بنام خدا

Subject: ....
Year: .... Month: .... Day: .... ( )

سوال ۱)

آ)

$$x_1(t) = 1 + \underbrace{\sin(\omega_0 t)}_{T_1} + 3\cos\underbrace{(2\omega_0 t + \tfrac{\pi}{4})}_{T_2}$$

$T_1 = \frac{2\pi}{\omega_0}$ $\quad T_2 = \frac{2\pi}{2\omega_0} = \frac{\pi}{\omega_0}$ $\longrightarrow T_t = \frac{2\pi}{\omega_0} \longrightarrow \omega_t = \frac{2\pi}{2\pi}\omega_0 = \omega_0$

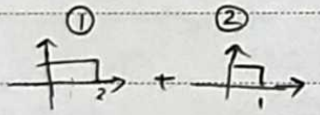
$$x_1(t) = 1 + \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + \frac{3}{2}\left[e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}\right]$$

$$= 1 \times e^{0j\omega_t t} + \frac{e^{j\omega_t t}}{2j} - \frac{e^{-j\omega_0 t}}{2j} + \frac{3}{2} e^{2j\omega_t t} e^{\frac{\pi}{4}j} +$$

$$\frac{3}{2} e^{-2j\omega_t t} e^{-j\frac{\pi}{4}}$$

$\Rightarrow a_0 = 1$, $a_1 = \frac{1}{2j}$, $a_{-1} = -\frac{1}{2j}$, $a_2 = \frac{3}{2} e^{\frac{\pi}{4}j}$

$a_{-2} = \frac{3}{2} e^{-\frac{\pi}{4}j}$

ب) با استفاده از خواص فوریه پیش می‌رویم : خروجی $\Leftarrow$ 

① : $T_1 = 1 \Rightarrow a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} = \frac{\sin(k\omega_0)}{k\pi}$

time shifting خاصیت : $x(t - t_0) \xrightarrow{FS} e^{-jk\omega_0 t_0} \times a_k \xrightarrow[t_0 = 1]{} e^{-jk\omega_0}$

$\Rightarrow a_{①} = e^{-jk\omega_0} \frac{\sin(k\omega_0)}{k\pi}$

② : $T_1 = \frac{1}{2} \Rightarrow b_k = \frac{\sin(k\omega_0/2)}{k\pi}$

time shifting $\rightarrow x(t - \frac{1}{2}) \xrightarrow{FS} e^{-jk\omega_0/2}$

 $a_{②} = e^{-jk\omega_0/2} \frac{\sin(k\omega_0/2)}{k\pi}$

خاصیت خطی بودن $\Rightarrow x_1 + x_2 \xrightarrow{FS} a_{①} + a_2 = e^{-jk\omega_0} \frac{\sin(k\omega_0)}{k\pi} + e^{-jk\frac{\omega_0}{2}} \frac{\sin(k\omega_0/2)}{k\pi}$

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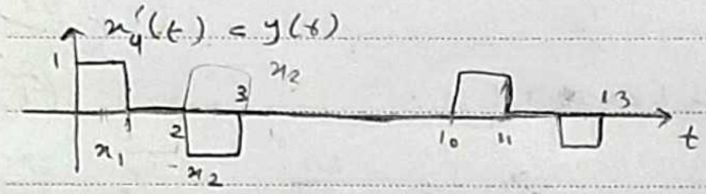

$$T = \frac{2\pi}{\omega_0} = 10 \rightarrow \omega_0 = \frac{2\pi}{10} = \frac{\pi}{5}$$

$$T_1 = \frac{1}{2} \rightarrow a_{k_1} = \frac{\sin(k\omega_0/2)}{k\pi} = \frac{\sin(\frac{k\pi}{10})}{k\pi}$$

$$x_1(t-t_0) \rightarrow a_{k_1} e^{jk\omega_0 t_0} \qquad t_0 = \frac{1}{2}$$

$$\Rightarrow a_{k_1} = \frac{\sin(k\pi/10)}{k\pi} \times e^{j\frac{k\pi}{10}}$$

$$T_2 = \frac{1}{2} \rightarrow a_{k_2} = \frac{\sin(k\pi/10)}{k\pi}$$

$$\Rightarrow x_1(t-\frac{5}{2}) \rightarrow a_{k_2} e^{jk\frac{\pi}{2}} = \frac{\sin(k\pi/10)}{k\pi} \times e^{j\frac{k\pi}{2}}$$

$$\Rightarrow y(t) = x_1 - x_2 \xrightarrow{FS} a_{1k} - a_{k2} = \frac{\sin(\frac{k\pi}{10})}{k\pi}\left(e^{j\frac{k\pi}{10}} - e^{j\frac{k\pi}{2}}\right)$$

