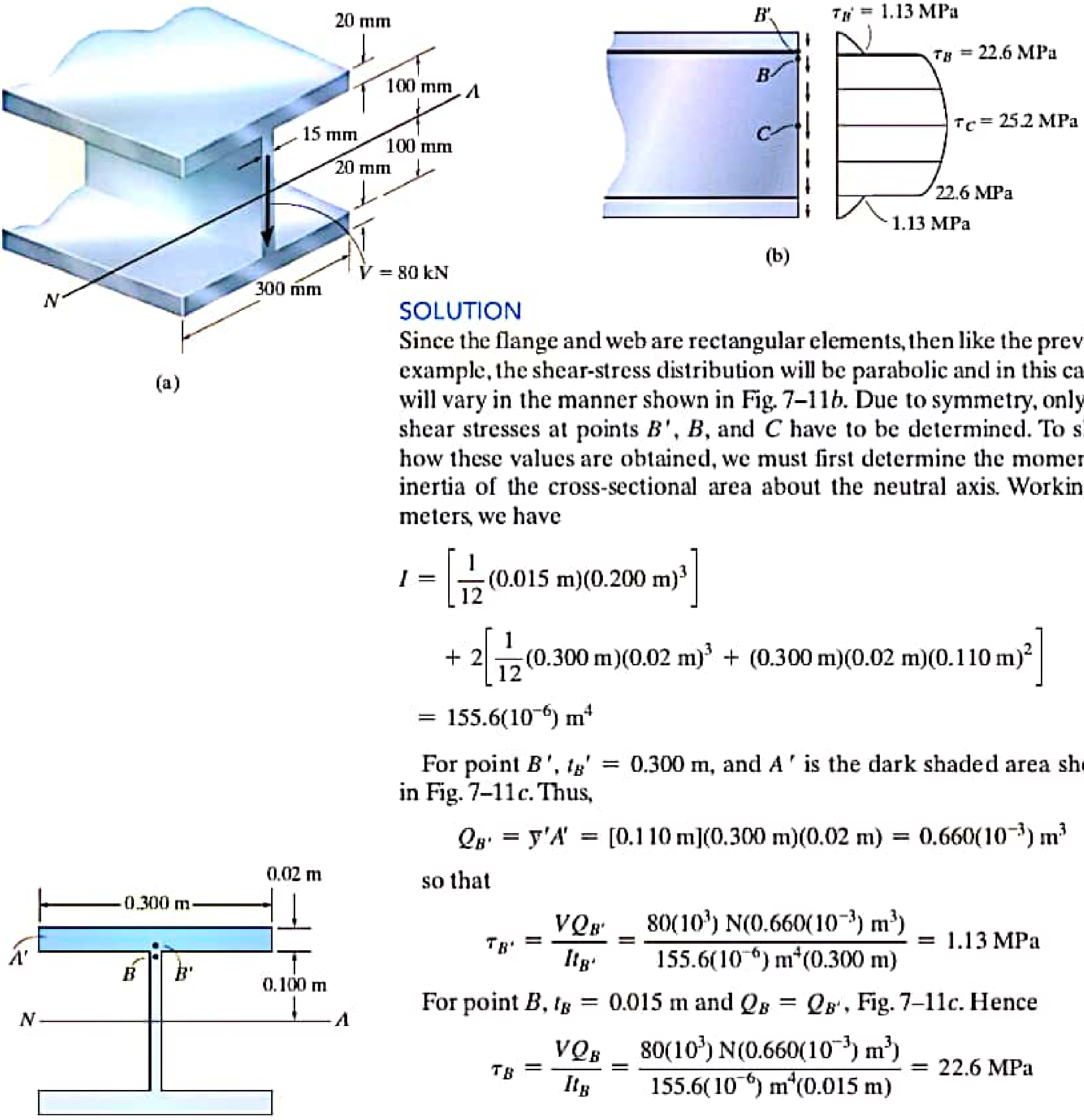
EXAMPLE 7.3

A steel wide-flange beam has the dimensions shown in Fig. 7–11*a*. If it is subjected to a shear of $V = 80$ kN, plot the shear-stress distribution acting over the beam's cross-sectional area.



(c)

Fig. 7–11

SOLUTION

Since the flange and web are rectangular elements, then like the previous example, the shear-stress distribution will be parabolic and in this case it will vary in the manner shown in Fig. 7–11*b*. Due to symmetry, only the shear stresses at points B', B, and C have to be determined. To show how these values are obtained, we must first determine the moment of inertia of the cross-sectional area about the neutral axis. Working in meters, we have

$$I = \left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3\right]$$

$$+ 2\left[\frac{1}{12}(0.300\text{ m})(0.02\text{ m})^3 + (0.300\text{ m})(0.02\text{ m})(0.110\text{ m})^2\right]$$

$$= 155.6(10^{-6})\text{ m}^4$$

For point B', $t_B' = 0.300$ m, and A' is the dark shaded area shown in Fig. 7–11*c*. Thus,

$$Q_{B'} = \bar{y}'A' = [0.110\text{ m}](0.300\text{ m})(0.02\text{ m}) = 0.660(10^{-3})\text{ m}^3$$

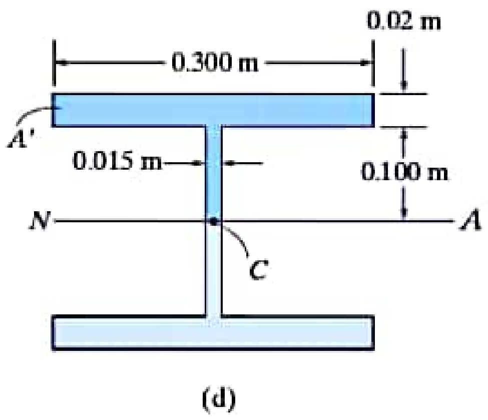
so that

$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80(10^3)\text{ N}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.300\text{ m})} = 1.13\text{ MPa}$$

For point B, $t_B = 0.015$ m and $Q_B = Q_{B'}$, Fig. 7–11*c*. Hence

$$\tau_B = \frac{VQ_B}{It_B} = \frac{80(10^3)\text{ N}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})} = 22.6\text{ MPa}$$

Note from the discussion of "Limitations on the Use of the Shear Formula" that the calculated values for both $\tau_{B'}$ and τ_B will actually be very misleading. Why?



(d)

Fig. 7–11 (cont.)

For point C, $t_C = 0.015$ m and A' is the dark shaded area shown in Fig. 7–11*d*. Considering this area to be composed of two rectangles, we have

$$Q_C = \Sigma \bar{y}'A' = [0.110\text{ m}](0.300\text{ m})(0.02\text{ m})$$
$$+ [0.05\text{ m}](0.015\text{ m})(0.100\text{ m})$$
$$= 0.735(10^{-3})\text{ m}^3$$

Thus,

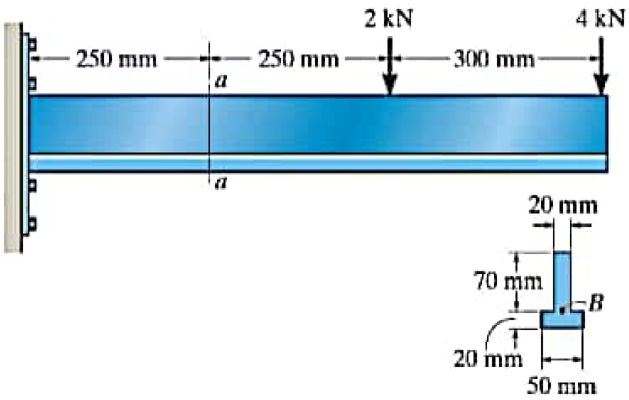
$$\tau_C = \tau_{\max} = \frac{VQ_C}{It_C} = \frac{80(10^3)\text{ N}[0.735(10^{-3})\text{ m}^3]}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})} = 25.2\text{ MPa}$$

NOTE: From Fig. 7–11*b*, note that most of the shear stress occurs in the web and is almost uniform throughout its depth, varying from 22.6 MPa to 25.2 MPa. It is for this reason that for design, some codes permit the use of calculating the *average* shear stress on the cross section of the web rather than using the shear formula; that is,

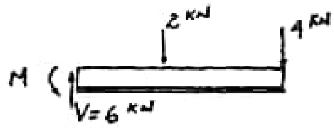
$$\tau_{\text{avg}} = \frac{V}{A_w} = \frac{80(10^3)\text{ N}}{(0.015\text{ m})(0.2\text{ m})} = 26.7\text{ MPa}$$

This will be discussed further in Chapter 11.

7–22. Determine the shear stress at point B on the web of the cantilevered strut at section a–a.



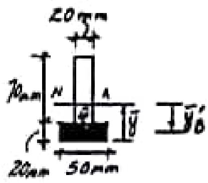
$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$



$$I = \frac{1}{12}(0.05)(0.02^3) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)(0.07^3) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625(10^{-6}) \text{ m}^4$$

$$\bar{y}'_B = 0.03625 - 0.01 = 0.02625 \text{ m}$$



$$Q_B = (0.02)(0.05)(0.02625) = 26.25(10^{-6}) \text{ m}^3$$

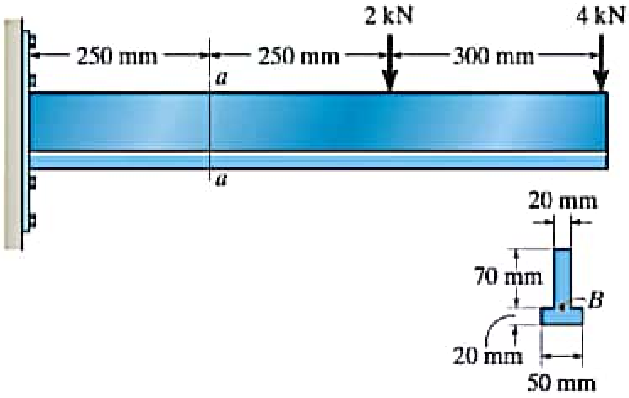
$$\tau_B = \frac{VQ_B}{I\,t} = \frac{6(10^3)(26.25)(10^{-6})}{1.78622(10^{-6})(0.02)}$$

$$= 4.41 \text{ MPa}$$

Ans.

Ans:

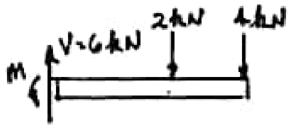
7–23. Determine the maximum shear stress acting at section a–a of the cantilevered strut.



$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$

$$I = \frac{1}{12}(0.05)(0.02^3) + (0.05)(0.02)(0.03625 - 0.01)^2$$

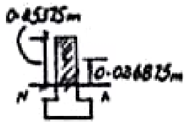
$$+ \frac{1}{12}(0.02)(0.07^3) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625(10^{-6}) \text{ m}^4$$



$$Q_{max} = \bar{y}'A' = (0.026875)(0.05375)(0.02) = 28.8906(10^{-6}) \text{ m}^3$$

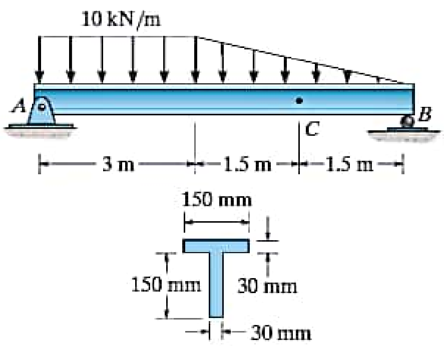
$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{6(10^3)(28.8906)(10^{-6})}{1.78625(10^{-6})(0.02)}$$

$$= 4.85 \text{ MPa}$$ **Ans.**



Ans:
$\tau_{max} = 4.85$ MPa

7–25. Determine the maximum shear stress in the T-beam at section C. Show the result on a volume element at this point.



Using the method of sections (Fig. *a*),

$$+\uparrow \Sigma F_y = 0; \qquad V_C + 17.5 - \frac{1}{2}(5)(1.5) = 0$$

$$V_C = -13.75 \text{ kN}$$

The neutral axis passes through centroid C of the cross section,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.075\,(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

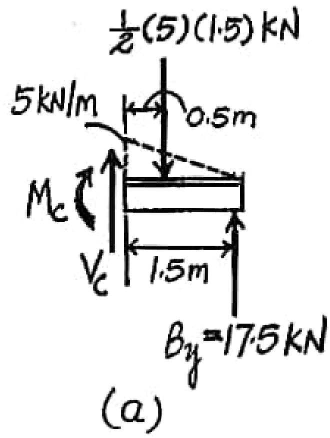
$$I = \frac{1}{12}(0.03)(0.15^3) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0\,(10^{-6}) \text{ m}^4$$

$$Q_{max} = \bar{y}'A' = 0.06\,(0.12)(0.03)$$

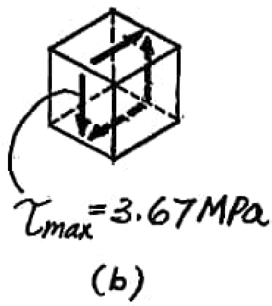
$$= 0.216\,(10^{-3}) \text{ m}^3 \text{ 490}$$



The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness $t = 0.03$ m is the smallest (Fig. *b*).

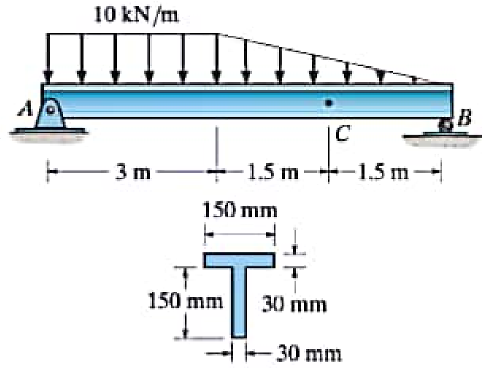
$$\tau_{max} = \frac{V_C Q_{max}}{It} = \frac{13.75(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})\,(0.03)}$$

$$= 3.667(10^6) \text{ Pa} = 3.67 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:

***7–24.** Determine the maximum shear stress in the T-beam at the critical section where the internal shear force is maximum.



The FBD of the beam is shown in Fig. *a*,

The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = 27.5$ kN

The neutral axis passes through centroid *c* of the cross section, Fig. *c*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.075(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

$$I = \frac{1}{12}(0.03)(0.15^3) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0(10^{-6}) \text{ m}^4$$

From Fig. *d*,

$$Q_{max} = \bar{y}'A' = 0.06(0.12)(0.03)$$

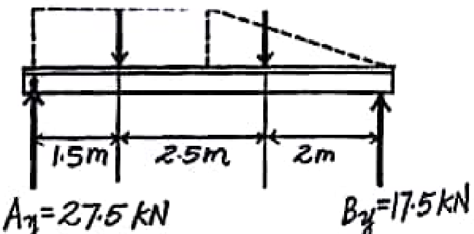
$$= 0.216\,(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness $t = 0.03$ m is the smallest.

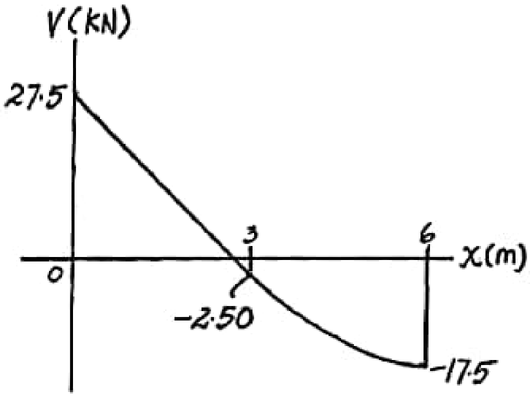
$$\tau_{max} = \frac{V_{max} Q_{max}}{I t} = \frac{27.5(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})(0.03)}$$

$$= 7.333(10^6) \text{ Pa}$$

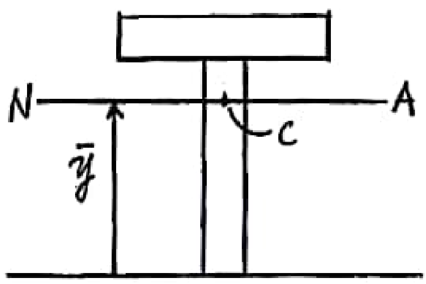
$$= 7.33 \text{ MPa}$$ **Ans.**



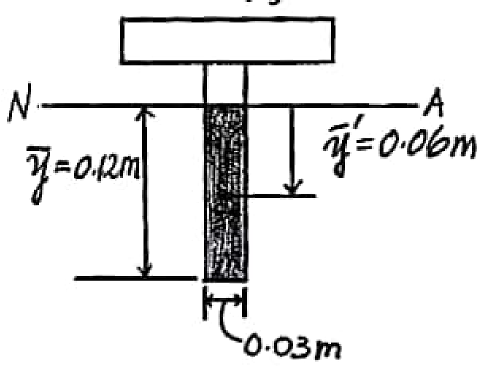
(a)



(b)

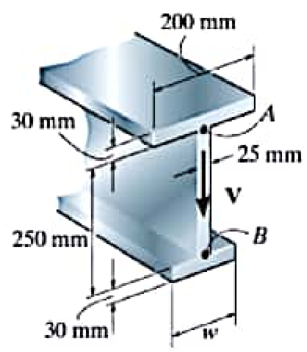


(C)



(d)

7–17. If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 125$ mm. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})$ m^4.



$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747 - 0.015)^2$$

$$+ \frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747 - 0.155)^2$$

$$+ \frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295 - 0.1747)^2 = 0.218182(10^{-3}) \text{ m}^4$$

$$Q_A = \bar{y}A'_A = (0.310 - 0.015 - 0.1747)(0.2)(0.03) = 0.7219(10^{-3}) \text{ m}^3$$

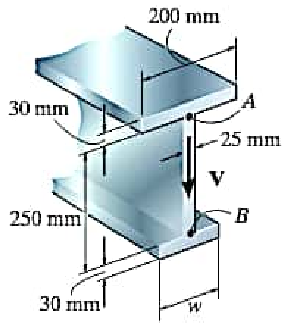
$$Q_B = \bar{y}A'_B = (0.1747 - 0.015)(0.125)(0.03) = 0.59883(10^{-3}) \text{ m}^3$$

$$\tau_A = \frac{VQ_A}{It} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})0.025} = 1.99 \text{ MPa}$$ **Ans.**

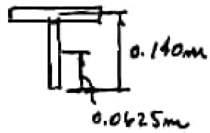
$$\tau_B = \frac{VQ_B}{It} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})0.025} = 1.65 \text{ MPa}$$ **Ans.**

Ans:
$\tau_A = 1.99$ MPa, $\tau_B = 1.65$ MPa

7–18. If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 200$ mm.



Section Properties:



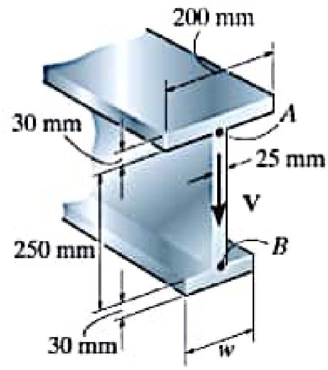
$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q_{\max} = \Sigma \bar{y}A = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3}\ \text{m}^3$$

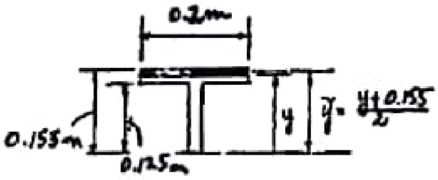
$$\tau_{\max} = \frac{VQ}{It} = \frac{30(10)^3(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62\ \text{MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 4.62$ MPa

7–19. If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the shear force resisted by the web of the beam. Set $w = 200$ mm.



