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Brief paper

Sliding mode observer for fault reconstruction of time-delay and sampled-output systems – a Time Shift Approach[☆]

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ABSTRACT

This paper presents a novel *Time Shift Approach* for actuator fault reconstruction of systems with output time-delay based on a sliding mode observer (SMO). Arbitrary output delay duration, possibly uncertain and time-varying, can be considered. Ideal sliding mode is made possible even in the presence of delay. This is particularly important since chattering problems can be circumvented so that the proposed approach is conveniently applicable to the case of sampled-output systems. In order to allow a smooth fault reconstruction using noncausal filters, an intentional output delay of one or more sampling periods is introduced. Numerical simulations illustrate the effectiveness of the proposed methodology.

1. Introduction

The increasing industry demand for better reliability of equipment has led to the development of fault detection and isolation (FDI) techniques, allowing diagnostics and prognostics of the system health, aiming at safer operations (Isermann, 2006). Strategies for condition-based maintenance (CBM), whereby the equipment health is continuously monitored, can also increase its availability and thus reduce its downtime by avoiding long degraded-state operation or unnecessary preventive maintenance stops (Jardine, Lin, & Banjevic, 2006).

Sliding mode observers (SMO) have long been proposed for control systems (Edwards & Spurgeon, 1994; Walcott & Zak, 1987) (for more references about the early stages of SMO see Spurgeon (2008)). Its application for model-based fault reconstruction has gained growing attention since the seminal paper by Edwards, Spurgeon, and Patton (2000). In the latter approach, the fault signal is reconstructed by manipulating the output error injection signal of the observer so that the amount of continuous control needed for the observer to remain on the sliding surface is achieved even in the presence of a fault. The injection signal can provide the fault signal reconstruction. Such method was further developed in Tan and Edwards (2002, 2003) where the observer gain optimization, based on the *Bounded Real Lemma* (Chilali & Gahinet, 1996), was proposed to improve the robustness with respect to model uncertainties. Also, a second-order sliding mode observer for fault reconstruction was proposed in Nagesh and Edwards (2014), using the Super Twisting Algorithm for multivariable systems. Recently, a cascade of two SMO were applied to reconstruct faults in a class of non-infinitely observable descriptor systems (Chan, Tan, Trinh, & Kamal, 2019; Chan, Tan, Trinh, Kamal, & Chiew, 2019).

Systems with time delays were considered by several authors in the recent past (Basin & Rodriguez-Gonzalez, 2005; Fridman, 2014a; Gu, Kharitonov, & Chen, 2003; Krstic, 2009) For a recent account of papers about this topic see the recent book (Malisoff, Pepe, Mazenc, & Karafyllis, 2016) and the bibliography therein.

To the best of our knowledge, in the literature on sliding mode observers for time-delay systems, only a few publications are dedicated to the reconstruction of faults in systems with measurement delays. In Han, Fridman, and Spurgeon (2014) the delay is motivated by sampled-output systems while in Han, Fridman, and Spurgeon (2016) more general output delay systems are considered. A common feature of the latter papers is that a delayed output error signal is employed to construct the observer. Unfortunately, this leads to undesirable chattering in the observer output injection term. The stability analysis relies on the well known Lyapunov–Krasovskii's functionals which in turn lead to LMI conditions, appropriate from a numerical

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computation viewpoint. A singular perturbation approach was invoked to obtain the observer gains guaranteeing observation error system stability and reduction of the chattering effects. However, this appears to limit the allowable measurement delays or sampling periods. In contrast, arbitrarily large delays can be allowed using a cascade of high-gain observers as in Ahmed-Ali, Cherrier, and Lamnabhi-Lagarrigue (2012), Coutinho, Oliveira, and Cunha (2014). In these papers, the aim was to predict the current system state with application of the separation principle for control design. However, no unknown disturbance was allowed in their approaches as is the case of interest for fault reconstruction purposes. The same limitation applies to the partial differential equation (PDE) approach to develop current state observers of delay systems described in Krstic (2009) so that its application for fault reconstruction is still an open problem.

In Hann and Ahmed-Ali (2014) and Folin, Ahmed-Ali, Giri, L. Burlion, and Lamnabhi-Lagarrigue (2016), adaptive observer structures for classes of nonlinear sampled-output systems presenting unknown constant parameters are designed. Although their method does not require output reconstruction filters like the zero-order hold, they use some knowledge of the disturbance dynamics to determine the unknown constant parameters.

In the present paper, one main contribution is the formalization of the *time-shift approach*. It allows the design of SMO for arbitrarily long time-varying delays of the output measurement. Without invoking the Lyapunov–Krasovskii functionals, the SMO can be used for fault reconstruction provided the time-shifted system satisfies the usual minimum phase and relative degree one assumptions. As a further advantage of the proposed approach, uncertainties of the delay are allowed without compromising the realization of ideal sliding mode, thus avoiding undesirable chattering effects present in existing fault reconstruction methods. The price to be paid is that the reconstructed fault signal is also delayed by the corresponding time-shift, which is acceptable for certain relevant applications, *e.g.*, for space or deep water systems when large communication delays are expected. For long delay durations, the delayed reconstruction can be applied in closed loop control in cases where the system bandwidth is not so large or some prediction method to advance the fault reconstruction can be used (Pinto, Oliveira, Hsu, & Krstic, 2018). Also, the ability to reconstruct the entire fault signal including its transient is interesting to identify short, intermittent faults.

Another contribution of our article is the application of the proposed method to sampled-output systems. By *deliberately* introducing a known constant delay, the output signal can be interpolated, and the fault can be reconstructed in continuous-time. The possibility of using higher-order interpolations allows reconstructing faults in systems with longer sampling periods. In Folin et al. (2016), Hann and Ahmed-Ali (2014) and Han et al. (2014), the maximum allowed sampling period is limited.

The remainder of this paper is organized as follows. In Section 2, the time-shift approach is described for systems with measurement delay which can be assumed arbitrarily long, time-varying and uncertain. Well known results for observer design are then applied in a straightforward way to the time-shifted system under mild assumptions. Section 3 presents the application of the proposed approach to develop a fault reconstruction method for the specific case of systems with sampled-output. Numerical simulations illustrate the effectiveness of the proposed methodology for smooth fault reconstruction.

For the notation in this paper, $\lambda_i(.)$, $\lambda_{\max}(.)$, $\lambda_{\min}(.)$ represent the ith, largest and smallest eigenvalue of a matrix, respectively, $\|\,.\,\|$ represents the 2-norm of a matrix or vector, $\mathbb{E}[.]$ represents the expected value.

2. Sliding mode observers for fault reconstruction with measurement delay

2.1. Problem statement

Consider the following linear system presenting a time-varying measurement delay $\tau(t) \geq 0$:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) \\ y(t) = Cx(t-\tau(t)) \end{cases} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $F \in \mathbb{R}^{n\times q}$. The input signal $u(t) \in \mathbb{R}^m$ is known, the system state $x(t) \in \mathbb{R}^n$ is not measurable, and $y(t) \in \mathbb{R}^p$ is the output signal. The unknown disturbance signal $f(t) \in \mathbb{R}^q$, $q \leq p < n$, represents the effect of an actuator fault.

The main objective of this paper is to develop a method to reconstruct the actuator fault signal $f(t)$ based only on the known input and output signal $y(t)$ which is measured with delay $\tau(t)$.

The following assumptions are made:

- **(A1)** Matrices A, B, C, F are known. B and F have full rank.
- **(A2)** The time-delay $\tau(t)$ is differentiable and satisfies $0 < \tau(t) \leq \bar{\tau}$ for some constant $\bar{\tau}$, and its time derivative $\dot{\tau}(t)$ satisfies:
$$\underline{d} \leq \dot{\tau}(t) \leq d < 1, \quad |\ddot{\tau}(t)| \leq r_d \tag{2}$$
where r_d, d and $\underline{d}$ are constants, with $\underline{d}$ being possibly negative. These conditions must be simultaneously satisfied.
- **(A3)** The unknown disturbance/fault $f(t)$ is bounded by:
$$\|f(t)\| \leq \alpha, \quad \alpha \in \mathbb{R}^+ \tag{3}$$
- **(A4)** The pair (A,C) is observable and the invariant zeros of the triple (A, F, C) lie in the left half plane.
- **(A5)** $rank(CF) = rank(F) = q$.
- **(A6)** The past values of $u(t)$ for $t \in [t-\tau, t]$ are available for the fault reconstruction method.

