Geometric Camera Calibration

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Objective

“ *This chapter addresses the problem of estimating the intrinsic and extrinsic parameters of a camera.”*

In particular, it introduces the Zhang’s calibration method .

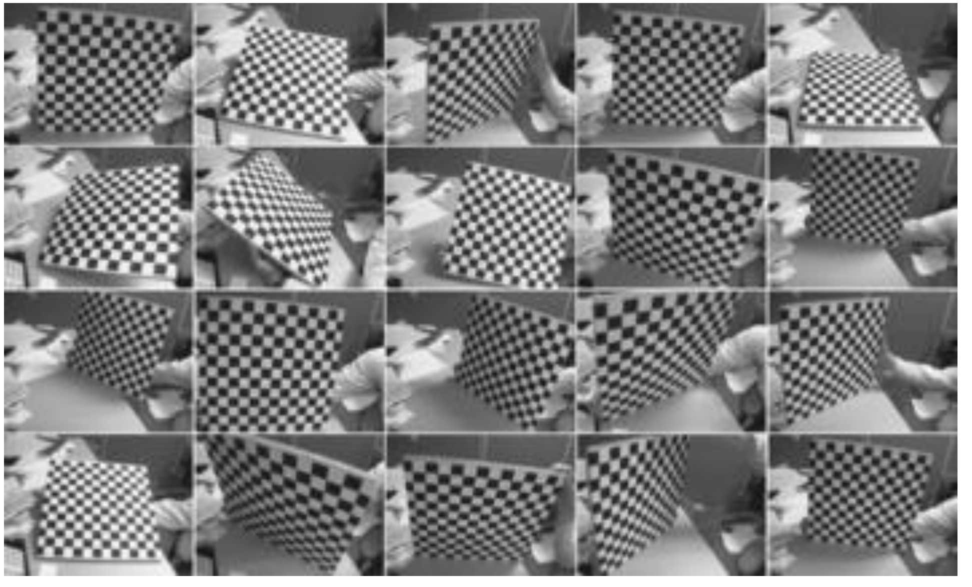
Geometric Camera Models

Contents

- Zhang's Camera Calibration method
- Accounting for distortions
- Assignments

Calibration with a planar rig

It uses multiple images of a checkerboard



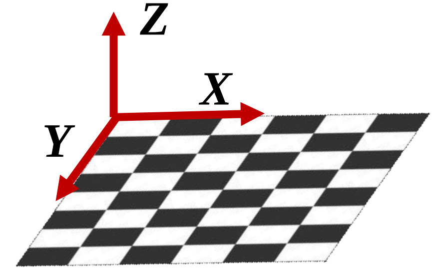
Enough point-correspondences (world $\leftrightarrow$ image).

Zhang's trick

From camera model

columns of $\mathcal{R}$

$$\begin{pmatrix}u\\ v\\ 1\end{pmatrix} = \frac{1}{z}\mathcal{M}\boldsymbol{P} = \frac{1}{z}\mathcal{K}\begin{pmatrix}\mathcal{R} & \boldsymbol{t}\end{pmatrix}\begin{pmatrix}X\\ Y\\ Z\\ 1\end{pmatrix} = \frac{1}{z}\mathcal{K}\begin{pmatrix}\boldsymbol{r}_1 & \boldsymbol{r}_2 & \boldsymbol{r}_3 & \boldsymbol{t}\end{pmatrix}\begin{pmatrix}X\\ Y\\ Z\\ 1\end{pmatrix}$$



Trick: without loss of generality, assume the plane is on $Z=0$.

$$\begin{pmatrix}u\\ v\\ 1\end{pmatrix} = \frac{1}{z}\mathcal{K}\begin{pmatrix}\boldsymbol{r}_1 & \boldsymbol{r}_2 & \boldsymbol{t}\end{pmatrix}\begin{pmatrix}X\\ Y\\ 1\end{pmatrix} \quad\Rightarrow\quad z\begin{pmatrix}u\\ v\\ 1\end{pmatrix} = \mathcal{H}\begin{pmatrix}X\\ Y\\ 1\end{pmatrix}$$

Homography!

From $\mathcal{H}$ to parameters

From each image we get a different $\mathcal{H}$

But we are interested on $\mathcal{K}$, $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{t}$, not on $\mathcal{H}$.

How do we extract them from these $\mathcal{H}$s?

