Digital Audio Processing
6. Audio Coding

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On the Origins of Audio Coding

The origins of audio coding

The Vocoder, abbreviation for “VOice enCODER”, is first system proposed for audio coding, originally applied to speech.

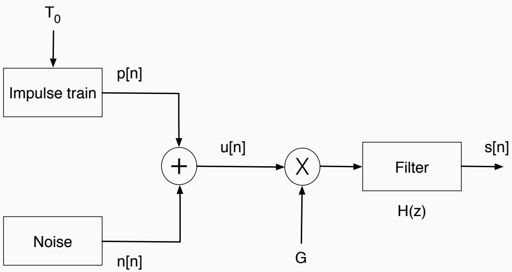
- Invented and patented by Homer Dudley at Bell Labs in 1935
- Objective: reducing the bandwidth necessary for transmitting speech through a communication network
- At analysis: coding a speech signal by measuring the frequency response a_k of a filterbank composed of K filters representing isolated frequency bands
- At synthesis: the speech signal is synthesized by passing an excitation signal through the previous filterbank

Source/Filter Model

The source/filter model is a simplified signal representation of sound production by a physical system, which is commonly used in audio coding (speech, music).

The source/filter model assumes that the audio signal $s[n]$ results from the excitation of a filter $H(z)$ by a source signal $u[n]$:

$$s[n] = (Gu * h)[n] \tag{1}$$



Schematic illustration of the source/filter model.

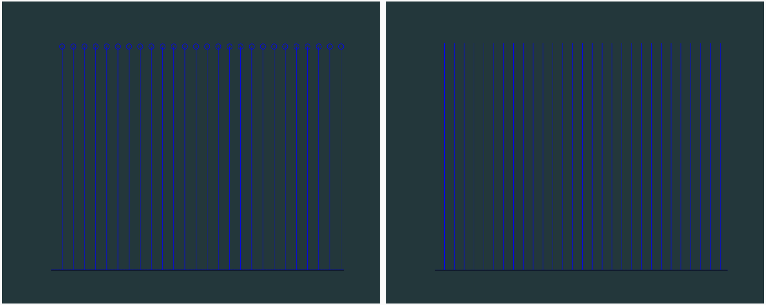
The source signal $u[n]$ is a mix of a periodic signal $p[n]$ and a noise signal $n[n]$:

$$u[n] = G(p[n] + n[n]) \tag{2}$$

- The periodic signal $p[n]$ is an impulse train with fundamental period T_0 (respectively, fundamental frequency F_0):

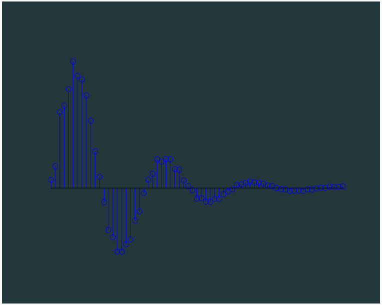
$$p[n] = \text{Ш}_{T_0}[n] \tag{3}$$

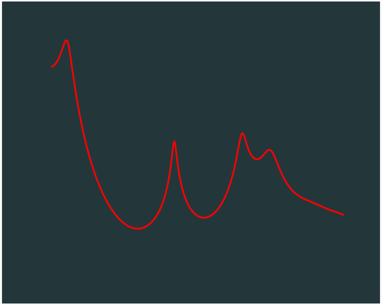
- The noise signal $n[n]$ is a random signal.



Impulse train $p[n]$ at T_0 (left), and corresponding TFD $P[k]$ at F_0 (right)

The filter is defined by its transfer function $H(z)$:





Filter: impulse response $h[n]$ (left), and frequency response $H(z)$(right).

The convolution in the time domain is a multiplication in the frequency domain:

$$S(z) = GU(z) \times H(z) \tag{4}$$

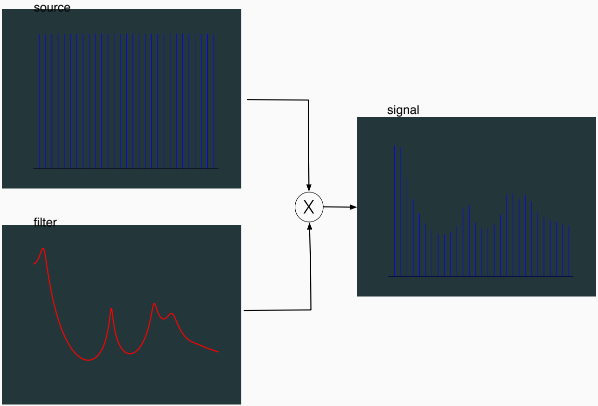


Illustration of the source/filter model in the frequency domain.