2.2. Time-shift approach for observer design

From **(A4)** and **(A5)**, it is assumed that the system (1) is already in the following canonical form suitable for observer design (Edwards & Spurgeon, 1998, Lemma 6.1):

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right], \quad B = \left[\begin{array}{c} B_1 \\ \hline B_2 \end{array}\right],$$
$$F = \left[\begin{array}{c} 0 \\ \hline F_2 \end{array}\right], \quad F_2 = \left[\begin{array}{c} 0 \\ \hline F_0 \end{array}\right]$$
$$C = \left[\begin{array}{c|c} 0 & T \end{array}\right], \quad T^T T = I \tag{4}$$

where $A_{11} \in \mathbb{R}^{(n-p)\times(n-p)}$, $T \in \mathbb{R}^{p\times p}$ is orthogonal, $B_2 \in \mathbb{R}^{p\times m}$, $F_2 \in \mathbb{R}^{p\times q}$. Matrix $F_0 \in \mathbb{R}^{q\times q}$ is non-singular.

Now, let us define the τ-delayed variables:

$$x_\tau(t) = x(t-\tau(t)),\ f_\tau(t) = f(t-\tau(t)),$$
$$u_\tau(t) = u(t-\tau(t)). \tag{5}$$

Then, applying the chain rule for derivatives, system (1) can be described using the delayed state variables as follows:

$$\begin{cases} \dot{x}_\tau(t) = (1-\dot{\tau}(t))\,[Ax_\tau(t) + Bu_\tau(t) + Ff_\tau(t)] \\ y(t) = Cx_\tau(t) \end{cases} \tag{6}$$

Notice that the time-shifted τ-delayed system has no (explicit) output delay.

The *objective* is to design an observer for the time-shifted system (6) that is capable of reproducing the τ-delayed disturbance signal $f_\tau(t)$ from the output signal $y(t)$ and from the input signal $u(t)$. To this end, a generalization of the existing sliding mode observers for time delays will be proposed.

Remark 1. The Time-Shift approach can be applied to standard nonlinear systems used in the literature of time-delay systems (Ahmed-Ali et al., 2012; Coutinho et al., 2014), broadening its applicability. As an example consider the system:

$$\begin{cases} \dot{x}(t) &= Ax(t) + \phi(x(t)) + Bu(t) + Ff(t) \\ y(t) &= Cx(t-\tau(t)) \end{cases}$$

where the known function $\phi(x(t))$ is Lipschitz in $x(t)$. By applying the method above, it is transformed to

$$\begin{cases} \dot{x}_\tau(t) &= (1-\dot{\tau}(t))\,[Ax_\tau(t) + \phi(x_\tau(t)) \\ & \quad +Bu_\tau(t) + Ff_\tau(t)] \\ y(t) &= Cx_\tau(t). \end{cases} \tag{7}$$

Notice that it would be very simple to extend all the results presented in this paper to obtain an appropriate SMO for fault reconstruction in (7).

2.3. Known time-delays

In this section we consider that the measurement delay $\tau(t)$ is known and, based on **(A6)**, that the delayed input $u_\tau(t) = u(t-\tau(t))$ is available.

Consider the following sliding mode observer:

$$\begin{cases} \dot{\hat{x}}_\tau(t) = (1-\dot{\tau}(t))\left[A\hat{x}_\tau(t) + Bu_\tau(t) - G_l e_y + G_n \nu\right] \\ \hat{y}(t) = C\hat{x}_\tau(t) \end{cases} \tag{8}$$

where the output observation error $e_y(t)$ and the state observation error $e(t)$ are defined as:

$$e(t) \triangleq \hat{x}_\tau(t) - x_\tau(t) \tag{9}$$

$$e_y(t) \triangleq \hat{y}(t) - y(t) = Ce(t) \tag{10}$$

From (6), (8) and (9), it can be seen that the observation error $e(t)$ is governed by:

$$\dot{e}(t) = (1-\dot{\tau}(t))\,[(A-G_lC)e(t) + G_n\nu - Ff_\tau(t)] \tag{11}$$

$$e_y(t) = Ce(t) \tag{12}$$

where G_l is chosen such that $A - G_lC$ is Hurwitz.

The discontinuous injection term ν is given by:

$$\nu = -\rho \frac{P_0 e_y}{\|P_0 e_y\|} \tag{13}$$

where $P_0 > 0$ is a design matrix. The modulation function or gain ρ is designed so that a sliding mode $e_y(t) \equiv 0$ is reached in finite time.

Remark 2. When the measurement delay $\tau(t)$ is constant ($\dot{\tau}(t) = 0$), Eqs. (8) and (11) for the observer proposed here reduces to those of Edwards et al. (2000) originally designed for fault reconstruction free of delays, but now estimating the delayed states rather than the current ones.

Matrix G_n is chosen to have the structure:

$$G_n = \begin{bmatrix} -L \\ I_{p\times p} \end{bmatrix} T^T, \tag{14}$$

where T is orthogonal as defined in (4). Matrix $L = [L_1 \quad 0] \in \mathbb{R}^{(n-p)\times p}$, with $L_1 \in \mathbb{R}^{(n-p)\times(p-q)}$, is chosen such that $LF_2 = 0$ and $A_{11} + LA_{21}$ is Hurwitz, due to **(A4)** (see (Alwi, Edwards, & Tan, 2011, Lemma 4.3)), for $p < q$.

Remark 3. Note that if $p = q$ it will not be possible to determine matrix L. In this case, it is necessary to consider $\det(CF) \neq 0$ in assumption **(A4)**. In this case, the system (A, F, C) is minimum phase and relative degree one, and there exists exactly $(n-p)$ invariant zeros, so that A_{11} is Hurwitz.

By a change of coordinates $e \mapsto \bar{e} = T_L e$, where T_L is a non-singular linear transformation defined as:

$$T_L = \begin{bmatrix} I_{n-p} & L \\ 0 & T \end{bmatrix}, \quad \bar{e} = \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix}, \quad e_y = \bar{e}_2, \tag{15}$$

the error dynamics can be described as:

$$\begin{cases} \dot{\bar{e}}_1(t) &= (1-\dot{\tau}(t))\left(\mathcal{A}_{11}\bar{e}_1(t) + (\mathcal{A}_{12} - \mathcal{G}_{l,1})e_y(t)\right) \\ \dot{e}_y(t) &= (1-\dot{\tau}(t))\left(\mathcal{A}_{21}\bar{e}_1(t) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})e_y(t) \right. \\ & \quad \left. +\nu - \mathcal{F}_2 f_\tau(t)\right) \end{cases} \tag{16}$$

where matrices $(\mathcal{A}, \mathcal{C}, \mathcal{F}, \mathcal{G}_l)$ correspond to the transformed matrices (A, C, F, G_l). Matrices $\mathcal{G}_{l,1}$ and $\mathcal{G}_{l,2}$ are partitions of the transformed matrix $\mathcal{G}_l = T_L G_l$ with the appropriate dimensions. The partitions of $\mathcal{A}$ are given by:

$$\mathcal{A}_{11} = A_{11} + LA_{21}, \quad \mathcal{A}_{12} = (A_{12} + LA_{22} - A_{11}L - LA_{21})T^T,$$

$$\mathcal{A}_{21} = TA_{21}, \quad \mathcal{A}_{22} = TA_{22}T^T - TA_{21}LT^T.$$

It can be seen that the sliding mode is governed by the eigenvalues of $\mathcal{A}_{11} = A_{11} + LA_{21}$, which is Hurwitz by design. Choosing $\mathcal{G}_{l,1} = \mathcal{A}_{12}$ and $(\mathcal{A}_{22} - \mathcal{G}_{l,2}) = \mathcal{A}_{22}^*$, where $\mathcal{A}_{22}^* \in \mathbb{R}^{p\times p}$ is any stable design matrix, the system (16) becomes triangular. Consequently, the eigenvalues of $A - G_lC$ will be those of $\mathcal{A}_{11}$ and $\mathcal{A}_{22}^*$. In the coordinate system of (4), G_l is then given by:

$$G_l = T_L^{-1} \begin{bmatrix} \mathcal{A}_{12} \\ \mathcal{A}_{22} - \mathcal{A}_{22}^* \end{bmatrix} \tag{17}$$

The symmetric design matrix $P_0 > 0$ of (13) must be selected satisfying $(\mathcal{A}_{22}^*)^T P_0 + P_0(\mathcal{A}_{22}^*) < 0$. Also, there exists a matrix $P > 0$, satisfying $(A - G_lC)^T P + P(A - G_lC) < 0$, with the following structure (Alwi et al., 2011):

$$P = T_L^T \begin{bmatrix} P_1 & 0 \\ 0 & P_0 \end{bmatrix} T_L \tag{18}$$

The convergence of the observer (8) to the τ-delayed system (6) can be proved in the next theorem.

