Introduction to Artificial Intelligence

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RECEIVER OPERATING CHARACTERISTICS

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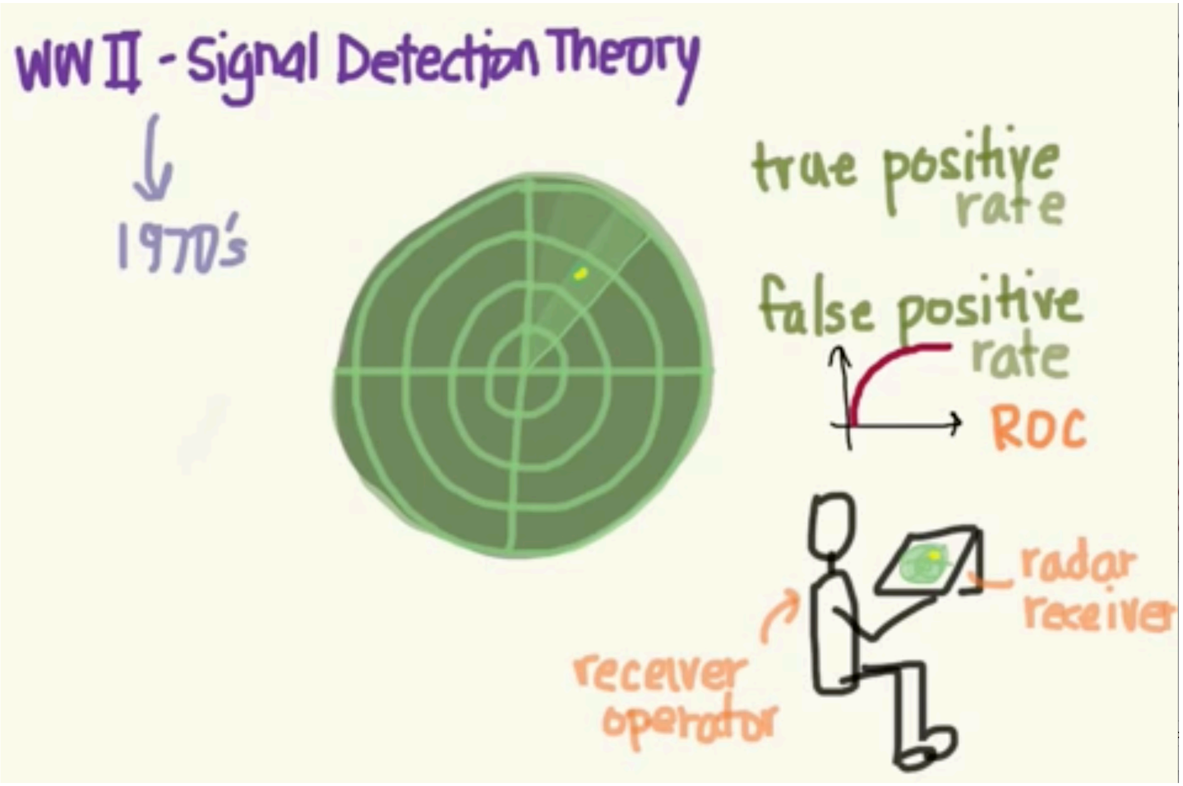
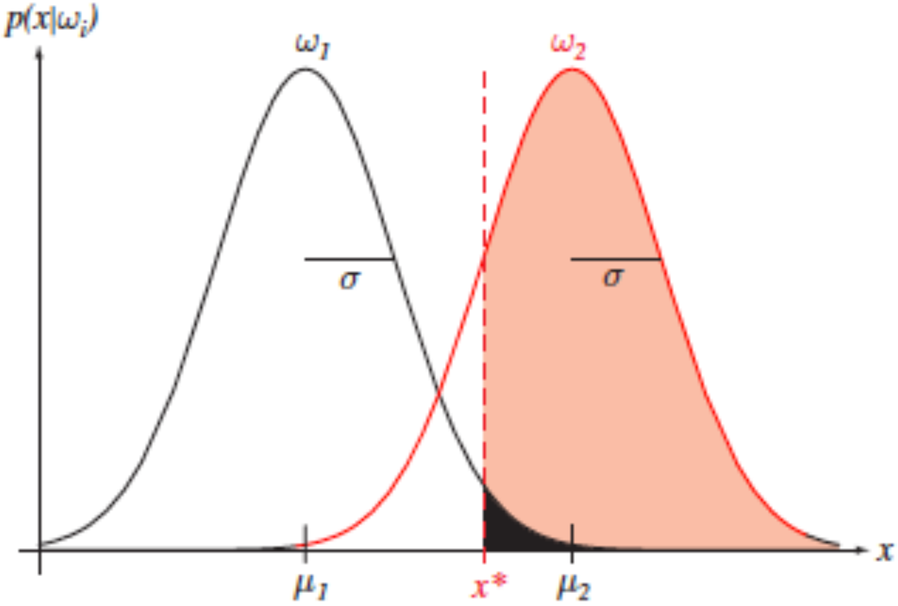
Origin of the ROC curve

