Feature Detection

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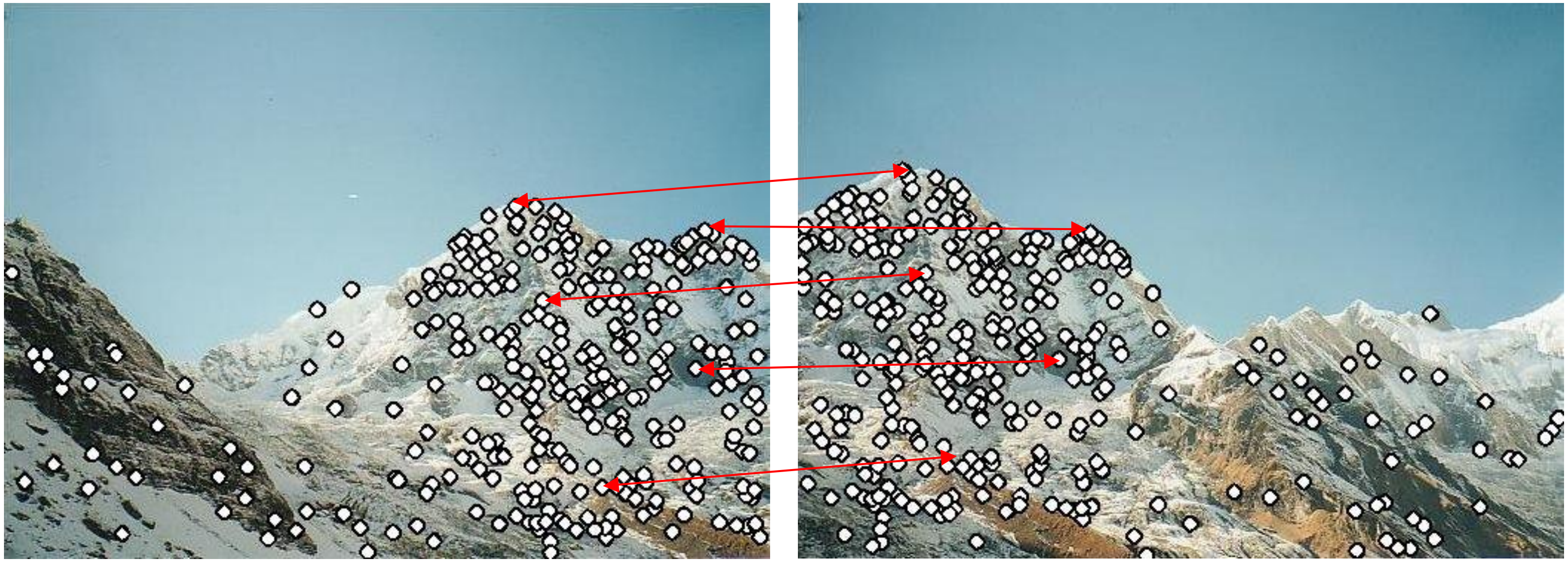
Objetive

This chapter discusses the correspondence problem and presents approaches to solve it.

Outline

- The Correspondence Problem
- Corner Detection
  - Basics
  - Kanade Lucas Tomasi corner detector
  - Harris corner detector
  - Alternative approaches
- SIFT
  - Overview
  - Keypoint detector
  - Descriptor
  - Matching
  - Alternative Approaches

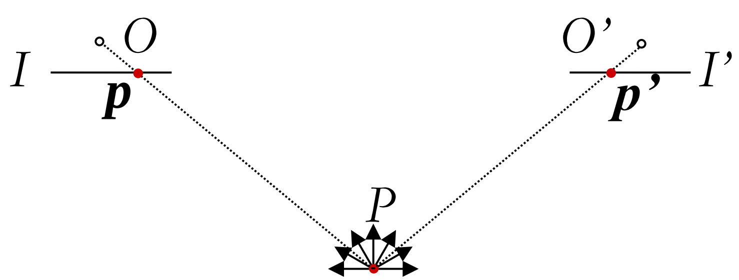
The Correspondence Problem



The Correspondence Problem

Introduction

- Assume the point P in space emits light with the same energy in all directions (i.e., the surface around P is Lambertian)



restrictive assumption

then

$$I(\boldsymbol{p}) = I'(\boldsymbol{p}')$$ *(brightness constancy constraint)*

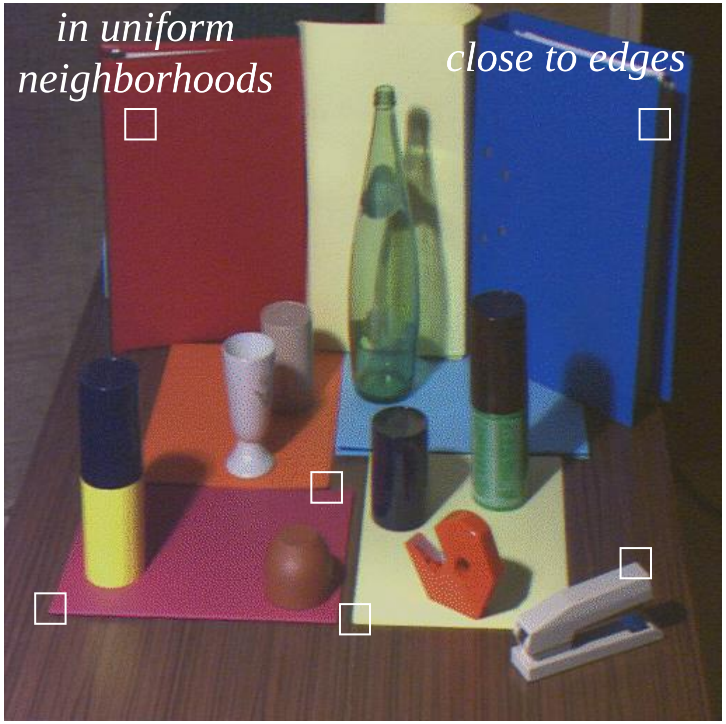
where $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the images of P in the views I and I'.

- The correspondence problem consists of establishing relationships between $\boldsymbol{p}$ and $\boldsymbol{p}'$, i.e.

$$I(\boldsymbol{p}) = I'(\,h(\boldsymbol{p})\,)$$ *($h \rightarrow$ deformation)*

The Correspondence Problem

Introduction: difficult to locate an accurate match …



The Correspondence Problem

Promising image entities

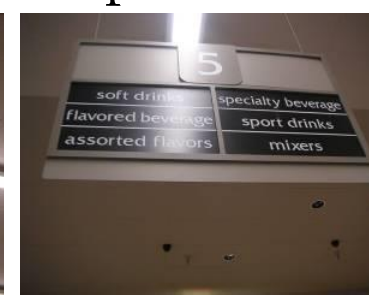
Before starting the search for correspondences, it is convenient to locate entities, (**interest points** or **keypoints**) with good characteristics for an accurate match:

1. **Distinctiveness**: locally separable
2. **Invariance:** identifiable under the expected geometric and radiometric distortions
3. **Stability**: appears in both images
4. **Seldomness**: globally separable

The Correspondence Problem

Types of invariance

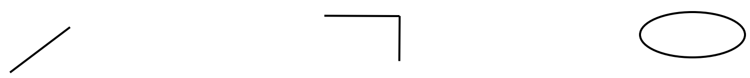

Illumination Scale Rotation Affine Full Perspective



The Correspondence Problem

Two groups of matching techniques

1. **Feature Based**: search for salient geometric features



2. **Area Based**: search for salient radiometric features

May be combined for improved accuracy.

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Corner Detection - Basics

Covariance matrix: definition

Let $\{(x_1,y_1),(x_2,y_2),\ldots,(x_N,y_N)\}$ be a set of N points in a bi-dimensional space. The covariance matrix for this data set is defined by

$$\mathbf{S} = \frac{1}{N-1}\begin{bmatrix} \sum_i (x_i - \bar{x})^2 & \sum_i (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_i (y_i - \bar{y})(x_i - \bar{x}) & \sum_i (y_i - \bar{y})^2 \end{bmatrix}$$

where

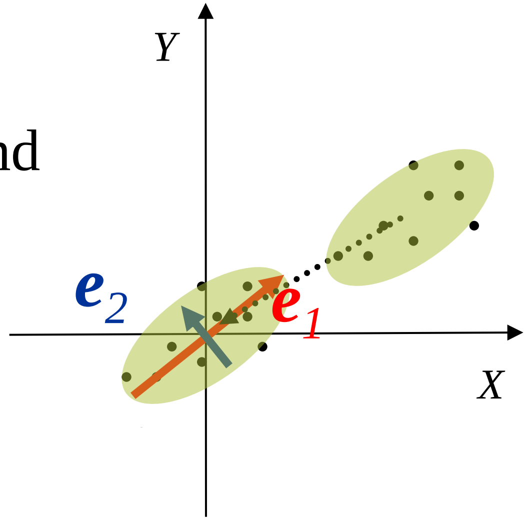
$$\bar{x} = \frac{1}{N}\sum_i x_i \quad \text{and} \quad \bar{y} = \frac{1}{N}\sum_i y_i$$

actually valid for any dimension

Corner Detection - Basics

Covariance matrix: geometric interpretation

- Let $X_i=(x_i-\bar{x})$ and $Y_i=(y_i-\bar{y})$. Clearly $\overline{X}=\overline{Y}=0$, and



$$(N\text{-}1)\boldsymbol{S}=\begin{bmatrix} \sum_i (x_i-\bar{x})^2 & \sum_i (x_i-\bar{x})(y_i-\bar{y}) \\ \sum_i (x_i-\bar{x})(y_i-\bar{y}) & \sum_i (y_i-\bar{y})^2 \end{bmatrix} = \begin{bmatrix} \sum_i X_i^{\,2} & \sum_i X_i Y_i \\ \sum_i X_i Y_i & \sum_i Y_i^{\,2} \end{bmatrix}$$

- Let $\lambda_1 \geq \lambda_2$ and $\mathbf{e}_1$ and $\mathbf{e}_2$ be the eingenvalues/eigenvectors of **S**.
- $\mathbf{e}_1$ points in the direction of maximum variance.
- $\mathbf{e}_2$ is perpendicular to $\mathbf{e}_1$.
- Each eigenvalue is proportional to the variance of the projections over the corresponding eigenvector.

