1 Distances and Time

1.1 Lookback Time

The lookback time t_L is the difference between the current age of the universe and the time at which the photons were emitted.

$$t_L = t_H \int_0^z \frac{dz'}{(1+z')E(z')}$$

With t_H the Hubble time and $E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\lambda}$

Considering the Einstein-de-Sitter model Universe ($\Omega_m = 1,\ \Omega_\lambda = 0$) we can easily compute the age of the Universe.

$$t_0 = t_H \int_0^\infty (1+z')^{-\frac{5}{2}} dz' = -\frac{2}{3} t_H (1+z')^{-\frac{3}{2}}]_0^\infty = \frac{2}{3} t_H$$

1.1.1 Question 2

We will consider four distinct models of the universe.

- Einstein-de-Sitter universe ($\Omega_m = 1,\ \Omega_\lambda = 0$)
- Classical closed universe ($\Omega_m = 2,\ \Omega_\lambda = 0$)
- Baryon dominated low density universe ($\Omega_m = 0.04,\ \Omega_\lambda = 0$)
- Currently Popular universe ($\Omega_m = 0.27,\ \Omega_\lambda = 0.73$)

A program has been written to tabulate and graph results for lookback times in each of these models. It can be found in the "Code" section of the project.

Redshift	Einstein-de-Sitter	Classical closed	Baryon Dominated	Currently Popular
0.1	12.06762	11.80719	12.34083	12.68661
1.0	58.55965	52.94232	67.42440	76.28055
2.0	73.15351	64.55459	89.44977	101.87216
4.0	82.48464	71.64913	106.62361	119.34947
6.7	86.34735	74.50169	115.30207	126.735608
10000	90.87743	77.90534	128.53490	135.05773

Table 1: Lookback times in 100 million years for various redshifts

Note that the last entry can be used as an approximation for the age of the universe as the redshift is so high the remaining part of the integral will be negligible.

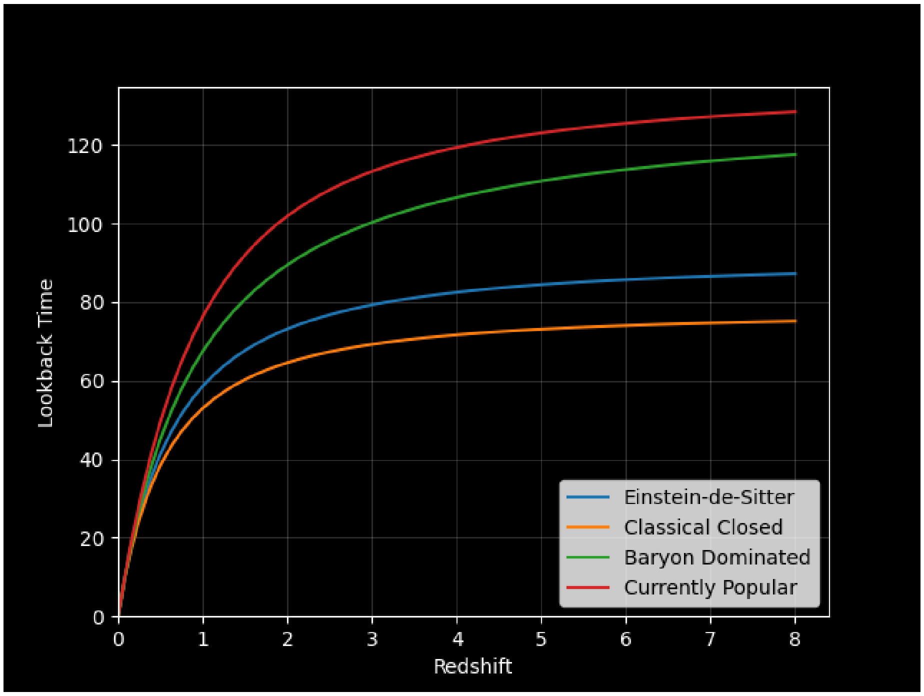

Figure 1: Lookback time graphed against redshift for various universes

1.2 Distance Measures

Considering the Einstein-de-Sitter model Universe ($\Omega_k = 0$) we get an expression for D_A in:

$$D_A = \frac{D_C}{1+z} = \frac{D_H}{1+z}\int_0^z \frac{dz'}{E(z')} = \frac{D_H}{1+z}\int_0^z (1+z')^{-\frac{3}{2}}dz' = \frac{2D_H}{1+z} - \frac{2D_H}{(1+z)^{\frac{3}{2}}}$$

