Midterm 25 October 2017, questions and answers

Applied Logic for Computer Science (The University of Western Ontario)

Problem 1. Multiple choice [10 points]

(1) Which of the following logic formulas is NOT a tautology?

(a) $(p \to q) \land p \to q$

(b) $(p \lor q) \land \neg p \to q$

(c) $(p \to q) \land (q \to r) \to (p \to r)$

(d) $(p \lor r) \land (q \lor \neg r) \to (p \lor q)$

(e) $\neg\neg q$

(f) $p \lor \neg p$

(2) Which of the following equivalence relations is NOT valid?

(a) $p \to q \equiv \neg q \to \neg p$

(b) $p \to q \equiv \neg p \lor q$

(c) $\neg(\forall x \in \mathbb{Z}\ Even(x)) \equiv \exists x \in \mathbb{Z}\ \neg Even(x)$

(d) $\exists x\, \forall y\, P(x,y,z) \equiv \forall y\, \exists x\, P(x,y,z)$

(e) $\forall x\, \exists y\, \exists z\, P(x,y) \land Q(x,z) \equiv \forall x\, \exists z\, \exists y\, P(x,y) \land Q(x,z)$

(3) If you prove $F(a)$ without any assumptions about a other than $a \in S$, then $\forall x \in S,\ F(x)$. This is called

(a) universal generalization (b) existential generalization (c) transitivity

(4) If you can find an element $a \in S$ such that $F(a)$, then $\exists x \in S,\ F(x)$. This is called

(a) universal generalization (b) existential generalization (c) transitivity

(5) Recall the *Pigeon Hole Principle:* For any n, if there are n+1 pigeons and n holes, then if every pigeon sits in some hole, then there is a hole with at least two pigeons. If we prove that "if every hole has at most one pigeon, then some pigeon is not sitting in any hole.", then the *Pigeon Hole Principle* is proved. This type of proof is

(a) direct proof (b) proof by cases (c) proof by contraposition (d) proof by contradiction

Answer sheets (Page 6 to 11)

Student Name : __________________________

Student Number: __________________________

2. (1)

p	q	$\neg q$	$p \vee q$	$p \wedge \neg q$	$p \vee q \to \neg q$	$(p \wedge \neg q) \to (p \vee q \to \neg q)$
1	1	0	1	0	0	1
1	0	1	1	1	1	1
0	1	0	1	0	0	1
0	0	1	0	0	1	1

(2) $(p \wedge \neg q) \to (p \vee q \to \neg q)$

$\equiv \neg(p \wedge \neg q) \vee (\neg(p \vee q) \vee \neg q)$

$\equiv \neg p \vee q \vee ((\neg p \wedge \neg q) \vee \neg q)$

$\equiv \neg p \vee q \vee \neg q$

$\equiv \neg p \vee T \equiv T$

(3) $\neg[(p \wedge \neg q) \to (p \vee q \to \neg q)]$

$\equiv (p \wedge \neg q) \wedge \neg(\neg(p \vee q) \vee \neg q)$

$\equiv p \wedge \neg q \wedge q$

$\equiv p \wedge F$

$\equiv F$

3. (a) $\forall x[C(x) \to S(x)]$

$\forall x[\neg G(x) \to D(x)]$

$\neg\exists x[G(x) \wedge \neg C(x)]$

(b) $\forall x[\neg C(x) \vee S(x)]$

$\forall x[G(x) \vee D(x)]$

$\forall x[\neg G(x) \vee C(x)]$

4. (1) $p=1 \quad q=0$

$p \to q = 0$

$q \to p = 1$

(2) $x = y = 0$

$z = 1$

(3) $x = 2 \quad x^2 = 4 \neq 2$

5. (a) $\neg p \vee q$

(b) $\neg r \to \neg q$

(c) $r \to p$

(d) p

(1)
$$\frac{\begin{array}{c} p \to q \\ p \end{array}}{q} \qquad \frac{\begin{array}{c} q \to r \\ q \end{array}}{r}$$

(2) $(\neg p \vee q) \wedge (r \vee \neg q) \wedge (\neg r \vee p) \wedge p$

q

r

(4) (3) $(\neg p \vee q) \wedge (\neg r \to \neg q) \wedge (r \to p) \wedge p \wedge \neg r$

$\equiv (\neg p \vee q) \wedge (r \vee \neg q) \wedge (\neg r \vee p) \wedge p \wedge \neg r$

$\equiv (\neg p \vee q) \wedge (r \vee \neg q) \wedge \neg r \wedge p$

$\equiv ((p \wedge \neg p) \vee (p \wedge q)) \wedge ((\neg r \wedge r) \vee (\neg r \wedge \neg q))$

$\equiv (p \wedge q) \wedge (\neg r \wedge \neg q)$

$\equiv p \wedge \neg r \wedge q \wedge \neg q \equiv p \wedge r \wedge F \equiv F$

(5) $(\neg p \vee q) \wedge (r \vee \neg q) \wedge (\neg r \vee p) \wedge p \wedge \neg r$

$\neg q$

$\neg p$

F