$y(t) = x'(t) \rightarrow x(t) = \int y(t)\,dt$ از خاصیت مشتق در ابتدا استفاده کردیم پس :

$$\int y(t)\,dt \xrightarrow{FS} \frac{1}{jk\omega_0} a_k$$

$$\Rightarrow x(t) \xrightarrow{FS} \frac{1}{jk\omega_0} \frac{\sin(\frac{k\pi}{10})}{k\pi}\left[e^{j\frac{k\pi}{10}} - e^{j\frac{k\pi}{2}}\right]$$

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$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = 1 + 2e^{j\omega_0 t} + 2e^{-j\omega_0 t} + e^{2j\omega_0 t} + e^{-2j\omega_0 t}$$

$$+ 2e^{3\omega_0 j t} + 2e^{-3j\omega_0 t} + e^{4j\omega_0 t} + e^{-4j\omega_0 t} + \cdots =$$

$$1 + 4\cos(\omega_0 t) + 2\cos(2\omega_0 t) + 4\cos(3\omega_0 t) + 2\cos(4\omega_0 t)$$

$$1 + 4\left[\cos(\omega_0 t) + \cos(3\omega_0 t) + \cos(5\omega_0 t) \cdots\right] + 2\left[\cos(2\omega_0 t) + \cos(4\omega_0 t) + \cdots\right]$$

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$$a_k = jk \quad -3 \leq k \leq 3 \quad , \quad a_k = 0 \quad k > 3 \; , \; k < -3$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \frac{\times 2j}{2j}\Big[je^{j\omega_0 t} - je^{-j\omega_0 t} + 2je^{2j\omega_0 t} - 2je^{-2j\omega_0 t}$$

$$+ 3je^{3j\omega_0 t} - 3je^{-3j\omega_0 t}\Big] = 2\sin(-\omega_0 t) + 4\sin(-2\omega_0 t)$$

$$+ 6\sin(-3\omega_0 t)$$

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time shifting خاصیت :

$$x(t-t_0) \xrightarrow{FS} a_k \times e^{-jk\omega_0 t_0}$$

$$x(t+t_0) \xrightarrow{FS} a_k \times e^{+jk\omega_0 t_0}$$

خطی بودن

خطی بودن

$$x(t-t_0) + x(t+t_0) \xrightarrow{FS} a_k\left[e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right]$$

$$= 2a_k \times \cos(k\omega_0 t)$$

ب) $$\mathrm{Even}[x(t)] = \frac{1}{2}\left[x(t) + x(-t)\right] = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$$

عکس پذیری : $x(-t) \xrightarrow{FS} a_{-k}$

خطی بودن : $\frac{1}{2}x(t) + \frac{1}{2}x(-t) \xrightarrow{FS} \frac{1}{2}a_k + \frac{1}{2}a_{-k}$

اگر $x(t)$ حقیقی باشد :

حاصل : $\frac{1}{2}(a_k + a^*_k)$

اگر زوج هم باشد :

حاصل : $\frac{1}{2}(a_k + a_k) = a_k$

ج) $$\mathrm{Re}[x(t)] = \frac{x(t) + x^*(t)}{2} \xrightarrow[\text{خطی بودن، خاصیت مزدوج}]{FS} \frac{1}{2}a_k + \frac{1}{2}a^*_{-k}$$

د) ① $\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$ $\qquad \frac{dx}{dt} \xrightarrow[\text{خاصیت مشتق}]{FS} jk\omega_0 . a_k$

① $\Rightarrow \frac{d}{dt}\left(\frac{dx}{dt}\right) \xrightarrow{FS} jk\omega_0\, jk\omega_0 \cdot a_k = -k^2\omega_0^2 a_k$