Calculation of intrinsic parameters

From each image you get $\mathcal{H} = \lambda(\boldsymbol{h}_1\ \boldsymbol{h}_2\ \boldsymbol{h}_3) = \mathcal{K}(\boldsymbol{r}_1\ \boldsymbol{r}_2\ \boldsymbol{t})$

Then, $\boldsymbol{r}_1 = \lambda\mathcal{K}^{-1}\boldsymbol{h}_1$ and $\boldsymbol{r}_2 = \lambda\mathcal{K}^{-1}\boldsymbol{h}_2$

Recall that $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are columns of a rotation matrix and, therefore, orthonormal vectors. Thus,

$$\left.\begin{aligned} \boldsymbol{h}_1^T\mathcal{K}^{-T}\mathcal{K}^{-1}\boldsymbol{h}_2 &= 0 \\ \boldsymbol{h}_1^T\mathcal{K}^{-T}\mathcal{K}^{-1}\boldsymbol{h}_1 &= \boldsymbol{h}_2^T\mathcal{K}^{-T}\mathcal{K}^{-1}\boldsymbol{h}_2 \end{aligned}\right\}$$

each image provides 2 equations on the elements of $\mathcal{K}$ <u>only</u>!!!

How many images do you need to compute $\mathcal{K}$?

Calculation of intrinsic parameters

Procedure to compute $\mathcal{K}$

Let $\mathcal{B} = \mathcal{K}^{-T} \mathcal{K}^{-1}$

1. Collect two equations from each image ($\mathcal{H}$) and build an equation system.
2. Solve the system for $\mathcal{B}$.

   Recall that $\mathcal{B}$ will be computed up to a scale factor (λ).

3. Apply Cholesky factorization to $\mathcal{B}$ to obtain $\mathcal{K}^{-T}$ and $\mathcal{K}^{-1}$
4. Invert the results.

What about λ?

Calculation of extrinsic parameters

For each image / homography you computed:

- $\boldsymbol{r}_1 = \mathcal{K}^{-1}\boldsymbol{h}_1 \big/ \left|\mathcal{K}^{-1}\boldsymbol{h}_1\right|$
- $\boldsymbol{r}_2 = \mathcal{K}^{-1}\boldsymbol{h}_2 \big/ \left|\mathcal{K}^{-1}\boldsymbol{h}_2\right|$
- $\boldsymbol{r}_3 = \boldsymbol{r}_1 \times \boldsymbol{r}_2$

Theoretically $|\mathcal{K}^{-1}\boldsymbol{h}_1|$ and $|\mathcal{K}^{-1}\boldsymbol{h}_2|$ should be equal, but may differ due to inaccuracies in the estimation procedure. So, for $\boldsymbol{t}$

- $\boldsymbol{t} = 2\mathcal{K}^{-1}\boldsymbol{h}_3 \big/ \left(\left|\mathcal{K}^{-1}\boldsymbol{h}_1\right| + \left|\mathcal{K}^{-1}\boldsymbol{h}_2\right|\right)$

Maximum likelihood refinement

The solution computed by the previous linear method can be refined by minimizing the functional

non linear optimization

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\|\underbrace{\boldsymbol{p}_{ij}-\widehat{\boldsymbol{p}}\left(\mathcal{K},\boldsymbol{R}_i,\boldsymbol{t}_i,\boldsymbol{P}_j\right)}_{\text{projection error}}\Big\|^2$$

where

- $\widehat{\boldsymbol{p}}\left(\mathcal{K},\boldsymbol{R}_i,\boldsymbol{t}_i,\boldsymbol{P}_j\right)$ is the projection of point $\boldsymbol{P}_j$ on image i, according to $\mathcal{H}_i$ (Zhang's trick).
- $\boldsymbol{R}_i$ is the rotation matrix $\mathcal{R}_i$ parameterized by a vector of 3 parameters, according to Rodrigues formula.

Radial Distortion

The radial aberration accounts for the radial distortion, that depends on the distance *d* separating the optical axis from the point of interest ***p*** (without distortion).

