به نام خدا

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$$a_k = \frac{1}{8} \sum_{n=-4}^{4} x[n]\, e^{-jk\frac{2\pi}{8}n} \qquad N = 8$$ (a (1

$$\Rightarrow a_0 = \frac{1}{8} \sum_{n=-4}^{4} x[n] \times 1 = \frac{1}{8}\left[1+2+3+4+3+2+1\right] = 2$$

$$a_1 = \frac{1}{8} \sum_{n=-4}^{3} x[n] \times e^{-jk\frac{2\pi}{8}n} = \frac{1}{8}\Big[1 \times e^{3j\frac{\pi}{4}k} + 2e^{2j\frac{\pi}{4}k} + 3e^{j\frac{\pi}{4}k} + 4e^{0k}$$

$$+ 3e^{-j\frac{\pi}{4}k} + 2e^{-2j\frac{\pi}{4}k} + 1e^{-3j\frac{\pi}{4}k}\Big] = \frac{1}{8}\Big[\cancel{\cos\left(\frac{3\pi k}{4}\right)} + j\sin\left(\frac{3\pi k}{4}\right) + \cancel{2\cos\left(\frac{\pi k}{2}\right)} +$$

$$2j\sin\left(\frac{\pi k}{2}\right) + \cancel{3\cos\left(\frac{\pi k}{4}\right)} + 3j\sin\left(\frac{\pi k}{4}\right) + 4 - \cancel{3\cos\left(\frac{\pi k}{4}\right)} + 3j\sin\left(\frac{\pi k}{4}\right)$$

$$- \cancel{2\cos\left(\frac{\pi k}{2}\right)} + 2j\sin\left(\frac{\pi k}{2}\right) - \cancel{\cos\left(\frac{3\pi k}{4}\right)} + j\sin\left(\frac{3\pi k}{4}\right)\Big]$$

$$= \frac{1}{2} + \frac{j}{2}\left[\sin\left(\frac{3\pi k}{4}\right) + 2\sin\left(\frac{\pi k}{4}\right) + \sin\left(\frac{\pi k}{2}\right)\right]$$

$$x[n] = \frac{1}{2j} e^{\frac{\pi}{6}jn} - \frac{1}{2j} e^{-\frac{\pi}{6}jn} + \frac{1}{2} e^{\frac{\pi}{4}jn} + \frac{1}{2} e^{-\frac{\pi}{4}jn}$$ (b

$$x[n] = \sum_{\langle N_T \rangle} a_k\, e^{jk\left(\frac{2\pi}{N}\right)n} \qquad N_1 = 12 \quad N_2 = 8 \rightarrow N_T = 24$$

$$\Rightarrow a_2 = \frac{1}{2j} \qquad a_{-2} = -\frac{1}{2j} \qquad a_3 = \frac{1}{2} \qquad a_{-3} = \frac{1}{2}$$

در یک دوره تناوب 24 واحدی این مقادیر غیرصفر و بقیه صفراند و :

$$a_k = a_{k+24}$$

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a) $x[n-n_0] \rightarrow e^{-jn_0\omega} X(e^{j\omega})$

$x[n_1-n]$ ابتدا $x[n_1+n] \rightarrow e^{n_1 j\omega} X(e^{j\omega})$

سپس $x[n_1-n] \rightarrow e^{-n_1 j\omega} X(e^{-j\omega})$

خاصیت خطی بودن تبدیل فوریه زمان گسسته :

$$x_1[n] \xrightarrow{D.T.F.T} e^{-jn_0\omega} X(e^{j\omega}) + e^{-jn_1\omega} X(e^{-j\omega})$$

b) $3n\,x[n] \xrightarrow{DT.FT} 3j\dfrac{dX(e^{j\omega})}{d\omega}$ خاصیت خطی و مشتق در حوزه فرکانس

c) $(n-n_1)^2 x[n] = n^2 x[n] - 2nn_1 x[n] + n_1^2 x[n]$

n_1 : عدد ثابت

$$\Rightarrow n^2 x[n] = -\frac{dX(e^{j\omega})}{d\omega}$$

$$-2nn_1 x[n] = -2n_1 j \frac{dX(e^{j\omega})}{d\omega}$$

$$n_1^2 x[n] = n_1^2 X(e^{j\omega})$$

$$\Rightarrow x_3[n] \xrightarrow{D.T.FT} -\frac{dX(e^{j\omega})}{d\omega} - 2n_1 j\frac{dX(e^{j\omega})}{d\omega} + n_1^2 X(e^{j\omega})$$