Theorem 1. *Consider the τ-delayed system (6), that satisfies **(A1)** to **(A6)** and its observer (8), with ν in (12), matrices G_n and G_l defined in (16)* and *(17), respectively. If the modulation function ρ of the discontinuous term (13) satisfies*

$$\rho \geq \alpha\|CF\| + \eta_0, \tag{19}$$

where η_0 is any positive scalar, then the state estimation error $e(t)$ converges exponentially to zero. The output error $e_y(t)$ reaches zero in finite time $t = t_s$ and a sliding mode starts on $\mathscr{S} = \{e : e_y = Ce = 0\}$, $\forall t \geq t_s$.

Proof. As for Proposition 4.1 in Alwi et al. (2011), where systems without delay were considered, the proof is based on the Lyapunov candidates $V_1(e) = e^T Pe$, with $e(t)$ from (11), for the state stability and convergence, and $V_2(e_y) = e_y^T P_0 e_y$, with $e_y(t)$ from (16), for sliding mode reachability. The only difference is the scalar factor $(1-\dot{\tau}(t))$, present in their time derivatives. Since such factor is always finite, positive and uniformly away from zero, due to assumption **(A2)**, the stability and convergence can be concluded from the Lyapunov analysis. □

Once the sliding mode is reached in finite time, then $e_y(t) \equiv \dot{e}_y(t) \equiv 0$ and from (16), the error $\bar{e}_1(t)$ will be governed by:

$$\begin{cases} \dot{\bar{e}}_1(t) &= (1-\dot{\tau}(t))\,(\mathcal{A}_{11}\bar{e}_1(t)) \\ 0 &= (1-\dot{\tau}(t))\left(\mathcal{A}_{21}\bar{e}_1(t) + \nu_{eq} - \mathcal{F}_2 f_\tau(t)\right) \end{cases} \tag{20}$$

where ν_{eq} is the equivalent output error injection signal (in the sense of equivalent control (Utkin, 1992)), which represents the continuous control signal needed to maintain the sliding mode.

Corollary 1. *Under the conditions in Theorem 1, the delayed disturbance signal $f_\tau(t)$ in (5) can be reconstructed, with arbitrary accuracy, as $\delta \to 0$, by the expression*

$$\hat{f}_\tau(t) = \mathcal{F}_2^{\dagger} \nu_\delta \tag{21}$$

where ν_δ is the continuous approximation of ν_{eq} defined as:

$$\nu_\delta = -\rho \frac{P_0 e_y}{\|P_0 e_y\| + \delta} \tag{22}$$

for a small scalar parameter $\delta > 0$, and $\mathcal{F}_2^{\dagger}$ is the left pseudo-inverse of $\mathcal{F}_2 = T_L F_2$, given by:

$$\mathcal{F}_2^{\dagger} \triangleq (\mathcal{F}_2^T \mathcal{F}_2)^{-1} \mathcal{F}_2^T. \tag{23}$$

Proof. Since, by assumption **(A5)**, the matrix F is full rank, the Moore–Penrose left pseudo-inverse for $\mathcal{F}_2$ exists as defined by (23). Therefore, the estimate $\hat{f}_\tau$ of the τ-delayed fault signal $f_\tau(t)$ is given by the expression (21). From Theorem 1, the sliding mode $e_y \equiv 0$ is reached in finite time and the state observation error $\bar{e}_1(t) \to 0$ exponentially, and according to (20), one has:

$$\nu_{eq} \to \mathcal{F}_2 f_\tau(t). \tag{24}$$

The equivalent output error injection ν_{eq} can be recovered by using the approximation ν_δ given by expression (22), with small enough choice of δ, to achieve any degree of accuracy (Burton & Zinober, 1986). □

The main advantage of the above time-shift approach is the possibility of determining the observer gains without appealing to specific methods for time-delay systems, like *Lyapunov–Krasovskii* (LK) or *Lyapunov–Razumikhin* (LR) approaches (Fridman, 2014a; Gu et al., 2003). From the analytical viewpoint, both LK or LR approaches can lead to conservative results which may restrict the maximum delay duration to some sufficiently small value. LR is well know to be generally more conservative than LK (Fridman, 2014b). On the other hand, to reduce the conservativeness of LK, increased complexity of the proper functionals is required. Obtaining the LMI associated to such functionals involve rather specialized mathematical tools (Liu, Park, & Guo, 2017), (Seuret & Gouaisbaut, 2013). Moreover, from the numerical viewpoint, the number of the LMI decision variables may greatly increase making the method less attractive (Seuret, Gouaisbaut, & Baudouin, 2016). In the present approach, an ideal sliding mode can theoretically be produced for arbitrarily long delays, which in practice also avoids low frequency chattering problems. The price to be paid is that the fault is identified with some delay. Thus a residual error exists for the estimated and the actual fault signal. It should be stressed that the LR and LK approaches also lead to residual errors and do not identify exactly the fault signal. In Han et al. (2016), the observer is designed with delayed output, according to the original (non-time-shifted) system, and the discontinuous term depends on the delayed error signal. An ideal sliding mode is not achievable and the reconstruction is subject to chattering effects (see (Han et al., 2016, Remark 3)).

The problem of noise was already considered by Edwards et al. (2000). They suggest that the equivalent control, which is used in the fault reconstruction, should be passed by a low-pass filter so that the true fault signal, usually of low frequency, could be extracted or separated from the high-frequency noise signal. We have verified the effectiveness of this measure in our method, provided a time separation is possible, *i.e.*, when the noise does not contain large enough low frequency components leading to false alarms.

2.4. Parametric and time-delay uncertainties

Full knowledge of the plant parameters as well of the time delay $\tau(t)$ is certainly unrealistic. Hence the accommodation of some uncertainty of the plant characteristics is pertinent. To this end, we show that such uncertainties can be lumped as an additional disturbance, thus reducing the fault reconstruction problem to a well known formulation (Tan & Edwards, 2003). First, let the delay be modeled by

$$\tau(t) = \hat{\tau} + \tau_v(t) \tag{25}$$

where $\hat{\tau}$ is some known constant estimate for the output delay and $\tau_v(t)$ is the time-varying delay uncertainty. In addition, let the parametric uncertainties be represented by the bounded lumped signal $D\varsigma(t)$ where $\varsigma \in \mathbb{R}^l$, and $D \in \mathbb{R}^{n\times l}$ is some distribution matrix. Then, system (1) becomes:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) + D\varsigma(t) \\ y(t) = Cx(t - \tau(t)) \end{cases} \tag{26}$$

Applying the time-shift presented in Section 2.2, the delayed system can be described as (see (5)):

$$\begin{cases} \dot{x}_\tau(t) &= (1 - \dot{\tau}(t))[Ax_\tau(t) + Bu_\tau(t) \\ & \quad + Ff_\tau(t) + D\varsigma_\tau(t)] \\ y(t) &= Cx_\tau(t) \end{cases} \tag{27}$$

where $\varsigma_\tau(t) = \varsigma(t - \tau(t))$.

Since $\dot{\tau}(t)$ is unknown, the observer considers only some constant (nominal) estimate of the delay $\hat{\tau}$ and the estimate of the delayed input signal $\hat{u}(t) = u(t - \hat{\tau})$:

$$\begin{cases} \dot{\hat{x}}_\tau(t) &= A\hat{x}_\tau(t) + B\hat{u}(t) - G_l e_y + G_n \nu \\ \hat{y}(t) &= C\hat{x}_\tau(t) \end{cases} \tag{28}$$

where $e_y(t)$ is defined in (10) and ν in (13). Matrices G_n and G_l are as determined in (14) and (17) respectively.

The state observation error then presents the following dynamics:

$$\begin{aligned} \dot{e}(t) &= (A - G_l C)e(t) + G_n \nu - Ff_\tau(t) + B\tilde{u}(t) - D\varsigma_\tau(t) \\ &+ \dot{\tau}(t)[Ax_\tau(t) + Bu_\tau(t) + Ff_\tau(t) + D\varsigma_\tau(t)] \end{aligned} \tag{29}$$

where $f_\tau = f(t - \tau(t))$, and $\tilde{u}(t) = \hat{u}(t) - u_\tau(t)$.

It can be seen in (29) that delay uncertainty terms in $u(t)$, $u_\tau(t)$ and $x_\tau(t)$ are present in the error dynamics. In order to limit their influence in the design of the modulation function ρ and in the performance of the observer, we assume:

- **(A7)** The system is stable or already stabilized, so that matrix A is Hurwitz.
- **(A8)** The signal $u(t)$ represents an external known (measurable) disturbance. Moreover, it is norm bounded by $\|u(t)\| \leq \beta_u$, for some constant $\beta_u > 0$, $\forall t$.

