

## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 1

#### Forces, Moments and FBD

1. Determine the internal normal force, shear force, and bending moment at point C in the beam.

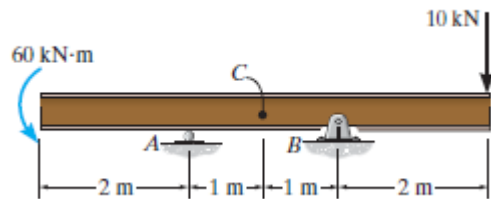


Fig. 1 (a)

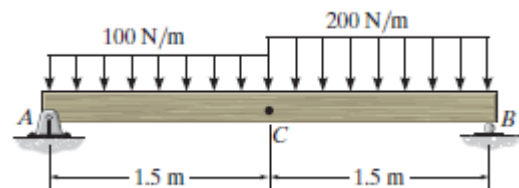


Fig. 1 (b)

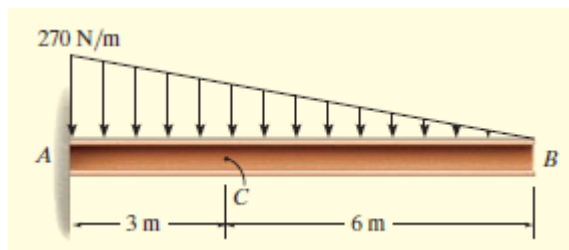


Fig. 1 (c)

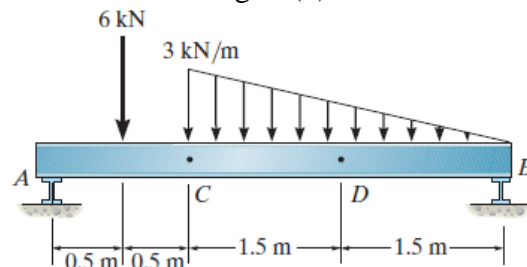


Fig. 1 (d)

2. Determine resultant internal loadings acting on section a–a and section b–b. Each section passes through the centreline at point C.

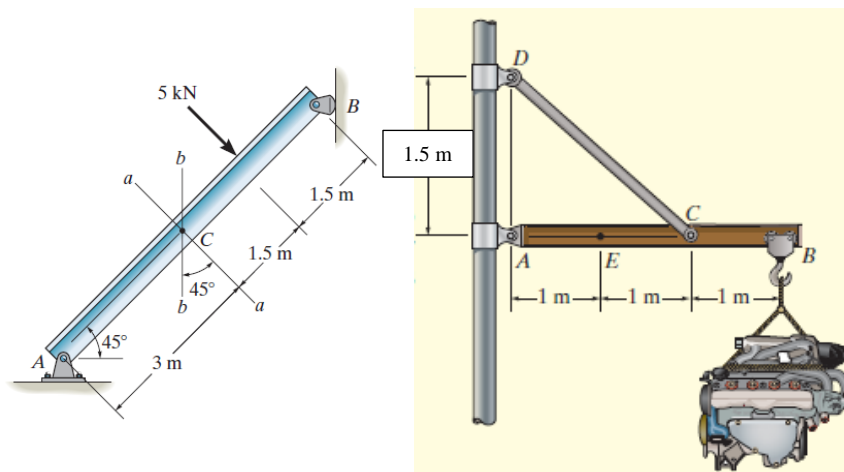


Fig. 2

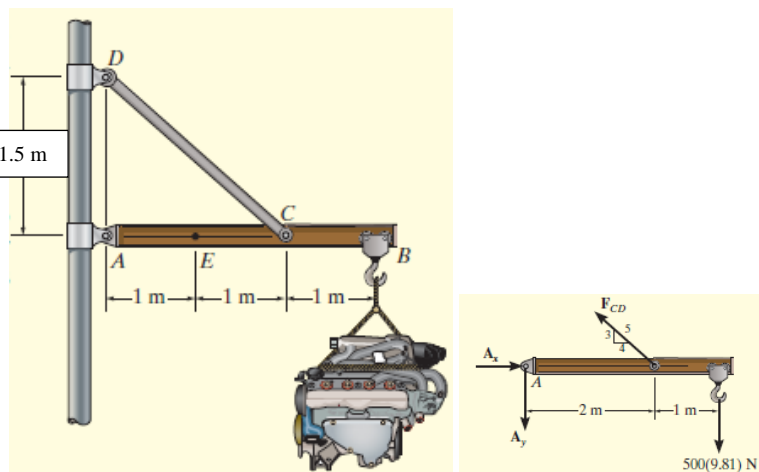


Fig. 3

3. The 500-kg engine is suspended from the crane boom in Fig. 3. Determine the resultant internal loadings acting on the cross section of the boom at point E.