$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$



$$Q = \left(\frac{0.155 + y}{2}\right)(0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

$$V_f = \int \tau_f\, dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2\, dy)$$

$$= 11.1669(10)^6 \left[0.024025y - \frac{1}{3}y^3\right]_{0.125}^{0.155}$$

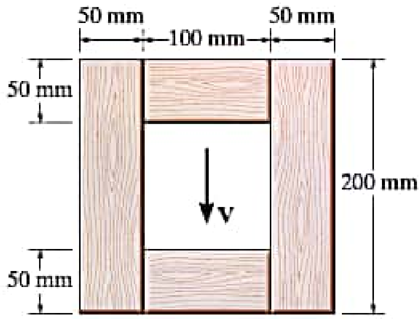
$$V_f = 1.457 \text{ kN}$$

$$V_w = 30 - 2(1.457) = 27.1 \text{ kN}$$ **Ans.**

6

Ans:
$V_w = 27.1$ kN

7–6. The wood beam has an allowable shear stress of $\tau_{allow} = 7$ MPa. Determine the maximum shear force V that can be applied to the cross section.

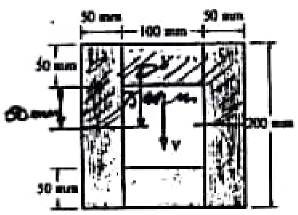


$$I = \frac{1}{12}(0.2)(0.2)^3 - \frac{1}{12}(0.1)(0.1)^3 = 125(10^{-6})\ \text{m}^4$$

$$\tau_{allow} = \frac{VQ_{max}}{It}$$

$$7(10^6) = \frac{V[(0.075)(0.1)(0.05) + 2(0.05)(0.1)(0.05)]}{125(10^{-6})(0.1)}$$

$V = 100$ kN **Ans.**



EXAMPLE 5.3

The pipe shown in Fig. 5–12*a* has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at *A* using a torque wrench at *B*, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.

SOLUTION

Internal Torque. A section is taken at an intermediate location *C* along the pipe's axis, Fig. 5–12*b*. The only unknown at the section is the internal torque **T**. We require

$$\Sigma M_y = 0;\quad 80\text{ N }(0.3\text{ m}) + 80\text{ N }(0.2\text{ m}) - T = 0$$

$$T = 40\text{ N}\cdot\text{m}$$

Section Property. The polar moment of inertia for the pipe's cross-sectional area is

$$J = \frac{\pi}{2}\left[(0.05\text{ m})^4 - (0.04\text{ m})^4\right] = 5.796(10^{-6})\text{ m}^4$$

Shear Stress. For any point lying on the outside surface of the pipe, $\rho = c_o = 0.05$ m, we have

$$\tau_o = \frac{Tc_o}{J} = \frac{40\text{ N}\cdot\text{m}(0.05\text{ m})}{5.796(10^{-6})\text{m}^4} = 0.345\text{ MPa} \qquad \textit{Ans.}$$

And for any point located on the inside surface, $\rho = c_i = 0.04$ m, so that

$$\tau_i = \frac{Tc_i}{J} = \frac{40\text{ N}\cdot\text{m}(0.04\text{ m})}{5.796(10^{-6})\text{m}^4} = 0.276\text{ MPa} \qquad \textit{Ans.}$$

NOTE: To show how these stresses act at representative points *D* and *E* on the cross-section, we will first view the cross section from the front of segment *CA* of the pipe, Fig. 5–12*a*. On this section, Fig. 5–12*c*, the resultant internal torque is equal but opposite to that shown in Fig. 5–12*b*. The shear stresses at *D* and *E* contribute to this torque and therefore act on the shaded faces of the elements in the directions shown. As a consequence, notice how the shear-stress components act on the other three faces. Furthermore, since the top face of *D* and the inner face of *E* are in stress-free regions taken from the pipe's outer and inner walls, no shear stress can exist on these faces or on the other corresponding faces of the elements.

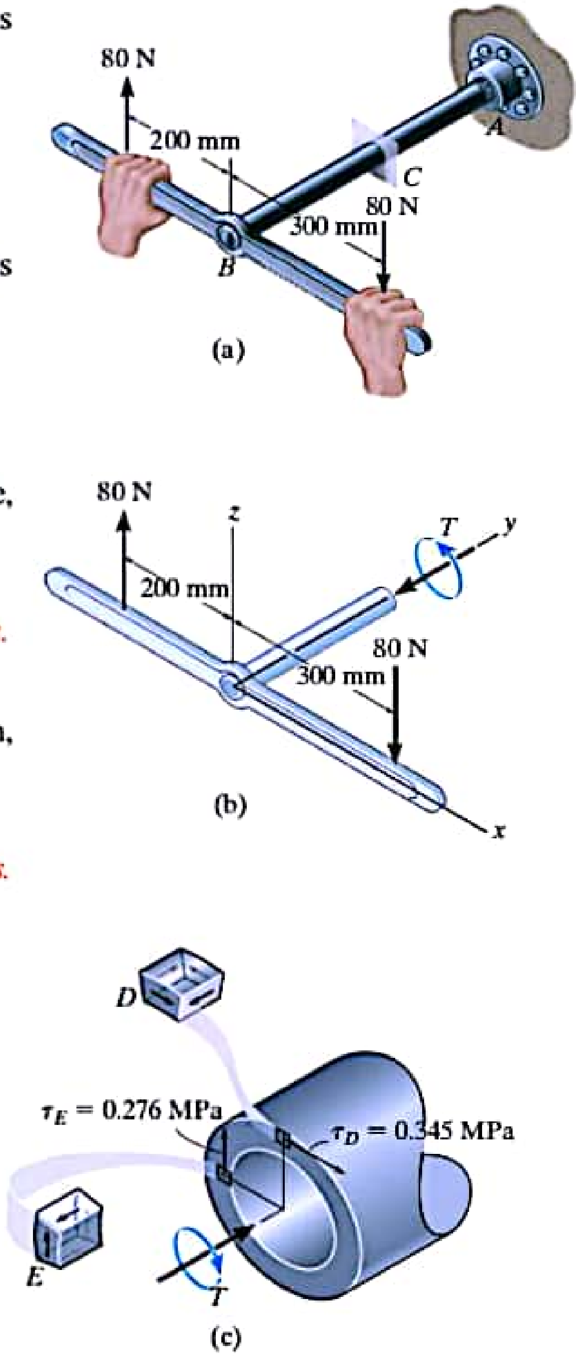


Fig. 5–12

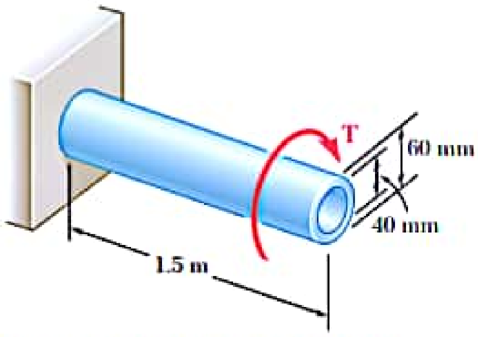


Fig. 3.15 Hollow, fixed-end shaft having torque T applied at end.

Concept Application 3.1

A hollow cylindrical steel shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm (Fig. 3.15). (*a*) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120 MPa? (*b*) What is the corresponding minimum value of the shearing stress in the shaft?

The largest torque **T** that can be applied to the shaft is the torque for which $\tau_{max} = 120$ MPa. Since this is less than the yield strength for any steel, use Eq. (3.9). Solving this equation for T,

$$T = \frac{J\tau_{max}}{c} \tag{1}$$

Recalling that the polar moment of inertia J of the cross section is given by Eq. (3.11), where $c_1 = \frac{1}{2}(40 \text{ mm}) = 0.02 \text{ m}$ and $c_2 = \frac{1}{2}(60 \text{ mm}) = 0.03 \text{ m}$, write

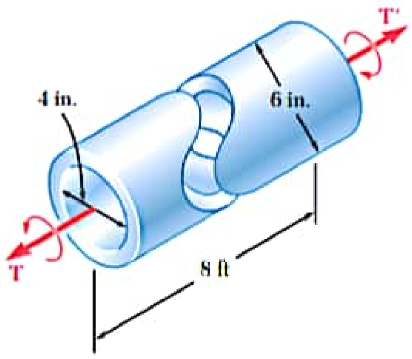
$$J = \tfrac{1}{2}\pi(c_2^4 - c_1^4) = \tfrac{1}{2}\pi(0.03^4 - 0.02^4) = 1.021 \times 10^{-6} \text{ m}^4$$

Substituting for J and τ_{max} into Eq. (1) and letting $c = c_2 = 0.03$ m,

$$T = \frac{J\tau_{max}}{c} = \frac{(1.021 \times 10^{-6} \text{ m}^4)(120 \times 10^6 \text{ Pa})}{0.03 \text{ m}} = 4.08 \text{ kN}\cdot\text{m}$$

The minimum shearing stress occurs on the inner surface of the shaft. Equation (3.7) expresses that τ_{min} and τ_{max} are respectively proportional to c_1 and c_2:

$$\tau_{min} = \frac{c_1}{c_2}\tau_{max} = \frac{0.02 \text{ m}}{0.03 \text{ m}}(120 \text{ MPa}) = 80 \text{ MPa}$$



Sample Problem 3.2

The preliminary design of a motor to generator connection calls for the use of a large hollow shaft with inner and outer diameters of 4 in. and 6 in., respectively. Knowing that the allowable shearing stress is 12 ksi, determine the maximum torque that can be transmitted by (*a*) the shaft as designed, (*b*) a solid shaft of the same weight, and (*c*) a hollow shaft of the same weight and an 8-in. outer diameter.

STRATEGY: Use Eq. (3.9) to determine the maximum torque using the allowable stress.

MODELING and ANALYSIS:

a. Hollow Shaft as Designed. Using Fig. 1 and setting $\tau_{all} = 12$ ksi, we write

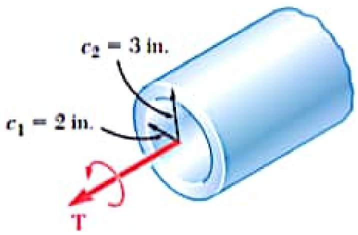


Fig. 1 Shaft as designed.

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(3\text{ in.})^4 - (2\text{ in.})^4] = 102.1\text{ in}^4$$

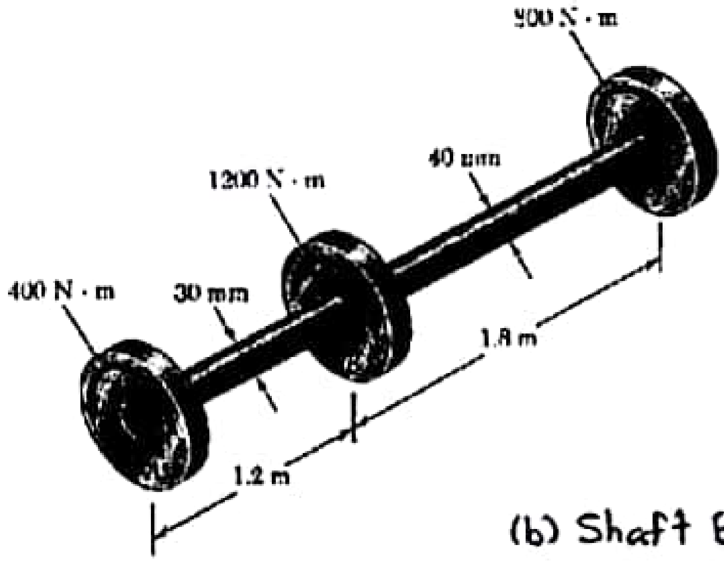
Using Eq. (3.9), we write

$$\tau_{max} = \frac{Tc_2}{J} \qquad 12\text{ ksi} = \frac{T(3\text{ in.})}{102.1\text{ in}^4} \qquad T = 408\text{ kip·in.} \blacktriangleleft$$

(continued)

Problem 3.12

3.12 The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 60 MPa, determine the smallest allowable diameter of (*a*) shaft *AB*, (*b*) shaft *BC*.



(a) Shaft AB: $T = 400$ N·m

$\tau_{max} = 60$ MPa $= 60 \times 10^6$ Pa

$J = \frac{\pi}{2} c^4 \qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(400)}{\pi(60 \times 10^6)}}$

$= 16.19 \times 10^{-3}$ m $= 16.19$ mm

$d_{AB} = 2c = 32.4$ mm ◄

(b) Shaft BC: $T = 800$ N·m

$\tau_{max} = 60$ MPa $= 60 \times 10^6$ Pa

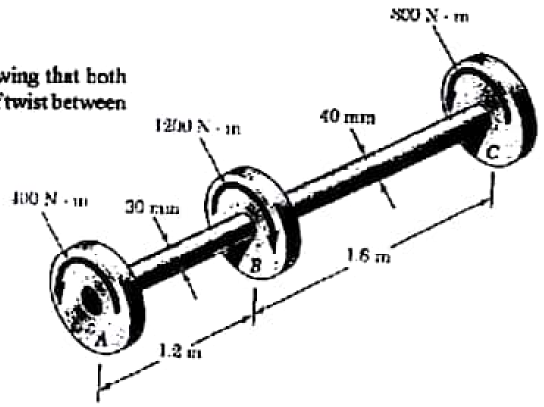
$c = \sqrt[3]{\frac{2T}{\pi c^3}} = \sqrt[3]{\frac{(2)(800)}{\pi(60 \times 10^6)}}$

$= 20.40 \times 10^{-3}$ m $= 20.40$ mm

$d_{BC} = 2c = 40.8$ mm ◄

Problem 3.35

3.35 The torques shown are exerted on pulleys *A*, *B*, and *C*. Knowing that both shafts are solid and made of brass ($G = 39$ GPa), determine the angle of twist between (*a*) *A* and *B*, (*b*) *A* and *C*.



(a) Angle of twist between A and B

$T_{AB} = 400$ N·m, $L_{AB} = 1.2$ m

$c = \frac{1}{2}d = 0.015$ m $\quad G = 39 \times 10^9$ Pa

$J_{AB} = \frac{\pi}{2}c^4 = 79.52 \times 10^{-9}$ m^4

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(400)(1.2)}{(39\times10^9)(79.52\times10^{-9})}$$

$= 0.154772$ rad $\circlearrowleft$

$\varphi_{A/B} = 8.87° \circlearrowleft$

(b) Angle of twist between A and C

$T_{BC} = 800$ N-m, $L_{BC} = 1.8$ m, $c = \frac{1}{2}d = 0.020$ m, $G = 39\times10^6$ Pa

$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.327 \times 10^{-9}$ m^4

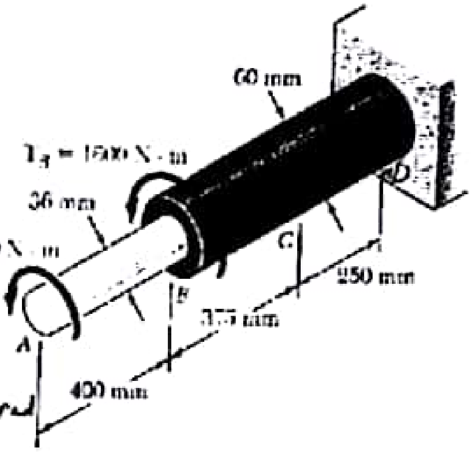
$$\varphi_{B/C} = \frac{(800)(1.8)}{(39\times10^9)(251.327\times10^{-9})} = 0.146912 \text{ rad } \circlearrowright$$

$\varphi_{A/C} = \varphi_{A/B} + \varphi_{B/C} = 0.154772 - 0.146912 = 0.007850$ rad $\circlearrowleft$

$\varphi_{C/A} = 0.450° \circlearrowleft$

Problem 3.37

3.37 The aluminum rod AB ($G = 27$ GPa) is bonded to the brass rod BD ($G = 39$ GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm determine the angle of twist at A.



Rod AB: $G = 27 \times 10^9$ Pa, $L = 0.400$ m

$T = 800$ N·m $\qquad c = \frac{1}{2}d = 0.018$ m

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9}$ m

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Part BC: $G = 39 \times 10^9$ Pa $\quad L = 0.375$ m, $\quad c = \frac{1}{2}d = 0.030$ m

$T = 800 + 1600 = 2400$ N-m, $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6}$ m^4

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part CD: $c_1 = \frac{1}{2}d_1 = 0.020$ m $\qquad c_2 = \frac{1}{2}d_2 = 0.030$ m, $\quad L = 0.250$ m

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6}$ m^4

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

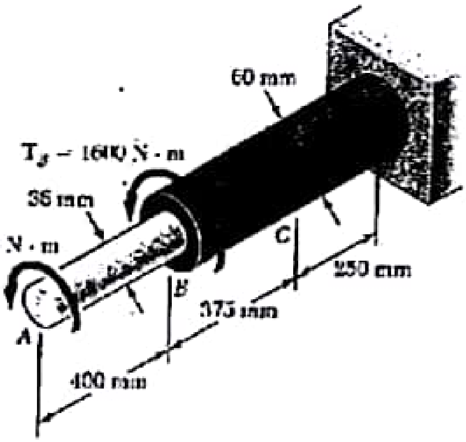
Angle of twist at A $\qquad \varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$

$= 105.080 \times 10^{-3}$ rad $\qquad\qquad \varphi_A = 6.02°$ ◄

Problem 3.38

3.38 Solve Prob. 3.37, assuming that portion BD is a solid 60-mm-diameter rod of length 625 mm.

3.37 The aluminum rod AB ($G = 27$ GPa) is bonded to the brass rod BD ($G = 39$ GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm determine the angle of twist at A.



Rod AB: $G = 27 \times 10^9$ Pa, $L = 0.400$ m

$T = 800$ N·m $\qquad c = \frac{1}{2}d = 0.18$ m

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9}$ m^4

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^6)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Rod BCD: $G = 39 \times 10^9$ Pa, $L = 0.375 + 0.250 = 0.625$ m, $c = \frac{1}{2}d = 0.030$ m

$T = 800 + 1600 = 2400$ N·m $\qquad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6}$ m^4

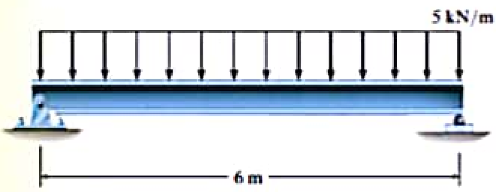
$$\varphi_{B/D} = \frac{TL}{GJ} = \frac{(2400)(0.625)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 30.229 \times 10^{-3} \text{ rad}$$

Angle of twist at A $\qquad \varphi_A = \varphi_{A/B} + \varphi_{B/D}$

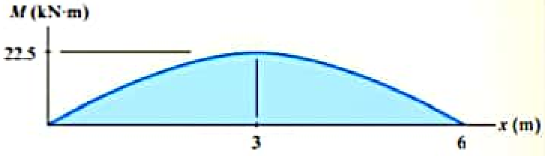
$= 102.104 \times 10^{-3}$ rad $\qquad\qquad \varphi_A = 5.85°$ ◄

EXAMPLE 6.12

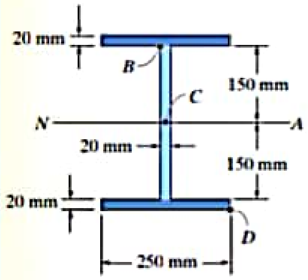
The simply supported beam in Fig. 6–26*a* has the cross-sectional area shown in Fig. 6–26*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



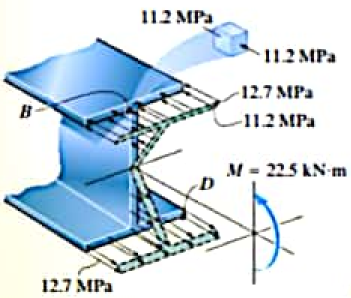
(a)



(c)



(b)



(d)

Fig. 6–26

SOLUTION

Maximum Internal Moment. The maximum internal moment in the beam, $M = 22.5\ \text{kN}\cdot\text{m}$, occurs at the center.

