrence relation: 2, 5, 15, 47, 147, 455, 1395, …

(iii) $a_n= -3a_{n-1} – 3a_{n-2} + a_{n-3,}$ $a_0=1$, $a_1=-2$, $a_2=-1$

$a_3= -3a_2 – 3a_1 + a_0$

$= -3(-1)-3(-2)+0$

$=10$

$a_4= -3a_3 – 3a_2 + a_1$

$= -3(10)-3(-1)+(-2)$

$= -29$

$a_5= -3a_4 – 3a_3 + a_2$

$= -3(-29)-3(10)+(-1)$

$= 56$

$a_6= -3a_5 – 3a_4 + a_3$

$= -3(56)-3(-29)+10$

$= -71$

∴ Recurrence relation: 1, -2, -1, 10, -29, 56, -71, …

2. (i) $a_{n+1}= 5a_{n-3}$, $a_1=k$

$a_2= 5a_1-3$

$= 5(k)-3$

$= 5k-3$

$a_3= 5a_2-3$

$= 5(5k-3)-3$

$= 25k-15-3$

$= 25k-18$

$a_4= 5a_3-3$

$= 5(25k-18)-3$

$= 125k-90-3$

$= 125k-93$

$\therefore a_4= 125k-93$

(ii) $a_4= 7$

$125k-93= 7$

$125k= 100$

$k= 0.8$

Question 2 : Basic Principle

1. (a) Number of ways = (5+3+2)!

= 10!

= 3628800 ways

(b) Number of ways to arrange distinct of books = 3! = 6

Number of ways to arrange same discipline of books

Computer Science = 5! = 120 ways

Mathematics = 3! = 6 ways

Art= 2! = 2 ways

$\therefore$ Total number of ways = 6 x 120 x 6 x 2 = 8640 ways

(c) 10 copies of books (identical) + 10 distinct books

When 10 copies of books (identical) and 0 distinct book are chosen,
Number of ways = ${}^{10}C_{10}$ = 1 ways

When 9 copies of books (identical) and 1 distinct book are chosen,
Number of ways = ${}^{10}C_9$ = 10 ways

When8 copies of books (identical) and 2 distinct books are chosen,
Number of ways = ${}^{10}C_8$ = 45 ways

Total number of ways = ${}^{10}C_{10} + {}^{10}C_9 + {}^{10}C_8 + {}^{10}C_7 + {}^{10}C_6 + {}^{10}C_5 + {}^{10}C_4 + {}^{10}C_3 + {}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0$
= 1024 ways

2. (a) Numbers = 200-5+1
= 196

(b) Tn = a+(n-1)d
200 = 5+(n-1)5
200 = 5+5n-5
200 = 5n
n = 40

∴ 40 numbers are divisible by 5

(c) T_1 : digits contain 7 from 5 to 100
= 7, 17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 87,97 (19 numbers)

T_2 : digits contain 7 from 101 to 200
= 107, 117, 127, 137, 147, 157, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178,179, 187,197 (19 numbers)

$T_1 + T_2$ = 19 + 19
= 38 numbers
∴ 38 numbers contain digit 7.

(d) T_1 : digits in strictly increasing order from 5 to 100
= 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 56, 57, 58, 59, 67, 68, 69, 78, 79, 89 (41 numbers)

T_2 : digits in strictly increasing order from 101 to 200
= 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 149, 156, 157, 158, 159, 167, 168, 169, 178, 179, 189 (28 numbers)

$T_1 + T_2$ = 41 + 28
= 69 numbers
∴ 69 numbers contain digits in strictly increasing order.

Question 3 : Permutation & Combination

1. (a) ACE, B, D

Number of ways = 3!

= 6 ways

(b) AE, B, C, D

Number of ways = 4!

= 24 ways

EA, B, C, D

Number of ways = 4!

