HW 5 , Answers

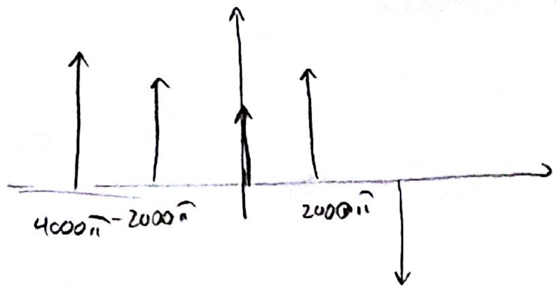
1)

a) $x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$

Nyquist rate $= 2 \times 4000\pi$

$= 8000\pi$

F.T. $\longleftrightarrow$

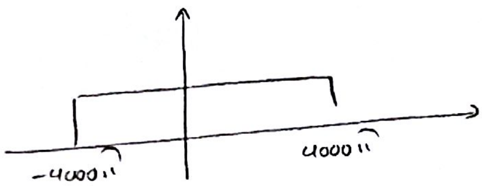


b) $\dfrac{\sin(4000\pi t)}{\pi t}$

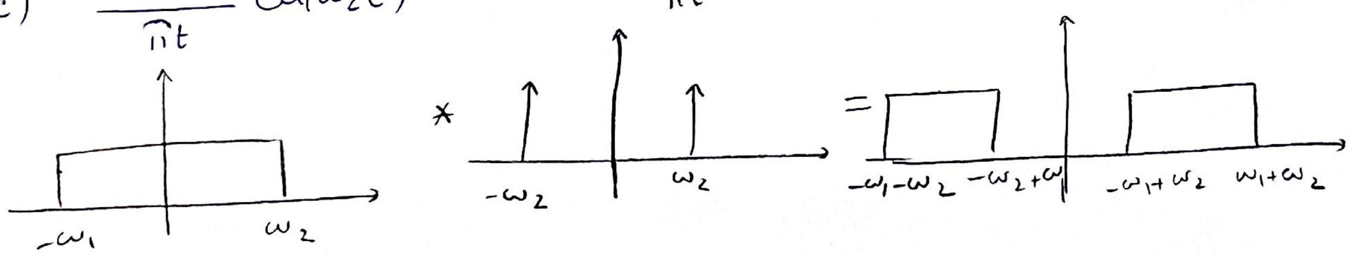
Nyquist rate $= 2 \times 4000\pi$

$= 8000\pi$

F.T. $\longleftrightarrow$



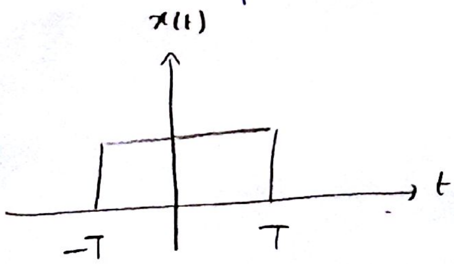
c) $\dfrac{\sin(\omega_1 t)}{\pi t}\cos(\omega_2 t) \xleftrightarrow{\text{F.T.}} F\left\{\dfrac{\sin(\omega_1 t)}{\pi t}\right\} * F\{\cos(\omega_2 t)\}$



Nyquist rate $= 2 \times (\omega_1 + \omega_2)$

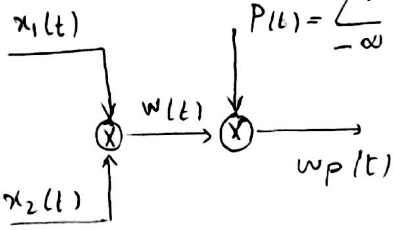
d) $x(t) = \begin{cases} 1 & \text{if } |t| < T \\ 0 & |t| \geq T \end{cases}$