Section Property. By reasons of symmetry, the neutral axis passes through the centroid C at the midheight of the beam, Fig. 6–26*b*. The area is subdivided into the three parts shown, and the moment of inertia of each part is calculated about the neutral axis using the parallel-axis theorem. (See Eq. A-5 of Appendix A.) Choosing to work in meters, we have

$$I = \Sigma(\bar{I} + Ad^2)$$

$$= 2\left[\frac{1}{12}(0.25\ \text{m})(0.020\ \text{m})^3 + (0.25\ \text{m})(0.020\ \text{m})(0.160\ \text{m})^2\right]$$

$$+ \left[\frac{1}{12}(0.020\ \text{m})(0.300\ \text{m})^3\right]$$

$$= 301.3(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I};\quad \sigma_{\max} = \frac{22.5(10^3)\ \text{N}\cdot\text{m}(0.170\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = 12.7\ \text{MPa} \qquad \textit{Ans.}$$

A three-dimensional view of the stress distribution is shown in Fig. 6–26*d*. Notice how the stress at points B and D on the cross section develops a force that contributes a moment about the neutral axis that has the same direction as **M**. Specifically, at point B, $y_B = 150$ mm, and so as shown in Fig. 6–26*d*,

$$\sigma_B = -\frac{My_B}{I};\qquad \sigma_B = -\frac{22.5(10^3)\ \text{N}\cdot\text{m}(0.150\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = -11.2\ \text{MPa}$$

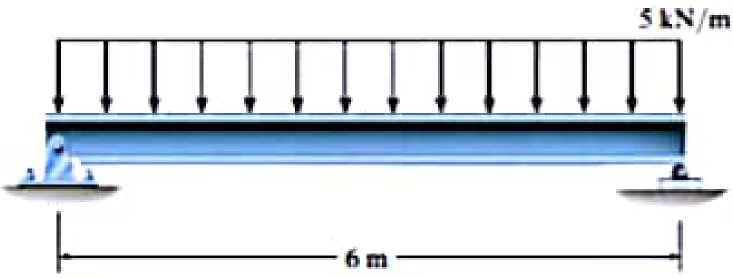
EXAMPLE 6.13

The beam shown in Fig. 6–27*a* has a cross-sectional area in the shape of a channel, Fig. 6–27*b*. Determine the maximum bending stress that occurs in the beam at section *a–a*.

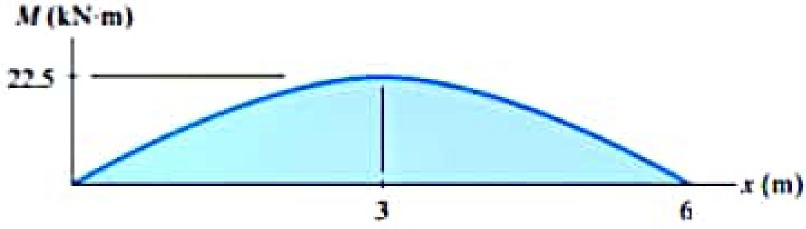
SOLUTION

EXAMPLE 6.12

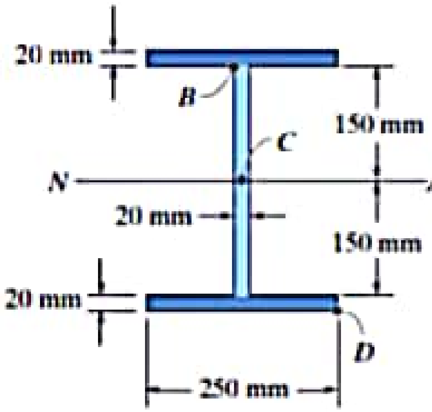
The simply supported beam in Fig. 6–26*a* has the cross-sectional area shown in Fig. 6–26*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



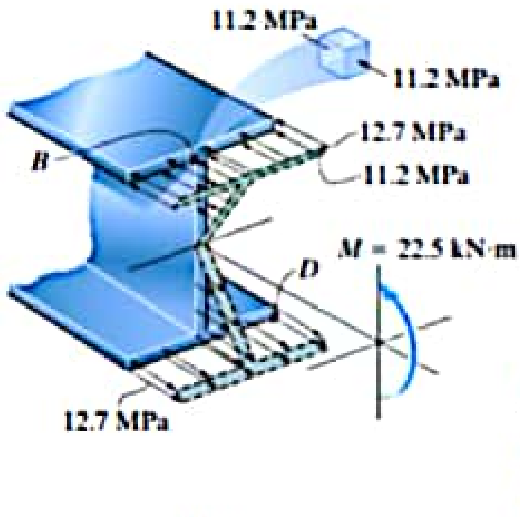
(a)



(c)



(b)



(d)

Fig. 6–26

SOLUTION

Maximum Internal Moment. The maximum internal moment in the beam, $M = 22.5\ \text{kN}\cdot\text{m}$, occurs at the center.

Section Property. By reasons of symmetry, the neutral axis passes through the centroid *C* at the midheight of the beam, Fig. 6–26*b*. The area is subdivided into the three parts shown, and the moment of inertia of each part is calculated about the neutral axis using the parallel-axis theorem. (See Eq. A-5 of Appendix A.) Choosing to work in meters, we have

$$I = \Sigma(\bar{I} + Ad^2)$$

$$= 2\left[\frac{1}{12}(0.25\text{ m})(0.020\text{ m})^3 + (0.25\text{ m})(0.020\text{ m})(0.160\text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.020\text{ m})(0.300\text{ m})^3\right]$$

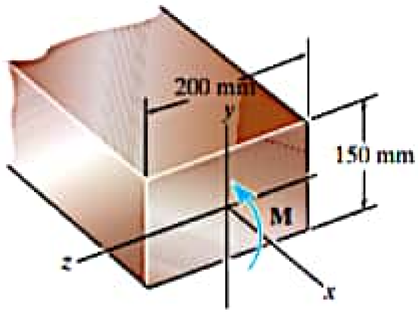
$$= 301.3(10^{-6})\text{ m}^4$$

$$\sigma_{max} = \frac{Mc}{I};\quad \sigma_{max} = \frac{22.5(10^3)\text{ N}\cdot\text{m}(0.170\text{ m})}{301.3(10^{-6})\text{ m}^4} = 12.7\text{ MPa} \quad \textit{Ans.}$$

A three-dimensional view of the stress distribution is shown in Fig. 6–26*d*. Notice how the stress at points *B* and *D* on the cross section develops a force that contributes a moment about the neutral axis that has the same direction as **M**. Specifically, at point B, $y_B = 150$ mm, and so as shown in Fig. 6–26*d*,

$$\sigma_B = -\frac{My_B}{I};\qquad \sigma_B = -\frac{22.5(10^3)\text{ N}\cdot\text{m}(0.150\text{ m})}{301.3(10^{-6})\text{ m}^4} = -11.2\text{ MPa}$$

6–47. A member having the dimensions shown is used to resist an internal bending moment of $M = 90\ \text{kN}\cdot\text{m}$. Determine the maximum stress in the member if the moment is applied (a) about the z axis (as shown) (b) about the y axis. Sketch the stress distribution for each case.



The moment of inertia of the cross-section about z and y axes are

$$I_z = \frac{1}{12}(0.2)(0.15^3) = 56.25(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.15)(0.2^3) = 0.1(10^{-3})\ \text{m}^4$$

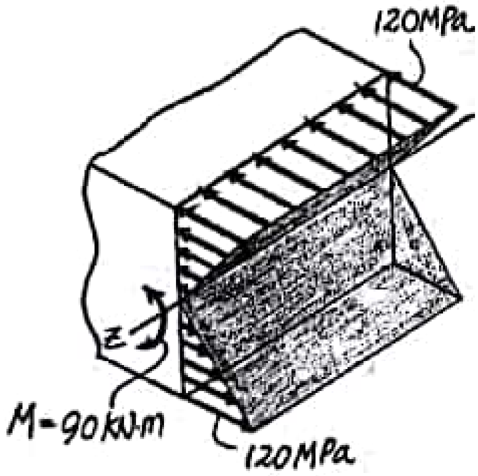
For the bending about z axis, $c = 0.075$ m.

$$\sigma_{\max} = \frac{Mc}{I_z} = \frac{90(10^3)\,(0.075)}{56.25\,(10^{-6})} = 120(10^6)\text{Pa} = 120\ \text{MPa} \qquad \textbf{Ans.}$$

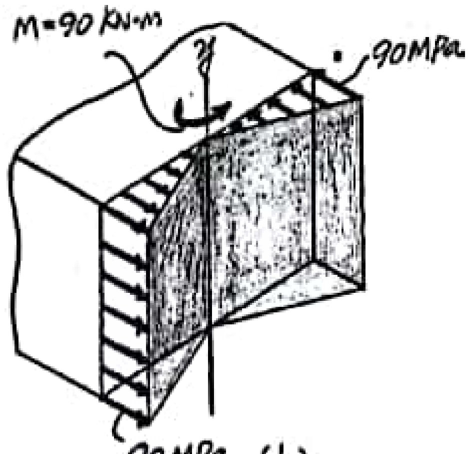
For the bending about y axis, $C = 0.1$ m.

$$\sigma_{\max} = \frac{Mc}{I_y} = \frac{90(10^3)\,(0.1)}{0.1\,(10^{-3})} = 90\,(10^6)\text{Pa} = 90\ \text{MPa} \qquad \textbf{Ans.}$$

The bending stress distribution for bending about z and y axes are shown in Fig. *a* and *b* respectively.

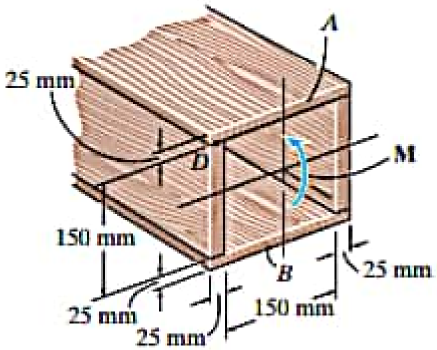


(a)



(b)

•6–53. Determine the moment **M** that should be applied to the beam in order to create a compressive stress at point D of $\sigma_D = 30$ MPa. Also sketch the stress distribution acting over the cross section and compute the maximum stress developed in the beam.



Section Property:

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.15)(0.15^3) = 91.14583(10^{-6})\ \text{m}^4$$

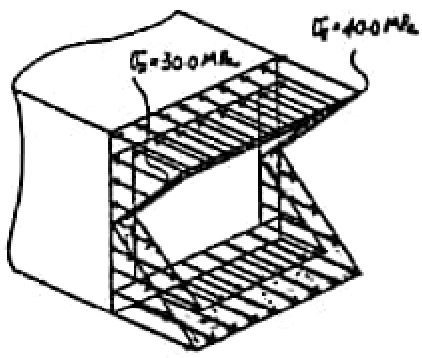
Bending Stress: Applying the flexure formula

$$\sigma = \frac{My}{I}$$

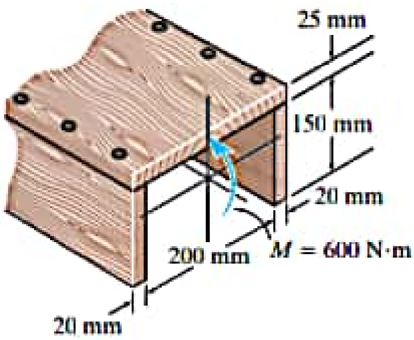
$$30(10^6) = \frac{M(0.075)}{91.14583(10^{-6})}$$

$$M = 36458\ \text{N}\cdot\text{m} = 36.5\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{36458(0.1)}{91.14583(10^{-6})} = 40.0\ \text{MPa} \qquad \textbf{Ans.}$$



6–54. The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.



$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2\,(0.1)(0.15)(0.2)}{0.24\,(0.025) + 2\,(0.15)(0.02)} = 0.05625\ \text{m}$$

$$I = \frac{1}{12}(0.24)(0.025^3) + (0.24)(0.025)(0.04375^2)$$

$$+ 2\left(\frac{1}{12}\right)(0.02)(0.15^3) + 2(0.15)(0.02)(0.04375^2)$$

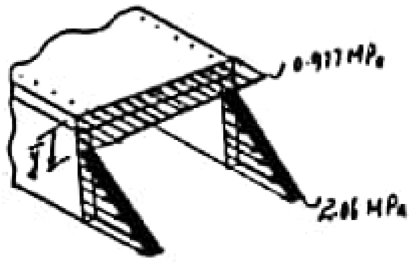
$$= 34.53125\,(10^{-6})\ \text{m}^4$$

$$\sigma_{max} = \sigma_B = \frac{Mc}{I}$$

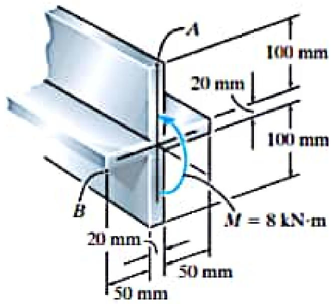
$$= \frac{600\,(0.175 - 0.05625)}{34.53125\,(10^{-6})}$$

$$= 2.06\ \text{MPa}$$ **Ans.**

$$\sigma_C = \frac{My}{I} = \frac{600\,(0.05625)}{34.53125\,(10^{-6})} = 0.977\ \text{MPa}$$

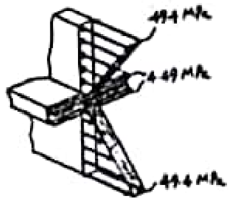


•6–57. The aluminum strut has a cross-sectional area in the form of a cross. If it is subjected to the moment $M = 8\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the beam, and sketch a three-dimensional view of the stress distribution acting over the entire cross-sectional area.



Section Property:

$$I = \frac{1}{12}(0.02)\left(0.22^3\right) + \frac{1}{12}(0.1)\left(0.02^3\right) = 17.8133\left(10^{-6}\right)\ \text{m}^4$$

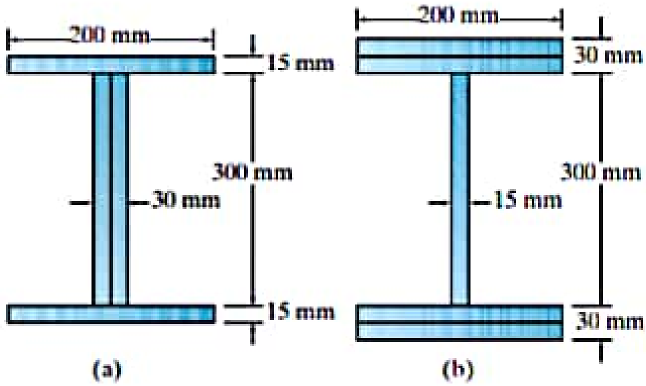


Bending Stress: Applying the flexure formula $\sigma_{\max} = \dfrac{Mc}{I}$ and $\sigma = \dfrac{My}{I}$.

$$\sigma_{\max} = \frac{8(10^3)(0.11)}{17.8133(10^{-6})} = 49.4\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y=0.01\text{m}} = \frac{8(10^3)(0.01)}{17.8133(10^{-6})} = 4.49\ \text{MPa}$$

•6–69. Two designs for a beam are to be considered. Determine which one will support a moment of $M = 150\ \text{kN}\cdot\text{m}$ with the least amount of bending stress. What is that stress?



Section Property:

For section (a)

$$I = \frac{1}{12}(0.2)\left(0.33^3\right) - \frac{1}{12}(0.17)(0.3)^3 = 0.21645(10^{-3})\ \text{m}^4$$

For section (b)

$$I = \frac{1}{12}(0.2)\left(0.36^3\right) - \frac{1}{12}(0.185)\left(0.3^3\right) = 0.36135(10^{-3})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{max} = \frac{Mc}{I}$

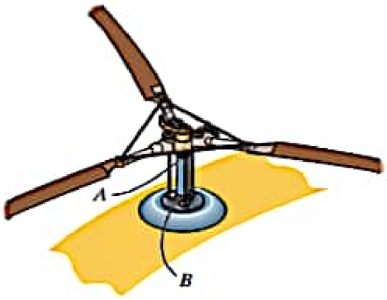
For section (a)

$$\sigma_{max} = \frac{150(10^3)(0.165)}{0.21645(10^{-3})} = 114.3\ \text{MPa}$$

For section (b)

$$\sigma_{max} = \frac{150(10^3)(0.18)}{0.36135(10^{-3})} = 74.72\ \text{MPa} = 74.7\ \text{MPa}$$ **Ans.**

5–51. The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 8$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{allow} = \frac{Tc}{J}$$

$$8(10^3) = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.3586 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4(11.0)(10^6)}$$

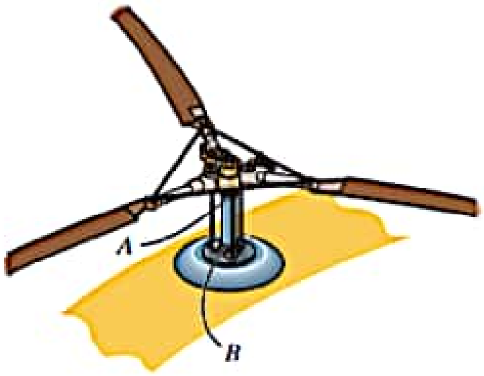
$$c = 0.967 \text{ in.}$$

Shear - stress failure controls the design.

$$d = 2c = 2\,(1.3586) = 2.72 \text{ in.}$$

Use $d = 2.75$ in. **Ans.**

•5–52. The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 10.5$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{allow} = 10.5(10)^3 = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.2408 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4(11.0)(10^6)}$$

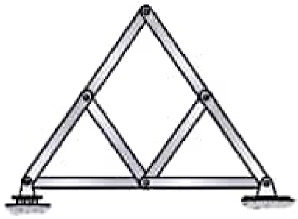
$$c = 0.967 \text{ in.}$$

Shear stress failure controls the design

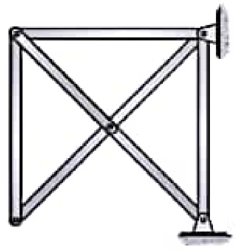
$$d = 2c = 2\,(1.2408) = 2.48 \text{ in.}$$

Use $d = 2.50$ in. **Ans.**

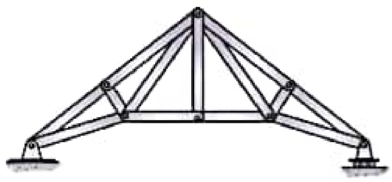
3–1. Classify each of the following trusses as statically determinate, statically indeterminate, or unstable. If indeterminate, state its degree.



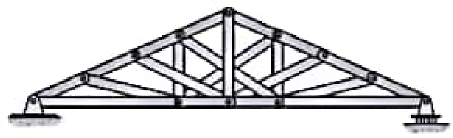
(a)



(b)



(c)



(d)

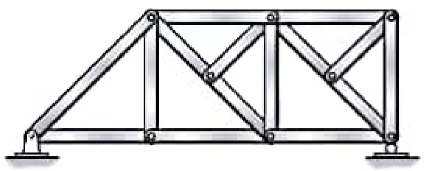
a) $b = 8,\ r = 3,\ j = 6$
$b + r = 2j$
$11 < 12$
Unstable. **Ans.**

b) $b = 7,\ r = 4,\ j = 5$
$b + r = 2j$
$11 > 10$
Statically indeterminate to 1°. **Ans.**

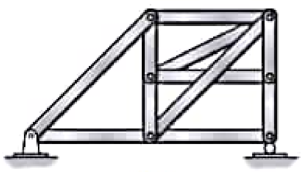
c) $b = 13,\ r = 3,\ j = 8$
$b + r = 2j$
$16 = 16$
Statically determinate. **Ans.**

d) $b = 21,\ r = 3,\ j = 12$
$b + r = 2j$
$24 = 24$
Statically determinate. **Ans.**

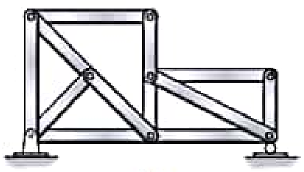
3–2. Classify each of the following trusses as stable, unstable, statically determinate, or statically indeterminate. If indeterminate, state its degree.



