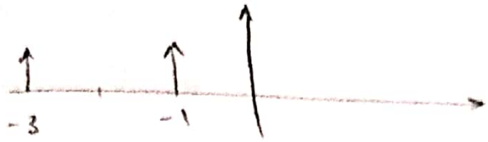
$x(2-3n)$:

$x(n) = \delta[n+1] + \delta[n-1]$

$x(n) \longrightarrow x(2+n)$



$x(n) \longrightarrow x(-3n)$

$x(2-3n)$

$\times$

$u(n)$

$\Rightarrow \quad x(2-3n) \cdot u(n)$

$= \delta[n-1]$

$$z(n) = \delta[n-1] * x[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot \delta[n-1-k]$$

$$= \sum_{k=-\infty}^{+\infty} \left(\delta[k+1] + \delta[k-1]\right) \cdot \delta[n-1-k] \overset{n=0}{=}$$

$$\sum_{k=-\infty}^{+\infty} \left(\delta[k+1] + \delta[k-1]\right) \cdot \delta[-1-k] = \delta[0] + \delta[-1]$$

$k+1 = -1-k \rightarrow 2k = -2 \longrightarrow k = -1 \quad (\delta[0] + \delta[-2])(\delta[0]) = \delta[0]$

$k-1 = -1-k \rightarrow 2k = 0 \longrightarrow k = 0 \quad (\delta[1] + \delta[-1])(\delta[-1]) = \delta[-1]$

$\Rightarrow \quad z[0] = \delta[0] + \delta[-1]$

$T_0 = 4 \rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$ ②

$$x(t) = \sum_{k=-\infty}^{k=+\infty} a_k e^{jk\frac{\pi}{2}t} = -1 - je^{j\frac{\pi}{2}t} + e^{j\pi t} + je^{j\frac{3\pi}{2}t}$$

$$x(t+1) = -1 - je^{j\frac{\pi}{2}} e^{j\frac{\pi}{2}t} + e^{j\pi} e^{j\pi t} + je^{j\frac{3\pi}{2}} e^{j\frac{3\pi}{2}t}$$

$$x(-t) = \underbrace{-1}_{a_0} \underbrace{- je^{j\frac{\pi}{2}}}_{a_{-1}} e^{-j\frac{\pi}{2}t} + \underbrace{e^{j\pi}}_{a_{-2}} e^{-j\pi t} + \underbrace{je^{j\frac{3\pi}{2}}}_{a_{-3}} e^{-j\frac{3\pi}{2}t}$$

$a_0 = -1$

$$a_{-1} = -je^{j\frac{\pi}{2}} = -j\left(\cancel{\cos\frac{\pi}{2}}^{0} + j\sin\frac{\pi}{2}^{1}\right) = -j^2 = 1$$

$$a_{-2} = e^{j\pi} = \cos\pi + j\cancel{\sin\pi}^{0} = -1$$

$$a_{-3} = je^{j\frac{3\pi}{2}} = j\left(\cancel{\cos\frac{3\pi}{2}}^{0} + j\sin\frac{3\pi}{2}^{-1}\right) = -j^2 = 1$$

-1

3

$y[n] = \frac{1}{4} y[n-1] + x[n]$

$\frac{1}{4} y(n-2) \quad \frac{1}{4} x[n-1]$

$y[n] = \sum_{0}^{n} (\frac{1}{4})^i x[n-i] \xrightarrow{F} \sum_{i=0}^{n} (\frac{1}{4})^i e^{+j\omega_0 i} \underbrace{\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 n}}_{\text{تبدیل فوریه } x[n]}$

الف)

$x[n] = \sin(\overbrace{\tfrac{3\pi}{4}}^{\omega_0} n) = \frac{e^{j\frac{3\pi}{4}t} - e^{-j\frac{3\pi}{4}t}}{2j}$ $\quad a_1 = \frac{1}{2j} \quad a_{-1} = -\frac{1}{2j} \quad a_k = 0 \quad k \neq \pm 1$

$y[n] = (\underbrace{\sum_{i=0}^{n} (\frac{1}{4})^i e^{j\frac{3\pi}{4}i}}_{Z})(-\frac{1}{2j} e^{-j\frac{3\pi}{4}n} + \frac{1}{2j} e^{j\frac{3\pi}{4}n})$ سری فوریه

$\Rightarrow a_1 = \frac{1}{2j} Z \qquad a_{-1} = -\frac{1}{2j} Z$ ضرایب فوریه :

ب)

$x[n] = \cos(\frac{\pi}{4} n) + 2\cos(\frac{\pi}{2} n)$

$= \frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2} + e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}$

$w = \frac{\pi}{4}$

$\xrightarrow{F} \underbrace{\frac{1}{2}}_{a_1} e^{jwn} + \underbrace{\frac{1}{2}}_{a_{-1}} e^{-jwn} + \underbrace{1}_{a_2} e^{j2wn} + \underbrace{1}_{a_{-2}} e^{-j2wn}$

$\Rightarrow y[n] = \underbrace{[\sum (\frac{1}{4})^i e^{j\frac{3\pi}{4}i}]}_{Z} (\frac{1}{2} e^{jwn} + \frac{1}{2} e^{-jwn} + e^{j2wn} + e^{-j2wn})$ سری فوریه

$\Rightarrow$

ضرایب فوریه : $a_1 = \frac{1}{2} Z \quad a_2 = \frac{1}{2} Z \quad a_3 = Z \quad a_4 = Z$