بازه فرکانسی نامحدود است



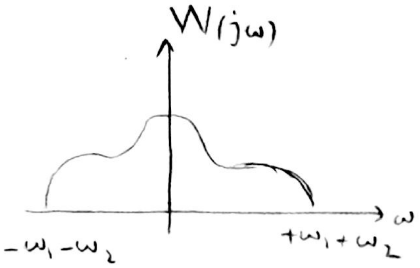
F.T. $\longleftrightarrow$ $\dfrac{2\sin(\omega t)}{\omega}$

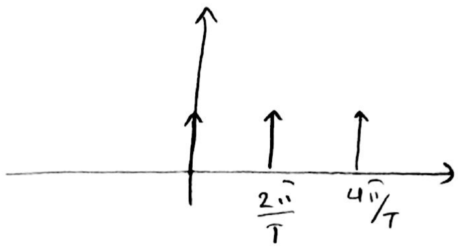
2) $P(t)=\sum_{-\infty}^{+\infty}\delta(t-nT)$



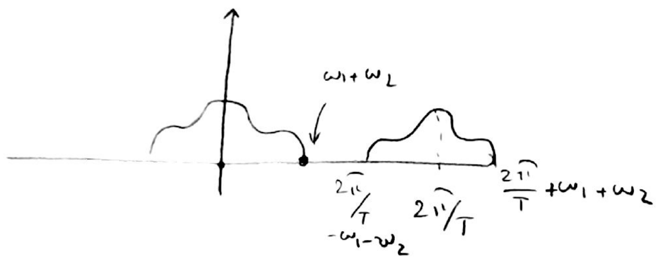
$$W_p(j\omega)=P(j\omega)*W(j\omega)$$

$$w(t)=x_1(t)\times x_2(t) \Rightarrow W(j\omega)=X_1(j\omega)*X_2(j\omega)$$





$$\Rightarrow$$



$$\omega_1+\omega_2 \leqslant \frac{2\pi}{T}-(\omega_1+\omega_2)$$

$$2(\omega_1+\omega_2)\leqslant \frac{2\pi}{T}$$

$$T_{max}=\frac{\pi}{\omega_1+\omega_2}$$

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3) a) $(-1)^n+\cos^2\left(\frac{\pi}{5}n+\pi/4\right)$ $\qquad \cos^2\theta=\frac{1+\cos(2\theta)}{2}$ $\qquad N=\frac{2\pi}{\pi/5}=10$

$$\Rightarrow x[n]=(-1)^n+\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2\pi}{5}n+\frac{\pi}{2}\right)$$

$$X=e^{j\pi n}+\frac{1}{2}+\frac{1}{2}\left(e^{j\left(\frac{2\pi}{5}n+\pi/2\right)}+e^{-j\left(2\pi/5\,n+\pi/2\right)}\right)$$

$$=e^{5j\frac{2\pi}{10}n}+\frac{1}{2}+\frac{1}{2}\left(e^{j\pi/2}e^{2\times\frac{2\pi}{10}jn}+e^{-j\pi/2}e^{-2j\frac{2\pi}{10}n}\right)$$

$$a_0=1/2,\quad a_5=1,\quad a_2=\frac{1}{2}j,\quad a_{-2}=-1/2\,j$$

b) $x[n] = \sum_{m=-\infty}^{+\infty} \{4\delta[n-4m] + 8\delta[n-1-4m]\}$

$$= \underbrace{\sum_{m=-\infty}^{+\infty} 4\delta[n-4m]}_{b_k} + \underbrace{\sum_{m=-\infty}^{+\infty} 8\delta[n-1-4m]\}}_{c_k}$$

$b_k = \frac{4}{N} = \frac{4}{4}$, $c_k = \frac{8}{4} \times e^{-jk\frac{2\pi}{4_2}}$

