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CHIRAG VARSHNEY

A1D - 29.

CEA - 1120 Strength of Materials

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Civil Department.

R.C. Hibbeler → "Mechanics of Materials"

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Mechanics of materials

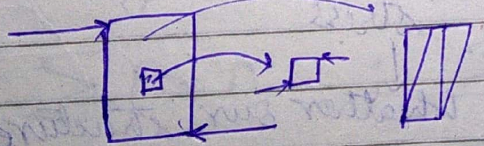
Beer & Jonston →
Subhramanyam
Timoshunk and young.

MECHANICS

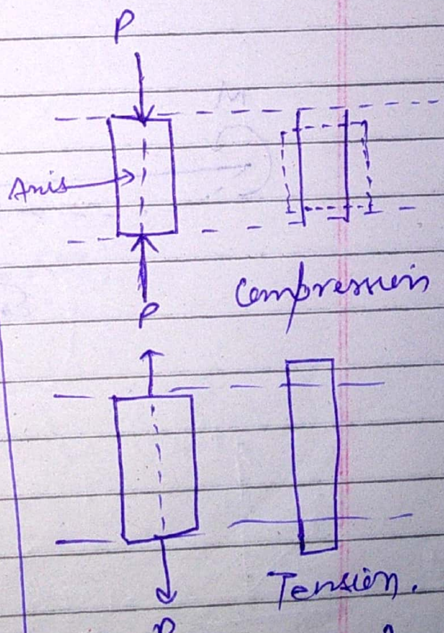
- Rigid body
  - MEA-1110
    - → statics
    - → Dynamics
  - Study of external forces on rigid/incompressible forces.
- Fluid Mechanics
- Strength/Mechanics of material
  - → for elastic bodies
  - → Deformation can be done.
  - → Deformable bodies.
  - Study of external forces on deformble bodies.

External forces/Loads.

- → Axial forces/loads. (along the axis of the body)
- → Shear (forces about an axis)
- → Moments (always occur in pair.)
  - → Bending
  - → Torsional

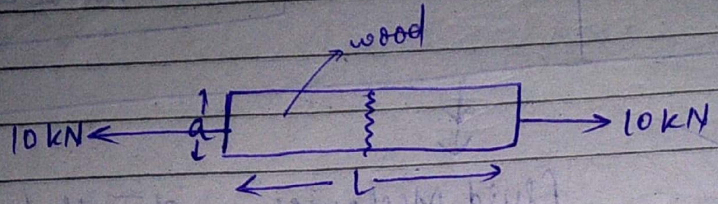


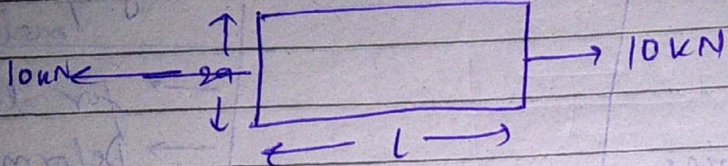
(shape change)



(Change in size.)

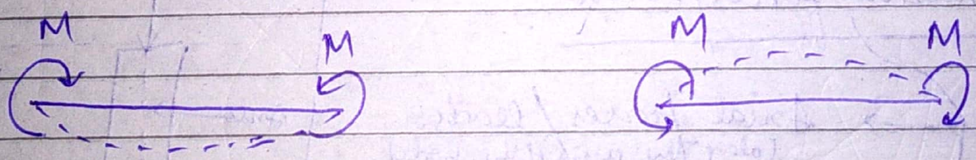
* Deformation is due to elasticity.





$$\boxed{f(\text{force}, \text{Area}) = \frac{\text{force}}{\text{Area}}}$$

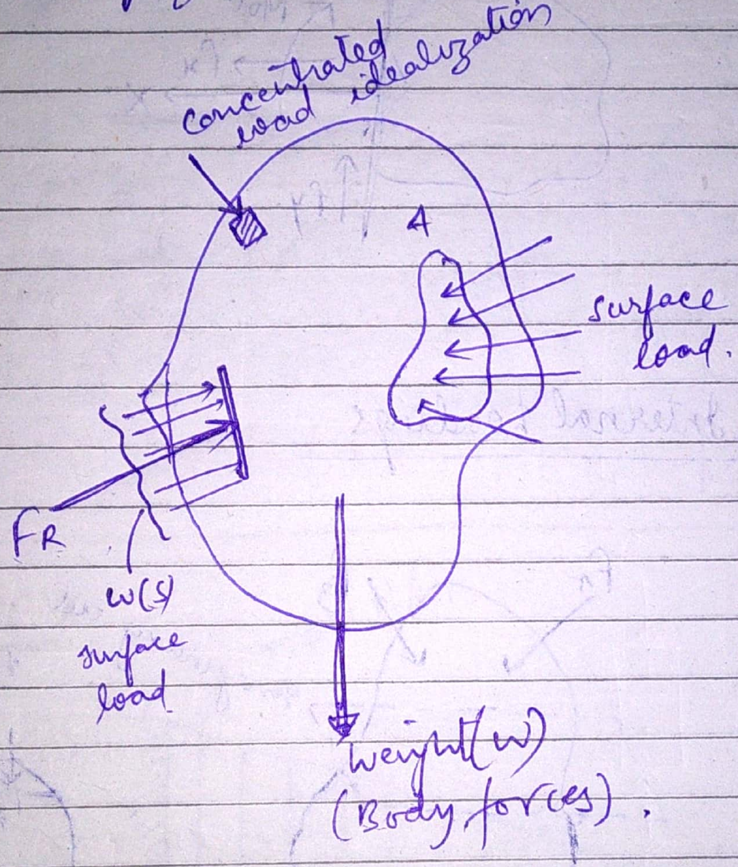
'f' is known as "stress".



External loads
↓
Deformations
↓
Internal resistance
↓
stress
↓
whether our structures are safe or not

Principles of statics :-

Equilibrium of deformable bodies.
— external forces / loads.
— surface forces
— Body forces.



Equations of Equilibrium

— Balance of forces.
— Balance of Moments

$$\sum F = 0 \quad ; \quad \sum M_o = 0$$

for 3D bodies — moments

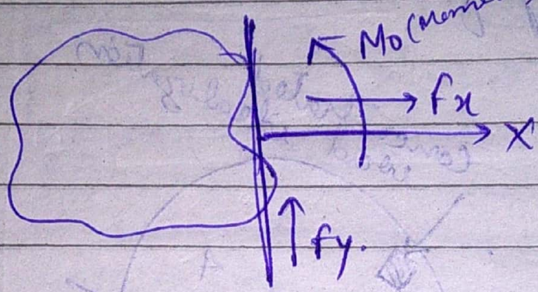
$\sum f_x = 0$, $\sum f_y = 0$, $\sum f_z = 0$; $\sum M_x = 0$ $\sum M_y = 0$

$\sum M_z = 0$

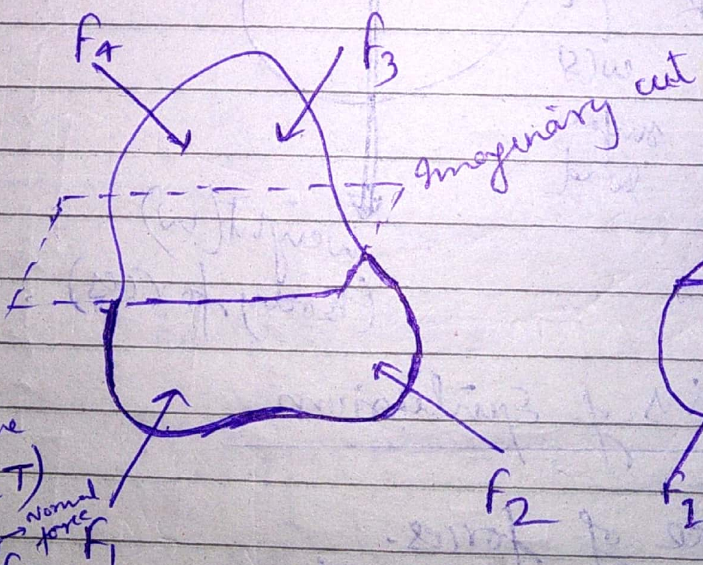
Coplanar forces..

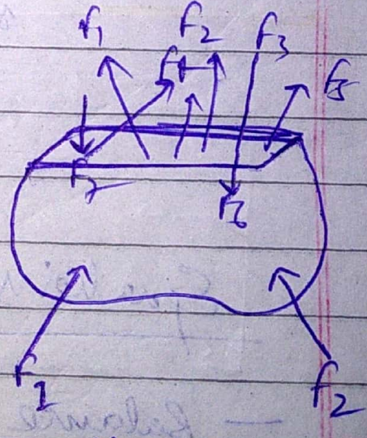
3- eqns

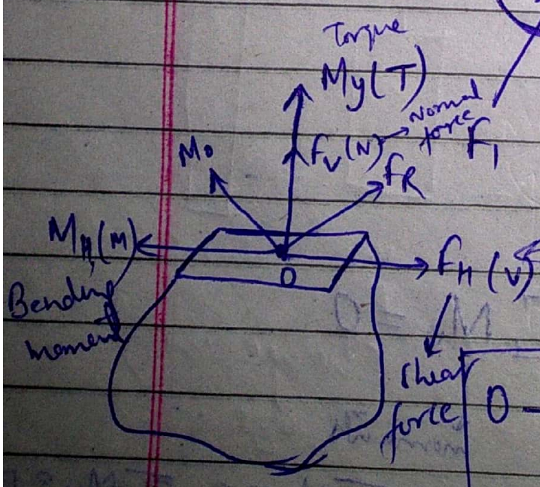
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



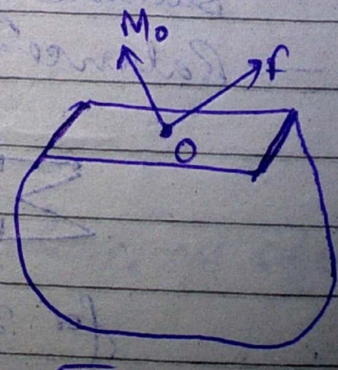
Internal Loadings







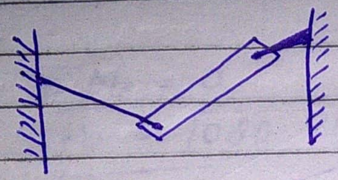
$\boxed{O \to \text{Geometric centre}}$



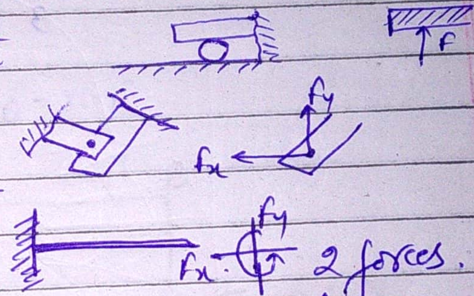
$\Sigma M_O = 0 ; \ \Sigma F = 0$

⟹ F.B.D (free body diagram)

⟹ Support reaction.

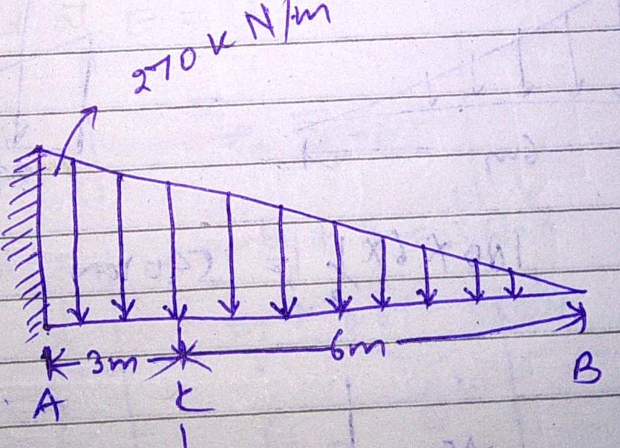


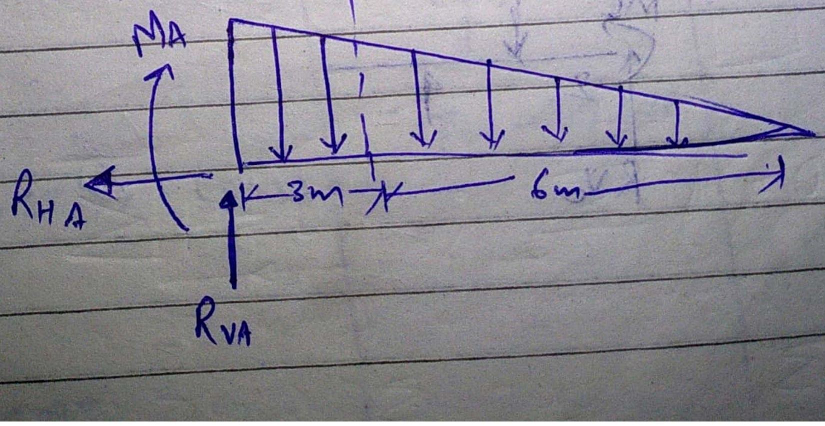
→ cable support
→ Roller support
→ internal pin
→ External pin
→ fixed support



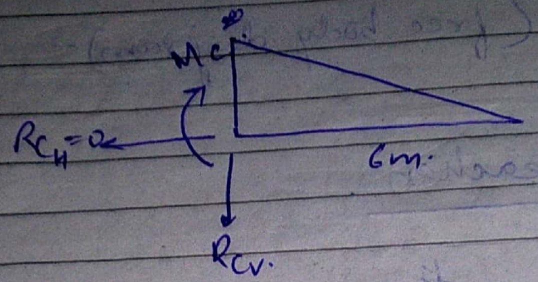
2 forces.
1 moments.

Aug, 10, 2018



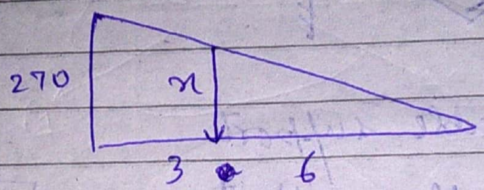


$\sum M_B = 0$



$\Sigma M_c = 0$

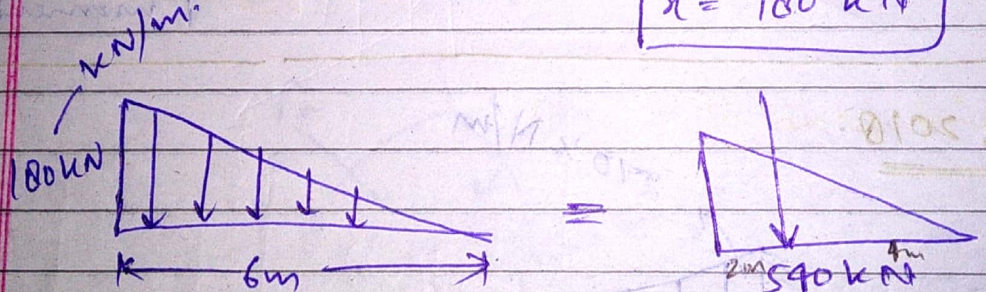
$\Sigma F_y = 0$



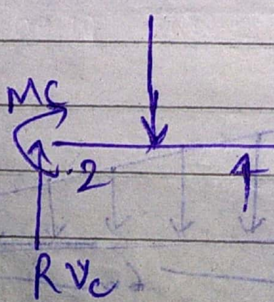
$$\frac{270}{x} = \frac{9}{6}$$

$$\frac{30}{1} = \frac{x}{6}$$

$$\boxed{x = 180 \text{ kN}}$$



$$180 \times 6 \times \frac{1}{2} = 540 \text{ kN}$$



$\Sigma f_v = 0$

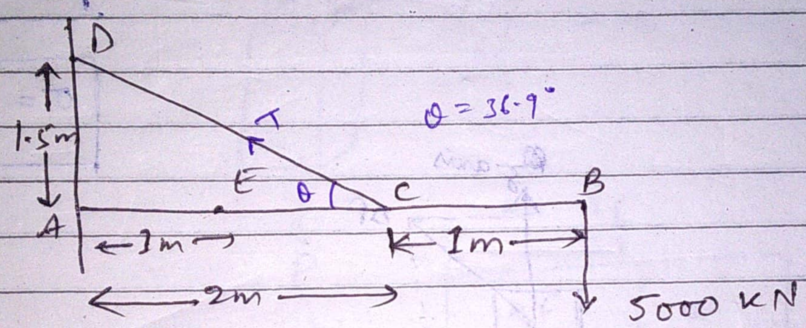
$R_{vc} - 540 = 0$

$\boxed{R_{vc} = 540 \text{ kN}}$

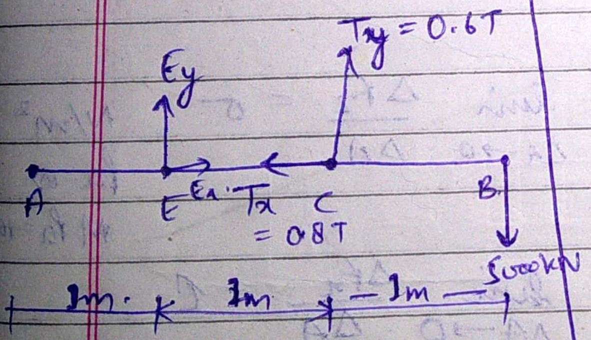
$\Sigma M_c = 0$ $\quad M_c = 540 \times 2 = 1080$ kNm

$\boxed{M_c = 1080 \text{ kN m.}}$ Ans.

Q.



find load at E = ?



$\Sigma M_A = 0$

$E_y \times 1 + (0.6T) \times 2 - 5\times10^6 \times 3 = 0$

$\Sigma f_x = 0$

$E_x = 0.8T$

$\Sigma F_y = 0$

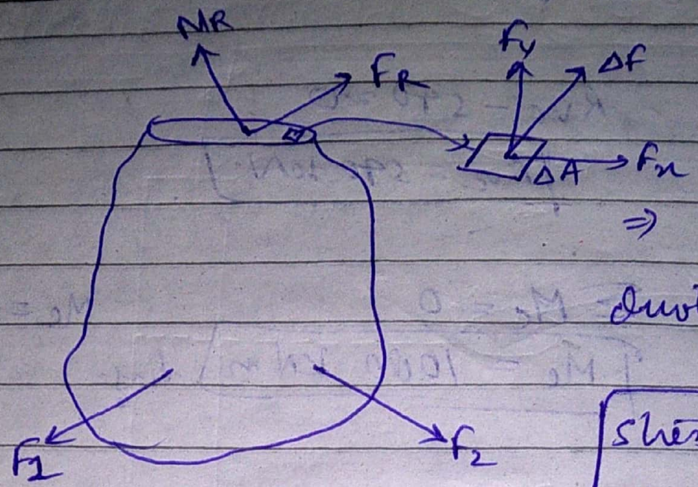
$E_y + T_y = 5\times10^6$ N

$E_y + 0.6T = 5\times10^6$ N

$E_y =$

~~T = 1667 kN.~~

Stress.



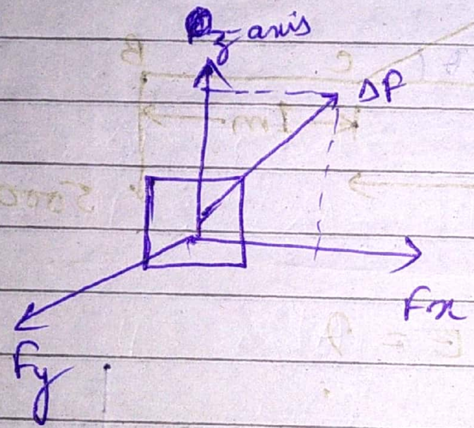
$\Delta A \to 0$

$\Rightarrow \Delta F \to 0$

Quotient $\frac{\Delta F}{\Delta A} \neq 0$

$$\boxed{\text{Stress} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}}$$

$$\boxed{\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}}$$



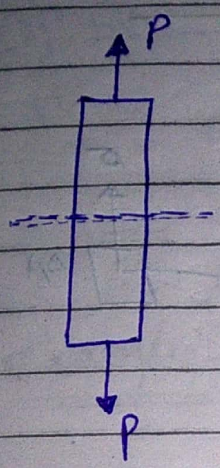
$$\text{Normal stress} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A} = \sigma$$

N/m^2
Pa or
$MPa = 10^6 Pa$

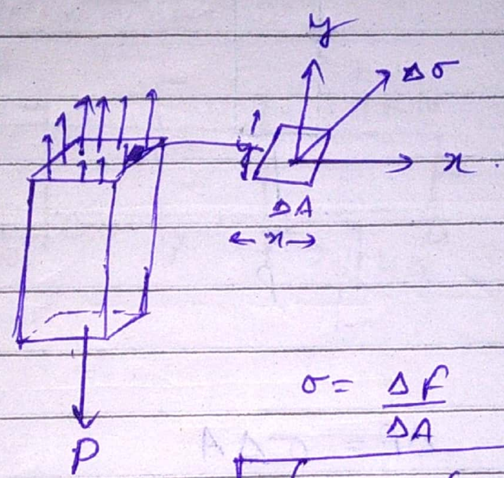
$$\text{Shear stress} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} = \tau$$

$$= \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} = \tau$$

Average Normal stress in axially loaded bar



→ Bar is prismatic → uniform cross section throughout.
→ Bar is homogeneous.
→ Bar should be material which is an isotrop
→



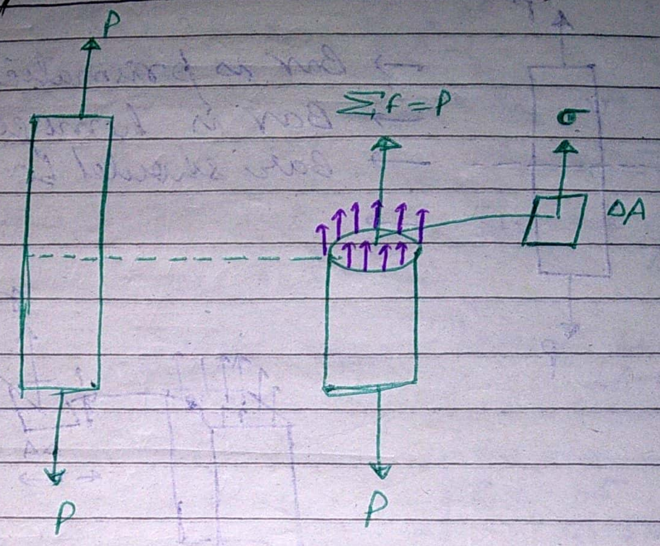
$$\sigma = \frac{\Delta F}{\Delta A}$$

$$\boxed{\int_A \Delta F = \int_A \sigma \Delta A}$$

A → whole ar

13 Aug, 2018.

AVERAGE NORMAL STRESS



$$\Delta f = \sigma \Delta A$$

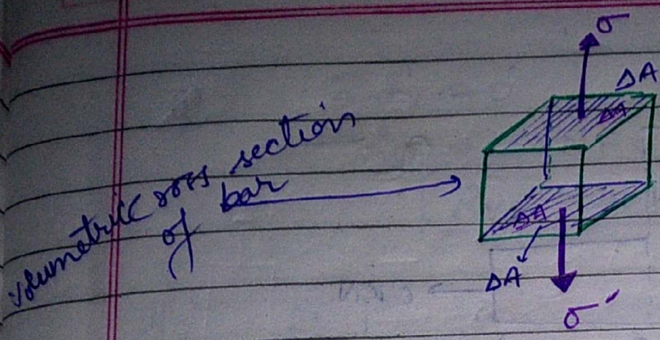
$\Delta f \to df$; $\Delta A \to dA$

$$df = \sigma\, dA \Rightarrow \int_A df = \int_A \sigma \cdot dA$$

$$P = \sigma \int_A dA$$

$$\boxed{P = \sigma A \Rightarrow \sigma = P/A}$$

Uniaxial state of stress



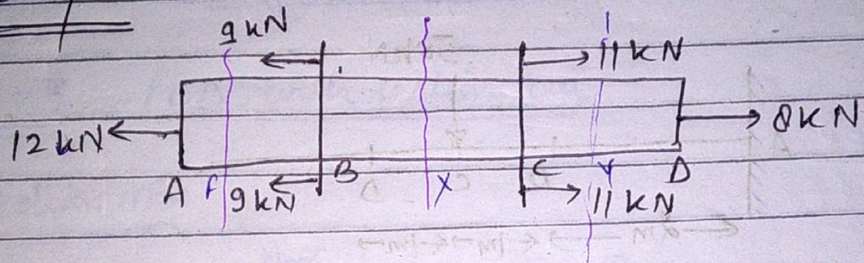
$\Sigma F = 0$

$\sigma \Delta A - \sigma' \Delta A = 0$

$\sigma \Delta A = \sigma' \Delta A$

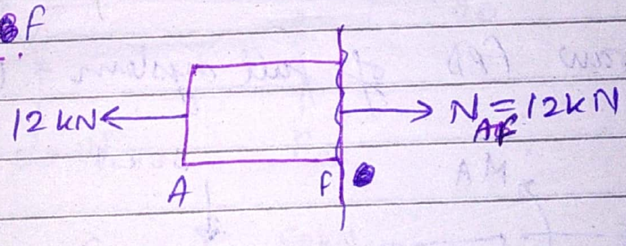
$\boxed{\sigma = \sigma'}$

Example

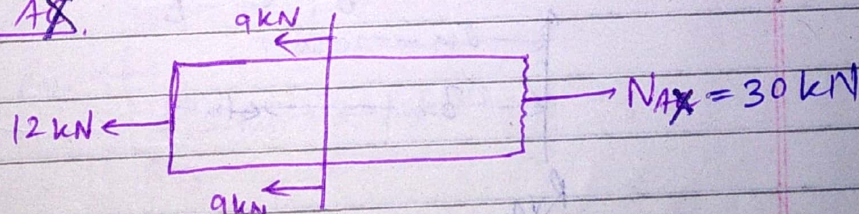


Sollution

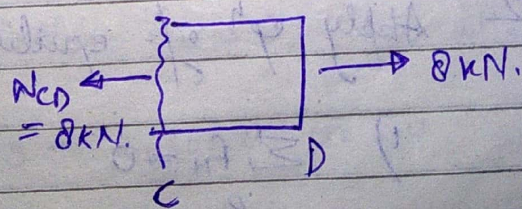
Section AF

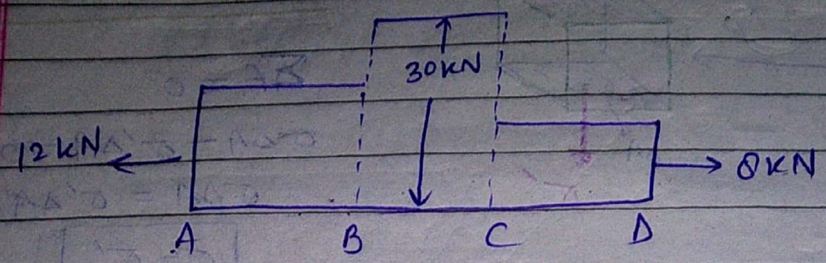


Section AX



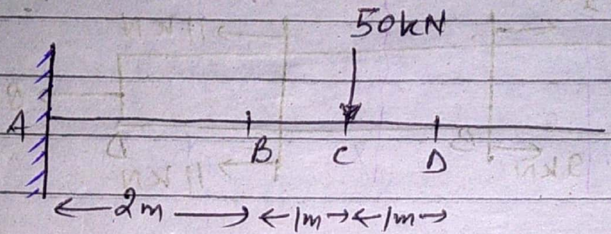
Section CD





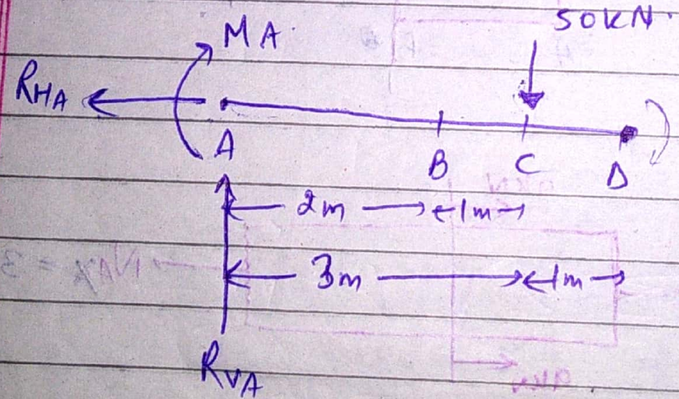
Normal force diagram.