For the estimation error $\tilde{u}(t)$, from **(A8)** we can also assume:

- **(A9)** The input error $\tilde{u}(t)$ is zero if the exact delay is known and increases with growing uncertainty $|\tau_v(t)| = |\hat{\tau} - \tau| < \epsilon_v$. Therefore, it can be assumed that a class $\mathcal{K}$ function (Khalil, 2002) $\beta_v(\epsilon_v)$ exists such that $\|\tilde{u}(t)\| < \beta_v(\epsilon_v)$.

Remark 4. The conditions presented in **(A7)** may arise when the system is stabilized locally but monitored in a remote location subject to delays, where the fault reconstruction is to take place. This is the price of considering uncertain and arbitrarily long delays. As discussed in the time-delays literature, if unstable systems are assumed, the uncertain delays must be sufficiently small so that the delayed term $x_\tau(t)$ can be properly compensated in the control design.

Let us denote, $M = \sqrt{\frac{\lambda_{\max}(P_x)}{\lambda_{\min}(P_x)}}$, $\kappa_1 = \|P_x B\|$, $\kappa_2 = \|P_x F\|$, and $\kappa_3 = \|P_x D\|$, where $P_x > 0$ is a symmetric matrix used in the Lyapunov function $V_x(x) = x^T P_x x$. Based on assumption **(A7)**, system (26) is BIBS (bounded-input,bounded-state) stable. From **(A3)**, **(A8)** and reminding that $\|\varsigma(t)\| \leq \beta_\varsigma$, it can be seen that $x(t)$ is ultimately bounded by:

$$\|x\| \leq M c_\epsilon, \tag{30}$$

where $c_\epsilon = 2(\kappa_1\beta_u + \kappa_2\alpha + \kappa_3\beta_\varsigma) + \epsilon$, for any arbitrarily small $\epsilon > 0$. Since $x(t)$ is ultimately bounded, and considering the bound r_d for $\dot{\tau}(t)$ in **(A2)**, it is possible to aggregate the three last terms of (29) as a norm bounded signal $\xi(t)$, $\|\xi(t)\| \leq \bar{\xi}$, such that:

$$\begin{aligned}\xi(t) &\triangleq B\tilde{u}(t) - D\varsigma_\tau(t) + \dot{\tau}(t)[Ax_\tau(t) + Bu_\tau(t) \\ &\quad + Ff_\tau(t) + D\varsigma_\tau(t)], \qquad (31)\\ \bar{\xi} &\triangleq \|B\|\beta_v(\epsilon_v) + \frac{r_d\|B\|\beta_u}{2} + 2r_d\|A\|Mc_\epsilon \\ &\quad + r_d\|F\|\alpha + (1+r_d)\|D\|\beta_\varsigma. \qquad (32)\end{aligned}$$

The partition of the uncertainty vector as $\xi = [\xi_1^T \ \ \xi_2^T]$ is made according to the partition of the error $\bar{e} = [\bar{e}_1^T \ \ \bar{e}_2^T]$. Hence, after the change of coordinates (15), the equations for the error dynamics becomes:

$$\begin{cases} \dot{\bar{e}}_1(t) &= \mathcal{A}_{11}\bar{e}_1(t) + (\mathcal{A}_{12} - \mathcal{G}_{l,1})\bar{e}_y(t) \\ &\quad +(\xi_1(t) + L\xi_2(t)), \\ \dot{e}_y(t) &= \mathcal{A}_{21}\bar{e}_1(t) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})e_y(t) + \nu \\ &\quad - \mathcal{F}_2 f_\tau(t) + T\xi_2(t). \end{cases} \tag{33}$$

Then, the next theorem can be stated:

Theorem 2. *Consider the uncertain τ-delayed system (27), satisfying **(A1)** to **(A9)**, and its observer (28), where matrix G_n is determined by (14) and G_n is given by (17). If the modulation function ρ of the discontinuous term (13) is chosen as:*

$$\rho \geq 2\frac{\|TA_{21}\|\mu_0}{\|P\|}\bar{\xi} + \|TF_2\|\alpha + \|TM_2\|\bar{\xi} + \eta_0 \tag{34}$$

where $M_2 = [0^{p\times(n-p)} \ \ I^{p\times p}]$, $\mu_0 = -\lambda_{\max}((A - G_lC)^T P + P(A - G_lC))$, with $P > 0$ as in (18), $\bar{\xi}$ is given in (32) and η_0 is any positive scalar, then the state estimation error $e(t)$ is ultimately bounded and the output error $e_y(t)$ reaches zero in finite time $t = t_s$ so that a sliding mode starts on $\mathscr{S} = \{e : e_y = Ce = 0\}$, $\forall t \geq t_s$.

Proof. Substituting (31) in (29), one has:

$$\dot{e}(t) = (A - G_lC)e(t) + G_n\nu - Ff_\tau(t) + \xi(t). \tag{35}$$

Since the unknown terms in (35) were grouped in a bounded signal $\xi(t)$, it is only needed to follow the steps of Tan and Edwards (2003, Lemma 1 and Proposition 2), using Lyapunov candidates $V_3(e) = e^T Pe$, with $e(t)$ from (35), for error boundedness, and $V_4(e_y) = e_y^T P_0 e_y$, with $e_y(t)$ from (33), for sliding mode reachability to conclude this proof. □

Observe that Theorem 2 only requires the norm bound $\bar{\xi}$ from (32) to determine the gain from the discontinuous term. Hence, this observer design does not require the complete knowledge of the delay, only its duration estimate $\hat{\tau}$ from (25) and the norm bound for the delay rate r_d from assumption **(A2)**.

On the sliding mode $e_y(t) \equiv 0$, error Eq. (33) becomes:

$$\begin{cases} \dot{\bar{e}}_1(t) &= \mathcal{A}_{11}\bar{e}_1(t) + (\xi_1(t) + L\xi_2(t)), \\ 0 &= \mathcal{A}_{21}\bar{e}_1(t) + \nu_{eq} - \mathcal{F}_2 f_\tau(t) + T\xi_2(t). \end{cases} \tag{36}$$

In contrast to the previous case, where $\xi(t) = 0$, here, the error $\bar{e}_1$ does not converge to zero but depends on the uncertainty term $\xi(t)$. However, its effect can be minimized according to the following procedure.

Consider a scaling matrix $W \triangleq \begin{bmatrix} W_1 & F_0^{-1} \end{bmatrix}$ where $W_1 \in \mathbb{R}^{q\times(p-q)}$ is a design matrix, and F_0 is the lower block of the matrix F as in (4) . The matrix W will be used to reconstruct the fault signal using the expression:

$$\hat{f}_\tau(t) = WT^T\nu_{eq} \tag{37}$$

Multiplying the second equation of (36) by W and rearranging its terms, we obtain:

$$\begin{cases} \dot{\bar{e}}_1(t) &= \mathcal{A}_{11}\bar{e}_1(t) + (\xi_1(t) + L\xi_2(t)) \\ \hat{f}_\tau(t) &= -WT^T\mathcal{A}_{21}\bar{e}_1(t) - W\xi_2(t) + f_\tau(t) \end{cases} \tag{38}$$

Therefore, the estimated value for the fault signal is given by:

$$\hat{f}_\tau(t) = f_\tau(t) + G(s)\xi(t) \tag{39}$$

where $G(s)$ is a transfer function between the signal $\xi(t)$ and the estimated fault signal, and it is defined as:

$$G(s) = -WT^T\mathcal{A}_{21}\left[(sI - \mathcal{A}_{11})^{-1}(M_1 + LM_2)\right] - WQ_2 \tag{40}$$

where $M_1 = [I^{(n-p)\times(n-p)} \ \ 0^{(n-p)\times p}]$ and $M_2 = [0^{p\times(n-p)} \ \ I^{p\times p}]$. The gain of the transfer function $G(s)$ can be then minimized by using the *Bounded Real Lemma* (Chilali & Gahinet, 1996) to find the appropriate scale matrix W.