Audio Coding

Introduction

Linear Predictive Coding (LPC) is historically the most widespread method for the estimation of the source/filter parameters in audio coding, recognition, and synthesis.

The basic assumption of LPC is that the current audio sample can be approximated as the linear combination of past P audio samples:

$$\hat{s}[n] = \sum_{p=1}^{P} \alpha_p s[n-p] \tag{5}$$

The corresponding TZ is:

$$\hat{S}[z] = \sum_{p=1}^{P} \alpha_p S[z] z^{-p} \tag{6}$$

J. Makhoul. Linear Prediction : A Tutorial Review. In Proceedings of the IEEE, volume 63, pages 561–580, Cambridge, Massachusetts, 1975.

LPC and the source/filter production model of speech

Firstly, we consider the source/filter model

Hypothesis: the transfer function of the vocal-tract $H(z)$ is a all-pole filter

$$H(z) = \frac{S(z)}{GU(z)} = \frac{1}{1 - \sum_{p=1}^{P} a_p z^{-p}} \tag{7}$$

This can be written in the time domain as:

$$s[n] = \sum_{p=1}^{P} a_p s[n-p] + Gu[n] \tag{8}$$

Prediction Model

Secondly, we consider the LPC model.

The current sample $\hat{s}[n]$ as predicted by LPC is:

$$\hat{s}[n] = \sum_{p=1}^{P} \alpha_p s[n-p] \tag{9}$$

The transfer function of the system defined by input $s[n]$ and by output $\hat{s}[n]$ is:

$$P[z] = \frac{\hat{S}(z)}{S(z)} = \sum_{p=1}^{P} \alpha_p z^{-p} \tag{10}$$

Prediction Error

The prediction error $e[n]$ is defined as the difference between the current sample $s[n]$ and the current sample $\hat{s}[n]$ as predicted by LPC:

$$e[n] = s[n] - \hat{s}[n] \tag{11}$$

By definition:

$$e[n] = s[n] - \sum_{p=1}^{P} \alpha_p s[n-p] \tag{12}$$

The error signal $e[n]$ is also know as the residual signal.

The transfer function of the system defined by input $s[n]$ and by output $e[n]$ is:

$$\boxed{\mathrm{A}[\mathrm{z}] = \frac{E(z)}{S(z)} = 1 - P(z) = 1 - \sum_{p=1}^{P} \alpha_p z^{-p}} \tag{13}$$

LPC and the source/filter production model of speech

Finally, by equating the source/filter model and the LPC model:

$$s[n] = \sum_{p=1}^{P} a_p s[n-p] + Gu[n] = \sum_{p=1}^{P} \alpha_p s[n-p] + e[n] \tag{14}$$

By identification:

$$Gu[n] = e[n] \tag{15}$$

$$\alpha[n] = a[n] \tag{16}$$

Then:

$$\boxed{\mathrm{H(z)} = \frac{1}{A(z)}} \tag{17}$$

Which means that $A(z)$ is the inverse filter of $H(z)$.

Estimation Issues

For stationary signals:

- Determining the source/filter parameters from the Discrete Fourier Transform (DFT).
- Estimating α_p from the Discrete Fourier Transform (DFT).
- The estimation is performed so as to minimize the mean-squared prediction error.
- Idealistically, the LPC is the actual filter.

For non-stationary signals:

- Determining the time-varying filter from the Short-Term Fourier Transform (STFT).

