به نام خدا

Subject: ....
Year: .... Month: .... Day: .... ( )

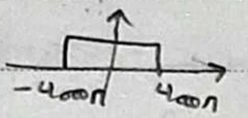
1)

آ)

$w_{m_{max}} = 4000\pi \rightarrow w_s = 8000\pi$

$f_m = 2000$ $\qquad f_s = 4000$

ب) از آنجا که $\frac{\sin(4000\pi t)}{\pi t}$ تبدیل فوریه موج مربعی



می باشد

$w_m = 4000\pi$

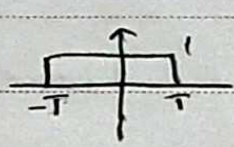
$\rightarrow w_s = 8000\pi \quad , \quad f_s = 4000$

موج مربعی مانند ب ←

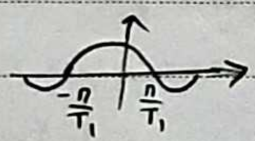
ج) $m_1(t) = \frac{\sin(w_1 t)}{\pi t} \qquad m_2(t) = \cos(w_2 t) \qquad m(t) = m_1(t) . m_2(t)$

$\rightarrow w_s = w_1 + w_2 \rightarrow f_s = \frac{w_1}{2\pi} + \frac{w_2}{2\pi}$

د)



$\xrightarrow{FT}$



$\Rightarrow w_m = \frac{\pi}{T_1} \qquad w_s = \frac{2\pi}{T_1} \rightarrow f_s = \frac{1}{T_1}$

2)

$x_1(t) . x_2(t) \xrightarrow{F.T} \frac{1}{2\pi} X_1(jw) * X_2(jw) = W(jw)$

از آنجا که کانولوشن از جمع کران های دو سیگنال محدوده بدست می آید $W(jw)$ نیز $|w| \geq |w_1 + w_2|$ برقرار است پس

$w_s = 2(w_1 + w_2) \quad , \quad w_m = w_1 + w_2$

$f_s = \frac{(w_1 + w_2)}{\pi} \Rightarrow T_s = \frac{\pi}{(w_1 + w_2)}$

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$$a_k = \frac{1}{N} \sum_{\langle N \rangle} \left(\underbrace{(-1)^n}_{I} + \underbrace{\cos^2\left(\frac{\pi}{5} n + \frac{\pi}{4}\right)}_{II} \right) e^{-jk\frac{2\pi}{N}n}$$

(T (3

$$N_I = 2 \qquad N_{II} = \frac{2\pi}{2\pi} \times 5 = 5 \qquad N_T = 10$$

$$\frac{1}{10} \sum_{n=1}^{10} \left[(-1)^n + \frac{1}{2} + \cos\left(\frac{2\pi}{5} n + \frac{\pi}{2}\right) \right] e^{-jk\frac{2\pi}{10}n}$$

$$= \frac{1}{10} \sum_{n=1}^{10} \left[e^{\pi j n} + \cos\left(\frac{2\pi}{5} n + \frac{\pi}{2}\right) \right] e^{-jk\frac{2\pi}{L}n} + \frac{1}{2}$$

$a_k^* = a_{-k} = -a_k$

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$a_{11} = a_{2\times5+1} = a_1 \xrightarrow{\text{فرد حقیقی}} a_{-1} = -a_1 = a_1^* = -j$ (4

$a_{13} = a_{2\times5+3} = a_3 \longrightarrow a_{-3} = -a_3 = -3j$

$a_{17} = a_{3\times5+2} = a_2 \longrightarrow a_{-2} = -a_2 = \frac{j}{2}$

$a_0 = -a_0 \rightarrow a_0 = 0$

• $\sum_{r=\langle N\rangle} x[r]\,x[n+\ell-r] = x[n] * x[n-\ell]$. (5

$x[n-\ell] \longrightarrow e^{+jk\frac{2\pi}{N}\ell}\, a_k$

خاصیت کانولوشن $\Rightarrow$ $x[n] * x[n-\ell] \rightarrow N a_k \cdot e^{jk\frac{2\pi}{N}\ell}\, a_k$