$a_k = b_k + C_k = 1 + 2e^{-jk\pi/2}$

c) $2 + 3\cos(2\pi/3 \, n) + \sin(\pi/3 \, n)$ $\quad N = \frac{2\pi}{\pi/3} = 6$

$$= 2 + \frac{3}{2}\left(e^{2\times\frac{2\pi}{6}jn} + e^{-2\times 2\pi/6 jn}\right) + \frac{1}{2j}\left(e^{j\frac{2\pi}{6}n} - e^{-j\frac{2\pi}{6}n}\right)$$

$a_0 = 2$, $a_2 = 3/2$, $a_{-2} = 3/2$, $a_1 = \frac{1}{2j}$, $a_{-1} = -\frac{1}{2j}$

d) $x[n] = \sum_{m=-\infty}^{+\infty} \underbrace{\delta[n-5m]}_{b_K} + \underbrace{2\delta[n-5m-1]}_{C_K} + \underbrace{4\delta[n-5m-2]}_{d_K}$

$b_K = \frac{1}{N} = \frac{1}{5}$ $\quad C_K = 2\times\frac{1}{5}\times e^{-jk\frac{2\pi}{5}}$ $\quad d_K = 4\times\frac{1}{5} e^{-jk\frac{2\pi}{5}\times 2}$

$x[n] \longleftrightarrow a_K = b_K + C_K + d_K = \frac{1}{5}\left(1 + 2e^{-jk\frac{2\pi}{5}} + \frac{4}{\cancel{7}} e^{-jk\frac{4\pi}{5}}\right)$

e) $N = 8$ $\quad a_K = \frac{1}{8}\sum_{n=0}^{7} x[n] e^{-jk\frac{2\pi}{8}n}$

$a_0 = \frac{1}{8}(5+4+3+2+1+2+3+4) = 3$

4)

$x[n]$ real and odd $\Rightarrow$ a_k purely img. and odd

$a_0 = 0$

$a_{11} = a_6 = a_1 = -a_{-1} \Rightarrow a_{-1} = -j$

$a_{13} = a_8 = a_3 = -a_{-3} \Rightarrow a_{-3} = -3j$

$a_{17} = a_{12} = a_7 = a_2 = -a_{-2} \Rightarrow a_{-2} = -j/2$

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5)

a) $\sum_{r=\langle N\rangle} x[r]\, x[n+l-r]$

$= N a_k \times e^{jk\frac{2\pi}{N}l} a_k = N a_k^2 e^{jk\frac{2\pi}{N}l}$

b) $-x[n] + x[n+1] + x[n-2]$

$a_k' = \left(-1 + e^{jk\frac{2\pi}{N}} + e^{-j\times 2k\frac{2\pi}{N}}\right) a_k$

c) $x^2[n]$

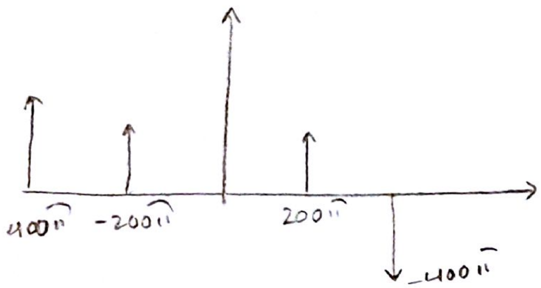
$\sum_{l=\langle N\rangle} a_l a_{k-l}$

6) $\omega(t) = g(t)\sin(400\pi t)$

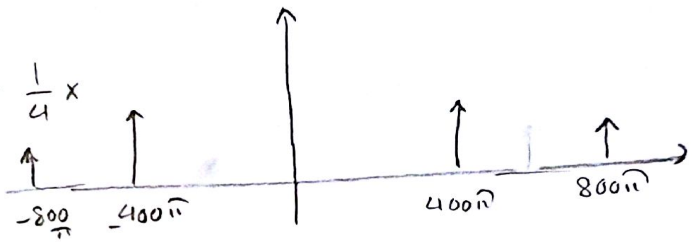
$$\omega(t) = x(t)\sin^2(400\pi t) = x(t)\left(\frac{1-\cos(800\pi t)}{2}\right)$$

$$= \frac{x(t)}{2} - \frac{x(t)\cos(800\pi t)}{2}$$

$$W(j\omega) = \frac{X(j\omega)}{2} - \frac{1}{4}\left(X(j\omega+800\pi) + X(j\omega-800\pi)\right)$$