(a)



(b)



(c)

(a) $r = 3$
$b = 15$
$j = 9$

$3 + 15 = 9(2)$

Statically determinate. **Ans.**

(b) $r = 3$
$b = 11$
$j = 7$

$3 + 11 = 7(2)$

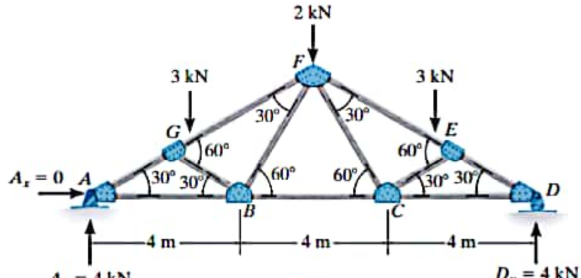
Statically determinate. **Ans.**

(c) $r = 3$
$b = 12$
$j = 8$

$3 + 12 < 8(2)$
$15 < 16$

Unstable. **Ans.**

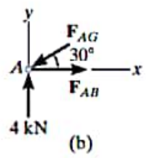
Determine the force in each member of the roof truss shown in the photo. The dimensions and loadings are shown in Fig. 3–20a. State whether the members are in tension or compression.



(a)

SOLUTION

Only the forces in half the members have to be determined, since the truss is symmetric with respect to *both* loading and geometry.

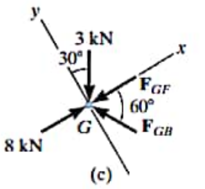


(b)

Joint A, Fig. 3–20b. We can start the analysis at joint A. Why? The free-body diagram is shown in Fig. 3–20b.

$$+\uparrow \Sigma F_y = 0; \quad 4 - F_{AG} \sin 30^\circ = 0 \qquad F_{AG} = 8 \text{ kN (C)} \qquad \textit{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} - 8 \cos 30^\circ = 0 \qquad F_{AB} = 6.928 \text{ kN (T)} \qquad \textit{Ans.}$$



(c)

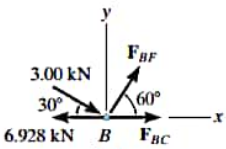
Joint G, Fig. 3–20c. In this case note how the orientation of the x, y axes avoids simultaneous solution of equations.

$$+\nwarrow \Sigma F_y = 0; \quad F_{GB} \sin 60^\circ - 3 \cos 30^\circ = 0$$

$$F_{GB} = 3.00 \text{ kN (C)} \qquad \textit{Ans.}$$

$$+\nearrow \Sigma F_x = 0; \quad 8 - 3 \sin 30^\circ - 3.00 \cos 60^\circ - F_{GF} = 0$$

$$F_{GF} = 5.00 \text{ kN (C)} \qquad \textit{Ans.}$$



(d)

Fig. 3–20

Joint B, Fig. 3–20d.

$$+\uparrow \Sigma F_y = 0; \quad F_{BF} \sin 60^\circ - 3.00 \sin 30^\circ = 0$$

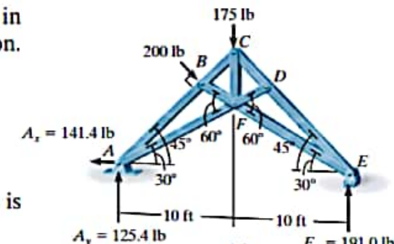
$$F_{BF} = 1.73 \text{ kN (T)} \qquad \textit{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} + 1.73 \cos 60^\circ + 3.00 \cos 30^\circ - 6.928 = 0$$

$$F_{BC} = 3.46 \text{ kN (T)} \qquad \textit{Ans.}$$

EXAMPLE 3.3

Determine the force in each member of the scissors truss shown in Fig. 3–21a. State whether the members are in tension or compression. The reactions at the supports are given.



(a)

SOLUTION

The truss will be analyzed in the following sequence:

Joint E, Fig. 3–21b. Note that simultaneous solution of equations is avoided by the x, y axes orientation.

$$+\nearrow \Sigma F_y = 0; \quad 191.0 \cos 30^\circ - F_{ED} \sin 15^\circ = 0$$

$$F_{ED} = 639.1 \text{ lb (C)} \qquad \textit{Ans.}$$

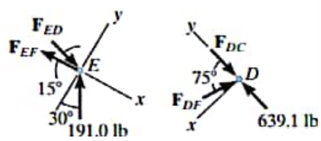
$$+\searrow \Sigma F_x = 0; \quad 639.1 \cos 15^\circ - F_{EF} - 191.0 \sin 30^\circ = 0$$

$$F_{EF} = 521.8 \text{ lb (T)} \qquad \textit{Ans.}$$

Joint D, Fig. 3–21c.

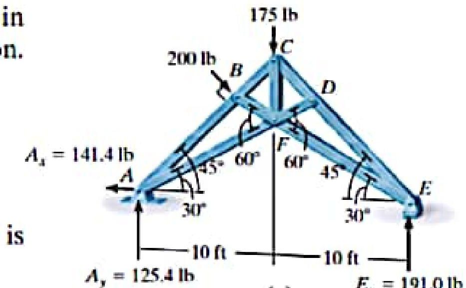
$$+\swarrow \Sigma F_x = 0; \quad -F_{DF} \sin 75^\circ = 0 \qquad F_{DF} = 0 \qquad \textit{Ans.}$$

$$+\nwarrow \Sigma F_y = 0; \quad -F_{DC} + 639.1 = 0 \qquad F_{DC} = 639.1 \text{ lb (C)} \qquad \textit{Ans.}$$



EXAMPLE 3.3

Determine the force in each member of the scissors truss shown in Fig. 3–21*a*. State whether the members are in tension or compression. The reactions at the supports are given.



(a)

SOLUTION

The truss will be analyzed in the following sequence:

Joint *E*, Fig. 3–21*b*. Note that simultaneous solution of equations is avoided by the *x*, *y* axes orientation.

$+\nearrow\Sigma F_y = 0;\quad 191.0\cos 30^\circ - F_{ED}\sin 15^\circ = 0$

$F_{ED} = 639.1 \text{ lb (C)}$ *Ans.*

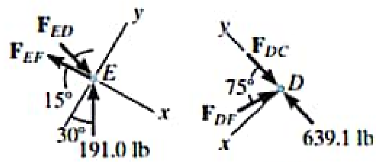
$+\searrow\Sigma F_x = 0;\quad 639.1\cos 15^\circ - F_{EF} - 191.0\sin 30^\circ = 0$

$F_{EF} = 521.8 \text{ lb (T)}$ *Ans.*

Joint *D*, Fig. 3–21*c*.

$+\swarrow\Sigma F_x = 0;\quad -F_{DF}\sin 75^\circ = 0 \qquad F_{DF} = 0$ *Ans.*

$+\nwarrow\Sigma F_y = 0;\quad -F_{DC} + 639.1 = 0 \qquad F_{DC} = 639.1 \text{ lb (C)}$ *Ans.*



(b) (c)

Joint *C*, Fig. 3–21*d*.

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{CB}\sin 45^\circ - 639.1\sin 45^\circ = 0$

$F_{CB} = 639.1 \text{ lb (C)}$ *Ans.*

$+\uparrow\Sigma F_y = 0;\quad -F_{CF} - 175 + 2(639.1)\cos 45^\circ = 0$

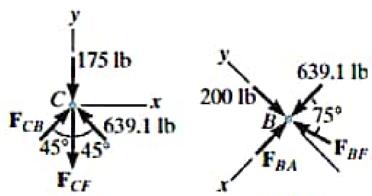
$F_{CF} = 728.8 \text{ lb (T)}$ *Ans.*

Joint *B*, Fig. 3–21*e*.

$+\nwarrow\Sigma F_y = 0;\quad F_{BF}\sin 75^\circ - 200 = 0 \qquad F_{BF} = 207.1 \text{ lb (C)}$ *Ans.*

$+\swarrow\Sigma F_x = 0;\quad 639.1 + 207.1\cos 75^\circ - F_{BA} = 0$

$F_{BA} = 692.7 \text{ lb (C)}$ *Ans.*



(d) (e)

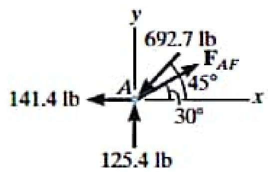
Joint *A*, Fig. 3–21*f*.

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{AF}\cos 30^\circ - 692.7\cos 45^\circ - 141.4 = 0$

$F_{AF} = 728.9 \text{ lb (T)}$ *Ans.*

$+\uparrow\Sigma F_y = 0;\quad 125.4 - 692.7\sin 45^\circ + 728.9\sin 30^\circ = 0$ check

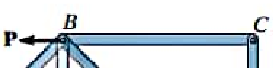
Notice that since the reactions have been calculated, a further check of the calculations can be made by analyzing the last joint *F*. Try it and find out.



(f)

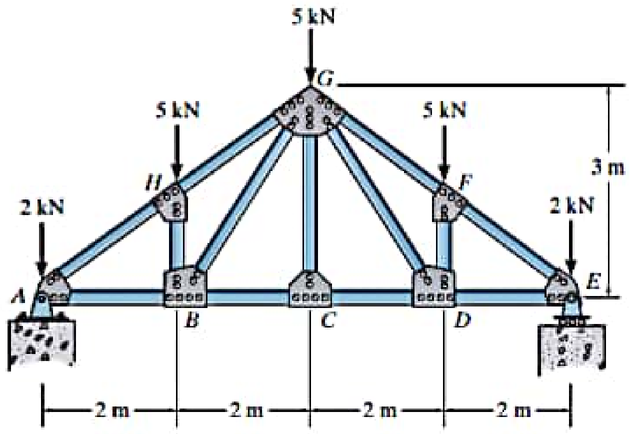
Fig. 3–21

3.4 Zero-Force Members



Truss analysis using the method of joints is greatly simplified if one is able to first determine those members that support *no loading*. These

3–21. The *Howe* truss is subjected to the loading shown. Determine the forces in members *GF*, *CD*, and *GC*. State if the members are in tension or compression. Assume all members are pin connected.

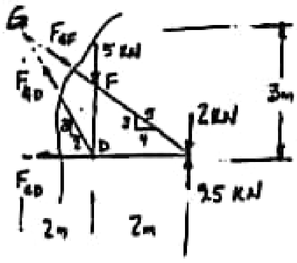


$\circlearrowleft + \sum M_G = 0;\quad F_{CD}(3) - 9.5(4) + 5(2) + 2(4) = 0$

$F_{CD} = 6.67 \text{ kN (T)}$ **Ans.**

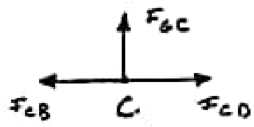
$\circlearrowleft + \sum M_D = 0;\quad -9.5(2) + 2(2) + \frac{4}{5}(1.5)\,F_{GF} = 0$

$F_{GF} = 12.5 \text{ kN (C)}$ **Ans.**

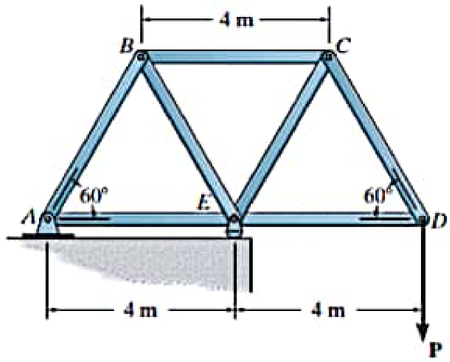


Joint *C*:

$F_{GC} = 0$ **Ans.**



3–7. Determine the force in each member of the truss. State whether the members are in tension or compression. Set $P = 8$ kN.

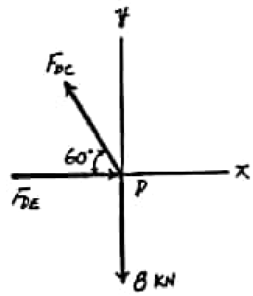


Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint *D*:

$+\uparrow\sum F_y = 0;\quad F_{DC}\sin 60° - 8 = 0$

$F_{DC} = 9.238\text{ kN (T)} = 9.24\text{ kN (T)}$ **Ans.**



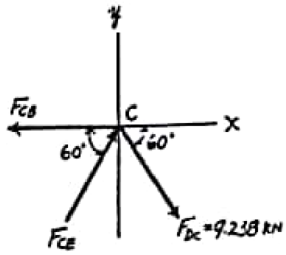
$\overset{+}{\rightarrow}\sum F_x = 0;\quad F_{DE} - 9.238\cos 60° = 0$

$F_{DE} = 4.619\text{ kN (C)} = 4.62\text{ kN (C)}$ **Ans.**

Joint *C*:

$+\uparrow\sum F_y = 0;\quad F_{CE}\sin 60° - 9.328\sin 60° = 0$

$F_{CE} = 9.238\text{ kN (C)} = 9.24\text{ kN (C)}$ **Ans.**



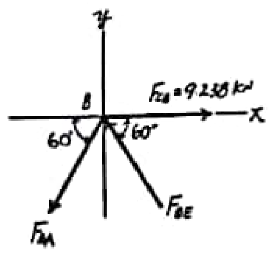
$\overset{+}{\rightarrow}\sum F_x = 0;\quad 2(9.238\cos 60°) - F_{CB} = 0$

$F_{CB} = 9.238\text{ kN (T)} = 9.24\text{ kN (T)}$ **Ans.**

Joint *B*:

$+\uparrow\sum F_y = 0;\quad F_{BE}\sin 60° - F_{BA}\sin 60° = 0$

$F_{BE} = F_{BA} = F$



$\overset{+}{\rightarrow}\sum F_x = 0;\quad 9.238 - 2F\cos 60° = 0$

$F = 9.238$ kN

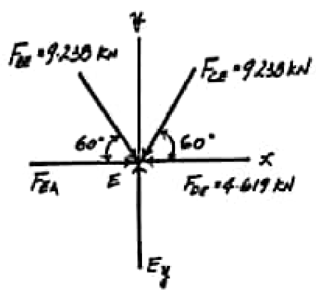
Thus, $F_{BE} = 9.24\text{ kN (C)}\quad F_{BA} = 9.24\text{ kN (T)}$ **Ans.**

Joint *E*:

$+\uparrow\sum F_y = 0;\quad E_y - 2(9.238\sin 60°) = 0\qquad E_y = 16.0\text{ kN}$

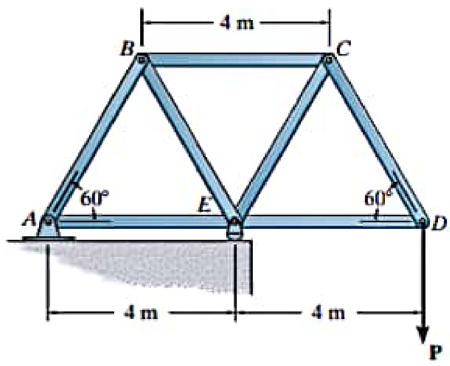
$\overset{+}{\rightarrow}\sum F_x = 0;\quad F_{BA} + 9.238\cos 60° - 9.238\cos 60° - 4.619 = 0$

$F_{EA} = 4.62\text{ kN (C)}$ **Ans.**



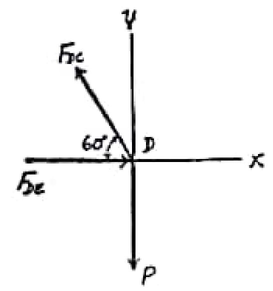
Note: The support reactions A_x and A_y can be determined by analyzing Joint A using the results obtained above.

*3–8. If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force P that can be supported at joint D.



Method of Joints: In this case, the support reactions are not required for determining the member forces.

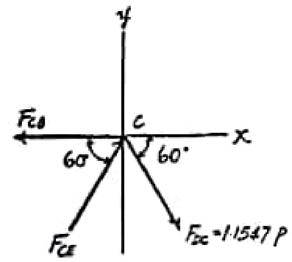
Joint *D*:



$+\uparrow\sum F_y = 0;\quad F_{DC}\sin 60° - P = 0\qquad F_{DC} = 1.1547P\ (T)$

$\xrightarrow{+}\sum F_x = 0;\quad F_{DE} - 1.1547P\cos 60° = 0\qquad F_{DE} = 0.57735P\ (C)$

Joint *C*:

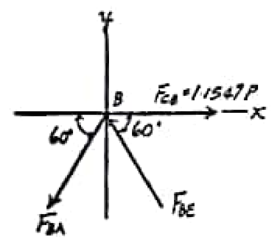


$+\uparrow\sum F_y = 0;\quad F_{CE}\sin 60° - 1.1547P\sin 60° = 0$

$F_{CE} = 1.1547P\ \ (C)$

$\xrightarrow{+}\sum F_x = 0;\quad 2(1.1547P\cos 60° - F_{CB} = 0\qquad F_{CB} = 1.1547P\ \ (T)$

Joint *B*:

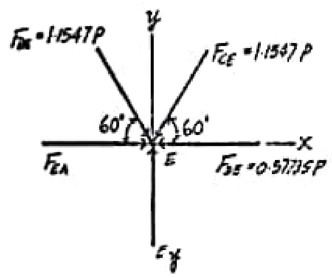


$+\uparrow\sum F_y = 0;\quad F_{BE}\sin 60° - F_{BE}\sin 60° = 0\qquad F_{BE} = F_{BA} = F$

$\xrightarrow{+}\sum F_x = 0;\quad 1.1547P - 2F\cos 60° = 0\qquad F = 1.1547P$

Thus, $\quad F_{BE} = 1.1547P\ (C)\qquad F_{BA} = 1.1547P\ \ (T)$

Joint *E*:



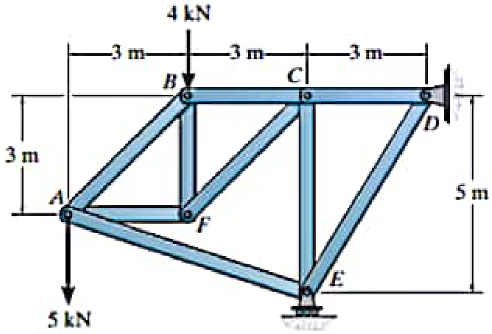
$\xrightarrow{+}\sum F_x = 0;\quad F_{EA} + 1.1547P\cos 60° - 1.1547P\cos 60° - 0.57735P = 0$

$F_{EA} = 0.57735P\ (C)$

From the above analysis, the maximum compression and tension in the truss members is $1.1547P$. For this case, compression controls which requires

$$1.1547P = 6$$
$$P = 5.20\text{ kN}\qquad \textbf{Ans.}$$

3–13. Determine the force in each member of the truss and state if the members are in tension or compression.



