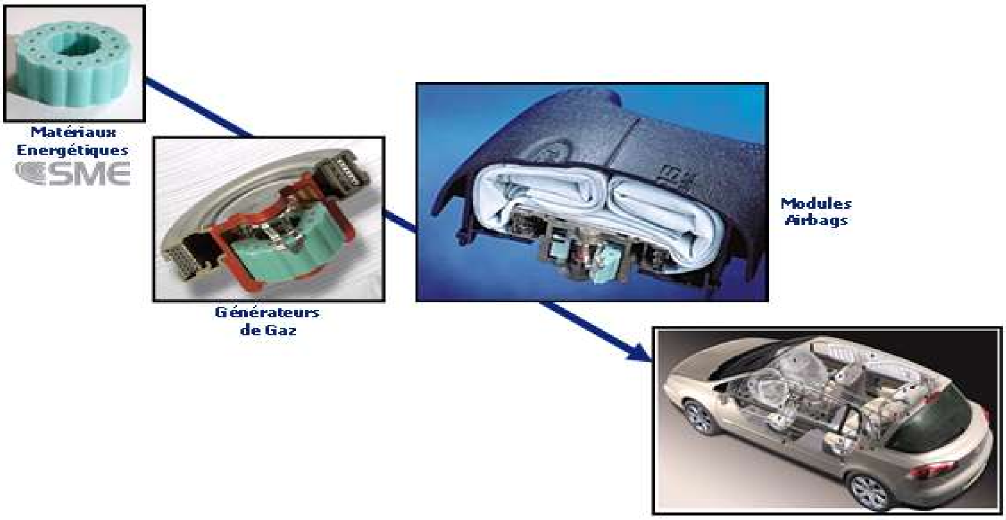
Project on mixed-integer linear programming

Modeling and solving an industrial production planning problem by mixed-integer linear programming

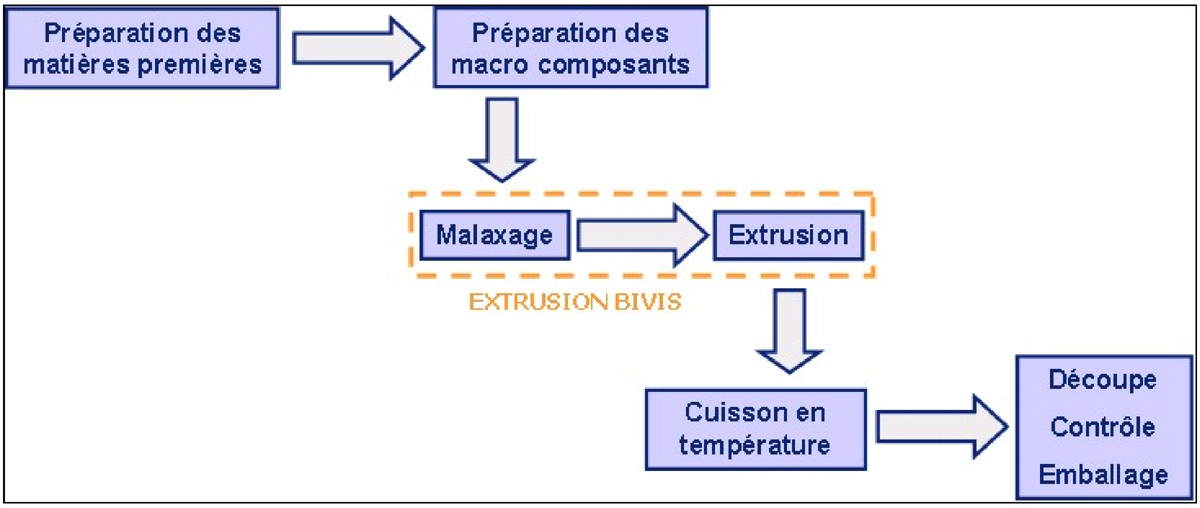
This case study is inspired by the master thesis of a former CentraleSupelec student.

We consider the problem of planning production for a plant producing energy blocks for gas generators. These ones are used in the automotive industry as components for airbags. In case of a car accident, the energy bloc explodes and provide the gas necessary to inflate the airbag.



Various types of energy blocks may be used according to the car model and the location of the airbag in the car (frontal or side airbag…). These blocs differ by their chemical composition, the form of their section and their height. The photo below shows a subset of the energy blocs produced in the studied plant.

The main production steps of an energy bloc are the following. We start by preparing and mixing the required raw materials. These ones are then extruded in the form of long tubes corresponding to a given chemical composition and section form. These tubes are then cooked and cut lengthwise to obtain energy blocks of the required height.



The optimization problem studied here focus on planning the production in the extruding unit which is the production bottleneck of the plant. It consists in determining which product (i.e., which type of tube) will be produced, on which extruding machine and at which time.