2.5. Simulation example

Consider the example proposed in Edwards et al. (2000), an inverted pendulum with a cart, linearized around the equilibrium point. The system matrices for the state space representation already transformed to the canonical form (4) are:

$$A = \begin{bmatrix} -0.1738 & 0 & 36.9771 & -6.2589 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ -0.0091 & 0 & 1.9333 & -1.9872 \end{bmatrix},$$
$$B = \begin{bmatrix} -1.0095 \\ 0 \\ 0 \\ -0.3205 \end{bmatrix}, F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.3205 \end{bmatrix}, C = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{41}$$

Assume that an input disturbance is given by u(t) = 5 sin(0.5t), which is assumed known but, due to the delay uncertainty, generates the input uncertainty term $\tilde{u}(t)$ term defined by (26). Then, the fault reconstruction observer is designed as follows:

1. Compute matrix $L = [L_1 \ \ 0] \in \mathbb{R}^{(n-p)\times p}$, with $L_1 \in \mathbb{R}^{(n-p)\times(p-q)}$, such that $LF_2 = 0$ and $A_{11} + LA_{21}$ is Hurwitz. The solution is nonunique. We have set $L = [0 \ -9.83 \ 0]$ which trivially makes $A_{11} + LA_{21} = [-10]$.
2. Using L and T (see (4)), compute T_L in (15) and obtain the transformed matrix $\mathcal{A} = T_L A T_L^{-1}$ and its partitions.
3. Choose the stable design matrix $\mathcal{A}_{22}^* \in \mathbb{R}^{(p\times p)}$ and compute matrix G_l in (17) using T_L the partitions of $\mathcal{A}$. This solution is also nonunique. We have set $\mathcal{A}_{22}^* = \mathrm{diag}(-15, -16, -17)$ to allocate the poles of $A - G_lC$ in $\{-10, -15, -16, -17\}$.
4. Then, from (14) and (17) we obtain:

$$G_n = \begin{bmatrix} 0 & 9.83 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \ G_l = \begin{bmatrix} 0 & 192.49 & 6.29 \\ -15 & 0 & -1 \\ 0 & 25.82 & 0 \\ 0 & 1.84 & 1 - 5.01 \end{bmatrix} \tag{42}$$

5. Choose P_0 satisfying $(\mathcal{A}_{22}^*)^T P_0 + P_0(\mathcal{A}_{22}^*) < 0$. Trivially, $P_0 = I$.

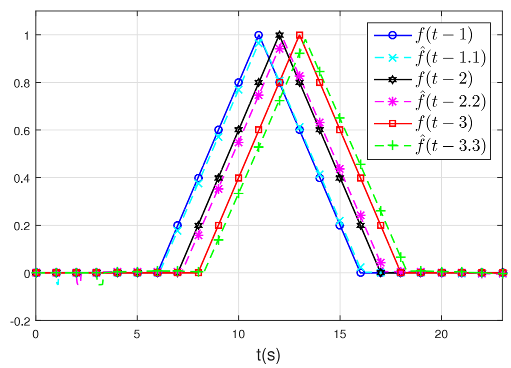


Fig. 1. Fault signal reconstruction for several measurement delay lengths, with 10% uncertainty on the duration.

6. Determine ρ satisfying (34). In our case, a constant $\rho = -3$ was used.
7. Determine the scaling matrix $W = [W_{1,1} \quad W_{1,2} \quad F_0^{-1}]$ to be used in the reconstruction equation (37). Since $\xi(t) = B\tilde{u}(t)$, Eq. (39) can be written as $\hat{f}_\tau(t) = f_\tau(t) + G_u(s)\tilde{u}(t)$, where $G_u(s) = G(s)B$. Substituting values, we obtain:

$$G_u(s) = -\frac{s - 1.01W_{1,2} + 9.97}{s + 10}. \tag{43}$$

From this transfer function, it can be seen that $W_{1,1}$ does not influence $G_u(s)$, hence it was set to zero. The value of $W_{1,2}$ was manually adjusted to $W_{1,2} = 7.5$ using the bode plot of $G_u(s)$ so that the matrix $W = \left[\begin{array}{ccc} 0 & 7.5 & -3.12 \end{array}\right]$ minimizes the reconstruction error of the fault signal to less than 1%.

In order to simulate the delay uncertainty, a 10% difference between the true and the nominal delay was introduced, i.e., $\tau = 1.1\hat{\tau}$. Fig. 1 presents the simulation results for reconstructing a triangular signal representing an actuator fault. The current delays were 1.1 s, 2.2 s, and 3.3 s, while the nominal delays to design the observer were 1 s, 2 s, and 3 s, respectively.

3. Sampled-output systems

This section considers sampled-output systems with constant sampling period h which occur in practice when the output signal measurement is not continuously available.

In Han et al. (2014), the sampled output is reconstructed by a Zero-Order Hold (ZOH) which is modeled as a time-varying output delay. In contrast, here the idea is to reconstruct the continuous-time signal that generates the sampled output, departing from the latter. Then, the original sampled-output system can be viewed as an unsampled continuous-time system with *constant* delay of one sampling time. To improve the reconstruction quality, one can wait for more samples and than use more effective interpolation methods. This is equivalent to *deliberately* reducing the problem to a system with constant output delays of $n_s \in \mathbb{N}$ sampling periods in the output signal, where n_s is the number of samples. Then, the time-shift method for observer design of Section 2.3 is applied to the reconstructed system.

3.1. Problem statement

Consider a linear time-invariant system with sampled output:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) \\ y(t) = Cx(t) \\ y_k = y(kh) \end{cases} \tag{44}$$

where $k \in \mathbb{N}$ is the sample index, and h is the sampling period. The output signal $y(t) \in \mathbb{R}^p$ is not directly available, only its sampled version y_k which is only valid at instants $t = kh$. The input signal $u(t) \in \mathbb{R}^m$ is measurable and the system states $x(t) \in \mathbb{R}^n$ are not measurable. The unknown disturbance signal $f(t) \in \mathbb{R}^p$, $q \le p < n$ represents the effect of an actuator fault. Consider that **(A1)–(A6)** are still valid.

Then, the objective is to estimate the continuous-time fault signal $f(t)$ from the measurable signals and the plant model.

3.2. Recovering the continuous-time signal from its sampled values

A well known reconstruction of the continuous-time output $y(t)$ is the ZOH (Zero Order Hold) filter. As a rule, the reconstructed output will present an approximation error. For the observers, this would appear as an output measurement disturbance. Even though SMO have some tolerance to disturbances, output disturbance can strongly deteriorate its performance and causal filters like ZOH may lead to unacceptable reconstruction error for SMO. Therefore, a suitable reconstruction filter must be chosen so that the approximation error is small enough. Interpolation reconstruction filters yield smaller reconstruction errors (Cleveland, 1976) but have the disadvantage of being non-causal.

Our strategy, as outlined above, is to intentionally allowing, say, $n_s \in \mathbb{N}$ sampling periods of delay in the reconstructed output signal y_R given by

$$y_R(t) = Cx(t - n_s h) + e_r(t). \tag{45}$$

where e_r is the reconstruction error.

From (44) and (45), the resulting system with continuous-time reconstructed output is given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) \\ y_R(t) = Cx(t - n_s h) + e_r(t) \end{cases} \tag{46}$$

Since the output delay now is constant and known, assumption **(A2)** is also valid with $\tau = n_s h$ and $\dot{\tau}(t) = 0$, and the time-shift method presented in Section 2.2 can be directly applied to reconstruct the delayed disturbance signal $f_\tau \triangleq f(t - n_s h)$. To this end, the SMO is given by

$$\begin{cases} \dot{\hat{x}}_\tau(t) & = & A\hat{x}_\tau(t) + Bu_\tau(t) - G_l e_y + G_n \nu \\ e_y(t) & = & \hat{y}_R(t) - y_R(t) \\ \hat{y}_R(t) & = & C\hat{x}_\tau(t) \end{cases} \tag{47}$$

where $\hat{x}_\tau(t) = \hat{x}(t - n_s h)$ is the estimate of the delayed state and $u_\tau(t) = u(t - n_s h)$.

The discontinuous injection term ν is given by:

$$\nu = -\rho \frac{P_0 e_y}{\|P_0 e_y\|} \tag{48}$$

The gain ρ is designed so that a sliding mode $e_y(t) \equiv 0$ is reached in finite time. Matrices $P_0 > 0$, G_l and G_n are designed as in Section 2.

3.3. Sampled-data reconstruction methods

The simplest reconstruction method is the zero-order hold (ZOH), in which the value presented by the system is held constant until a new value is available:

$$y_{ZOH}(t) = y(kh), \quad \forall t \in [kh, (k+1)\,h)\,. \tag{49}$$

Since this filter is causal and of simple implementation, it is commonly used in most applications.