$= N e^{jk\frac{2\pi}{N}\ell} (a_k)^2$

• $x[n+1] - x[n] + x[n-2]$

$e^{-jk\frac{2\pi}{N}} a_k$ $\qquad$ $e^{jk\frac{4\pi}{N}} a_k$

خاصیت خطی بودن:

$x[n+1] - x[n] + x[n-2] \longrightarrow e^{-jk\frac{2\pi}{N}} a_k - a_k + e^{jk\frac{4\pi}{N}} a_k$

• $x^2[n] = x[n] \cdot x[n] \xrightarrow{DS} a_k * a_k$ خاصیت ضرب در ضرایب سری فوریه زمان گسسته

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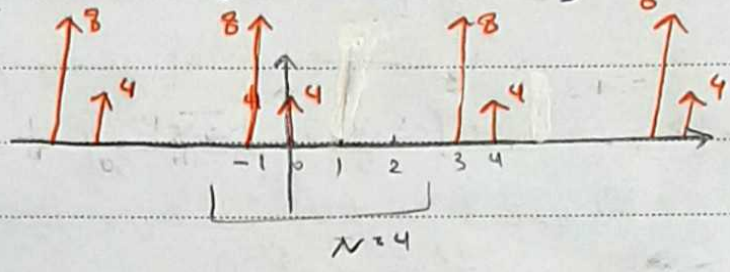
$x[n] = \sum_{m=-\infty}^{+\infty} \{ 4\delta[n-4m] + 8\delta[n-1-4m] \}$ (ب) (3



$N=4 \rightarrow$

$a_k = \frac{1}{4}\left[8e^{-jk\frac{2\pi}{4}(-1)} + 4e^{-jk\frac{2\pi}{4}(0)}\right] = 2e^{+jk\frac{\pi}{2}} + 1$

$= 2\cos(\frac{k\pi}{2}) + 2j\sin(\frac{k\pi}{2}) + 1$

$x[n] = 2 + 3\cos(\frac{2\pi}{3}n) + \sin(\frac{\pi}{3}n)$ (ج (2

$N_1 = 3$ $N_2 = 6$ $N_T = 6$

$a_k = \frac{1}{6}\sum_{0}^{5}\left[2 + 3\cos(\frac{2\pi}{3}n) + \sin(\frac{\pi}{3}n)\right]e^{-jk\frac{2\pi}{6}n} = \frac{1}{3}$

$(3+0)\times 1 + (3\cos\frac{2\pi}{3} + \sin\frac{\pi}{3})e^{-jk\frac{2\pi}{6}} + (3\cos(\frac{4\pi}{3}) + \sin(\frac{2\pi}{3}))e^{-jk\frac{2\pi}{3}}$

$+ (3\cos(2\pi) + \sin(\pi))e^{-jk\pi} + (3\cos(\frac{8\pi}{3}) + \sin(\frac{4\pi}{3}))e^{-jk\frac{4}{3}\pi}$

$+ (3\cos(\frac{10\pi}{3}) + \sin(\frac{5\pi}{3}))e^{-jk\frac{10}{6}\pi}$

$= 3 + (\frac{\sqrt{3}-3}{2})e^{-jk\frac{2\pi}{6}} + (-\frac{1}{2} + \frac{\sqrt{3}}{2})e^{-jk\frac{2\pi}{3}} + 3e^{-jk\pi}$

$= (\frac{3}{2} + \frac{\sqrt{3}}{2})e^{-jk\frac{4}{3}\pi} + (-\frac{1}{2} + \frac{\sqrt{3}}{2})e^{-jk\frac{10}{6}\pi}$

می دانیم $a_k = a_{N+k}$ پس در تناوب های بعدی همین ضرایب a_0 تا a_5 تکرار می شوند

