

This is surely different from the previously constructed Mealy machine in the number of states and in transactions.

But these two Mealy machines are equivalent. This can be proved by finding the equivalent partitions of the second machine.

The tabular format of the previous machine is

Present State	Next State,Z	
	X=0	X=1
A	A,0	B,0
B	C,0	B,0
C	A,0	D,0
D	E,1	B,0
E	A,0	D,0

$$P_0 = A, B, C, D, E$$

$$P_1 = A, b, C, ED$$

$$P_2 = A, BC, ED$$

$$P_3 = ABC, ED$$

$$P_4 = ABC, ED$$

Rename the states as S_1, S_2, S_3 , and S_4 .

Present State	Next State,Z	
	X=0	X=1
$S_1(\mathbf{A})$	$S_1,0$	$S_2,0$
$S_2(\mathbf{B})$	$S_3,0$	$S_2,0$
$S_3(\mathbf{C},\mathbf{E})$	$S_1,0$	$S_4,0$
$S_4(\mathbf{D})$	$S_3,1$	$S_2,0$

The machine is same as the first Mealy machine.

24. Minimize the following DFA M using the Myhill–Nerode theorem.

Present State	Next State	
	I/P=a	I/P=b
A	A	D
B	C	F
C	D	E
D	A	F
E	A	G
F	B	E
G	B	D

A is the initial state, and G is the final state.

[WBUT 2003]

114 | Introduction to Automata Theory, Formal Languages and Computation

Solution:

Step I: Divide the states of DFA into two subsets: final (F) and non-final (Q-F).

$$F = \{G\}, \text{ Q-F} = \{A, B, C, D, E, F\}$$

Make a two-dimensional matrix labelled at the left and bottom by the states of the DFA.

A	-	-	-	-	-	-	-
B		-	-	-	-	-	-
C			-	-	-	-	-
D				-	-	-	-
E					-	-	-
F						-	-
G							-
	A	B	C	D	E	F	G

Step II:

i) The following combinations are the combination of the beginning and final states.

(A,B), (B,G), (C,G), (D,G), (E,G), (F,G). Put X in these combinations of states.

The modified matrix is

A	–	–	–	–	–	–	–
B		–	–	–	–	–	–
C			–	–	–	–	–
D				–	–	–	–
E					–	–	–
F						–	–
G	X	X	X	X	X	X	–
	A	B	C	D	E	F	G

ii) The pair combination of non-final states are (A, B), (A, C), (A, D), (A, E), (A, F), (B, C), (B, D), (B, E), (B, F), (C, D), (C, E), (C, F), (D, E), (D, F), and (E, F).

$r = \delta(A, b) \rightarrow Ds = \delta(B, b) \rightarrow F$, in the place of (DF), there is neither X nor x. So, in the place of (A, B), there will be 0.

Similarly,

$(r, s) = \delta((A, C), b) \rightarrow (D, E)$ (neither X nor x). In the place of (A, C), there will be 0.

$(r, s) = \delta((A, D), b) \rightarrow (DF)$ (neither X nor x). In the place of (A, D), there will be 0.

$(r, s) = \delta((A, E), b) \rightarrow (DG)$ (there is X). In the place of (AE), there will be x.

All there processes will result in 0, except (AE), (BE), (CE), (DE) and (EF) (there are x).

Solution:

Step I: Divide the states of DFA into two subsets: final (F) and non-final (Q-F).

$$F = \{G\} , Q-F = \{A,B,C,D,E,F\}$$

Make a two-dimensional matrix labelled at the left and bottom by the states of the DFA.

A	-	-	-	-	-	-	-
B		-	-	-	-	-	-
C			-	-	-	-	-
D				-	-	-	-
E					-	-	-
F						-	-
G							-
	A	B	C	D	E	F	G

Step II:

i) The following combinations are the combination of the beginning and final states.

(A,B), (B,G), (C,G), (D,G), (E,G), (F,G). Put X in these combinations of states.

The modified matrix is

A	-	-	-	-	-	-	-
B		-	-	-	-	-	-
C			-	-	-	-	-
D				-	-	-	-
E					-	-	-
F						-	-
G	X	X	X	X	X	X	-
	A	B	C	D	E	F	G

ii) The pair combination of non-final states are (A, B), (A, C), (A, D), (A, E), (A, F), (B, C), (B, D), (B, E), (B, F), (C, D), (C, E), (C, F), (D, E), (D, F), and (E, F).

