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Quantitative Management Modelling

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Solving LP Model - Graphically

1)

a)

Minimize Z = 56X1A + 48X1B + 56X2A + 48X2B + 56X3A + 48X3B + 48X4B

Subject to:

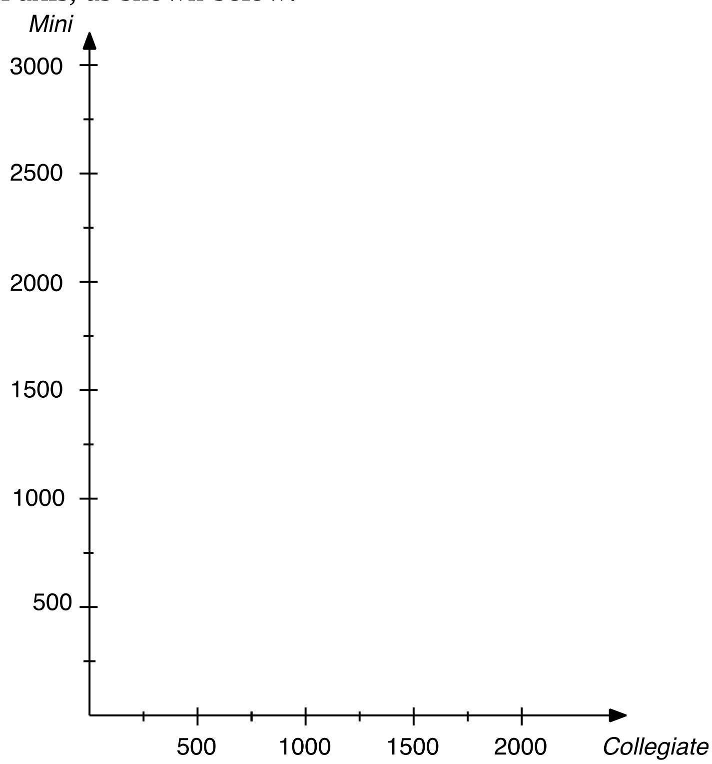
X1 + X1				≥ 4	8:00 AM – 12:00 Noon
A B					
X1 +	X2 + X2B			≥ 8	12:00 Noon – 4:00 PM
A	A				
	X2	+ X3 + X3B		≥ 1	4:00 PM – 8:00 PM
	A	A			
		X3	X4B	≥ 6	8:00 PM – 12:00 MN
		A			
X1 – 2X				≥ 0	8:00 AM – 12:00 Noon
A 1B					
X1	X2 – 2X2			≥ 0	12:00 Noon – 4:00 PM
A	A B				
	X2	X3 – 2X3		≥ 0	4:00 PM – 8:00 PM
	A	A B			
		X3 -	2X4	≥ 0	8:00 PM – 12 MN
		A	B		
X1A, X1B, X2A, X2B, X3A, X3B, X4B				≥ 0	Non-negativity

b)

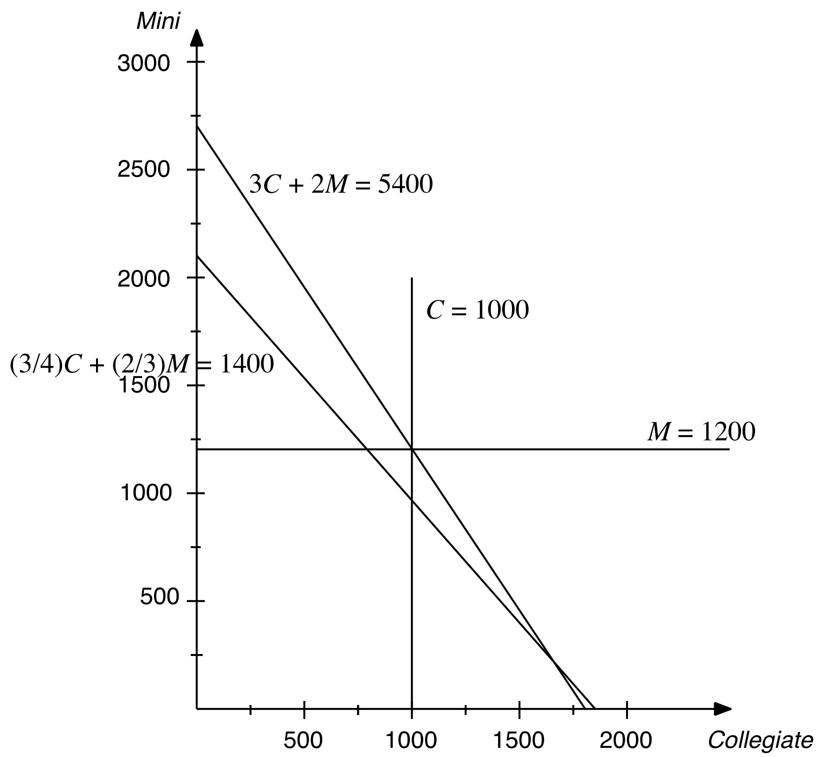
The optimal solution is to engage full-time consultants only for each of the three full-time shifts. The minimum cost involved is $784.