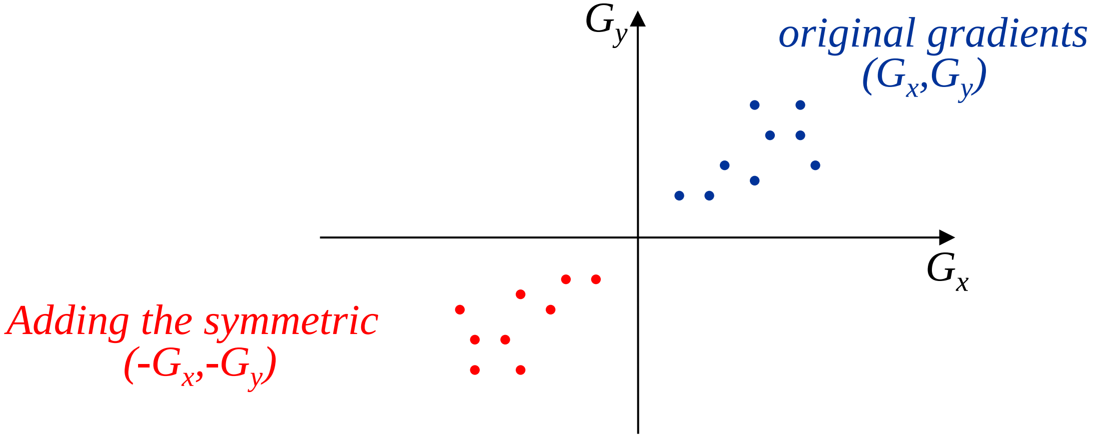
Corner Detection - Basics

Mathematical Description

- Let $[G_x\ G_y]^T$ denote the gradient components on a pixel p and **C** the matrix defined as

$$\mathbf{C} = \sum_Q \begin{bmatrix} G_x^2 & G_x G_y \\ G_x G_y & G_y^2 \end{bmatrix}$$

  for each point $\boldsymbol{p}$ in the image, where the sums are taken over a small neighborhood Q.
- This corresponds (up to a scale) to the covariance matrix of the following dataset



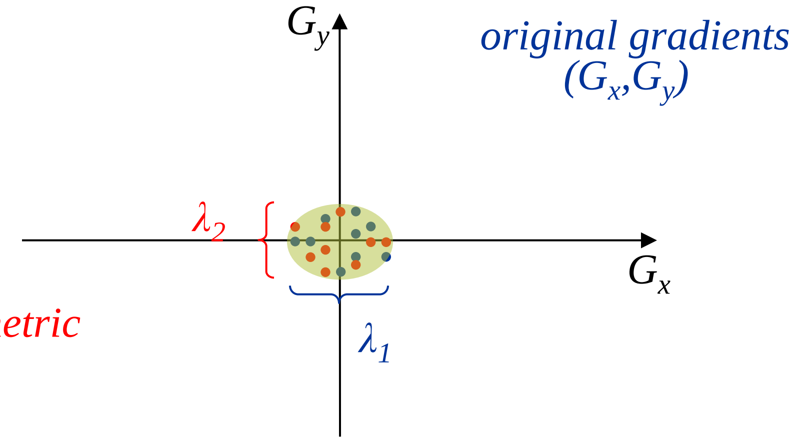
Corner Detection - Basics

Local Gradient Behavior

- On uniform regions

  magnitude of the gradient is small

Q



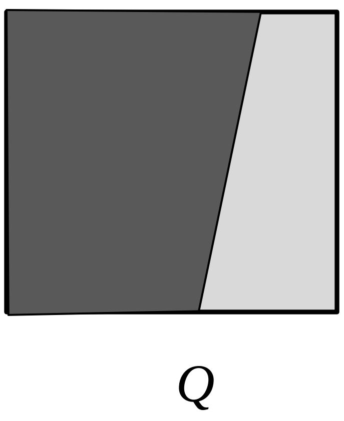
both eigenvalues λ_1 and λ_2 of **C** will be small.

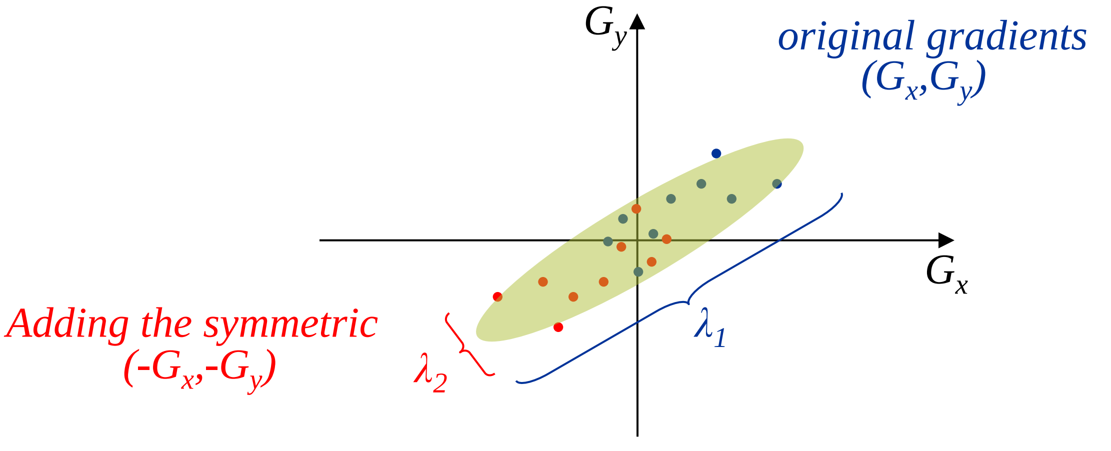
Corner Detection - Basics

Local Gradient Behavior

- On edges

  significant changes in magnitude but not in direction





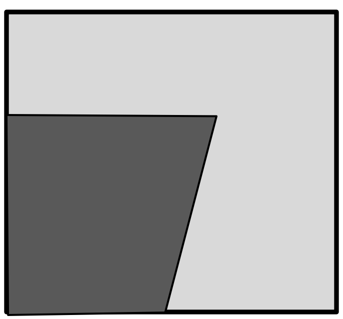
One eigenvalue (λ_1) of **C** will be large and the other (λ_2) small.

Corner Detection - Basics

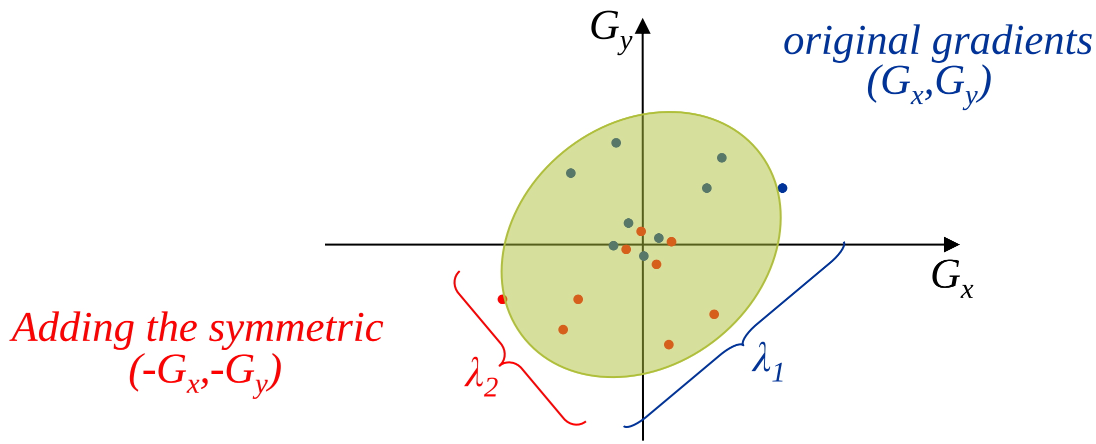
Local Gradient Behavior

- On corners

  significant changes in magnitude as well as in direction



Q



both eigenvalues λ_1 and λ_2 of **C** will be large.

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Corner Detection (Kanade Lucas Tomasi)

Geometric interpretation of the eigenvalues

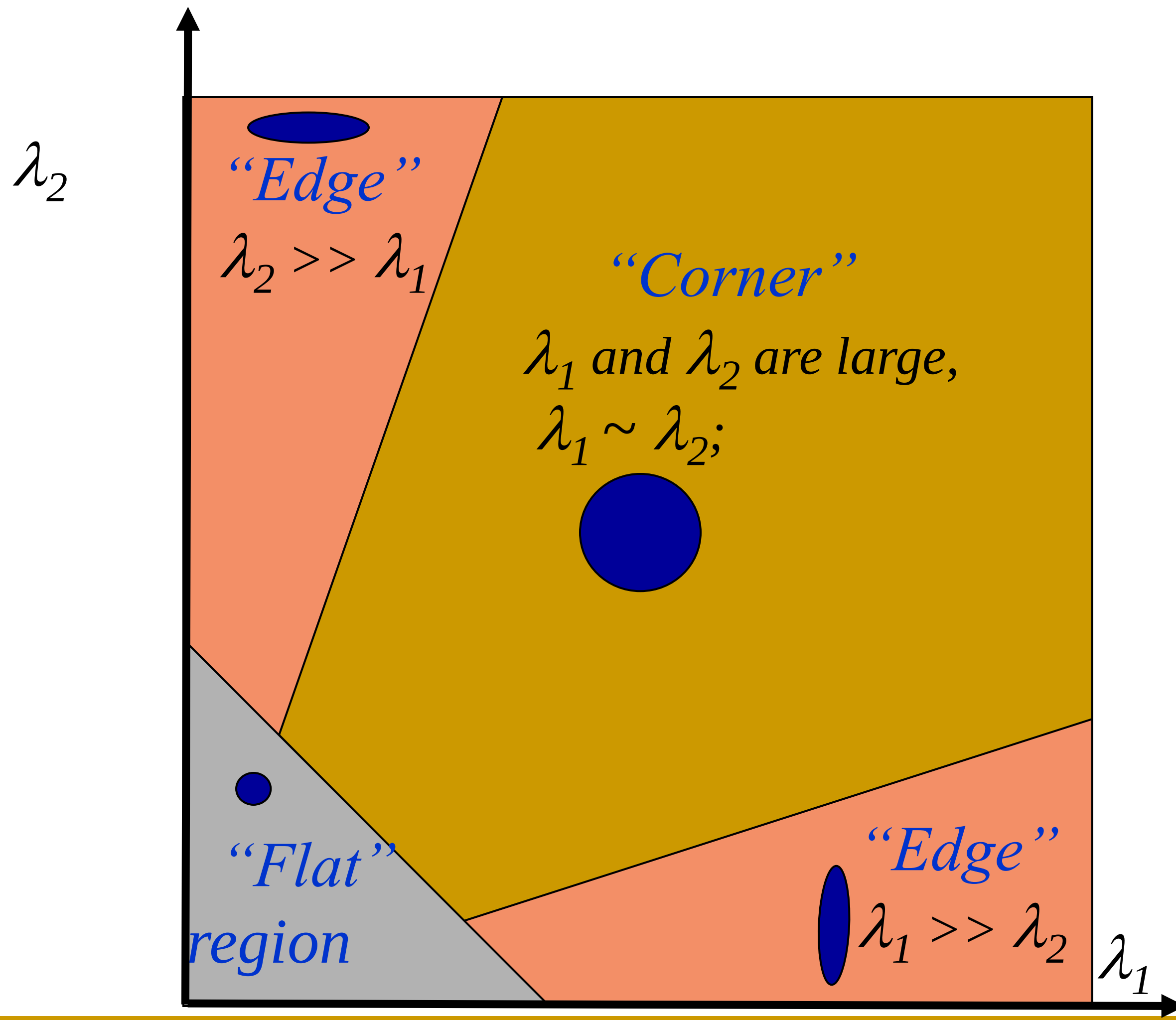
- neighborhood Q on a uniform region $\rightarrow \lambda_2 \approx \lambda_1 \approx 0$
- neighborhood Q over an ideal edge $\rightarrow \lambda_2 \approx 0, \lambda_1 > 0$
- neighborhood Q over an ideal corner $\rightarrow \lambda_1 \geq \lambda_2 > 0$