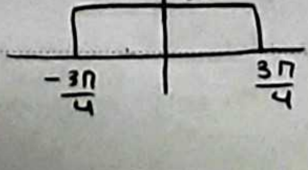
a) $x[n] = 6 + \dfrac{1}{2j} e^{(\frac{\pi}{4}n + \frac{1}{2})j} - \dfrac{1}{2j} e^{-(\frac{\pi}{4}n+\frac{1}{2})j}$ (3

پریودیک : $N = 8$

$$DTFT\{x[n]\} = 2\pi \sum_{\langle N\rangle} a_k \delta(\omega - k\frac{2\pi}{N})$$

$$= 2\pi\left(6\delta(\omega) + \frac{1}{2j} e^{\frac{j}{2}} \delta(\omega - \frac{2\pi}{8}) - \frac{1}{2j} e^{-\frac{j}{2}} \delta(\omega + \frac{2\pi}{8})\right) = 12\delta(\omega) + \frac{\pi}{j} e^{\frac{j}{2}} \delta(\omega - \frac{\pi}{4}) - \frac{\pi}{j} e^{-\frac{j}{2}}\delta(\omega + \frac{\pi}{4})$$

b) $3\dfrac{\sin(\frac{3\pi}{4}n)}{\pi n} \xrightarrow{DTFT} X(e^{j\omega}) = \begin{cases} 3 & |\omega| < \frac{3\pi}{4} \\ 0 & o\omega \end{cases}$

$=$ 

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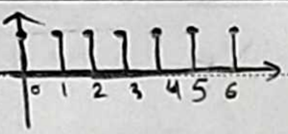
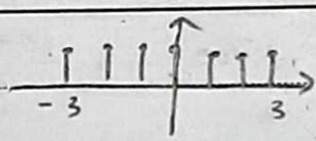
c)

$x[n]$



$x_2[n]$



$\xrightarrow{D.T.F.T} \dfrac{\sin(\frac{7}{2}w)}{\sin(w/2)}$

$$x[n] = x_2[n-3] \xrightarrow{D.T.F.T} e^{-3jw}\,\frac{\sin(\frac{7}{2}w)}{\sin(w/2)}$$

d) $(n+1)\,\alpha^n u[n] = n\,\alpha^n u[n] + \alpha^n u[n]$

خطی بودن و مشتق $\xrightarrow{D.T.F.T} j\dfrac{d}{dw}\left(\dfrac{1}{1-\alpha e^{-jw}}\right) + \dfrac{1}{1-\alpha e^{-jw}}$

$$= \frac{-j\alpha e^{-jw}}{(1-\alpha e^{-jw})^2} + \frac{1}{1-\alpha e^{-jw}} = \frac{1}{(1-\alpha e^{-jw})^2}$$

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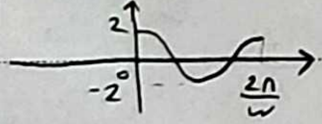
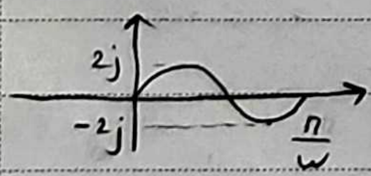
$$x[n] = \delta[n-1] + \delta[n+1] \xrightarrow{D.T.F.T} e^{-jw} + e^{jw} = 2\cos(w)$$



$$x[n] = \delta[n+2] - \delta[n-2] \xrightarrow{D.T.F.T} e^{2jw} - e^{-2jw} = 2j\sin(2w)$$



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$$x[n] = \sum_{k=-2}^{2} a_k e^{jk\frac{2\pi}{5}n} \qquad a_{-2} = 2e^{j\frac{\pi}{6}} \quad a_{-1} = e^{j\frac{\pi}{3}} \quad a_0 = 2$$

$$a_1 = e^{j\frac{\pi}{3}} \quad a_2 = 2e^{j\frac{\pi}{6}}$$

$$2e^{j\frac{\pi}{6}} e^{-\frac{4\pi}{5}nj} + e^{j\frac{\pi}{3}} e^{-\frac{2\pi}{5}nj} + 2e^{0} + e^{j\frac{\pi}{3}} e^{\frac{2\pi}{5}nj} + 2e^{j\frac{\pi}{6}} e^{\frac{4\pi}{5}nj}$$

$$= \underset{k=0}{2e^{0}} + \underset{k=1}{e^{j\frac{\pi}{3}}\left[e^{\frac{2\pi}{5}nj} + e^{-\frac{2\pi}{5}nj}\right]} + \underset{k=2}{2e^{j\frac{\pi}{6}}\left[e^{\frac{4\pi}{5}nj} + e^{-\frac{4\pi}{5}jn}\right]} + \underset{k=\dots}{0 + 0}$$

$$= 2 + 2e^{j\frac{\pi}{3}} \times \cos\frac{2\pi}{5}n + 2\times 2e^{j\frac{\pi}{6}} \times \cos\left(\frac{4\pi}{5}n\right)$$

$$= 2 + 2\,a_1 \times \sin\left(\frac{\pi}{2} + \frac{2\pi}{5}n\right) + 2\times a_2 \times \sin\left(\frac{\pi}{2} + \frac{4\pi}{5}n\right)$$