Example



Determine the internal forces at B.

soll^n Draw FBD of full system (step 1)



(removing support)

Step-2 Apply eq^n of equilibrium.

i) $\Sigma F_H = 0$

$R_{HA} = 0$.

ii) $\Sigma f_v = 0$

$R_{VA} - 50\,kN = 0$

$$\boxed{R_{VA} = 50\,kN}$$

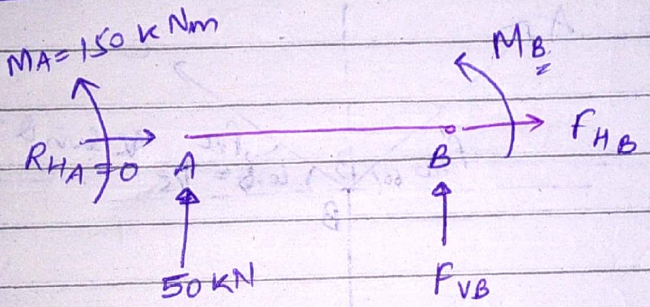
iii) $\Sigma M_D = 0$

$$M_A + 50\times4 - 50\times1 = 0$$

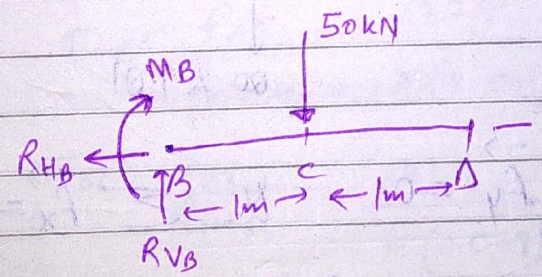
$$\boxed{M_A = -150\,kNm}$$

Step-3. FBD with section cut.

Section AB.



Section BD

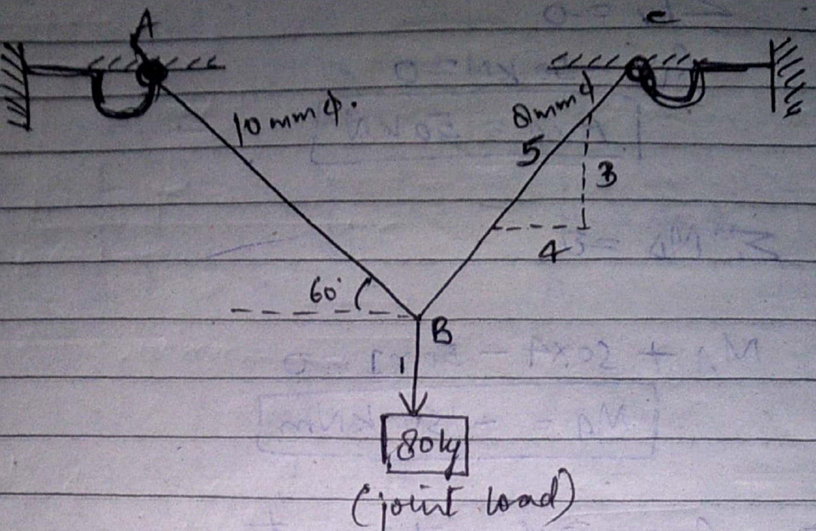


$\Sigma \vec{M_D} = 0$; $M_B + R_{VB}\times(2) + 150\times1 = 0$

$R_{VB} = 50\,kN$; $R_{HB} = 0$

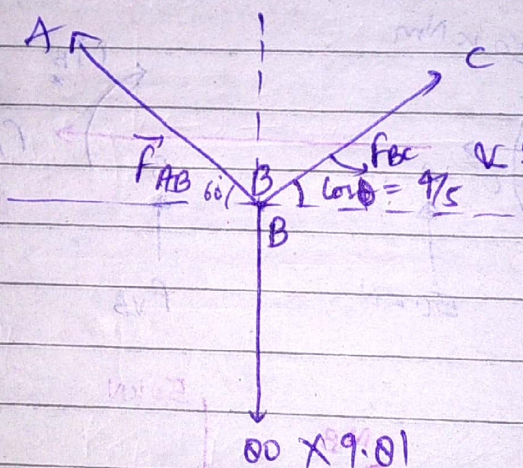
$$M_B + 2(50\,kN) + 150 = 0$$

Q/1



ϕ → diameter

FBD



$\sin\theta = 3/5$

$\cos\theta = 4/5$

$\sum \vec{F_y} = 0$ & $\sum \vec{F_x} = 0$

$F_{AB} \sin 60° + (F_{BC} \times 3/5) = 0$

$F_{AB} \times \sqrt{3}/2 = -F_{BC} \times 3/5$

$F_{AB} = -F_{BC} \cdot \frac{2\sqrt{3}}{5}$

$F_{AB} = -F_{BC}(0.69)$ —— (1)

$$\sum \vec{F_x} = 0.$$

$$\vec{F_{AB}} \times \cos 60 - F_{BC}\left(4/5\right) = 0.$$

$$\vec{F_{AB}} \times \frac{1}{2} = F_{BC} \times \frac{4}{5}.$$

$$\vec{F_{AB}} = F_{BC} \times 8/5.$$

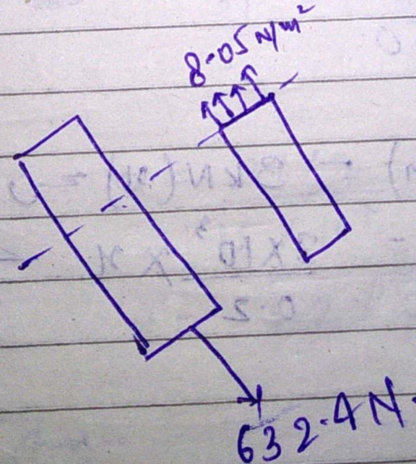
$$\vec{F}_{AB} = \vec{F}_{BC}\,(1.6) \text{ ——— } (2).$$

A. $\sigma = F/A$

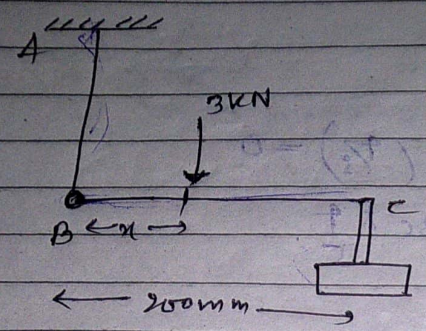
Rod AB; $\sigma_{AB} = \dfrac{F_{AB}}{\text{Area }AB} = \dfrac{6324}{\pi(0.005)^2} = 8.05 \times 10^6\ N/m^2$

Rod BC; $\sigma_{BC} = \dfrac{F_{BC}}{\text{Area}_{BC}} = \dfrac{395.2}{\pi(0.004)^2} = 7.86 \times 10^6\ N/m^2.$

(MPa)
mega Pascal.



Q 2



Area at C = 650 mm²
Area of rod AB = 400 mm²

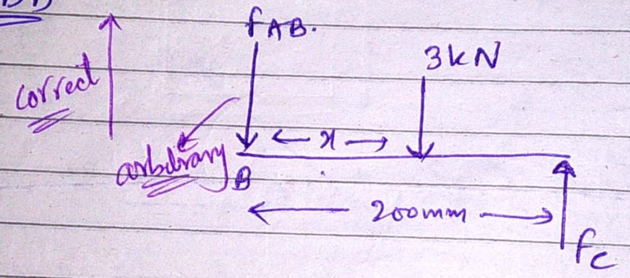
Find x = ? such that the tension in AB is equal to the compression at C.

Tensile stress

stress

Sol^n

FBD



$\sum F_y = 0.$

$\cancel{F_{AB} = F_c \leftarrow (1)}$

$\vec{F}_{AB} + 3\,kN = \vec{F}_c$ ——— (1)

$\sum M_B = 0.$

$F_c\,(0.2\,m) - 3\,kN\,(x) = 0$

$F_c = \dfrac{3 \times 10^3}{0.2} \times x$ ⟵ (2)

Tensile stress at Rod$/_{AB}$ = $\frac{f_{AB}}{400\ mm^2}$.

compressive stress at C = $\frac{f_c}{650\ mm^2}$.

$$\frac{f_{AB}}{40\cancel{0}\ \cancel{mm^2}} = \frac{f_c}{65\cancel{0}\ \cancel{mm^2}}$$

$$f_{AB} = f_c \times \frac{40}{65}$$

$$f_{AB} = f_c \times \frac{8}{13} \quad \text{———} \quad (3)$$

$$\frac{8 f_c}{13} + 3kN = f_c$$

$$\frac{13-8}{13} f_c = 3kN$$

$$f_c = \frac{3 \times 13}{5}$$

$$\boxed{f_c = 39/5}$$

$$\frac{39}{5} = \frac{3 \times 10^4}{2} \times x$$

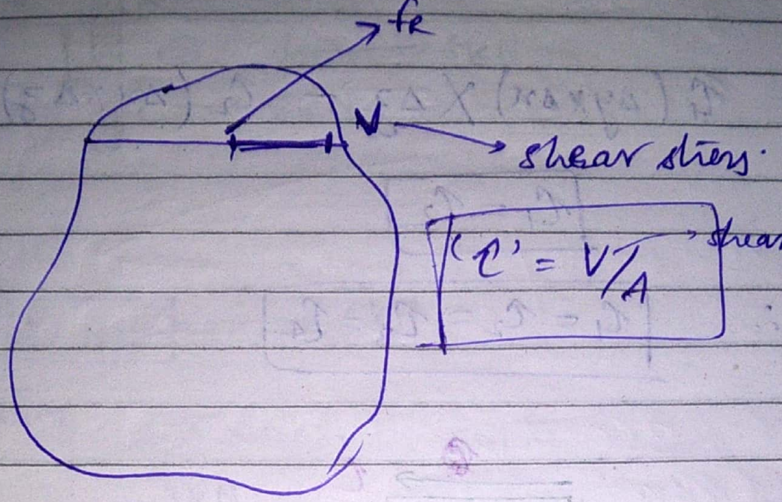
$$x = \frac{39 \times 2}{15 \times 10^4}$$

$$x = 5.2 \times 10^{-4}\ m.$$

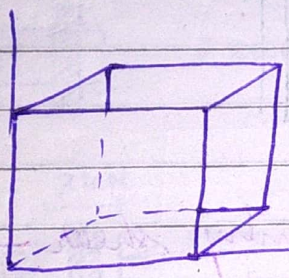
$\boxed{x = 1.24\ mm}$ Ans

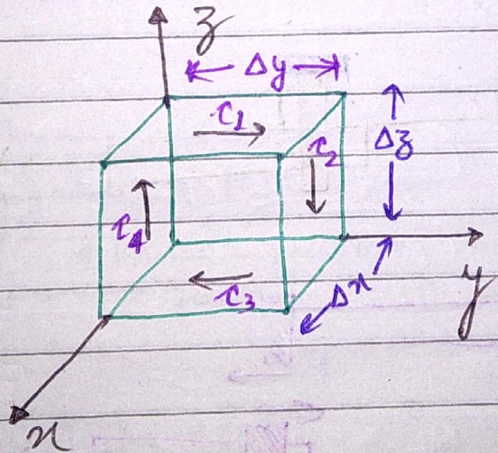
Aug, 31, 2018.

Average Shear Stress



$$\boxed{\tau = V/A}$$ shear force.





for Eq^b,

$$\sum f_y = 0$$

$$\tau_1(\Delta y \times \Delta x) - \tau_2(\Delta y \times \Delta x) = 0$$

$$\boxed{\tau_1 = \tau_2}$$

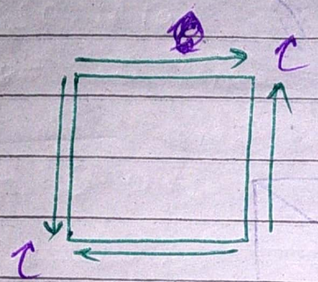
$$\sum f_z = 0 \quad \boxed{\tau_2 = \tau_4}$$

$$\sum M_x = 0$$

$$\tau_1(\Delta y \times \Delta x) \times \Delta z - \tau_2(\Delta x \times \Delta z) \times \Delta y = 0$$

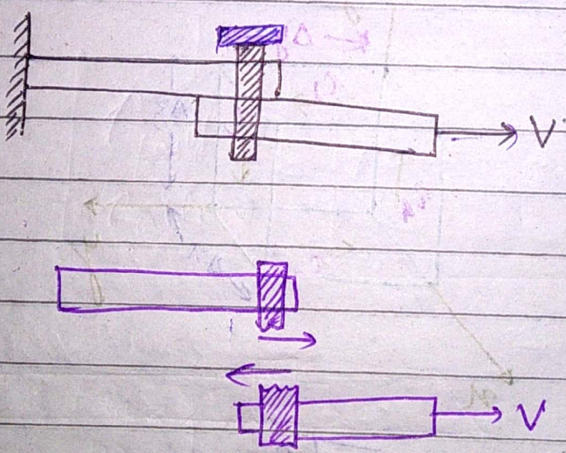
$$\boxed{\tau_1 = \tau_2}$$

$$\therefore \boxed{\tau_1 = \tau_2 = \tau_3 = \tau_4}$$

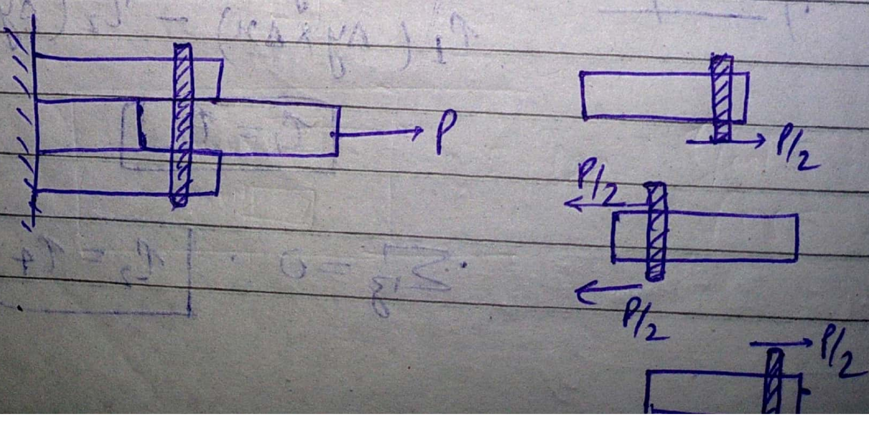


Complementry shear $\longrightarrow$ Pure shear.

e.g.

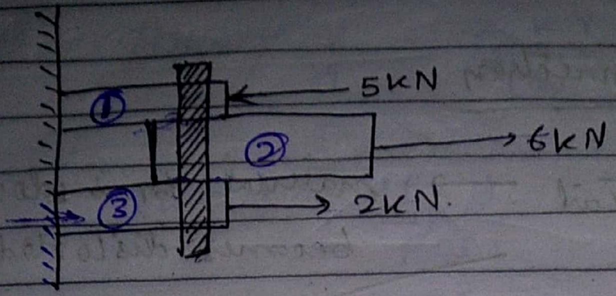


e.g.

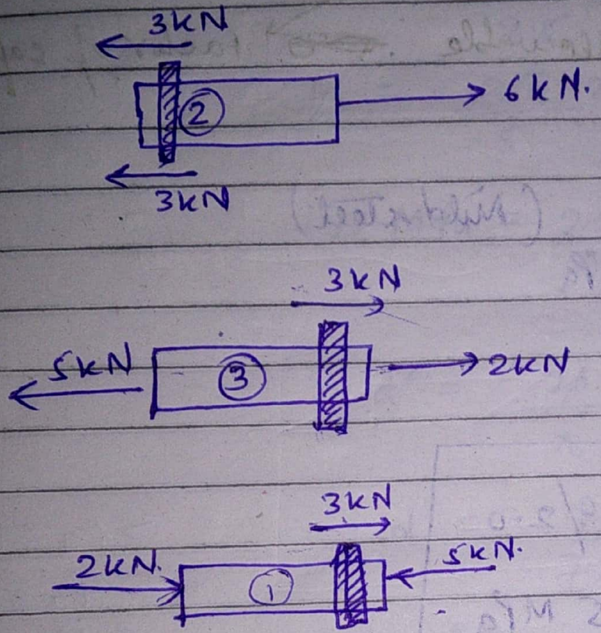


Q

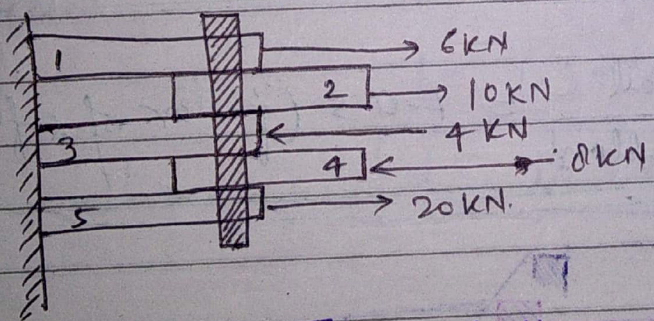
Find the shear force.

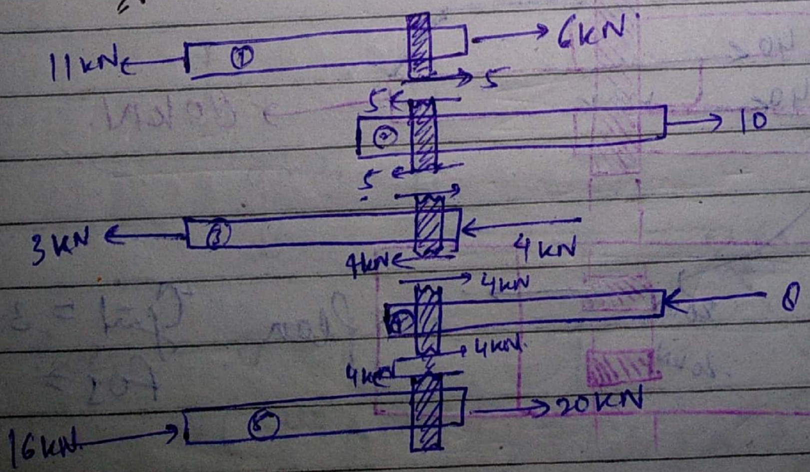


Soln



Q





Design of Connection

$\tau_{fail} / \sigma_{fail} \longrightarrow$ material breakdown or becomes distorted.

$\tau_{allowable} / \sigma_{allowable} \longrightarrow$ factor of safety.

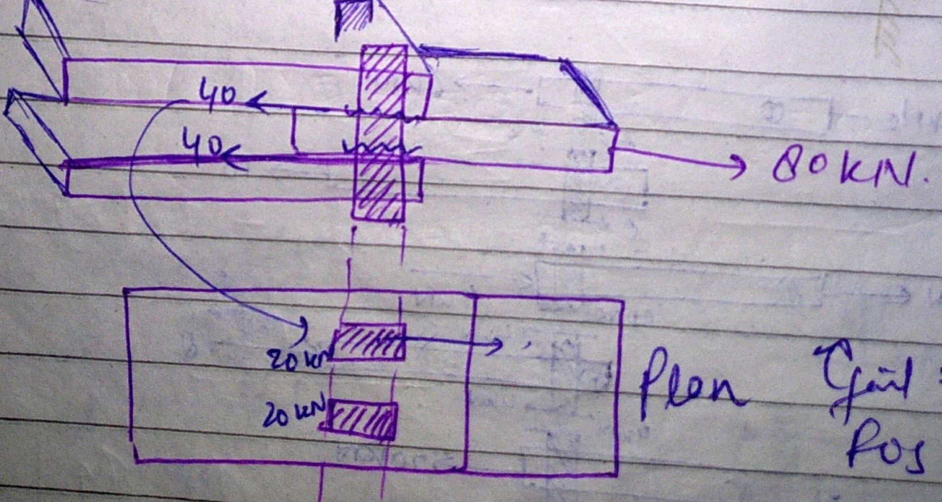
eg: 350 MPa (Mild steel)

$\tau_{fail} = 350$ MPa

FOS = 2.0

$$\boxed{\begin{aligned} \tau_{allow} &= 350/2.0 \\ &= 1.75 \text{ MPa} \end{aligned}}$$

$$\boxed{\frac{\tau_{fail}}{\tau_{allow}} = \text{F.O.S (factor of safety)}}$$



Plan $\tau_{fail} = 350$ MPa

FOS = 2.5.

$$\tau_{allow} = \frac{V}{A} = \frac{20\,kN}{\pi(d/2)^2}$$

$$\tau_{allow} = \frac{\tau_{fail}}{FOS}$$

$$= \frac{350}{2.5}$$

$$= \frac{350 \times 2}{5} = 140\,MPa$$

$$140\,MPa = \frac{20\,kN}{\pi/4\,d^2}$$

$$d^2 = \frac{20 \times 10^3 \times 4}{\pi \times 140 \times 10^6}$$

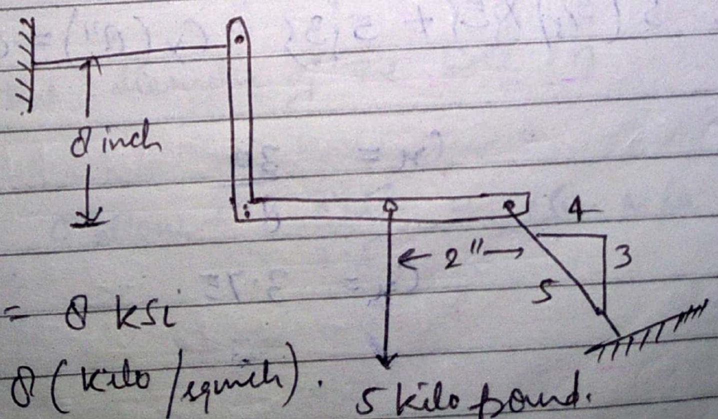
$$d^2 = \frac{40}{7 \times \pi} \times 10^{-4}$$

$$d = \sqrt{\frac{40}{7\pi}} \times 10^{-2}$$

$$d = 1.34 \times 10^{-2}\,m$$

$$\boxed{d = 13.4\,mm}$$

Q2

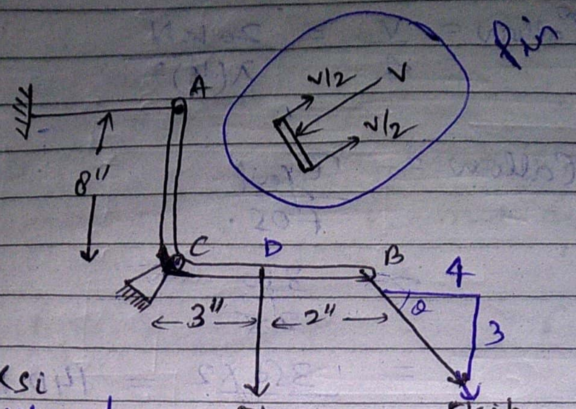


$\tau_{allow} = 8$ ksi

8 (kilo/squinch).

7 sept, 2018 :

Q. 2



$\tau_{allow} = 8\,ksi$

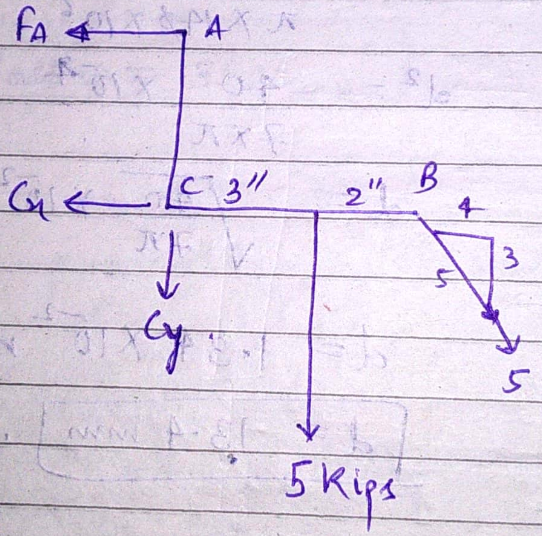
8 kilo pound / squarr inch.

5 kilo pound

Determine the diameter of the pin?

soln

Free body diagram :-



$\Sigma F_x = 0$

$\Sigma F_y = 0$

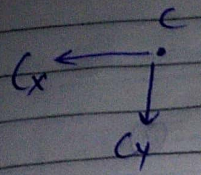
$\Sigma M_A / \Sigma M_B = 0.$

$\Sigma_A = 0$

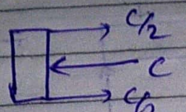
$$5(3/5) \times \{5\} + 5\{3\} + C_x(8'') = 0.$$

$$C_x = \frac{30}{8''}$$

$$C_x = 3.75$$



$C = \sqrt{C_x^2 + C_y^2}$; $\boxed{C = 8.1}$



$\tau_{allow} = \dfrac{C/2}{A}$

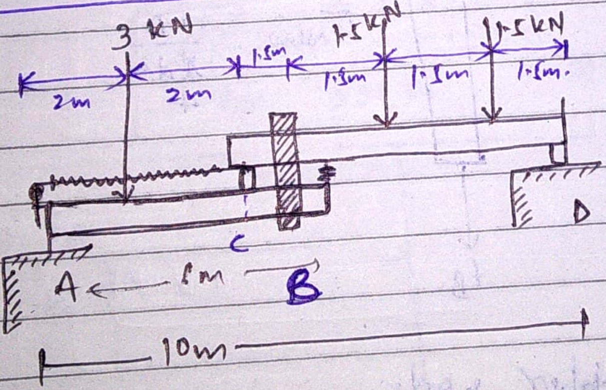
$$8 = \frac{C/2}{\frac{\pi}{4} d^2}$$

$$8 = \frac{4C/2}{\pi d^2} \Rightarrow 4 = \frac{C}{\pi d^2}$$

$$d = \sqrt{\frac{C}{4\pi}} = \sqrt{\frac{8.1}{4\pi}} = 0.80$$

$\boxed{d = 0.8 \text{ inches}}$

Q. Imp



P.G. 76

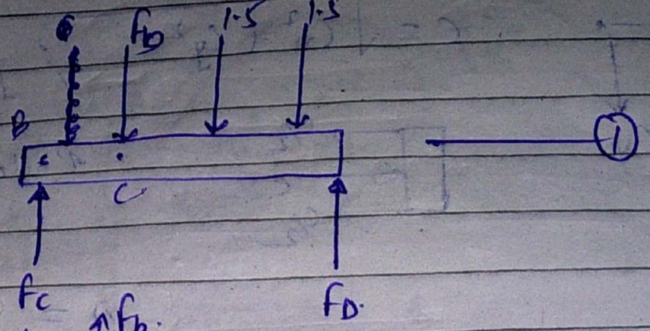
$\sigma_{fail} = 375$ MPa

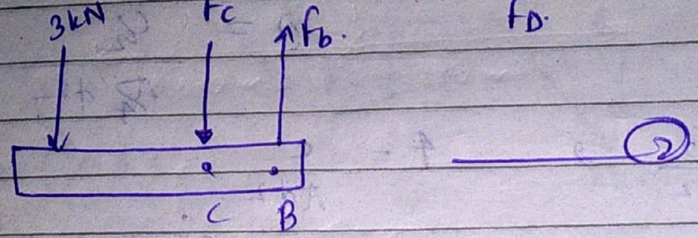
FOS = 5/2

Find the diameter of the bolt = ?

$$\sigma_{allow} = \frac{375}{2.5} = 150 \text{ MPa}.$$

Sol^n



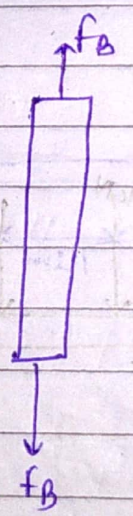


i) FBD

ii) Take moments $\Sigma M_A = 0$

$\Sigma M_D = 0$

iii) Simultaneous eq^n in f_B & f_c



$$\sigma_{allow} = \frac{f_B}{\frac{\pi d^2}{4}}$$

1) In the upper rod:

$\Sigma \vec{M_D} = 0 \Rightarrow \quad 1.5 \times 1.5 + 1.5 \times 3 + f_b \times 4.5 - f_c \times 6 = 0$

$6 f_c - 4.5 f_b = 6.75$ —— ①

ii) In the lower block.