Finding stationary points:

$$\frac{dD_A}{dz} = -\frac{2D_H}{(1+z)^2} + \frac{3D_H}{(1+z)^{\frac{5}{2}}} = 0$$

$$\implies 2(1+z)^{\frac{1}{2}} = 3 \implies 4+4z = 9 \implies z = 1.25$$

Proving it is maxima:

$$\frac{d^2D_A}{dz^2} = \frac{4D_H}{(1+z)^3} - \frac{15D_H}{2(1+z)^{\frac{7}{2}}}$$

$$\frac{d^2D_A}{dz^2}|_{z=1.25} = D_H(\frac{256}{729} - \frac{320}{729}) = -D_H\frac{64}{729} < 0$$

1.2.1 Question 4

A program has been written to compute D_A and D_L given a redshift and universe. It is also designed to plot the values of D_A/D_H and D_L/D_H against redshift. It can be found in the "Code" section of the project.

	Einstein		Baryon		Popular	
Redshift	D_A/D_H	D_L/D_H	D_A/D_H	D_L/D_H	D_A/D_H	D_L/D_H
1.0	0.29260	1.17040	0.36969	1.47874	0.39244	1.56976
1.25	0.29600	1.49850	0.39401	1.99467	0.40981	2.07465
2.0	0.28148	2.53336	0.43112	3.88008	0.41366	3.72297
4.0	0.22089	5.52235	0.45001	11.25031	0.34575	8.64375