4. Determine the resultant internal loadings acting on the cross section at B of the pipe shown in Fig. 4. The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a couple moment of 70 N·m at its end A. It is fixed to the wall at C.

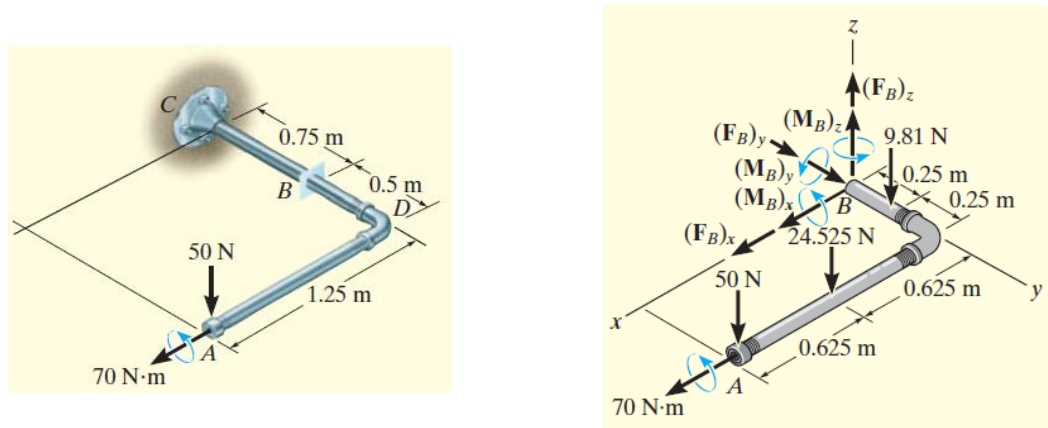


Fig. 4

5. The pipe has a mass of 12 kg/m. If it is fixed to the wall at A, determine the resultant internal loadings acting on the cross section at B. Neglect the weight of the wrench CD.

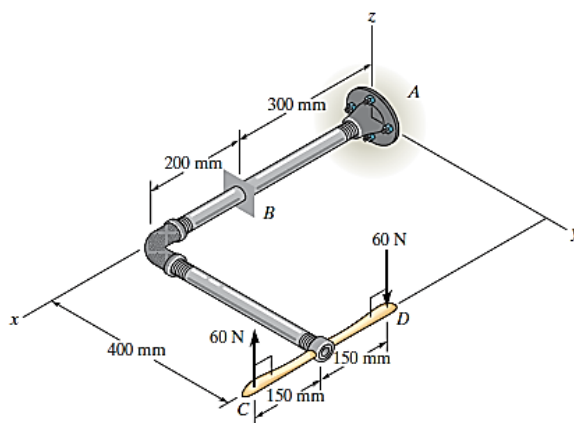


Fig. 5

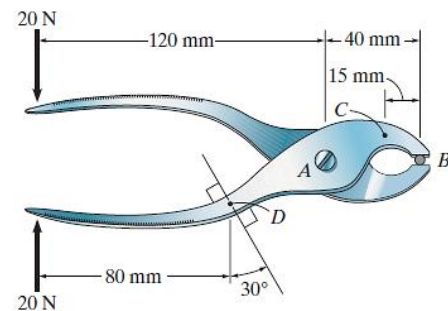


Fig. 6

6. Determine the resultant internal loading on the cross section through point C of the pliers. There is a pin at A, and the jaws at B are smooth. Determine the resultant internal loading on the cross section through point D of the pliers.

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### CEA 1120 – Tutorial Sheet # 2

#### Normal and Shear Stress

1. The bar in Fig. 1a has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.

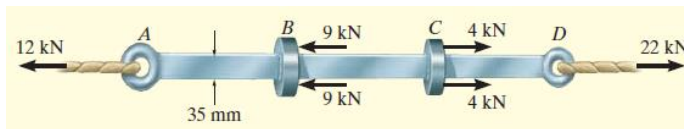


Fig. 1 (a)

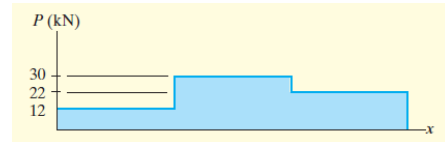


Fig. 1 (b)

2. The 80-kg lamp is supported by two rods AB and BC as shown in Fig. 2a. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod.