$A_0 = 2$, $A_k = 2a_k$, $\phi_k = \frac{\pi}{2}$ $\Rightarrow$ $x[n] = 2 + 2\sum_{k=1} a_k \sin(\frac{\pi}{2} + \omega_k$

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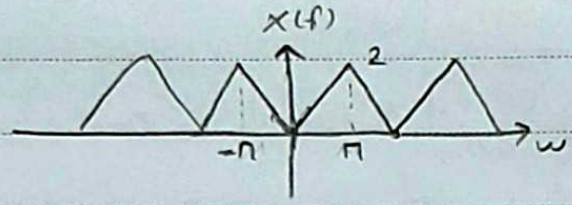

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega =$$

$$\frac{1}{2\pi}\int_{-\pi}^{0} \left(\frac{2}{\pi}\omega\right)^2 d\omega + \frac{1}{2\pi}\int_{0}^{\pi} \left(\frac{2}{\pi}\omega\right)^2 d\omega = \frac{2}{\pi^3}\left(\frac{\omega^3}{3}\Big|_{-\pi}^{0} + \frac{\omega^3}{3}\Big|_{0}^{\pi}\right)$$

$$= \frac{2}{\pi^3}\left(0 + \frac{\pi^3}{3} + \frac{\pi^3}{3}\right) = \frac{4\pi^3}{3\pi^3} = \frac{4}{3}$$

$$\sum_{n=1}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2 - \sum_{n=-\infty}^{1} |x[n]|^2 - (x[0])^2$$

سیگنال زوج درنتیجه $\sum_{n=1}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{1} |x[n]|^2$

$$\Rightarrow 2\sum_{n=1}^{\infty} |x[n]|^2 = \frac{4}{3} - (x[0])^2$$

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = \frac{1}{2\pi}\left[-\frac{\omega^2}{\pi}\Big|_{-\pi}^{0} + \frac{\omega^2}{\pi}\Big|_{0}^{\pi}\right]$$

$$\frac{1}{2\pi}\left[0 + \pi + \pi - 0\right] = 1$$

$$\Rightarrow 2\sum_{n=1}^{\infty} |x[n]|^2 = \frac{4}{3} - 1 = \frac{1}{3} \rightarrow \sum_{n=1}^{\infty} |x[n]|^2 = \frac{1}{6}$$

$$X(e^{j\omega}) = |X(e^{j\omega})|\, e^{j\measuredangle(X(e^{j\omega}))} = e^{+\frac{3\omega}{2}j} \quad |\omega| < \frac{\pi}{4}$$ (7

$$\Rightarrow x[n] = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{+\frac{3\omega}{2}j} e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{j\omega(\frac{3}{2}+n)}$$

$$= \frac{1}{j2\pi(\frac{3}{2}+n)} e^{j\omega(n+\frac{3}{2})}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{e^{\frac{\pi}{4}(n+\frac{3}{2})j} - e^{-\frac{\pi}{4}(n+\frac{3}{2})j}}{2\pi j(n+\frac{3}{2})} =$$

$$= \frac{\sin\left(\frac{\pi}{4}(n+\frac{3}{2})\right)}{\pi(n+\frac{3}{2})}$$

$$H(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}} \qquad X(e^{j\omega}) = \pi\left(\delta(\omega+\frac{3\pi}{5}) + \delta(\omega-\frac{3\pi}{5})\right)$$ (8

$$Y(e^{j\omega}) = H(e^{j\omega})\cdot X(e^{j\omega}) = \frac{\pi\,\delta(\omega+\frac{3\pi}{5})}{1-\alpha e^{+j\frac{3\pi}{5}}} + \frac{\pi\,\delta(\omega-\frac{3\pi}{5})}{1-e^{-j\frac{3\pi}{5}}}$$

$$\sum a_k e^{jk\frac{2\pi}{N}n} \xrightarrow{DTFT} 2\pi\sum_{k=-\infty}^{+\infty} a_k\, \delta(\omega-\frac{2\pi}{N}k)$$

TANDIS

$$N=10 \rightarrow \quad y[n] = \frac{e^{j\frac{2\pi}{10}\times 3}}{2(1-\alpha e^{j\frac{3\pi}{5}})} + \frac{e^{-j\frac{2\pi}{10}\times 3}}{2(1-\alpha e^{-\frac{3\pi}{5}})}$$