Table 2: Table of distance ratios according to redshift

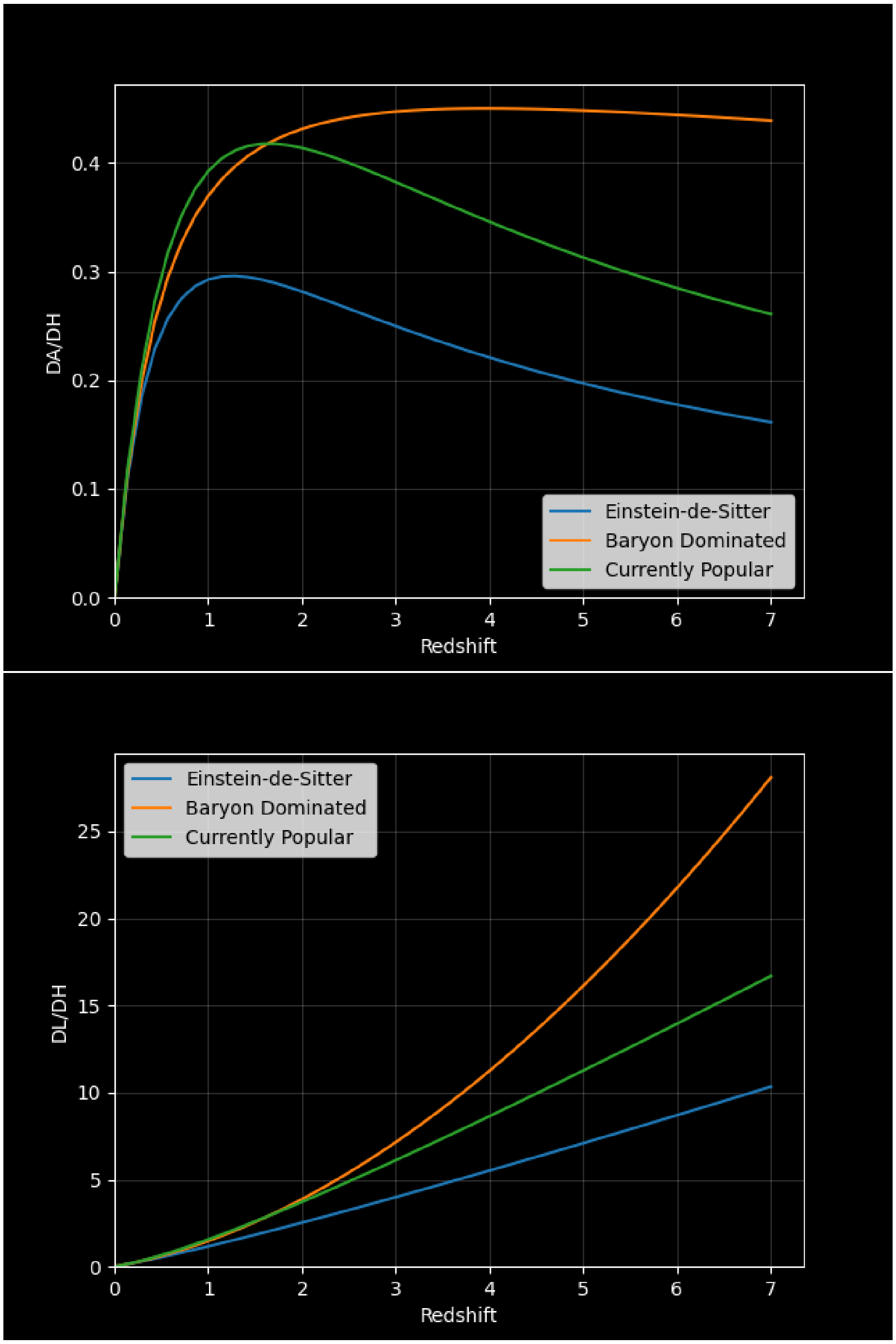

Figure 2: Plots of D_A/D_H and D_L/D_H against redshift

2 Comoving Volume

2.1 Uniform Comoving Distribution

Finding $< V/V_{max} >$ we must average the population weighted according to V/V_{max}. It is clear that we should use the given formula for number of objects to create a probability distribution. We then see:

$$
\begin{aligned}
< V/V_{max} > &= \frac{\int_0^\infty \Phi(L) \int_0^{V_{max(L)}} V/V_{max} dV dL}{\int_0^\infty \Phi(L) \int_0^{V_{max}(L)} dV dL} \\
&= \frac{\int_0^\infty \Phi(L) \frac{1}{2} V_{max} dL}{\int_0^\infty \Phi(L) V_{max} dL} = \frac{1}{2}
\end{aligned}
$$

[[Not exctly sure if this is right but we move come back to later i guess]]

2.2 Computing V/V_{max}

2.2.1 Question 6

A program has been written to compute V and V_{max} for arbitrary redshift z and ratio between detected and minimum flux f/f_0. It works by computing the maximum redshift z_0 corresponding to a flux of f_0 by binary search and then using z and z_0 along with the formulas provided to compute D_C in each case, yielding V and V_{max}. The program can be found in the "Code" section of the project.

In an extension of this program it generates random numbers for Ω_m, z and f/f_0 with z small and plots $(f/f_0)^{-\frac{3}{2}}$ against V/V_{max} from the graph it is clear that in the euclidean limit they are directly proportional.

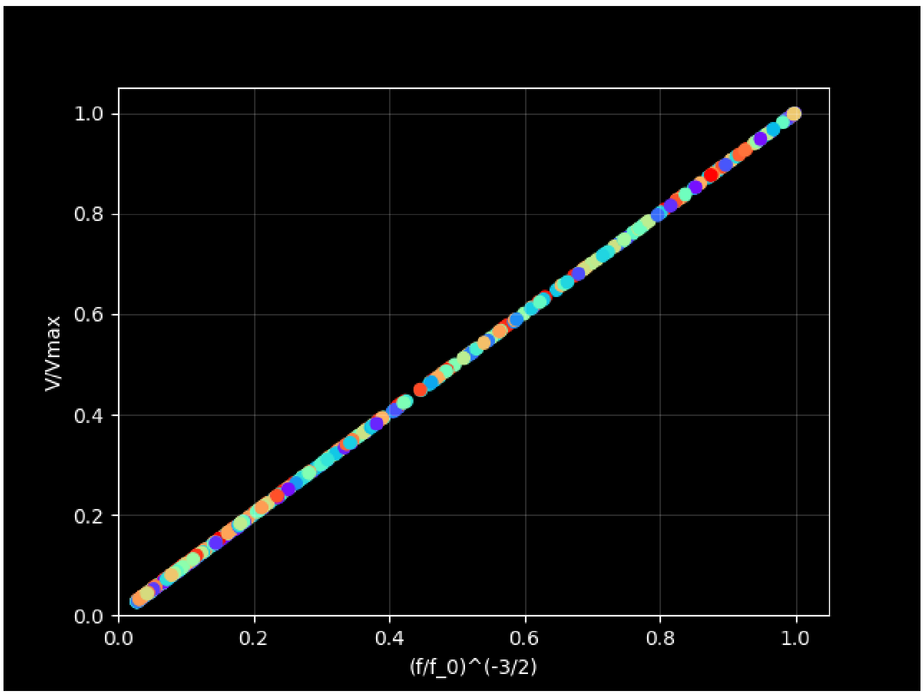

Figure 3: Plot of $(f/f_0)^{-\frac{3}{2}}$ against V/V_{max}