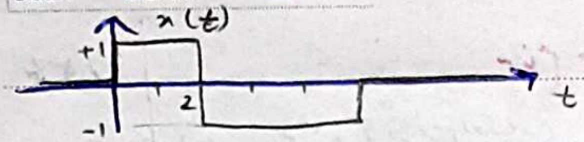
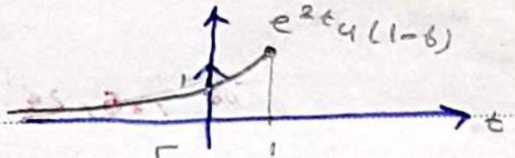
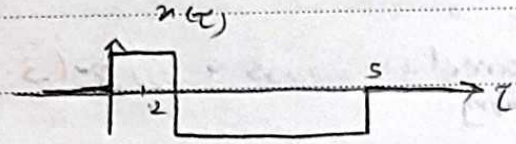
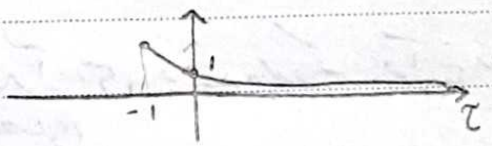


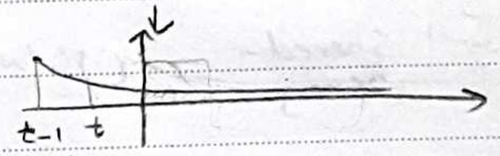


$t \to \tau$, $h[-\tau]$ ↳ $e^{-2\tau} u(1+\tau)$

↓ $t \to \tau$







$t \leq 1$

if $\boxed{t-1 \leq 0} \to y(t) = \int_0^2 e^{+2(t-\tau)} d\tau = -\frac{e^{2t}}{2} e^{-2(\tau+t)} \Big|_0^2$

$= -\frac{1}{2}\left[e^{-4+4t} - 1\right]$

if $\boxed{t-1>0}$ && $\boxed{t-1<2} \Rightarrow y(t) = \int_{t-1}^{2} e^{2(-\tau+t)} d\tau =$

$-\frac{e^{2t}}{2}\left[e^{+2(-\tau+t)}\Big|_{t-1}^{2}\right] = -\frac{e^{2t}}{2}\left[e^{+2(-2+t)} - e^{2}\right]$

$= -\frac{e^{2t}}{2}\left[e^{2t-4} - e^{2}\right] = -\frac{1}{2}\left[e^{4t-4} - e^{2t+2}\right]$

if $\boxed{t-1>2}$ && $\boxed{t-1<5} \to y(t) = \int_{t-1}^{5} -e^{-2(\tau+t)} = \frac{1}{2} e^{2t}\left(e^{-10} - e^{-2t+2}\right)$

if $\boxed{t-1 \geq 5} \to y(t) = 0$

$$\Rightarrow x(t) * h(t) = \begin{cases} -\frac{1}{2}\left[-1 + e^{-4+4t}\right] & t \leq 1 \\ -\frac{1}{2}\left[e^{+4t-4} - e^{2+2t}\right] & 1 < t < 3 \\ +\frac{1}{2}\left[e^{2t-10} - e^{2}\right] & 3 \leq t < 6 \\ 0 & t \geq 6 \end{cases}$$

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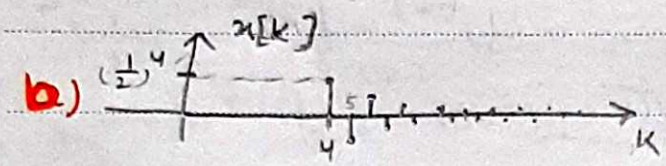
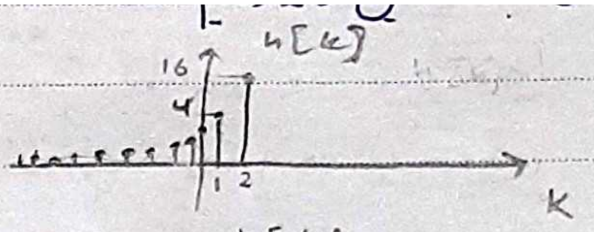
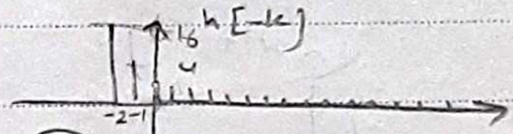
b)







