$$X[k] = \sum_{n=0}^{N-1} x[n]\, W^{nk}$$; N - dĺžka postupnosti

$$W = e^{j\frac{2\pi}{N}}$$

Výpočtová náročnosť:

$\forall k$: $N-1$ kompl. sčítaní a N kompl. násobení => pre N-bodovú DFT : $(+)\ N^2 - N$

$(\times)\ N^2$

pri FFT $N^2 \to N \log N$

1)

$$W = e^{-j\frac{2\pi}{N}}$$

Periodicita:

$$W^{k+N} = e^{-j\frac{2\pi}{N}(k+N)} = \cos\left(\frac{2\pi k}{N} + 2\pi\right) - j\sin\left(\frac{2\pi k}{N} + 2\pi\right) = \cos\frac{2\pi k}{N} - j\sin\frac{2\pi k}{N} = e^{-j\frac{2\pi k}{N}} = W^{k}$$

Symetria:

$$W^{k+\frac{N}{2}} = e^{-j\frac{2\pi}{N}(k+\frac{N}{2})} = \cos\left(\frac{2\pi k}{N} + \pi\right) - j\sin\left(\frac{2\pi k}{N} + \pi\right) = -\cos\frac{2\pi k}{N} + j\sin\frac{2\pi k}{N} = -e^{-j\frac{2\pi k}{N}} = -W^{k}$$

$$W^{kN/2} = e^{-j\frac{2\pi}{N}\frac{kN}{2}} = \cos k\pi - j\sin k\pi = \cos k\pi = (-1)^{k}$$

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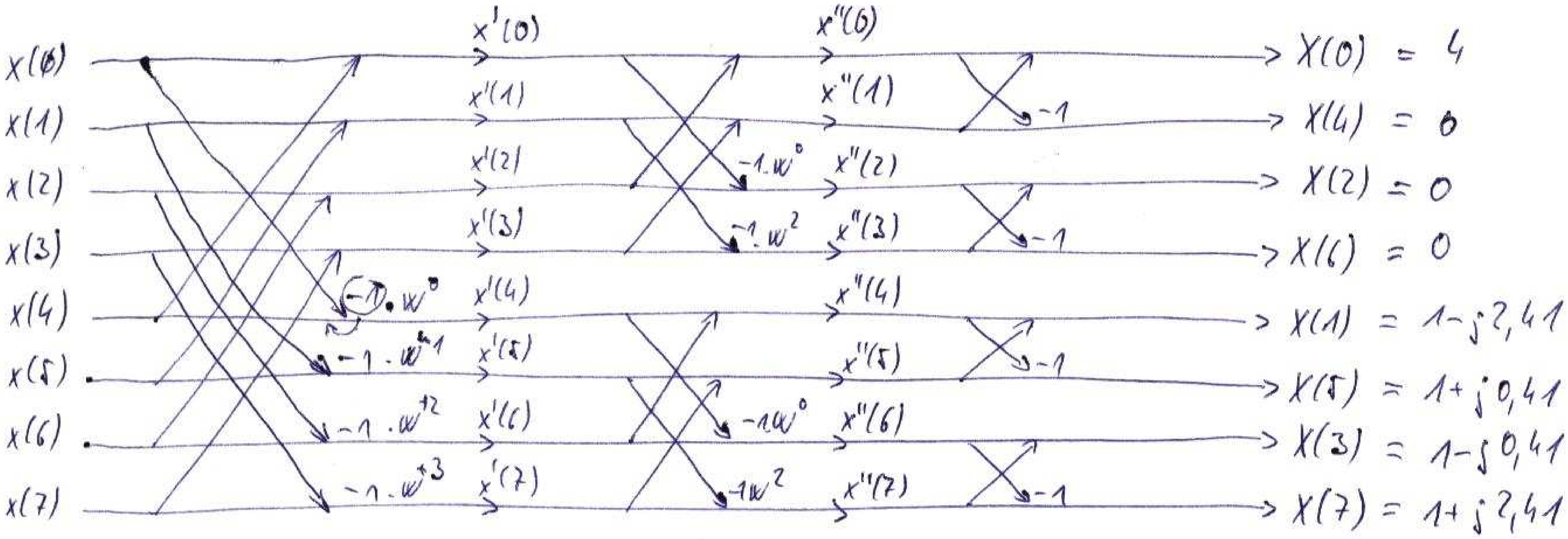
2)



$$\boxed{N = 8}$$

$$W^{0} = e^{-j\frac{2\pi}{N}\cdot 0} = 1$$

$$W^{1} = e^{-j\frac{2\pi}{8}\cdot 1} = e^{-j\frac{\pi}{4}} = 0{,}707 - j0{,}707$$

$$W^{2} = e^{-j\frac{2\pi}{8}\cdot 2} = e^{-j\frac{\pi}{2}} = -j$$

$$W^{3} = e^{-j\frac{2\pi}{8}\cdot 3} = e^{-j\frac{3\pi}{4}} = -0{,}707 - j0{,}707$$

$$X(k) = \sum_{n=0}^{7} x[n]\, W^{nk}$$