Defining **cornerness** $R(x_{\boldsymbol{p}}, y_{\boldsymbol{p}})$ of a point $\boldsymbol{p}$ as:

$$R\left(x_{\boldsymbol{p}}, y_{\boldsymbol{p}}\right) = \min(|\lambda_1|, |\lambda_2|)$$

A corner is a location where $R(x_{\boldsymbol{p}}, y_{\boldsymbol{p}})$ is large enough.

Corner Detection (Kanade Lucas Tomasi)



Corner Detection (Kanade Lucas Tomasi)

Algorithm

1. Compute the image gradient over the entire image
2. For each image point $\boldsymbol{p}$
   a) Form the matrix **C** over a $(2N+1) \times (2N+1)$ neighborhood Q.
   b) Compute $R(x_{\boldsymbol{p}}, y_{\boldsymbol{p}})$.
   c) If $R(x_{\boldsymbol{p}}, y_{\boldsymbol{p}}) > \tau_{\mathrm{KLT}}$, save the coordinates of $\boldsymbol{p}$ into a list L.
3. Go through the list L in decreasing order of $R(x_{\boldsymbol{p}},y_{\boldsymbol{p}})$; if it does not fall within the minimum separation space of any previously selected corner, then select it; otherwise discard it.

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Corner Detection (Harris)

Harris uses a smoothed version of the matrix **C**, given by

$$\mathbf{M} = \sum_{Q} w_Q \begin{bmatrix} G_x^2 & G_x G_y \\ G_x G_y & G_y^2 \end{bmatrix}$$

where $w_Q\,(u,v)$ is a smooth circular window (usually a Gaussian).

Corner Detection (Harris)

Harris proposes another **cornerness** measure.

If λ_1 and λ_2 are the eigenvalues of matrix **M**, then R is given by:

$$R = \lambda_1 \lambda_2 - k\left(\lambda_1 + \lambda_2\right)^2 = \det(\mathbf{M}) - k\left[\mathrm{tr}(\mathbf{M})\right]^2$$

(*k* – empirical constant, *k* = 0.04-0.06)

which should be interpreted in the following way:

- R is large for a corner,
- R is negative with large magnitude for an edge, and
- $|R|$ is small for a flat region.

Corner Detection (Harris)

Algorithm

1. Compute the image gradient over the entire image
2. For each image point ***p***
   a) Form the matrix **M** over a $(2N+1) \times (2N+1)$ neighborhood Q.
   b) Compute $R(x_{\boldsymbol{p}}, y_{\boldsymbol{p}})$.
   c) If $R(x_{\boldsymbol{p}}, y_{\boldsymbol{p}}) > \tau_{\text{Harris}}$, save the coordinates of p into a list L.
3. Find points with large corner response : $R > \tau_{\text{Harris}}$,
4. Take only the points of local maxima of R.

Corner Detection (Kanade Lucas Tomasi)

For $\tau_{KLT} = 0.01$

Corner Detection (Kanade Lucas Tomasi)

For $\tau_{KLT} = 0.1$

Corner Detection (Harris)

For τ_{Harris} = 0.001

Corner Detection (Harris)

For τ_{Harris} = 0.01

Corner Detection so far

Some Properties

- Rotation invariant: eigenvectors rotate with the image

- Partially invariant to affine intensity change
  - invariant to intensity shift ($I = I + b$): only derivatives count
  - partially invariant to intensity scale ($I = aI$)
- Non-invariant to image scale

corner ?

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Alternative Corner Detectors

- **Förstner**

  FÖRSTNER, W. and GÜLCH, E. A fast operator for detection and precise location of distinct points, corners and circular features. Intercommission Conf. on Fast Processing of Photogrammetric Data, p. 281-305, 1987.

- **SUSAN**

  SMITH, S.M.; BRADY, J.M., SUSAN – a new approach to low level image processing. Journal of Computer Vision, v.23, n.1, p. 45-78, 1997.

- **Harris-Laplacian**

  MIKOLAJCZYK, K.; SCHMID, C. Indexing based on scale invariant interest points, Proceedings Eighth IEEE International Conference on Computer Vision - ICCV 2001., v. 1, pp.525 - 531, 2001

- **Multi-scale**

  BROWN, M.; SZELISKI, R.; WINDER, S. Multi-image matching using multi-scale oriented patches, Conf. on Computer Vision and Pattern Recognition - CVPR 2005, v. 1, p 510-517, 2005.

- **…**

Assignment on Corner Detectors

Download the program that implements the Harris and KLT Corner Detectors. Read the help, choose some image and test it.

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SIFT

Scale Invariant Feature Transform

*David G. Lowe, "**Distinctive image features from scale-invariant keypoints,**" International Journal of Computer Vision, 60, 2 (2004), pp. 91-110.*

SIFT

Overview: Advantages

- **Invariance** to
  - Image scale,
  - Rotation on the plane.
- **Robustness** to
  - Affine distortion,
  - Change in 3D viewpoint,
  - Addition to noise.
- **Locality:** feature are local, so robust to occlusion and clutter
- **Distinctiveness**: individual features can be matched to a large database of objects
- **Quantity**: many features can be generated for even small objects.

SIFT

Example:

SIFT

Basic Steps:

1. Scale-space extrema detection
2. Keypoint localization
3. Orientation assignment
4. Keypoint descriptor construction
5. Matching keypoints

Interest-point detection (steps 1–2)

Description (steps 3–4)

Matching (step 5)

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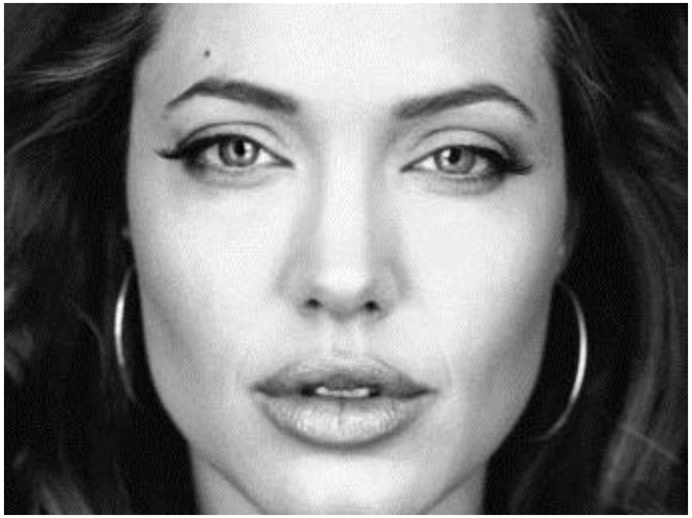
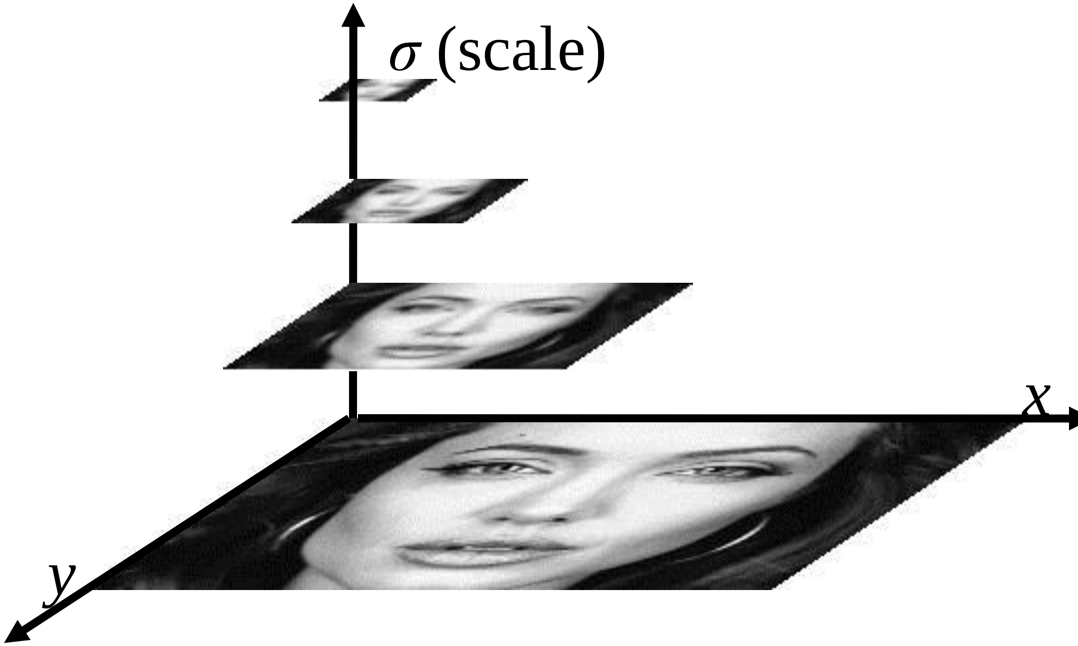
Image Pyramid

The scale-space representation:

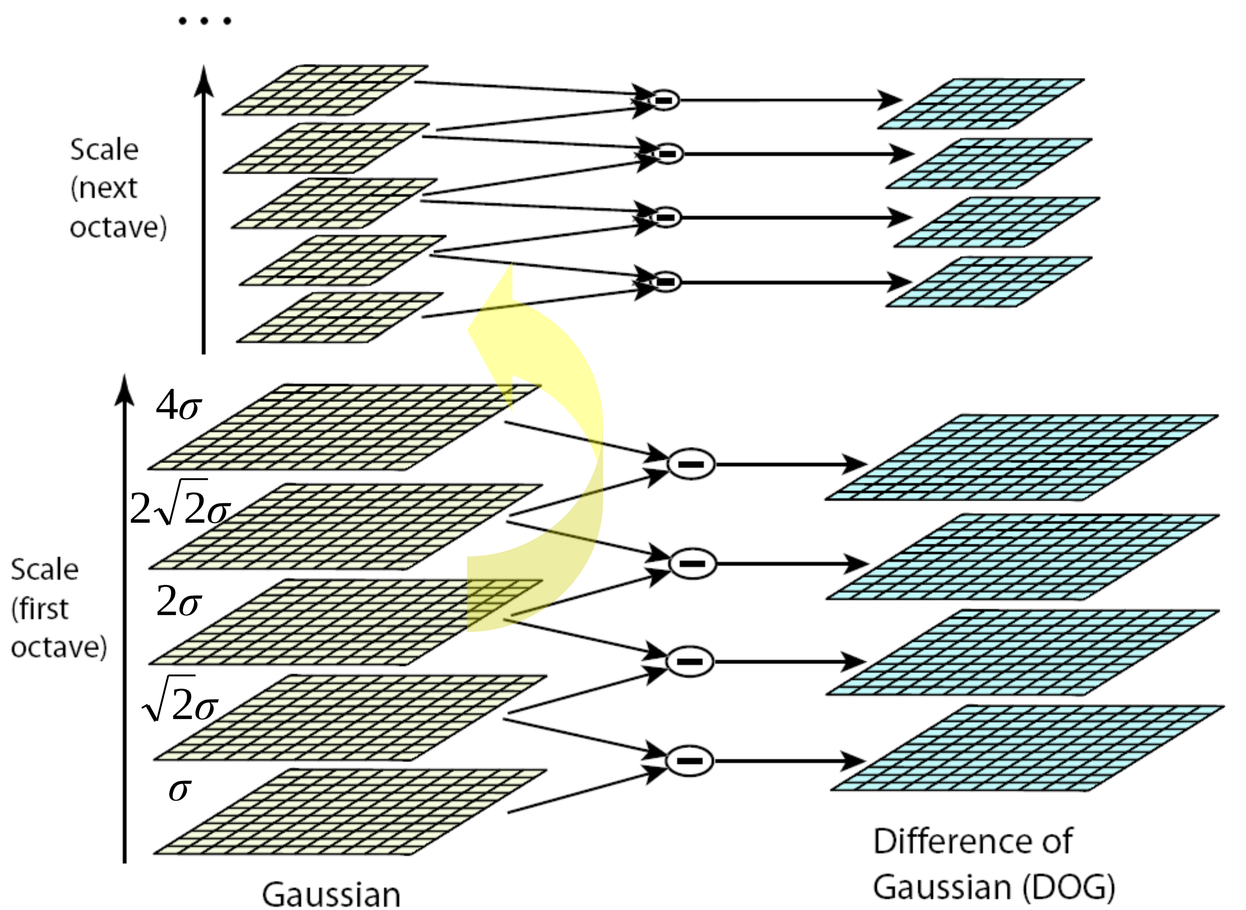
The input image is successively smoothed with a Gaussian kernel and subsampled, forming an image pyramid.





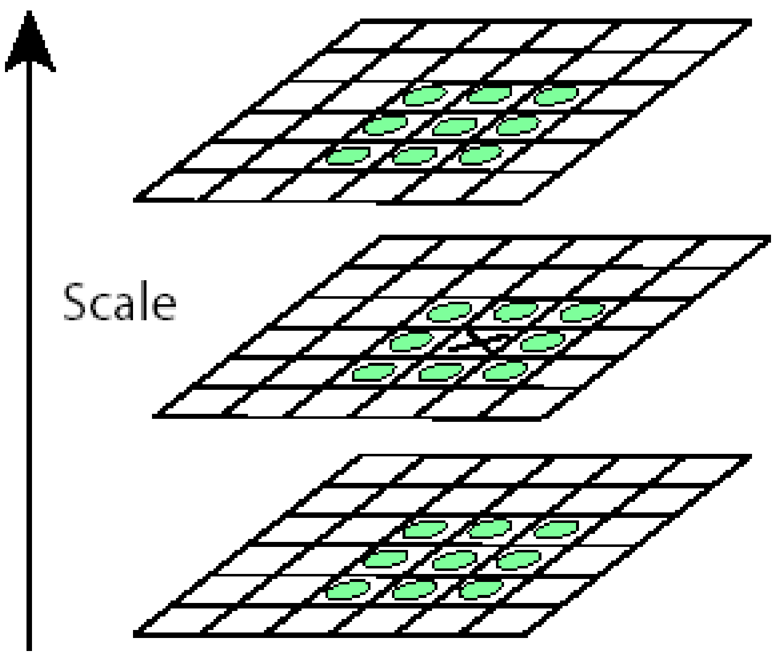
SIFT Keypoint Detector

Overview:



SIFT Keypoint Detector

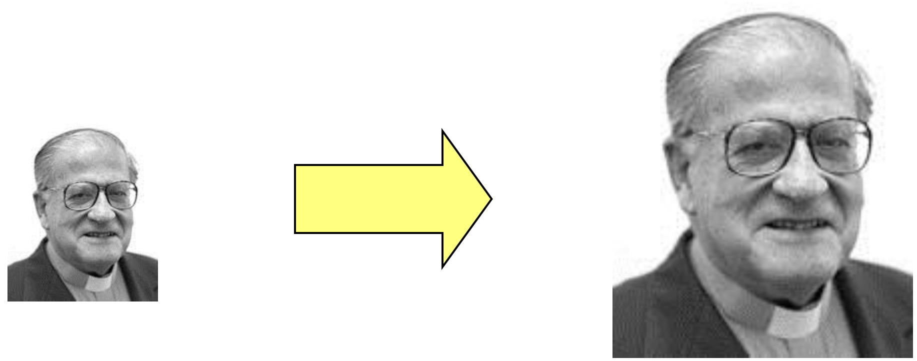
Detecting Extrema:



SIFT Keypoint Detector

Algorithm:

1. The image size is doubled

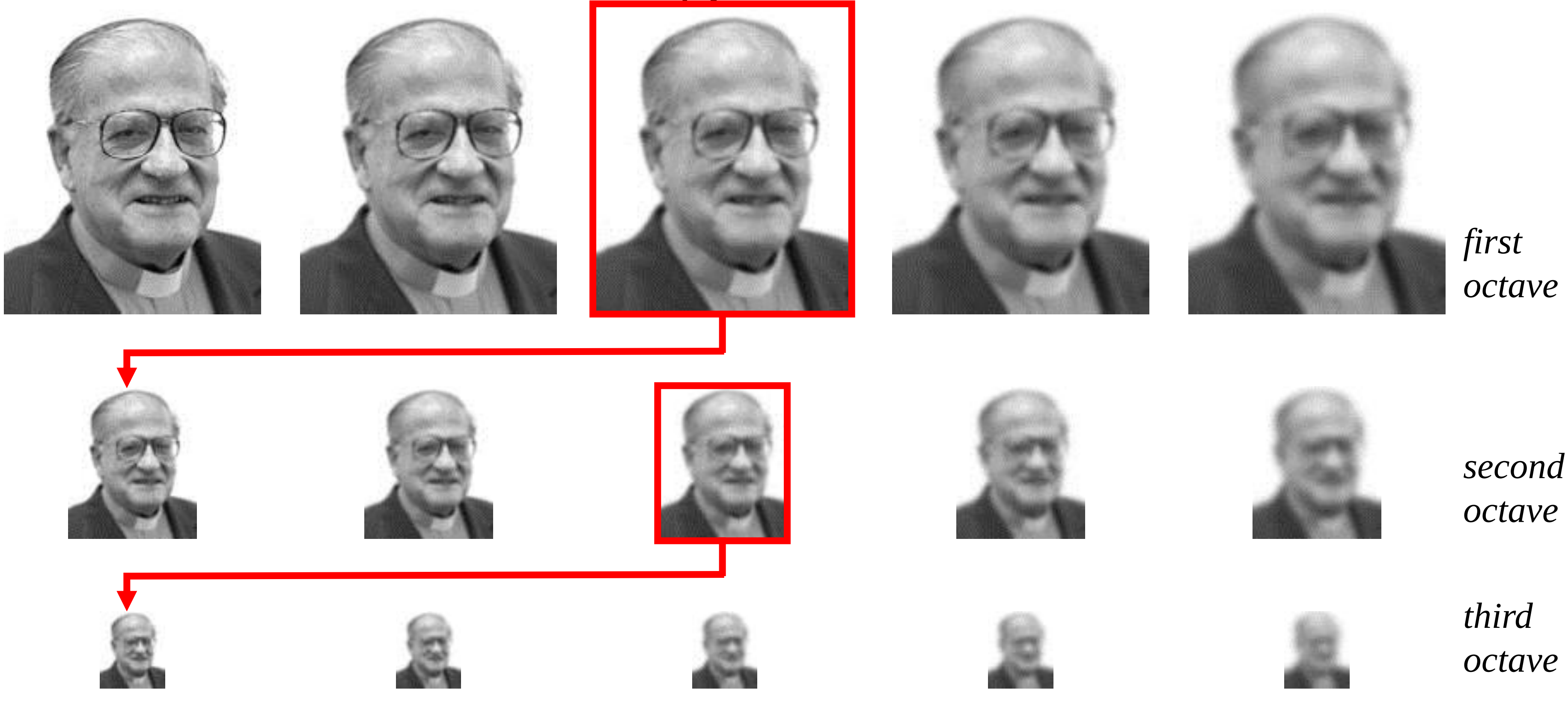


This implies in a smoothing corresponding to $\sigma=0.5$. Thus, the first smoothing should use $\sigma=0.5$ and from then on $\sigma=1.6$.