Estimation

The mean squared error is:

$$
\begin{aligned}
E &= \sum_{n=1}^{N} e^2[n] && (18)\\
&= \sum_{n=1}^{N} (s[n] - \hat{s}_n[n])^2 && (19)\\
&= \sum_{n=1}^{N} (s[n] - \sum_{p=1}^{P} \alpha_p s[n-p])^2 && (20)
\end{aligned}
$$

The solution $\{\hat{\alpha}_p\}_{p=1}^{P}$ is determined so as to minimize the short-term mean squared error:

$$
\hat{\alpha}_p = \arg\min_{\alpha_p} E \qquad (21)
$$

Estimation

The solution $\{\hat{\alpha}_p\}_{p=1}^{P}$ is determined by using numerical optimization.

Set partial derivatives to 0 with respect to α_i:

$$\frac{\partial E}{\partial \alpha_i} = 0, \qquad \forall i \in [1, P] \tag{22}$$

This leads to a set of equations:

$$\sum_{n=1}^{N}(s[n-i](s[n] - \sum_{p=1}^{P}\hat{\alpha}_p s[n-p])) = 0, \qquad \forall i \in [1, P] \tag{23}$$

Estimation

After expansion:

$$\sum_{p=1}^{P} \alpha_p \left(\sum_{n=1}^{N} s[n-i]s[n-p] \right) = \sum_{n=1}^{N} s[n]s[n-p] \qquad \forall i \in [1, P] \tag{24}$$

This is a set of P equations with P unknown parameters $\{\alpha_p\}_{p=1}^{P}$

Let consider $R(p)$ the auto-correlation function defined as:

$$R(p) = \sum_{n=1}^{N} s[n]s[n+p] \tag{25}$$

Finally, this can be written as a set of Yule-Walker equations:

$$\sum_{p=1}^{P} \alpha_p R(|i-p|) = R(i) \qquad \forall i \in [1, P] \tag{26}$$

Estimation

This can be expressed in a matrix form:

$$\mathbf{R}\alpha = \mathbf{r} \tag{27}$$

i.e:

$$\begin{bmatrix} R(0) & R(1) & \cdots & R(P-1) \\ R(1) & R(0) & \cdots & R(P-2) \\ \vdots & \vdots & \vdots & \vdots \\ R(P-1) & R(P-2) & \cdots & R(0) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_P \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \\ \vdots \\ R(P) \end{bmatrix} \tag{28}$$

This is a Toeplitz matrix (symmetric with all diagonal elements equal)

The solution is obtained by matrix inversion:

$$\alpha = \mathbf{R}^{-1}\mathbf{r} \tag{29}$$

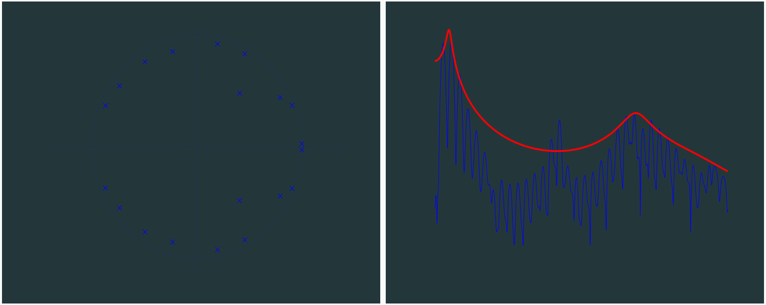
The matrix inversion can be efficiently solved with the Levinson-Durbin algorithm.

Interpretation in the Frequency Domain

LPC approximates the filter response $H(z)$ with an all-pole filter:

$$H(z) = \frac{1}{1 - \sum_{p=1}^{P} \alpha_p z^{-p}} \tag{30}$$

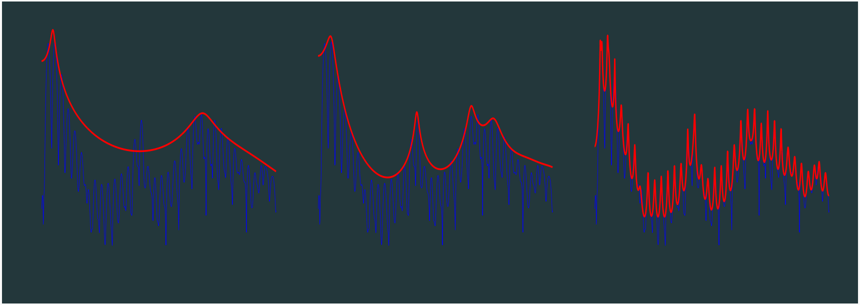
Each pair of poles represents a resonance in the magnitude response of the filter.