ه) $x(3t-1)$

① $x(t-1) \xrightarrow{FS} e^{jk\omega_0} a_k$ $\qquad$ دوره تناوب T

② $x(3t-1) \xrightarrow{FS} e^{jk\omega_0} a_k$ $\qquad$ دوره تناوب $T/3$ $\quad$ در سیگما تعریف اسم تاثیری ندارد

↳ در واقع $e^{jk\frac{2\pi}{T}\times 3} a_k \xrightarrow{K'=3K} e^{jK'\omega_0} a_{K'/3} = e^{jK\omega_0} a_{K/3}$

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$$x(t) = \frac{3\pi}{2}\cos\underbrace{\left(\frac{3\pi}{2}t+\pi\right)}_{T_1}\sin\underbrace{\left(\frac{3\pi}{4}t+\frac{\pi}{2}\right)}_{T_2}$$

$$T_1 = \frac{2\pi}{3\pi}\,2 = \frac{4}{3} \qquad T_2 = \frac{2\pi}{3\pi}\times 4 = \frac{8}{3} \longrightarrow T_0 = \frac{8}{3} \rightarrow \omega_0 = \frac{3\pi}{4}$$

$$x(t) = \frac{3\pi}{2}\left[\frac{e^{j\left(\frac{3\pi}{2}t+\pi\right)} + e^{-j\left(\frac{3\pi}{2}t+\pi\right)}}{2}\right]\left[\frac{e^{j\left(\frac{3\pi}{4}t+\frac{\pi}{2}\right)} - e^{-j\left(\frac{3\pi}{4}t+\frac{\pi}{2}\right)}}{2j}\right]$$

$$= \frac{3\pi}{4j}\left[e^{j(2\omega_0 t+\pi)} + e^{-j(2\omega_0 t+\pi)}\right]\left[e^{j\left(\omega_0 t+\frac{\pi}{2}\right)} - e^{-j\left(\omega_0 t+\frac{\pi}{2}\right)}\right]$$

$$= \frac{3}{4}\frac{\pi}{j}\left[e^{3j\omega_0 t}e^{\frac{3\pi}{2}j} - e^{j\omega_0 t}e^{\frac{\pi}{2}j} + e^{-j\omega_0 t}e^{-\frac{\pi}{2}j} - e^{-3\omega_0 t}e^{-\frac{3\pi}{2}j}\right]$$

$$\Rightarrow a_3 = \frac{3}{4}\frac{\pi}{j}e^{\frac{3\pi}{2}j},\quad a_1 = -\frac{3}{4}\frac{\pi}{j}e^{\frac{\pi}{2}j},\quad a_{-1} = \frac{3}{4}\frac{\pi}{j}e^{-\frac{\pi}{2}j}$$

$$,\ a_{-3} = -\frac{3}{4}\frac{\pi}{j}e^{-\frac{3\pi}{2}j}$$

$$P_{\text{متوسط}} = \frac{1}{T}\int_T |x(t)|^2\,dt = \sum_{k=-\infty}^{+\infty}|a_k|^2 =$$

$$-\frac{9}{16}\pi^2\left[1\right] = -\frac{9}{16}\pi^2$$

e^{xj} هستند اندازه آن برابر 1 است

$$e^{xj} = \cos x + j\sin x \Rightarrow |e^{xj}| = \sqrt{\cos^2 x+\sin^2 x} = 1$$

T

Subject: $T=4$ $T=8$ $T=1$ $T_0=8 \rightarrow \omega_0=\frac{\pi}{4}$

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$$y(t) = x(t)\left[\frac{e^{\frac{\pi}{4}jt}+e^{-\frac{\pi}{4}jt}}{2}\right] + x(4t)$$ (5

$$= \frac{1}{2}x(t)\,e^{\frac{\pi}{4}jt} + \frac{1}{2}x(t)\,e^{-\frac{\pi}{4}jt} + x(4t)$$

$x(t)\,e^{\omega_0 tj\times 1} \xrightarrow{FS} a_{k-1}$: frequency shifting خاصیت