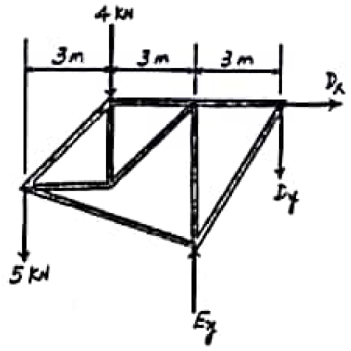
Support Reactions:

$\circlearrowleft + \sum M_D = 0;\quad 4(6) + 5(9) - E_y(3) = 0 \qquad E_y = 23.0\text{ kN}$

$+\uparrow \sum F_y = 0;\quad 23.0 - 4 - 5 - D_y = 0 \qquad D_y = 14.0\text{ kN}$

$\overset{+}{\rightarrow} \sum F_x = 0;\qquad D_x = 0$

Method of Joints:



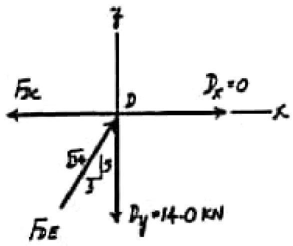
Joint *D*:

$+\uparrow \sum F_y = 0;\qquad F_{DE}\left(\frac{5}{\sqrt{34}}\right) - 14.0 = 0$

$F_{DE} = 16.33\text{ kN (C)} = 16.3\text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;\qquad 16.33\left(\frac{3}{\sqrt{34}}\right) - F_{DC} = 0$

$F_{DC} = 8.40\text{ kN (T)}$ **Ans.**



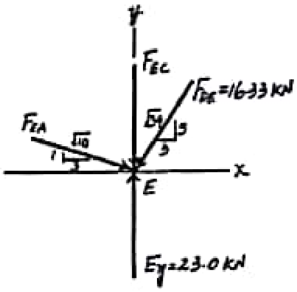
Joint *E*:

$\overset{+}{\rightarrow} \sum F_x = 0;\qquad F_{EA}\left(\frac{3}{\sqrt{10}}\right) - 16.33\left(\frac{3}{\sqrt{34}}\right) = 0$

$F_{EA} = 8.854\text{ kN (C)} = 8.85\text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0;\qquad 23.0 - 16.33\left(\frac{5}{\sqrt{34}}\right) - 8.854\left(\frac{1}{\sqrt{10}}\right) - F_{EC} = 0$

$F_{EC} = 6.20\text{ kN (C)}$ **Ans.**



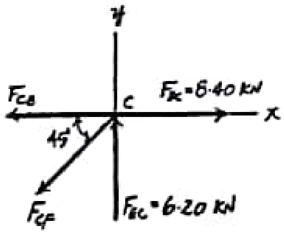
Joint *C*:

$+\uparrow \sum F_y = 0;\qquad 6.20 - F_{CF}\sin 45^\circ = 0$

$F_{CF} = 8.768\text{ kN (T)} = 8.77\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;\qquad 8.40 - 8.768\cos 45^\circ - F_{CB} = 0$

$F_{CB} = 2.20\text{ kN (T)}$ **Ans.**



Joint *B*:

$\overset{+}{\rightarrow} \sum F_x = 0;\qquad 2.20 - F_{BA}\cos 45^\circ = 0$

$F_{BA} = 3.111\text{ kN (T)} = 3.11\text{ kN (T)}$ **Ans.**

$+\uparrow \sum F_y = 0;\qquad F_{BF} - 4 - 3.111\sin 45^\circ = 0$

$F_{BF} = 6.20\text{ kN (C)}$ **Ans.**

Joint *F*:

$+\uparrow \sum F_y = 0;\qquad 8.768\sin 45^\circ - 6.20 = 0\text{ (Check!)}$

$\overset{+}{\rightarrow} \sum F_x = 0;\qquad 8.768\cos 45^\circ - F_{FA} = 0$

$F_{FA} = 6.20\text{ kN (T)}$ **Ans.**

3.5 The Method of Sections

If the forces in only a few members of a truss are to be found, the method of sections generally provides the most direct means of obtaining these forces. The *method of sections* consists of passing an *imaginary section* through the truss, thus cutting it into two parts. Provided the entire truss is in equilibrium, each of the two parts must also be in equilibrium; and as a result, the three equations of equilibrium may be applied to either one of these two parts to determine the member forces at the "cut section."

When the method of sections is used to determine the force in a particular member, a decision must be made as to how to "cut" or section the truss. Since only *three* independent equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$) can be applied to the isolated portion of the truss, try to select a section that, in general, passes through not more than *three* members in which the forces are unknown. For example, consider the truss in Fig. 3–25*a*. If the force in member *GC* is to be determined, section *aa* will be appropriate. The free-body diagrams of the two parts are shown in Figs. 3–25*b* and 3–25*c*. In particular, note that the line of action of each force in a sectioned member is specified from the *geometry* of the truss, since the force in a member passes along the axis of the member. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part—Newton's third law. As shown, members assumed to be in *tension* (*BC* and *GC*) are subjected to a "pull," whereas the member in *compression* (*GF*) is subjected to a "push."

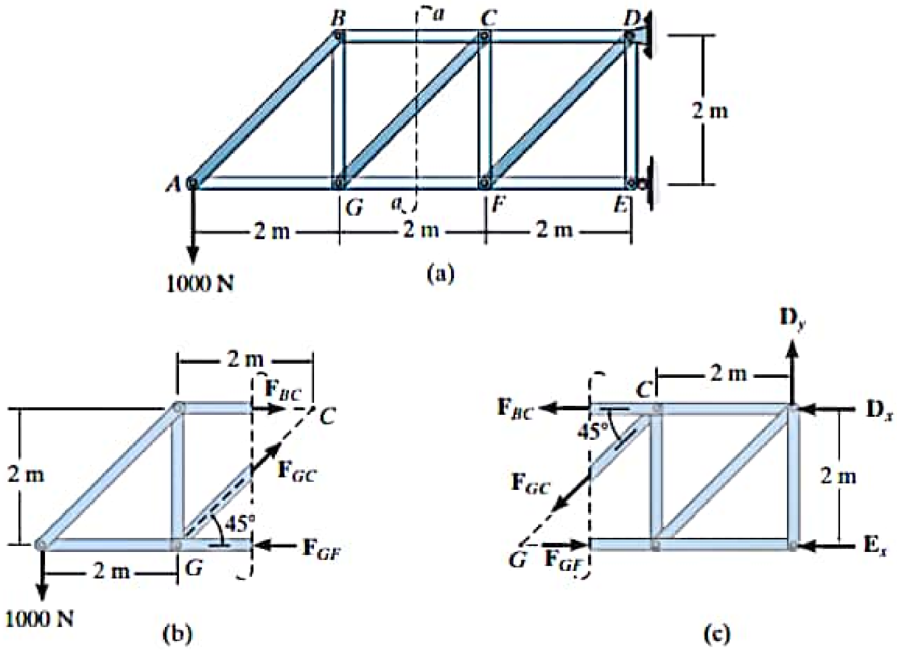
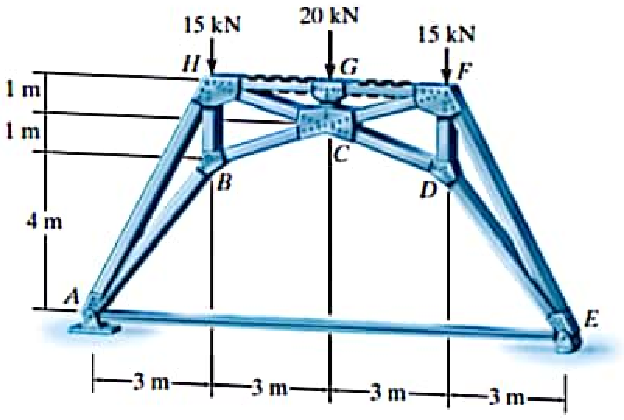
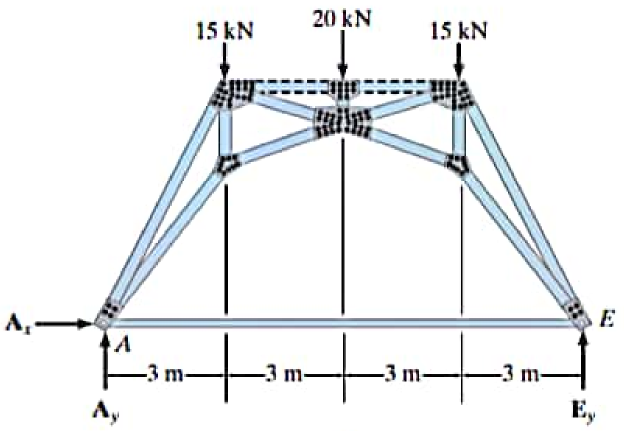


Fig. 3–25

The three-hinged tied arch is subjected to the loading shown in Fig. 5–11*a*. Determine the force in members *CH* and *CB*. The dashed member *GF* of the truss is intended to carry no force.



(a)



(b)

Fig. 5–11

SOLUTION

The support reactions can be obtained from a free-body diagram of the entire arch, Fig. 5–11*b*:

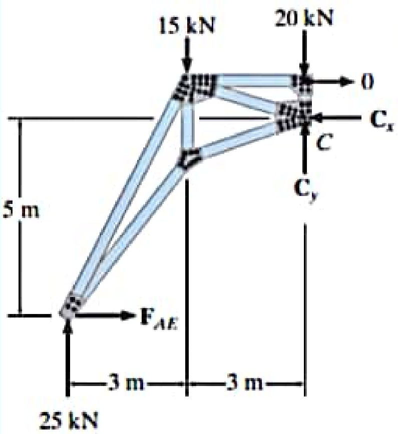
$$\curvearrowleft + \Sigma M_A = 0; \quad E_y(12\text{ m}) - 15\text{ kN}(3\text{ m}) - 20\text{ kN}(6\text{ m}) - 15\text{ kN}(9\text{ m}) = 0$$

$$E_y = 25\text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 15\text{ kN} - 20\text{ kN} - 15\text{ kN} + 25\text{ kN} = 0$$

$$A_y = 25\text{ kN}$$



(c)

The force components acting at joint *C* can be determined by considering the free-body diagram of the left part of the arch, Fig. 5–11*c*. First, we determine the force:

$$\curvearrowleft + \Sigma M_C = 0; \quad F_{AE}(5\text{ m}) - 25\text{ kN}(6\text{ m}) + 15\text{ kN}(3\text{ m}) = 0$$

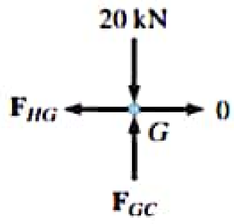
$$F_{AE} = 21.0\text{ kN}$$

Then,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -C_x + 21.0 \text{ kN} = 0, \quad C_x = 21.0 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad 25 \text{ kN} - 15 \text{ kN} - 20 \text{ kN} + C_y = 0, \quad C_y = 10 \text{ kN}$

To obtain the forces in CH and CB, we can use the method of joints as follows:

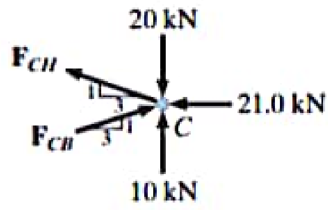


(d)

Joint G; Fig. 5–11d,

$+\uparrow \Sigma F_y = 0; \quad F_{GC} - 20 \text{ kN} = 0$

$F_{GC} = 20 \text{ kN (C)}$



(e)

Joint C; Fig. 5–11e,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{CB}\left(\frac{3}{\sqrt{10}}\right) - 21.0 \text{ kN} - F_{CH}\left(\frac{3}{\sqrt{10}}\right) = 0$

$+\uparrow \Sigma F_y = 0; \quad F_{CB}\left(\frac{1}{\sqrt{10}}\right) + F_{CH}\left(\frac{1}{\sqrt{10}}\right) - 20 \text{ kN} + 10 \text{ kN} = 0$

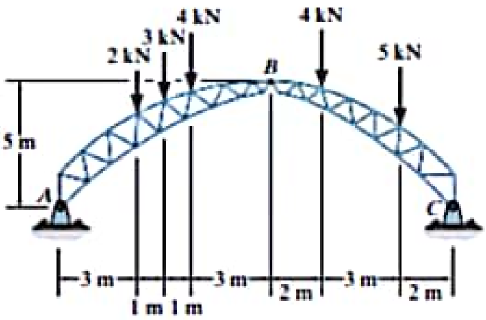
Thus,

$F_{CB} = 26.9 \text{ kN (C)}$ *Ans.*

$F_{CH} = 4.74 \text{ kN (T)}$ *Ans.*

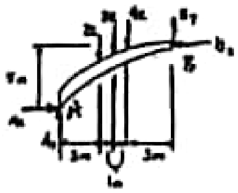
Note: Tied arches are sometimes used for bridges. Here the deck is supported by suspender bars that transmit their load to the arch. The deck is in tension so that it supports the actual thrust or horizontal force at the ends of the arch.

5–22. Determine the resultant forces at the pins A, B, and C of the three-hinged arched roof truss.



Member AB:

$\curvearrowleft + \sum M_A = 0;$ $\quad B_x(5) + B_y(8) - 2(3) - 3(4) - 4(5) = 0$

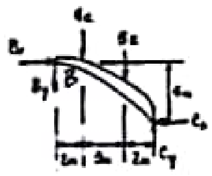


Member BC:

$\curvearrowleft + \sum M_C = 0;$ $\quad -B_x(5) + B_y(7) + 5(2) + 4(5) = 0$

Soving,

$B_y = 0.533 \text{ k}, \qquad B_x = 6.7467 \text{ k}$



Member AB:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad A_x = 6.7467 \text{ k}$

$+\uparrow \sum F_y = 0;$ $\quad A_y - 9 + 0.533 = 0$

$A_y = 8.467 \text{ k}$

Member BC:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad C_x = 6.7467 \text{ k}$

$+\uparrow \sum F_y = 0;$ $\quad C_y - 9 + 0.533 = 0$

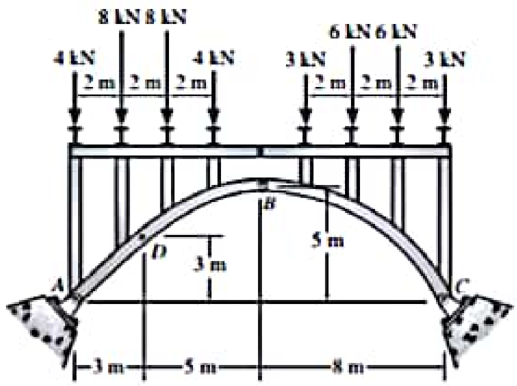
$C_y = 9.533 \text{ k}$

$F_B = \sqrt{(0.533)^2 + (6.7467)^2} = 6.77 \text{ k}$ **Ans.**

$F_A = \sqrt{(6.7467)^2 + (8.467)^2} = 10.8 \text{ k}$ **Ans.**

$F_C = \sqrt{(6.7467)^2 + (9.533)^2} = 11.7 \text{ k}$ **Ans.**

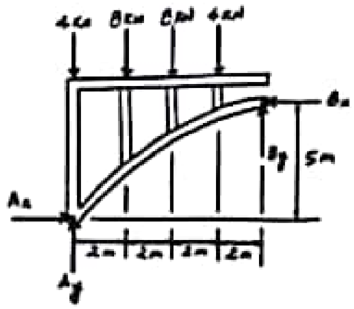
5–23. The three-hinged spandrel arch is subjected to the loading shown. Determine the internal moment in the arch at point D.



Member AB:

$\zeta + \sum M_A = 0;\qquad B_x(5) + B_y(8) - 8(2) - 8(4) - 4(6) = 0$

$$B_x + 1.6B_y = 14.4 \qquad (1)$$



Member CB:

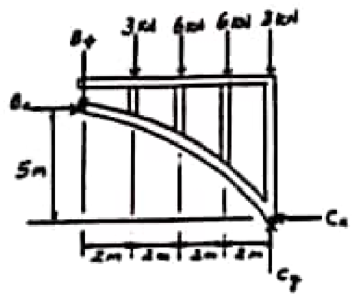
$\zeta + \sum M_C = 0;\qquad B_{(y)}(8) - B_x(5) + 6(2) + 6(4) + 3(6) = 0$

$$-B_x + 1.6B_y = -10.8 \qquad (2)$$

Soving Eqs. (1) and (2) yields:

$$B_y = 1.125 \text{ kN}$$

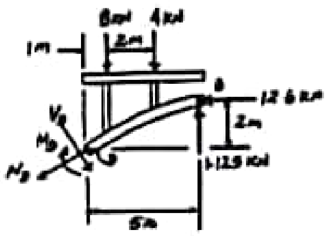
$$B_x = 12.6 \text{ kN}$$



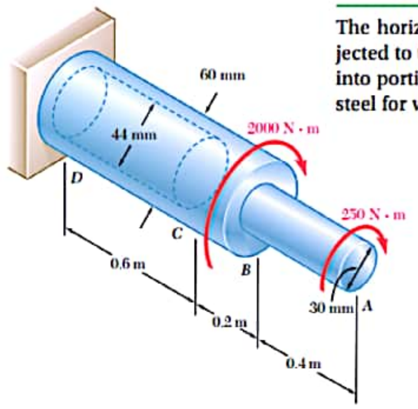
Segment BD:

$\zeta + \sum M_D = 0;\qquad -M_D + 12.6(2) + 1.125(5) - 8(1) - 4(3) = 0$

$$M_D = 10.825 \text{ kN}\cdot\text{m} = 10.8 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Sample Problem 3.3



The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which $G = 77$ GPa, determine the angle of twist at end A.

STRATEGY: Use free-body diagrams to determine the torque in each shaft segment AB, BC, and CD. Then use Eq. (3.16) to determine the angle of twist at end A.

MODELING:

Passing a section through the shaft between A and B (Fig. 1), we find

$$\Sigma M_x = 0: \qquad (250\ \text{N·m}) - T_{AB} = 0 \qquad T_{AB} = 250\ \text{N·m}$$

Passing now a section between B and C (Fig. 2) we have

$$\Sigma M_x = 0: (250\ \text{N·m}) + (2000\ \text{N·m}) - T_{BC} = 0 \qquad T_{BC} = 2250\ \text{N·m}$$

Since no torque is applied at C,

$$T_{CD} = T_{BC} = 2250\ \text{N·m}$$

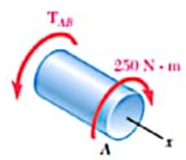


Fig. 1 Free-body diagram for finding internal torque in segment AB.

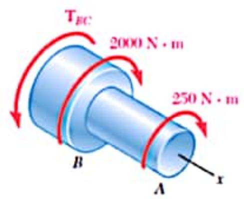


Fig. 2 Free-body diagram for finding internal torque in segment BC.

(continued)

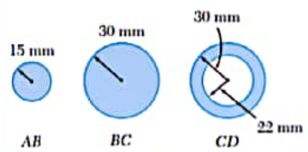


Fig. 3 Dimensions for three cross sections of shaft.

ANALYSIS:

Polar Moments of Inertia

Using Fig. 3

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015\ \text{m})^4 = 0.0795 \times 10^{-6}\ \text{m}^4$$

$$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030\ \text{m})^4 = 1.272 \times 10^{-6}\ \text{m}^4$$

$$J_{CD} = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.030\ \text{m})^4 - (0.022\ \text{m})^4] = 0.904 \times 10^{-6}\ \text{m}^4$$

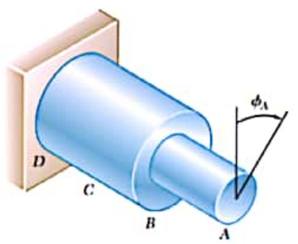


Fig. 4 Representation of angle of twist at end A.

Angle of Twist. From Fig. 4, using Eq. (3.16) and recalling that $G = 77$ GPa for the entire shaft, we have

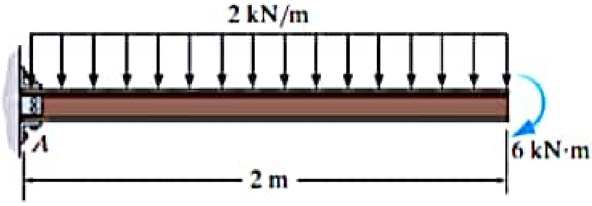
$$\phi_A = \sum_i \frac{T_i L_i}{J_i G} = \frac{1}{G}\left(\frac{T_{AB}L_{AB}}{J_{AB}} + \frac{T_{BC}L_{BC}}{J_{BC}} + \frac{T_{CD}L_{CD}}{J_{CD}}\right)$$

$$\phi_A = \frac{1}{77\ \text{GPa}}\left[\frac{(250\ \text{N·m})(0.4\ \text{m})}{0.0795 \times 10^{-6}\ \text{m}^4} + \frac{(2250)(0.2)}{1.272 \times 10^{-6}} + \frac{(2250)(0.6)}{0.904 \times 10^{-6}}\right]$$

$$= 0.01634 + 0.00459 + 0.01939 = 0.0403\ \text{rad}$$

$$\phi_A = (0.0403\ \text{rad})\frac{360°}{2\pi\ \text{rad}} \qquad \phi_A = 2.31°$$

***6–4.** Draw the shear and moment diagrams for the cantilever beam.



The free-body diagram of the beam's right segment sectioned through an arbitrary point shown in Fig. *a* will be used to write the shear and moment equations of the beam.