Pole positions and transfer function of a LPC filter ($P = 20$)

LPC order

The choice of the LPC order P is crucial for source/filter deconvolution.



From left to right: under-estimated (P = 20), correctly-estimated (P = 50), over-estimated (P = 500) transfer function.

Main issue: no optimal solution for the choice of the order.

Introduction

The source/filter model assumes that the audio signal is the convolution of a source by a filter:

$$s[n] = (e * h)[n] \tag{31}$$

The frequency response of this system is then a point-wise multiplication:

$$S[z] = E(z).H(z) \tag{32}$$

One solution for the source/filter deconvolution is to determine some homomorphic transform $\blacksquare$ for which the convolution is replaced by a sum:

$$s[n] = (e * h)[n] \rightarrow \blacksquare \rightarrow \hat{s}[k] = \hat{e}[k] + \hat{h}[k] \tag{33}$$

If so,

$$\hat{e}[k] \rightarrow \blacksquare^{-1} \rightarrow e[n] \tag{34}$$

$$\hat{h}[k] \rightarrow \blacksquare^{-1} \rightarrow h[n] \tag{35}$$

Complex Cepstrum

The TZ is an homorphic transform which replaces a convolution by a product:

$$s[n] = (e * h)[n] \rightarrow S(z) = E(z).H(z) \tag{36}$$

Then, a simple log transform replaces a product by an sum:

$$S(z) = E(z).H(z) \rightarrow \log \rightarrow \log(S(z)) = \log(E(z)) + \log(H(z)) \tag{37}$$

The inverse TZ preserves the sum, so that:

$$\log(S(z)) = \log(E(z)) + \log(H(z)) \rightarrow TZ^{-1} \rightarrow \hat{s}[k] = \hat{e}[k] + \hat{h}[k] \tag{38}$$

Finally, one homomorphic transform can be defined as:

$$\blacksquare = TZ^{-1}(.) \circ log(.) \circ TZ(.) \tag{39}$$

$$\blacksquare^{-1} = TZ^{-1}(.) \circ exp(.) \circ TZ(.) \tag{40}$$

The DFT version of this transform is: the complex cepstrum.

Real Cepstrum

The main issue of the complex cepstrum is the phase: after the log transform, the phase is defined $\mod (2\pi)$, so that the inverse transform is difficult.

The solution to this issue is to ignore the phase during transforms.

This is the real cepstrum:

$$\blacksquare = DFT^{-1}(.) \circ log(|.|) \circ DFT(.) \tag{41}$$

$$\blacksquare^{-1} = DFT^{-1}(.) \circ exp(|.|) \circ DFT(.) \tag{42}$$

The cepstrum is defined as the DFT of a DFT (!)

Interpretation

For a signal, the DFT is the representation of the signal as a sum of frequency contributions.

Thus, the cepstrum is the representation of the spectrum as a sum of frequency contributions:

- low cepstral bins represent slow variations (low frequencies) in the spectrum
- high cepstral bins represent fast variations (high frequencies) in the spectrum

The cepstrum terminology is just constructed from back slang:

$$\begin{aligned} \textit{spec} - \textit{trum} &= \textit{ceps} - \textit{trum} \\ \textit{freque} - \textit{ncy} &= \textit{quefre} - \textit{ncy} \\ \textit{filt} - \textit{ering} &= \textit{lift} - \textit{ering} \end{aligned}$$

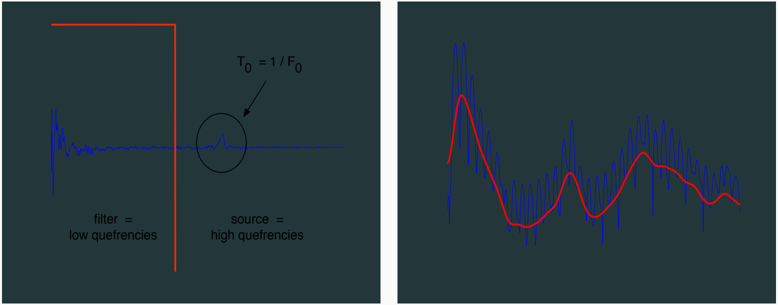
B. P. Bogert, M. J. R. Healy, and J. W. Tukey.
The Quefrency Alanysis of Time Series for Echoes: Cepstrum, Pseudo Autocovariance, Cross-Cepstrum and Saphe Cracking. Proceedings of the Symposium on Time Series Analysis, Chapter 15, 209-243. New York: Wiley, 1963.