SIFT Keypoint Detector

Algorithm:

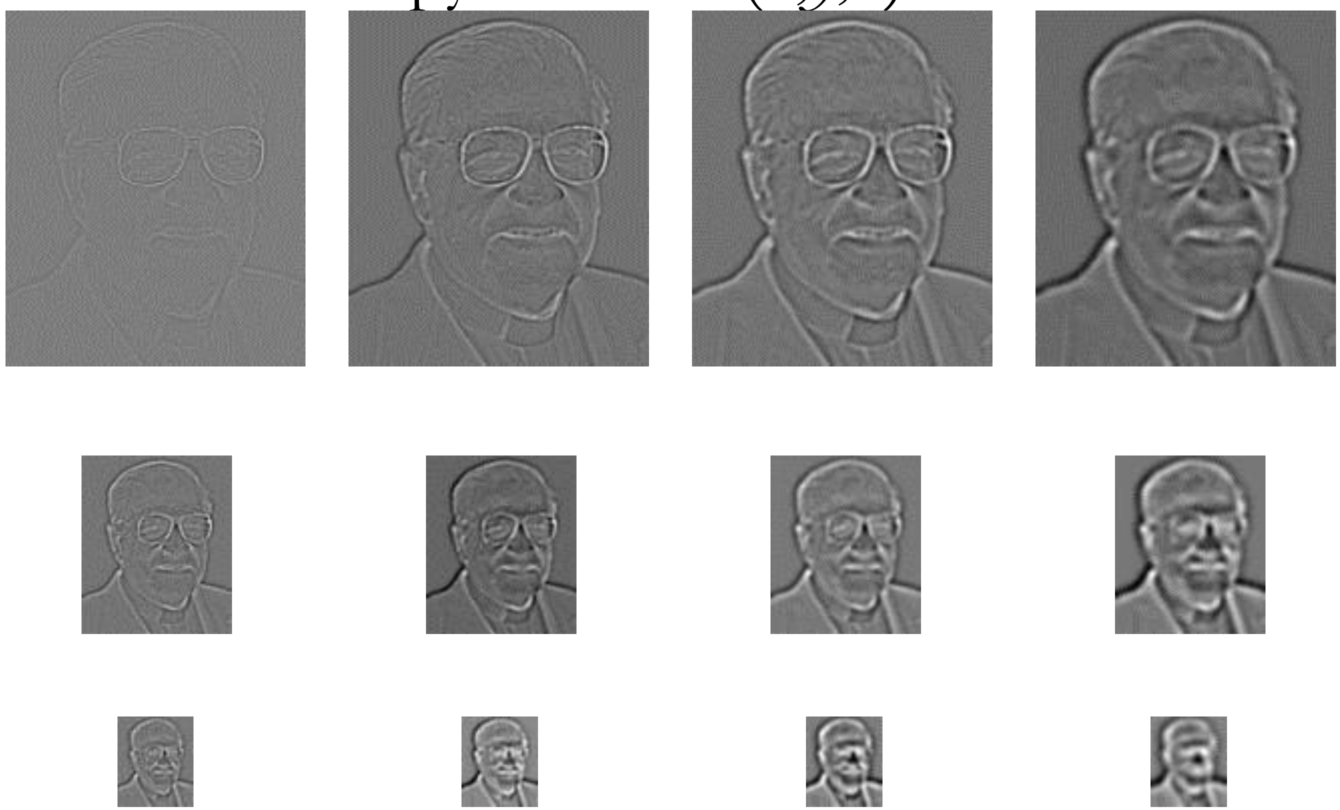
2. Build the Gaussian pyramid



SIFT Keypoint Detector

Algorithm:

3. Build the DoG pyramid - $D(x,y,\sigma)$



first octave

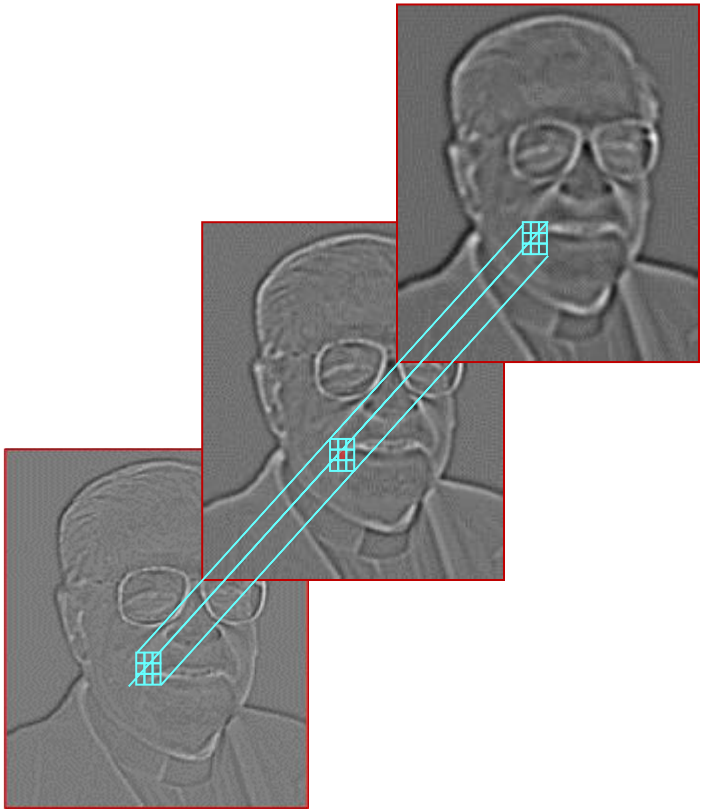
second octave

third octave

SIFT Keypoint Detector

Scale-space extrema detection - algorithm :

4. Extrema in 3D



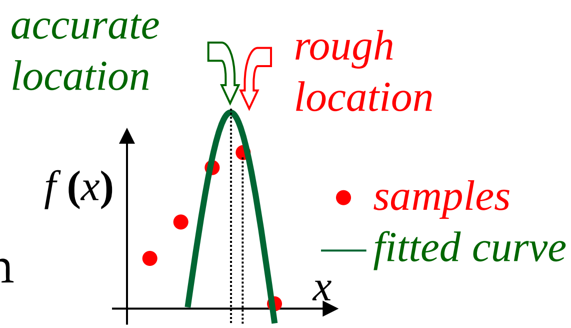
SIFT Keypoint Detector

Accurate Keypoint Localization

- From difference-of-Gaussian local extrema detection we obtain approximate values for keypoints.
- Originally these approximations were used directly.
- For an improvement in matching and stability, fitting to a 3D quadratic function is used.
- It is specially important to localize keyponts detected on higher octaves.

SIFT Keypoint Detector

Accurate Keypoint Localization



- A quadratic function is fitted to the sample points and its maximum is the correct location

- For the 3D DoG function $D(x,y,\sigma)$,
  1. Take Taylor Series Expansion of scale-space function

$$D(\mathbf{x}) = D + \frac{\partial D^T}{\partial \mathbf{x}}\mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x}$$

  where

  D and its derivatives are evaluated at the sample point and $\mathbf{x} = (x, y, \sigma)$ is the offset from this point.

  2. Use up to quadratic terms

SIFT Keypoint Detector

Accurate Keypoint Localization

- For the 3D DoG function $D(x,y,\sigma)$,
  3. take derivative of

$$D(\mathbf{x}) = D + \frac{\partial D^T}{\partial \mathbf{x}}\mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x}$$

  and set to 0; this yields

$$\frac{\partial D}{\partial \mathbf{x}} + \frac{\partial^2 D}{\partial \mathbf{x}^2}\hat{\mathbf{x}} = 0 \qquad \rightarrow \qquad \boxed{\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}}$$

  where $\hat{\mathbf{x}}$ is the offset from the sample point.

SIFT Keypoint Detector

Filtering out weak keypoints

- **Keypoints with low contrast are discarded**
  - Evaluate the function (DoG) value at the location and scale of the keypoint (replace the formula for $\hat{\mathbf{x}}$ in the Taylor approximation of $D(\mathbf{x})$)

$$D(\hat{\mathbf{x}}) = D + \frac{1}{2}\frac{\partial D^T}{\partial \mathbf{x}}\hat{\mathbf{x}}$$

  - If $\left| D(\hat{\mathbf{x}}) \right| < 0.03$, discard it, whereby the image intensities are normalized in [0 1].

SIFT Keypoint Detector

Filtering out weak keypoints

- **Keypoints along edges are discarded**
  - Evaluate the Hessian function at the extrema

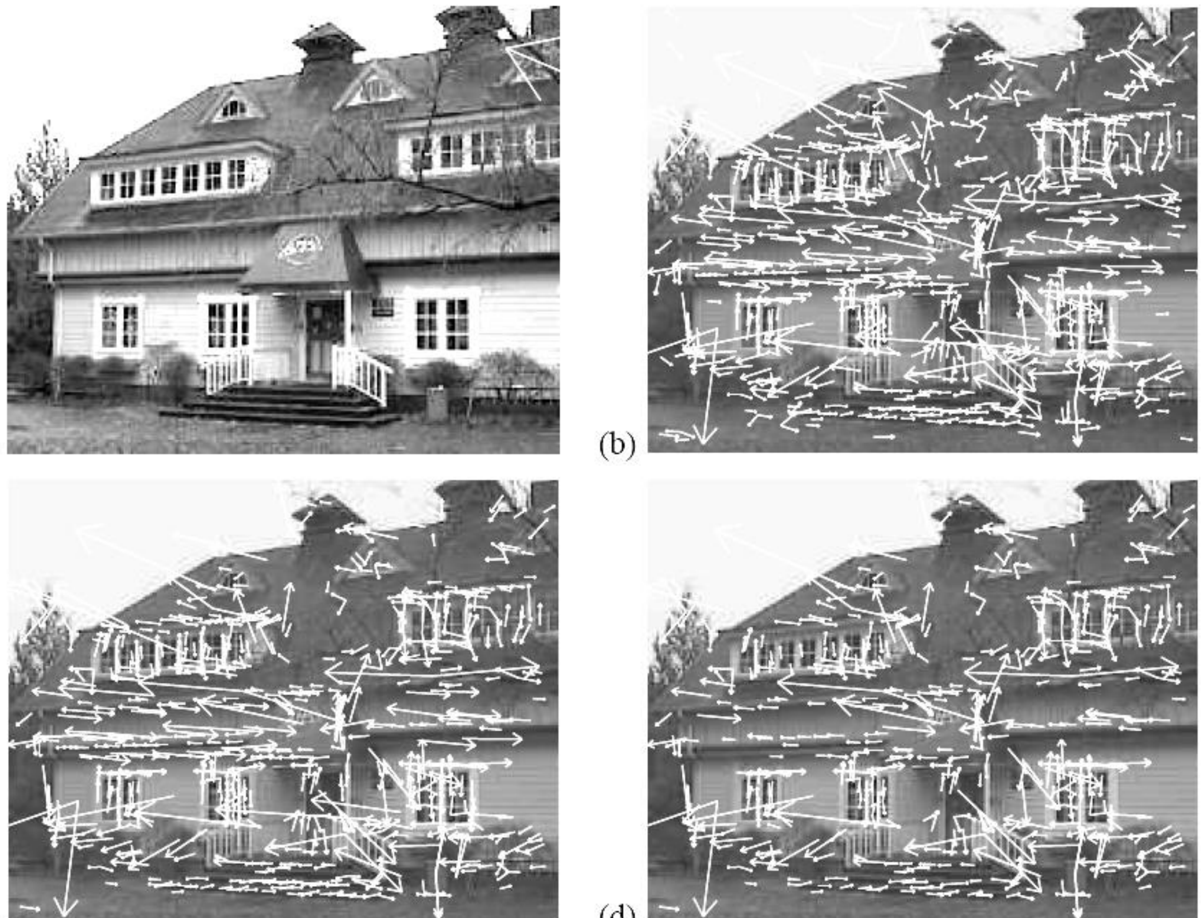
$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

  - If λ_1 and λ_2 are the eigenvalyes of **H**, and $r = \lambda_1/\lambda_2 > 1$
  - We know that $\frac{\mathrm{tr}(\mathbf{H})^2}{\det(\mathbf{H})} = \frac{(\lambda_1 + \lambda_2)^2}{\lambda_1 \lambda_2} = \frac{(r+1)^2}{r}$
  - If $\frac{\mathrm{tr}(\mathbf{H})^2}{\det(\mathbf{H})} > \frac{(r_{max}+1)^2}{r_{max}}$ for r_{max} =10, discard it.

