Robotic Vision HW3

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I want detailed feedback.

Part 1:

Task 1.1:

K & G : same origin → the vector between p & o is the same direction & magnitude.

Task 1.2:

diffrent origins → starting point is diffrent → direction & magnitude diffrent.

Part 2:

Task 2.1:

$$\vec{r}' = R(\theta) \cdot \vec{r}$$

$$R_{K \to G} := \vec{K}\,\vec{G} = \begin{bmatrix} \vec{k_1} \cdot \vec{g_1} & \vec{k_1} \cdot \vec{g_2} & \vec{k_1} \cdot \vec{g_3} \\ \vec{k_2} \cdot \vec{g_2} & \vec{k_2} \cdot \vec{g_2} & \vec{k_2} \cdot \vec{g_3} \\ \vec{k_3} \vec{g_1} & \vec{k_3} \cdot \vec{g_2} & \vec{k_3} \cdot \vec{g_3} \end{bmatrix} = \begin{bmatrix} 1\times1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$k_i \cdot g_i = |k_i| \times |g_i| \times \cos\theta$$

$$\left.\begin{matrix} k_1 \cdot g_2 \\ k_1 \cdot g_3 \\ k_2\ g_2 \\ k_3 g_1 \end{matrix}\right\} 90^\circ \quad \cos(90) = 0 \to 0$$

$$\vec{k_2} \cdot \vec{g_2} = 1 \times 1 \times \cos\theta = \cos\theta$$

$$\vec{k_2} \cdot \vec{g_3} = 1 \times 1 \cos(90+\theta) = -\sin(\theta)$$

$$\vec{k_3} \cdot \vec{g_2} = 1 \times 1 \times \cos(90-\theta) = \sin\theta$$

$$\vec{k_3}\ \vec{g_3} = 1 \times 1 \times \cos(\theta) = \cos\theta$$

Task 2.2:

$$\vec{r}' = R(\theta) \cdot \vec{r}$$

$$R_{K \to G} := \vec{K}\,\vec{G} = \begin{bmatrix} \vec{k_1} \cdot \vec{g_1} & \vec{k_1} \cdot \vec{g_2} & \vec{k_1} \cdot \vec{g_3} \\ \vec{k_2} \cdot \vec{g_2} & \vec{k_2} \cdot \vec{g_2} & \vec{k_2} \cdot \vec{g_3} \\ \vec{k_3} \vec{g_1} & \vec{k_3} \cdot \vec{g_2} & \vec{k_3} \cdot \vec{g_3} \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix}$$

$$\vec{k_i} \cdot \vec{g_i} = |k_i| \times |g_i| \times \cos\theta$$

$$\vec{k_1} \cdot \vec{g_1} = 1 \times 1 \times \cos\varphi = \cos\varphi$$

$$\left.\begin{matrix} \vec{k_2} \cdot \vec{g_2} \\ \vec{k_1} \cdot \vec{g_2} \\ \vec{k_2}\ \vec{g_3} \\ \vec{k_3} \vec{g_2} \end{matrix}\right\} 90^\circ \quad \cos(90) = 0 \to 0$$

$$\vec{k_2} \cdot \vec{g_2} = 1 \times 1 \cos(0) = 1$$

$$\vec{k_1} \cdot \vec{g_3} = 1 \times 1 \cos(180 - 2\varphi) = \sin\varphi$$

$$\vec{k_3} \cdot \vec{g_1} = 1 \times 1 \times \cos(\varphi - 90) - \sin\varphi$$

$$\vec{k_3}\ \vec{g_3} \quad 1 \times 1 \times \cos(\varphi) = \cos\varphi$$

$\vec{K_3}\cdot\vec{g_2}$

$$\vec{K_3}\cdot\vec{g_1} = 1\times1\times\cos(\varphi-90) - \sin\varphi$$
$$\vec{K_3}\,\vec{g_3} = 1\times1\times\cos(\varphi) = \cos\varphi$$