Interpretation

Beyond cepstrum, one assumes that source and filter can be linearly separated in the cepstrum domain:

- source: the source has fast frequency variations, with an upper bound defined by fundamental frequency F_0 of the the signal (if periodic).
- filter: the filter has slow frequency variations.

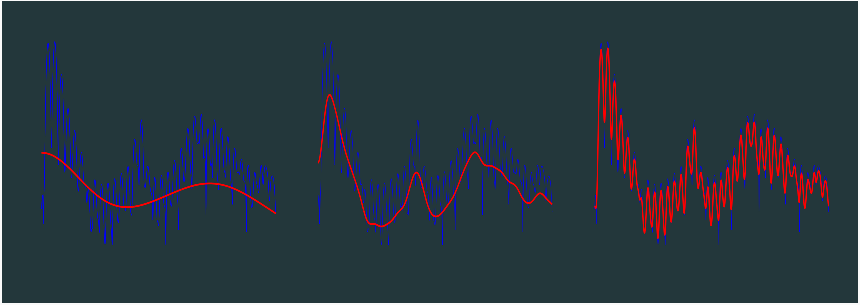
The source/filter deconvolution is obtained through low-quefrency liftering.



Cepstrum Order

Advantage: the optimal solution to the cepstrum order can be directly determined by:

$$P_{opt} = \frac{F_s}{2F_0} \tag{43}$$



From left to right: under-estimated ($P = 25$), correctly-estimated (P_{opt}), over-estimated ($P = 1024$) filter response.

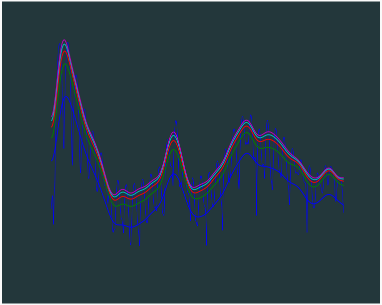
This requires the a priori estimation of the fundamental frequency (F_0)

Limitations

LPC over-estimates the frequency response of the filter.

Limitation: because of averaging, the cepstrum under-estimates the frequency response of the filter.

Solution: the True-Enveloppe method provides iterative estimation of the filter response.

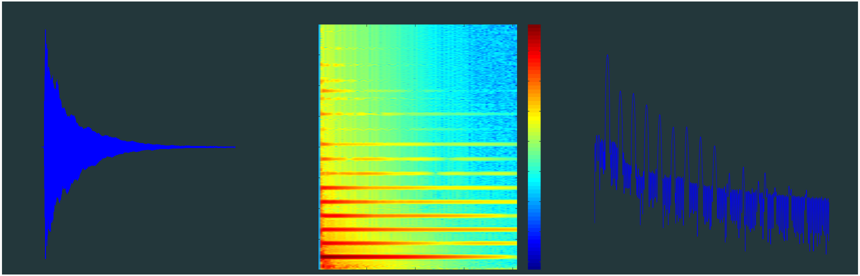


S. Imai et Y. Abe. Spectral Envelope Extraction by Improved Cepstral Method.

Example: Sound Analysis by Synthesis

For a piano, the source/filter model can be interpreted as:

- source
  a noise produced by the strike of the touch on the keyboard, and the strike of the hammer on the chord.
  a periodic impulse train at $F_0 = 440Hz$ produced by dump oscillations of a fixed-length chord after hammer strike on the chord.
- filter
  a filter which is due to the resonance table of the piano