- Receiver Operating Characteristics



Signal Detection Theory

- A fundamental way of analysing a classifier:
  - Suppose we are interested in detecting a single pulse.
  - We can read an internal signal x.

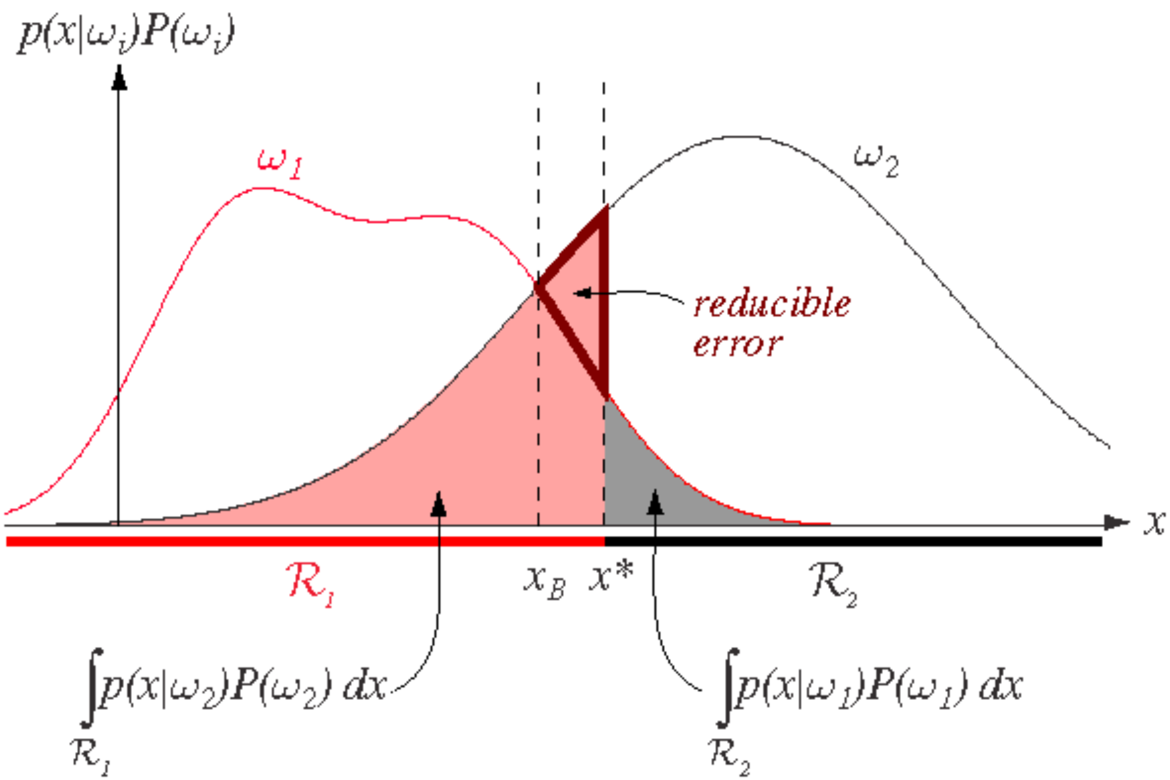


- The signal is distributed
  - around mean μ_2 when an external signal **is** present
  - and around mean μ_1 when **no** external signal is present.
- Assume the distributions have the same variances:

$$p(x|\omega_i) \sim N(\mu_i, \sigma^2)$$

Error Probabilities and Integrals

- Components of the probability of error for equal priors and non-optimal decision point x^*. The optimal point x_B minimizes the total shaded area and gives the Bayes error rate.



Signal Detection Theory

- The detector uses x^* to decide if the external signal is present.
- **Discriminability** characterizes how difficult it will be to decide if the external signal is present without knowing x^*.

$$d' = \frac{|\mu_2 - \mu_1|}{\sigma}$$

- Even if we do not know μ_1, μ_2 , σ or x^*, we can find d' by using a **receiver operating characteristic** or ROC curve, as long as we know the state of nature for some experiments.

Receiver Operating Characteristics
Definitions

- A **Hit** is the probability that the internal signal is above x^* given that the external signal is present. $$P(x > x^* | x \in \omega_2)$$
- A **Correct Rejection** is the probability that the internal signal is below x^* given that the external signal is not present. $$P(x < x^* | x \in \omega_1)$$
- A **False Alarm (False Positive)** is the probability that the internal signal is above x^* despite there being no external signal present. $$P(x > x^* | x \in \omega_1)$$
- A **Miss-Detection (False Negative)** is the probability that the internal signal is below x^* given that the external signal is present. $$P(x < x^* | x \in \omega_2)$$

Receiver Operating Characteristics

Confusion Matrix

- Consider the two-category case and define
  - ω_1: target is present,
  - ω_2: target is not present.
- Confusion Matrix

		Predicted condition (Test Outcome)	
		ω_1 (Predicted Condition positive)	ω_2 (Predicted Condition negative)
True condition (Gold Standard)	ω_1 (Condition positive)	✓ Hit (TP)	✧ False Negative (FN)
	ω_2 (Condition negative)	• False Positive (FP)	✓ Correct Rejection (TN)

Receiver Operating Characteristics

		Predicted condition (Test Outcome)	
		ω_1 (Predicted Condition positive)	ω_2 (Predicted Condition negative)
True condition (Gold Standard)	ω_1 (Condition positive)	✓ Hit (TP)	✧ False Negative (FN)
	ω_2 (Condition negative)	• False Positive (FP)	✓ Correct Rejection (TN)

- Sensitivity (True Positive Rate = Recall): TPR= TP/P=TP/(TP+FN)
  - MED: test sensitivity is the ability of a test to correctly identify those **with the disease**
- Specificity (True Negative Rate): SPC = TN/N=TN/(TN+FP)
  - MED: ability of the test to correctly identify those **without the disease**
- Fall-out (False Positive Rate): FPR=FP/N=FP/(FP+TN)=1-SPC
- Precision (Positive Predictive Value): PPV=TP/(TP+FP)
- Accuracy: ACC= (TP+TN)/(TP+FP+FN+TN)
- F1 score: F1=2TP/(2TP+FP+FN)
  - Harmonic mean of precision and sensitivity