$n-2 \leq 4 \rightarrow \sum_{k=-2}^{4} \sum_{i=k}^{\infty} (\frac{1}{2})^{i+4-k} 4^{n-i}$ (I)

$n-2 > 4 \rightarrow y[n] = \sum_{4}^{\infty} x[k] \cdot h[n-k] = \sum_{k=4}^{\infty} 4^{n-k} (-\frac{1}{2})^{k}$

$= \sum_{k=4}^{\infty} (-\frac{1}{4})^{k} 4^{n} = 4^{n} \times \frac{4}{16\times5} = \frac{4^{n+1}}{20}$

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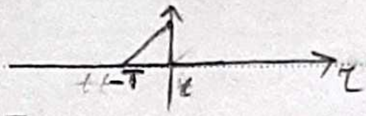
$I = \sum_{k=-2}^{4} \sum_{i=k}^{\infty} (-\frac{1}{2})^{i+4-k} 4^{n-i} = \sum_{k=-2}^{4} \sum_{i=k}^{\infty} 4^{n} (-\frac{1}{4})^{i} (-\frac{1}{2})^{4-k}$

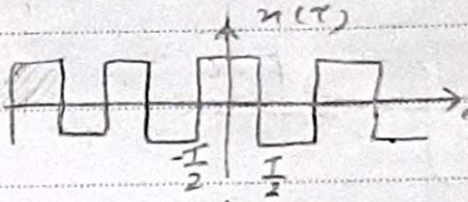
$\sum_{k=-2}^{4} (-\frac{1}{2})^{4-k} \frac{(-\frac{1}{4})^{k} 4^{n}}{1+\frac{1}{4}} = \frac{4}{5} \times 4^{n} \times \frac{1}{4^{16}} \sum_{k=-2}^{4} (-\frac{3}{4})^{k} = 4^{n} \times \frac{1}{20} \times \frac{9}{16} \frac{(1-(-\frac{3}{4})^{7})}{1+\frac{3}{4}}$

$= \frac{9}{80\times7} (1+(\frac{3}{4})^{7}) 4^{n} = 4^{n} \frac{9}{560} \times (\frac{2184}{16384}+1) \sim \frac{81}{4480} 4^{n} \sim 0/018 \times 4^{n}$

$h(-\tau) = \tau + 1$

c)





$-\frac{T}{2} < t < \frac{T}{2}$

$$y(t) = \int_{-\frac{T}{2}}^{t} (\tau + t)\, d\tau = \left(\frac{\tau^2}{2} + \tau t\right)\Big|_{-\frac{T}{2}}^{t} = \frac{1}{2}\left(t+\frac{1}{2}\right)^2 + \left(t+\frac{1}{2}\right)t$$

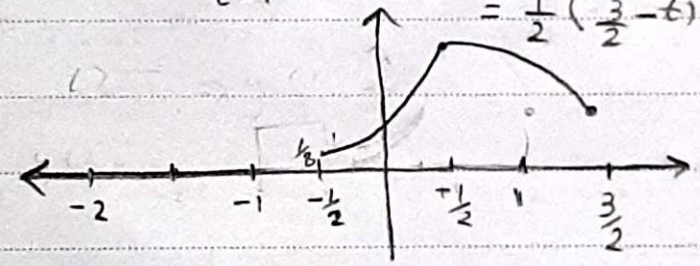
$\frac{T}{2} < t < \frac{3}{2}T \Rightarrow$

$$= \frac{3}{2}t^2 + t + \frac{1}{8}$$

$$y(t) = \int_{t-T}^{\frac{T}{2}} (\tau + t)\, d\tau = \left(\frac{\tau^2}{2} + \tau t\right)\Big|_{t-T}^{\frac{T}{2}} = \frac{T^2}{8} + \frac{Tt}{2} - \frac{(t-T)^2}{2} + (t-T)t$$

$$= \frac{1}{2}\left(\frac{3}{2} - t\right)^2 + t\left(\frac{3}{2} - t\right)$$

در شکل مشاهده می‌کنیم که $T = 1$



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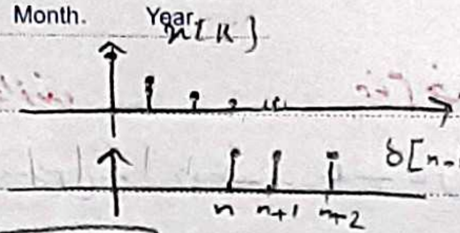
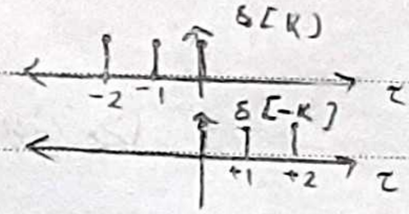