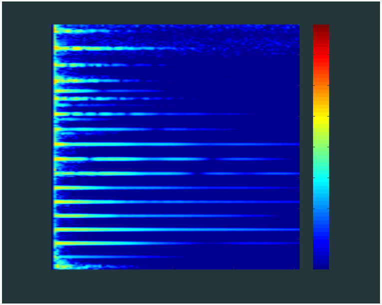
Example: Sound Analysis by Synthesis

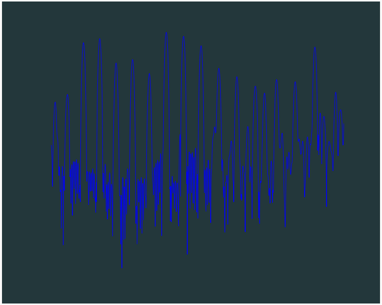
Once the filter $H(z)$ is determined, inverse filtering can be processed to obtain the residual signal $e[n]$.

When the filter $H(z)$ is correctly estimated, the residual equals the source signal:

$$e[n] = u[n] = p[n] + n[n] \tag{44}$$

The residual contains: “white” harmonic comb at F_0, and noise.





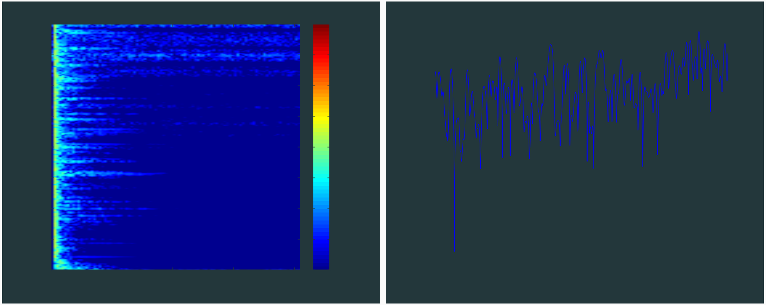
Example: Sound Analysis by Synthesis

When the filter $H(z)$ is over-estimated, then the filter contains the periodic part $p[n]$ of the source signal $u[n]$.

Then, the residual contains the noise part of the source signal:

$$e[n] = n[n] \tag{45}$$

One can clearly hear the attack of the piano (strike of the touch on the keyboard)!



Residual after LPC inverse filtering (P = 500): spectrogram (left), and DFT of a frame (right).

Audio Compression

Why compression?

Compression is important for stocking, transferring, and streaming data

Example for digital audio: Fs = 44,100 Hz, 16 bits, 2 channels

- Data rate: sampling rate * quantification bits * channels
- Without compression: 1.4 Mb / second, about 5 Gb / hour
- With compression: transparent at 128 or 256 Kb / seconds!!!

Existing audio formats

- Without compression: wav, aiff, flac, etc...
- With compression: mp3, aac, etc...

Lossless vs. lossy audio compression

Lossless compression

- Principle: Do not encode what is redundant.
- Example : Entropic enconding, Huffmann algorithm.
- Details: Minimise the entropy of the encoded data, using observation probability distribution of symbols. Less bits to frequent symbols, more bits to rare symbols. Variable-length encoding (vs. fixed-length encoding)

The compression is lossless in the sense that the compressed data is strictly equivalent to the raw data

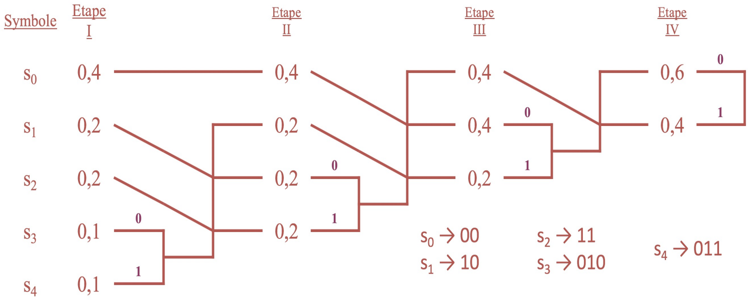
Lossy compression

- Principle: Do not encode what cannot be perceived.
- Example : MPEG encoding.
- Details: Requires a psycho-acoustics model of human audition.

