Graph traversal methods

- Narges BabaAhmadi
- Niloufar BabaAhmadi
- Zahra Khatibi
- Mahyar Mohammadi Matin
- Niko Rokni

Teacher : Dr.M.Nori

TA : Sh.Peyghambari

Summer 202۰

Introduction

Graph traversal is a technique used for a searching vertex in a graph. The graph traversal is also used to decide the order of vertices is visited in the search process. A graph traversal finds the edges to be used in the search process without creating loops. That means using graph traversal we visit all the vertices of the graph without getting into looping path.

There are two graph traversal techniques and they are as follows :

1. DFS (Depth First Search)
2. BFS (Breadth First Search)

Here in this article we are going to explain both of these techniques plus their differences and where each of them is being used.

DFS :

Depth-first search (**DFS**) is an algorithm for traversing or searching tree or graph data structures. The algorithm starts at the root node (selecting some arbitrary node as the root node in the case of a graph) and explores as far as possible along each branch before **backtracking.** Here, the word backtrack means that when you are moving forward and there are no more nodes along the current path, you move backwards on the same path to find nodes to traverse. All the nodes will be visited on the current path till all the unvisited nodes have been traversed after which the next path will be selected.

Time and space :

The time and space analysis of DFS differs according to its application area. In theoretical computer science, DFS is typically

used to traverse an entire graph, and takes time O(|v| + |E|), linear in the size of the graph. In these applications it also uses space O(|v|) in the worst case to store the stack of vertices on the current search path as well as the set of already-visited vertices.

- **Usage :**

  For applications of DFS in relation to specific domains, such as searching for solutions in artificial intelligence or web-crawling, the graph to be traversed is often either too large to visit in its entirety or infinite (DFS may suffer from non-termination). In such cases, search is only performed to a limited depth; due to limited resources, such as memory or disk space, one typically does not use data structures to keep track of the set of all previously visited vertices. When search is performed to a limited depth, the time is still linear in terms of the number of expanded vertices and edges (although this number is not the same as the size of the entire graph because some vertices may be searched more than once and others not at all) but the space complexity of this variant of DFS is only proportional to the depth limit, and as a result, is much smaller than the space needed for searching to the same depth using breadth-first search.

- **DFS algorithm :**

  A standard DFS implementation puts each vertex of the graph into one of two categories:

  1. Visited
  2. Not Visited

The purpose of the algorithm is to mark each vertex as visited while avoiding cycles.

The DFS algorithm works as follows:

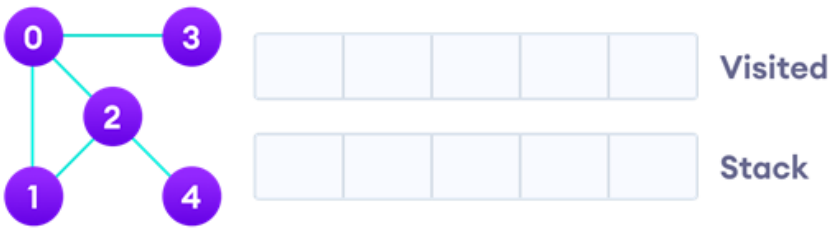
1. Start by putting any one of the graph's vertices on top of a stack.
2. Take the top item of the stack and add it to the visited list.
3. Create a list of that vertex's adjacent nodes. Add the ones which aren't in the visited list to the top of the stack.

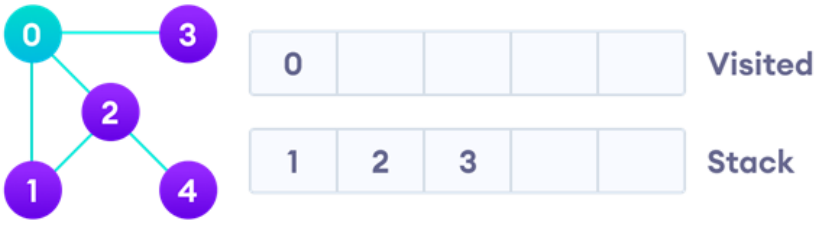
Keep repeating steps 2 and 3 until the stack is empty.

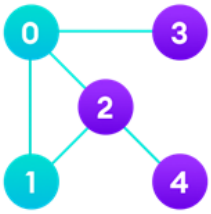
Here's a code that we prepare that calculate all ways between two vertex: (http://s13.picofile.com/file/8403847168/DFS.cpp.html)

- **DFS example:**

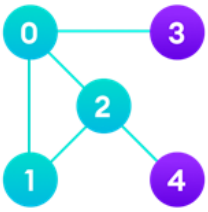
Let's see how the Depth First Search algorithm works with an example. We use an undirected graph with 5 vertices.