$\boxed{\Sigma M_A = 0}$.

$30 \times 2 + f_c \times 4 - f_B \times 6 = 0$

$3f_B - 2f_c = 3$ —— ②

$f_c = \frac{3f_B - 3}{2}$.

Putting it into i, ①

$6\left(\frac{3f_B - 3}{2}\right) - \frac{9}{2} f_B = 6.75$

$9f_B - 9 - \frac{9}{2} f_B = 6.75$.

$f_B = -\frac{2.25}{4.5}$

$\boxed{f_B = -0.5\ kN}$.

$f_B = 500\ N$ $\Rightarrow$ $f_c = 733.5\ N$.

$\sigma_{allow} = \frac{f_B}{\frac{\pi}{4} d^2}$

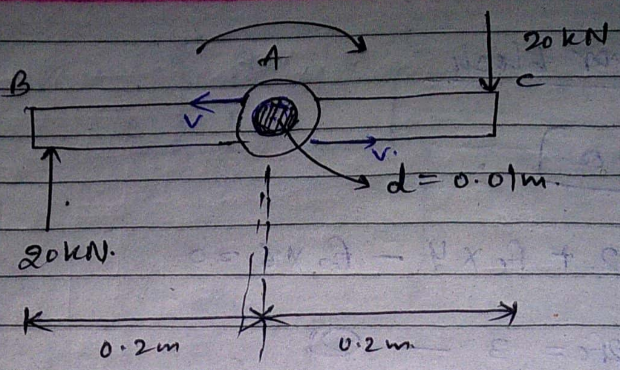
$15\cancel{0} \times 10^6 = \frac{50\cancel{0}}{\frac{\pi}{4} d^2}$.

$d = \sqrt{\frac{200 \times 10^{-6}}{\pi \times 15}}$

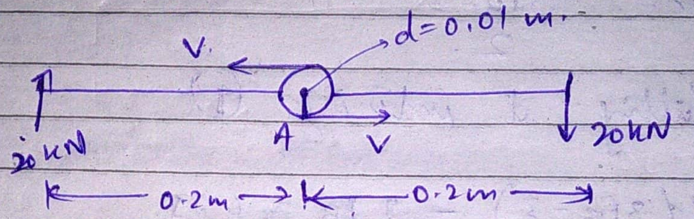
$d = 2.06 \times 10^{-3}\ m$.

$\boxed{d = 2.06\ mm}$.

Q. 2



Soln FBD



$$\Sigma M_A = 0.$$

$$20 \times 0.2 + 20 \times 0.2 + \underbrace{\left(V \times \frac{0.01}{2} + V \times \frac{0.01}{2} \right)}_{\text{bolt}} = 0.$$

$$4 + 4 + 0.01V = 0$$

$$V = \frac{-8}{0.01}$$

$$\boxed{V = -800 \text{ kN}}.$$

$$\sigma = \frac{800 \times 10^3}{\frac{\pi}{4}(0.01)^2} = \frac{800 \times 4 \times 100 \times 100 \times 10^3}{\pi \times 1 \times 1}$$

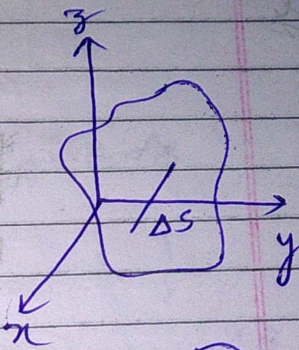
$$= \frac{32 \times 10^6}{\pi}.$$

$$\boxed{\sigma = 10.18 \text{ GPa}}$$

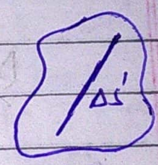
So, 10.18 GPa of stress will develop at the bolt.

STRAINS

$$\epsilon = \frac{\text{change in length}}{\text{original length}}$$



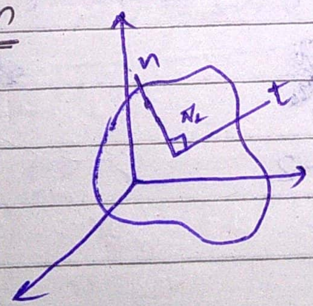
$$\boxed{\epsilon = \frac{\Delta s' - \Delta s}{\Delta s'}}$$

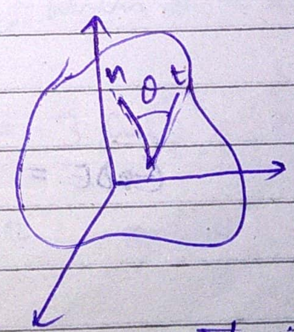


⇒ Normal strain is unitless,
mm/mm, m/m, ...

1 % = 0.01 mm/mm.

Shear strain





change in angle is called shear strain.

$$\boxed{\gamma_{nt} = \pi/2 - \theta}$$ Radians.

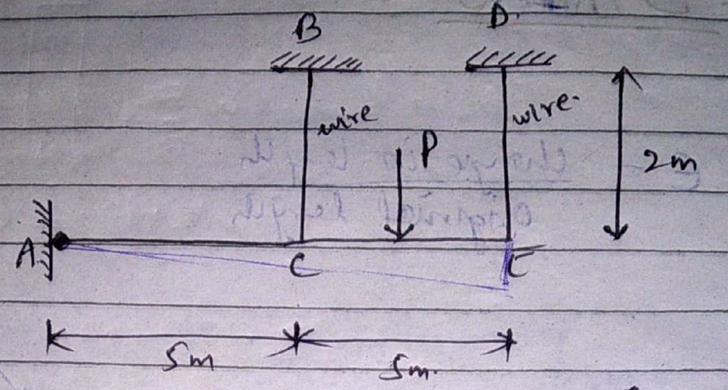
$\theta > \pi/2$ γ_{nt} -ve
$\theta < \pi/2 \rightarrow \gamma_{nt}$ +ve

γ_{nt} = +ve compressive
γ_{nt} = -ve tension

+ve → compression ; $\frac{y}{nt} < \frac{h}{2}$

-ve → tension ; $\frac{y}{nt} > \frac{h}{2}$

Q.



$\epsilon_{DE} = \frac{2.01-2}{2}$

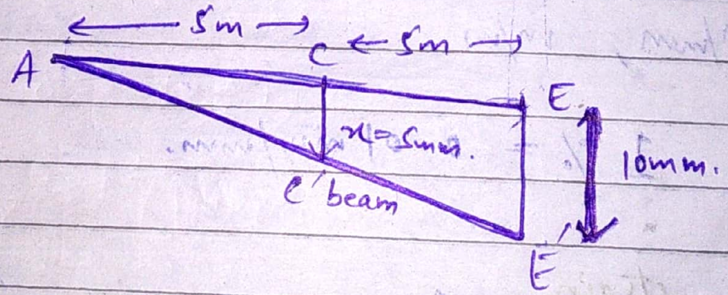
$= \frac{0.01}{2}$

$= 0.005$

$\delta E = 10mm$

Determine the strain wire BC and DE. ?

Soln F.B.D



$\frac{10}{x} = \frac{10}{5}$

$\boxed{x = 5}$

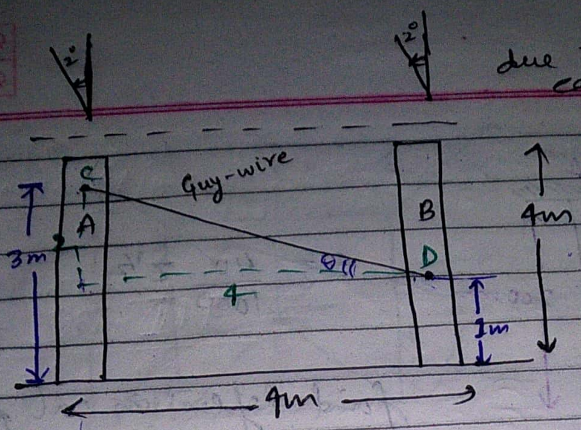
$\epsilon DE = \frac{2.01-2}{2}$

Change BC $= \frac{2.005-2}{2}$

$\epsilon BC = \frac{0.05}{2}$

$= 0.025$

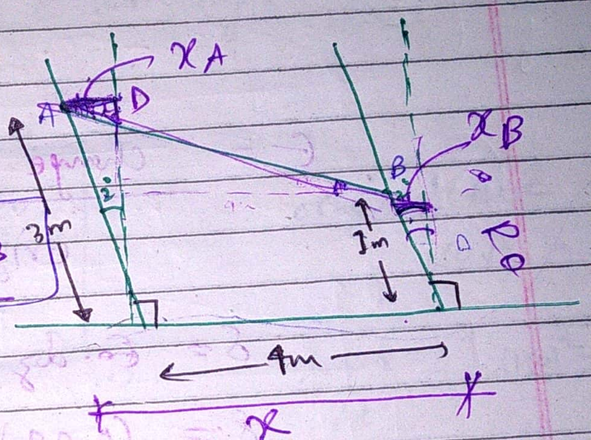
Q. (doubt)



Determine the normal strain in wire AB.

Soln

$$\boxed{\text{Strain} = \frac{\Delta L}{L}}$$



$$\boxed{x = 4 + x_A - x_B}$$

$CD = \sqrt{4^2 + 2^2}$

$= \sqrt{20}$

$= 4.48\ m$ → original length

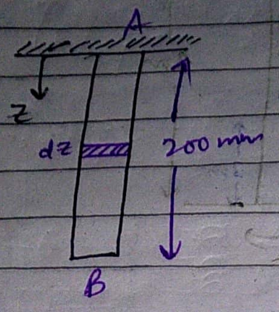
$\boxed{\tan\theta \simeq \theta \simeq \sin\theta}$; when θ is small

$AD = 3 \sin 2 = \underline{0.104}$

$\tan 2^\circ = \frac{AD}{}\ 2 = \underline{0.104}$.

Sin

Q.



$$\epsilon = \frac{40}{1000} z^{1/2}$$

find deflection at point B = ?

Sol

$$dz' \doteq dz + \delta.$$

$$\epsilon = \frac{\text{change in length } (\delta)}{\text{original length } (dz)}$$

$$\delta = \epsilon_0 \cdot dz$$

$$\delta = (0.04)\, z^{1/2}\, dz$$

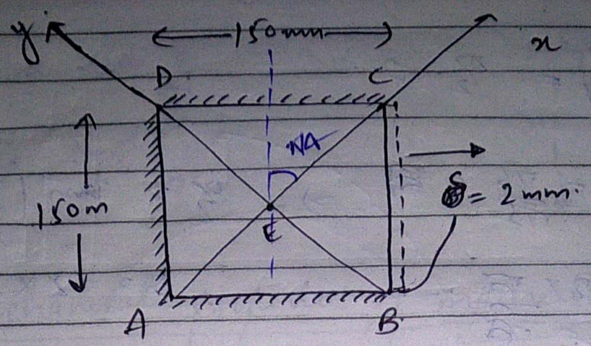
$$dz' = \left(1 + 0.04 z^{1/2}\right) dz$$

$$= \left[z + 0.04 \times \frac{2}{3} \times z^{3/2} \right]_0^{0.2\,m}$$

$$= 0.2 + 0.04 \times \frac{2}{3} \times (0.2)^{3/2}$$

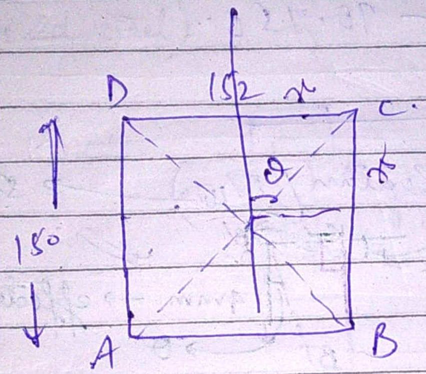
$$\boxed{dz' = 0.202}$$

Q.



i) Determine ε normal in Diagonal AC and BD
ii) Determine shear strain at E w.r.t xy.

Soln



original AC = $150\sqrt{2} = 212.14$

final AC' $= \sqrt{(150)^2 + (152)^2}$

$= \sqrt{22500 +}$

$= 213.56$

$\epsilon = \frac{AC' - AC}{AC}$

$\epsilon = \frac{213.56 - 212.14}{212.14}$

$= \frac{1.42}{212.14}$

$= 0.0066$

$\boxed{\epsilon = 6.7 \text{ mm/mm}}$

ii) $\gamma_{E/xy} = \frac{\pi}{2} - \theta$

$\tan\left(\frac{\theta}{2}\right) = \frac{76}{75}$ $\Rightarrow$ $\frac{\theta}{2} = \tan^{-1}\left(\frac{76}{75}\right)$

$\theta = 2 \times$ ———— $\theta = 2 \times (\quad)$

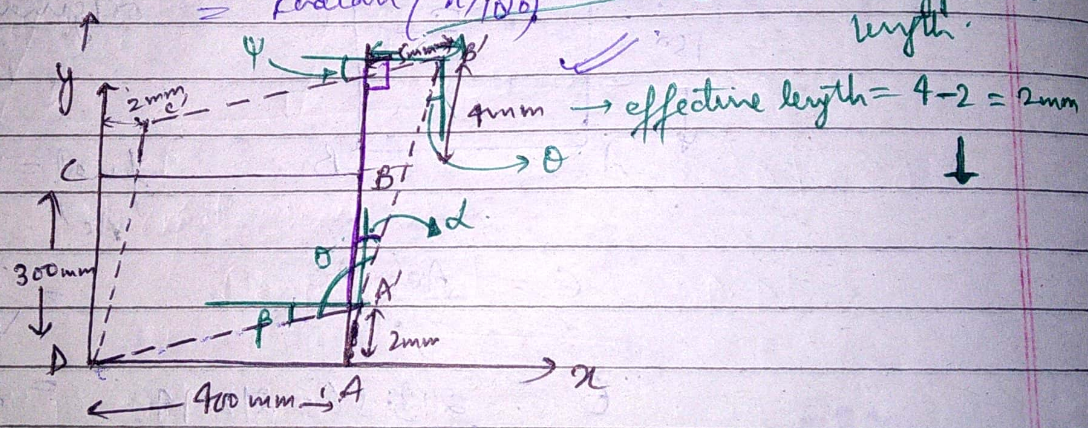
$= 90.758$

$= 90 - 90.758$

$=$

$=$ Radian $(\pi/180)$

Q.



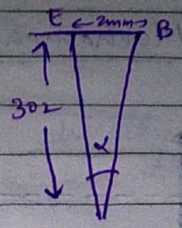
(i) Determine shear stress at A and B. ?

Q/2 ~~change in angle~~ $= \theta - \pi/2$.

length $A'B' = 302$ mm.

$\alpha \simeq \tan\alpha = \frac{2}{302}$

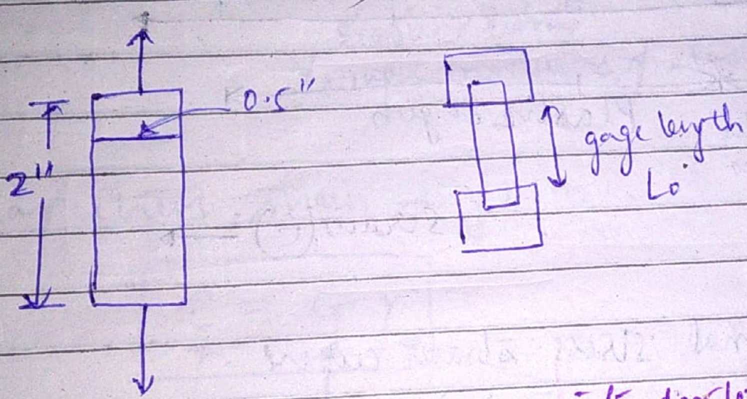
$\psi = \alpha = \tan^{-1}\left(\frac{2}{302}\right)$



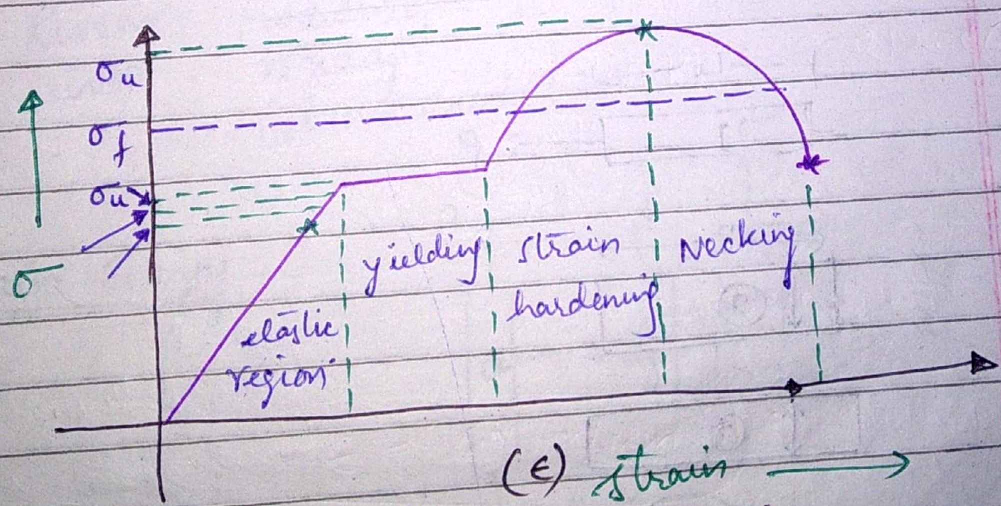
Mechanical properties of materials

- Tensile test diagram.

- steel (Mild steel)



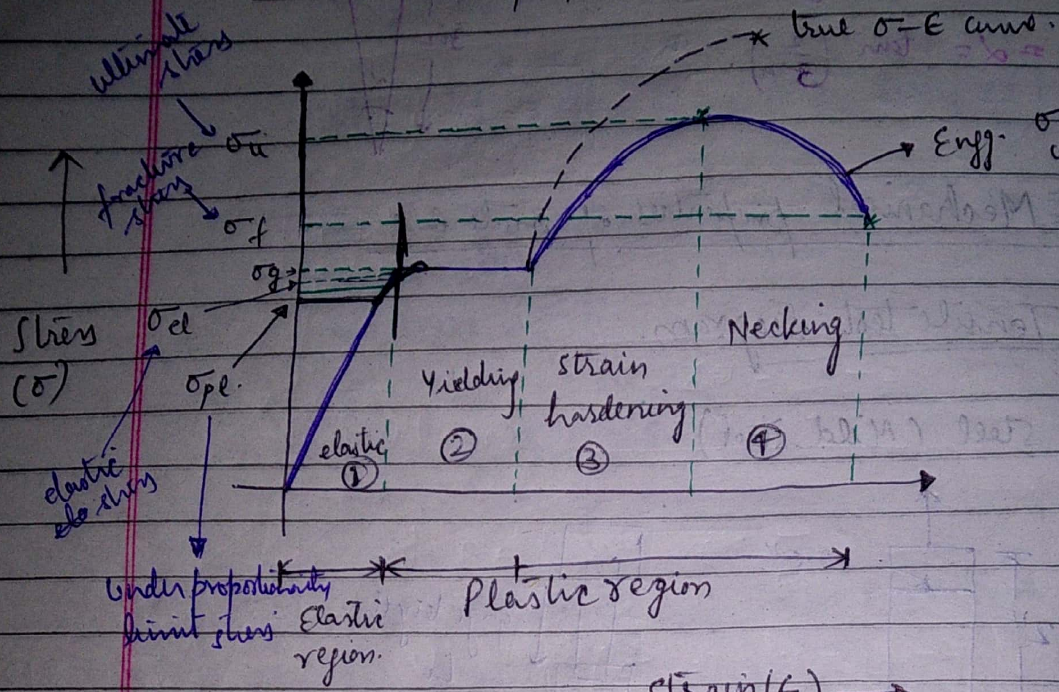
ASTM. (American society for testing of materials)



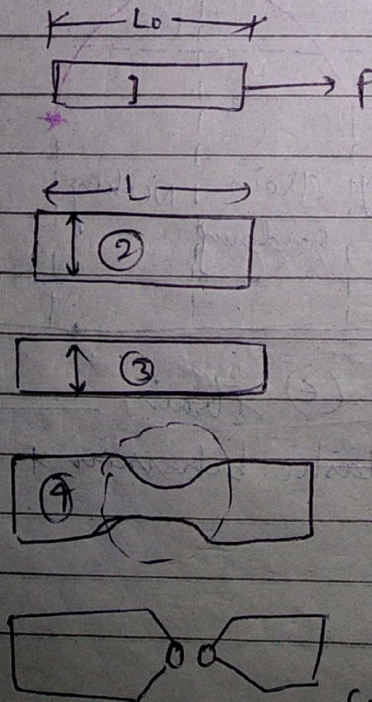
elastic behavior — Plastic behaviour

12 sept, 2018.

Mechanical properties



Normal stress-strain curve (Engineering)



cup & cone failure.

Hooke's Law

Within the elastic limit, stress is directly proportional to strain, i.e

$$\sigma \propto \epsilon$$

$$\boxed{\sigma = E\epsilon}$$

strain

stress

~~Elastic limit~~
Young's modulus of elasticity

i) Shear stress strain daigram.

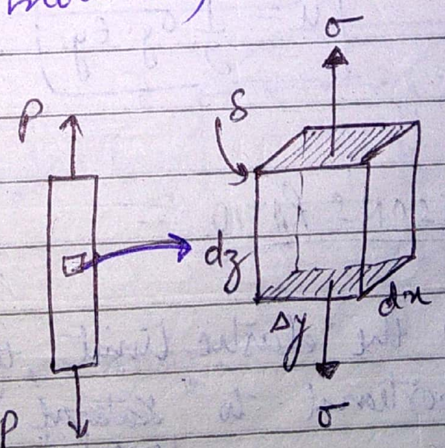
$$\boxed{\tau = G \cdot \gamma}$$

shear strain

Shear stress

Modulus of rigidity (shear modulus)

Strain Energy:



$$\text{workdone} = \frac{1}{2} \Delta f \times \delta$$

$$\epsilon = \frac{\delta}{dz} \Rightarrow \boxed{\delta = \epsilon \cdot dz}$$

$\delta \rightarrow$ change in dz.

$$W = \frac{1}{2}\left[\sigma(dxdy)\right] \cdot (\epsilon \cdot dz)\Big]$$

$$W = \frac{1}{2}\sigma \cdot \epsilon \; dx \cdot dy \cdot dz.$$

$$\frac{W}{dx \cdot dy \cdot dz} = \frac{1}{2}\sigma \cdot \epsilon$$

$$\boxed{u \triangleq \frac{\Delta W}{\Delta V} = \frac{1}{2}\sigma\epsilon}$$

work done per unit volume.

$$\boxed{u = \frac{\delta U}{\delta V} = \frac{1}{2}\sigma\epsilon}$$

$$\sigma = E\epsilon \Rightarrow \epsilon = \frac{\sigma}{E}$$

$$u = \frac{1}{2}\sigma\left(\frac{\sigma}{E}\right) \Rightarrow \boxed{u = \frac{1}{2}\frac{\sigma^2}{E}}$$

Modulus of Resilience

$$\boxed{u = \frac{1}{2}\frac{\sigma_y^2}{E}}$$ $y \rightarrow$ yielding

$$\boxed{u = \frac{1}{2}\sigma_y \cdot \epsilon_y}$$

R.C. Hibbeler:

$$\boxed{\mu_r = \frac{1}{2}\sigma_{pl} \cdot \epsilon_{pl}}$$

POISSON'S RATIO

Within the elastic limit, the ~~lateral~~ lateral ~~longitudinal~~ ~~stress~~ strain is ~~dir~~ directly proportional to ~~lateral~~ longitudinal strain.

$$\epsilon_{long} = \nu \, \epsilon_{lateral}$$

$$\boxed{\nu = -\frac{\epsilon_{lateral}}{\epsilon_{longitudinal}}}$$

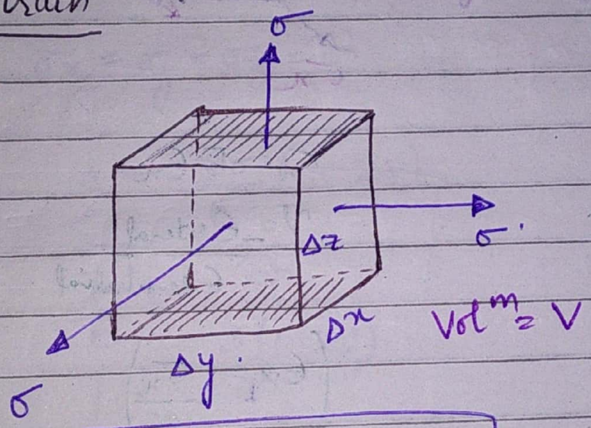
$$\boxed{0 < \nu < 0.5}$$

ν (steel) = 0.3

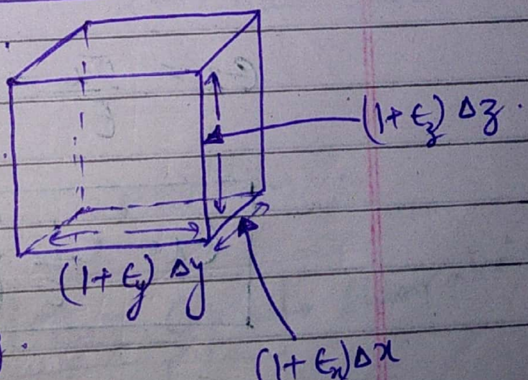
$\nu_{concrete}$ = 0.15 cd go upto 0.2

ν(strain rubber) = 0.5

Volumetric strain



$$\boxed{\delta V = \Delta x \cdot \Delta y \cdot \Delta z}$$



$$\delta V' = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z)\Delta x \cdot \Delta y \cdot \Delta z$$

$$\delta V' = (1 + \epsilon_x + \epsilon_y + \epsilon_z)\,\Delta x \cdot \Delta y \cdot \Delta z$$

$$\text{Vol}^m = V + \delta V$$

$$\boxed{V + \delta V' = (1 + \epsilon_x + \epsilon_y + \epsilon_z)\,\delta V}$$

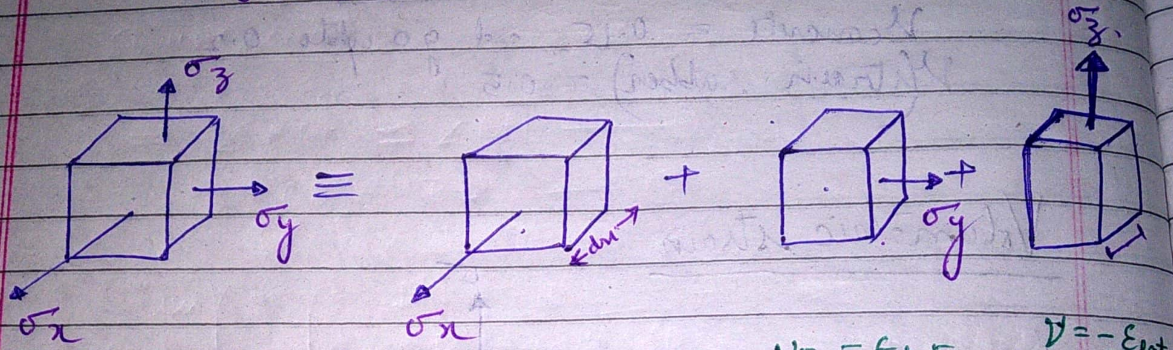
$$\cancel{V} + \delta V = \cancel{V} + V(\epsilon_x + \epsilon_y + \epsilon_z)$$

$$\frac{d V}{V} = (\epsilon_x + \epsilon_y + \epsilon_z)$$

$$\boxed{e = \frac{\delta V}{V} = (\epsilon_x + \epsilon_y + \epsilon_z)}$$

Generalised Hooke's law

$\sigma = E \cdot \epsilon$



$\sigma_x = E \cdot \epsilon_x$

$\nu = \frac{-\epsilon_{lateral}}{\epsilon_{longitudinal}}$

$\left(\epsilon_{x_1} = \frac{\sigma_x}{E}\right)$

$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$

$\nu = \frac{-\epsilon_{x_2}}{\epsilon_y}$

$\epsilon_{x_2} = -\nu \epsilon_y$

$\epsilon_{x_2} = -\nu\left(\frac{\sigma_y}{E}\right)$

$\nu = -\frac{\epsilon_{lat}}{\epsilon_z}$

$\epsilon_{x_3} = -\left(\frac{\sigma_z}{E}\right)$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E}$$

$$\boxed{\epsilon_x = \frac{1}{E}\left\{\sigma_x - \nu(\sigma_y + \sigma_z)\right\}} \longrightarrow (1)$$

Similarly

$$\epsilon_y = \frac{1}{E}\left\{\sigma_y - \nu(\sigma_x + \sigma_z)\right\} \quad \text{--- (2)}$$

$$\epsilon_z = \frac{1}{E}\left\{\sigma_z - \nu(\sigma_x + \sigma_y)\right\} \quad \text{--- (3)}$$

Adding (1) + (2) + (3), we have,

$$\boxed{\epsilon_x + \epsilon_y + \epsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\nu)} \quad \text{--- (4)}$$

Assuming $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$e = \frac{3\sigma}{E}(1-2\nu)$$

$$\frac{\sigma}{e} = \frac{E}{3(1-2\nu)}$$

$$\boxed{K = \frac{E}{3(1-2\nu)}}$$

$K \longrightarrow$ bulk modulus.