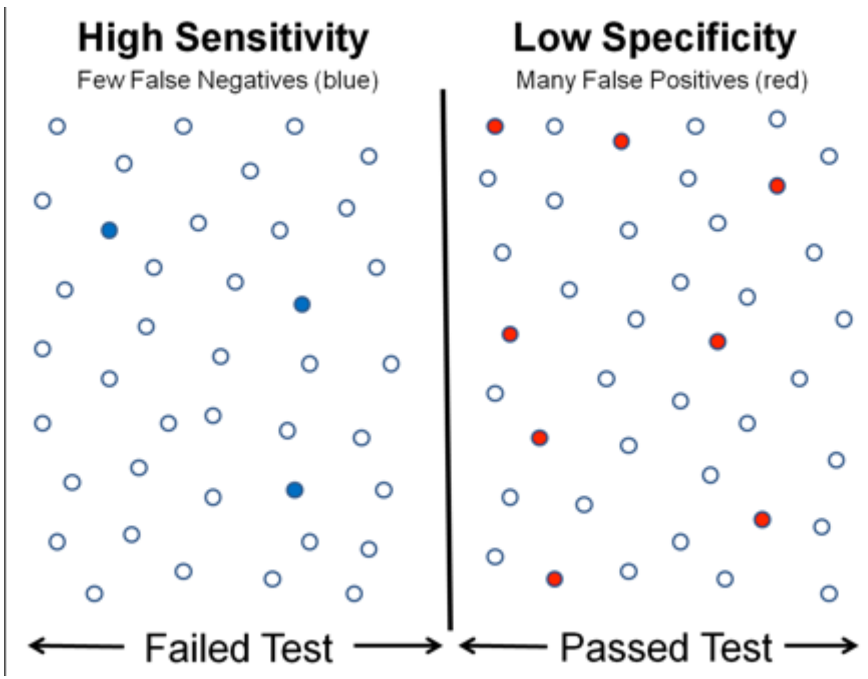
the diagnostic power of any test is determined by both its sensitivity and its specificity !!!

Receiver Operating Characteristics

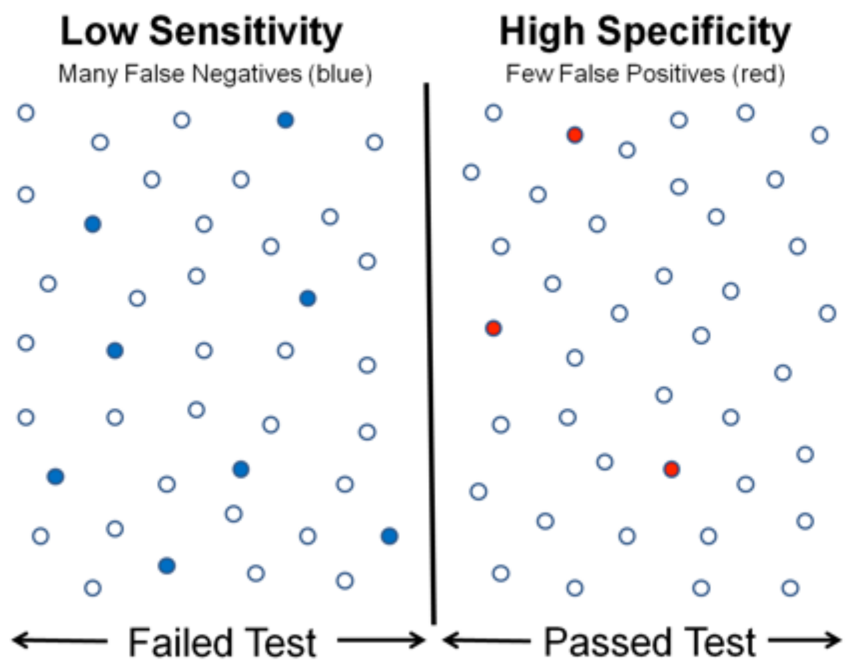
		Predicted condition (Test Outcome)		
		ω_1 (Predicted Condition positive)	ω_2 (Predicted Condition positive)	
True condition (Gold Standard)	ω_1 (Condition positive)	✓ **Hit (TP)**	✧ False Negative (FN = Type II error)	✧ **Sensitivity** (True Positive Rate): TPR= TP/P=TP/(TP+FN)
	ω_2 (Condition negative)	• False Positive (FP-Type I error)	✓ Correct Rejection (TN)	✓ **Specificity** (True Negative Rate): SPC = TN/N=TN/(TN+FP)