Another causal method is the first-order hold (FOH):

$$y_{FOH}(t) = y\,(kh) + [y\,(kh) - y\,((k-1)\,h)]\,\frac{t - kh}{h}, \qquad \forall t \in [kh, (k+1)\,h)\,. \tag{50}$$

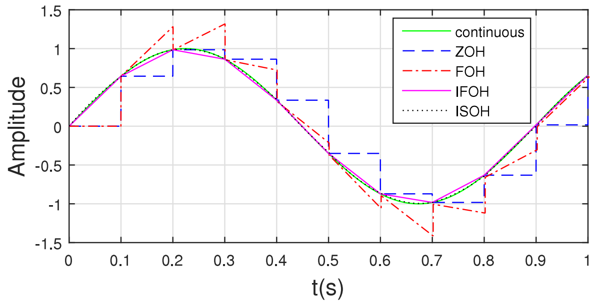


Fig. 2. Comparison between ZOH, FOH, IFOH, and ISOH reconstruction methods for the signal $x = \sin(5t)$ sampled at a 0.1 s period.

The interpolation first-order hold (IFOH) method can also be used to reconstruct sampled data, by means of the linear interpolation between the current sample and the next one (Cleveland, 1976). Since both samples are part of the interpolation, the error is zero at every sample and there are no discontinuous jumps. On the other hand, because of its dependency from the next sample, the IFOH is a non-causal process. However, if a time-delay of one sample is allowed, the IFOH it can be implemented as a causal algorithm:

$$y_{IFOH}(t) = y\left((k-1)\,h\right) + \left[y\left(kh\right) - y\left((k-1)\right)h\right]\frac{t-kh}{h}, \quad \forall t \in \left[kh, (k+1)\,h\right). \tag{51}$$

In Zhang and Chong (2007), a second-order hold (SOH) based on Taylor series expansion to increase reconstruction accuracy is presented. This SOH is also extrapolative and presents discontinuous jumps at every new sample.

Here, we propose the use of an interpolation second-order hold (ISOH), by adding a two-sample time delay and using Lagrange's polynomial interpolation (Süli & Mayers, 2003) to reconstruct the signal. Since all sample points in the interval are used for interpolation, there will be no discontinuous jumps. The expression for ISOH can be written in a matrix form as:

$$y_{ISOH}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & h & h^2 \\ 1 & 2h & (2h)^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} y\left((k-2)\,h\right) \\ y\left((k-1)\,h\right) \\ y\left(kh\right) \end{bmatrix} \cdot \begin{bmatrix} 1 & t & t^2 \end{bmatrix}, \quad \forall t \in \left[kh, (k+2)\,h\right) \tag{52}$$

Fig. 2 presents a sampled sinusoidal signal reconstructed using the interpolation methods compared to the original signal and to the ZOH.

3.4. Influence of the reconstruction error

From (46) and (47), the state observation error $e(t) = \hat{x}_\tau(t) - x_\tau(t)$ is governed by:

$$\dot{e}(t) = Ae(t) - G_l e_y(t) + G_n \nu - Ff_\tau(t), \tag{53}$$

where $x_\tau(t) = x(t - n_s h)$ and $f_\tau = f(t - n_s h)$, are the delayed state and the delayed fault signal, respectively.

Applying the change of coordinates $\bar{e} = T_L e(t)$ as in (15), the error dynamics can be written as:

$$\begin{cases} \dot{\bar{e}}_1(t) &= \mathcal{A}_{11}\bar{e}_1(t) + \mathcal{A}_{12}\bar{e}_2(t) - \mathcal{G}_{l,1}e_y(t) \\ \dot{\bar{e}}_2(t) &= \mathcal{A}_{21}\bar{e}_1(t) + \mathcal{A}_{22}\bar{e}_2(t) - \mathcal{G}_{l,2}e_y(t) \\ & \quad + \nu - \mathcal{F}_2 f_\tau(t) \end{cases} \tag{54}$$

From the structures of C in (4) and T_L in (15) it can be seen that $\bar{e}_2(t) = Ce(t)$. Then, from Eq. (47), it can be seen that:

$$e_y(t) = Ce(t) - e_r(t) = \bar{e}_2(t) - e_r(t) \tag{55}$$

$$\dot{e}_y(t) = \dot{\bar{e}}_2(t) - \dot{e}_r(t) \tag{56}$$

Substituting (55) and (56) into (54), one obtains:

$$\begin{cases} \dot{\bar{e}}_1(t) &= \mathcal{A}_{11}\bar{e}_1(t) + \mathcal{A}_{12}(e_y(t) + e_r(t)) \\ & \quad - \mathcal{G}_{l,1}e_y(t) \\ \dot{e}_y(t) + \dot{e}_r(t) &= \mathcal{A}_{21}\bar{e}_1(t) + \mathcal{A}_{22}(e_y(t) + e_r(t)) \\ & \quad - \mathcal{G}_{l,2}e_y(t) + \nu - \mathcal{F}_2 f(t) \end{cases} \tag{57}$$

Rearranging terms, the error dynamics are described as:

$$\begin{cases} \dot{\bar{e}}_1(t) &= \mathcal{A}_{11}\bar{e}_1(t) + (\mathcal{A}_{12} - \mathcal{G}_{l,1})e_y(t) + \mathcal{A}_{12}e_r \\ \dot{e}_y(t) &= \mathcal{A}_{21}\bar{e}_1(t) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})e_y(t) + \nu \\ & \quad - \mathcal{F}_2 f(t) + \mathcal{A}_{22}e_r(t) - \dot{e}_r(t) \end{cases} \tag{58}$$

To analyze the influence of the reconstruction error, the following assumption will be adopted:

(A10) For the n-order interpolation reconstruction filter, the output $y(t)$ and its $(n+1)$ derivative, $y^{(n+1)}(t)$, are continuous in the interval $t \in [(k-n)h,\ kh]$ and $y^{(n+1)}(t)$ is norm bounded by a known constant $\bar{y}^{(n+1)}$,

$$\left\| \frac{d^{(n+1)}y(t)}{dt^{(n+1)}} \right\| \le \bar{y}^{(n+1)}. \tag{59}$$

Then, a bound for the reconstruction error e_r(45) for a polynomial interpolation of order n can be determined by Süli and Mayers (2003, Theorem 6.2):

$$\|e_r(t)\| \le \frac{M_{n+1}}{(n+1)!}\pi_{n+1}(t), \quad \forall t \in [(k-n)h, kh], \tag{60}$$

where

$$\pi_{n+1}(t) = (t-kh)(t-(k-1)h)\dots(t-(k-n)h), \quad M_{n+1} = \max_{\zeta \in ((k-n)h, kh)} \|y^{(n+1)}(\zeta)\|.$$

Also, for $n \ge 1$, a bound for $\dot{e}_r(t)$ can be written as (Süli & Mayers, 2003, Corollary 6.1):

$$\|\dot{e}_r(t)\| \le \frac{(nh)^n M_{n+1}}{n!}, \quad \forall t \in [(k-n)h, kh]. \tag{61}$$

For the zero-order hold (ZOH), applying $n = 0$, it can be seen that:

$$\|e_r(t)\| \le h \max_{\zeta \in (kh,(k+1)h)} \|\dot{y}(\zeta)\| \le h\bar{y}^{(1)}. \tag{62}$$

The derivative of the reconstruction error $\dot{e}_r$ is not norm bounded in the ZOH because the error shows a discontinuous jump at every new sample.

In the case of the interpolation first-order hold (IFOH), the reconstruction error is given by:

$$\|e_r(t)\| \le \frac{h^2}{8} \max_{\zeta \in ((k-1)kh, kh)} \|\ddot{y}(\zeta)\| \le \frac{h^2}{8}\bar{y}^{(2)}, \tag{63}$$

$$\|\dot{e}_r(t)\| \le h\bar{y}^{(2)}. \tag{64}$$

For the interpolation second-order hold (ISOH), we have:

$$\|e_r(t)\| \le \frac{h^3}{9\sqrt{3}} \max_{\zeta \in ((k-2)kh, kh)} \|y^{(3)}(\zeta)\| \le \frac{h^3}{9\sqrt{3}}\bar{y}^{(3)}, \tag{65}$$

$$\|\dot{e}_r(t)\| \le 2h^2\bar{y}^{(3)}. \tag{66}$$

Since the relative degree from $f(t)$ to $y(t)$ is one, differentiating the output $y(t)$ an upper bound can be obtained from the norm bound of $f(t)$ in (A3). Consequently, the nth derivatives of $y(t)$ can be norm bounded by the norm bounds for the $(n-1)$ derivatives of $f(t)$, which exist for sufficiently smooth faults. Considering that the relative degree from $u(t)$ to $y(t)$ is n^*, an upper bound can be obtained from the norm bound for the $\max(0, n - n^*)$ derivatives of $u(t)$, where order 0 means just $u(t)$ bounded.