Relation b/w E & G.

$$\boxed{\sigma = E\epsilon}$$

σ → Normal stress

E → Young's Modulus.

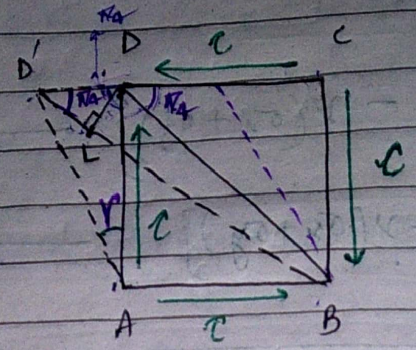
ϵ → Normal/direct/longitudinal strain

$$\boxed{\tau = G\gamma}$$

τ → shear stress

γ → shear strain

G → Shear Modulus.



Strains are small.

$$\boxed{\Rightarrow \angle BD'C \cong \angle BDC \quad \text{—— ①} \qquad BD \cong BL \quad \text{—— ②}}$$

$$\tan\gamma \approx \gamma = \frac{D'D}{AD} = \frac{D'L}{\cos 45 \times AD} = \frac{D'L}{\cos 45 \times BD \cos \pi/4}$$

$$\boxed{\gamma = \frac{2D'L}{BD}} \quad \text{—— (1)}$$

from normal strain's definition

$$\epsilon_l = \frac{D'L}{BD} \quad \text{—— (2)}$$

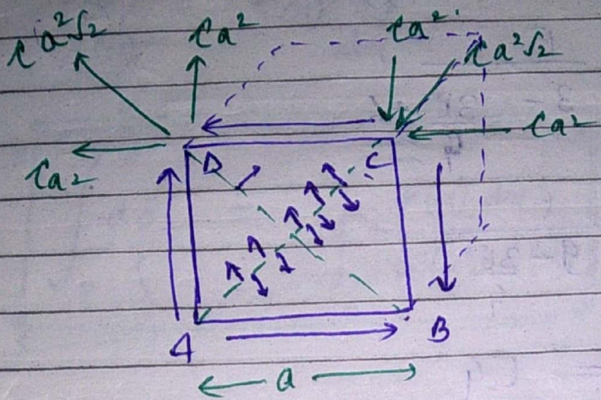
$$\epsilon_l = \gamma/2 \quad \text{—— (3)}$$

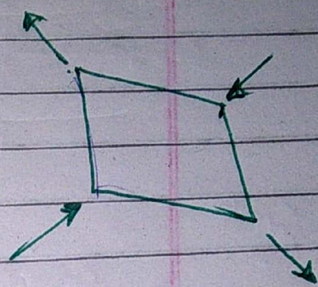
from Hooke's law

$$\tau = G\gamma$$

$$\gamma = \frac{\tau}{G}$$

or $\boxed{\epsilon_l = \frac{\tau}{2G}}$





$$\epsilon_e = \frac{\tau}{E} + \frac{\nu\tau}{E}$$

$$\epsilon_l = \frac{\tau}{E}(1+\nu) \quad \text{——— } (5)$$

(4) = (5)

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\nu)$$

$$\boxed{E = 2G(1+\nu)} \quad ***$$

Result: $\boxed{\text{Relationship b/w } E, K \text{ and } G}$

$$K = \frac{E}{3(1-2\nu)} \quad \text{——— } (1)$$

$$E = 2G(1+\nu) \quad \text{——— } (2)$$

Putting ν from (2) in (1).

$$\frac{E}{2G} - 1 = \nu.$$

$$K = \frac{E}{3\left(1 - 2\left(\frac{E}{2G} - 1\right)\right)}$$

$$K = \frac{E}{3 - \frac{3E}{G} + 6}$$

$$K = \frac{E}{9 - \frac{3E}{G}}$$

$$K = \frac{EG}{9G - 3E}$$

$$K(9G - 3E) = EG$$

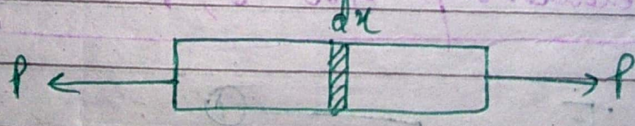
$$9KG = 3KE + EG$$

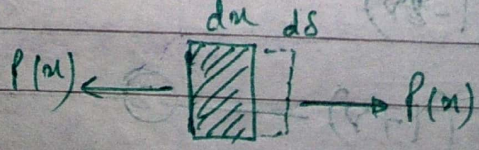
$$9KG = E(3K + G)$$

$$\boxed{E = \frac{9KG}{3K + G}}$$

***

Axial deformations

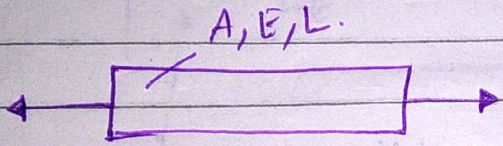




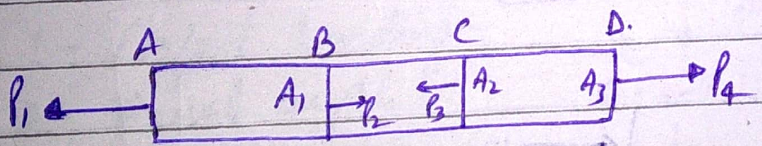
$$\sigma = E\epsilon$$

$$\frac{P(x)}{A(x)} = E(x)\frac{d\delta}{dx}.$$

$$\boxed{\int_0^{\delta} d\delta = \int_0^{L} \frac{P(x)\,dx}{E(x)\cdot A(x)}}$$



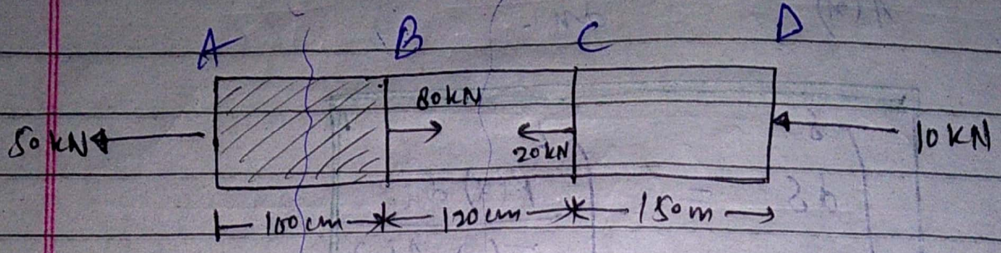
$$\boxed{\delta = \frac{PL}{AE}}$$



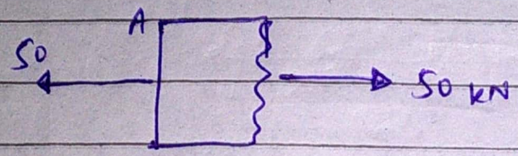
$$\delta = \Sigma \frac{PL}{AE}$$

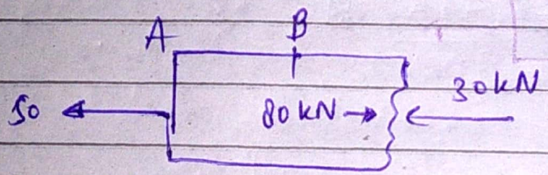
$$\boxed{\delta_{AB} = \frac{PL}{A_1E}\Big|_{AB} + \frac{PL}{A_2E}\Big|_{BC} + \frac{PL}{A_3E}}$$

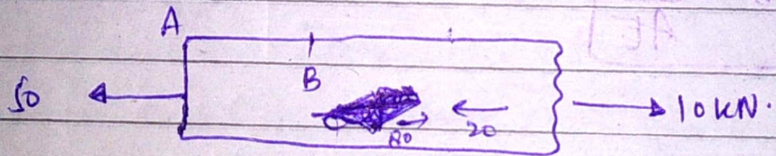
Q. Area is constt, but different forces are applied, Determine $\delta_{AD} = ?$

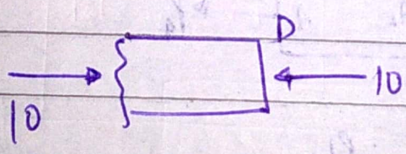


Soll^n









17 sept, 2018.

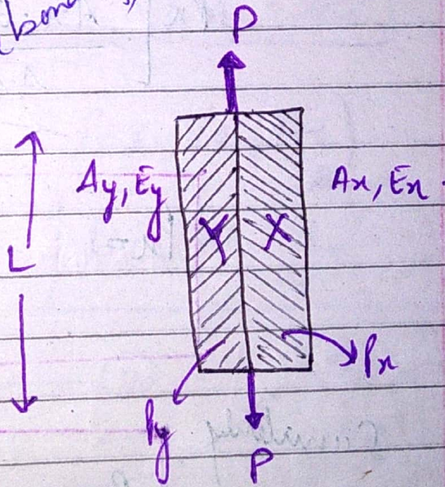
Axial Deformation of Compound Bar

$\boxed{\sigma = E\epsilon}$ $\boxed{\tau = G\gamma}$

$$\boxed{\nu = -\frac{\epsilon_{lateral}}{\epsilon_{longitudinal}}}$$

$$\delta = \frac{PL}{AE}, \quad \delta = \int_0^L \frac{P_x L_x}{A_x E_x}.$$

Deformation.



Assumption =

1- Perfect bonding (no relative bonding)
2- Equal length (L)
3- $\boxed{\delta_y = \delta_x}$

$$\frac{\delta_y}{L} = \frac{\delta_x}{L}$$

$\Rightarrow \boxed{\epsilon_y = \epsilon_x}$ —— (1)

*** $\boxed{P_x + P_y = P}$ —— (2)

$$\delta_y = \delta_x$$

$$\frac{P_y \cancel{L_y}}{A_y E_y} = \frac{P_x \cancel{L_x}}{A_x E_x} \qquad \because \boxed{L_x = L_y = L}$$

$$P_y = P_x \left(\frac{A_y \cdot E_y}{A_x \cdot E_x} \right) \quad \text{--- (3)}$$

eq^n (2) & (3)

$$P_x + P_x \frac{A_y E_y}{A_x \cdot E_x} = P \qquad \text{from (2)}$$

$$P_x \left[\frac{A_x \cdot E_x + A_y \cdot E_y}{A_x \cdot E_x} \right] = P$$

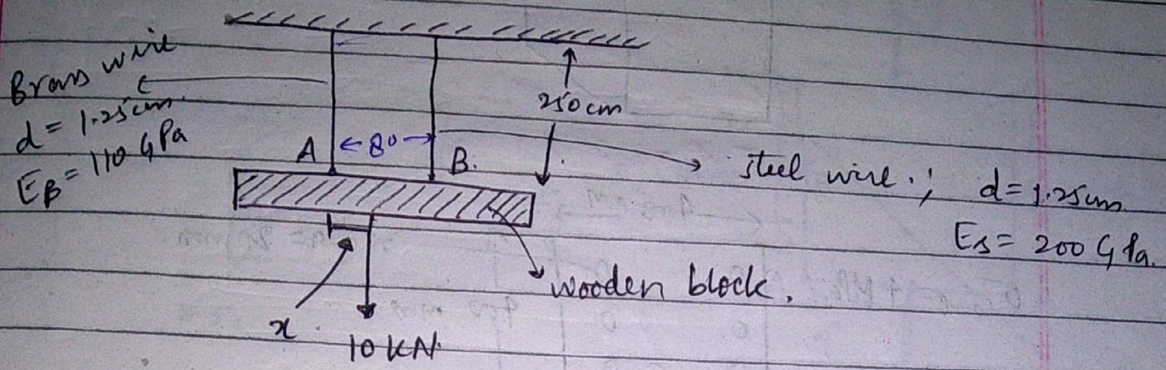
$$\boxed{P_x = P \left[\frac{A_x \cdot E_x}{A_x \cdot E_x + A_y \cdot E_y} \right]}$$

Similarly

$$\boxed{P_y = P \left[\frac{A_y \cdot E_y}{A_x E_x + A_y \cdot E_y} \right]}$$

$$\sigma_x = \frac{P_x}{A_x} \quad ; \quad \sigma_y = \frac{P_y}{A_y}.$$

Q.



Find x = ? such that block remains horizontal = ?

Sol^n

(1) $\delta_B = \delta_s$ ——— (1)

$$\frac{P_s}{A_s \cdot E_s} = \frac{P_B}{A_B E_B}. \qquad \boxed{A_s = A_B}$$

$$\frac{P_s}{E_s} = \frac{P_B}{E_B} \Rightarrow P_s = P_B\left(\frac{E_s}{E_B}\right)$$

$$\boxed{P_s = P_B \times \frac{20\cancel{0}}{11\cancel{0}} = \frac{20}{11} P_B}$$

P

(2) $P_s + P_B = 10 \times 10^3$ ——— (2) $\qquad P_B = \frac{11 P_s}{20}$

(3) $\Sigma M_A = 0.$

$$10 \times 10^3 x = 80 P_s.$$

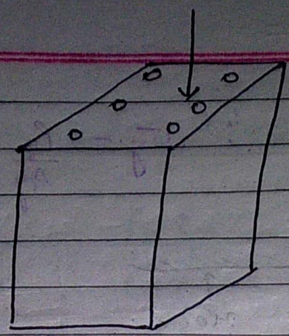
$$x = \left(\frac{80}{10000}\right) P_s \quad \text{cm.}$$

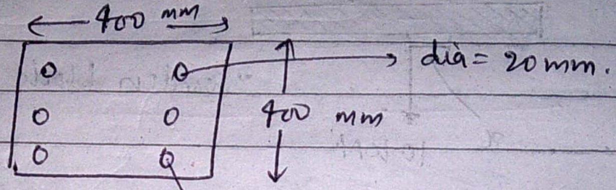
$$P_s\left(1 + \frac{11}{20}\right) = 10000$$

$$P_s = \frac{2 \times 10^5}{31} \quad ; \quad x = \frac{8\cancel{0}}{1000\cancel{0}} \times \frac{2 \times 10^5}{31}$$

$\boxed{P = 10.23 \text{ MPa}} \Leftarrow 10.23 \times 10^3$ $\qquad x = \frac{16}{31} \times 10^2 \quad ; \quad \boxed{x = 51.6 \text{ cm}}$

Q.





$\sigma_{conc} = 4\,MPa$

$\dfrac{E_{steel}}{E_{conc}} = 15$

$P = ?$

Area of Rebars (steel) $= \frac{\pi}{4}(20)^2\ mm^2$

Soln:

Area of concrete $= \left[(400 \times 400) - 6 \times \frac{\pi}{4}(20)^2\right] mm^2$

0.4

$0.16 - \frac{6\pi}{4}$

$$\sigma_c = \frac{P_c}{A_c}$$

$$4 \times 10^6 \times A_c = P_c.$$

$\boxed{\epsilon_s = \epsilon_c}$ → by equating strains, we have,

$$P_s = P_c\left(\frac{E_s}{E_B}\right)$$

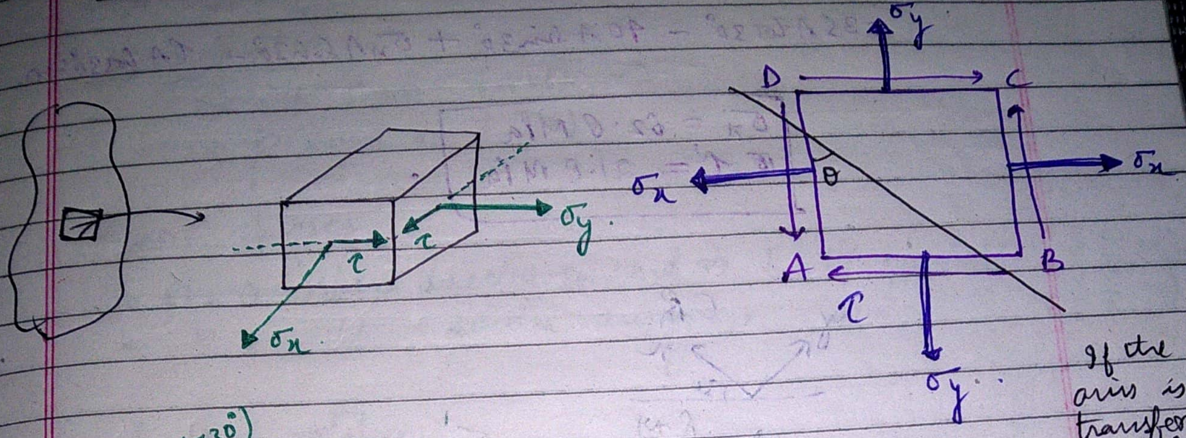
$$P_s = P_c(15)$$

$$P = P_s + P_c.$$
$$= P_c \times 16.$$

$$\boxed{P = 4 \times 10^6 \times 16 \times \left(0.16 - \frac{3\pi}{2}(0.02)^2\right)}\ Pa$$

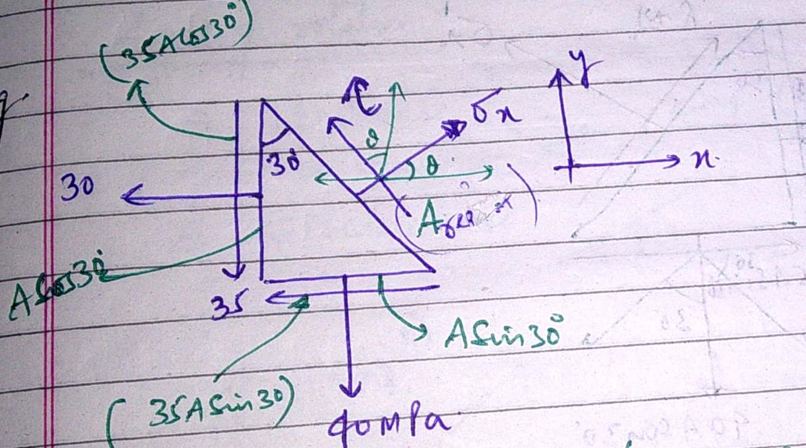
$P = 10.23 \times 10^6 \Rightarrow \boxed{P = 10.23\ MPa}$

Plane Stress Transformation



If the axis is transformed by θ.

e.g.



$$\sigma = 40 \times 10^6 \times A \sin 30^\circ$$

$$\boxed{\sigma = 20 \times 10^6 A}$$

$$\Sigma \vec{F_x} = 0.$$

$$-30 \cancel{A} \cos 30^\circ - 35 \cancel{A} \sin 30^\circ + \sigma_x \cancel{A} \cos 30^\circ - \tau \cancel{A} \sin 30^\circ = 0.$$

$$\sigma_x \times 0.86 - \frac{\tau}{2} = 25.98 + 17.5$$

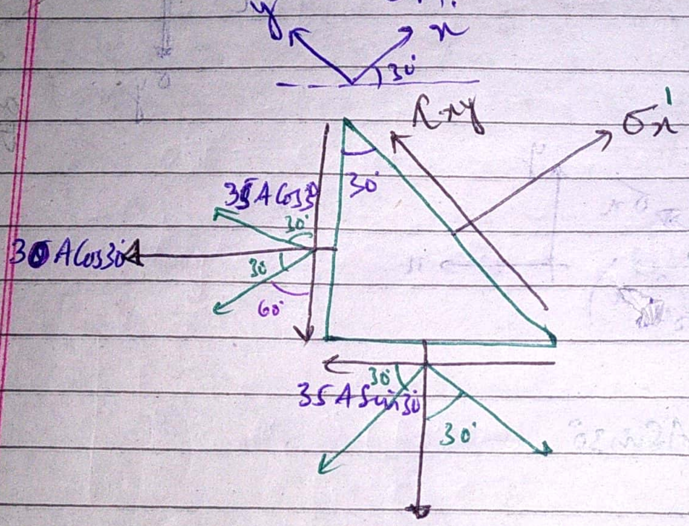
$$0.86 \sigma_x - \frac{\tau}{2} = 43.48 \longleftarrow (1).$$

$$\Sigma F_y = 0.$$

$$-35A\cos 30^\circ - 40A\sin 30^\circ + \sigma_x A\sin 30^\circ + \tau A\cos 30^\circ = 0.$$

$$\boxed{\begin{aligned}\sigma_x &= 62.0 \text{ MPa} \\ \tau' &= 21.0 \text{ MPa}\end{aligned}}$$

OR.



$$\Sigma F_x = 0$$

$$-30A\cos 30^\circ \cos 30^\circ - 35A\cos 30^\circ \cos 60^\circ$$

Sign Convention

Normal stresses.

Tensile stress (+ve)

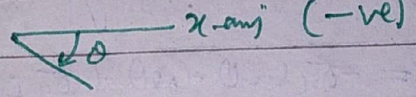
compressive " (−ve)

Shear stresses.

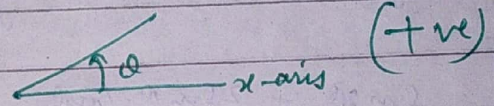
- Shear stress directed toward in I & III quadrant → (+ve).
- " " directed away " " " → (−ve).

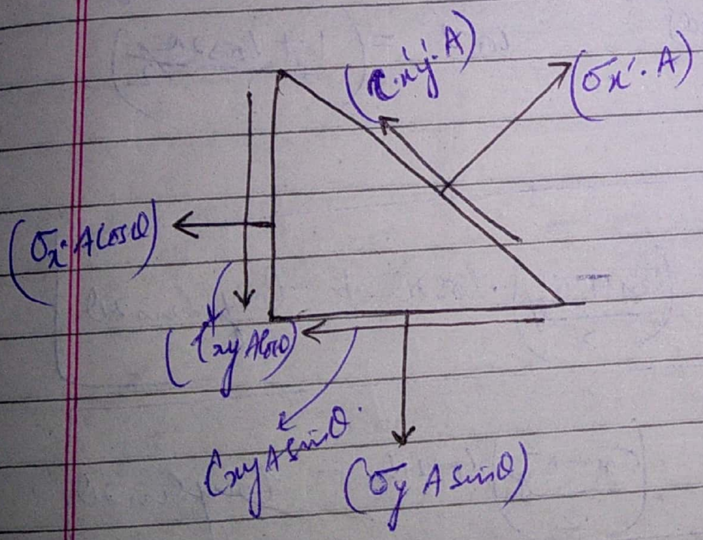
Angle of rotation

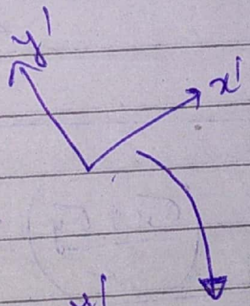
θ – clockwise

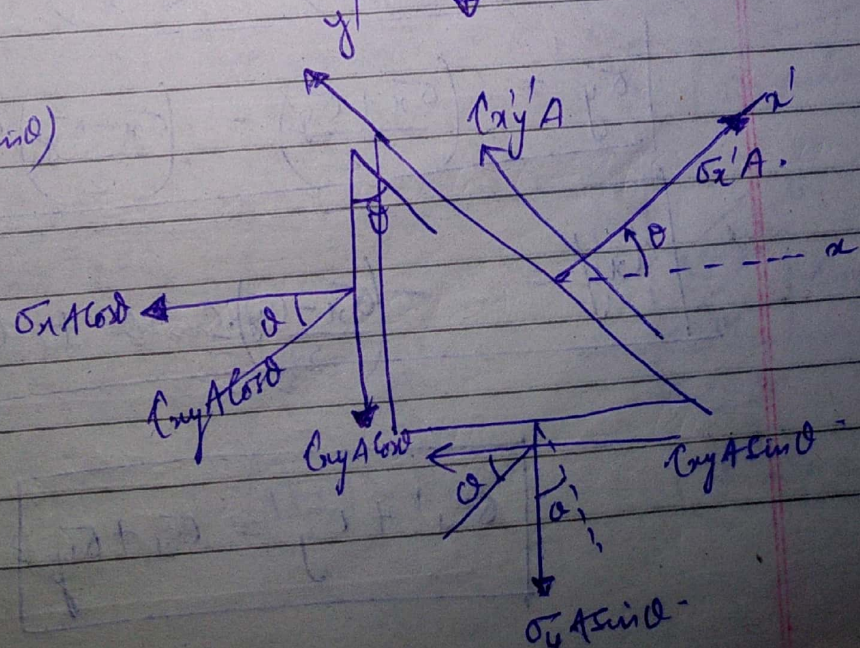


θ – anticlockwise









$$\sum F_{x'} = -\sigma_x A \cos\theta \cos\theta - \tau_{xy} A \cos\theta \sin\theta$$
$$- \tau_{xy} A \sin\theta \cos\theta - \sigma_y A \sin\theta \sin\theta$$
$$+ \sigma_{x'} \cdot A = 0 \longleftarrow (1)$$

$$\sum F_{y'} = \sigma_x A \cos\theta \cdot \sin\theta - \tau_{xy} A \cos\theta \cdot \cos\theta +$$
$$\tau_{xy} A \sin\theta \cdot \sin\theta - \sigma_y A \sin\theta \cos\theta + \tau_{x'y'} A = 0 \longleftarrow (2)$$

from (1) & (2), we have,

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cdot \cos\theta .$$

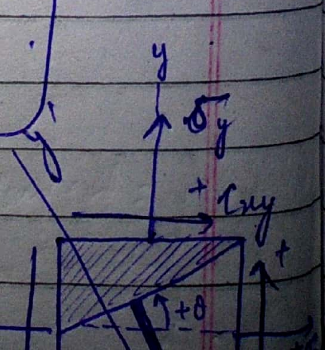
$$\tau_{x'y'} = -\sigma_x \sin\theta \cdot \cos\theta + \sigma_y \sin\theta \cdot \cos\theta + \tau_{xy} \cos^2\theta - \tau_{xy} \sin^2\theta$$

$$\sin 2\theta = 2\sin\theta \cdot \cos\theta ; \quad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin^2\theta = \frac{(1 - \cos 2\theta)}{2} \quad \cos^2\theta = \left(\frac{1 + \cos 2\theta}{2}\right)$$

$$\boxed{\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x + \sigma_y}{2}\right) \cdot \cos 2\theta + \tau_{xy} \sin 2\theta}$$

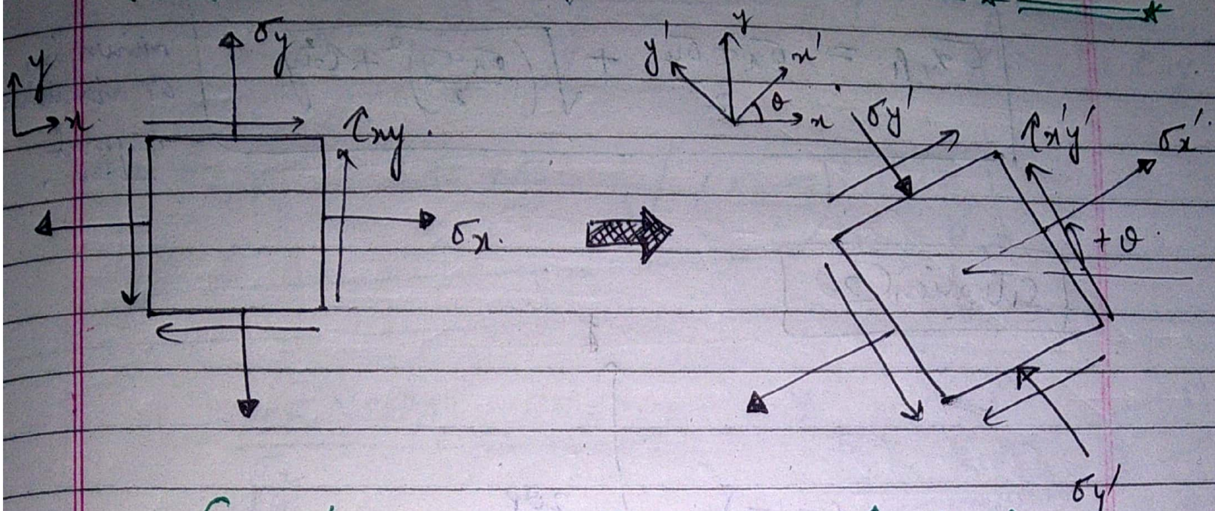
$$\boxed{\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta - \tau_{xy} \sin 2\theta .}$$

$$\boxed{\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \sin 2\theta + \tau_{xy} \cos 2\theta .}$$



$$\boxed{\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y}$$

PRINCIPAL & MAXIMUM SHEAR STRESSES.