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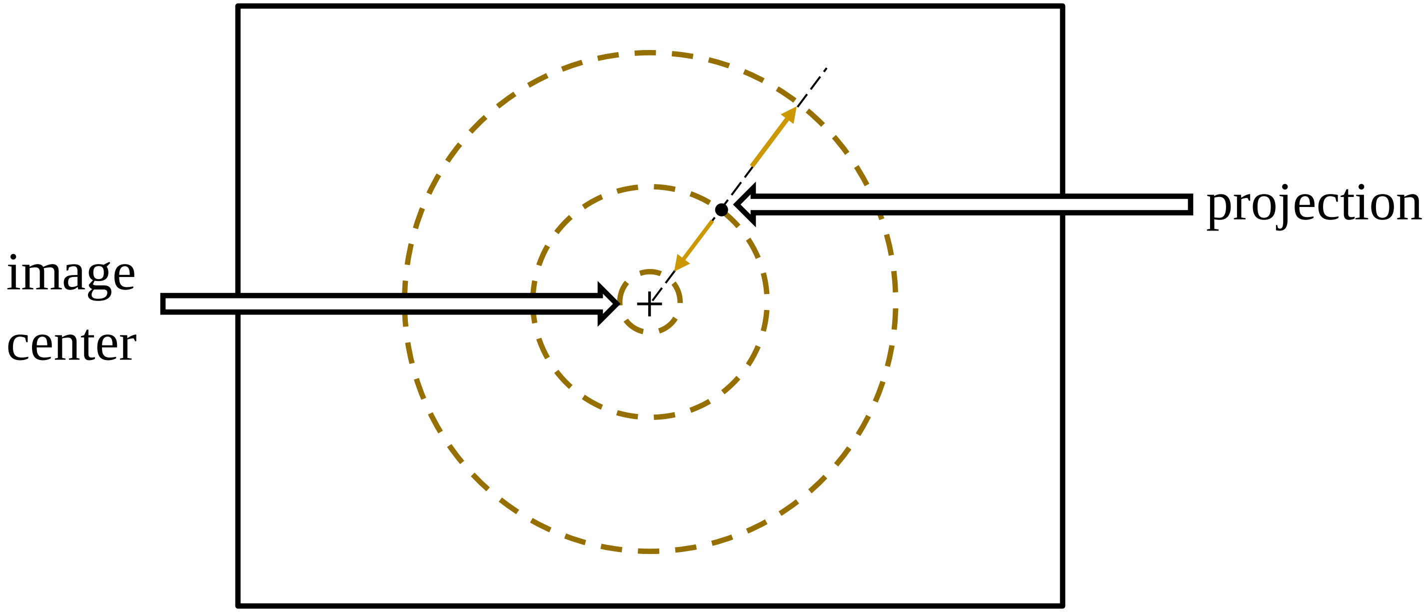
Radial Distortion

The radial aberration moves the projection along the line connecting it to the image center.



Radial Distortion
Basic Equations

Assuming that the image center is known we can take $u_0 = v_0 = 0$, and the projection process is modeled by

$$\boldsymbol{p} = \frac{1}{z}\begin{pmatrix} 1/\eta & 0 & 0 \\ 0 & 1/\eta & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{M} \boldsymbol{P}$$

where

- $\eta = 1 + \kappa_1 d^2 + \ldots + \kappa_q d^{2q}$ ($q \leq 3$ and κ_i are small),
- d is the distance of projection $\hat{\boldsymbol{p}}$ on the normalized image plane to the image center, i.e.,

$$d^2 = \hat{u}^2 + \hat{v}^2 \Rightarrow \boxed{d^2 = \frac{u^2}{\alpha^2} + \frac{v^2}{\beta^2} + 2\frac{uv}{\alpha\beta}\cos\theta}$$