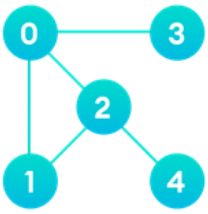




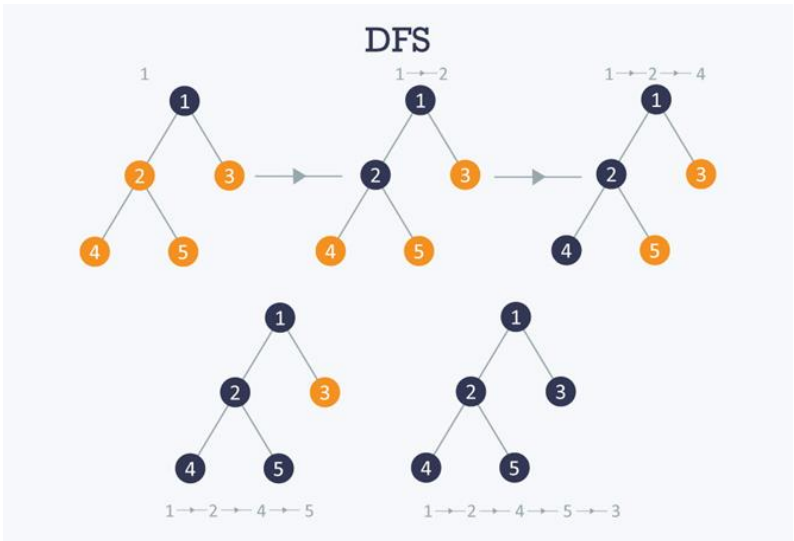
0	1				Visited
2	3				Stack



0	1	2			Visited
4	3				Stack



0	1	2	4	3	Visited
					Stack



DFS Algorithm Applications:

1. For finding the path (in a tree)
2. To test if the graph is bipartite
3. For detecting cycles in a graph
4. solving a maze using DFS

1. finding the path in a tree :

Use DFS but we cannot use visited [] to keep track of visited vertices since we need to explore all the paths. visited [] is used avoid going into cycles during iteration. (That is why we have a condition in this problem that graph does not contain cycle)

1. Start from the source vertex and make a recursive call to all it adjacent vertices.
2. During recursive call, if reach the destination vertex, increment the result (no of paths).
3. See the code in the link below for more understanding.

(https://algorithms.tutorialhorizon.com/graph-count-all-paths-between-source-and-destination/)

2. test if a graph is bipartite :

Given a connected graph, check if the graph is bipartite or not. A bipartite graph is possible if the graph coloring is possible using two colors such that vertices in a set are colored with the same color.

- It is not possible to color a cycle graph with an odd cycle using two colors.
- Use a *color[]* array which stores 0 or 1 for every node which denotes opposite colors.

- Call the function DFS from any node.
- If the node u has not been visited previously, then assign! color[v] to color[u] and call DFS again to visit nodes connected to u.
- If at any point, color[u] is equal to !color[v], then the node is bipartite.
- Modify the DFS function such that it returns a boolean value at the end.

See the code in the link below:
(https://www.geeksforgeeks.org/check-if-a-given-graph-is-bipartite-using-dfs)

- **3.detecting cycles in a graph:**

- **Detecting cycle in directed graphs using depth-first search (DFS) algorithm:**

  Cycle in directed graphs can be detected easily using a depth-first search traversal. While doing a depth-first search traversal, we keep track of the nodes visited in the current traversal path in addition to the list of all the visited nodes. During the traversal if we visit a node that was already in the current path of the traversal a cycle is found.

  Algorithm :

  1.Mark the source_node as visited.

  2.Mark the source_node as in_path node.

  3.For all the adjacent nodes to the source_node do

  4.If the adjacent node has been marked as in_path node.

5.Cycle found. Return.

6.If the adjacent node has not been visited

7.DFS Detect Cycle In A Directed Graph (adjacent_node)

8.Now that we are back-tracking unmark the source_node in in_path as it might be revisited.

(https://algotree.org/algorithms/tree_graph_traversal/dfs_detecting_cycles_in_graphs/cycle_detection_in_directed_graphs/)

- **Detecting cycle in an undirected graph using depth-first search (DFS) algorithm:**

  Cycle in undirected graphs can be detected easily using a depth-first search traversal. While doing a depth-first search traversal, we keep track of the visited node's parent along with the list of visited nodes. During the traversal, if an adjacent node is found visited that is not the parent of the source node, then we have found a cycle in the current path.

  Algorithm :

  1.Mark the source_node as visited.

  2.For all the adjacent nodes to the source_node do

  3.If the adjacent_node has not been visited

  4.Set parent [adjacent_node] = source_node.

  5.DFS Detect Cycle In An Undirected Graph (adjacent_node)

  6.Else If the parent[source_node] != adjacent_node

  7.Cycle found. Return.