$x(t)\,e^{-\omega_0 tj\cdot 1} \xrightarrow{FS} a_{k+1}$

$x(4t) \xrightarrow{FS} a_k \quad (T = \frac{T_x}{4} = \frac{T_0}{8})$: scaling خاصیت

61 $$x(4t) = \sum_{k=-\infty}^{+\infty} a_k\, e^{4jk\omega_0 t} \underset{k'=4k}{=} \sum_{k'=-\infty}^{+\infty} a_{k'/4}\, e^{jk'\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_{k/4}\, e^{jk\omega_0 t}$$

$y(t) \xrightarrow{FS} \frac{1}{2}a_{k-1} + \frac{1}{2}a_{k+1} + a_{k/4}$:خاصیت خطی بودن

$$y(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{2}\left[j^{k-1} e^{jk\omega_0 t} + j^{k+1} e^{jk\omega_0 t}\right] + j^{k/4} e^{j\omega_0 kt}$$

$$= \sum_{k=-\infty}^{+\infty}\left[\frac{1}{2}j^{k-1} + \frac{1}{2}j^{k+1} + j^{\frac{k}{4}}\right] e^{j\omega_0 kt}$$

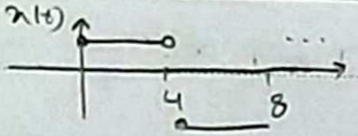
$k=1 \qquad b_k = \frac{1}{2} + (-\frac{1}{2}) + \sqrt[4]{j} = \sqrt[4]{j}$

$k=2 \qquad b_k = \frac{1}{2}j - \frac{1}{2}j + j^{\frac{1}{2}} = \sqrt[2]{j}$

$k=3 \qquad b_k = -\frac{1}{2} + \frac{1}{2} + j^{\frac{3}{4}} = \sqrt[4]{-j}$

$k=4 \qquad b_k = 0 + j^{\frac{4}{4}} = j$

$k=5 \qquad b_k = 0 + j^{\frac{5}{4}} = j\sqrt[4]{j}$



$T = 8 \rightarrow \omega_0 = \frac{\pi}{4}$ (6

$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t} = \frac{1}{8}\int_0^4 1\, e^{-jk\omega_0 t}\,dt - \frac{1}{8}\int_4^8 e^{-jk\omega_0 t}\,dt$

$= -\frac{1}{8 j k \frac{\pi}{4}}\left[e^{-4jk\frac{\pi}{4}} - 1\right] + \frac{1}{8jk\frac{\pi}{4}}\left[e^{-8jk\frac{\pi}{4}} - e^{-jk\pi}\right]$

$= -\frac{1}{2jk\pi}\left[2e^{-jk\pi} - 1 - e^{-2jk\pi}\right] = \frac{1}{2k\pi} j\left[e^{-jk\pi} - 1\right]^2$

$a_0 = \frac{1}{8}\int_0^4 1\,dt - \frac{1}{8}\int_4^8 dt = 0$

اگر $x(t)$ متناوب و ورودی سیستم LTI باشد:

$y(t) = \sum_{k=-\infty}^{+\infty} a_k H(jk\omega_0)\, e^{jk\omega_0 t}$

$a_0 = 0 \quad a_1 = \frac{1}{2} j\left[\overset{-1}{e^{-j\pi}} - 1\right]^2 = 2j \qquad a_2 = 16j\left[e^{-2j\pi} - 1\right]^2 = 0$

$a_3 = \frac{1}{6} j\left[e^{-3j\pi} - 1\right]^2 = \frac{2}{3} j \qquad a_4 = \frac{1}{8} j[0] = 0$

$a_5 = \frac{1}{2\times 5} j \times 4 = \frac{2}{5} j \quad a_6 = 0 \quad a_7 = \frac{2}{7} j$

$y(t) = 3j \times \frac{\overset{0}{\overline{\sin(\pi)}}}{\frac{\pi}{4}} e^{j\frac{\pi}{4}t} + 0 + \frac{128}{3} j \frac{\overset{0}{\overline{(\sin 12\pi)}}}{3\pi} e^{3j\pi t} + 0 + \cdots$

$\sin(k\pi)$ به ازای $k\pi$ همواره صفر است در نتیجه $y(t) = 0$