$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

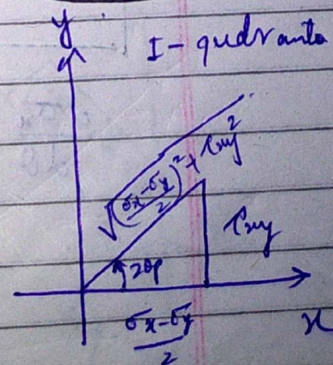
$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\boxed{\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta} \quad \text{——— (1)}$$

Situation I

$$\frac{d\sigma_x}{d\theta} = -(\sigma_x-\sigma_y)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$\boxed{\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}}$$



$$\sin 2\theta_p = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}} \; ; \; \cos 2\theta_p = \frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}}$$

$(\tan 2\theta_p = +ve)$

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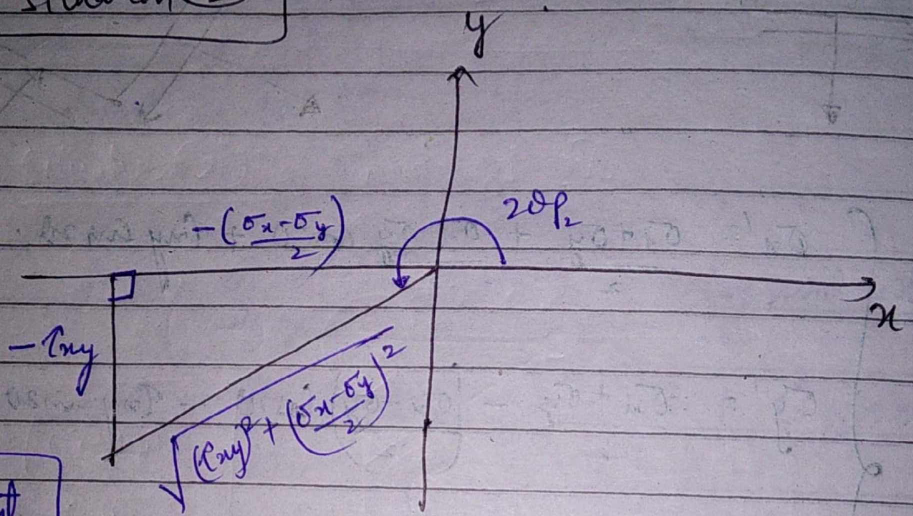
$$\boxed{\sigma_{x,p_1} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$ minimum or maximum normal stress.

$\boxed{\text{situation 2}}$



$\boxed{\text{II quadrant}}$

$$\boxed{2\theta_{p_2} = 2\theta_{p_1} + 180^\circ}$$

$(\tan 2\theta_p \text{ is } +ve) \Rightarrow \tan 2\theta_p$ is $(-ve)$ in quadrant II & IV.

$$\boxed{\sigma_x' = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cdot\cos 2\theta + \tau_{xy}\sin 2\theta}$$

$$\frac{d\sigma_x}{d\theta} = 0$$

$$-(\sigma_x-\sigma_y)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

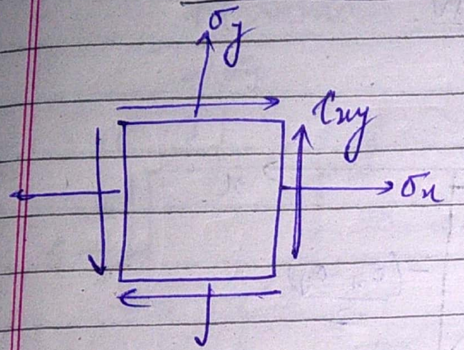
$$\boxed{\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}}$$

tangent: periodic fx^n with period π.

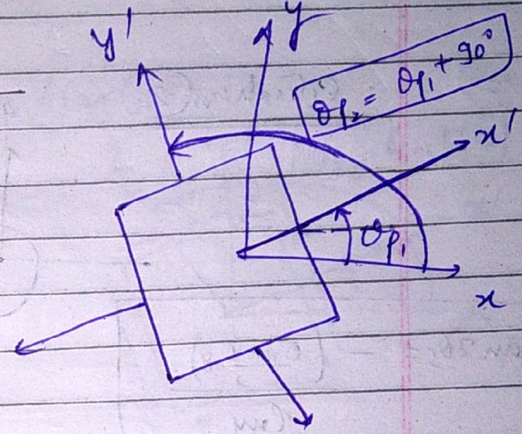
$\sin 2\theta_p = \dfrac{-\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$; $\cos 2\theta_p = \dfrac{-\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$.

$$\boxed{\sigma_{x,P_2} = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}.$$

PRINCIPAL STRESS



$\Rightarrow$



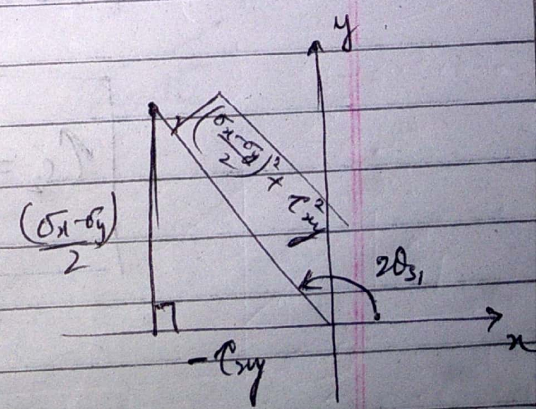
$$\boxed{\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}}.$$

$$\boxed{\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

→ Maximum & minimum stress.

$$\boxed{\tau_p = 0}.$$

Situation ①, Quadrant II.



$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right).\sin 2\theta + \tau_{xy}\cos 2\theta$.

$\frac{d}{d\theta}(\tau_{x'y'}) = 0$

$= -(\sigma_x-\sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$

$$\boxed{\tan 2\theta_s = -\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\tau_{xy}}}$$

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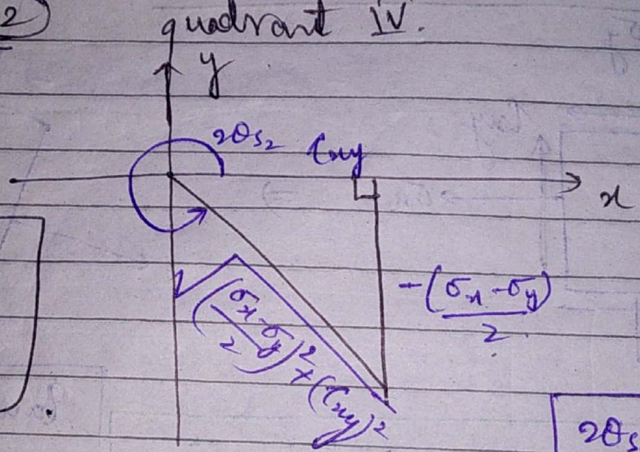
$$\sin 2\theta_{s_1} = \frac{(\sigma_x - \sigma_y)/2}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}} \; ; \quad \cos 2\theta_{s_1} = \frac{-\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\boxed{\tau_{s_1} = -\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

<u>Situation (2)</u> quadrant IV.



$$\boxed{\tan 2\theta_s = \frac{-\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\tau_{xy}}}$$

$$\boxed{2\theta_{s_2} = \pi + 2\theta_{s_1}}$$

$$\boxed{\theta_{s_2} = \pi/2 + \theta_{s_1}}$$

II quadrant. I

$$\sin 2\theta_{s_2} = \frac{-\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}} \; ; \quad \cos 2\theta_{s_2} = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\boxed{\tau_{s_2} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

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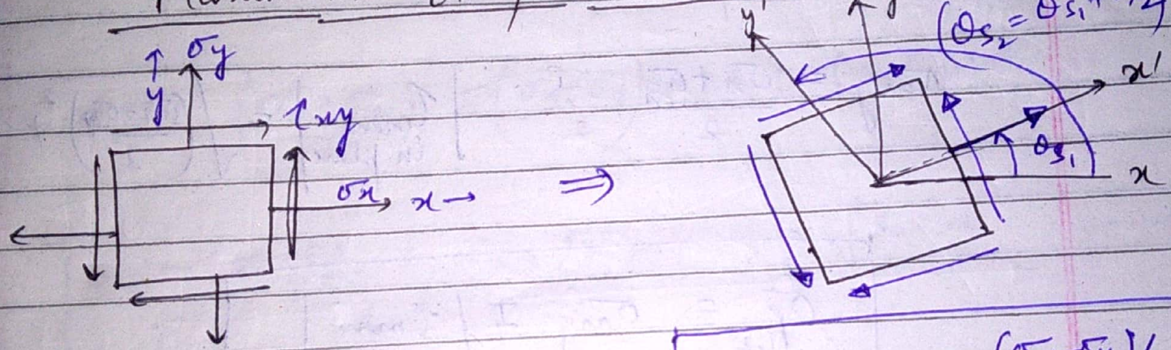
$$\tan 2\theta_s = -\frac{(\sigma_x-\sigma_y)/2}{\tau_{xy}}$$

$$\sigma_{x}' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\boxed{\sigma_{x,s_1} = \sigma_{x,s_2} = \frac{\sigma_x+\sigma_y}{2} = \sigma_{av}}$$

Maximum in-plane shear stress



$$\boxed{\tan 2\theta_s = \frac{-(\sigma_x-\sigma_y)/2}{\tau_{xy}}}$$

$$\boxed{\left|\tau_{\substack{max\\ in\ plane}}\right| = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\boxed{\sigma_{1,2} = \sigma_{av}}$$

Results.

~~Principal~~

Results

(Principal stresses)

$$\boxed{\sigma_{p_{1,2}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \boxed{\tau_p = 0}$$

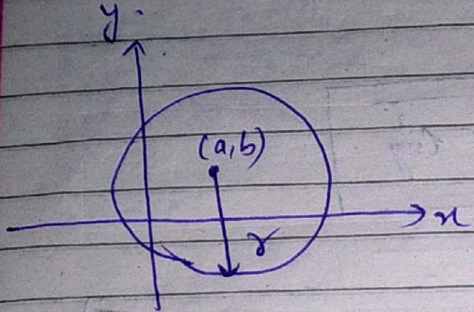
Max^m & min^m stress — Minim^m shear stress.

Maximum in-plane shear stress

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \; ; \quad \left|\tau_{\substack{max \\ in\ plane}}\right| = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\boxed{\sigma_{p_{1,2}} = \sigma_{avg} \pm \left|\tau_{\substack{max \\ in\ plane}}\right|}$$

MOHR'S CIRCLE FOR PLANE STRESS



General eqn.

$$(x-a)^2+(y-b)^2 = r^2$$

Centre $\equiv (a,b)$

Radius $\equiv r$.

$$\sigma_x' = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\left[\sigma_x' - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right] = \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{—— (1)}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\cdot\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{—— (2)}$$

squaring (1) & (2) and adding, we have,

$$\left(\sigma_x' - \frac{\sigma_x+\sigma_y}{2}\right)^2 + (\tau_{x'y'})^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

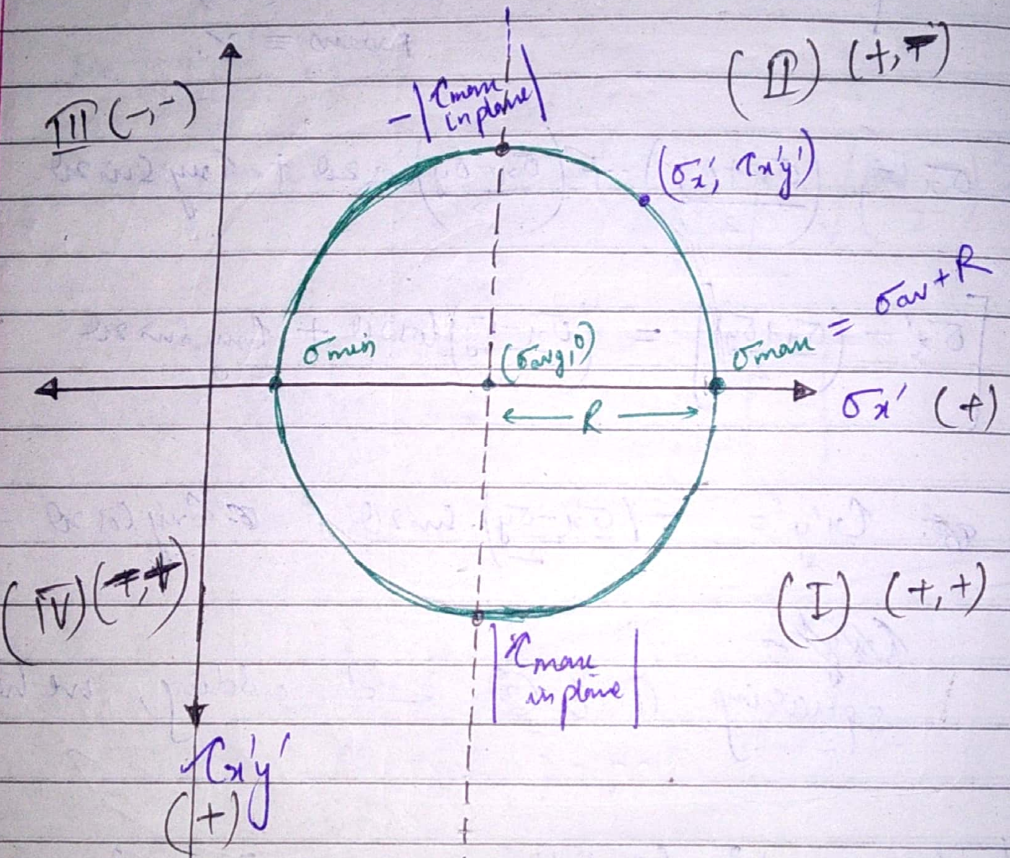
as $\sin^2\theta + \cos^2\theta = 1$ $\quad \boxed{\forall \theta \in R}$

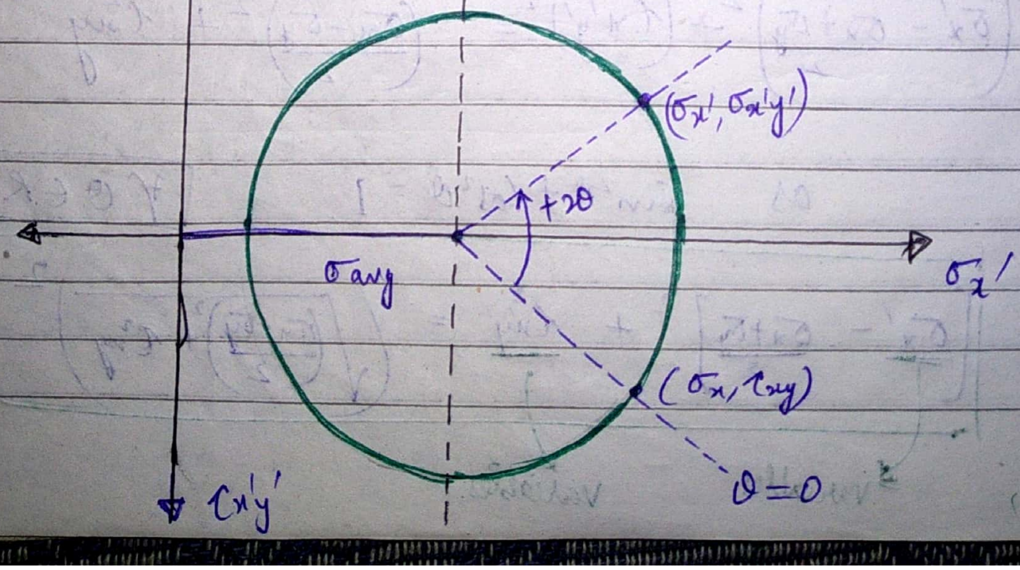
$$\boxed{\Big[\underbrace{\sigma_x'}_{\text{variables}} - \frac{\sigma_x+\sigma_y}{2}\Big]^2 + \underbrace{\tau_{x'y'}^2}_{\text{variables}} = \left(\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}\right)^2} \quad \text{—— (1)}$$

On comparing with the general equation of the circle, we have,

$$\boxed{C \equiv \left(\frac{\sigma_x + \sigma_y}{2}, 0 \right)}$$

$$\boxed{\text{Radius} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}}$$





Eg. 1. Determine the stresses for the rotated orientation, the principal stresses and the maximum in-plane shear stress.

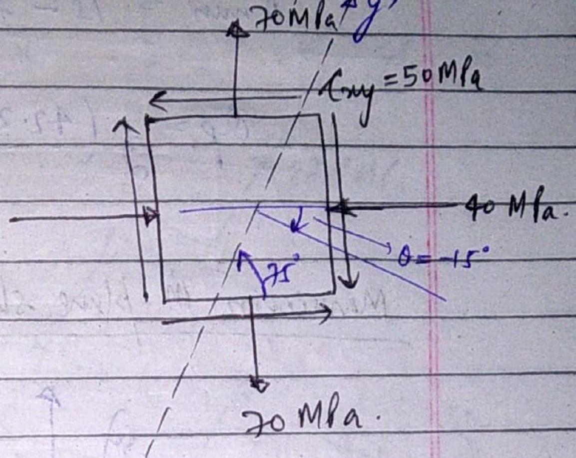
Soln

$\sigma_x = -40\ MPa$

$\tau_{xy} = -50\ MPa$

$\sigma_y = 70\ MPa$

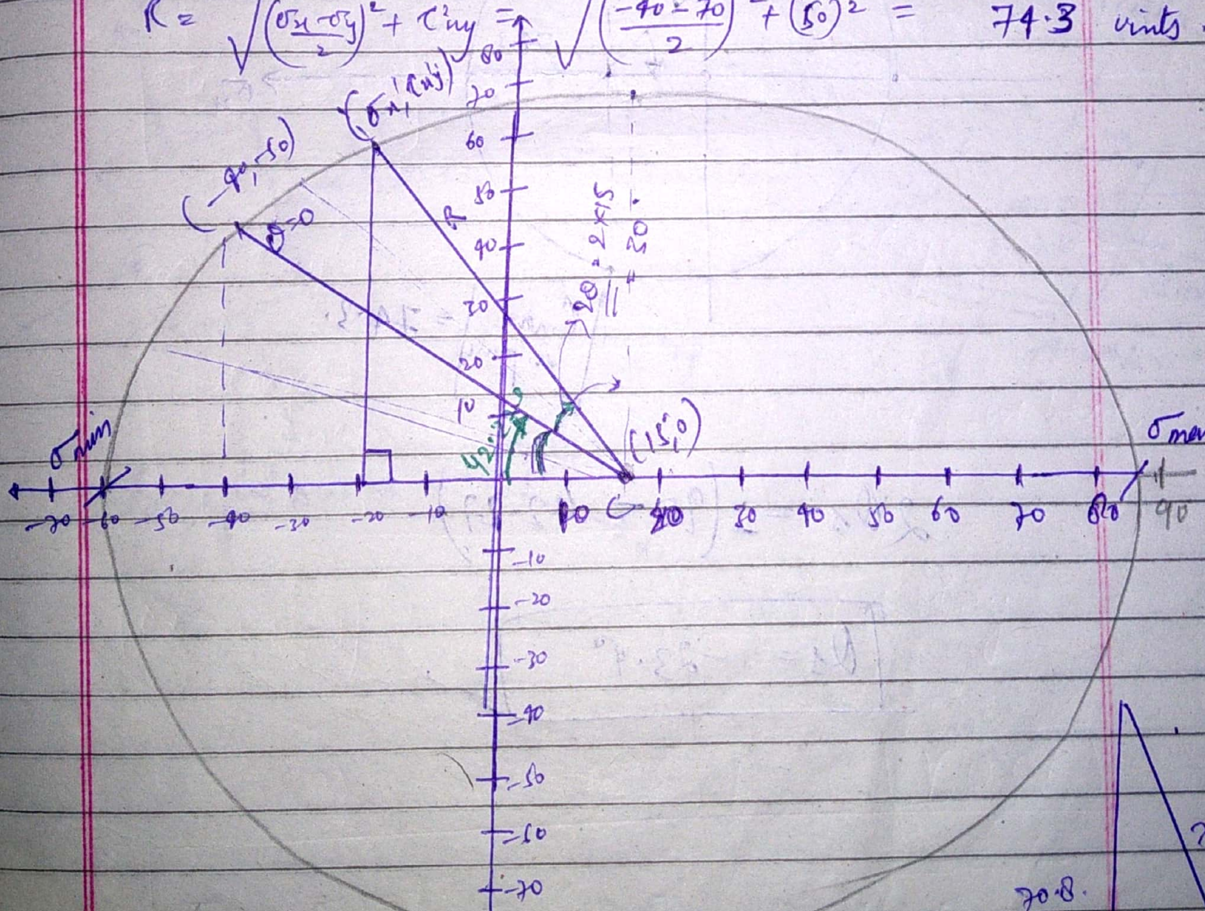
$\theta = -15°$

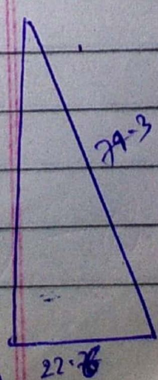


$C \equiv (\sigma_{av}, 0) \equiv \left(\frac{-40+70}{2}, 0\right)$

$\equiv (15, 0).$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-40-70}{2}\right)^2 + (50)^2} = 74.3$ units.





$\sigma_{x'} = -(22.6 - 15) = -7.6\ MPa$; $\tau_{x'y'} = -70.8\ MPa$

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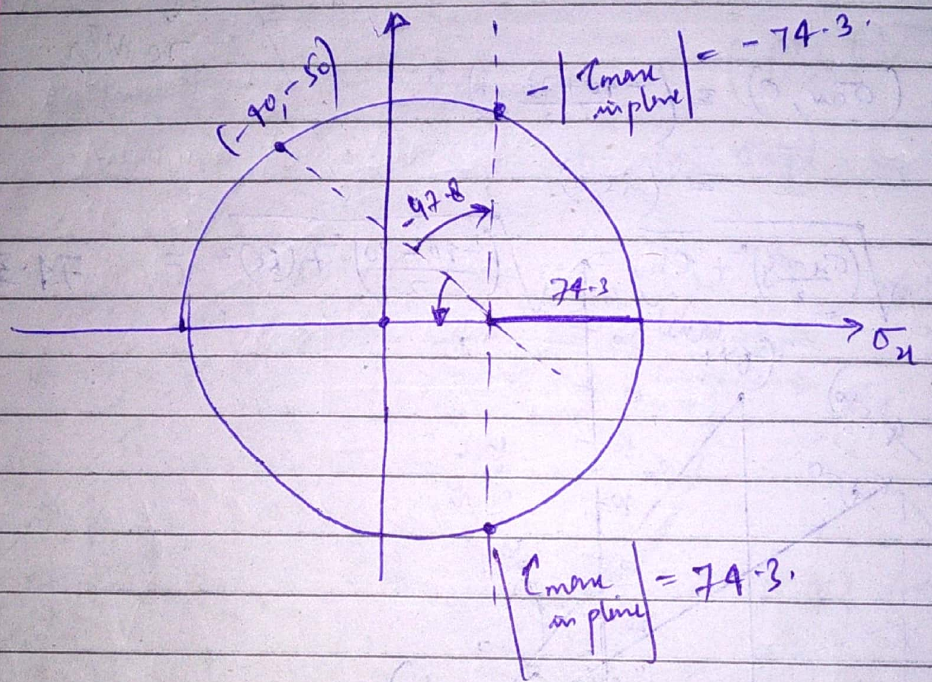
$\sigma_{max} = 15 + 74.3 = 89.3$

$\sigma_{min} = 15 - 74.3 = -59.3$

$\theta_p = \frac{1}{2}(42.27^\circ) = 21.1^\circ$

Maximum in-plane shear stress :-



$2\theta_s = -(90 - 42.27)$

$\boxed{\theta_s = -23.9^\circ}$

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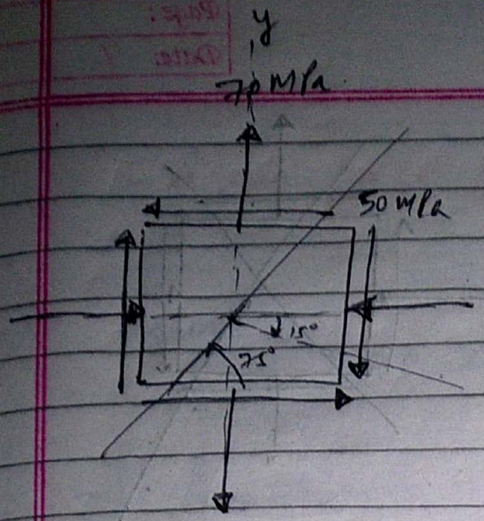
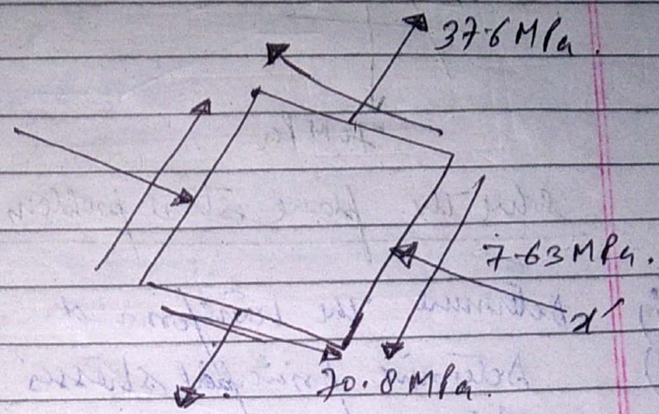
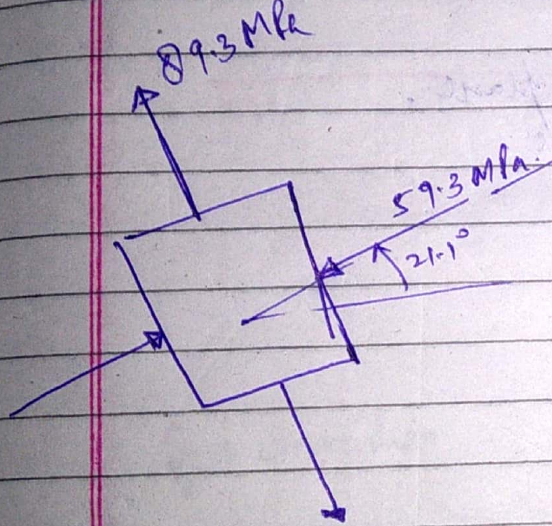
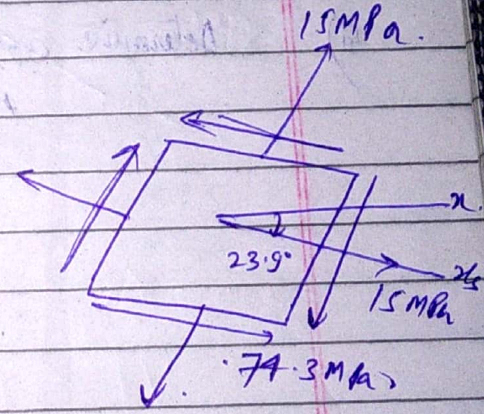
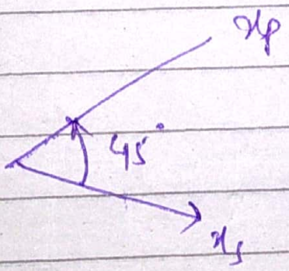






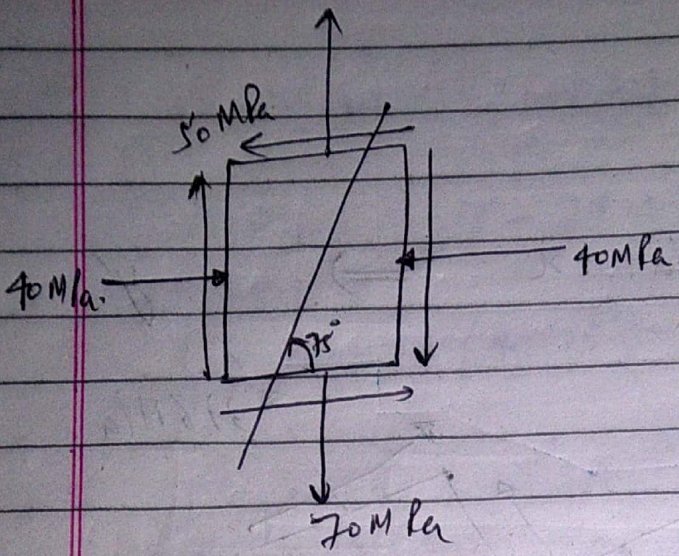


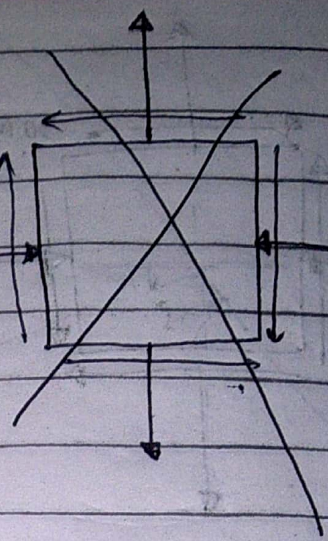
Principal stresses



5 Oct, 2018.

Q. 2





Solve the plane stress problem using Mohr's circle method :-

i) Determine the transformed stresses

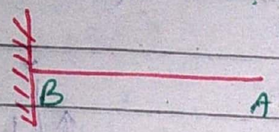
ii) Determine principal stresses & max^m shear stress.