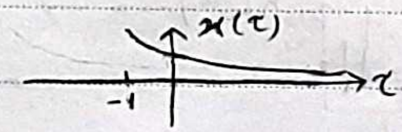


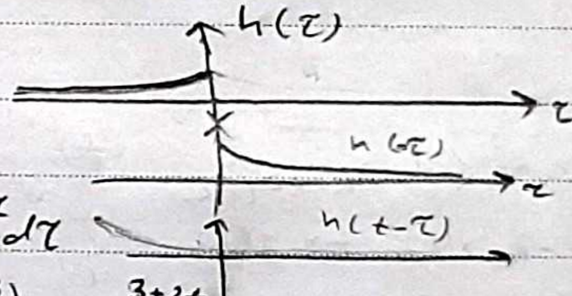
if $\boxed{n+2<0} \rightarrow y[n]=0$

$\boxed{n \geq 0} \rightarrow y[n]=\sum_{k=n}^{n+2}\left(\frac{1}{4}\right)^k=\left(\frac{1}{4}\right)^{n+2}+\left(\frac{1}{4}\right)^{n+1}+\left(\frac{1}{4}\right)^{n}$

$\boxed{n=-2} \rightarrow y[n]=\left(\frac{1}{4}\right)^{n+2}$ / $\boxed{n=-1} = \left(\frac{1}{4}\right)^{n+2}+\left(\frac{1}{4}\right)^{n+1}$

e)





$\boxed{t-1 \leq -1} \Rightarrow y(t)=\int_{-1}^{\infty} e^{+2(t-\tau)}e^{-\tau}d\tau$

$=-\frac{e^{2t}}{3}\left(e^{-3\tau}\right)\Big/_{-1}^{\infty}=-\frac{e^{2t}}{3}\left(0-e^{3}\right)=\frac{e^{3+2t}}{3}$

$\boxed{t-1 \geq -1}$ $y(t)=\int_{t}^{\infty} e^{2(t-\tau)}e^{-\tau}d\tau$

$=-\frac{e^{2t}}{3}\left(e^{-3\tau}\right)\Big/_{t}^{\infty}=+\frac{e^{-t}}{3}$

$$y(t)=\begin{cases}\frac{e^{3+2t}}{3} & t<-2\\ \frac{e^{-t}}{3} & t\geq 0\end{cases}$$

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برای راحتی شکل ها پیوسته رسم شده و گرنه سیگنال ها گسسته اند :

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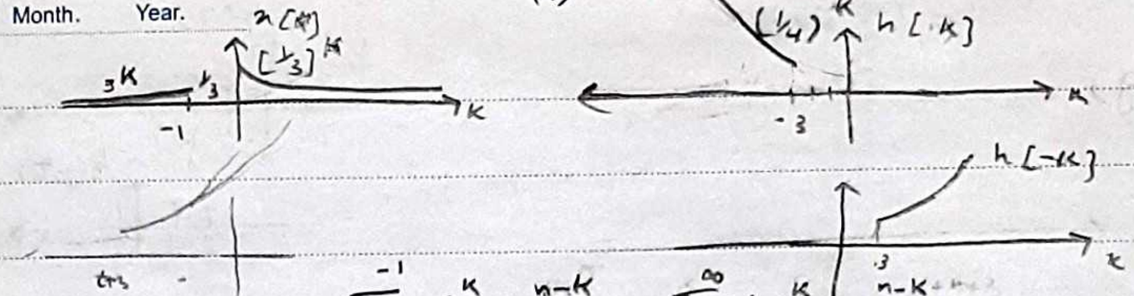

Ⅰ $n+3 \leq -1 \rightarrow y[n] = \sum_{k=n+3}^{-1} 3^k \left(\frac{1}{4}\right)^{n-k} + \sum_{k=0}^{\infty} \left[\frac{1}{3}\right]^k \left(\frac{1}{4}\right)^{n-k}$

$n+3 \geq -1 \;\&\&\; n+3 \leq 0 \rightarrow y[n] = 0$

Ⅱ $n+3 \geq 0 \rightarrow y[n] = \sum_{0}^{\infty} \left(\frac{1}{3}\right)^k \left(\frac{1}{4}\right)^{n-k}$