$-\frac{1}{4}\times$



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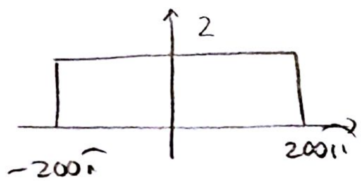


$\Rightarrow$

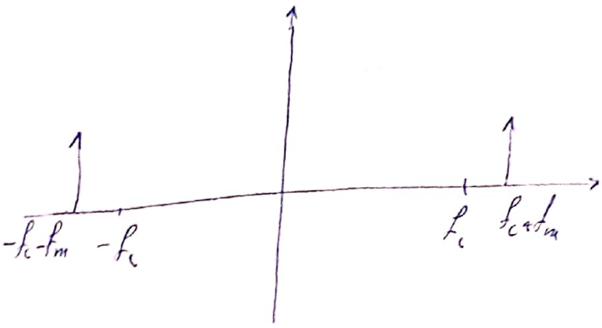
$\cos(200\pi t)$

7: SSB-AM

$$m(t) = \cos 2\pi f_m t \longrightarrow \hat{m}(t) = \sin 2\pi f_m t$$

$$\Rightarrow u_u(t) = A_c \cos 2\pi f_m t \cos 2\pi f_c t - A_c \sin 2\pi f_m t \sin 2\pi f_c t$$

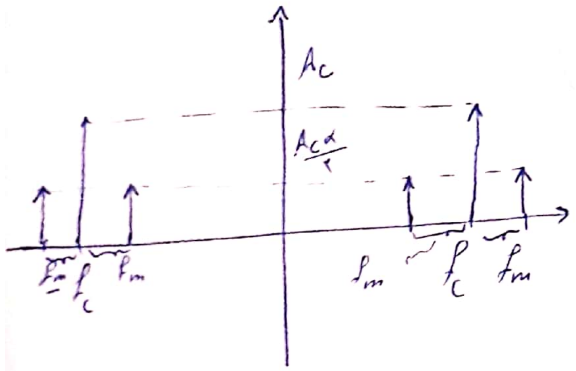
$$= A_c \cos 2\pi (f_c + f_m) t$$



DSB-AM:

$$u(t) = A_c (1 + \alpha \cos 2\pi f_m t) \cos 2\pi f_c t$$

$$= A_c \cos(2\pi f_c t) + \frac{A_c \alpha}{2} \cos(2\pi (f_c - f_m) t) + \frac{A_c \alpha}{2} (\cos 2\pi (f_c + f_m) t)$$



8.

1) $u_1(t) = A_c m(t) \cos(2\pi f_c t) + A_c \hat{m}(t) \sin(2\pi f_c t)$

$= 10(\cos 2\pi 1000 t + 2\sin(2\pi 1000 t)) \cos 2\pi f_c t$

$+ 10(\underbrace{\sin 2\pi 1000 t - 2\cos(2\pi 1000 t)}_{\hat{m}(t)}) \sin 2\pi f_c t$

$= 10 \cos(2\pi(f-1000)t) - 20 \sin(2\pi(f_c-1000)t)$

2) $U_l(f) = 5(\delta(f-f_c+1000) + \delta(f+f_c-1000))$

$+ 10j(\delta(f-f_c+1000) - \delta(f+f_c-1000))$

$= (5+10j)\delta(f-f_c+1000) + (5-10j)\delta(f+f_c-1000)$

$\Rightarrow |U_l(f)| = \sqrt{125}\,(\delta(f-f_c+1000) + \delta(f+f_c-1000))$