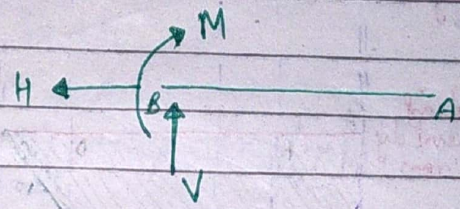
iii) Determine angles for principal plane.
max^m shear plane.

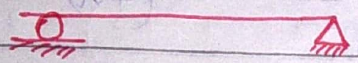
Shear force & Bending Moment Diagram

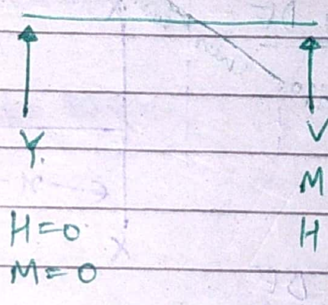
Beams → Cantilever beam.
Beams → Simply supported beams.

— Beams with internal hinges.



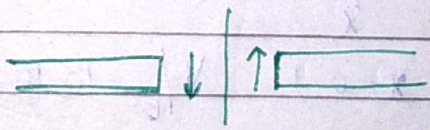




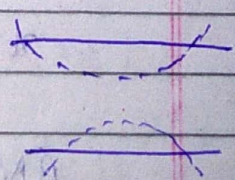


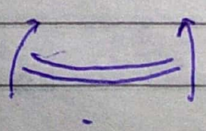
Sign convention

⟹ Shear force

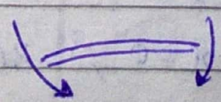


⟹ Bending moment → Sagging (+)
→ Hogging (−)





Sagging moment (+ve).



Hogging moment (−ve)

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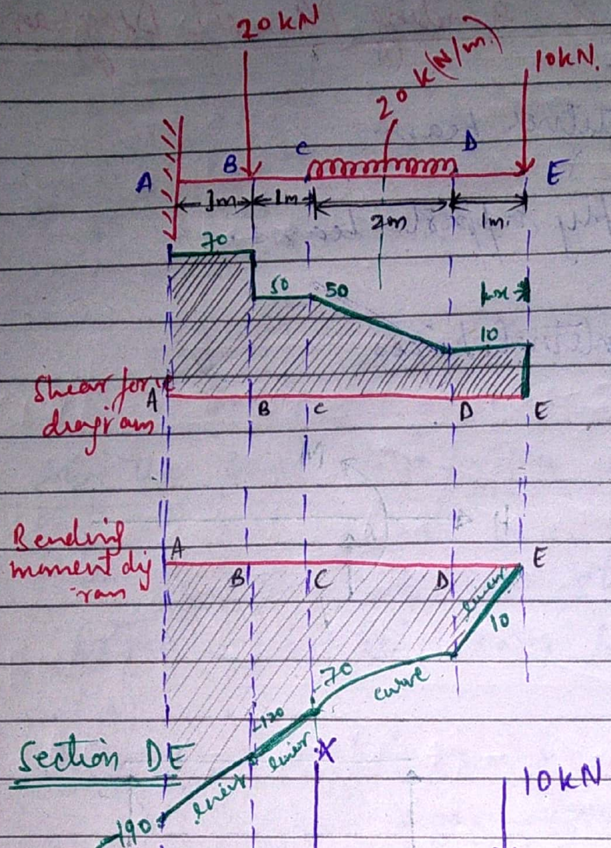
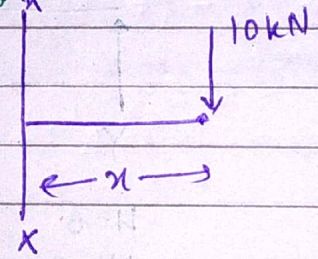
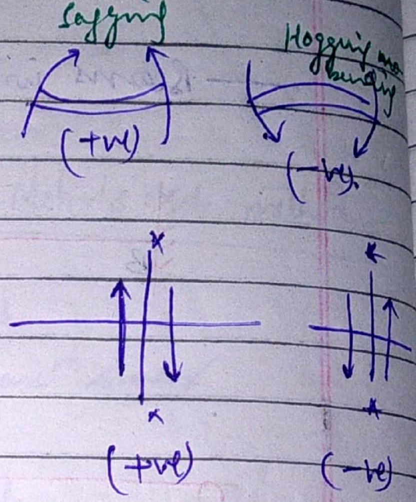
Eg.



Section DE





section DE

$$V\Big|_{x}^{x} = 10\ kN$$

$x = 0$; $V\Big|_E = 10\ kN$

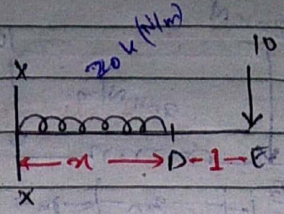
$x = 1$; $V\Big|_D = 10\ kN$

$$BM\Big|_{x}^{x} = -10x$$

$x = 0$; $BM\Big|_E = -0$

$x = 1$; $BM\Big|_D = -10\ kNm$

Section CD.



$$V\Big|_x^x = 10 + 20x.$$

$x=0$; $V_D = 10$ kN.

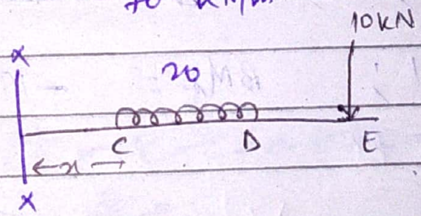
$x=2$; $V_D = 50$ kN

$$BM\Big|_x^x = -10(x+1) - 20\times x\times\left(\frac{x}{2}\right).$$

$x=0$; $BM|_D = -10$ kN/m.

$x=2$; $BM|_C = \cancel{-100}$ ~~kN/m~~.
-70 kN/m

Section BC ;



$$V\Big|_x^x = +(10 + 20\times 2)$$
$$= +(50 \text{ kN})$$

$$BM\Big|_x^x = -(3+x)10 - 20\times 2(x+1)$$

$x=0$; $BM_C = -70$ kN/m

$x=1$; $BM_B = -120$ kN/m.

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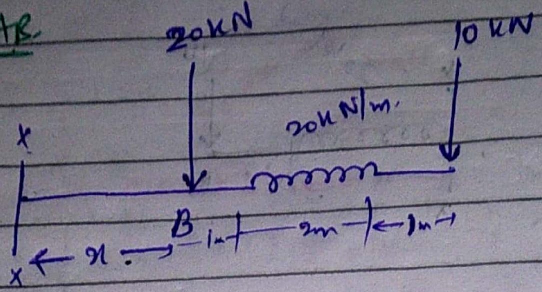
Section AB



$V\Big|_x^x = \cancel{10 + (20\times2)(\frac{x+3}{2})}$ $20\times2+20+10 = 70kN$

$x=0$; $V_B = 70$ kN.

$x=1$; $V_A = 70$ kN.

$B.M \Rightarrow -10\times(x+4) - 20\times2\left(x+1+\frac{2}{2}\right) - 20x$.

$x=0$, $BM_B = -120$ kNm.

$x=1$; $BM_A = -190$ kNm.

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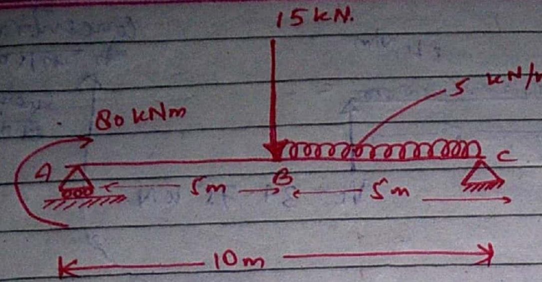
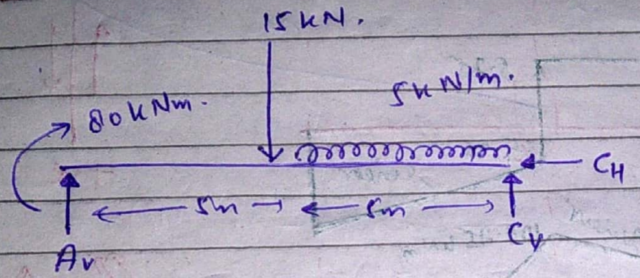




75.0
62.5
137.5

625

$$\curvearrowleft + \sum \vec{M_c} = 80 + A_v \times 10 - 15 \times 5 - 2.5 \times 25 = 0.$$

$\boxed{\cancel{A_v = 42 kN}}$ $\boxed{A_v = 5.75}$

$C_H = 0.$

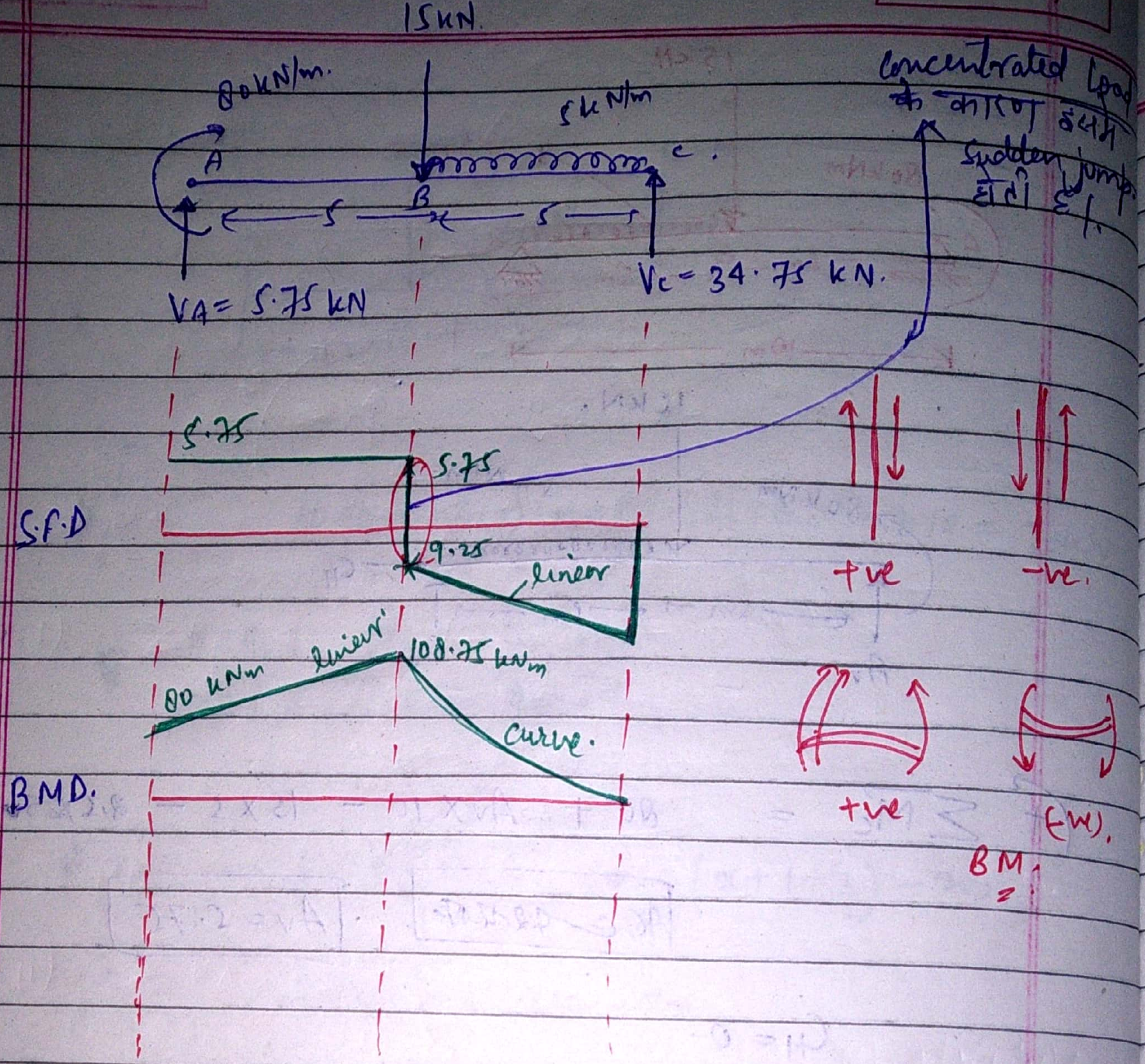
$\sum f_y = 0$ $\cancel{A_v + 15 - 150 + 112 = 0}$

$\cancel{A_v =}$

$-A_v + 15 + 25 - C_v = 0$

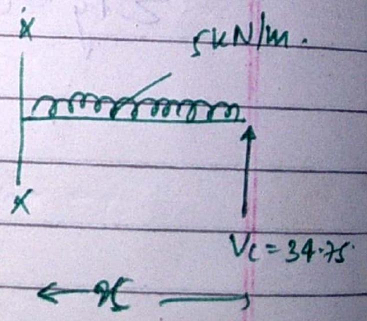
$C_v = A_v - 40$

$\boxed{C_v = 34.25 \text{ kN}}$.



Section BC

$V\Big|_{x}^{x} = \cancel{34.25 - 5x}$



$\cancel{M\Big|_{x}^{x} =}$

$\cancel{V\Big|_{x}^{x} = -34.25 + 5x}$

$\cancel{x = 0 \;,\; BM\big|_C = 0}$

$\cancel{x = 5 \;\; BM\big|_B = 108.75.}$

$$V\Big|_x = -34.27 + 5x$$

$x = 0$; $V_C = -34.25$ kN

$x = 5$; $V_B = -9.25$ kN

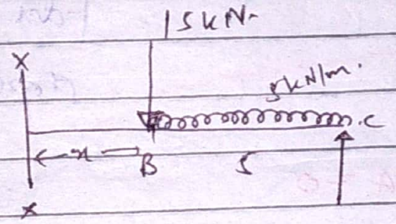
$$BM\Big|_x = 34.25x - 5 \cdot x \cdot \frac{x}{2}$$

$x = 0$; $BM|_C = 0.$

$x = 5$; $BM|_B = 108.75$ kN.

$x = 5/2$; $BM\Big|_{5/2} = 70.$

<u>Section AB.</u>



$$V\Big|_x = -34.25 + 5 \times 5 + 15$$

$x = 0$; $V_B = 5.75$ kN.

$x = 5$; $V_A = 5.75$ kN.

$$B \cdot M\Big|_x = 34.25(x+5) - 5 \times 5(x + 5/2) - 15x.$$

$x = 0$ $BM|_B = 108.75$ kNm.

$x = 5$; $BM|_C = 80$ kN.m.

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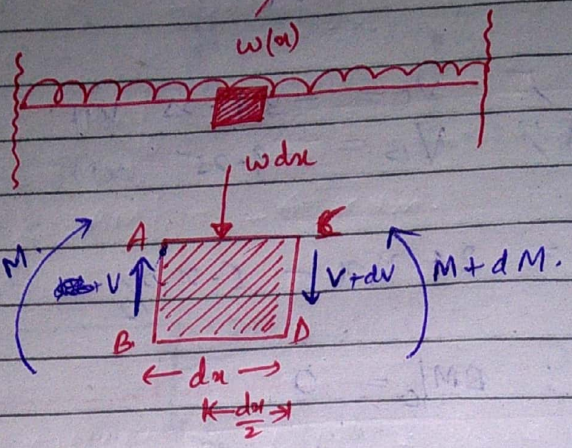
distributed load intensity $w = \frac{dV}{dx}$

=)



i) $\sum F_y = 0$ $V - w\,dx - (V+dV) = 0$

$dV = -w\,dx$

$$\boxed{\frac{dV}{dx} = -w}$$

slope of shear force diagram

ii) $\sum M_A = 0$

$$\cancelto{0}{w\,dx \times \left(\frac{dx}{2}\right)} + M - (M+dM) + (V+dV)\,dx = 0$$

ignoring this highly small term

$$\cancelto{0}{w\,dx} - dM + V\,dx + \cancelto{0}{dV\,dx} = 0$$

ignoring

$$\boxed{\frac{dM}{dx} = V}$$

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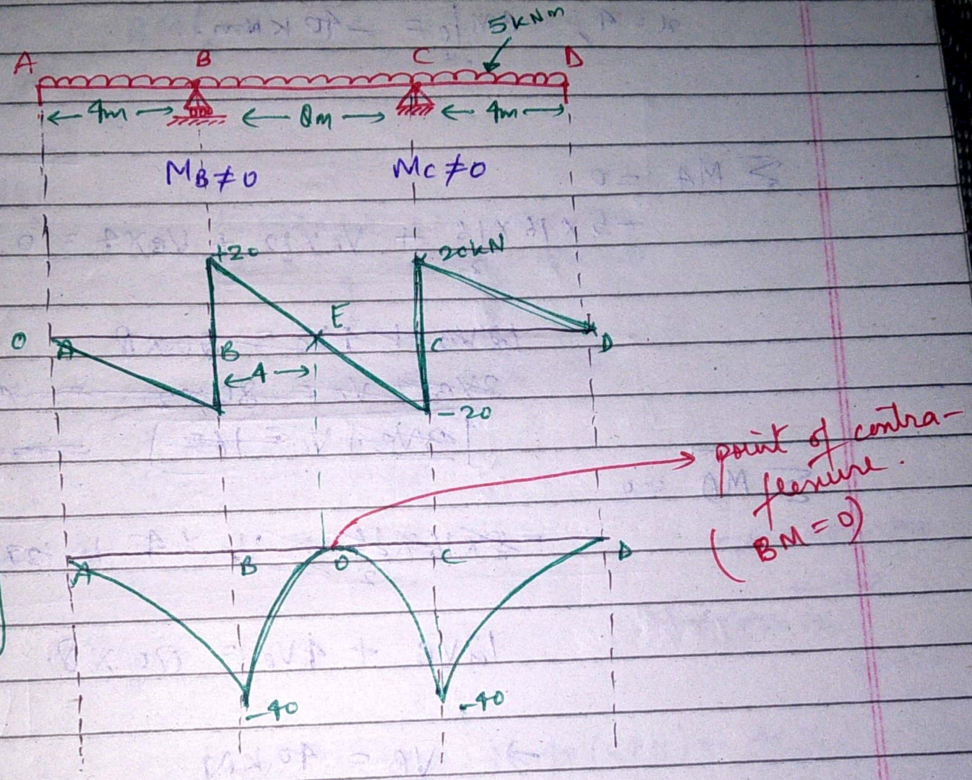
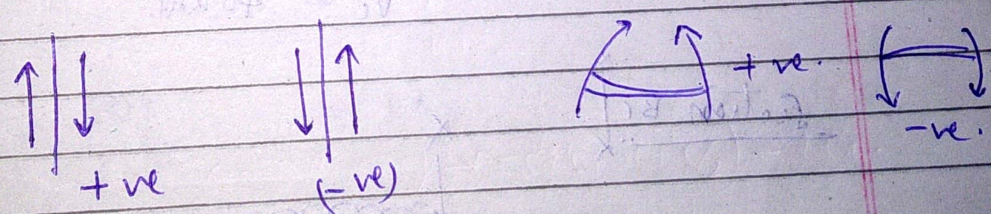
Overhang Beam

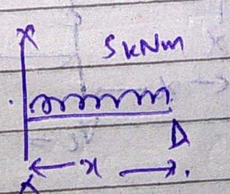


$x = 4;\ V\Big|_{x E}^{x} = 0$

$x = 4;\ BM\Big|_{x E}^{x} = 0$



Section · CD.



$$V\Big|_{x}^{x} = 5x.$$

$x = 0;\ V\Big|_{D} = 0$

$x = 4;\ V\Big|_{C} = 20\ kN.$

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$$BM\Big|_x^x = -(5 \cdot x) \cdot \frac{x}{2}.$$

$x = 0;\quad BM\big|_D = 0$

$x = 4;\quad BM\big|_C = -40\ kNm.$

$\Sigma M_A = 0$

$$-5 \times 16 \times \frac{16}{2} + V_C \times 12 + V_B \times 4 = 0. \quad \text{——— (1)}$$

$$\cancel{12V_B + 4V_C = 80 \times 8}$$

$$\cancel{3V_B + V_C = 80 \times 2} \quad \text{——— (2)}$$

$$\boxed{\cancel{3V_B + V_C = 160}}$$

$\Sigma \overrightarrow{M_D} = 0$

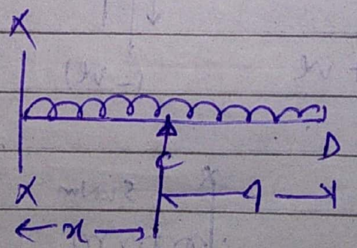
$$\cancel{-5 \times 16 \times \frac{16}{2}} + V_C \times 4 + \cancel{12 \times V_C}$$

$$12V_B + 4V_C = 80 \times 8.$$

$\Rightarrow \quad V_B = 40\ kN$

$\quad\quad V_C = 40\ kN.$

Section BC $\Big|_x^x$



$V_C = 40\ kN.$

$$V\Big|_x^x = 5(4 + x) - 40$$

$x = 0;\quad V\big|_C = 20 - 40 = -20\ kN$

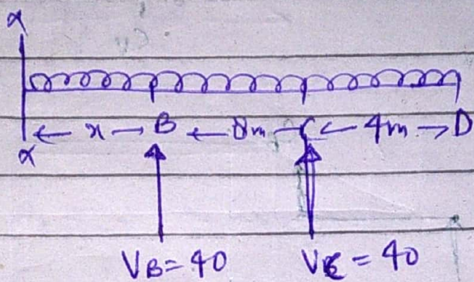
$x = 8;\quad V\big|_B = 60 - 40 = 20\ kN$

$$BM\Big|_x^x = -5(x+4)\frac{(x+4)}{2} + 40x.$$

$x = 0$; $BM\big|_C = -40$ kNm

$x = 8$; $BM\big|_B = -40$ kNm.

Section AB $\Big|_x^x$



$$V\Big|_x^x = 5(x+8+4) - 80$$

$x = 0$; $V\big|_B = -20$ kN.

$x = 4$ $V\big|_A = 0$

~~-384+84~~ 480

← 160

16[-48+30+10]

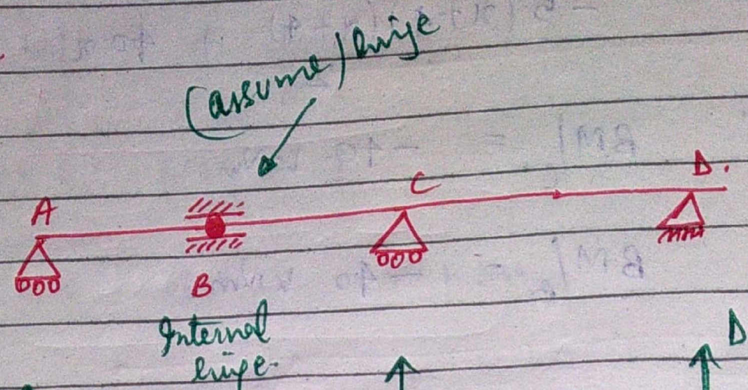
4 320

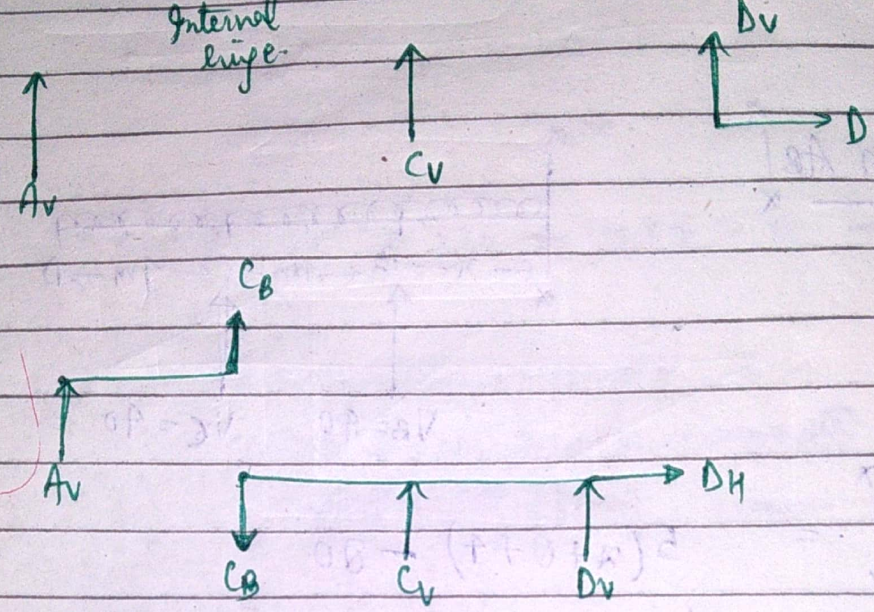
$$BM\Big|_x^x = -5\frac{(x+12)(x+12)}{2} + 40(x+8) + 40x.$$

$x = 0$; $BM\big|_B = -40$ kNm

$x = 4$; $BM\big|_A = 0$ kNm.

Example.