  (https://algotree.org/algorithms/tree_graph_traversal/dfs_detecting_cycles_in_graphs/cycle_detection_in_undirected_graphs/)

- **4.solving a maze using DFS :**

  Depth-first search is a common way that many people naturally approach solving problems like mazes. First, we select a path in the maze (for the sake of the example, let's choose a path according to some rule we lay out ahead of time) and we follow it until we hit a dead end or reach the finishing point of the maze. If a given path doesn't work, we backtrack and take an alternative path from a past junction, and try that path. Below is an animation of a DFS approach to solving this maze.You can see the code in the link below:

  (https://brilliant.org/wiki/depth-first-search-dfs/)

- **BFS :**

one of the simplest and most basic algorithms for navigating and searching the graph is "first-level search algorithm (first search) or BFS".
This algorithm has lots of usage. We can also state that many other algorithms for different kinds of problems , like finding the shortest way ,use the similar algorithm .
the reason of its name is that a boundary is drawn between the traversed vertices, and the traversed vertices are uniformly drawn along the crossing boundary level. In other words, you will visit all vertices with distance k from vertex x before vertices with distance K + 1 from vertex x.
from the name of the representative algorithm, we can understand that layers are visited one by one ,and each vertex goes to queue after getting visited.

- **How BFS works:**

  The algorithm starts at the root [in graphs and trees without roots, the arbitrary vertex is selected as the root] and sets it to zero. At each stage, it puts all unvisited neighbors of the lastly visited vertex to the next level. This process ends when all neighbors of the vertices of the last level have been seen.

  To investigate this, we can use a binary array .we first create an array with the same size as the number of vertices of the graph and set all its elements to zero. By scrolling and visiting each vertex, we change its value from zero to one in our array .The algorithm continues until the array contains no zero.

  In some cases, solving the problem requires finding a specific vertex. In this case, the algorithm visits all the neighbors of one vertex each time and then goes to the next vertex, and thus the graph is scrolled level by level. This search continues until the specific vertex is found. A queue is used to implement this algorithm.

  First, the root is placed in the queue, then at each step, the initial element of the queue is pulled out, its neighbors are checked, and any neighbor that has not been visited yet , is added to the queue (this can be done with the help of an array Examined the aforementioned binary).

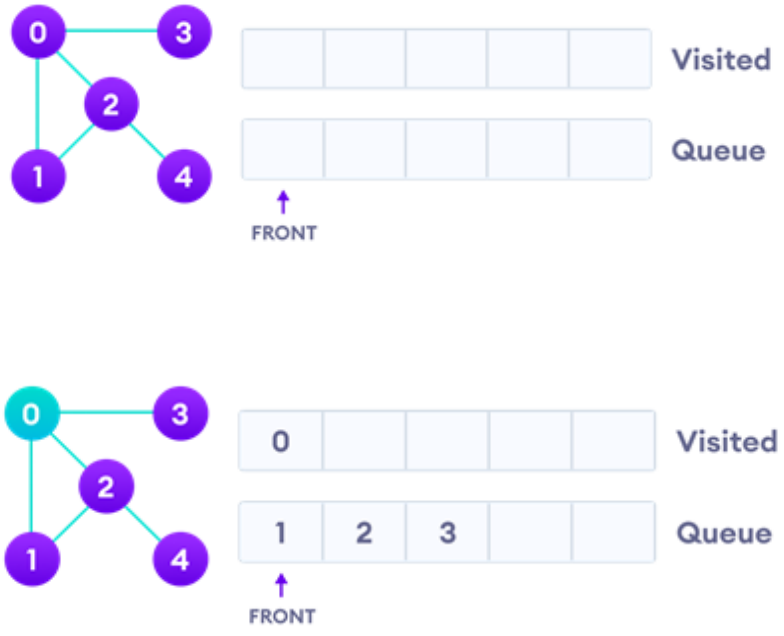
  For better visualization, the BFS algorithm can be explained as the spread of fire in a graph. In this example, the first vertex catches fire first, and at each stage it burns all neighbors of lastly visited vertex on the next floor.