$r = \delta(A, b) \rightarrow D$ $s = \delta(B, b) \rightarrow F$, in the place of (DF), there is neither X nor x. So, in the place of (A, B), there will be 0.

Similarly,

$(r, s) = \delta((A, C), b) \rightarrow (D, E)$ (neither X nor x). In the place of (A, C), there will be 0.

$(r, s) = \delta((A, D), b) \rightarrow (DF)$ (neither X nor x). In the place of (A, D), there will be 0.

$(r, s) = \delta((A, E), b) \rightarrow (DG)$ (there is X). In the place of (AE), there will be x.

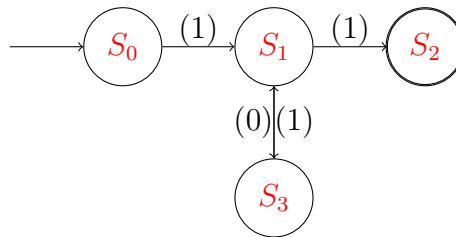
All there processes will result in 0, except (AE), (BE), (CE), (DE) and (EF) (there are x).

The modified matrix is

A	–	–	–	–	–	–	–
B	0	–	–	–	–	–	–
C	0	0	–	–	–	–	–
D	0	0	0	–	–	–	–
E	x	x	x	x	–	–	–
F	0	0	0	0	x	–	–
G	X	X	X	X	X	X	–
	A	B	C	D	E	F	G

The combination of entries 0 are the states of the modified machine. The states of the minimized machine are [AB], [AC], [AD], [AF], [BC], [BD], [BF], [CD], [CF], and [DF], which are simplified to [ABCDF]. It denotes that the machine is already minimized.

24. Use the Myhill–Nerode theorem to minimize the following FA:



[WBUT 2006]

Solution: The tabular format of the finite automata is

Present State	Next State	
	i=0	i=1
S_0	--	S_1
S_1	S_3	S_3
S_2	S_1	--
S_3	--	--

Solution: Divide the states of the DFA into two subsets, final (F) and non-final (Q-F).

$$F = \{S_3\}, \text{ Q-F} = \{S_0, S_1, S_2\}$$

Make a two-dimensional matrix labelled at the left and bottom by the states of the DFA.

S_0	S_1	S_2	S_3
S_0	—	—	—
S_1		—	—
S_2			—
S_3			
	S_0	S_1	S_2

Step II:

i) The following combinations are combination of beginning and final states (S_0, S_3) , (S_1, S_3) , (S_2, S_3) .

Put X in these combinations of states. The modified matrix becomes

S_0	S_1	S_2	S_3
S_0	—	—	—
S_1		—	—
S_2			—
S_3	X	X	X
	S_0	S_1	S_2

The pair combination of non-final states are (S_0, S_1) , (S_0, S_2) , (S_1, S_2) .

$r = \delta(S_0, 0) \rightarrow \text{notmentioned}$ $s = \delta(S_1, 0) \rightarrow S_3$, in the place of all combination of S_3 , there

is X. So, in the place of (S_0, S_1) , there will be x.

$r = \delta(S_0, 0) \rightarrow \text{notmentioned}$ $s = \delta(S_2, 0) \rightarrow S_1$, as S_1 is a non-final state, we shall consider

that $\delta(S_0, 0)$ will produce a non-final state. We are considering that (S_0, S_1) is filled by 0.

$r = \delta(S_1, 0) \rightarrow S_3$ $s = \delta(S_2, 1) \rightarrow S_1$, in the place of (S_1, S_3) , there is X. So, in the place of (S_1, S_2) , there will be x.

The modified matrix becomes

S_0	–	–	–	–
S_1	x	–	–	–
S_2	0	x	–	–
S_3	X	X	X	–
	S_0	S_1	S_2	S_3

The combination (S_0, S_2) is filled by 0. But another two states S_1 and S_3 are left. The states of the minimized machine are (S_0, S_2) , S_1 , S_3 , where the final state is S_3 . The minimized DFA becomes

Present State	Next State	
	i=0	i=1
(S_0, S_2)	S_1	S_1
S_1	S_3	(S_0, S_2)
S_3	–	–