These questions demonstrate some basic domain knowledge needed by any contributing member of Software/Algorithm team. We expect you to be able to solve problems like these, and hopefully much more complicated ones, before you join our team. The scenarios presented here are vastly simplified from what we do in real life.

# Radar Signal

We're going to deal with a single stationary radar transceiver (transmitter and receiver) producing measurements of stationary point-like targets. What we present here is a greatly simplified signal model of FMCW radar. **You do not need to know anything about the system besides what we present here.** For a point-like reflective target at distance $R$ from the radar transceiver (which for this problem is also assumed to be a single point), the observed radar response is a pure tone signal in the form of

$$s(t) = A \exp\left(2\pi j \frac{2R}{c}(f_0 + \alpha t)\right), \tag{1}$$

where $t$ is time, $f_0$ is the starting frequency of the radar, $\alpha$ is the *chirp rate*, $c$ is the speed of light ($2.997\,924\,580 \times 10^8\,\mathrm{m/s}$), and $j = \sqrt{-1}$. $A$ is the characteristic *reflectivity* of the target, encoding how much signal it reflects back to the radar. $A$ is not necessarily real, i.e., it has both magnitude and phase. For this problem, the signal model above is independent of the direction of the target, i.e., the radar unit transmits perfectly spherical waves, and its receiver's output is direction-independent.

For a scene containing multiple point-like targets, the radar response is simply the superposition of the signals for the individual targets:

$$s(t) = \sum_{n=1}^{N} A_n \exp\left(2\pi j \frac{2R_n}{c}(f_0 + \alpha t)\right), \tag{2}$$

where $N$ is the number of targets in the scene.

# 1 Where are the targets?

Can you find the distances from all visible targets to the radar? The radar parameters are $f_0 = 77 \times 10^9$ Hz and $\alpha = 60 \times 10^{12}$ Hz/s. You are given the discretely sampled signal(see attached signal.csv file with real and imaginary parts)

$$(s(t_0), ..., s(t_{M-1})),$$

where $t_m = t_0 + m\Delta t$, where $t_0 = 5.7 \times 10^{-6}\,\mathrm{s}$, $\Delta t = 5.33 \times 10^{-8}\,\mathrm{s}$, and $M = 1024$.

Note that we don't know a priori how many targets there are.

# 2 Where is the radar?

Now you are given the list

$$[(x_1, y_1, z_1), ..., (x_N, y_N, z_N)]$$

of precise positions of the $N$ targets in some fixed but arbitrary coordinate system, as well as the distances

$$[R_1, ..., R_N]$$

from each respective target to the radar (perhaps having solved a variant of the previous problem). Can you now find the position of the radar? As a concrete example, please consider the following measurements

| x | y | z | R |
|---|---|---|---|
| 10.000000 | 0.000000 | 0.000000 | 18.369529 |
| 0.000000 | 20.000000 | 0.000000 | 18.219923 |
| 1.000000 | -1.000000 | -20.000000 | 31.468964 |
| 30.000000 | -30.000000 | 30.000000 | 53.089849 |
| -4.500000 | 5.500000 | 6.000000 | 4.351196 |

Note that our radar has a range measurement error of 5cm.

# Guidelines

- If you need to make assumptions beyond what is given, please state them.
- If you write any code, please share it as well. You can use any programming language.