To determine a modulation function ρ that allows a sliding mode to be reached on the set $\mathcal{S} = \{e_y : e_y = Ce = 0\}$, a First

Order Approximation Filter (FOAF) can be used to estimate an upper bound for the state $\bar{e}_1$ of the error system (58).

Under assumption **(A10)**, an instantaneous upper bound for the reconstruction error $\|e_r(t)\| \leq \bar{e}_r$ is known and from (60), its derivative is also bounded, $\|\dot{e}_r(t)\| \leq \bar{\dot{e}}_r$. Let γ_0 be the stability margin of $G_y(s) \triangleq (sI - \mathcal{A}_{11})^{-1}\left(\mathcal{A}_{12} - \mathcal{G}_{l,1}\right)$ and γ_R be the stability margin of $G_R(s) \triangleq (sI - \mathcal{A}_{11})^{-1}\mathcal{A}_{12}$. From **(A8)**, it can be assumed that $\gamma_0 \leq \gamma_R$. Let $\gamma \triangleq \gamma_0 - \delta_0$, with $\delta_0 > 0$ being an arbitrary constant. Applying (Hsu, Costa, & da Cunha, 2003, Lemma 2) to the equation of $\bar{e}_1(t)$ in (58), it can be shown that:

$$\|\bar{e}_1\| \leq \beta_{e_1} \triangleq e^{-\gamma t} * \left(c_1\|e_y(t)\| + c_2\bar{e}_r(t)\right) + c_3 e^{-(\lambda_0-\delta_0)}\|e_1(0)\| \tag{67}$$

where $\lambda_0 := \min_j\{-Re(\lambda_j)\}$ is the stability margin of $\mathcal{A}_{11}$, and

$$c_1 = \frac{\|P_e(\mathcal{A}_{12} - \mathcal{G}_{l,1})\|}{\lambda_{\max}(P_e)},\quad c_2 = \frac{\|P_e\mathcal{A}_{12}\|}{\lambda_{\max}(P_e)},\quad c_3 = \sqrt{\frac{\lambda_{\max}(P_e)}{\lambda_{\min}(P_e)}}, \tag{68}$$

where $P_e > 0$ satisfies the Lyapunov equation $(\mathcal{A}_{11}^T P_e + P_e\mathcal{A}_{11}) = -2I$.

Since $\bar{e}_1$ is bounded, a sliding mode $e_y \equiv 0$ can be achieved in finite time. Then a reconstruction of the actuator fault can be obtained, added by a filtered signal from the reconstruction error. This highlights the importance of choosing an appropriate order for the interpolation filter to minimize reconstruction error.

Now, for a sampled-output system (44) with sampling period h and the number of samples n_s corresponding to the total time-shift $\tau = n_s h$ required by the continuous-time output reconstruction algorithm (see above (45)), the following theorem can be stated.

Theorem 3. *Consider the observer (47) designed to estimate the states of the plant (46), satisfying **(A1)** to **(A6)**, and the observer error dynamics in (58). Under assumption **(A10)**, the reconstruction error is bounded by $\|e_r(t)\| \leq \bar{e}_r$ from (60) and its derivative is also bounded by $\|\dot{e}_r(t)\| \leq \bar{\dot{e}}_r$ according to (61). Choosing the modulation function of the discontinuous term ν (48) of the observer to satisfy*

$$\rho \geq \|\mathcal{A}_{21}\|\beta_{e_1} + \|\mathcal{A}_{22}\|\bar{e}_r + \|\mathcal{F}_2\|\alpha + \bar{\dot{e}}_r + \eta, \tag{69}$$

modulo exponentially decaying terms due to the transient terms of the FOAFs in (67), and $\eta > 0$ being arbitrary constant, then an ideal sliding mode will be reached on the set $\mathcal{S} = \left\{e_y \in \mathbb{R}^p : e_y = 0\right\}$ in finite time, where $e_y(t)$ is given by (47). Moreover, on the sliding mode the delayed fault $f_\tau(t) = f(t - n_s h)$ can be estimated by :

$$\hat{f}_\tau(t) = \mathcal{F}_2^\dagger \nu_\delta \tag{70}$$

as in Corollary 1. Such estimate obeys, with arbitrary accuracy as $\delta \to 0$, the expression:

$$\hat{f}_\tau(t) = f_\tau(t) - \mathcal{F}_2^\dagger\left(\mathcal{A}_{21}\bar{e}_1(t) + \mathcal{A}_{22}e_r - \dot{e}_r\right). \tag{71}$$

Proof. Consider the Lyapunov candidate function for the output observation error $V_y = e_y^T P_0 e_y$, where $P_0 > 0$ is defined in (18). From assumption **(A8)** and considering the structure of P in (18), P_0 is a solution for the Lyapunov equation $P_0\left(\mathcal{A}_{22} - \mathcal{G}_{l,2}\right) + \left(\mathcal{A}_{22} - \mathcal{G}_{l,2}\right)^T P_0 = -I$. The derivative of V_y along the trajectories of (58) is:

$$\dot{V}_y = -e_y^T e_y + 2e_y^T P_0\left(\mathcal{A}_{21}\bar{e}_1 + \mathcal{A}_{22}e_r + \mathcal{F}_2 f_\tau - \dot{e}_r\right) - 2\rho\|e_y^T P_0\| \tag{72}$$

Applying the bounds β_{e_1} in (67), $\bar{e}_r$ in (60), and $\bar{\dot{e}}_r$ in (61), and α from assumption **(A3)**, the following inequality can be written:

$$\dot{V}_y \leq -\|e_y\|^2 + 2\|P_0 e_y\|\left(\|\mathcal{A}_{21}\|\beta_{e_1} + \|\mathcal{A}_{22}\|\bar{e}_r + \|\mathcal{F}_2\|\alpha + \bar{\dot{e}}_r - \rho\right) \tag{73}$$

It can be seen that, if ρ is given by (69), (73) becomes:

$$\dot{V}_y \leq -\|e_y\|^2 - 2\eta\|P_0 e_y\| \tag{74}$$

showing that the set $\mathcal{S}$ is reached in finite time.

When the sliding mode $e_y \equiv \dot{e}_y \equiv 0$ is initiated, from (58) one has:

$$\nu_{eq} = -\mathcal{A}_{21}\bar{e}_1(t) + \mathcal{F}_2 f_\tau(t) - \mathcal{A}_{22}e_r + \dot{e}_r \tag{75}$$

Then according to Corollary 1, Section 2.3, the fault can be estimated from the equivalent control using

$$\hat{f}_\tau(t) = \mathcal{F}_2^\dagger \nu_\delta. \tag{76}$$

Rearranging the terms of (75), one obtains the expression for the reconstructed fault (71), as $\delta \to 0$. □

From this result it can be seen that a filtered signal from the reconstruction error will additively influence the fault estimate signal. Note that in the sliding mode $e_y \equiv 0$, from (58)$\dot{\bar{e}}_1(t) = \mathcal{A}_{11}\bar{e}_1 + \mathcal{A}_{12}e_r(t)$, so $\bar{e}_1$ depends only on the reconstruction error *modulo* an exponentially decaying term.

The reconstruction error e_r and its derivative $\dot{e}_r$ can be made arbitrarily small by choosing a small sampling period h in (60) and (61). Therefore, it can be seen from (71) that $\hat{f}_\tau(t)$ approximates $f_\tau(t)$ as $t \to +\infty$.

Using n_s samples, it is possible to use a polynomial of degree n_s by the same method. In fact, it can also be used with splines or other interpolation methods considering the effect of the reconstruction error in Theorem 3.

3.5. Numerical example

Case 1: Consider a system with the same dynamics of the example of Section 2.5, but now with sampled output as in (44). Let the system be disturbed with a sinusoidal fault signal. An observer (47) is designed with the same G_l and G_n as in Section 2.5 since these gains do not depend on the delay duration. A constant ρ is selected so that the inequality (69) is satisfied. Numerical simulations were performed for sampling periods of 20 ms, 60 ms and 100 ms, comparing the performance of the observer for the three reconstruction filters proposed (zero-order hold (ZOH), interpolation first-order hold (IFOH) and interpolation second-order hold (ISOH)). In order to obtain a quantitative comparison, the Root Mean Squared Error Percentage (RMSE%) and the Mean Absolute Error Percentage (MAE%) are defined as:

$$RMSE\%_{f_\tau(t)} = \frac{\sqrt{\mathbb{E}\left[(f_\tau(t) - \hat{f}_\tau(t))^2\right]}}{\sqrt{\mathbb{E}\left[f_\tau^2(t)\right]}} \times 100\%, \tag{77}$$

$$MAE\%_{f_\tau(t)} = \frac{\mathbb{E}\left[\left|f_\tau(t) - \hat{f}_\tau(t)\right|\right]}{\mathbb{E}\left[|f_\tau(t)|\right]} \times 100\%. \tag{78}$$

Table 1 shows that the ISOH (second order filter) presents better performance in the fault reconstruction of the fault. Figs. 3 to 5 present the fault reconstruction for all developed strategies, assuming 60 ms sampling period.