SIFT Keypoint Detector

Example from Lowe

original image



(a) (b) (c) (d)

The initial 832 keypoints locations at extrema of DoG. Vectors indicating scale, orientation, and location.

After applying a threshold on minimum contrast, 729 keypoints remain.

The final 536 keypoints after threshold on ratio of principal curvatures.

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SIFT Descriptor

Assigning an Orientation

- Take the image *L* at the scale closest to the keypoint.
- magnitude and orientations of gradient *around* the key point are calculated according to

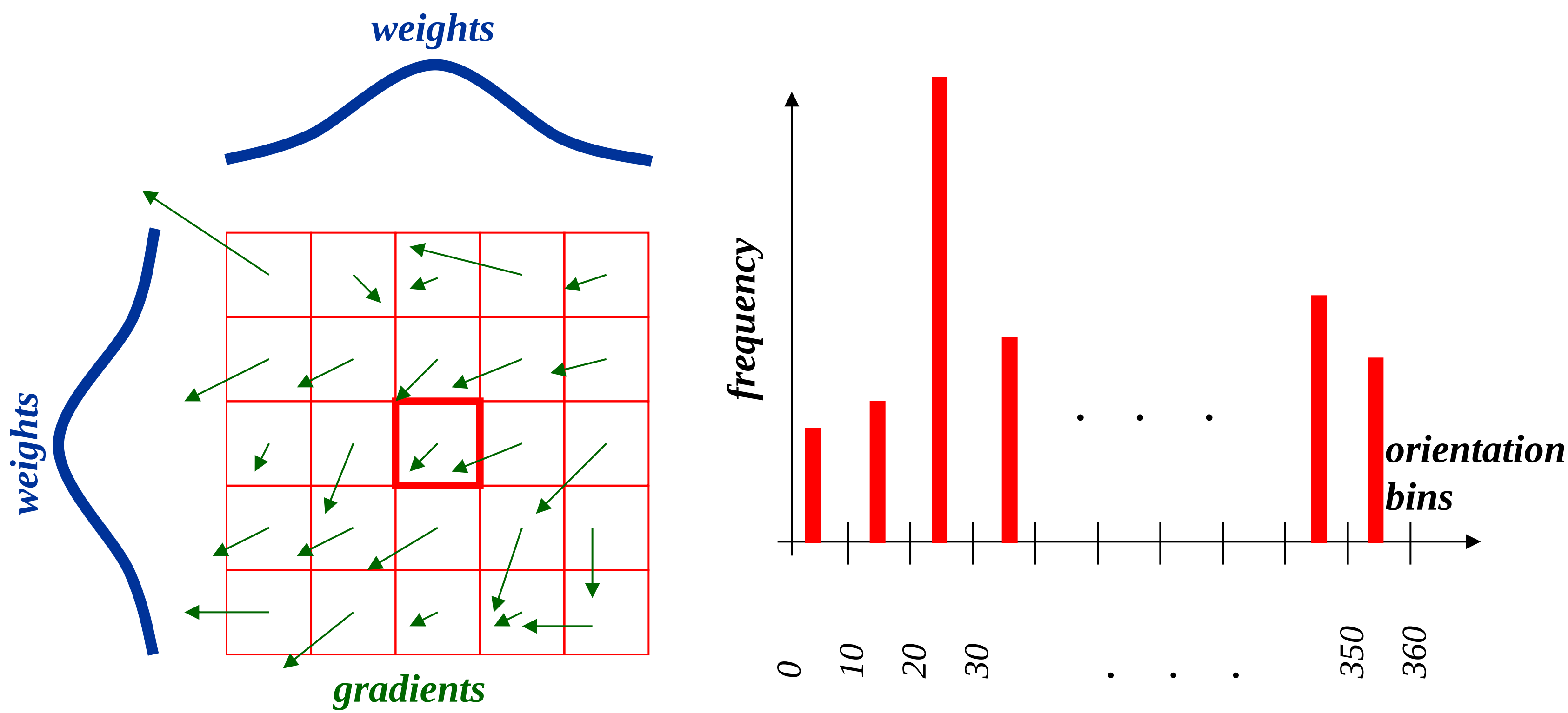
$$m(x,y)=\sqrt{(L(x+1,y)-L(x-1,y))^2+(L(x,y+1)-L(x,y-1))^2}$$

$$\theta(x,y)=\tan^{-1}((L(x,y+1)-L(x,y-1))/(L(x+1,y)-L(x-1,y)))$$

- Compute the *orientation histogram* with 36 bins. Each sample is weighted by
  - gradient magnitude and
  - Gaussian circular window with a σ equal to 1.5 times scale of keypoint

SIFT Descriptor

Assigning an Orientation



SIFT Descriptor

Assigning an Orientation

- Highest peak in orientation histogram is found along with *any other peaks within 80% of highest peak* → more than one orientation may be assigned to a key point.
- A parabola is fit to the 3 closest histogram values to each peak and its maximum is taken → more accurate peak detection.
- Thus each keypoint has 4 dimesions:
  - x location,
  - y location,
  - σ scale, and
  - orientation

SIFT Descriptor

Building the Keypoint Descriptor

1. Compute the gradient magnitude and orientation at each image sample point around the keypoint. These are weighted by a Gaussian window, with σ =1/2 width of the descriptor window.

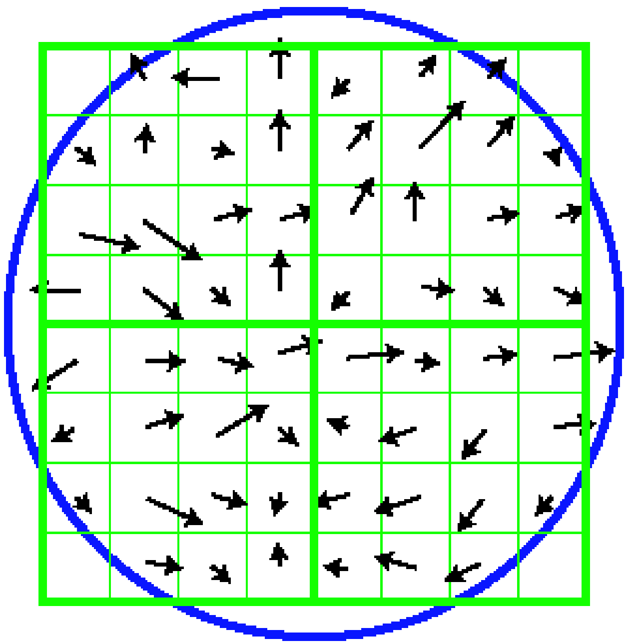


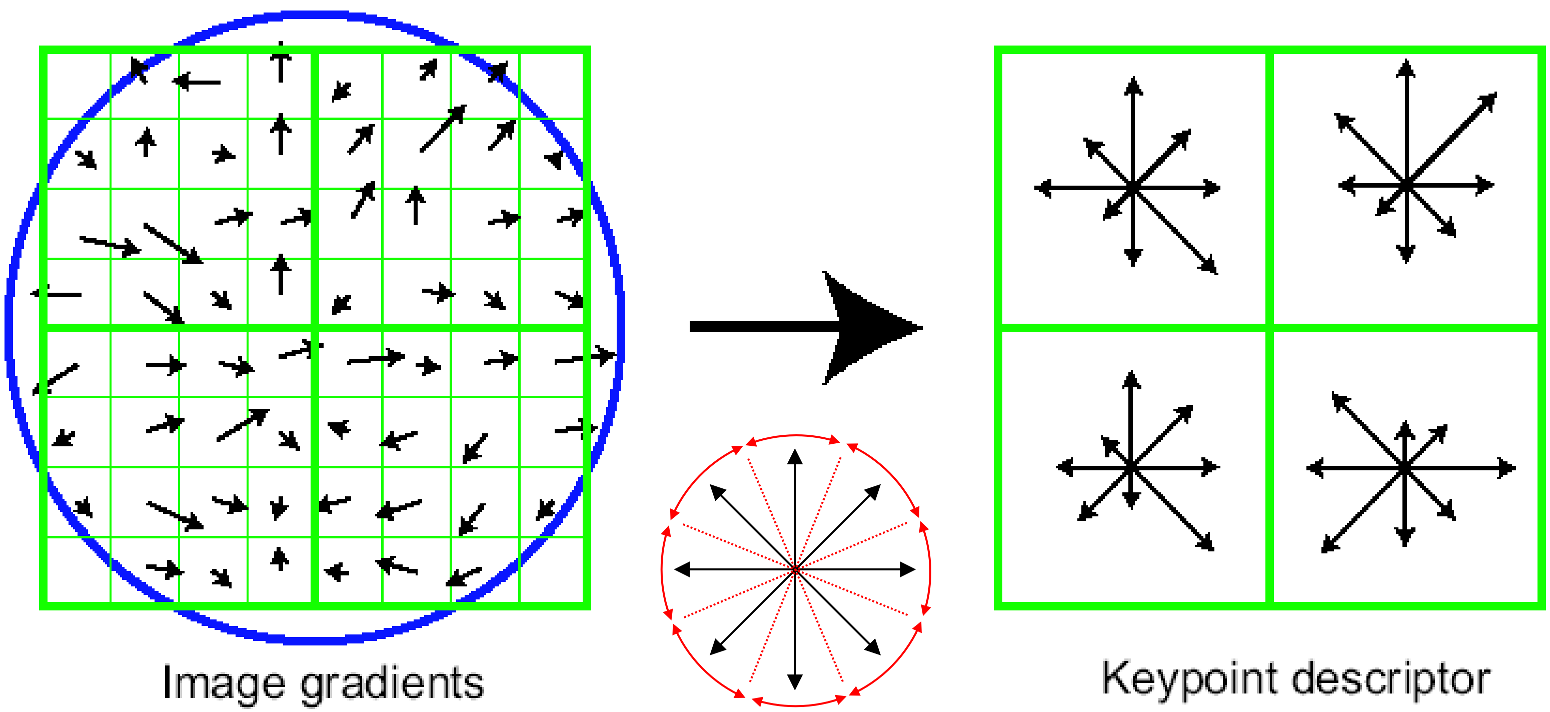
Image gradients

- *to achieve orientation invariance, the coordinates of the descriptor and the gradient orientations are rotated relative to the keypoint orientation.*

SIFT Descriptor

Building the Keypoint Descriptor

2. Form *orientation histograms* summarizing the contents over 4x4 subregions, with the *length* of each arrow given by the sum of the gradient magnitudes near that direction within the region.