2)

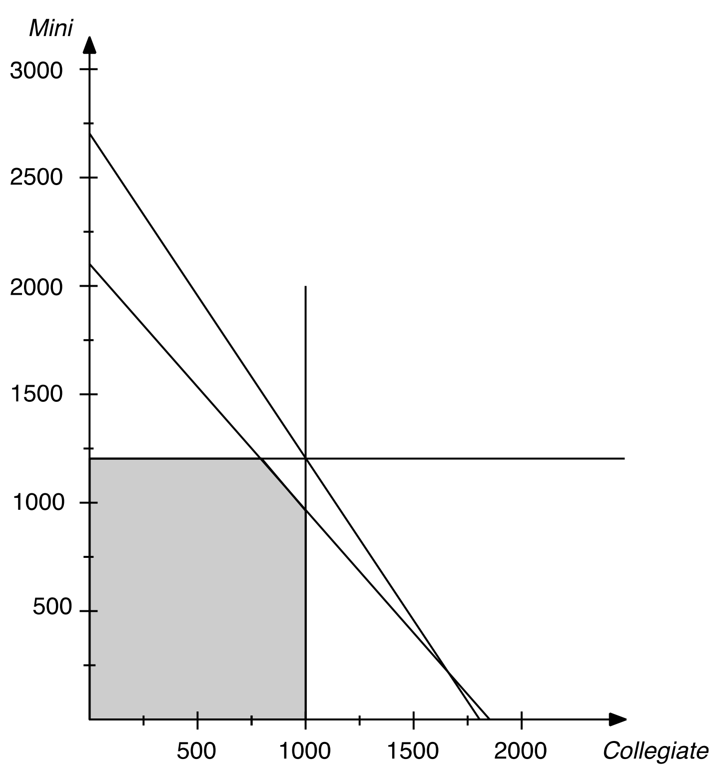
Start by plotting a graph with Collegiates (C) on the horizontal axis and Minis (M) on the vertical axis, as shown below.



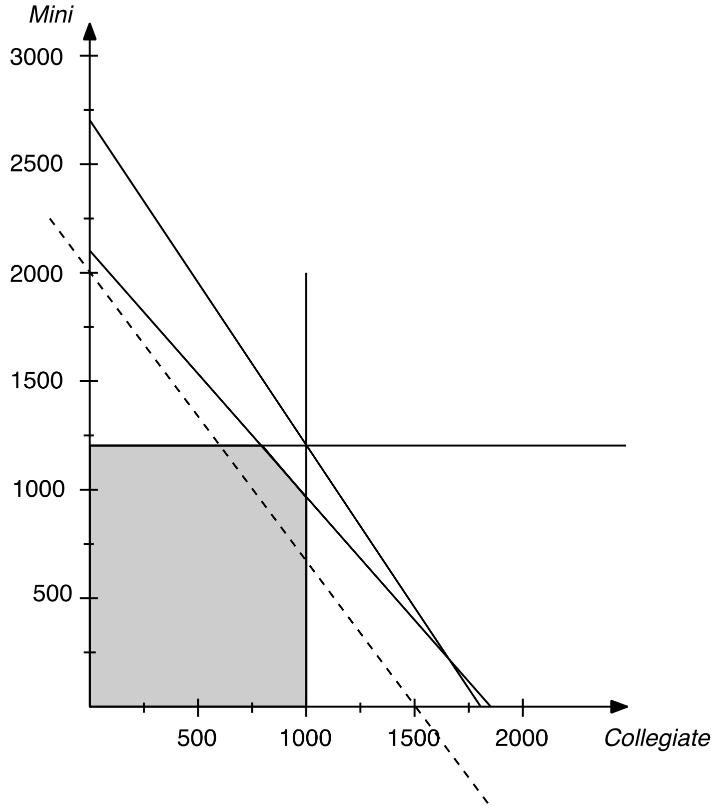
Next, the four constraint boundary lines (where the left-hand-side of the constraint exactly equals the right-hand-side) need to be plotted. The easiest way to do this is by determining where these lines intercepts the two axes. For the Nylon constraint boundary line ($3C + 2M = 5400$), setting $M = 0$ yields a C-intercept of 1800 while setting $C = 0$ yields an M-intercept of 2700. For the Labor constraint boundary line ($(3/4)C + (2/3)M = 1400$), setting $M = 0$ yields a C-intercept of 1866.67 while setting $C = 0$ yields an M-intercept of 2100. The sales forecast constraints are a horizontal line at $M = 1200$ and a vertical line at $C = 1000$. These constraint boundary lines are plotted below.



A feasible solution must be below and/or to the left of all four of these constraints while being above the Collegiate axis (since $C \geq 0$) and to the right of the Mini axis (since $M \geq 0$). This yields the feasible region shown below.