Ⅰ $= \left(\frac{1}{4}\right)^n \left[\sum_{k=n+3}^{-1} 12^k + \sum_{k=0}^{\infty} \left(\frac{4}{3}\right)^k\right] = 4^{-n}\left(12^{n+3}\left(\frac{12^{-n-3}-1}{11}\right) + \infty\right) = \infty$

Ⅱ $= 4^{-n}\left(\sum_{0}^{\infty} \left(\frac{4}{3}\right)^k\right) = \infty$

$$\rightarrow y[n] = \begin{cases} \sum_{k=n+3}^{-1} 3^k \left(\frac{1}{4}\right)^{n-k} + \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k \left(\frac{1}{4}\right)^{n-k} & n \leq -4 \\ 0 & -4 < n \leq -3 \\ \sum_{0}^{\infty} \frac{1}{3}^k \left(\frac{1}{4}\right)^{n-k} & n \geq -3 \end{cases}$$

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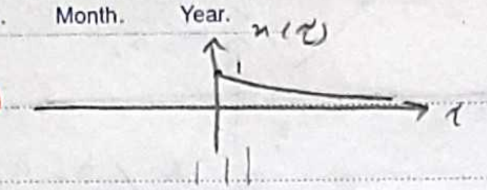
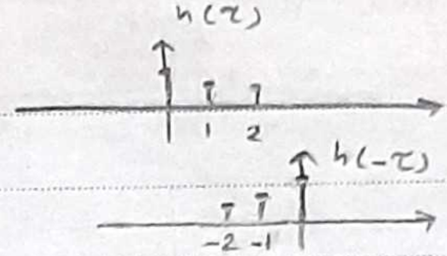
g)





$\boxed{t<0} \rightarrow y(t) = 0$

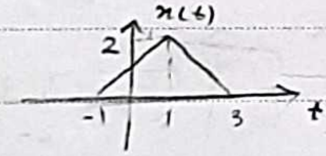
$\boxed{t \geq 2} \rightarrow y(t) = \int_0^{\infty} e^{-\tau}\left(\delta(t-\tau) + 0.5\,\delta(t-1-\tau) + 0.3\,\delta(t-\tau-2)\right) d\tau$

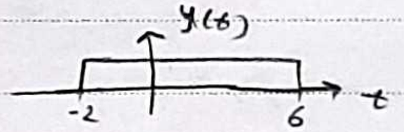
$= -\left(e^{t} + 0.5\,e^{t-1} + 0.3\,e^{t}\right)$

$\boxed{t=0} \rightarrow y(t) = \int_0^{\infty} e^{-\tau}\left(\delta(t-\tau)\right) d\tau = -e^{t}$

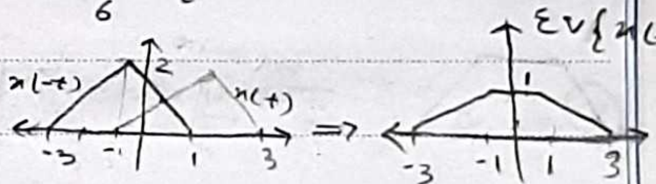
$\boxed{t=1} \rightarrow y(t) = \int_0^{\infty} e^{-\tau}\left(\delta(t-\tau) + 0.5\,\delta(t-\tau)\right) d\tau = -e^{t} - 0.5\,e^{t-1}$

$x(t) * h(t) = y(t) =$ (2





$Ev\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$



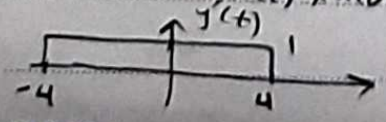
$y_2(t) = Ev\{x(t)\} * h(t) = \frac{1}{2}\left[x(t) + x(-t)\right] * h(t)$ خاصیت توزیع‌پذیری کانولوشن

$= \frac{1}{2}\,x(t) * h(t) + \frac{1}{2}\,x(-t) * h(t) = \frac{1}{2}\,y(t) + \frac{1}{2}\,x(-t) * h(t)$