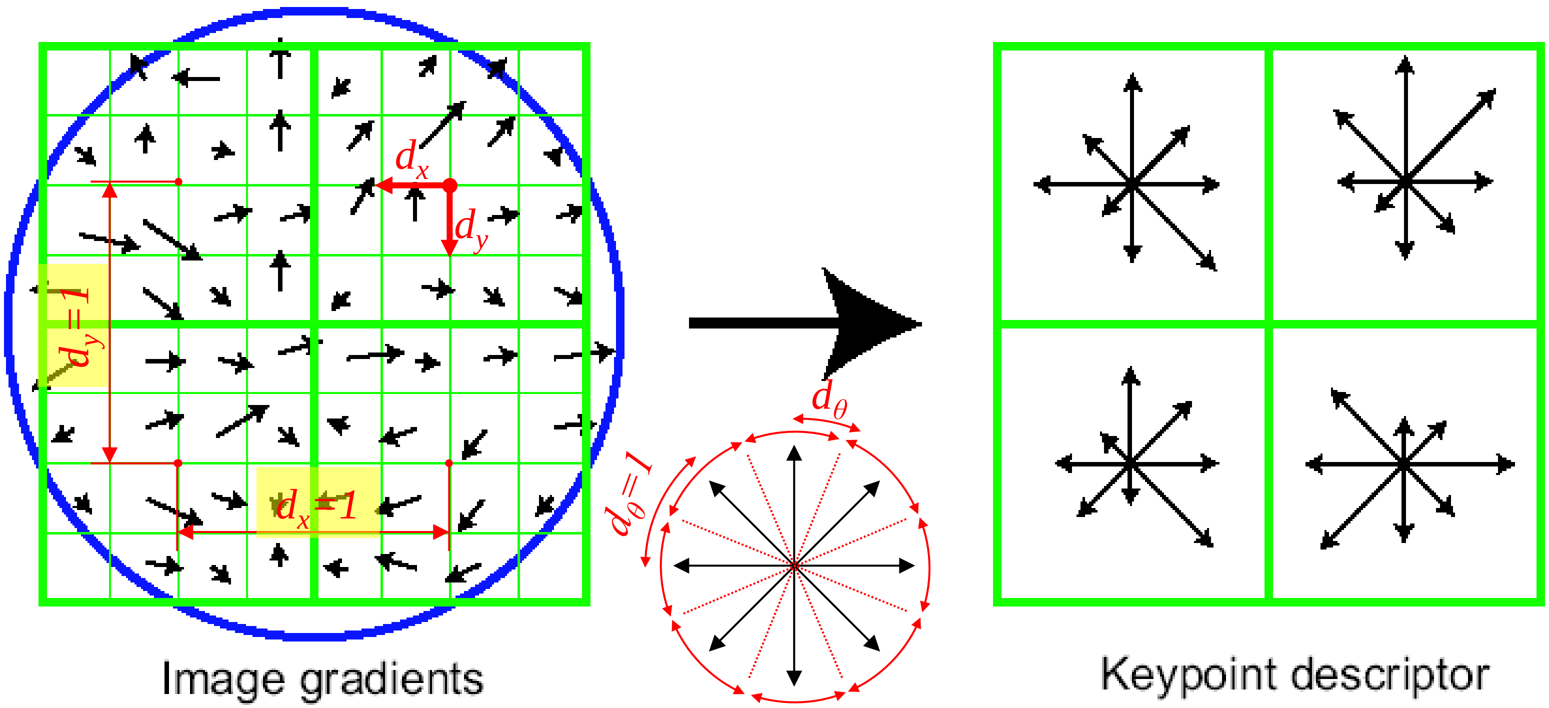


SIFT Descriptor

Building the Keypoint Descriptor

3. Avoiding boundary effects between histograms

- Weight equal to $1 - d$, for each of the 3 dimensions where d is the distance of a sample to the center of a bin