The compression is lossy in the sense that there is a loss of information, but this loss is assumed to be imperceptible by a human

Lossless coding

Le code de Huffman :



$$\bar{n} = 0{,}4\times 2 + 0{,}2\times 2 + 0{,}2\times 2 + 0{,}1\times 3 + 0{,}1\times 3 = 2{,}2$$

$$\boxed{H(S) \le \bar{n} < H(S)+1}$$

$$H(S) = -0{,}4.\log_2(0{,}4) - 2\times 0{,}2.\log_2(0{,}2) - 2\times 0{,}1.\log_2(0{,}1) = 2{,}12\,\text{bits}$$

The MPEG audio format

Moving Pictures Experts Group (MPEG, created in 1988): a group of experts in information technology who collaborate on the definition of international **ISO** standards for the coding of audio and video contents.

- First psycho-acoustic masking code, by Murray Hill (AT&T, Bell Labs., 1978)

- MPEG-1 audio: lossy compression of audio
- MPEG-1 audio (1992), MPEG-2 audio (1994), MPEG-4, MPEG-7
- MPEG-1 audio, layer I, II, III (aka MP3, 1993)
- AAC (Advanced Audio Coding): included in MPEG-4 (aka, MP4), allows streaming over internet

- Napster (1999): MP3 peer-to-peer sharing

Optimal coding

Objective: Distribute optimally R bits within K channels

This is formally equivalent to minimize the distortion D defined as:

$$D(R_1, \ldots, R_K) =< e^2 >= \frac{1}{K}\sum_{k=1}^{K} < e_k^2 > \tag{46}$$

under the constraint:

$$R = \frac{1}{K}\sum_{k=1}^{K} R_k \tag{47}$$

The quantification error caused by allocating R_k bits to the k-th channel is:

$$< e_k^2 > \approx < x_k^2 > /(3 \times 2^{R_k}) \tag{48}$$

This is strictly equivalent to the fact that the error is reduced by 3 dB for each bit allocated.

Optimal coding

Objective: Distribute optimally R bits within K channels

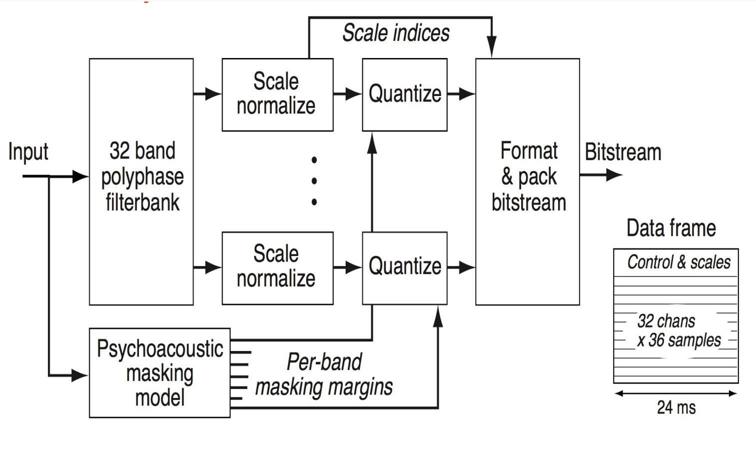
This is an **optimization under constraint** which can be solved by using Lagrange multipliers.

The quantification error caused by allocating R_k bits to the k-th channel is:

$$R_k = R + \frac{1}{2}\log_2 < x_k^2 > - \frac{1}{2}\log_2 \left(\prod_{j=1}^{K} \right)^{1/K} \tag{49}$$

This is equivalent to the fact that the error is reduced by 3 dB for each bit allocated.

MPEG-1, Layer-1 : Architecture



MPEG-1, Layer-1 : Perceptual coding

- Based on psychoacoustics, as long as the quantization noise is lower than the masking threshold at any frequency, the noise will be inaudible.