= 24 ways

AE, EA, B, C, D

Number of ways = 0

∴ Total number of ways = 24 + 24

= 48 ways

(c) _J_J_J_J_J_J_J_J_

T_1: arrange 5 Martians to 9 gaps

= $^9P_5$

= 15120 ways

T_2: arrange 8 Martians

= 8!

= 40320 ways

Total number of ways = 15120 x 40320

= 609638400 ways

(d) Number of ways = 10!

= 3628800 ways

2. If committees have different jobs,

Number of ways = $^{11}P_3$

= 990 ways

If committees have same jobs,

Number of ways = $^{11}C_3$

= 165 ways

3. T_1 : 4 speciality pizza
= 4 ways
T_2 : 0 topping
= $^{17}C_0$
= 1 ways

T_3 : 1 topping
= $^{17}C_1$
= 17 ways

T_4 : 2 topping
= $^{17}C_2$
= 136 ways

T_5 : 3 topping
= $^{17}C_3$
= 680 ways
Total number of ways = 4 + 1 + 17 + 136 + 680
= 838 ways

4. X= {a,b,c,d}
Number of ways = 4C3
= 4! ÷ (3! X 1!)
= 4 ways

Question 4 : Pigeonhole Principle (1^{st},2^{nd} 3^{rd})

1. Pigeons : Number of students in class
Pigeonholes : Scale of grade 0-100
|Pigeonholes|= 100+1
= 101
To have at least 2 students with same score,
$$\left\lceil \frac{\text{pigeons}}{\text{pigeonholes}} \right\rceil$$
= 2
Pigeons > Pigeonholes
According to Pigeonhole Principle, if there are 101 scales of grade (pigeonholes) and (101+1) number of students in class (pigeons), there always be at least two students received the same score on the final exam.
∴ Number of student = 101 + 1
= 102

2. Pigeons, n : Number of students

Pigeonholes, m : Letter grade (A, B, C, D, F)

To have at least 6 students receive same letter grade,

$$\left\lceil \frac{n}{m} \right\rceil = 6$$

$$\left\lceil \frac{n}{5} \right\rceil = 6$$

$$When\ \frac{n}{m} = 5, n = 25$$

$$If\ \left\lceil \frac{n}{m} \right\rceil = 6, 5 < \frac{n}{5} \leq 6$$

$\frac{n}{m}$ must exceed 5, n>25

In this situation, we need the number of students to be large enough that at least 1 pigeonhole has 6 students. Suppose placing 5 students in each grade (pigeonhole), there would be 25 students with no grade having more than 5 students.

According to Pigeonhole Principle, adding one more student to one grade (pigeonholes) to become total of 26 students so that the grade must have at least 6 students.

∴ The minimum number of students required = 25 + 1

= 26

3. Pigeons : 35 students

Pigeonholes : Letters (A - Z)

|Pigeons| = 35 |Pigeonholes| = 26

To have at least 2 students having same first letter of name,

$$\left\lceil \frac{\text{pigeons}}{\text{pigeonholes}} \right\rceil$$

= 2

There are 26 first letters of name and 35 students in class. Thus, at most 26 students in class having the different first letters of name, the remaining students must have the same first letter of name with the first 26 students.

According to Pigeonhole Principle, as long as there are 26 possible first letters of name (pigeon) and 35 students in class (pigeonhole), there will always be at least two students having same first letter of name.

4. Pigeons : 13 person

Pigeonholes : First name (Devvis, Evita, Ferdinand) and last name (Oh, Pietro, Quine, Rostenkowski)

|Pigeons| = 13 |Pigeonholes| = 3 x 4

= 12

To have at least 2 persons having the same first name and last name,

$$\left\lceil \frac{\text{pigeons}}{\text{pigeonholes}} \right\rceil$$

= 2

There are 12 combinations of first and last name, and 13 persons. Thus, at most 12 persons will have the different first and last name, the 13th person will having the same first name and last name with one of the 12 persons.

According to Pigeonhole Principle, as long as there are 13 persons (pigeon) and 12 combinations of first name and last name (pigeonhole) , there will always be at least two persons having the same first name and last name.