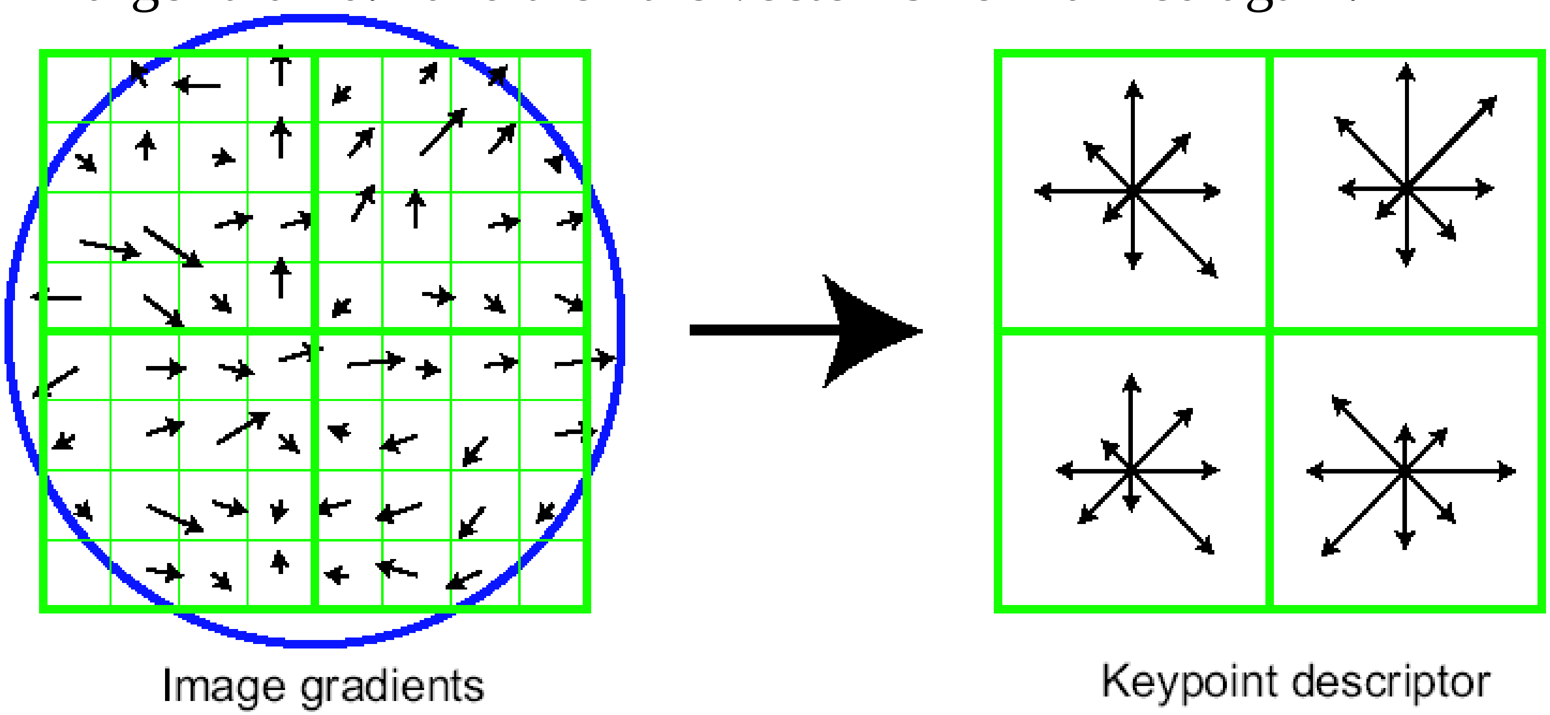


SIFT Descriptor

Building the Keypoint Descriptor

4. Ensuring invariance to illumination

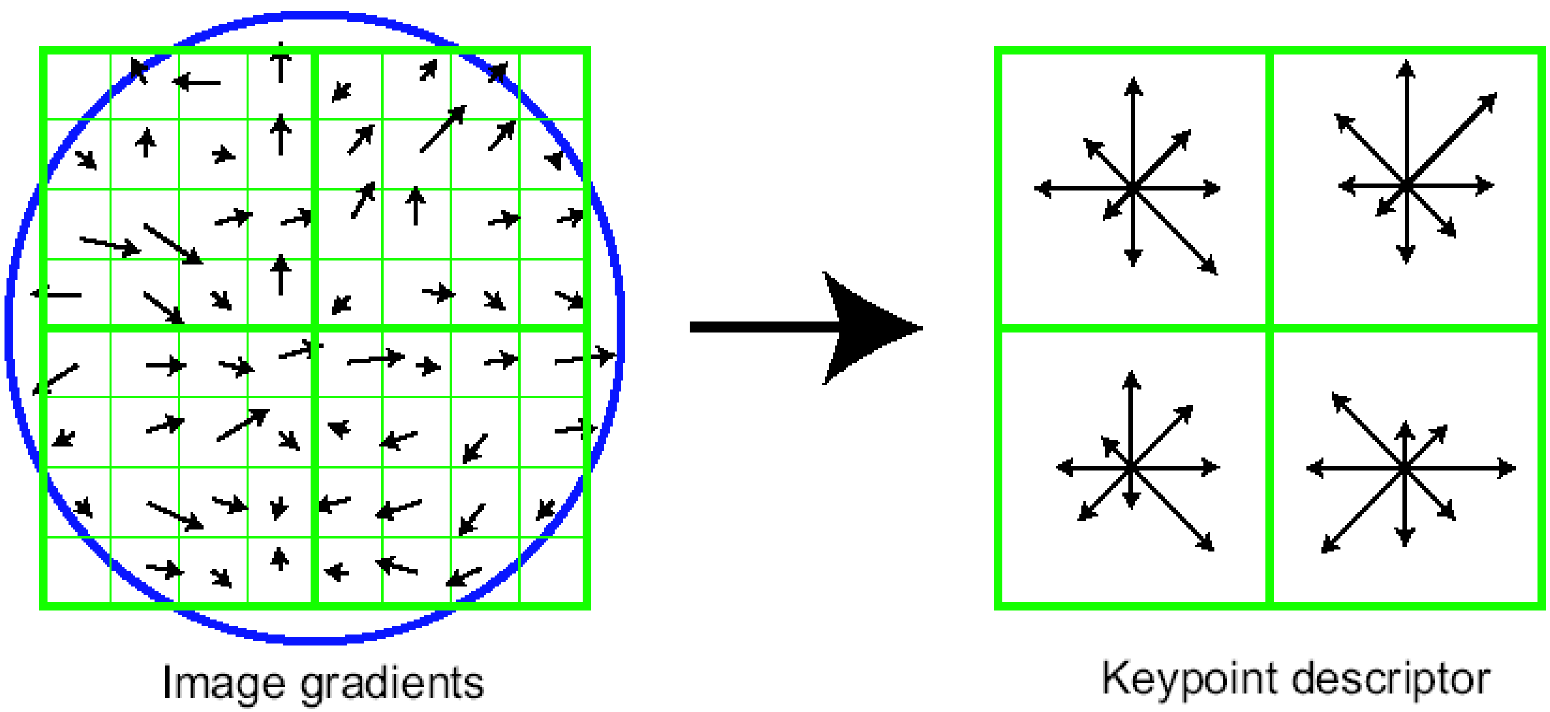
- Vector is normalized to 1; its components are thresholded to no larger than 0.2 and then the vector is normalized again.



SIFT Descriptor

Building the Keypoint Descriptor

- *"This figure shows a 2×2 descriptor array computed from an 8×8 set of samples, whereas the experiments in this paper use 4×4 descriptors computed from a 16×16 sample array."* Lowe



SIFT Descriptor

Keypoint Descriptor provides invariance to

- Scale (by using the DoG pyramid)
- Illumination (by using the gradients + normalization)
- Rotation (by rotating the description relative to the main direction)
- 3D camera viewpoint (to a certain extent)

Outline

- The Correspondence Problem
- Corner Detection
  - Basics
  - Kanade Lucas Tomasi corner detector
  - Harris corner detector
  - Alternative approaches
- SIFT
  - Overview
  - Keypoint detector
  - Descriptor
  - Matching
  - Alternative Approaches

SIFT- Keypoint Matching

Up to this point we have:

- Found rough approximations for features by looking at the difference-of-Gaussians
- Localized the keypoint more accurately
- Thresholded poor keypoints
- Determined the orientation of a keypoint
- Calculated a 128 feature vector for each keypoint

How to find corresponding keypoints on a pair (or more) of images?

SIFT- Keypoint Matching

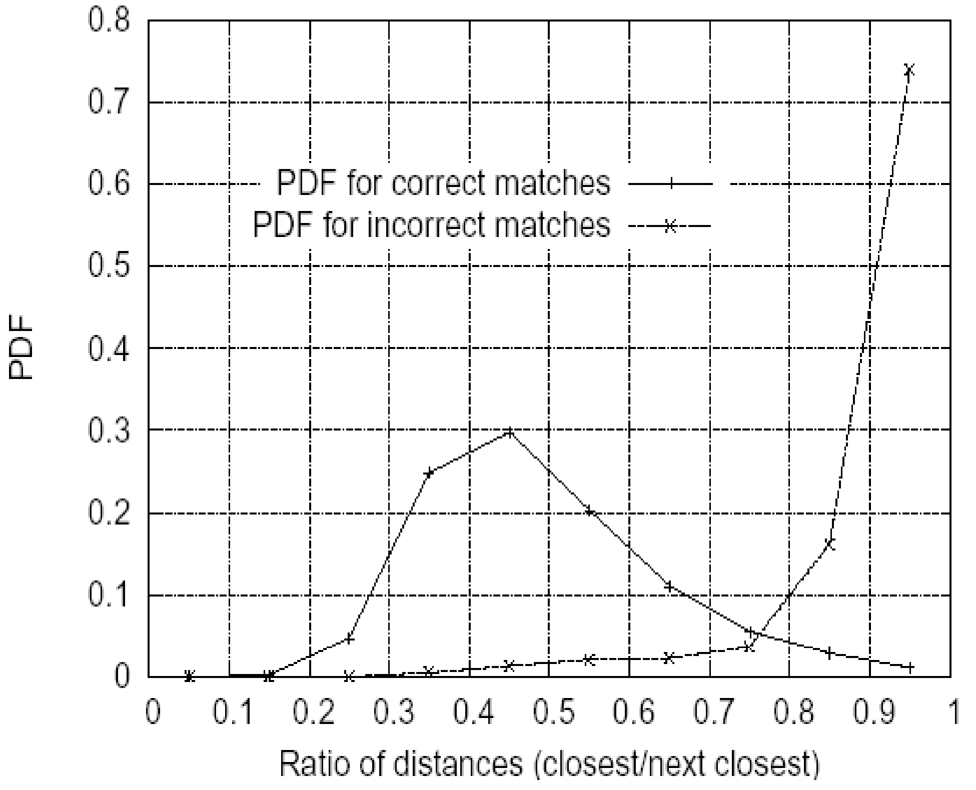
Keypoint Matching

- The *dissimilarity measure* is given by the Euclidean distance between descriptors.
- Independently match all keypoints in all octaves in one image with all keypoints in all octaves in other image.
- Take the closest neighbor.
- If the ratio of closest nearest neighbor with second closest nearest neighbor, is greater than 0.8, discard them.

SIFT- Keypoint Matching

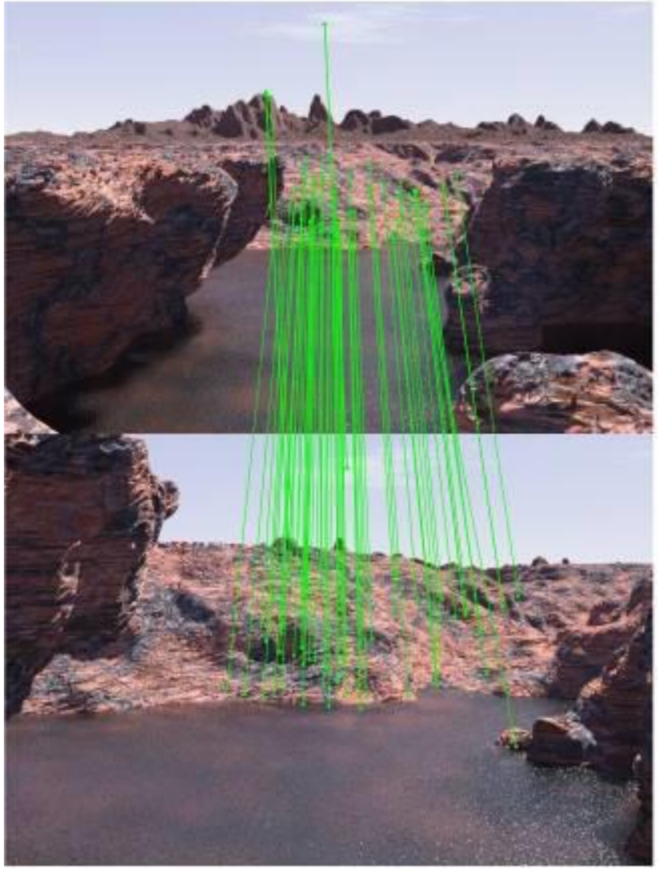
False × True Match distributions

- Threshold at 0.8
  - Eliminates 90% of false matches
  - Eliminates less than 5% of correct matches



SIFT- Keypoint Matching

Example of Matching



Outline

- The Correspondence Problem
- Corner Detection
  - Basics
  - Kanade Lucas Tomasi corner detector
  - Harris corner detector
  - Multi scale Harris corner detector
  - Matching
  - Alternative approaches
- SIFT
  - Overview
  - Keypoint detector
  - Descriptor
  - Matching
  - Alternative Approaches

SIFT Related References

- **SIFT**
  - LOWE, D.G. Distinctive image features from scale-invariant keypoints, International Journal of Computer Vision, v. 60, n. 2, pp. 91-110, 2004.
  - SUSAN, S., JAIN, A., SHARMA, A., VERMA, A., JAIN, S., Fuzzy match index for scale-invariant feature transform (SIFT) features with application to face recognition with wek supervision, IET Image Processing, v. 9, n. 11, pp. 951-958m 2915
- **SURF**
  - BAY, H.; ESS, A.; TUYTELAARS, T.; VAN GOOL, L. Speeded-up robust features (SURF). Journal of Computer Vision and Image Understanding, v.110, n.3, p. 346-359, 2008.
  - SANGINETO, E., Pose and Expression Independent Facil Landmark Localization Using Dense-SURF and the Hausdorff Distance, IEEE Trans. Pattern Analysis and Machine Intelligence, v. 35, n. 3, p. 624-637, 2013.
- **DAISY**
  - TOLA, E.; LEPETIT, V.; FUA, DAISY: An efficient dense descriptor applied to wide-baseline stereo. IEEE Transactions on Pattern Analysis and Machine Intelligence, v.32, n.5, p. 825-830, 2009.
- **HoG**
  - DALAL, N. and B. TRIGGS. Histograms of Oriented Gradients for Human Detection, IEEE Computer Society Conference on Computer Vision and Pattern Recognition, Vol. 1 (June 2005), pp. 886–893., v.32, n.5, p. 825-830, 2009.
  - DÉNIZ, O, BUENO, G., SALIDO, J. Dela TORRE, F., Face recognition using Histograms of Oriented Gradients, Pattern Recognition Letters, v. 32, n.12, (September 2011), pp. 1598-1603.

Assignment on SIFT

Write an extension to the program you developed for the Assignment on Homographies so that you use the

a) SIFT, and
b) SURF.

algorithms to automatically select pairs of corresponding points. Test your program with different sets of images and discuss the results. (*hint:* use the packages available here for SIFT, and SURF).

Next Topic

Cameras