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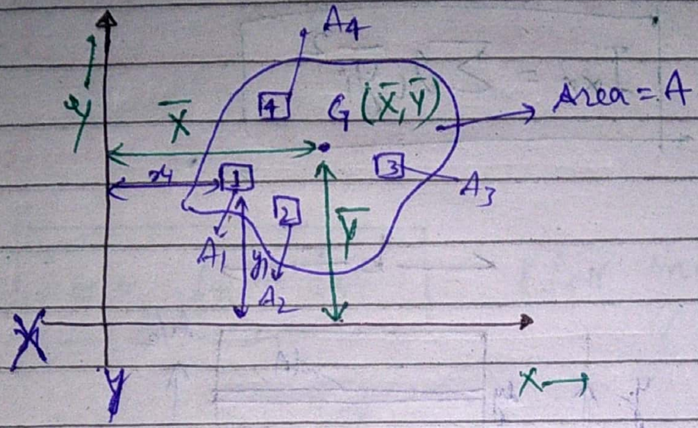
Unit II

Centroid & Second Moment of Area (moment of Inertia I)



$\boxed{1}$ $(x_1 y_1)$

$\boxed{2}$ $(x_2 y_2)$.

$$A = a_1 + a_2 + a_3 - \!-\!- a_n.$$
$$= \sum_{i=1}^{n} a_i$$

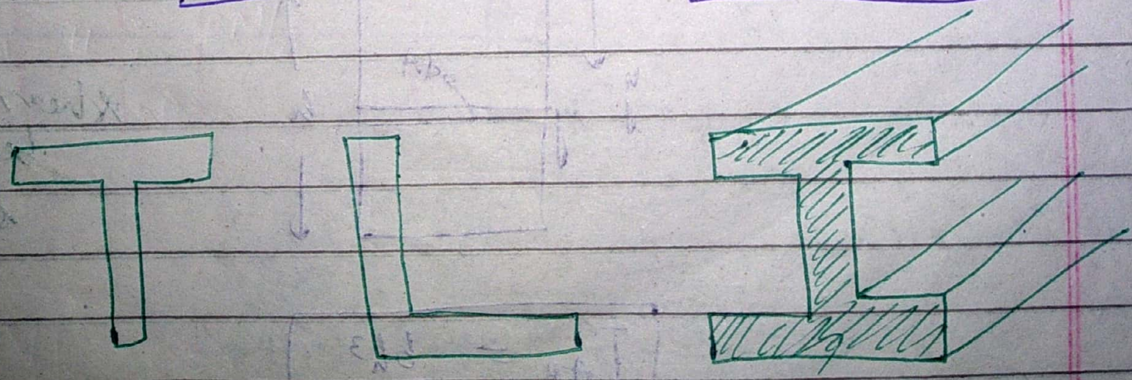
Moments from x-x axis

$$= a_1\bar{y}_1 + a_2\bar{y}_2 + a_3\bar{y}_3 + - -$$
$$= \sum_{i=1}^{n} a_i\bar{y}_i$$

$$A\bar{Y} = \sum a_i\bar{y}_i$$

first moment of area → Basis for shear stresses

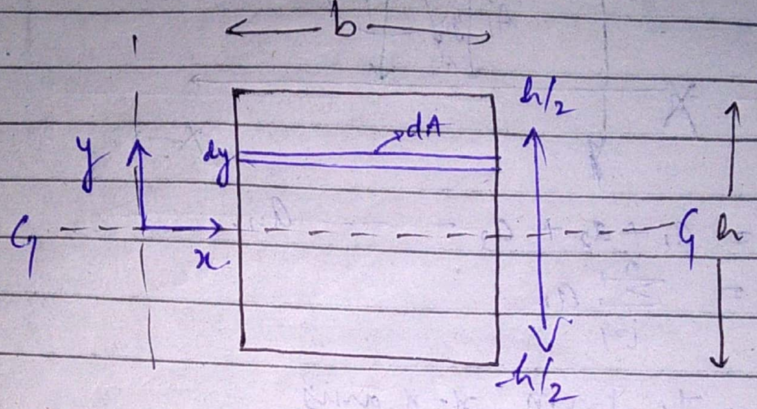
$$\boxed{\bar{Y} = \frac{\sum a_i\bar{y}_i}{A}} \quad \& \quad \boxed{\bar{X} = \frac{\sum a_i\bar{x}_i}{A}}$$



Second Moment of Area = I.

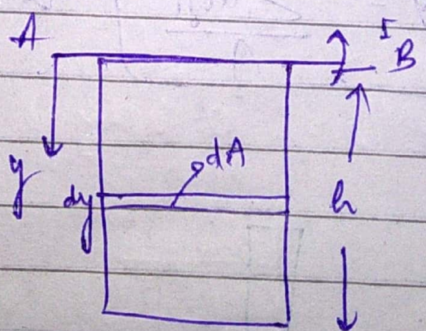
$$\boxed{I_{xx} = \sum a_i \bar{y}_i^{\,2}}$$

eg.



$$I_{GG} = \int_{-h/2}^{h/2} b\,dy \cdot y^2$$

$$\boxed{I_{GG} = \frac{bh^3}{12}}$$



shear and bending stresses.

$$\boxed{I_{AB} = \frac{bh^3}{3}}$$

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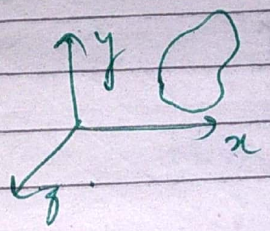
1). Parallel Axis theorem

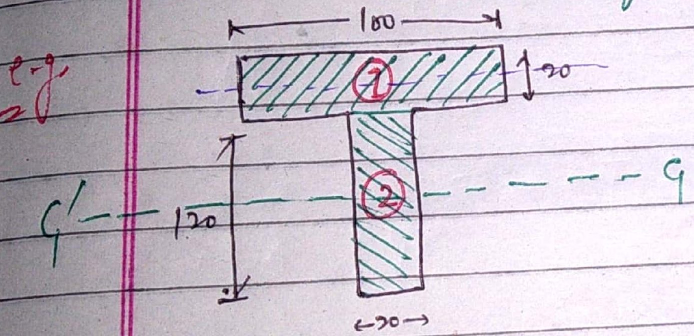
$$\boxed{I_{AB} = I_{xx/gg} + A\bar{y}^2}$$

2) Perpendicular axis theorem :-

$$\boxed{I_z = I_x + I_{yy}}$$

Polar moment of Inertia (Torsion).





(1) S.No	(2) Area a_i	(3) $\bar{y}_i$	(4) $a_i\bar{y}$	(5) $a_i\bar{y}^2$	(6) I_{gg} self	(7) I_{AB} self
1	20×100	20/2	2×10^4	2×10^5	$\frac{100\times(20)^3}{12}$	(5)+(6)
2	20×100	$20+\frac{100}{2}$			$\frac{20\times(100)^3}{12}$	(5)+(6)
Total						Σtotal =

$$I_{AB} = I_{xx/gg} + A\bar{y}^2$$

$$\boxed{I_{xx/gg} = I_{AB} - A\bar{y}^2}$$

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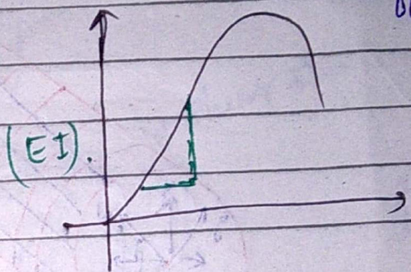
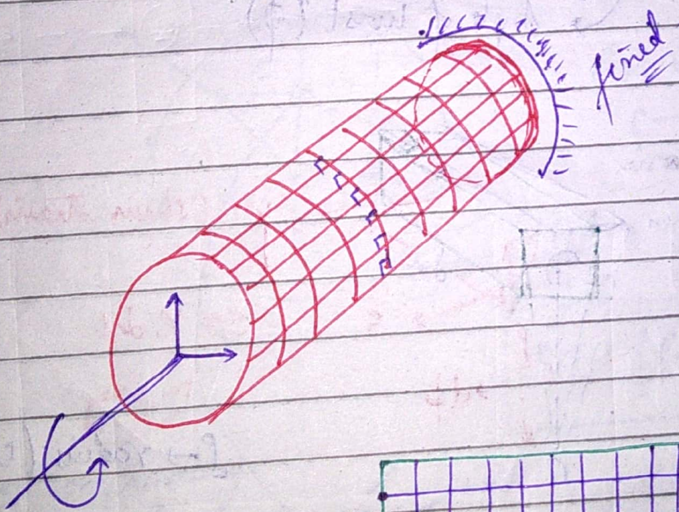
TORSION

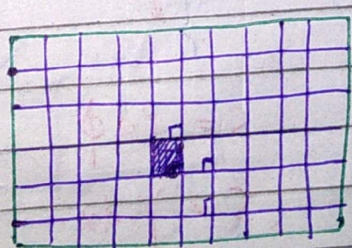
second moment of area (I) –
(I) Geometric stiffness

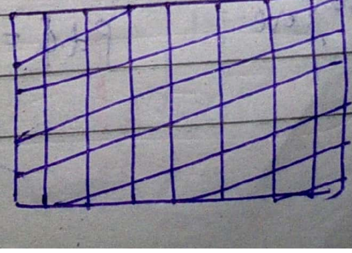
E → materialic stiffness.





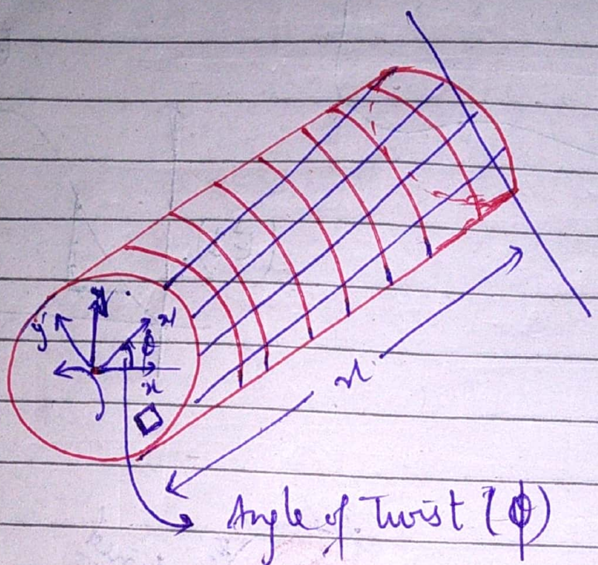
befor twisting



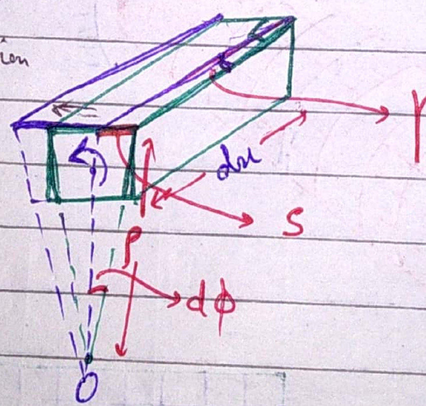


for small angles.

→ Circular sections remain circular.
→ No change in length of shaft
→ Longitudinal lines remains straight but twisted.



Green pen → before torsion
Blue pen → after torsion



γ (shear strain)

$S = \gamma \cdot dx$

$\rho \rightarrow$ radius (Unknown)

$$S = \rho\, d\phi$$

$$S = \gamma \cdot dx$$

Therefore $\rho\, d\phi = \gamma \cdot dx$

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$$\boxed{\rho \cdot \frac{d\phi}{dx} = \gamma}$$

$\left(\frac{d\phi}{dx}\right)$ will be constt.

$\rho \rightarrow$ unknown radius.

therefore

$$\gamma \propto \rho$$

from Hooke's law

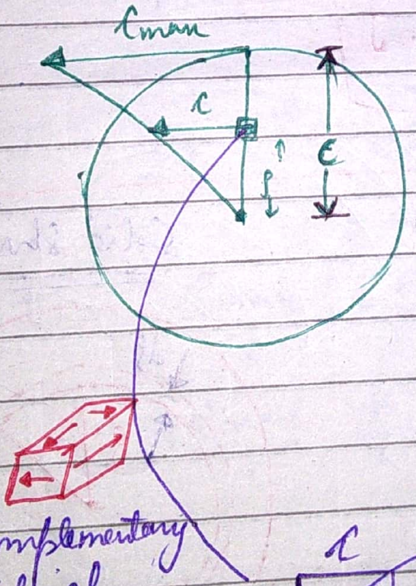
$$\tau = G\gamma$$

$$\tau \propto \gamma$$

shear strain.

therefore $\boxed{\tau \propto \rho}$.

$\rho \rightarrow$ unknown radius.



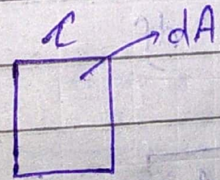
By similar traingle property

$c \rightarrow$ radius

$$\frac{\tau}{\tau_{max}} = \frac{\rho}{c}.$$

$$\boxed{\tau = \left(\frac{\tau_{max}}{c}\right) \cdot \rho}.$$

Because of complementary shear longitudinal shear stresses are also developed.



$$dF = \tau \cdot dA.$$

$$dM = \rho \cdot dF$$

$$\boxed{\int_A dM = \int_A \rho(\tau dA)}$$

$$\int_A dM = \int_A \rho \cdot \left(\frac{\tau_{max}}{c}\right) \cdot \rho \cdot dA$$

$$\underset{\text{(Torque/ Torsional moment)}}{T} = \frac{\tau_{max}}{c} \int_A \rho^2 \cdot dA$$

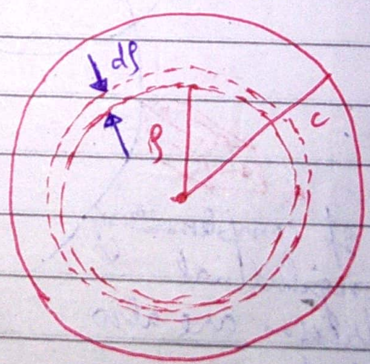
second moment of area (J)

$$\boxed{T = \frac{\tau_{max}}{c} \cdot J} \quad \text{where } J = \int_A \rho^2 \cdot dA$$

$$\frac{\tau_{max}}{c} = \frac{\tau}{\rho} \Rightarrow \boxed{\frac{\tau_{max}}{c} = \frac{\tau}{\rho}}$$

$$\boxed{T = \frac{\tau}{\rho} \cdot J}$$

Solid shaft



$$J = \int \rho^2 \cdot dA$$

$$J = \int \rho^2 \cdot 2\pi \rho \cdot d\rho$$

$$J = 2\pi \int_0^c \rho^3 \cdot d\rho$$

$$\boxed{J = \frac{\pi}{2} c^4}$$

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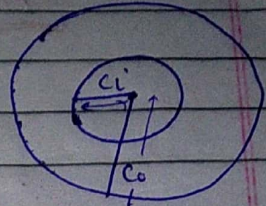
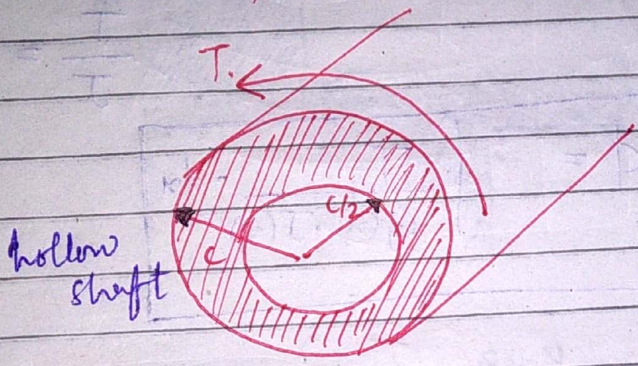
for hollow shaft:

$$\boxed{J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)}$$



Q. A solid shaft is subjected to a torque T. Determine the fraction of Torque T, that is resisted by the material contained with in the outer core of shaft, which has an inner radius of $c/2$ and outer radius c.



$$J = \frac{\pi}{2}\left[(c)^4 - \left(\frac{c}{2}\right)^4\right]$$

$$T' = \frac{\tau_{max}}{c} J$$

$$= \frac{\tau_{max}}{c} \cdot \frac{\pi}{2} \times \frac{15c^4}{16}$$

for solid shaft:

$$T = \frac{\tau_{max}}{c} \cdot J$$

$$T = \frac{\tau_{max}}{c} \times \frac{\pi}{2} c^4$$

$$\frac{T'}{T} = \frac{\frac{\tau_{max}}{\cancel{c}} \times \frac{15\pi c^4}{2 \times 16}}{\frac{\tau_{max}}{\cancel{c}} \times \frac{\pi}{2} c^4}$$

$$\boxed{T' = \frac{15T}{16}} \simeq 94\%$$

{economically feasible}.

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Angle of twist (ϕ)

$$\gamma dx = \rho \cdot d\phi$$

$$d\phi = \frac{\gamma \cdot dx}{\rho}$$

$$d\phi = \frac{\tau/G}{\rho} \cdot dx$$

$$\frac{T}{J} = \frac{\tau}{\rho}$$

$$\boxed{\phi = \int \frac{T}{G(x) \cdot J(x)} \cdot dx}$$

for constant torque and constant areas.

$$\cancel{\phi = \frac{TL}{GJ}} \qquad \boxed{\phi = \frac{TL}{GJ}}$$

for multiple torques

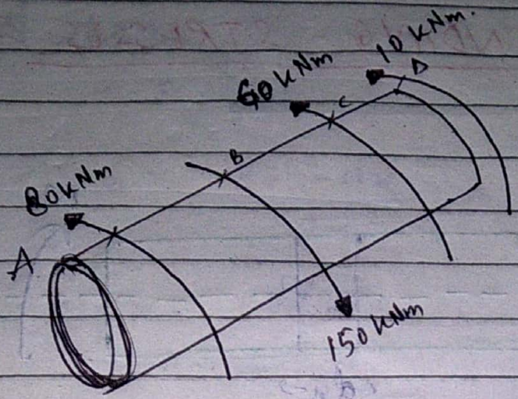
on comparing with axial deformation

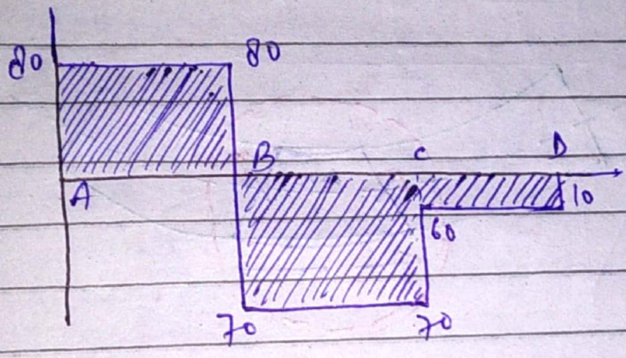
$$\boxed{\phi = \sum \frac{TL}{GJ}}$$

$$\boxed{\delta = \frac{PL}{AE}}$$

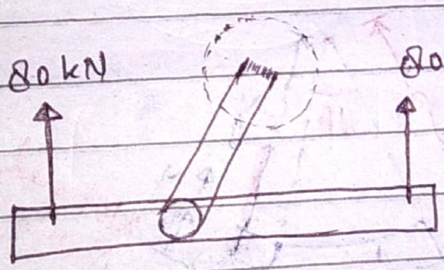
$$\boxed{\delta = \sum \frac{PL}{AE}}$$

Q.





Q.



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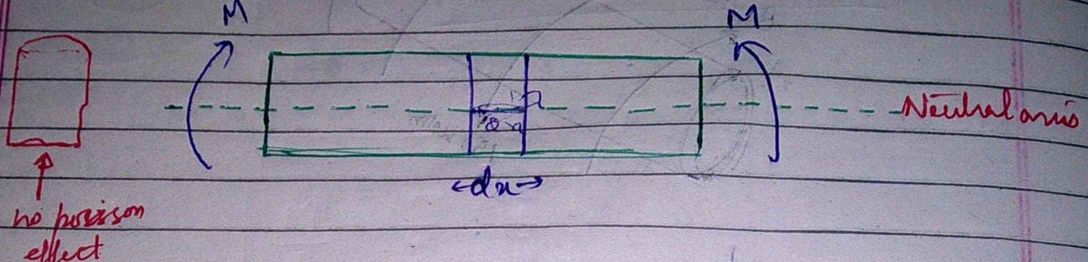
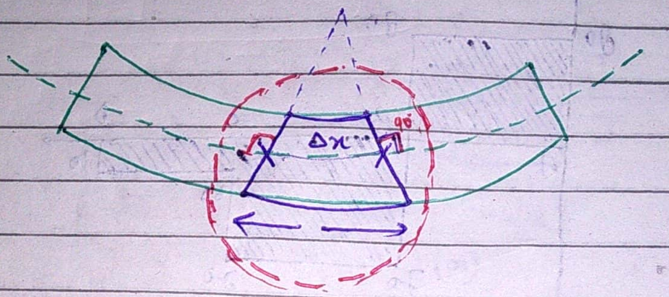
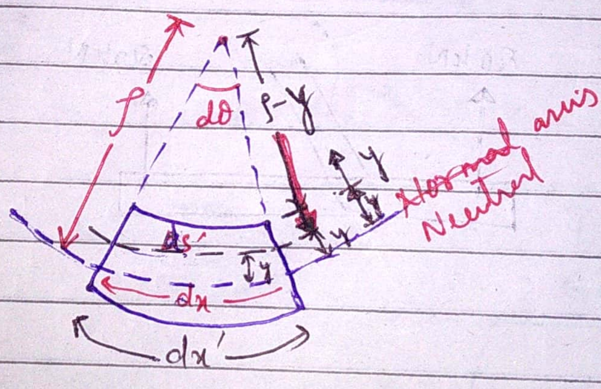
BENDING STRESSES







Assumptions

→ length of neutral axis remains unchanged.

→ cross-section remains straight but, rotated & remains at 90° w.r.t NA.

→ Poisson effect is neglected

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Longitudinal strain

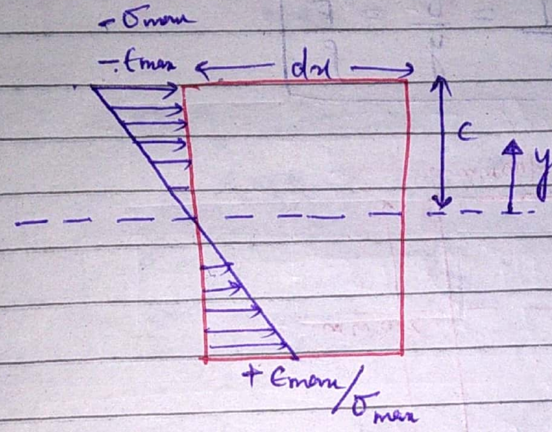
$$\epsilon = \frac{\Delta l}{l}$$

$$\epsilon = \frac{ds' - dx}{dx} = \frac{(\rho - y)d\theta - \rho . d\theta}{\rho d\theta}$$

$$\boxed{\epsilon = -y/\rho}$$

$$\sigma = E \cdot \epsilon = E \cdot y/\rho$$

$$\sigma = \frac{E}{\rho} \cdot y$$



$$\frac{\epsilon_{max}}{\epsilon} = \frac{c}{y}$$

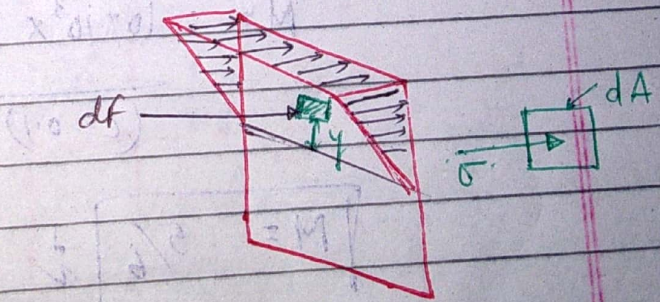
$$\epsilon = \epsilon_{max} \times (y/c)$$

$$\sigma = \sigma_{max} (y/c)$$

$$\epsilon_{max} = -\frac{c}{\rho}$$

$$\sigma = \epsilon \cdot E$$

$$\boxed{\sigma \propto \epsilon}$$



$$df = \sigma dA$$

$$dM = df \cdot y$$

$$= \sigma dA \cdot y$$

$$\int dM = \int \left(\frac{\sigma_{max}}{c}\right) y \cdot y \cdot dA$$

$$\int dM = \frac{\sigma_{max}}{c} \int y^2 dA$$

Second moment of Area (I)

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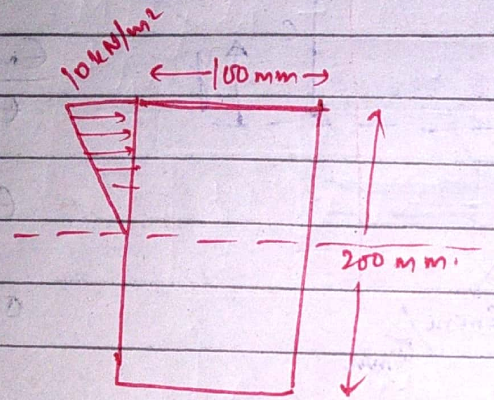
$$\boxed{M = \frac{\sigma_{max} \cdot I}{c}}$$

or

$$\boxed{M = \frac{\sigma}{y} I}$$

or $$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{\rho}}$$

Eg.



what will be the max^m moment from this eq^n?

$$M = \frac{\sigma_{max}}{\cancel{y}} I$$

$$I = \frac{bh^3}{12}$$

$$M = \frac{10 \times 10^3 \times \frac{1}{12} \times 10^{-4}}{(c = 0.1)}$$

$h = 100$ mm
$= 0.1$ m.

$b = 0.1$

$\cancel{y} = 0.1$

$$\boxed{M = 5/6}$$

$$I = \frac{0.1 \times (0.1)^3}{12}$$

$$I = \frac{1}{12} \times 10^{-4}.$$

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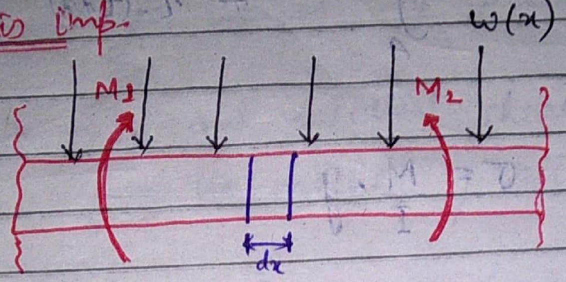
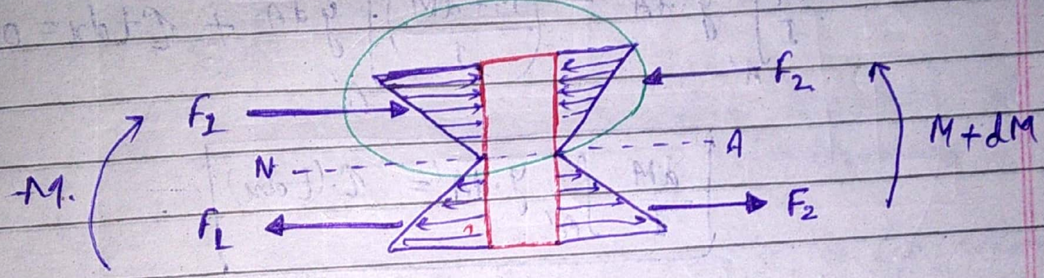
SHEAR STRESSES

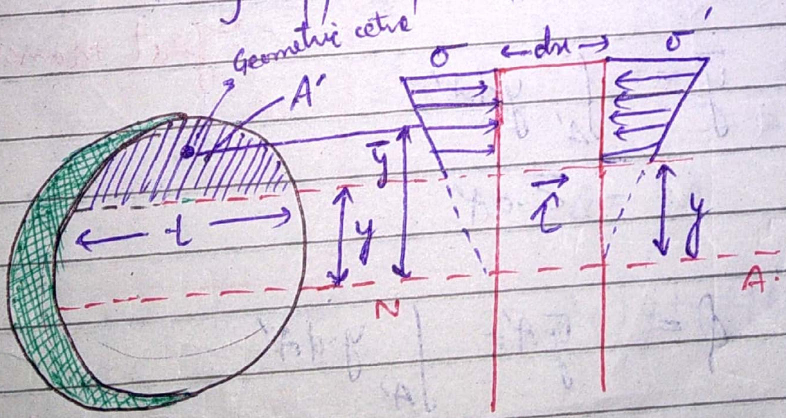
Derivation is imp.