A first study provided the following information about this extruding unit.

- The unit comprises M=2 identical extruding machines.
- There are P=12 types of tubes to be produced. The tube types may be divided into F=2 product families. Each family correspond to a given chemical composition and comprises several types of tubes differing by their section form. We have $\mathcal{P}_1$={1,…, 6} and $\mathcal{P}_2$={7,…, 12} the set of products belonging to Family 1 and 2 respectively.
- The unit works continuously 5 days a week (from Monday 6am to Fridy 10pm. Each day is divided into three 8-hours shifts. There are thus 14 shifts per week. We would like to plan production for a planning horizon spanning two weeks and thus comprising S=28 shifts.
- On each extruding machine, we may produce a single type of tube per shift. Moreover, the extruding machines are critical resources that should be always used at their maximum capacity. For this reason, the production manager requires that the quantity of tubes produced in a shift should be always equal to the maximal capacity of the extruding machine. For each tube type p, the maximum quantity C_p that an extruding machine can produce (in kg per shift) is provided below.
- A changeover between two types of tubes corresponding to the same product family requires a short operation of the machine: the corresponding time may be neglected in the problem modeling. However, this changeover incurs a material waste, the cost of which is estimated at f=2360€.
- A changeover between two types of tubes corresponding to different product families is dangerous as it involves that an employee directly works on the extruding machine in proximity of the exploding material. Moreover, the extruding machine must be stopped for a full shift, the cost in terms of material waste is estimated at F=3540€.
- Due to the low flexibility of the extruding unit, an inventory of semi-finished products (tubes) must be held to be able to feed the cooking and cutting units. The corresponding unit inventory holding cost h_p (in € per kg per shift) is given below for each type of tube.
- The demand for each type of tube over the next two weeks is computed thanks to sales forecasts. Let D_{ps} be the demand for product p to be satisfied at the end of shift s.
- The products of family f=1 represent around 60% of the total demand. The production manager would like to dedicate the first extruding machine to the production of products

of family 1 and to produce the remaining demand (for tubes of families 1 and 2) on the second extruding machine.

	Family f=1					
Product index	1	2	3	4	5	6
Inv. holding cost h_p	0.68	0.45	0.40	0.49	0.54	1
Production capacity C_p	402	473	544	447	329	351

	Family f=2					
Product index	7	8	9	10	11	12
Inv. holding cost h_p	0.70	0.63	0.54	0.45	0.62	0.48
Production capacity C_p	427	510	605	348	388	507

Question 1

We first consider the simple case with a single extruding machine and a single family (family 1) comprising P=6 product types.

Propose a mixed-integer linear programming formulation of this production planning problem and use CPLEX solver to solve the two provided instances of the problem.

- The first instance corresponds to a planning horizon of S=14 shifts. The data are provided in the file 'Project_Question1_Instance1.dat'.
- The second instance corresponds to a planning horizon of S=28 shifts. The data are provided in the file 'Project_Question1_Instance2.dat'.

Each data file recalls the value of P, S, C, f and h. It then gives the index p_0 of the product for which the extruding machine is setup at the beginning of shift 1, $Istart_p$ the inventory at the beginning of shift 1 for each product p and D_{ps} the demand for product p at the end shift s.

For each instance, give the optimal production plan, together with its cost.

Question 2

We then consider the more complicated case with two extruding machines, 2 product families and 12 product types. To simplify the problem modeling, we neglect the changeover time needed to change the setup of an extruding machine from one product family to the other.

Propose a mixed-integer linear programming formulation of this production planning problem and use CPLEX solver to solve the two provided instances of the problem.

- The first instance corresponds to a planning horizon of S=14 shifts. The data are provided in the file 'Project_Question2_Instance1.dat'.
- The second instance corresponds to a planning horizon of S=28 shifts. The data are provided in the file 'Project_Question2_Instance2.dat'.

Each data file recalls the value of P, S, C, f and h. It then gives the index p_{0m} of the product for which extruding machine m is setup at the beginning of shift 1, $Istart_p$ the inventory at the beginning of shift 1 for each product p and D_{ps} the demand for product p at the end shift s.

For each instance, give the best production plan found within 10 minutes of computation, together with its cost.

Remark: To fix a maximum computation time to CPLEX, we may add the following lines of code at the beginning of your .mod file.

```
execute timeTermination {
    cplex.tilim = 600;   // set time model stop (second)
 }
```

Documents to be handed in

- a report describing the problem modeling, the formulation of the mixed-integer linear programs and the optimal /best found production plan found for each instance.
- the .mod files used for the resolution by ILOG CPLEX.

Send your documents by mail to celine.gicquel@lri.fr before Decembre 5th, 2022