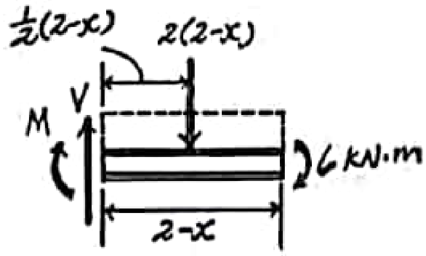
$+\uparrow\Sigma F_y = 0;\qquad V - 2(2-x) = 0 \qquad V = \{4 - 2x\}\ \text{kN},\quad (1)$

$\circlearrowleft + \Sigma M = 0;\ -M - 2(2-x)\left[\frac{1}{2}(2-x)\right] - 6 = 0 \quad M = \{-x^2 + 4x - 10\}\text{kN}\cdot\text{m},(2)$

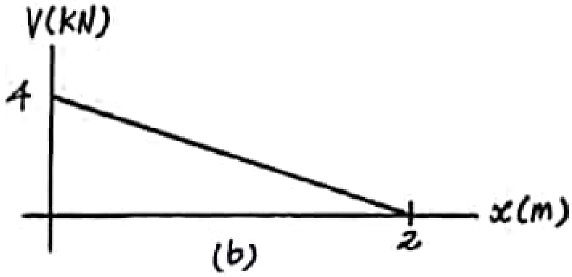
The shear and moment diagrams shown in Figs. *b* and *c* are plotted using Eqs. (1) and (2), respectively. The value of the shear and moment at $x = 0$ is evaluated using Eqs. (1) and (2).

$$V\big|_{x=0} = 4 - 2(0) = 4\ \text{kN}$$

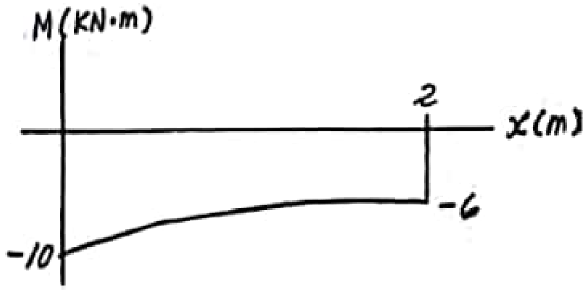
$$M\big|_{x=0} = \left[-0 + 4(0) - 10\right] = -10\text{kN}\cdot\text{m}$$



(a)



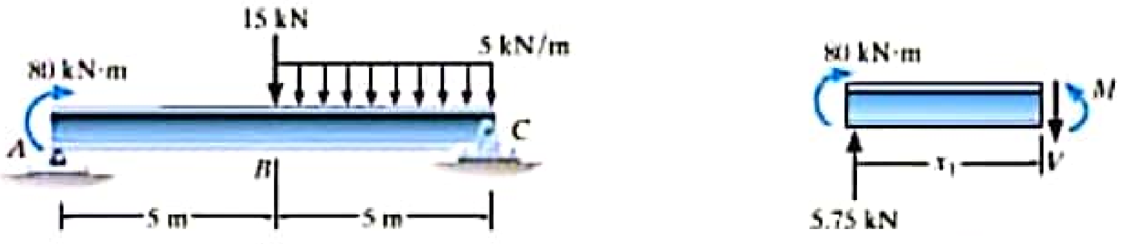
(b)



(c)

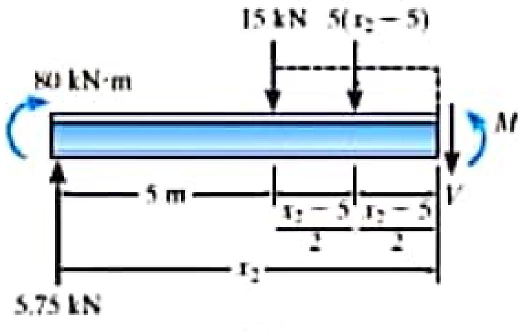
EXAMPLE 6.4

Draw the shear and moment diagrams for the beam shown in Fig. 6–7*a*.



SOLUTION

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. 6–7*d*.



Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of x must be considered in order to describe the shear and moment functions for the entire beam.

$0 \le x_1 < 5$ m, Fig. 6–7*b*:

$$+\uparrow \Sigma F_y = 0; \qquad 5.75 \text{ kN} - V = 0$$

$$V = 5.75 \text{ kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_1 + M = 0$$

$$M = (5.75x_1 + 80) \text{ kN}\cdot\text{m} \qquad (2)$$

$5 \text{ m} < x_2 \le 10$ m, Fig. 6–7*c*:

$$+\uparrow \Sigma F_y = 0; \quad 5.75 \text{ kN} - 15 \text{ kN} - 5 \text{ kN/m}(x_2 - 5 \text{ m}) - V = 0$$

$$V = (15.75 - 5x_2) \text{ kN} \qquad (3)$$

$$\circlearrowleft + \Sigma M = 0; \quad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_2 + 15 \text{ kN}(x_2 - 5 \text{ m})$$

$$+ 5 \text{ kN/m}(x_2 - 5 \text{ m})\left(\frac{x_2 - 5 \text{ m}}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5) \text{ kN}\cdot\text{m} \qquad (4)$$

These results can be checked in part by noting that $w = dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN·m; when $x_2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–7*d*.

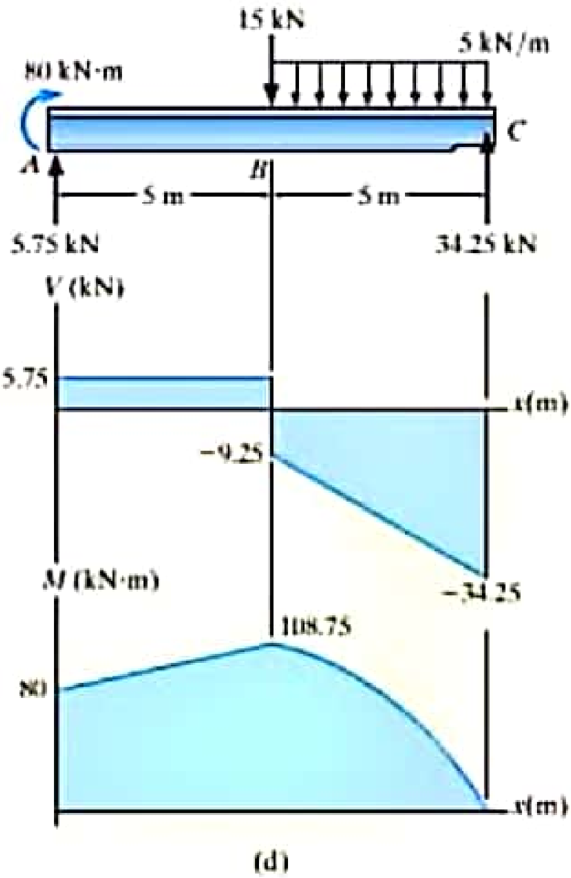
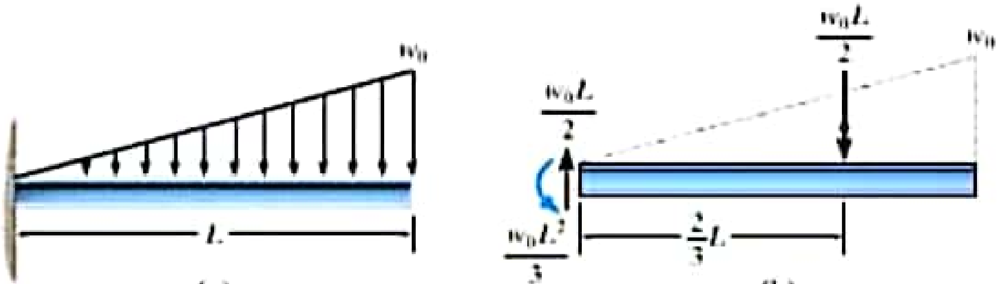


Fig. 6–7

Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. 6–7*d*.

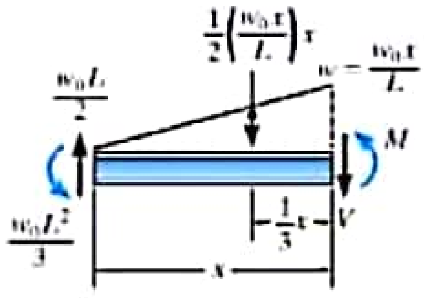


(a) (b)

SOLUTION

Support Reactions. The distributed load is replaced by its resultant force and the reactions have been determined as shown in Fig. 6–5*b*.

Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–5*c*. Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram. Thus,



(c)

$$+\uparrow\Sigma F_y = 0; \qquad \frac{w_0L}{2} - \frac{1}{2}\left(\frac{w_0x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \qquad (1)$$

$$\circlearrowleft+\Sigma M = 0; \qquad \frac{w_0L^2}{3} - \frac{w_0L}{2}(x) + \frac{1}{2}\left(\frac{w_0x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

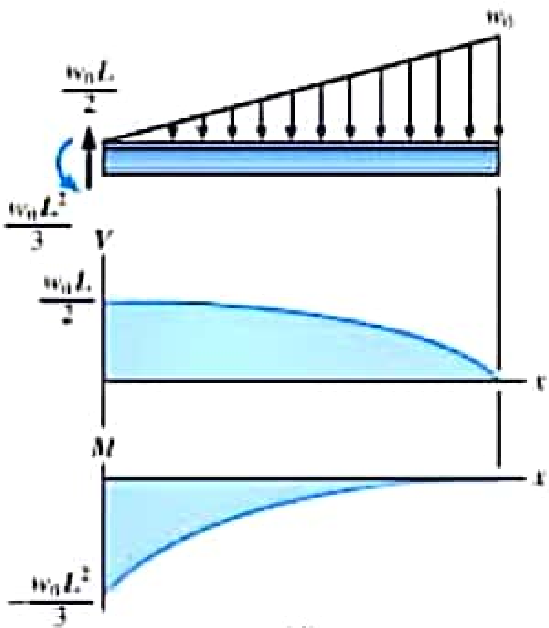
$$M = \frac{w_0}{6L}(-2L^3 + 3L^2x - x^3) \qquad (2)$$

These results can be checked by applying Eqs. 6-1 and 6-2 of Sec. 6.2, that is,

$$w = \frac{dV}{dx} = \frac{w_0}{2L}(0 - 2x) = -\frac{w_0x}{L} \qquad \text{OK}$$

$$V = \frac{dM}{dx} = \frac{w_0}{6L}(0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$

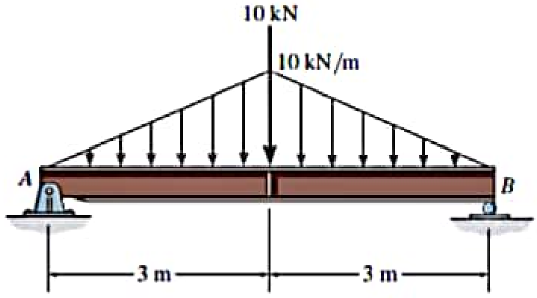
Shear and Moment Diagrams. The graphs of Eqs. 1 and 2 are shown in Fig. 6–5*d*.



(d)

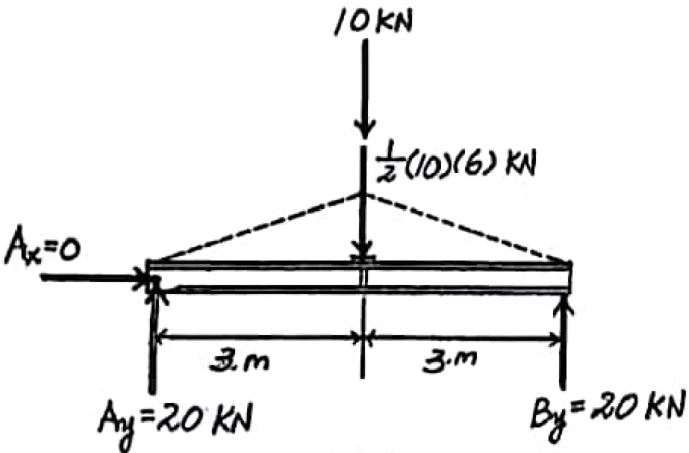
Fig. 6-5

***6–20.** Draw the shear and moment diagrams for the simply supported beam.

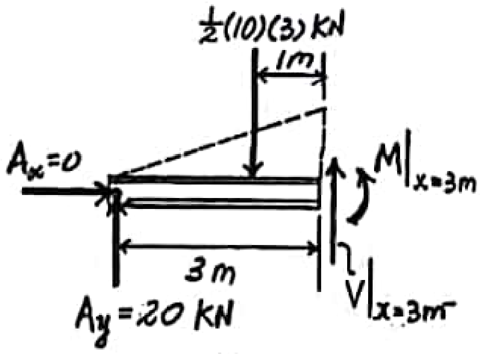


Since the area under the curved shear diagram can not be computed directly, the value of the moment at $x = 3$ m will be computed using the method of sections. By referring to the free-body diagram shown in Fig. *b*,

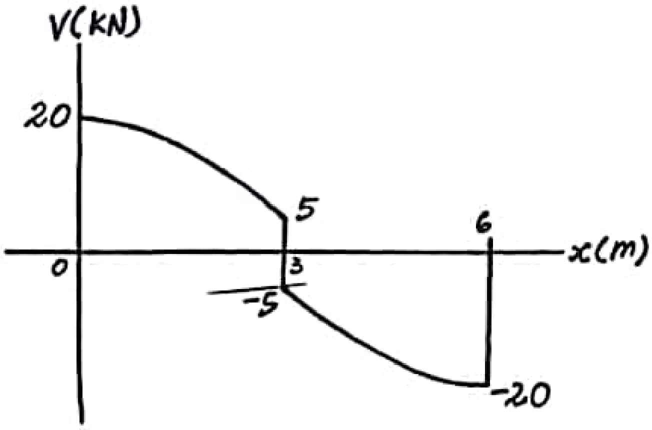
$$\circlearrowleft + \Sigma M = 0;\quad M\big|_{x=3\,\mathrm{m}} + \frac{1}{2}(10)(3)(1) - 20(3) = 0 \qquad M\big|_{x=3\,\mathrm{m}} = 45\ \mathrm{kN\cdot m} \qquad \textbf{Ans.}$$



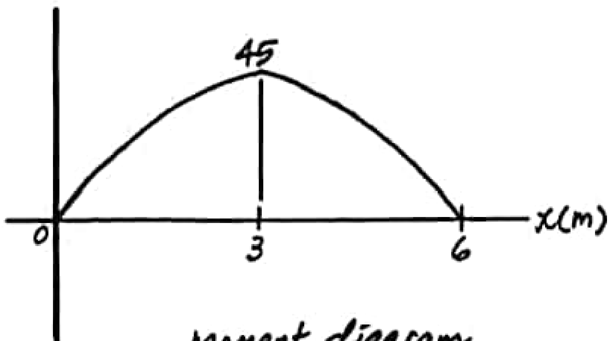
(a)



(b)

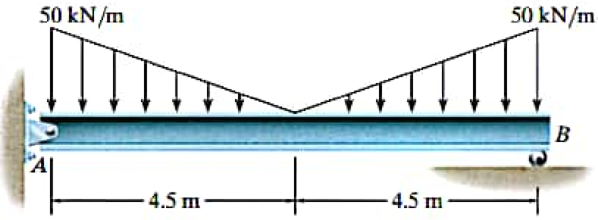


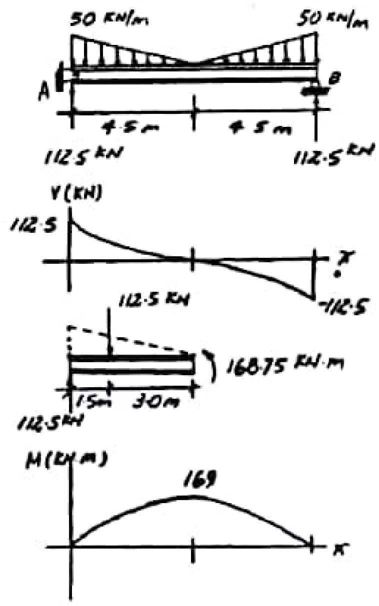
shear diagram
(c)

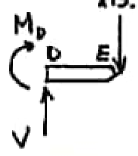


Moment diagram
(d)

6–37. Draw the shear and moment diagrams for the beam.







Bending moment at D

$+\circlearrowleft \Sigma M_D = 0$

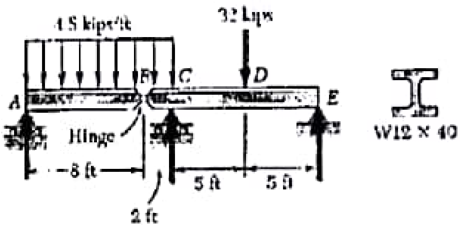
$-M_D - (0.2)(213.33) = 0$

$M_D = -25.33$ N·m

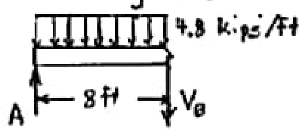
[redacted]

Problem 5.31

5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Free body AB



$\circlearrowleft \Sigma M_B = 0$

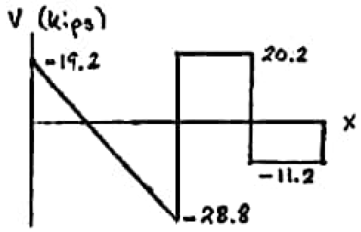
$(4.8)(8)(4) - 8A = 0$

$A = 19.2$ kips

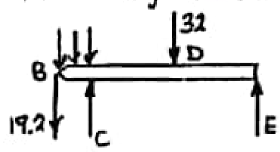
$\circlearrowleft M_A = 0$

$-(4.8)(8)(4) - 8V_B = 0$

$V_B = -19.2$ kips



Free body BCDE



$+\circlearrowleft \Sigma M_E = 0$

$(19.2)(12) + (4.8)(2)(11)$

$- 10C + (32)(5) = 0$

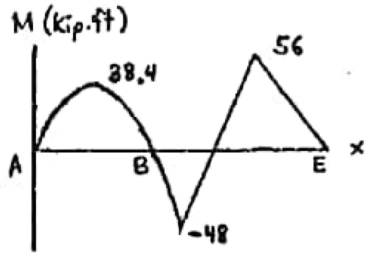
$C = 49.2$ Kips

$+\circlearrowleft M_C = 0$

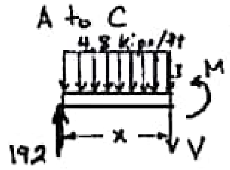
$(19.2)(2) + (4.8)(2)(1)$

$-(32)(5) + 10E = 0$

$E = 11.2$ kips



A to C $\quad 0 < x < 10$ ft.



$+\uparrow \Sigma F_y = 0$

$19.2 - 4.8x - V = 0$

$V = 19.2 - 4.8x$ kips.

$\circlearrowleft M_J = 0$

$-19.2x + (4.8x)\frac{x}{2} + M = 0$

$M = 19.2x - 2.4x^2$ kip·ft

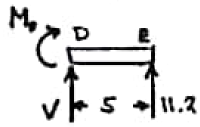
At C^- $\quad x = 10 \quad V = 19.2 - (4.8)(10) = -28.8$ Kips

At C $\quad x = 10 \quad M_C = (19.2)(10) - (2.4)(10)^2 = -48$ kip·ft

C to D $\quad V = 19.2 - (4.8)(10) + 49.6 = 20.8$ kips.

D to E $\quad V = -11.2$ kips

Bending moment at D



$\circlearrowleft \Sigma M_D = 0$

$-M_D + (11.2)(5) = 0$

$M_D = 56$ kip·ft

max $|M| = 56$ kip·ft $= 672$ kip·in

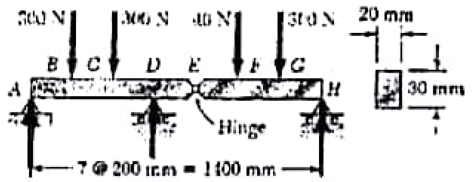
For W 12× 40 rolled steel section $\quad S = 51.9$ in³

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{672}{51.9} = 12.95$ ksi

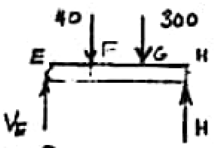
[redacted]

Problem 5.30

5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Free body EFGH



Note that $M_E = 0$ due to hinge.