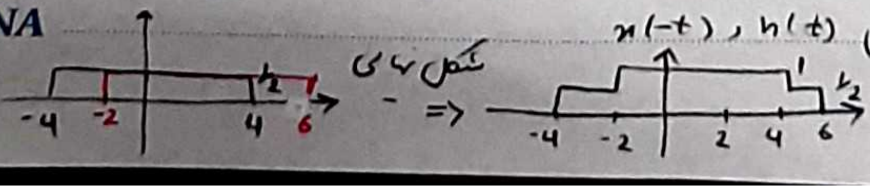
در $x(-t) * h(t)$:

می‌دانیم بازه‌ی $h(t)$ بین $(-1, 3)$ است زیرا همواره بازه‌ی $y(t)$ از جمع بازه‌های $x(t)$ و $h(t)$ است

از آنجا که حاصل $h(t) * x(-t)$ از نظر شکلی همان $x(t) * h(t)$ است و فقط بازه تعریف آن متفاوت است، بازه‌ی آن را پیدا می‌کنیم:



← از جمع بازه‌های $x(-t)$ و $h(t)$



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3/ a) از آنجا که $h[n] = k\,u[n]$ نیست و برای مقادیر منفی هم غیرصفر است

علی نیست

مطلقاً جمع پذیر نیست و $\sum_{k=-\infty}^{+\infty} |h[n]| > \infty$ (به خاطر عبارت $(1/01)^n$)

پس پایدار نیست

b) علی است (برای مقادیر $n < 0$: $h[n] = 0$)

پایدار است $\sum_{k=-\infty}^{+\infty} |h[n]| = \sum_{k=-\infty}^{+\infty} |(\frac{1}{3})^n n| < \infty$

c) علی نیست

$$\int_{-\infty}^{+\infty} e^{-6|\tau|} d\tau = \int_{0}^{+\infty} e^{-6\tau} d\tau = -\frac{1}{6} e^{-6\tau} \Big|_0^{\infty} = -\frac{1}{6}(0-1) = \frac{1}{6} < \infty$$

پایدار است

d) علی است ($h(t) = k\,u(t)$)

پایدار نیست $\int_{-\infty}^{+\infty} |\tau e^{-\tau}| d\tau = |(-t-1)e^{-t}| \Big|_{-\infty}^{+\infty} > \infty$

e) علی است $h(t) = t\,u(t)$

پایدار نیست $\int_{-\infty}^{+\infty} |t u(t)| dt = \int_0^{\infty} t\, dt = \frac{t^2}{2} \Big|_0^{\infty} > \infty$

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$y[n] = A z_0^{n} \rightarrow y[n-1] = A z_0^{n-1}$

$A z_0^{n} + 2 A z_0^{n-1} = 0 \xrightarrow{\div A z_0^{n-1}} z_0 + 2 = 0 \rightarrow z_0 = -2$

فرض می‌کنیم: $x[n] = K e^{jwn} u[n]$; $y[n] = Y e^{jwn} u[n]$

$Y e^{jwn} + 2 Y e^{jw(n-1)} = K e^{jwn} + 2 K e^{jw(n-2)}$

$Y + \frac{2Y}{e^{jw}} = K + \frac{2K}{e^{2jw}} \rightarrow Y = \frac{K + \frac{2K}{e^{2jw}}}{1 + \frac{2}{e^{jw}}}$

$= \frac{K(1 + 2K e^{-2jw})}{1 + 2e^{-jw}} = \frac{K((1 + 2K\cos 2w) - j(2K\sin 2w))}{(1 + 2\cos w) - j(2\sin w)}$

$\underbrace{\frac{K(\sqrt{1 + 4K^2 + 4K\cos 2w}}{\sqrt{1 + 4 + 4\cos w}}}_{A} \frac{e^{j\theta_1}}{e^{j\theta_2}}$

$\begin{cases} \theta_1 = \tan^{-1}\left(\frac{2K\sin 2w}{1 + 2K\cos 2w}\right) \\ \theta_2 = \tan^{-1}\left(\frac{2\sin w}{1 + 2\cos w}\right) \end{cases}$

$y[n] = Re[Y e^{jwn} u[n]] = Re[A e^{j[(\theta_1 - \theta_2) + wn]}] = A\cos(\theta_1 - \theta_2 + wn)$

$= \frac{K\sqrt{1 + 4K^2 + 4K\cos 2w}}{\sqrt{5 + 4\cos w}} \cos(\theta_1 - \theta_2 + wn)$

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$y[-2]+2y[-3] = 1+0$ $\quad y[-3] = 0$ (rest) $\rightarrow y[-2] = 1$