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$N=5 \rightarrow a_k = \frac{1}{5}\left[1e^{+jk\frac{2\pi}{5}\times 10} + 2e^{jk\frac{2\pi}{5}\times 9} + 4e^{jk\frac{2\pi}{5}\times 8}\right]$ (>

$a_0 = \frac{1}{5}[e^0 + 2e^0 + 4e^0] = \frac{7}{5}$ $a_1 = \frac{1}{5}\left[e^{j4\pi} + 2e^{j\frac{18}{5}\pi} + 4e^{j\frac{16}{5}\pi}\right] =$

$a_2 = \frac{1}{5}\left[e^{8\pi j} + 2e^{\frac{36}{5}\pi j} + 4e^{\frac{32}{5}\pi j}\right]$ $a_3 = \frac{1}{5}\left[e^{12\pi j} + 2e^{\frac{54}{5}\pi j} + 4e^{\frac{48}{5}\pi j}\right]$

$a_4 = \frac{1}{5}\left[e^{16\pi j} + 2e^{\frac{72}{5}\pi j} + 4e^{\frac{64}{5}\pi j}\right]$ $a_5 = \frac{1}{5}\left[e^{2\pi j} + e^{18\pi j} + 4e^{16\pi j}\right]$

$N=8 \rightarrow a_k = \frac{1}{8}\left[3e^{jk\frac{2\pi}{8}\times 10} + 4e^{jk\frac{2\pi}{8}\times 9} + 5e^{jk2\pi} + (\infty\right.$

$\left. 4e^{jk\frac{2\pi}{8}\times 7} + 3e^{jk\frac{2\pi}{8}\times 6} + 2e^{jk\frac{2\pi}{8}\times 5} + e^{jk\frac{2\pi}{8}\times 4} + 2e^{jk\frac{2\pi}{8}\times 3}\right]$ 1/2

$= \frac{1}{8}\left(3e^{jk\frac{2\pi}{8}\times 8}\left[e^{jk\frac{2\pi}{8}\times 2} + e^{jk\frac{2\pi}{8}\times(-2)}\right] + 4e^{jk\frac{2\pi}{8}\times 8}\left[e^{jk\frac{2\pi}{8}} + e^{-j\frac{2\pi}{8}}\right]\right.$

$\left. + 5e^{jk2\pi} + 2e^{jk\frac{2\pi}{8}\times 4}\left[e^{jk\frac{2\pi}{8}} + e^{-jk\frac{2\pi}{8}}\right]\right)$

$= \frac{1}{8}\left(6e^{2\pi jk}\cos\left(\frac{\pi}{2}k\right) + 8e^{2\pi jk}\cos\left(\frac{\pi}{4}k\right) + 5e^{2\pi jk} + \right.$

$\left. 4e^{2\pi jk}\cos\left(\frac{\pi}{4}k\right)\right) = e^{2\pi jk}\left[\frac{3}{4}\cos\left(\frac{\pi}{2}k\right) + \frac{3}{2}\cos\left(\frac{\pi}{4}k\right) + \frac{5}{8}\right]$

$a_0 = \frac{23}{8}$ $a_1 = \frac{5}{8} + \frac{3\sqrt{2}}{4}$ $a_2 = \left[-\frac{3}{4} + \frac{5}{8}\right] = -\frac{1}{8}$

$a_3 = -\frac{3\sqrt{2}}{4} + \frac{5}{8}$ $a_4 = \frac{3}{4} - \frac{3}{2} + \frac{5}{8} = -\frac{1}{8}$ $a_5 = -\frac{3\sqrt{2}}{4} + \frac{5}{8}$

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$a_6 = -\frac{3}{4} + \frac{5}{8} = -\frac{1}{8}$ $a_7 = \frac{3\sqrt{2}}{4} + \frac{5}{8}$, $a_k = a_{8+k}$

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$w(t) = g(t)\sin(400\pi t) = x(t)\sin^2(400\pi t) = \cos(200\pi t)\sin^2(400\pi t)$ (6