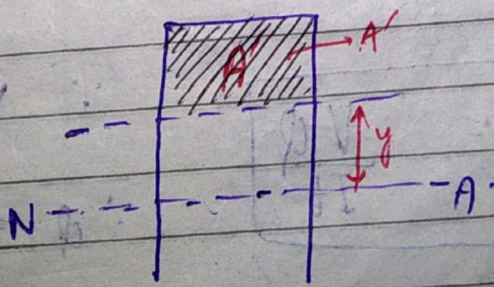




$$\sum_A \vec{F_x} = 0 \Rightarrow \boxed{F_1 = F_2}$$

Taking upper part of the cross section.





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$$\sum f_x = 0$$

$$\int_{A'} \sigma \, dA' - \int_{A} \sigma' dA' + \tau \cdot (t\,dx) = 0$$

$$\sigma = \frac{M}{I} \cdot y$$

$$\frac{M}{I}\int_{A'} y \cdot dA' - \left(\frac{M+dM}{I}\right)\int_{A'} y\,dA' + \tau t\,dx = 0.$$

$$\boxed{\frac{dM}{I}\int_{A'} y \cdot dA' = \tau \cdot (t\,dx)}$$

$$\boxed{\tau = \frac{dM}{dx} \cdot \frac{1}{I \cdot t} \underbrace{\int_{A'} y\,dA'}_{\text{first moment of inertia}}} = Q.$$

$$\bar{y} = \frac{\int_{A'} y\,dA'}{\Sigma\, dA'}$$

$$Q = \bar{y}A' = \int_{A'} y \cdot dA'$$

Imp

$$\boxed{\tau = \frac{VQ}{It}}$$

τ → shear stress

$\because V = \frac{dM}{dx}$ → shear force.

$\because Q = \int_{A'} y \cdot dA'.$

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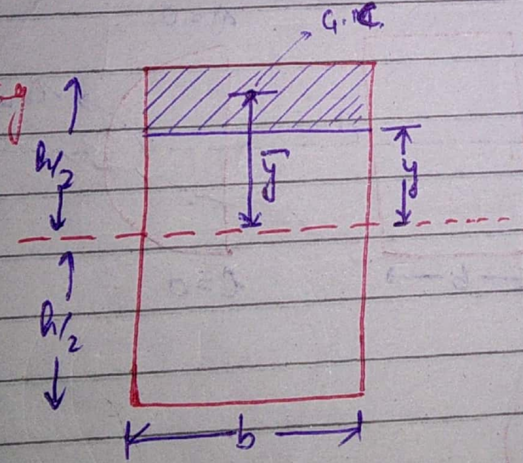
τ – shear stresses in the beam at a distance 'y' from the normal axis (NA). This stress is assumed to be constant and therefore average across 't' (thickness)

'V' → internal resultant shear force; determined from the method of sections & eqn.

I → second moment of area of the centre of entire cross section about normal axis (NA)

t → width of beam cross section measured at a point where 'τ' is to be determined

Q → $\bar{y}A'$ where A' is the area of the top (bottom) portion of the beam of cross section above or below the plane where t is measured.



$$\tau = \frac{VQ}{It} \quad t \to b$$

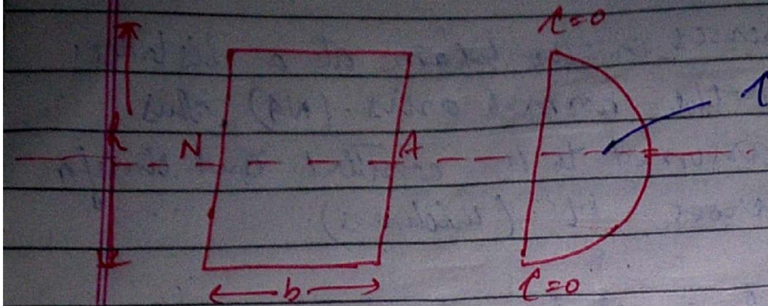
$$I_{NA} = \frac{bh^3}{12}$$

$$Q = \bar{y}A'$$

$$Q = \left\{y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right\} \cdot \left(\frac{h}{2} - y\right) \cdot b.$$

$$\boxed{\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)}$$

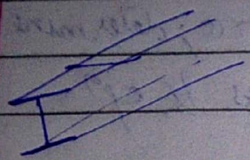
Scanned by CamScanner

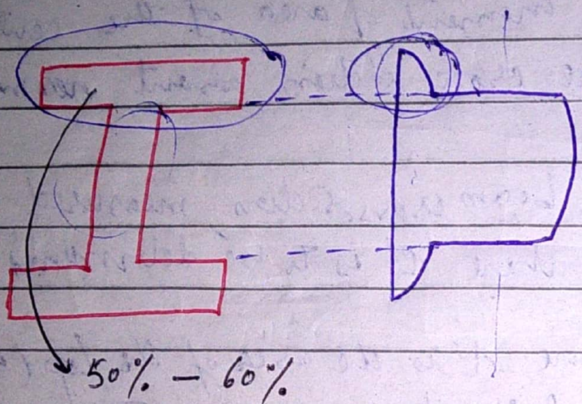


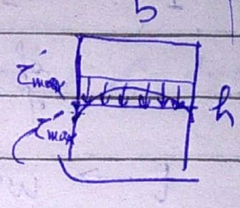
$$\tau_{max} = 1.5\frac{V}{bh} = 1.5\frac{V}{A}$$

$$\tau = \frac{V}{A}$$

$$\boxed{\tau_{max} = 1.5\tau}$$



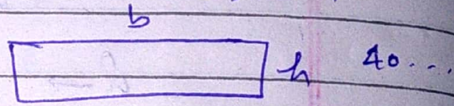


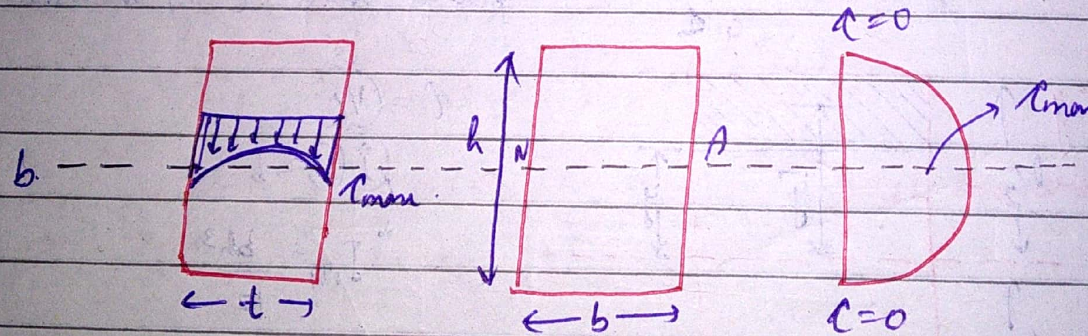


$b = \frac{h}{2}$

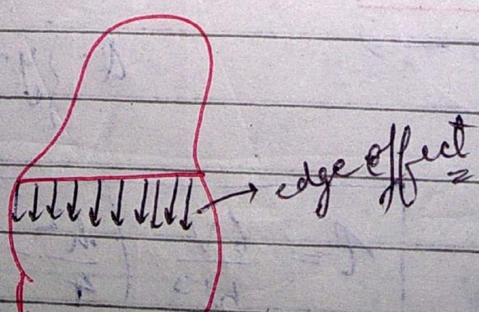
Cross-2-3

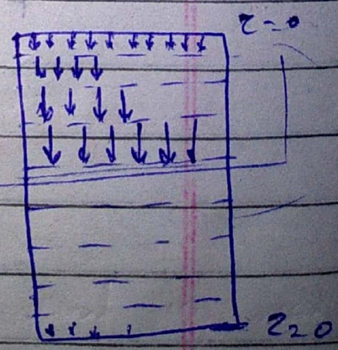
$b = 5h$



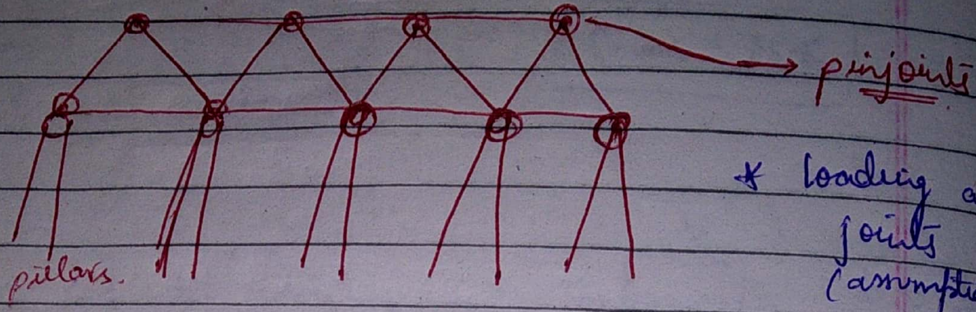


$\tau_{max} = 1.5\frac{V}{bh}$

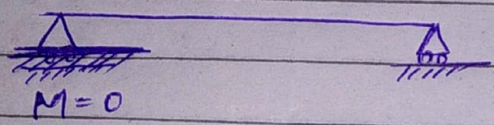


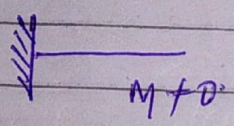


TRUSSES

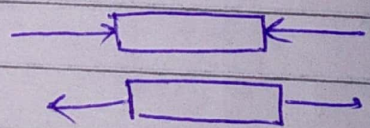


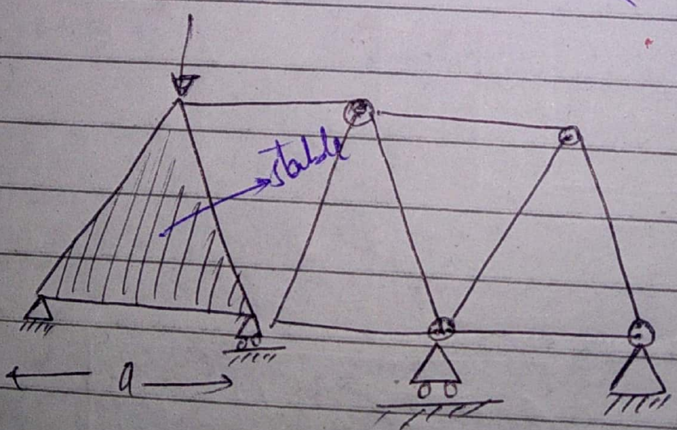
* loading at joints (assumption)
* Axial loads etc only.
* Weight of the members are ignored.





- Compression
- Tension.





by applying one pin, we can joint two members

Stability and Determinacy.

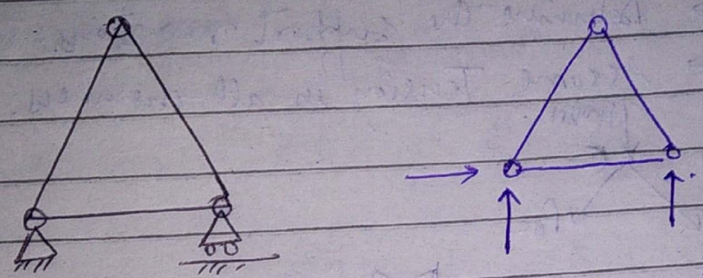
$\boxed{b + r = 2j}$ → Determinate

no. of members. no. of reactions due to support. no. of joints

If. $b + r < 2j$ → unstable (Internally unstable)

$b + r > 2j$ → Indeterminate.

e.g.



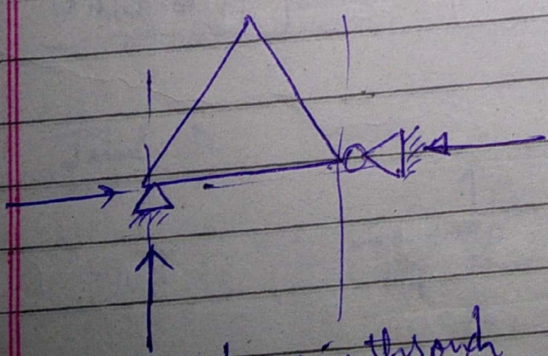
$b = 3$; $r = 3$; $j = 3$.

$b + r = 2j$ ✓

Determinate.

External unstability

Assumptions
- → support reactions are parallel
- → support reactions are collinear (passing through same point)



passing through a single point.

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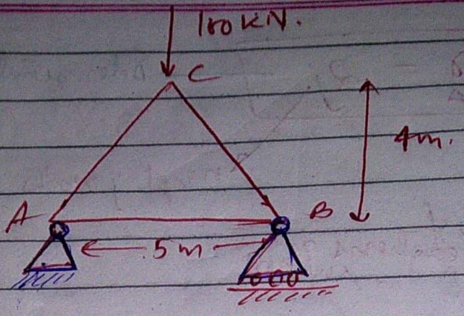
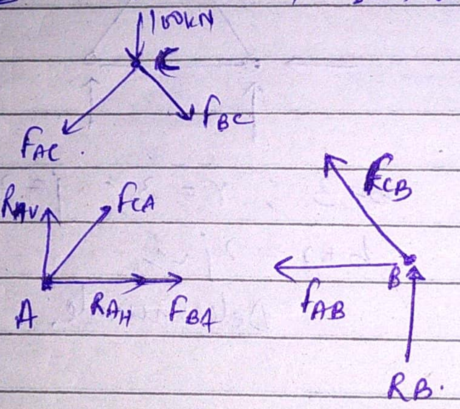


→ Method of joints
→ Method of sections

→ step 1 → Determine the support reactions.
→ step 2 → Assume Tension in all members. (FCB + FBA).



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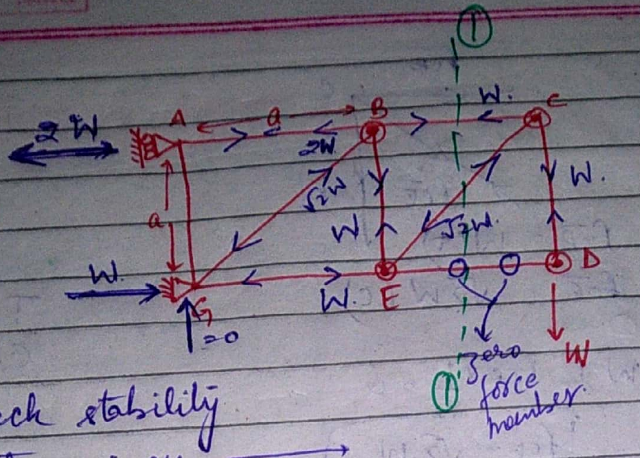


a) Check stability
b) Determinacy ⟶ Method of joints.
c)
d) Method of section

$$\underset{\text{members}}{b} + \underset{\text{reactions}}{r} = \underset{\text{joints}}{2j}$$
$$\lessgtr$$
$$9 + 3 = 2 \times 6$$
$$12 = 12$$

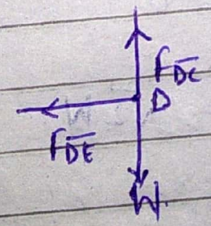
System is determinate

* LHS > 0
Internal structure is stable.
* ↳ Externally unstable.

If $b + r > 2j$
eg. $9 + 4 > 2 \times 6$
$13 > 12$
$13 - 12 = 1$
structure is indeterminate of order 1

Methods of joint

Joint - D.

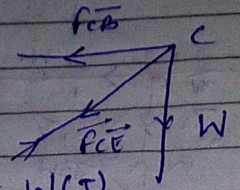


assuming the Tension in the member.

$\sum V = 0$; $W = F_{DC}$

$\sum H = 0$; $F_{DE} = 0$

Joint C



$\Sigma V = 0$; $\Sigma H = 0$;

$\vec{F_{CB}} = W (T)$

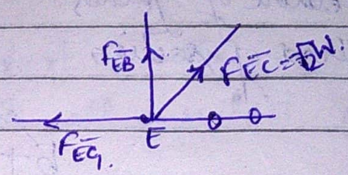
$\vec{F_{CE}} = \sqrt{2} W (C)$

T → Tension
C → Compression

$W + F_{CE} \times \frac{1}{\sqrt{2}} = 0$

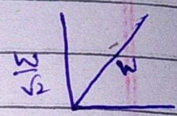
$\boxed{F_{CE} = \sqrt{2} W}$

Joint E



$\Sigma V = 0$

$\Sigma H = 0$

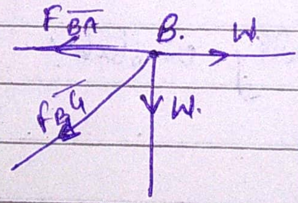


~~$F_{EC} = W$~~

$F_{EB} = \frac{\sqrt{2} W}{\sqrt{2}} = W$ (Tension)

$F_{EG} = \frac{\sqrt{2} \times W}{\sqrt{2}}$ (compression)

Joint B



$F_{BA} + \frac{\vec{F_{BG}}}{\sqrt{2}} = W$ —— ①

$\frac{F_{BG}}{\sqrt{2}} + W = 0.$

$F_{BG} = -\sqrt{2} W.$; $\boxed{F_{BG} = \sqrt{2} W \, (C)}$

$F_{BA} + \frac{-\sqrt{2} W}{\sqrt{2}} = W$

$\boxed{F_{BA} = 2W \, (T)}$

At most three member

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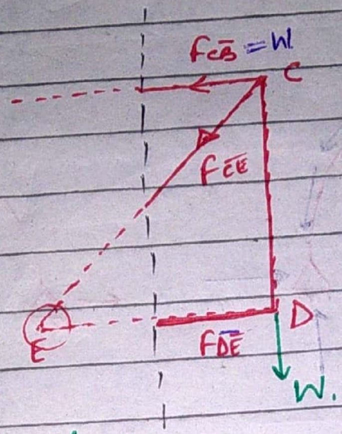
Method of section

(Atmost 3 members should be exposed).

Section (1).



Sum of Moment about E = 0

$$\sum M_E = 0.$$

$$f_{\overline{CB}} \cdot a - W \cdot a = 0$$

$$\boxed{W = f_{CB}} \text{ (Tension)}$$

$$\sum M_C = 0. \qquad f_{\overline{DE}} \cdot a = 0$$

$$\boxed{f_{DE} = 0}.$$

$$\sum H = 0. \qquad \frac{f_{CE}}{\sqrt{2}} + f_{CB} = 0$$

$$f_{CE} = -\sqrt{2}\, f_{CB}$$

$$= -\sqrt{2}\, W$$

$$\boxed{f_{CE} = \sqrt{2}\, W \text{ (compression)}}.$$

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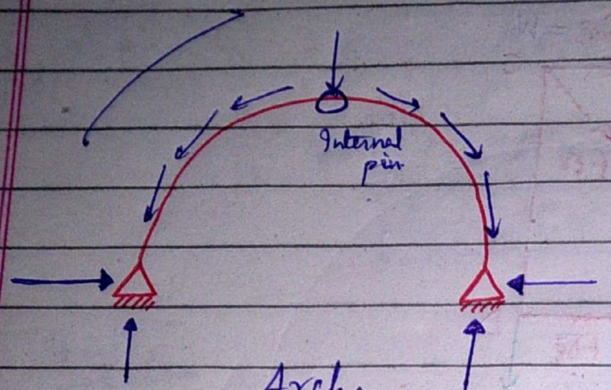
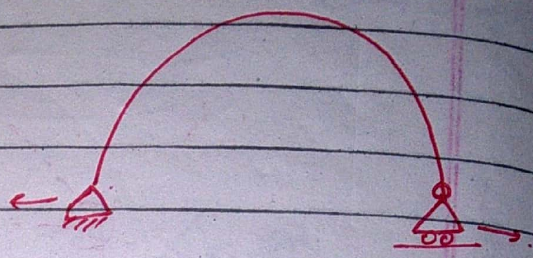
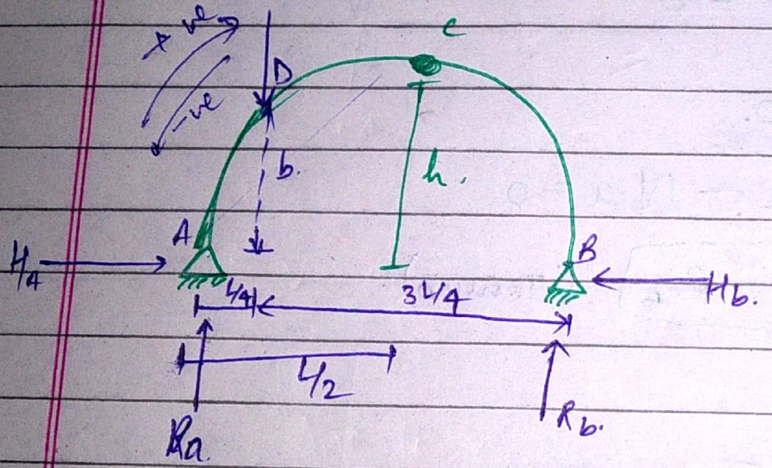
ARCHES



Arch.



Curved beam.



$$\sum M_B = 0$$

$$R_A \times L - P \cdot \frac{3L}{4} = 0$$

$$R_A = \frac{3P}{4}$$

$$\sum V = 0 \; ; \quad \cancel{R_B = 1/4}$$

$$R_A + R_B = P$$

$$R_B + \frac{3P}{4} = P$$

$$\boxed{R_B = P/4}$$

$\Sigma M_C = 0.$

$R_a \times L/2 - H_a \cdot h - PL/4 = 0.$

$$H_a = \frac{PL}{8h}.$$

$M_D = R_A \cdot L/4 - H_a \cdot b.$

$= \left(\frac{3P}{4}\right) L/4 - \left(\frac{PL}{8h}\right) \cdot b.$

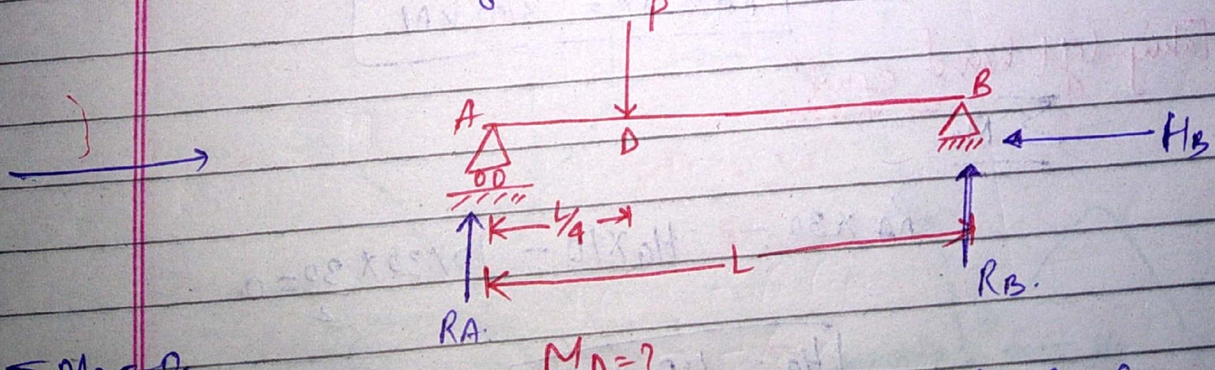
$= \frac{3PL}{16} - \frac{PLb}{8h}$

$\cancel{M_D = \frac{6PLh - PLb}{16h}}$

$= \frac{3PL}{16} - \frac{PL}{8}\left(\frac{b}{h}\right)$

if let $(b/h = 1/2)$.

$M_D = \frac{PL}{8} = 0.125\,PL$



$M_D = ?$

$\Sigma M_B = 0.$

$R_A \cdot L = P \times \frac{3L}{4}.$

$R_A = 3P/4$

$R_B = P/4.$

$P = R_A + R_B.$

$H_B = 0.$

$M_D = R_B \times \frac{3L}{4} + R_A \times \frac{L}{4}.$

$M_D = \frac{3PL}{16} + \frac{PL}{16} \quad = 3\left(\frac{PL}{8}\right)$

almost ...

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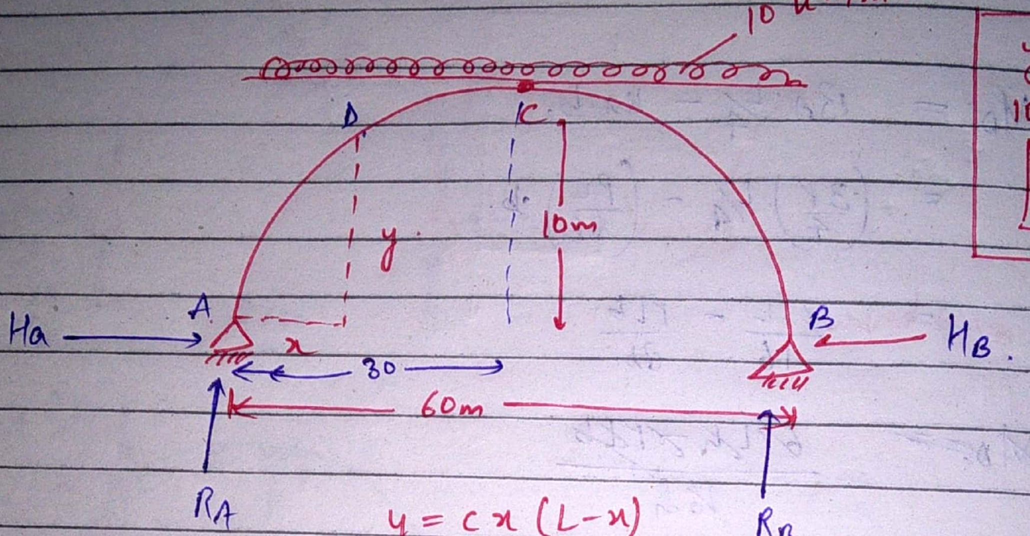
* जब straight बीम पर कोई load काम करता तो उससे produce moment 3 गुना होना होता as compare to the moment produced by the same force on an arche.

Q. 2



$y = cx(L-x)$

$10 = c \times 30(60-30)$

$\boxed{c = \frac{1}{90}}$

$y = cx(L-x)$

$\Sigma V = 0.$

$R_a + R_B = \frac{10 \times 60}{\cancel{2}}$

$C = 10 \times 60$

$C = 600$ kN.

Due to symmetry

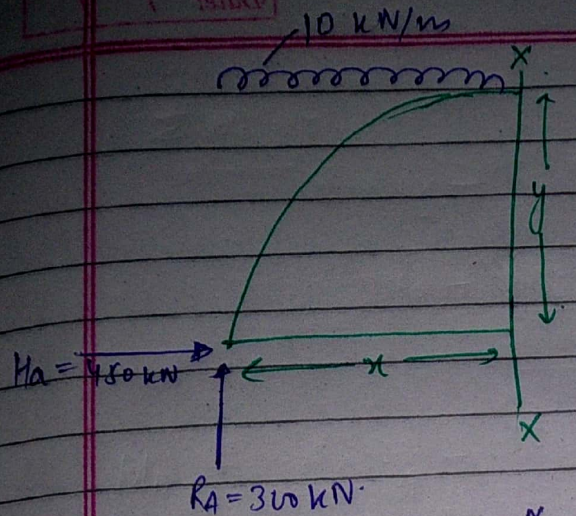
$\boxed{R_A = R_B = 300 \text{ kN}}$

"Taking left hand equil^n"

ΣM_C

$R_A \times 30 - H_a \times 10 - 10 \times 30 \times \frac{30}{2} = 0.$

$\boxed{H_a = 450 \text{ kN}}$



$$M\Big|_{x}^{x} = Ra\cdot x - Ha\cdot y - 10\cdot x\cdot \frac{x}{2}$$

$$M\Big|_{x}^{x} = 300x - 450\cdot\frac{1}{90}x\cdot(60-x) - \frac{10x^2}{2}$$

$$= 300x - 5x(60-x) - 5x^2$$

$$= 300x - 300x + 5x^2 - 5x^2$$

$$= 0$$

Therefore, moment at any point on the arch will be zero.