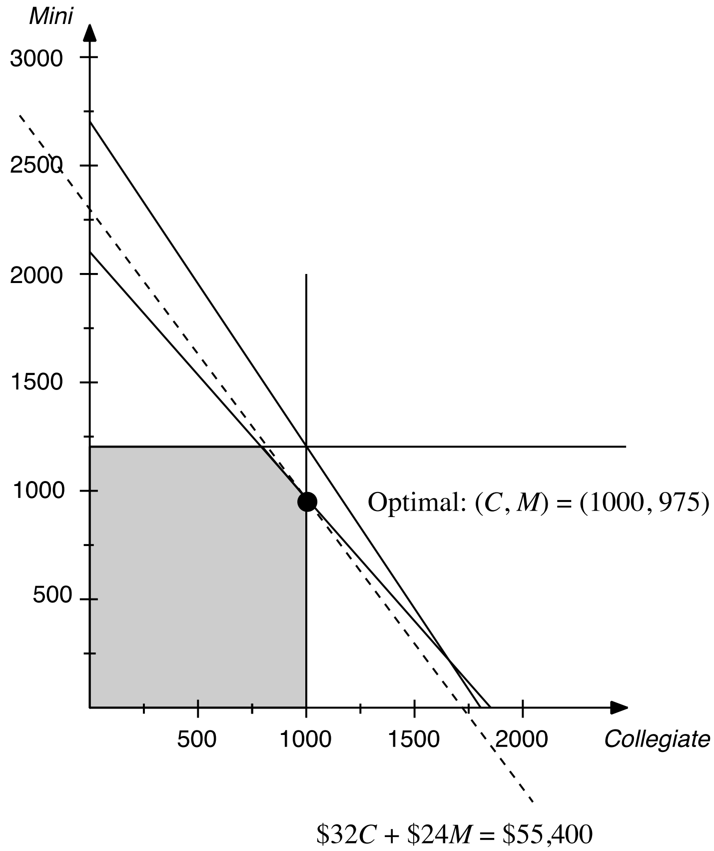


To find the optimal solution, an objective function line is plotted by setting the objective function equal to a value. For example, the objective function line when the value of the objective function is $48,000 is plotted as a dashed line below.



$\$32C + \$24M = \$48{,}000$

All objective function lines will be parallel to this one. To find the feasible solution that maximizes profit, slide this line out as far as possible while still touching the feasible region. This occurs when the profit is $55,400, and the objective function line intersect the feasible region at the single point with (C, M) = (1000, 975) as shown below.



Therefore, the optimal solution is to produce 1000 Collegiates and 975 Minis, yielding a total profit of $55,400.

3)

a)

x_{P1L} = number of large units produced per day at Plant 1,

x_{P1M} = number of medium units produced per day at Plant 1,

x_{P1S} = number of small units produced per day at Plant 1,

x_{P2L} = number of large units produced per day at Plant 2,

x_{P2M} = number of medium units produced per day at Plant 2,

x_{P2S} = number of small units produced per day at Plant 2,

x_{P3L} = number of large units produced per day at Plant 3,

x_{P3M} = number of medium units produced per day at Plant 3,

x_{P3S} = number of small units produced per day at Plant 3.

b)

Letting P (or Z) denote the total net profit per day, the linear programming model for this problem is,

Maximize $P = 420\, x_{P1L} + 360\, x_{P1M} + 300\, x_{P1S} + 420\, x_{P2L} + 360\, x_{P2M} + 300\, x_{P2S}$

$+ 420\, x_{P3L} + 360\, x_{P3M} + 300\, x_{P3S}$,

subject to

$$x_{P1L} + x_{P1M} + x_{P1S} \le 750$$

$$x_{P2L} + x_{P2M} + x_{P2S} \le 900$$

$$x_{P3L} + x_{P3M} + x_{P3S} \le 450$$

$$20\, x_{P1L} + 15\, x_{P1M} + 12\, x_{P1S} \le 13000$$

$$20\, x_{P2L} + 15\, x_{P2M} + 12\, x_{P2S} \le 12000$$

$$20\, x_{P3L} + 15\, x_{P3M} + 12\, x_{P3S} \le 5000$$

$$x_{P1L} + x_{P2L} + x_{P3L} \le 900$$

$$x_{P1M} + x_{P2M} + x_{P3M} \le 1200$$

$$x_{P1S} + x_{P2S} + x_{P3S} \le 750$$

$$\frac{1}{750}(x_{P1L} + x_{P1M} + x_{P1S}) - \frac{1}{900}(x_{P2L} + x_{P2M} + x_{P2S}) = 0$$

$$\frac{1}{750}(x_{P1L} + x_{P1M} + x_{P1S}) - \frac{1}{450}(x_{P3L} + x_{P3M} + x_{P3S}) = 0$$

and

$$x_{P1L} \ge 0,\ x_{P1M} \ge 0,\ x_{P1S} \ge 0,\ x_{P2L} \ge 0,\ x_{P2M} \ge 0,\ x_{P2S} \ge 0,$$

$$x_{P3L} \ge 0,\ x_{P3M} \ge 0,\ x_{P3S} \ge 0.$$

The above set of equality constraints also can include the following constraint:

$$\frac{1}{900}(x_{P2L} + x_{P2M} + x_{P2S}) - \frac{1}{450}(x_{P3L} + x_{P3M} + x_{P3S}) = 0.$$

However, any one of the three equality constraints is redundant, so any one (say, this one) can be deleted.