$\circlearrowleft \Sigma M_E = 0$

$0.6\,H - (0.2)(40) - (0.4)(300) = 0$

$H = 213.33\ N$

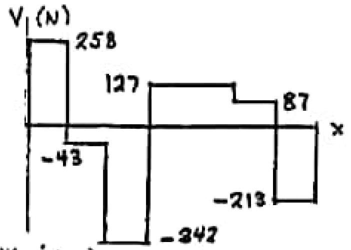
$+\uparrow \Sigma F_y = 0 \qquad V_E - 40 - 300 + 213.33 = 0$

$V_E = 126.67\ N$

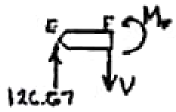
Shear: E to F $\quad V = 126.67\ N\cdot m$

F to G $\quad V = 86.67\ N\cdot m$

G to H $\quad V = -213.33\ N\cdot m$



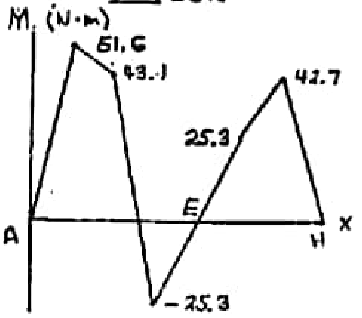
Bending moment at F



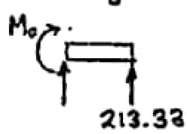
$\circlearrowleft \Sigma M_F = 0$

$M_F - (0.2)(126.67) = 0$

$M_F = 25.33\ N\cdot m$



Bending moment at G

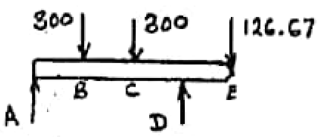


$\circlearrowleft \Sigma M_G = 0$

$-M_G + (0.2)(213.33) = 0$

$M_G = 42.67\ N\cdot m$

Free body ABCDE

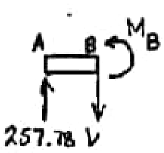


$+\circlearrowleft \Sigma M_B = 0 \qquad -0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$

$A = 257.78\ N$

$+\circlearrowleft \Sigma M_A = 0 \qquad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6\,D = 0$

$D = 468.89\ N$

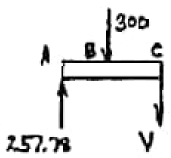


Bending moment at B

$\circlearrowleft \Sigma M_B = 0$

$-(0.2)(257.78) + M_B = 0$

$M_B = 51.56\ N\cdot m$

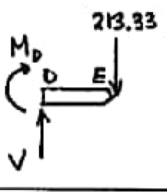


Bending moment at C

$+\circlearrowleft \Sigma M_C = 0$

$-(0.4)(257.78) + (0.2)(300) + M_C = 0$

$M_C = 43.11\ N\cdot m$



Bending moment at D

$+\circlearrowleft \Sigma M_D = 0$

$-M_D - (0.2)(213.33) = 0$

$M_D = -25.33\ N\cdot m$

max $|M| = 51.56\ N\cdot m$

$S = \frac{1}{6} b h^2 = \frac{1}{6}(20)(30)^2$

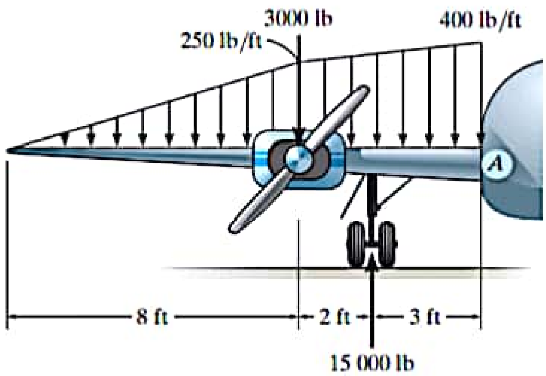
$= 3 \times 10^3\ mm^3 = 3 \times 10^{-6}\ m^3$

Normal stress

$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6\ Pa$

$= 17.19\ MPa$ ◄

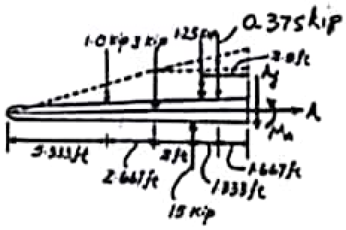
6–38. The dead-weight loading along the centerline of the airplane wing is shown. If the wing is fixed to the fuselage at A, determine the reactions at A, and then draw the shear and moment diagram for the wing.



Support Reactions:

$+\uparrow \Sigma F_y = 0;\quad -1.00 - 3 + 15 - 1.25 - 0.375 - A_y = 0$

$A_y = 9.375$ kip **Ans.**



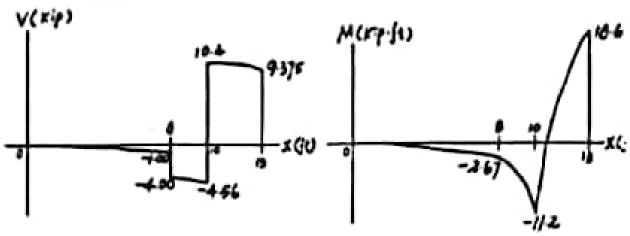
$\circlearrowleft +\Sigma M_A = 0;\quad 1.00(7.667) + 3(5) - 15(3)$

$+ 1.25(2.5) + 0.375(1.667) + M_A = 0$

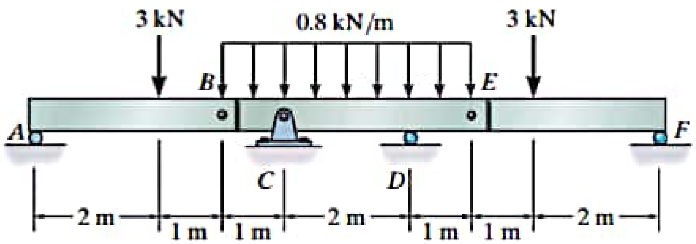
$M_A = 18.583$ kip · ft $= 18.6$ kip · ft **Ans.**

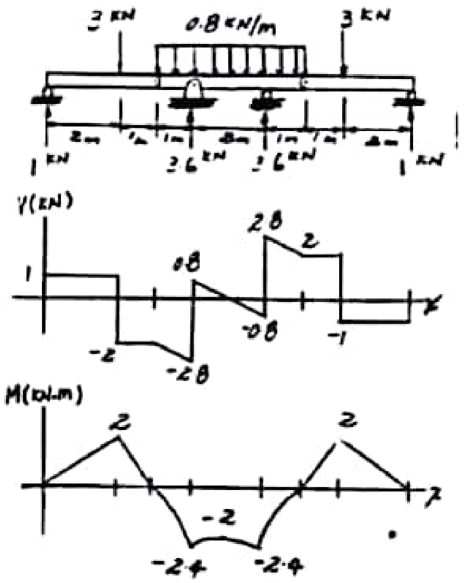
$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$ **Ans.**

Shear and Moment Diagram:

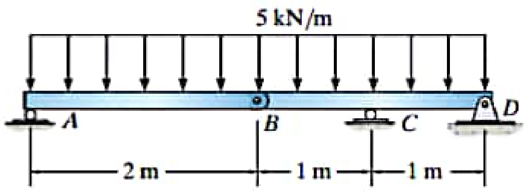


6–41. Draw the shear and moment diagrams for the compound beam. The three segments are connected by pins at B and E.





6–42. Draw the shear and moment diagrams for the compound beam.



Support Reactions:

From the FBD of segment AB

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(2) - 10.0(1) = 0 \qquad B_y = 5.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 10.0 + 5.00 = 0 \qquad A_y = 5.00 \text{ kN}$$

From the FBD of segment BD

$$\circlearrowleft + \Sigma M_C = 0; \qquad 5.00(1) + 10.0(0) - D_y(1) = 0$$

$$D_y = 5.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y - 5.00 - 5.00 - 10.0 = 0$$

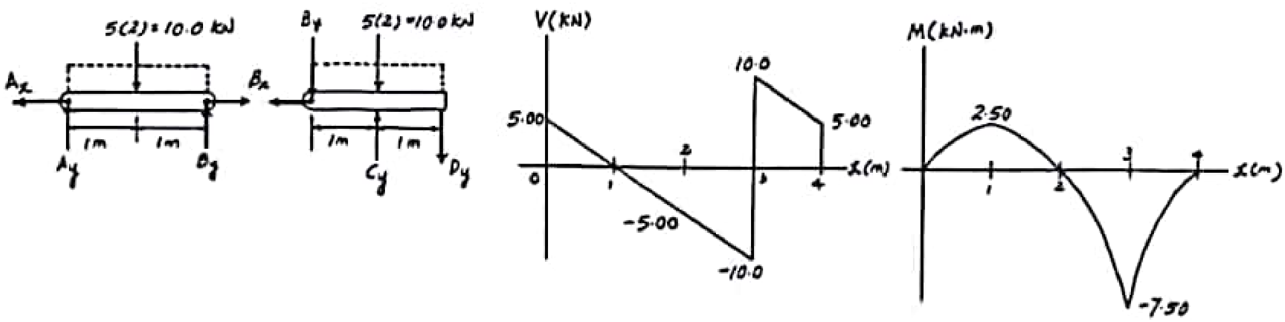
$$C_y = 20.0 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x = 0$$

From the FBD of segment AB

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$$

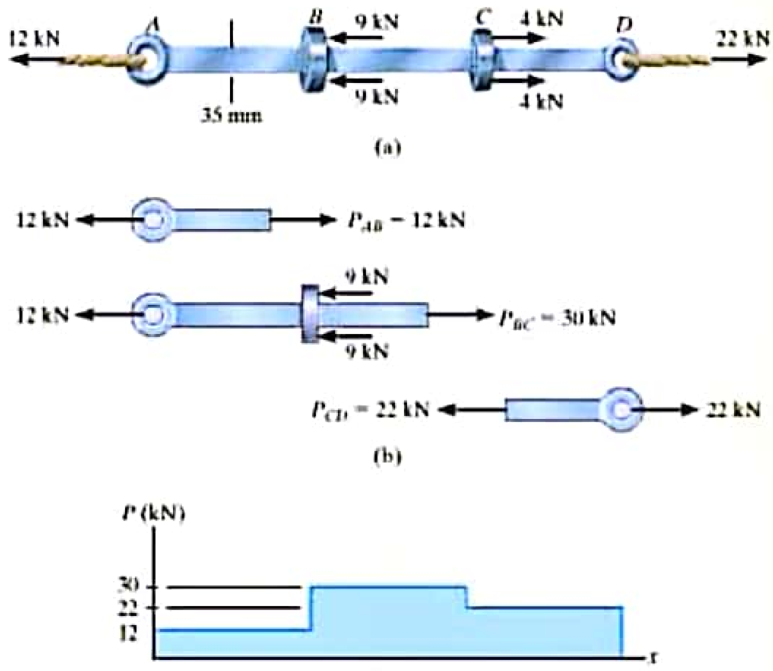
Shear and Moment Diagram:



1

EXAMPLE 1.6

The bar in Fig. 1–16a has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



SOLUTION

Internal Loading. By inspection, the internal axial forces in regions AB, BC, and CD are all constant yet have different magnitudes. Using the method of sections, these loadings are determined in Fig. 1–16b; and the normal force diagram which represents these results graphically is shown in Fig. 1–16c. The largest loading is in region BC, where $P_{BC} = 30$ kN. Since the cross-sectional area of the bar is *constant*, the largest average normal stress also occurs within this region of the bar.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{ N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa} \qquad \textit{Ans.}$$

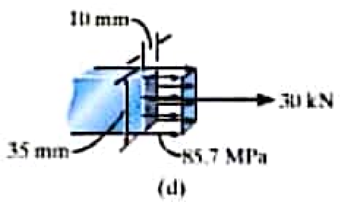


Fig. 1–16

NOTE: The stress distribution acting on an arbitrary cross section of the bar within region BC is shown in Fig. 1–16d. Graphically the *volume* (or "block") represented by this distribution of stress is equivalent to the load of 30 kN; that is, 30 kN = (85.7 MPa)(35 mm)(10 mm).

EXAMPLE 1.7

The 80-kg lamp is supported by two rods AB and BC as shown in Fig. 1–17a. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod.

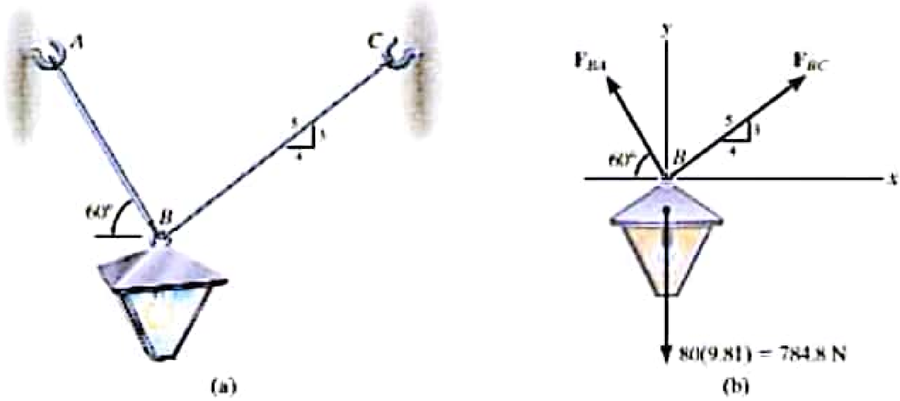


Fig. 1–17

SOLUTION

Internal Loading. We must first determine the axial force in each rod. A free-body diagram of the lamp is shown in Fig. 1–17b. Applying the equations of force equilibrium,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

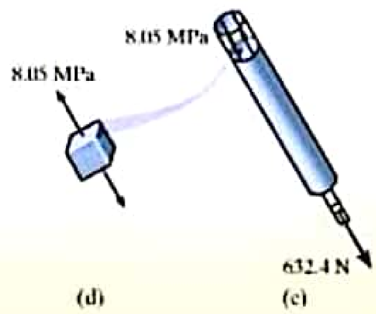
$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

By Newton's third law of action, equal but opposite reaction, these forces subject the rods to tension throughout their length.

Average Normal Stress. Applying Eq. 1–6,

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

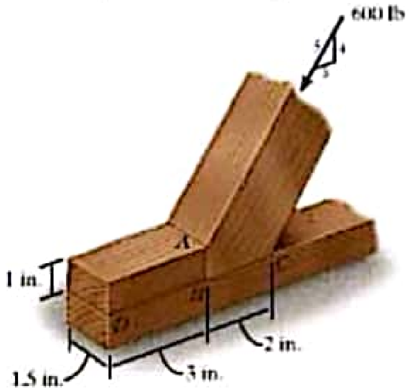
$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa} \qquad \textit{Ans.}$$



NOTE: The average normal stress distribution acting over a cross section of rod AB is shown in Fig. 1–17c, and at a point on this cross section, an element of material is stressed as shown in Fig. 1–17d.

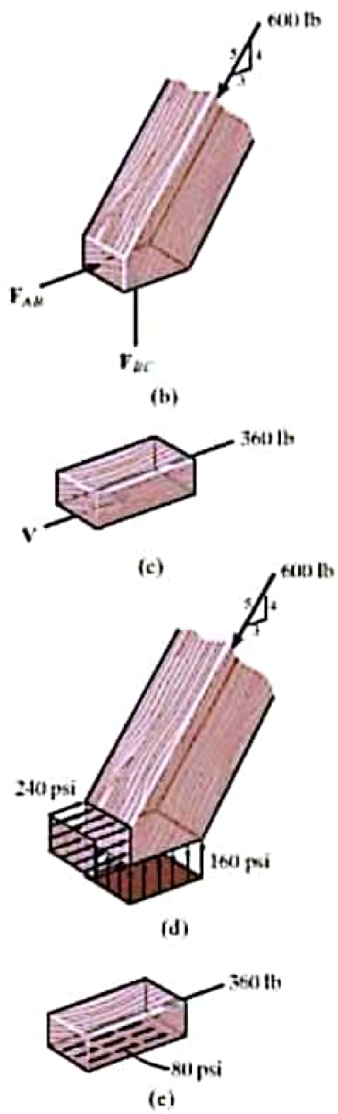
EXAMPLE 1.12

The inclined member in Fig. 1–24*a* is subjected to a compressive force of 600 lb. Determine the average compressive stress along the smooth areas of contact defined by *AB* and *BC*, and the average shear stress along the horizontal plane defined by *DB*.



(a)

Fig. 1–24



SOLUTION

Internal Loadings. The free-body diagram of the inclined member is shown in Fig. 1–24*b*. The compressive forces acting on the areas of contact are

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB} - 600\text{ lb}\left(\tfrac{3}{5}\right) = 0 \qquad F_{AB} = 360\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} - 600\text{ lb}\left(\tfrac{4}{5}\right) = 0 \qquad F_{BC} = 480\text{ lb}$$

Also, from the free-body diagram of the top segment *ABD* of the bottom member, Fig. 1–24*c*, the shear force acting on the sectioned horizontal plane *DB* is

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V = 360\text{ lb}$$

Average Stress. The average compressive stresses along the horizontal and vertical planes of the inclined member are

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{360\text{ lb}}{(1\text{ in.})(1.5\text{ in.})} = 240\text{ psi} \qquad \textit{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{480\text{ lb}}{(2\text{ in.})(1.5\text{ in.})} = 160\text{ psi} \qquad \textit{Ans.}$$

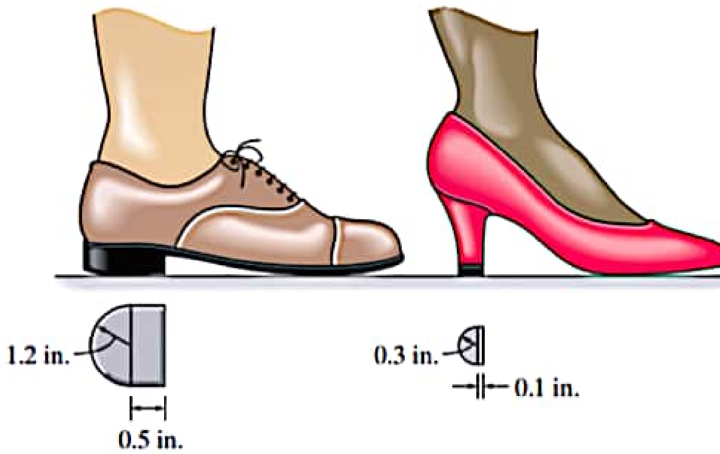
These stress distributions are shown in Fig. 1–24*d*.

The average shear stress acting on the horizontal plane defined by *DB* is

$$\tau_{avg} = \frac{360\text{ lb}}{(3\text{ in.})(1.5\text{ in.})} = 80\text{ psi} \qquad \textit{Ans.}$$

This stress is shown uniformly distributed over the sectioned area in Fig. 1–24*e*.

*1–44. A 175-lb woman stands on a vinyl floor wearing stiletto high-heel shoes. If the heel has the dimensions shown, determine the average normal stress she exerts on the floor and compare it with the average normal stress developed when a man having the same weight is wearing flat-heeled shoes. Assume the load is applied slowly, so that dynamic effects can be ignored. Also, assume the entire weight is supported only by the heel of one shoe.



Stiletto shoes:

$$A = \frac{1}{2}(\pi)(0.3)^2 + (0.6)(0.1) = 0.2014 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{0.2014 \text{ in}^2} = 869 \text{ psi}$$ **Ans.**

Flat-heeled shoes:

$$A = \frac{1}{2}(\pi)(1.2)^2 + 2.4(0.5) = 3.462 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{3.462 \text{ in}^2} = 50.5 \text{ psi}$$ **Ans.**

A bar made of A-36 steel has the dimensions shown in Fig. 3–22. If an axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.