  Here's a code that we prepare that calculate shortest way between two vertex : (http://s12.picofile.com/file/8403846950/BFS.cpp.html)

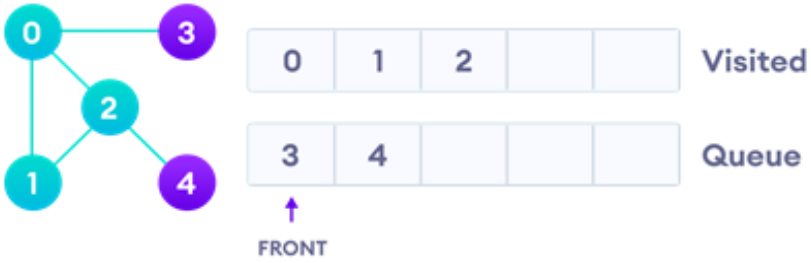
- **Temporal complexity :**

A vertex enters the queue if it is not visited, so each vertically accessible vertex enters the queue exactly once. Each time a vertex enters the queue, O(1) is performed, and assuming that the graph is connected, the queue operations will take O(n). Also, each edge in the directionless graph, is traversed exactly twice, and each edge in a directional graph is traversed exactly once .If the graph is not connected, the time complexity of the order is $O(m_s + n_s)$, where n_s and m_s are the number of vertices and the number of edges that are accessible from s.

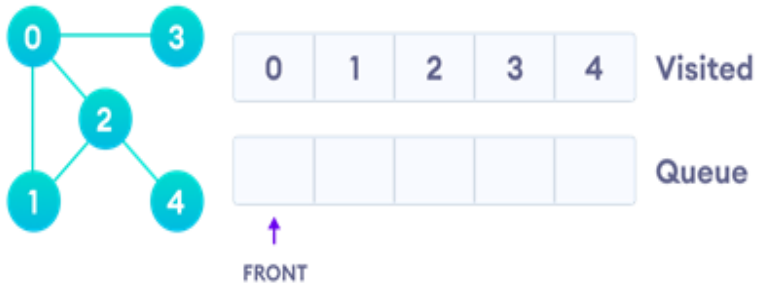
- **BFS example**




0
3
2
1
4
0
1
Visited
2
3
Queue
FRONT


0
3
2
1
4
0
1
2
Visited
3
4
Queue
FRONT


0
3
2
1
4
0
1
2
3
Visited
4
Queue
FRONT


0
3
2
1
4
0
1
2
3
4
Visited
Queue
FRONT

Difference between BFS and DFS:

- **Difference between BFS and DFS Binary Tree**

BFS	**DFS**
BFS finds the shortest path to the destination.	DFS goes to the bottom of a subtree, then backtracks.
The full form of BFS is Breadth-First Search.	The full form of DFS is Depth First Search.
It uses a queue to keep track of the next location to visit.	It uses a stack to keep track of the next location to visit.
BFS traverses according to tree level.	DFS traverses according to tree depth.
It is implemented using FIFO list.	It is implemented using LIFO list.
It requires more memory as compared to DFS.	It requires less memory as compared to BFS.
This algorithm gives the shallowest path solution.	This algorithm doesn't guarantee the shallowest path solution.

There is no need for backtracking in BFS.	There is a need for backtracking in DFS.
You can never be trapped into finite loops.	You can be trapped into infinite loops.
If you do not find any goal, you may need to expand many nodes before the solution is found.	If you do not find any goal, the leaf node backtracking may occur.

- **Applications of BFS:**

  - **1.Un-weighted Graphs:**

    The BFS algorithm can easily create the shortest path and a minimum spanning tree to visit all the vertices of the graph in the shortest time possible with high accuracy.

  - **2.P2P Networks:**

    BFS can be implemented to locate all the nearest or neighboring nodes in a peer to peer network. This will find the required data faster.

- **3.Web Crawlers:**

  Search engines or web crawlers can easily build multiple levels of indexes by employing BFS. BFS implementation starts from the source, which is the web page, and then it visits all the links from that source.

- **4.Network Broadcasting:**

  A broadcasted packet is guided by the BFS algorithm to find and reach all the nodes it has the address for.

Applications of DFS:

- **Weighted Graph:**

  In a weighted graph, DFS graph traversal generates the shortest path tree and minimum spanning tree.

- **Path Finding:**

  We can specialize in the DFS algorithm to search a path between two vertices.

- **Detecting a Cycle in a Graph:**

  A graph has a cycle if we found a back edge during DFS. Therefore, we should run DFS for the graph and verify for back edges.

- **Searching Strongly Connected Components of a Graph:**

  It is used in DFS graphs when there is a path from each and every vertex in the graph to other remaining vertices.

- **Topological Sorting:**

  It is primarily used for scheduling jobs from the given dependencies among the group of jobs. In computer science, it is used in instruction scheduling, data serialization, logic synthesis, determining the order of compilation tasks.

- **Solving Puzzles with Only One Solution:**

  The DFS algorithm can be easily adapted to search all solutions to a maze by including nodes on the existing path in the visited set.

❖ at last, we answer this problem in codeforces about Graph that solved with recursive DFS: http://s12.picofile.com/file/8403847200/FinalCode.cpp.html

Reference :

- geeksforgeeks.org
- programiz.com
- research gate
- algorithm.tutorialhorizon.com
- Algotree.org
- Brilliant.org
- wikipedia.org