Graphical illustration

- High sensitivity and low specificity

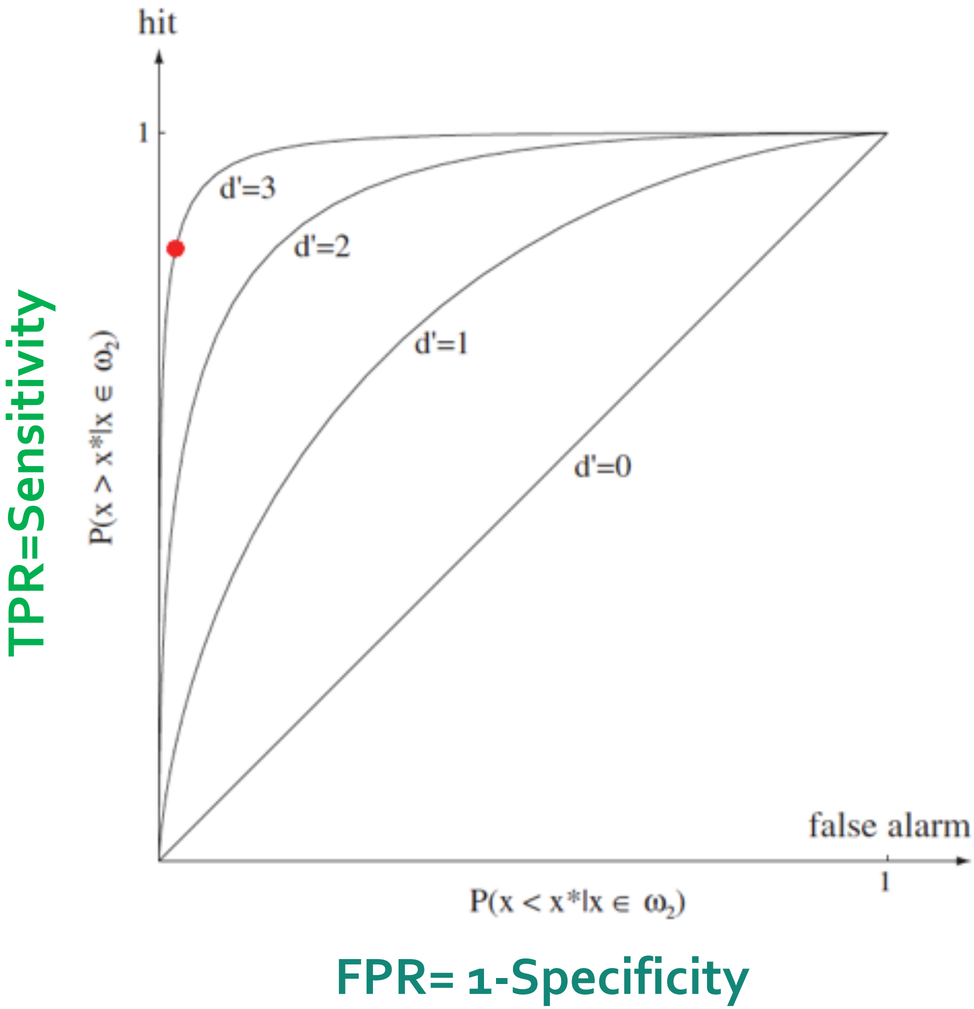


- Low sensitivity and high specificity

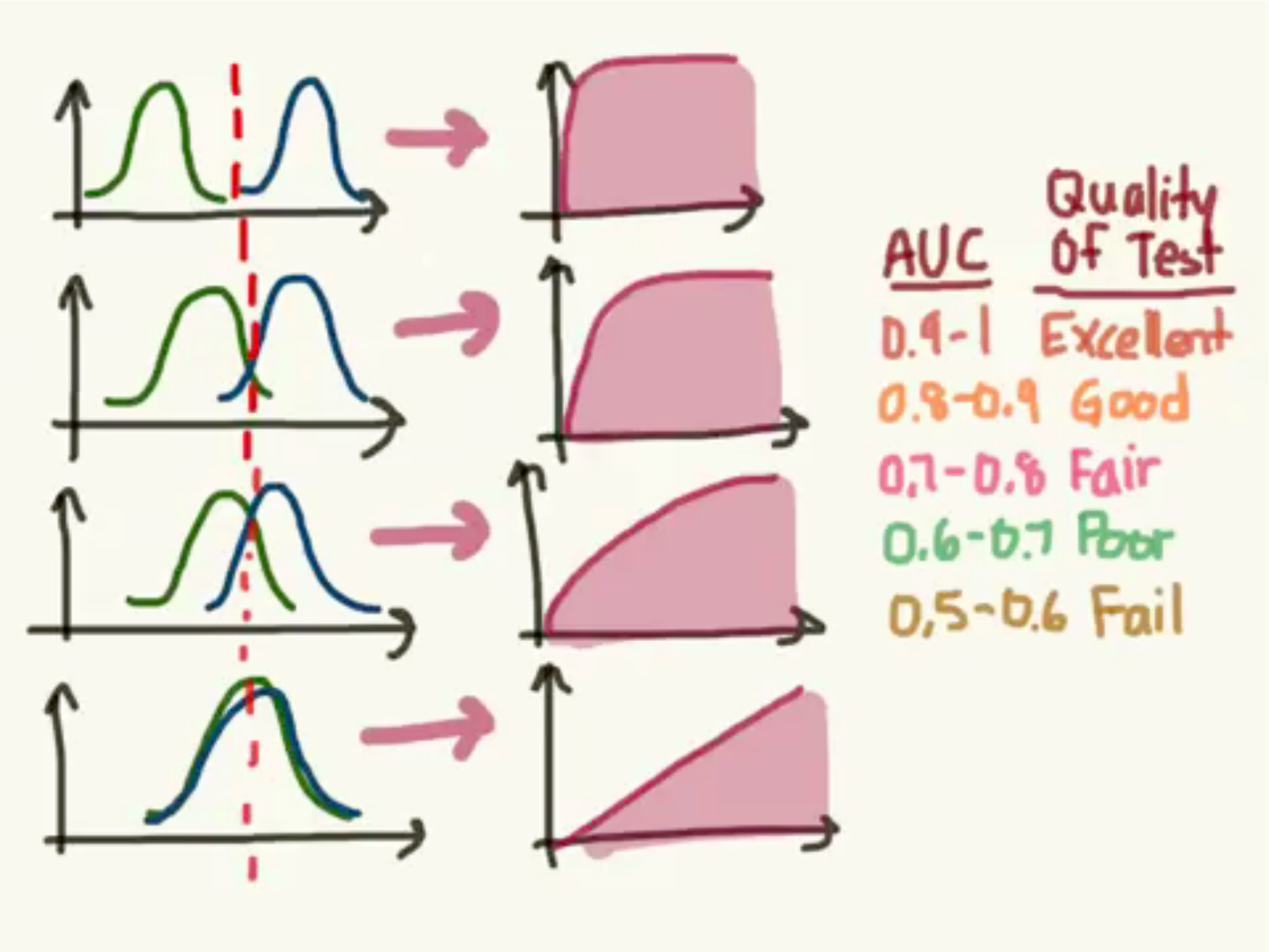


Receiver Operating Characteristics

- We can experimentally determine the rates, in particular the Hit-Rate and the False-Alarm-Rate.
- Basic idea is to assume our densities are fixed (reasonable) but vary our threshold x^*, which will thus change the rates.
- The receiver operating characteristic plots the hit rate against the false alarm rate.
- What shape curve do we want?



A gentle introduction to ROC



Role of the ROC

- The trade-off between Specificity and Sensitivity is explored in ROC analysis as a trade off between TPR and FPR (Recall and respectively Fall-out)
- Giving them equal weight optimizes

  Informedness = Specificity+Sensitivity-1 = TPR-FPR,

  the magnitude of which gives the probability of an informed decision between the two classes:

  - > 0 represents and appropriate use of information,
  - 0 represents a chance-level performance,
  - < 0 represents a perverse use of information.