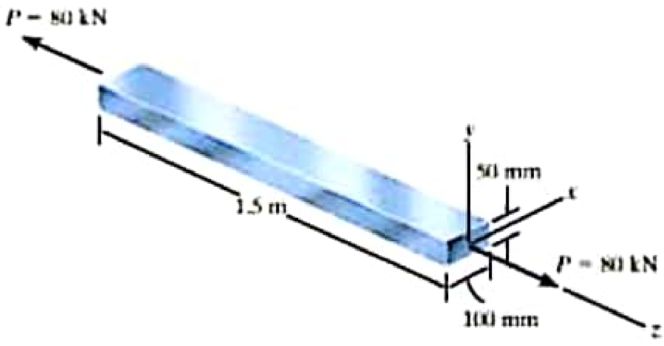


Fig. 3-22

SOLUTION

The normal stress in the bar is

$$\sigma_z = \frac{P}{A} = \frac{80(10^3)\text{ N}}{(0.1\text{ m})(0.05\text{ m})} = 16.0(10^6)\text{ Pa}$$

From the table on the inside back cover for A-36 steel $E_{st} = 200$ GPa, and so the strain in the z direction is

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\text{ Pa}}{200(10^9)\text{ Pa}} = 80(10^{-6})\text{ mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\text{ m}) = 120\ \mu\text{m} \qquad \textit{Ans.}$$

Using Eq. 3-9, where $\nu_{st} = 0.32$ as found from the inside back cover, the lateral contraction strains in *both* the x and y directions are

$$\epsilon_x = \epsilon_y = -\nu_{st}\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are

$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\text{ m}) = -2.56\ \mu\text{m} \qquad \textit{Ans.}$$

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\text{ m}) = -1.28\ \mu\text{m} \qquad \textit{Ans.}$$

3

EXAMPLE 3.6

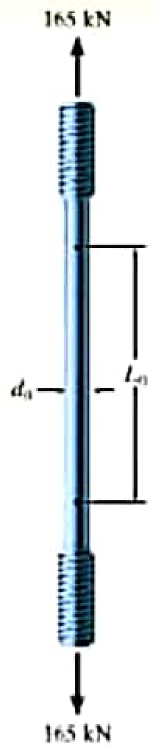


Fig. 3-26

An aluminum specimen shown in Fig. 3-26 has a diameter of $d_0 = 25$ mm and a gauge length of $L_0 = 250$ mm. If a force of 165 kN elongates the gauge length 1.20 mm, determine the modulus of elasticity. Also, determine by how much the force causes the diameter of the specimen to contract. Take $G_{al} = 26$ GPa and $\sigma_Y = 440$ MPa.

SOLUTION

Modulus of Elasticity. The average normal stress in the specimen is

$$\sigma = \frac{P}{A} = \frac{165(10^3)\text{ N}}{(\pi/4)(0.025\text{ m})^2} = 336.1\text{ MPa}$$

and the average normal strain is

$$\epsilon = \frac{\delta}{L} = \frac{1.20\text{ mm}}{250\text{ mm}} = 0.00480\text{ mm/mm}$$

Since $\sigma < \sigma_Y = 440$ MPa, the material behaves elastically. The modulus of elasticity is therefore

$$E_{al} = \frac{\sigma}{\epsilon} = \frac{336.1(10^6)\text{ Pa}}{0.00480} = 70.0\text{ GPa} \qquad \textit{Ans.}$$

Contraction of Diameter. First we will determine Poisson's ratio for the material using Eq. 3-11.

$$G = \frac{E}{2(1+\nu)}$$

$$26\text{ GPa} = \frac{70.0\text{ GPa}}{2(1+\nu)}$$

$$\nu = 0.347$$

Since $\epsilon_{long} = 0.00480$ mm/mm, then by Eq. 3-9,

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

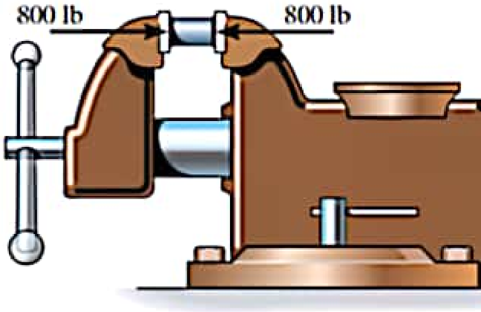
$$0.347 = -\frac{\epsilon_{lat}}{0.00480\text{ mm/mm}}$$

$$\epsilon_{lat} = -0.00166\text{ mm/mm}$$

The contraction of the diameter is therefore

$$\delta' = (0.00166)(25\text{ mm})$$
$$= 0.0416\text{ mm} \qquad \textit{Ans.}$$

3–26. The short cylindrical block of 2014-T6 aluminum, having an original diameter of 0.5 in. and a length of 1.5 in., is placed in the smooth jaws of a vise and squeezed until the axial load applied is 800 lb. Determine (a) the decrease in its length and (b) its new diameter.



a)

$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}(0.5)^2} = 4074.37 \text{ psi}$$

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{-4074.37}{10.6(10^6)} = -0.0003844$$

$$\delta = \varepsilon_{long} L = -0.0003844\,(1.5) = -0.577\,(10^{-3}) \text{ in.}$$ **Ans.**

b)

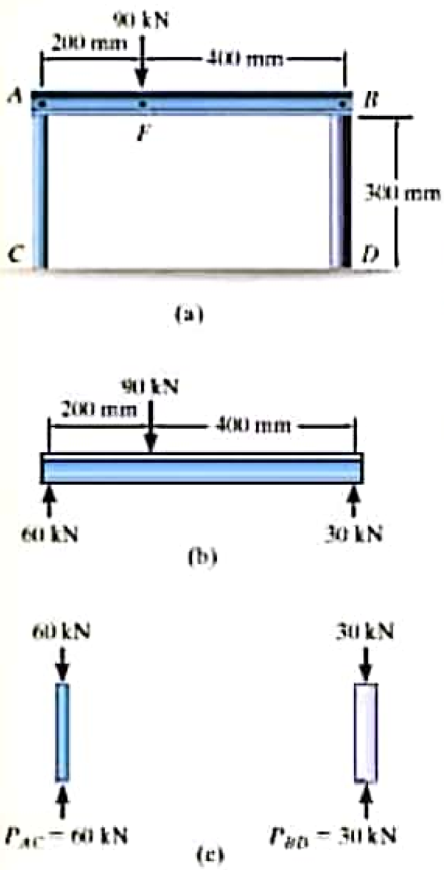
$$V = \frac{-\varepsilon_{lat}}{\varepsilon_{long}} = 0.35$$

$$\varepsilon_{lat} = -0.35\,(-0.0003844) = 0.00013453$$

$$\Delta d = \varepsilon_{lat}\, d = 0.00013453\,(0.5) = 0.00006727$$

$$d' = d + \Delta d = 0.5000673 \text{ in.}$$ **Ans.**

EXAMPLE 4.3



Rigid beam AB rests on the two short posts shown in Fig. 4–8a. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.

SOLUTION

Internal Force. The compressive forces acting at the top of each post are determined from the equilibrium of member AB, Fig. 4–8b. These forces are equal to the internal forces in each post, Fig. 4–8c.

Displacement. The displacement of the top of each post is

Post AC:

$$\delta_A = \frac{P_{AC}L_{AC}}{A_{AC}E_{st}} = \frac{[-60(10^3)\text{ N}](0.300\text{ m})}{\pi(0.010\text{ m})^2[200(10^9)\text{ N/m}^2]} = -286(10^{-6})\text{ m}$$

$$= 0.286\text{ mm}\downarrow$$

Post BD:

$$\delta_B = \frac{P_{BD}L_{BD}}{A_{BD}E_{al}} = \frac{[-30(10^3)\text{ N}](0.300\text{ m})}{\pi(0.020\text{ m})^2[70(10^9)\text{ N/m}^2]} = -102(10^{-6})\text{ m}$$

$$= 0.102\text{ mm}\downarrow$$

A diagram showing the centerline displacements at A, B, and F on the beam is shown in Fig. 4–8d. By proportion of the blue shaded triangle, the displacement of point F is therefore

$$\delta_F = 0.102\text{ mm} + (0.184\text{ mm})\left(\frac{400\text{ mm}}{600\text{ mm}}\right) = 0.225\text{ mm}\downarrow \qquad \textit{Ans.}$$

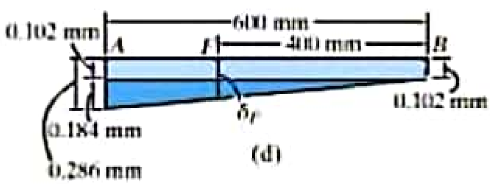
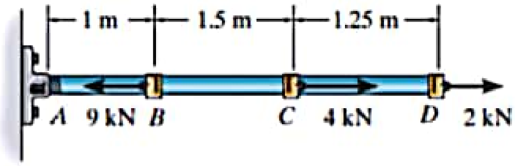
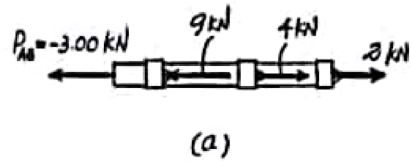


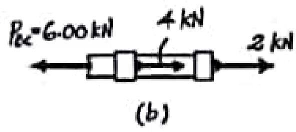
Fig. 4–8

***4–4.** The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 50 mm², determine the displacement of *C*. Neglect the size of the couplings at *B*, *C*, and *D*.



The normal forces developed in segments *AB* and *BC* are shown the *FBDS* of each segment in Fig. *a* and *b*, respectively. The cross-sectional area of these two segments are $A = \left(50\text{ mm}^2\right)\left(\dfrac{1\text{ m}}{10.00\text{ mm}}\right)^2 = 50.0\,(10^{-6})\text{ m}^2$. Thus,





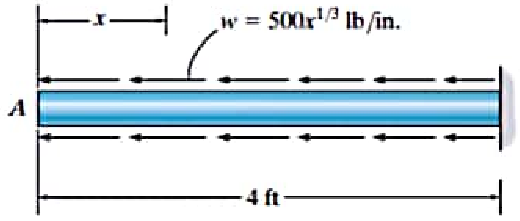
$$\delta_C = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{1}{A\,E_{SC}}\left(P_{AB}\,L_{AB} + P_{BC}\,L_{BC}\right)$$

$$= \frac{1}{50.0(10^{-6})\left[200(10^9)\right]}\left[-3.00(10^3)(1) + 6.00(10^3)(1.5)\right]$$

$$= 0.600\,(10^{-3})\text{ m} = 0.600\text{ mm} \qquad \textbf{Ans.}$$

The positive sign indicates that coupling *C* moves away from the fixed support.

4–6. The bar has a cross-sectional area of 3 in^2, and $E = 35(10^3)$ ksi. Determine the displacement of its end A when it is subjected to the distributed loading.



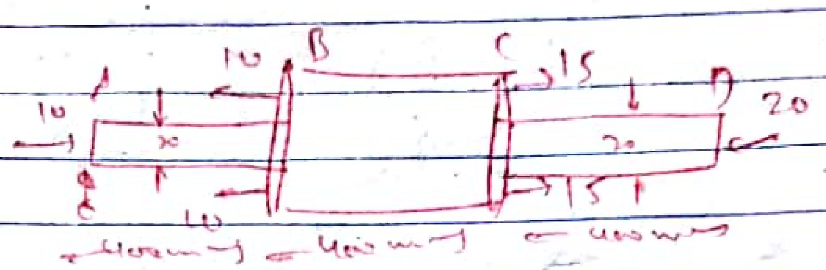
$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \frac{1500}{4}x^{\frac{4}{3}}$$

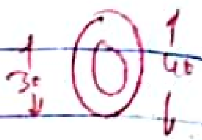
$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \frac{1}{(3)(35)(10^6)}\int_0^{4(12)} \frac{1500}{4}x^{\frac{4}{3}}\,dx = \left(\frac{1500}{(3)(35)(10^6)(4)}\right)\left(\frac{3}{7}\right)(48)^{\frac{7}{3}}$$

$\delta_A = 0.0128$ in. **Ans.**

10-2

②

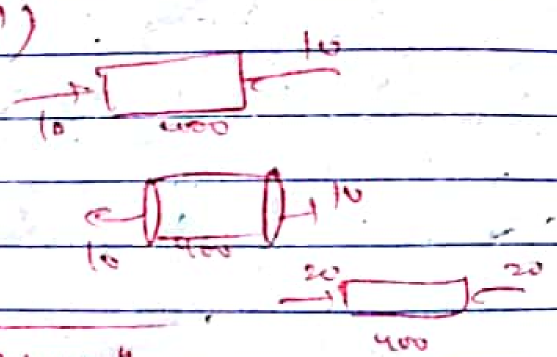




displacement in D wrt A

FBD

$$\Delta_{AB} = \frac{PL}{AE} = \frac{10\times10^3\times(0.4)}{3.14\times(0.01)^2\times(68.9\times10^9)}$$



$$= 1.85\times10^{-4}$$ (contraction)

$$\Delta_{BC} = \frac{PL}{AE} = \frac{10\times10^3\times0.4}{3.14\left(0.02^2-0.015^2\right)\times68.9\times10^9}$$

$\pi r_2^2 - \pi r_1^2$

$\pi(r_2^2 - r_1^2)$

$$= 1.056\times10^{-4}$$ (elongation)

$$\Delta_{CD} = \frac{PL}{AE} = \frac{20\times10^3\times0.4}{3.14\times(0.01)^2\times68.9\times10^9}$$

$$= 3.7\times10^{-4}$$ contraction

$$\Delta_{AD} = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$

$$= -4.4917\times10^{-4}$$

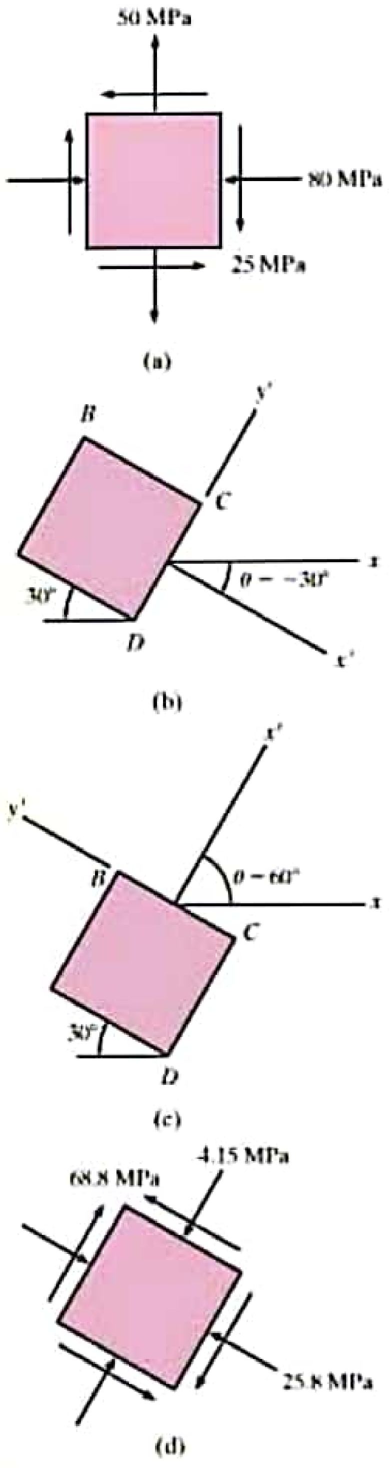


Fig. 9-7

The state of plane stress at a point is represented by the element shown in Fig. 9–7*a*. Determine the state of stress at the point on another element oriented 30° clockwise from the position shown.

SOLUTION

This problem was solved in Example 9.1 using basic principles. Here we will apply Eqs. 9-1 and 9-2. From the established sign convention, Fig. 9-5, it is seen that

$$\sigma_x = -80\text{ MPa} \qquad \sigma_y = 50\text{ MPa} \qquad \tau_{xy} = -25\text{ MPa}$$

Plane *CD*. To obtain the stress components on plane *CD*, Fig. 9-7*b*, the positive x' axis is directed outward, perpendicular to *CD*, and the associated y' axis is directed along *CD*. The angle measured from the x to the x' axis is $\theta = -30°$ (clockwise). Applying Eqs. 9-1 and 9-2 yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80+50}{2} + \frac{-80-50}{2}\cos 2(-30°) + (-25)\sin 2(-30°)$$

$$= -25.8\text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-80-50}{2}\sin 2(-30°) + (-25)\cos 2(-30°)$$

$$= -68.8\text{ MPa} \qquad \textit{Ans.}$$

The negative signs indicate that $\sigma_{x'}$ and $\tau_{x'y'}$ act in the negative x' and y' directions, respectively. The results are shown acting on the element in Fig. 9-7*d*.

Plane *BC*. In a similar manner, the stress components acting on face *BC*, Fig. 9-7*c*, are obtained using $\theta = 60°$. Applying Eqs. 9-1 and 9-2,* we get

$$\sigma_{x'} = \frac{-80+50}{2} + \frac{-80-50}{2}\cos 2(60°) + (-25)\sin 2(60°)$$

$$= -4.15\text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -\frac{-80-50}{2}\sin 2(60°) + (-25)\cos 2(60°)$$

$$= 68.8\text{ MPa} \qquad \textit{Ans.}$$

Here $\tau_{x'y'}$ has been calculated twice in order to provide a check. The negative sign for $\sigma_{x'}$ indicates that this stress acts in the negative x' direction, Fig. 9-7*c*. The results are shown on the element in Fig. 9-7*d*.

*Alternatively, we could apply Eq. 9-3 with $\theta = -30°$ rather than Eq. 9-1.

Notice how the failure plane is at an angle (23.7°) due to tearing of the material, Fig. 9–11c.

The state of plane stress at a failure point on the shaft is shown on the element in Fig. 9–11*a*. Represent this stress state in terms of the principal stresses.

SOLUTION

From the established sign convention, we have

$$\sigma_x = -20 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 60 \text{ MPa}$$

Orientation of Element. Applying Eq. 9–4,

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{60}{(-20 - 90)/2}$$

Solving, and referring to this root as θ_{p_2}, as will be shown below, yields

$$2\theta_{p_2} = -47.49^\circ \qquad \theta_{p_2} = -23.7^\circ$$

Since the difference between $2\theta_{p_1}$ and $2\theta_{p_2}$ is 180°, we have

$$2\theta_{p_1} = 180^\circ + 2\theta_{p_2} = 132.51^\circ \qquad \theta_{p_1} = 66.3^\circ$$

Recall that θ is measured positive *counterclockwise* from the x axis to the outward normal (x' axis) on the face of the element, and so the results are shown in Fig. 9–11*b*.

Principal Stress. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-20 + 90}{2} \pm \sqrt{\left(\frac{-20 - 90}{2}\right)^2 + (60)^2}$$

$$= 35.0 \pm 81.4$$

$\sigma_1 = 116$ MPa *Ans.*

$\sigma_2 = -46.4$ MPa *Ans.*

The principal plane on which each normal stress acts can be determined by applying Eq. 9–1 with, say, $\theta = \theta_{p_2} = -23.7^\circ$. We have

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-20 + 90}{2} + \frac{-20 - 90}{2}\cos 2(-23.7^\circ) + 60\sin 2(-23.7^\circ)$$

$$= -46.4 \text{ MPa}$$

Hence, $\sigma_2 = -46.4$ MPa acts on the plane defined by $\theta_{p_2} = -23.7^\circ$, whereas $\sigma_1 = 116$ MPa acts on the plane defined by $\theta_{p_1} = 66.3^\circ$. The results are shown on the element in Fig. 9–11c. Recall that no shear stress acts on this element.

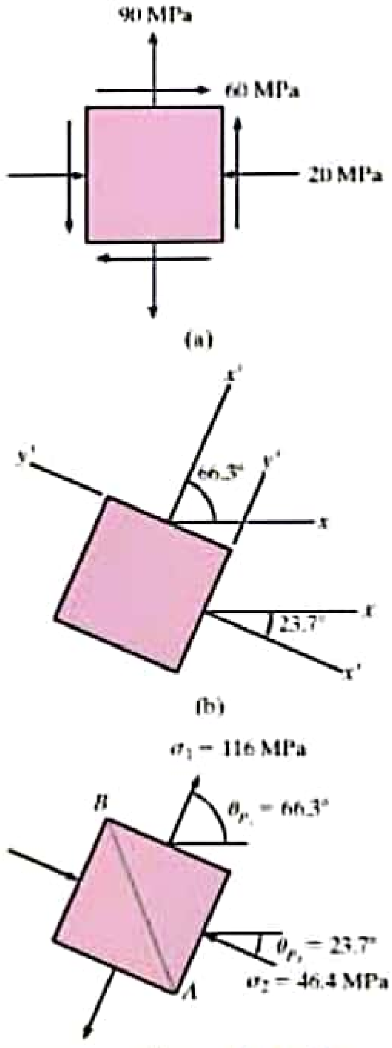


Fig. 9–11