Task 2.3:

$$\vec{r}' = R(\theta)\cdot\vec{r}$$

$$R_{K\to G} := \vec{K}\,\vec{G} = \begin{bmatrix} \vec{k_1}\cdot\vec{g_1} & \vec{k_1}\cdot\vec{g_2} & \vec{k_1}\cdot\vec{g_3} \\ \vec{k_2}\cdot\vec{g_2} & \vec{k_2}\cdot\vec{g_2} & \vec{k_2}\cdot\vec{g_3} \\ \vec{k_3}\cdot\vec{g_1} & \vec{k_3}\cdot\vec{g_2} & \vec{k_3}\cdot\vec{g_3} \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{k_i}\cdot\vec{g_i} = |k_i|\times|g_i|\times\cos\theta$$

$$\left.\begin{matrix}\vec{k_1}\cdot\vec{g_3}\\ \vec{k_2}\cdot\vec{g_3}\\ \vec{k_3}\,\vec{g_1}\\ \vec{k_3}\cdot\vec{g_2}\end{matrix}\right\} 90^\circ \quad \cos(90) = 0 \to 0$$

$$\vec{K_1}\cdot\vec{g_1} = 1\times1\times\cos\varphi = \cos\varphi$$
$$\vec{K_3}\,\vec{g_3} = 1\times1\times\cos(0) = 1$$
$$\vec{K_2}\cdot\vec{g_2} = 1\times1\,\cos(180-2\varphi) = \sin\varphi$$
$$\vec{K_1}\cdot\vec{g_2} = 1\times1\times\cos(\varphi-90) - \sin\varphi$$
$$\vec{K_2}\,\vec{g_2} = 1\times1\times\cos(\varphi) = \cos\varphi$$

Task 2.4:

$$R_{\vec{K_1}}\left(\theta=\frac{\pi}{6}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\frac{\pi}{6}) & -\sin(\frac{\pi}{6}) \\ 0 & \sin(\frac{\pi}{6}) & \cos(\frac{\pi}{6}) \end{bmatrix} \qquad R_{\vec{K_2}}\left(\phi=\frac{\pi}{6}\right) = \begin{bmatrix} \cos(\frac{\pi}{6}) & 0 & \sin\frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{6} & 0 & \cos(\frac{\pi}{6}) \end{bmatrix}$$

$$R_{\vec{K_3}}\left(\phi=\frac{\pi}{6}\right) = \begin{bmatrix} \cos(\frac{\pi}{6}) & -\sin\frac{\pi}{6} & 0 \\ \sin(\frac{\pi}{6}) & \cos(\frac{\pi}{6}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{\vec{K_3}}(\phi)R_{\vec{K_2}}(\psi)R_{\vec{K_1}}(\theta) = \begin{bmatrix} 0.75 & -0.2165 & 0.625 \\ 0.433 & 0.875 & -0.2165 \\ -0.5 & 0.433 & 0.75 \end{bmatrix}$$

$$R_{\vec{K_1}}(\theta)R_{\vec{K_2}}(\varphi)R_{\vec{K_3}}(\phi) = \begin{bmatrix} 0.75 & -0.433 & 0.5 \\ 0.6495 & 0.625 & -0.433 \\ -0.125 & 0.6495 & 0.75 \end{bmatrix}$$

$$R_{\vec{K_3}}(\phi)R_{\vec{K_2}}(\psi)R_{\vec{K_1}}(\theta) \neq R_{\vec{K_1}}(\theta)R_{\vec{K_2}}(\varphi)R_{\vec{K_3}}(\phi)$$

Task 2.5:

→ first one: rotation direction: light green

→ second one: rotation direction: dark green

Intrinsic: around the object's own axes. & moves with it and moves with it.

extrinsic rotations around the stationary axes of an external frame, unaffected by the object's orientation.

Task 2.6:

$$[\vec{g_1}\ \vec{g_2}\ \vec{g_3}] \cancel{\begin{bmatrix} r_1^G \\ r_2^G \\ r_3^G \end{bmatrix}} = [\vec{K_1}\ \vec{K_2}\ \vec{K_3}] \overbrace{\begin{bmatrix} r_1^K \\ r_2^K \\ r_3^K \end{bmatrix}}^{\text{equation 3}} \rightarrow = [\vec{K_1}\ \vec{K_2}\ \vec{K_3}] R_{K,G} \cancel{\begin{bmatrix} r_1^G \\ r_2^G \\ r_3^G \end{bmatrix}}$$

$$\rightarrow [\vec{g_1}\ \vec{g_2}\ \vec{g_3}] = [\vec{K_1}\ \vec{K_2}\ \vec{K_3}] R_{K,G}$$

Task 2.7:

$$\theta_r = \arctan2(r_{3,2}, r_{3,3}) = \arctan2(\cos\varphi\sin\theta, \cos\varphi\cos\theta)$$

$\varphi = \frac{\pi}{2} \rightarrow r_{3,2}, r_{3,3} = 0 \rightarrow \arctan2(0,0)$ undefined. (Singularity) → Can't recover.