$y[-1]+2y[-2] = 2+0 \rightarrow y[-1] = 0$

$y[0]+2y[-1] = 3+2 \rightarrow y[0] = 5$

$y[1]+2y[0] = 2+4 \rightarrow y[1] = -4$

$y[2]+2y[1] = 2+6 \rightarrow y[2] = 16$

$y[3]+2y[2] = 1+4 \rightarrow y[3] = -27$

$y[4]+2y[3] = 0+4 \rightarrow y[4] = 58$

$y[5]+2y[4] = 0+2 \rightarrow y[5] = -114$

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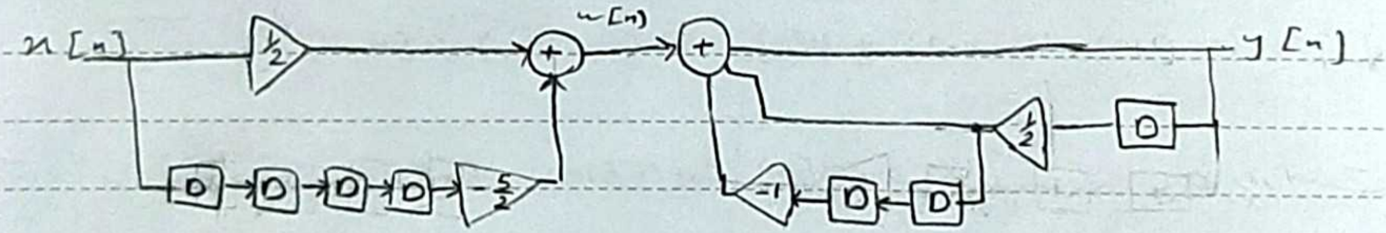
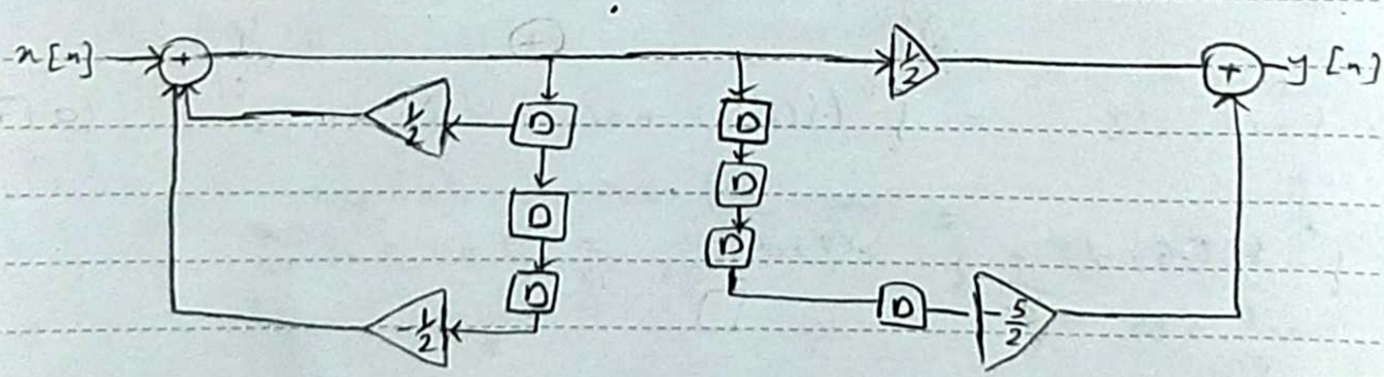
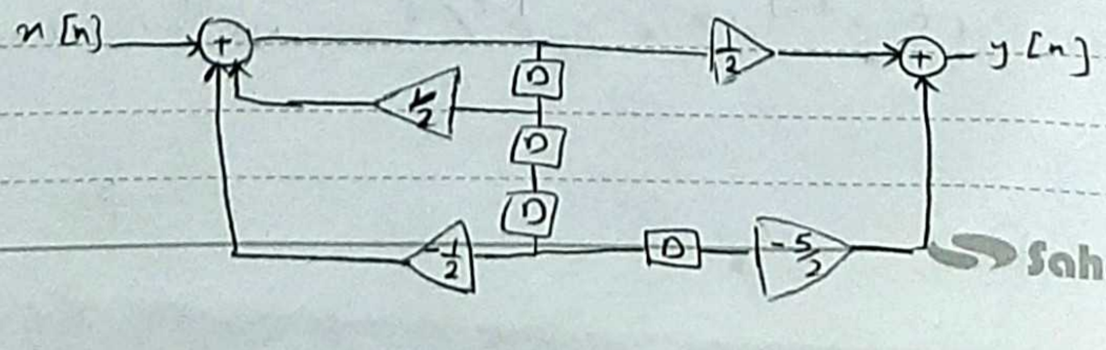
$$y[n] = \frac{1}{2}y[n-1] - \frac{1}{2}y[n-3] + \frac{1}{2}x[n] - \frac{5}{2}x[n-4]$$