$+ 2\sin^3(400\pi t) = \frac{1}{2}\cos(200\pi t)\left(1 - \cos(800\pi t)\right) + \sin(400\pi t)(1 - \cos 800\pi t)$

$= \frac{1}{2}\cos(200\pi t) - \frac{1}{4}\left[\sin(1000\pi t) - \sin(600\pi t)\right] + \sin(400\pi t)$

$\frac{1}{2}\left[\sin(1200\pi t) - \sin(400\pi t)\right]$

با فیلتر پایین گذر با پهنای باند 400π :

$\text{Output} = \frac{1}{2}\cos(200\pi t)$

$m(t) = \cos(2\pi f_m t)$ $\quad C(t) = \cos(2\pi f_c t)$ (7

$M(jw) = \pi\left[\delta(w + 2\pi f_m) + \delta(w - 2\pi f_m)\right]$

$C(jw) = \pi\left[\delta(w + 2\pi f_c) + \delta(w - 2\pi f_c)\right]$

$S_{SSB} = \frac{AC}{2}\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]$

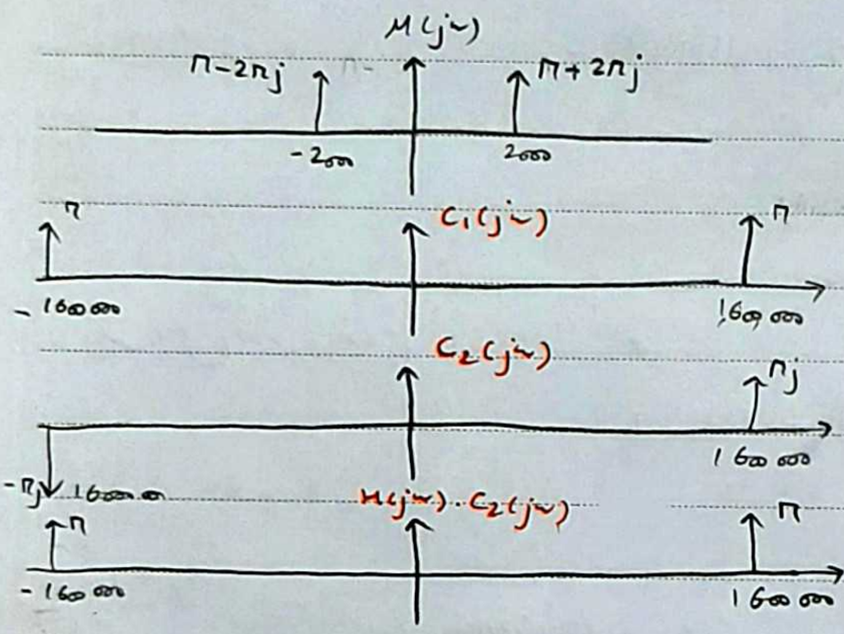
$\Rightarrow m(t) \xrightarrow{FT} M(jw) = \pi\left(\delta(2\pi f_m + w) + \delta(w - 2\pi f_m)\right]$

$y(t) = m(t) * c_1(t) + h(t) * c_2(t)$ الف (8

$Y(j\omega) = M(j\omega) . C_1(j\omega) + H(j\omega) . C_2(j\omega)$

$c_1(t) = 10\cos(1600^k \pi t)$

$c_2(t) = 10\sin(1600^k \pi t)$



$\rightarrow M(j\omega) . C_1(j\omega) = 0$

$Y(j\omega) = 0 + H(j\omega) . C_2(j\omega) \longrightarrow y(t) = \cos(1600k\pi t)$

$\cos(1600k\pi t)$ $f = 1600{,}000\ \pi$ ب)

$LSSB : [\omega + \omega_m - \omega_c , \omega + \omega_c - \omega_m]$

دامنه طیف فرکانسی $= [\omega_m - \omega_c , \omega_c - \omega_m] = [-15980000 , 15980000]$