- In order to make the reconstruction error least detectable by the human ear, **the NMR in each channel must be minimised**

$$NMR(k) = SMR(k) - SNR(k) \tag{50}$$

where:

NMR is the Noise-to-Mask Ratio (in dB)

SMR is the Signal-to-Mask Ratio (in dB), as computed by the psycho-acoustics model
SNR is the Signal-to-Noise Ratio (in dB), corresponding to the quantification error due the the number of bits allocated

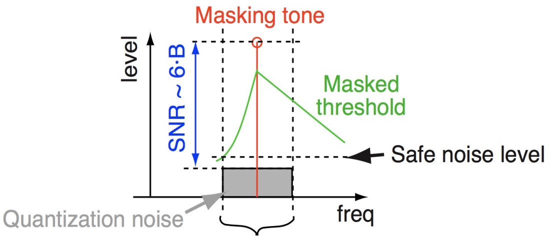
- After bit allocation, the scale factor of the channel is encoded with 3 bits (similar to the gain envelope of the amplitude spectrum)
- Each normalized channel is quantized accordingly to the bit allocation

MPEG-1, Layer-1 : Optimizing the Mask-to-Noise Ratio

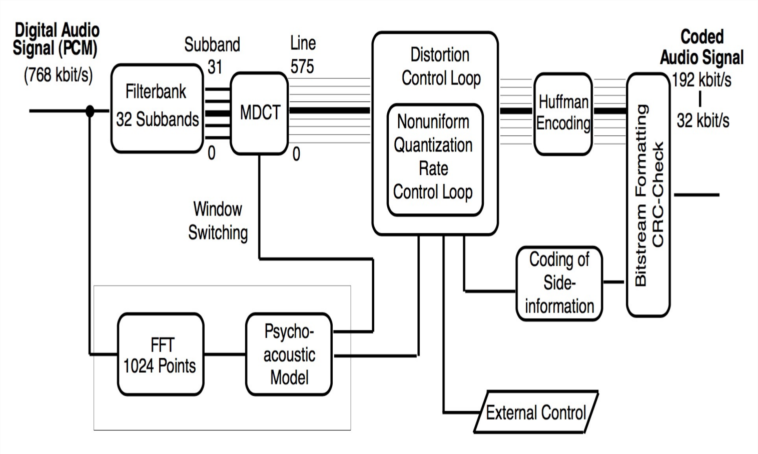
Algorithm 1 *

- **initialization** : 0 bits for each sub-band
- **while** $R > 0$ or $NMR(k) > 0, \forall k$ **do**
  - **Compute NMR** in each channel
  - $\widehat{k} = \arg\max_k (MNR)$
  - Find channel with **higher NMR**
  - Improve that channel by **allocating 1 bit** **(3 dB in SNR reduction)**
  - Update number of remaining bits available R
- **end while**

An iterative algorithm for the bit allocation in MPEG-1, Layer 1



MPEG-1, Layer-3 : Architecture



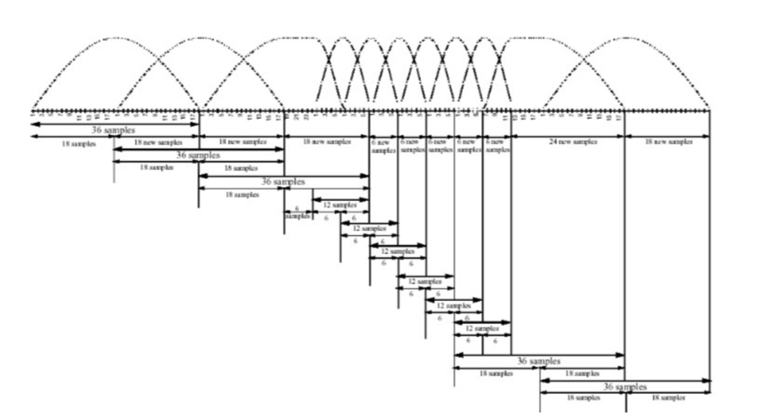
MPEG-1 layer-3

Some of the key advances of the Layer-3 format

- MDCT (Modified Discrete Cosine Transform)
  Transform with real values, 4 times less values than the STFT!
- Variable length Windows
  Short / long windows are used to encode fast / slow time variations. The choice of the window is decided after measuring the entropy of the audio signal.
- Entropic encoding
  An entropic encoding, based on the Huffman algorithm, is used following the perceptual coding, for additional lossless compression
- Bits reservoir
  Bits are not necessarily used uniformingly at each time. Unused bits are stored and can be further used forward

Variable window length

- Short window (12 samples)
  Good time resolution, non-stationary/transients segments
- Long window (36 samples)
  Good frequency resolution, stationary segment



Bits reservoir

- Bit reservoir is used to encode the next time frames