Case 2: In order to compare the reconstruction method presented in this paper with the existing literature, simple system was used. Consider a first-order system:

$$\dot{x}(t) = -5x(t) + u(t) + f(t) \tag{79}$$

$$y_k = x(kh) \tag{80}$$

with a sampling time $h = 50$ ms, and a fault signal $f(t) = 0.6\sin(2t)$.

Table 1
Error metrics for the reconstructed delayed fault in Case 1.

Hold type	h (ms)	RMSE%	MAE%
ZOH	20	37.02	30.72
	60	57.03	52.26
	100	71.16	61.60
IFOH	20	2.00	1.78
	60	6.52	6.07
	100	12.51	12.00
ISOH	20	0.21	0.18
	60	1.07	0.94
	100	3.32	3.15

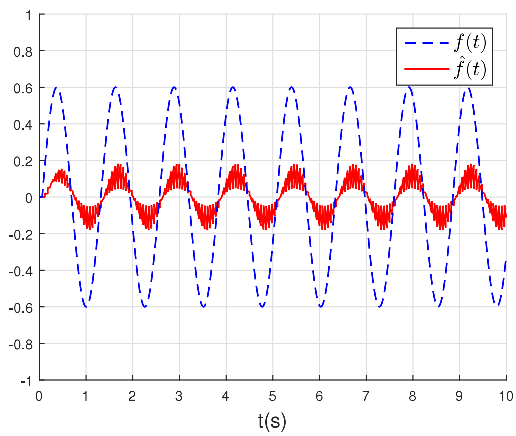


Fig. 3. Fault reconstruction of a sampled data system with 60 ms sampling time using a zero-order hold.

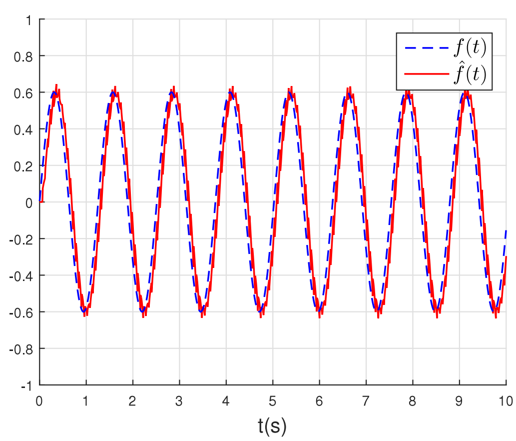


Fig. 4. Fault reconstruction of a sampled data system with 60 ms sampling time using an interpolation first-order hold.

The fault reconstruction is performed using the method presented in Section 2.3, with ISOH (52) as the output reconstruction filter:

$$\dot{x}(t) = -5x(t) + u(t) + f(t) \tag{81}$$

$$y_{ISOH}(t) = x(t-0.1) + e_r(t). \tag{82}$$

with $n_s = 2$ and $\tau = 0.1$. The observer is given by:

$$\begin{cases} \dot{\hat{x}}_\tau(t) &= -5\hat{x}_\tau(t) + u_\tau(t) - 2e_y + \nu \\ e_y(t) &= \hat{y}_{ISOH}(t) - y_{ISOH}(t) \\ \hat{y}_{ISOH}(t) &= \hat{x}_\tau(t) \end{cases} \tag{83}$$

where the discontinuous injection term $\nu = -2e_y/\|e_y\|$. The reconstruction is performed using (21) which in this example is

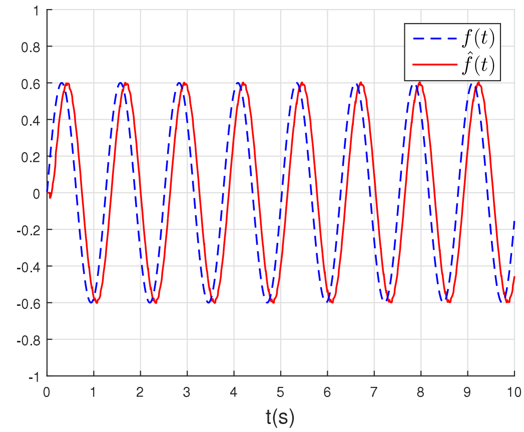


Fig. 5. Fault reconstruction of a sampled data system with 60 ms sampling time using an interpolation second-order hold.

given by $\hat{f}_\tau(t) = \nu_\delta$ where $\nu_\delta = -2e_y/(\|e_y\| + \delta)$, with $\delta = 10^{-4}$. Fig. 6(a) presents that the reconstructed signal is continuous, it reproduces closely the fault signal delayed by $\tau = n_s h = 0.1$ after some initial transient.

As a comparison result, an observer in the form proposed in Han et al. (2014) was also tested:

$$\dot{\hat{x}}(t) = -5\hat{x} + \frac{1}{\mu}\left(x(kh) - \hat{x}(kh)\right) - \nu(kh) \tag{84}$$

$$\hat{y}_k = x(kh) \tag{85}$$

$$\nu(kh) = -M_\beta \text{sign}\left(x(kh) - \hat{x}(kh)\right) \tag{86}$$

$$M_\beta = \alpha + \delta_1\alpha \tag{87}$$

$$\|f(t)\| \le \alpha, \quad \alpha > 0. \tag{88}$$

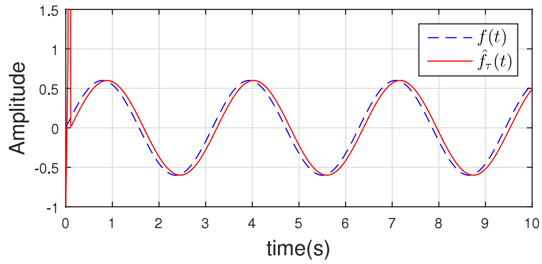
The sampling kh is implemented as a time-varying delay, where $t - \tau(t) = kh$, $\forall t \in [kh, (k+1)h)$. Using the method in Han et al. (2014), the observer is feasible with $\mu = 0.05$ and $\delta_1 = 0.1$. The fault reconstruction shown in Fig. 6(b) is performed by Han et al. (2014, eq. (31)):

$$\hat{f}_\tau(t) = \nu_\delta(kh) - \frac{1}{\mu}\left(x(kh) - \hat{x}(kh)\right) \tag{89}$$

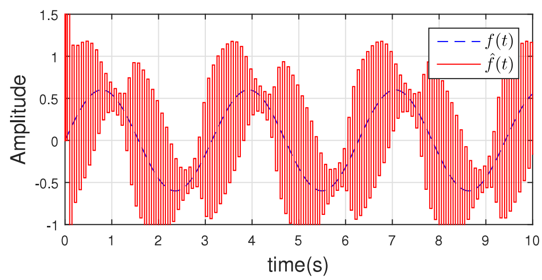
where $\nu_\delta = -2e_y/(\|e_y\| + \delta)$, with $\delta = 0.01$. The result presents significant chattering that could only be reduced by removing the discontinuous term $\nu(kh)$, in which case the observer becomes purely linear. The method presented in this paper manipulates the system so that there is no delay in the observer feedback loop, allowing it to achieve an ideal sliding mode, avoiding chattering.

4. Conclusion

A time-shift approach to design sliding mode observer has been proposed to reconstruct actuator fault signals for systems with arbitrary measurement delays. By the proposed method, ideal sliding mode can be theoretically achieved, thus avoiding chattering problems even for arbitrarily long time-varying delay. The price to be paid is that the reconstructed fault signal is also delayed by the corresponding time-shift, which is acceptable for certain relevant applications, *e.g.*, for space or deep water systems when large communication delays are expected. A fault reconstruction method for sampled-output systems was shown to be possible using the time-shift approach together with continuous-time interpolation methods. Simulations and comparison with existing methods illustrate the efficacy of the new method.



(a) Comparison between the fault signal and its estimate using the method presented in section 2.3 using the ISOH.



(b) Comparison between the fault signal and its estimate using the method presented in (Han et al., 2014).

Fig. 6. Comparison of the reconstruction of a sinusoidal fault signal on a first order system.

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