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6) a) در سیستم بدون حافظه $h(t) = k\delta(t)$ یعنی فقط $h[n] \neq 0$ می باشد

$$y(t) = \int_{-\infty}^{+\infty} x(t) e^{3(t-\tau)} u(-1+t+\tau)\, d\tau =$$ پس به زمان بعد از $1-t$ وابسته می شود

← حافظه دار است. ← به ازای هر زمان های بعد از $1-t$ غیرصفر است

به ازای زمان های بعد $1-t$ غیر صفر است

b) $u(t)$ از آنجا که برای $t>0$ غیرصفر است و $\sin(5\pi t)$ هم غیر صفر است حافظه دار است

$$y(t) = \int_{-\infty}^{+\infty} x(t) \sin 5\pi(t-\tau)\, u(t-\tau)\, d\tau = \int_{0}^{t} x(t) \sin 5\pi(t-\tau)\, d\tau$$

← به زمان های قبل از t هم وابسته است

← حافظه دار است

c)

$$\sum_{k=-\infty}^{+\infty} x[k] \cos((n-k)\pi)\, u[n-k+5] = \dots + x[5] \cdot \cos((n-5)\pi)\, u[n] + \cos(n-6)\pi\, x[6] \cdot u[n-1]$$

$$+ x[7] \cos((n-7)\pi)\, u[n-2] + x[8] \cos((n-8)\pi)\, u[n-3] + \dots$$

از آنجا که فقط عبارت $x[5] \cos(n-5)\pi\, u[n]$ غیر صفر نیست حافظه دار است

7) a)

$$g(t) = \int_{0}^{t} h(\tau)\, d\tau = \int_{0}^{t} (\delta(\tau-5) + \delta(\tau))\, d\tau =$$

$$\int_{0}^{t} \delta(\tau-5)\, d\tau + \int_{0}^{t} \delta(\tau)\, d\tau = 1 + 1 = 2$$

if $t>5$ $= 1$
else $= 0$

b)

$$g(t) = \int_{0}^{t} e^{-|\tau|}\, d\tau = -e^{-\tau}\Big|_{0}^{t} = -(e^{-t}-1) = 1 - e^{-t}$$

$y(t) = y_1(t) + y_3(t) + y_4(t) = x(t) * h_1(t) + w(t) * h_3(t)$ (8

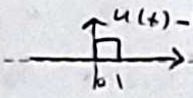
$+ w(t) * h_4(t) = x(t) * h_1(t) + (x(t) * h_2(t)) * h_3(t) + (x(t) * h_2(t)) * h_4(t)$

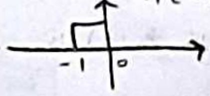
$\xrightarrow{\text{خاصیت شرکت پذیری}} y(t) = x(t) * h_1(t) + x(t) * (h_2(t) * h_3(t)) + x(t) * (h_2(t) * h_4(t))$

$\xrightarrow[\text{توزیع پذیری (کانولوشنی)}]{\text{برعکس خاصیت}} y(t) = x(t) * \underbrace{\left(h_1(t) + (h_2(t) * h_3(t)) + (h_2(t) * h_4(t))\right)}_{heq}$

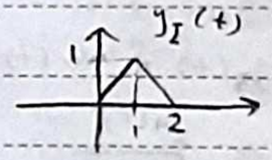
$heq = e^{-t}u(t) + \overbrace{\left((u(t) - u(t-1)) * (u(t) - u(t-1))\right)}^{\mathrm{I}} + \overbrace{\left((u(t) - u(t-1)) * \delta(t-1)\right)}^{\mathrm{II}}$

I:

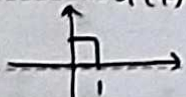


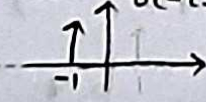


$$\left.\begin{cases} t<0 \rightarrow y_{\mathrm{I}}(t) = 0 \\ 0<t<1 , \quad t-1<0 \rightarrow y_{\mathrm{I}}(t) = \int_0^t 1\, dt = t \\ t>1 \quad 0<t-1<1 \rightarrow y_{\mathrm{I}}(t) = \int_t^1 1\, dt = 1-t \\ t-1>1 \rightarrow y_{\mathrm{I}}(t) = 0 \end{cases}\right\} \rightarrow$$



II:





$\rightarrow y_{\mathrm{II}} = u(t) - u(t-1)$

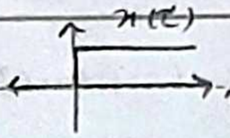
کانولوشن هر تابع با ضربه واحد خود آن تابع است

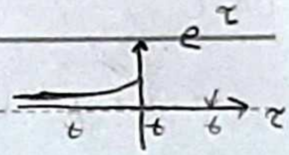
$\Rightarrow heq = e^{-t}u(t) + t(u(t) - u(t-1)) + (1-t)(u(t-1) - u(t-2))$

$+ u(t) - u(t-1) = u(t)(e^{-t} + t + 1) + u(t-1)(-t + 1 - t - 1) + u(t-2)(t-1)$

$-2t$

$$y_1(t) = \underbrace{u(t)}_{x} * \underbrace{e^{-t}u(t)}_{h}$$

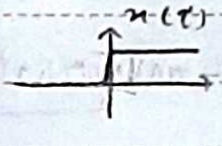


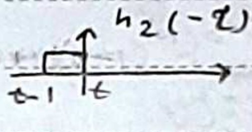


$t<0 \rightarrow y_1(t) = 0$

$t>0 \rightarrow y_1(t) = \int_0^t e^{\tau+t}\, d\tau = e^t e^{\tau} \Big/_0^t = e^{2t} - e^t$

$$w(t) = u(t) * (u(t) - u(t-1))$$





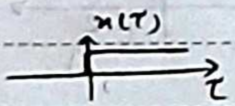
$t<0 \rightarrow w(t) = 0.$

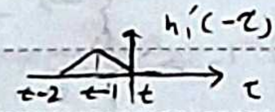
$t>0,\ t-1<0 \rightarrow w(t) = \int_0^t d\tau = t$

$t-1>0 \rightarrow w(t) = \int_{t-1}^t d\tau = 1$

$$y_3(t) = w(t) * h_3(t) = (x(t) * h_2(t)) * h_3(t) = x(t) * (\underbrace{h_2(t) * h_3(t)}_{h_1'(t)})$$

که در بخش قبل بدست آوردیم





$t<0 \rightarrow y_3(t) = 0$

$t \geq 0,\ t-1<0 \rightarrow y_3(t) = \int_0^t -\tau\, d\tau = -\frac{\tau^2}{2}\Big/_0^t = -\frac{t^2}{2}$

$t-1 \geq 0,\ t-2<0 \rightarrow y_3(t) = \int_0^{t-1} \tau\, d\tau + \int_{t-1}^t -\tau\, d\tau = \frac{\tau^2}{2}\Big/_0^{t-1} - \frac{\tau^2}{2}\Big/_{t-1}^{t}$

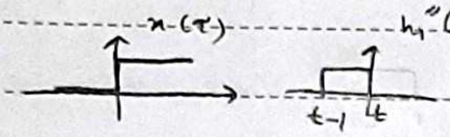
$= \frac{(t-1)^2}{2} - \frac{t^2}{2} + \frac{(t-1)^2}{2} = (t-1)^2 - \frac{t^2}{2}$

$t-2 \geq 0 \rightarrow y_3(t) = \int_{t-2}^{t-1} \tau\, d\tau + \int_{t-1}^t -\tau\, d\tau = \frac{(t-1)^2 - (t-2)^2}{2} - \frac{t^2 - (t-1)^2}{2}$

$= -\frac{t^2}{2} + (t-1)^2 - \frac{(t-2)^2}{2}$

DATE / / SUBJECT:

$$y_4(t) = x(t) * (h_2(t) * h_4(t)) = x(t) * \underbrace{(u(t) - u(t-1))}^{h_1''(t)}$$



$t<0 \rightarrow y_4(t) = 0$

$t \geq 0 \rightarrow t-1<0 \rightarrow y_4(t) = \int_0^t 1\, d\tau = t$

$t-1 \geq 0 \rightarrow y_4(t) = \int_{t-1}^{t} 1\, d\tau = 1$