Radial Distortion

Calibration Algorithm

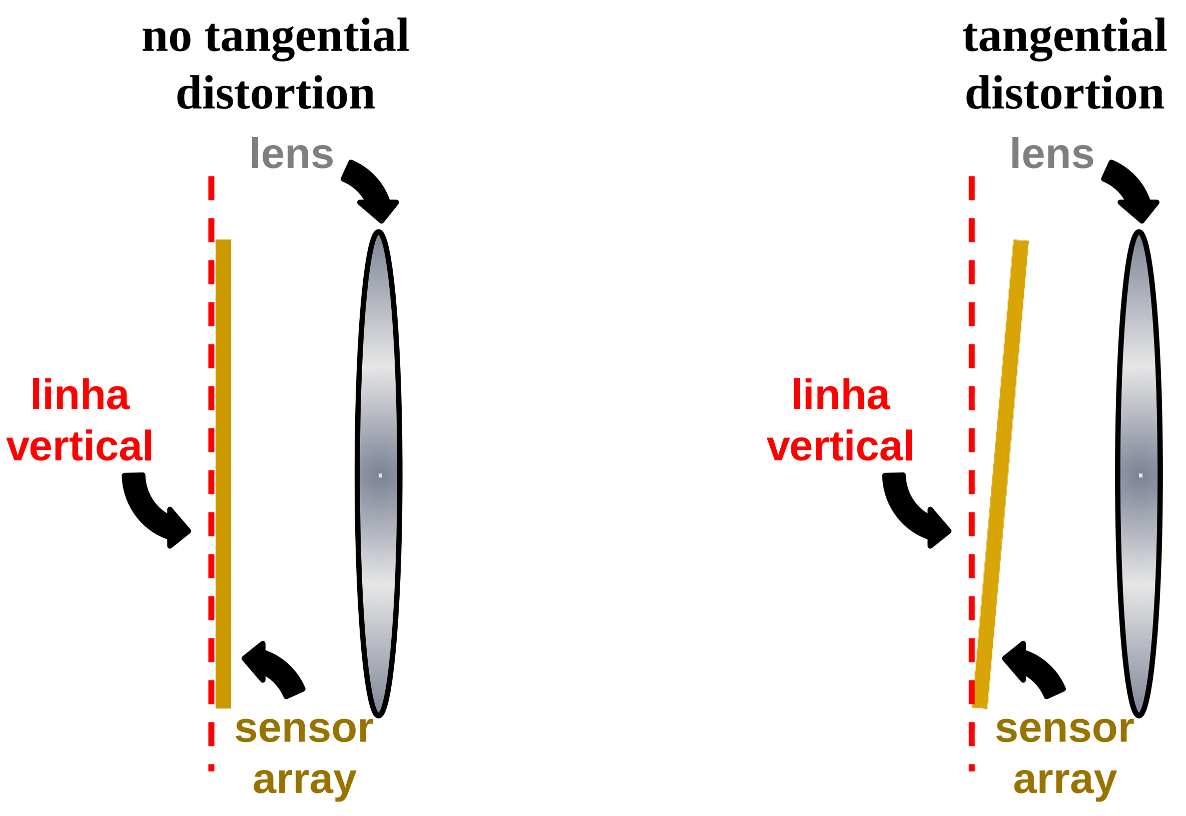
Algorithm

1. Estimate $\mathcal{M}_0$ with the non linear algorithm,
2. Extend the model by including the radial distortion
3. Using $\mathcal{M}_0$ and $\kappa_{i0}=0$, for $i=1,2,\ldots$ as initial solution, apply a non linear least square method (e.g. Lavenberg Marquardt) to estimate $\mathcal{M}$ and κ_i, for $i=1,2,\ldots,$
4. Compute intrinsic and extrinsic parameter from $\mathcal{M}$.

Once the calibration is done, how would you correct the radial distortion?

Tangential Distortion

Occurs when the lens is not paralel to the sensor array.



See references for further details.

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Assignment 1

This assignment consists of the following steps

a) Learn how to use the MATLAB calibration toolbox from OpenCV,
b) Create a planar calibration rig (chessboard),
c) Apply the calibration tool from OpenCV - use the Zhang's algorithm.
d) Show the accuracy of your result.

Be ready to demonstrate to your colleagues the whole procedure in the next class.

Assignment 2

This assignment consists of

a) Read the article "What is Camera Calibration" from Mathworks.
b) Create a planar calibration rig (chessboard),
c) Follow all steps of the MATLAB "Single Camera Calibration App".
d) Show the accuracy of your result.

Be ready to demonstrate the whole procedure in the next class.

Assignment 3

This assignment consists of the following steps

a) Download the latest release of OpenCV
b) Learn how to use a C++ calibration tool (preferably the implementation of Zhang's algorithm
c) Follow all steps to calibrate a camera using that tool
d) Show the accuracy of your result.

Be ready to demonstrate to your colleagues the whole procedure in the next class.

References

- Zhang, Z. "A Flexible New Technique for Camera Calibration." *IEEE Transactions on Pattern Analysis and Machine Intelligence*. Vol. 22, No. 11, 2000, pp. 1330–1334.
- Bouguet, J. Y. "Camera Calibration Toolbox for Matlab." Computational Vision at the California Institute of Technology. Camera Calibration Toolbox for MATLAB.
- MathWorks, “What is Camera Calibration”, available at https://www.mathworks.com/help/vision/ug/camera-calibration.html?requestedDomain=www.mathworks.com, (last access, May 5th, 2017).

Next Topic

Geometry

of

Multiple Views