$\varphi < \frac{\pi}{2} \rightarrow$ fine ✓

$\varphi > \frac{\pi}{2} \rightarrow$ fine ✓ $\}$ $\theta_r \overset{?}{=} \theta \rightarrow$ depend on the value of φ

Task 2.8:

$$\varphi_r = \arctan2(-r_{3,1}, \sqrt{r_{3,2}^2 + r_{3,3}^2}) = \arctan2(\sin\varphi, |\cos\varphi|)$$

$|\varphi| < \frac{\pi}{2} \rightarrow$ square term positive (cos) $\rightarrow \varphi_r = \varphi$

$|\varphi| > \frac{\pi}{2} \rightarrow \cos < 0 \rightarrow$ maybe not recover.

Task 2.9:

$$\phi_r = \arctan2(r_{2,1}, r_{1,1}) = \arctan2(\sin\phi\cos\varphi, \cos\phi\cos\varphi)$$

$\varphi = \frac{\pi}{2} \rightarrow$ not recover

else: depends on sign & magnitudes of $r_{2,1}, r_{1,1}$ and ϕ, φ, θ

Part 3:

Task 3.1:

$$p^K = R_{K,G}\, p^G + t^K_{K,G}$$

p^G $[P_1^G$ ...

$p^K = R_{K,G}\, p^G + t_{K,G}$

$$p^G = \begin{bmatrix} p_1^G \\ p_2^G \\ p_3^G \\ 1 \end{bmatrix} \rightarrow T_{K,G} = \begin{bmatrix} R_{K,G} & t_{K,G}^K \\ 0 & 1 \end{bmatrix} \rightarrow p^K = T_{K,G}\, p^G = \begin{bmatrix} R_{K,G} & t_{K,G}^K \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_1^G \\ p_2^G \\ p_3^G \\ 1 \end{bmatrix}$$

$$\rightarrow p^K = \begin{bmatrix} R_{K,G}\, p^G + t_{K,G}^K \\ 1 \end{bmatrix} = \begin{bmatrix} R_{K,G}\, p_1^G + t_{K,G_1}^K \\ R_{K,G}\, p_2^G + t_{K,G_2}^K \\ R_{K,G}\, p_3^G + t_{K,G_3}^K \\ 1 \end{bmatrix}$$

Task 3.2:

$t_{K,G}^K \rightarrow$ translation from origin of frame K to the origin of frame G

$t_{K,G}^G \rightarrow$ expressed in the coordinates of G

1. $R_{K,G}$ based on rotate points of frame G to K.

2. translation not aligned with frame and axes of G not frame K.

3. $t_{K,G}^G \neq t_{K,G}^K \rightarrow$ mathematically not valid.

Task 3.3:

$$p^G = R_{K,G}\, p^K + t_{K,G}^G \rightarrow t_{K,G}^G = p^G - R_{K,G}\, p^K$$ $\times (R_{K,G})^T$ to both sides

$$(R_{K,G})^T t_{K,G}^G = (R_{K,G})^T (p^G - R_{K,G}\, p^K)$$

$$= (R_{K,G})^T p^G - \underbrace{(R_{K,G})^T R_{K,G}}_{I}\, p^K = I p^K = p^K$$

$\times -1$

$$\rightarrow (R_{K,G})^T t_{K,G}^G = (R_{K,G})^T p^G - p^K \quad \times -1$$

$$\rightarrow -(R_{K,G})^T t_{K,G}^G = p^K - (R_{K,G})^T p^G \quad *$$

$$t_{K,G}^G = p^G - R_{K,G}\, p^K \text{ (from eq (16))} \quad **$$

$$\rightarrow t_{K,G}^G = \underline{-(R_{K,G})^T t_{K,G}^K} \quad ***$$ (based on translation from K to G in K coordinates.)

$$*,\ **,\ *** \Rightarrow t_{K,G}^G = -(R_{K,G})^T t_{K,G}^K$$