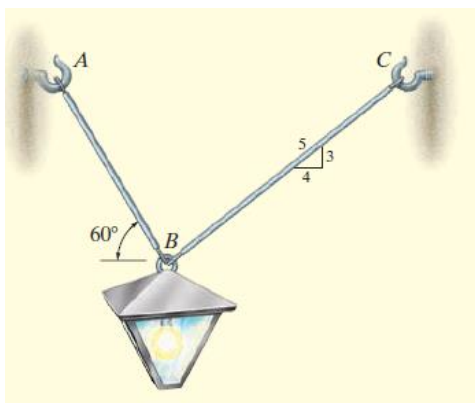


Fig. 2a

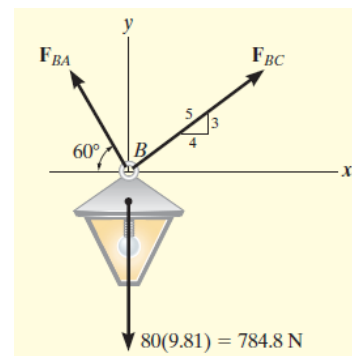


Fig. 2b

3. Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam in Fig. 3a.

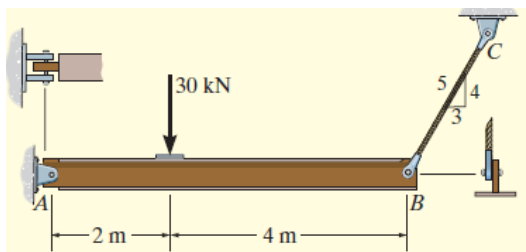


Fig. 3a

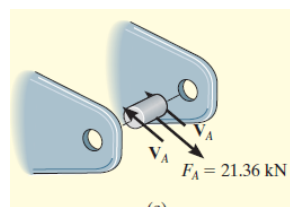


Fig. 3b

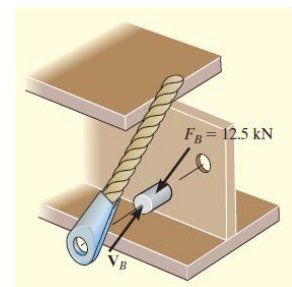


Fig. 3c

4. The inclined member in Fig. 4a is subjected to a compressive force of 600 lb. Determine the average compressive stress along the smooth areas of contact defined by AB and BC, and the average shear stress along the horizontal plane defined by DB.

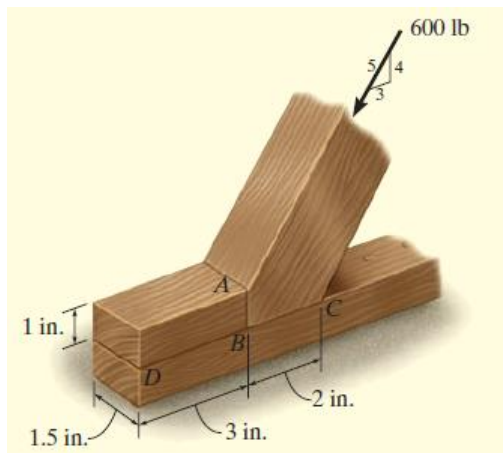


Fig. 4a

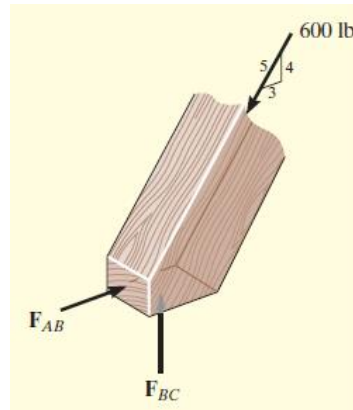


Fig. 4b

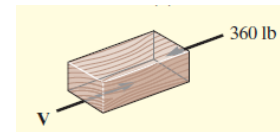


Fig. 4c

5. If the wood joint in Fig. 5a has a width of 150 mm, determine the average shear stress developed along shear planes a-a and b-b. For each plane, represent the state of stress on an element of the material.

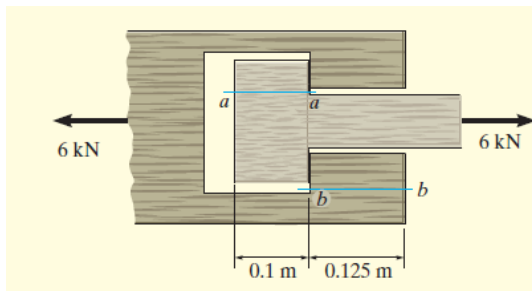


Fig. 5

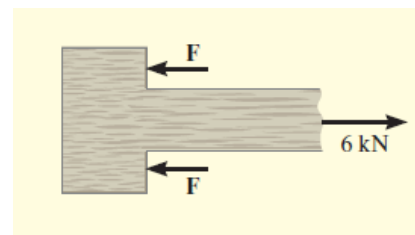


Fig. 5b

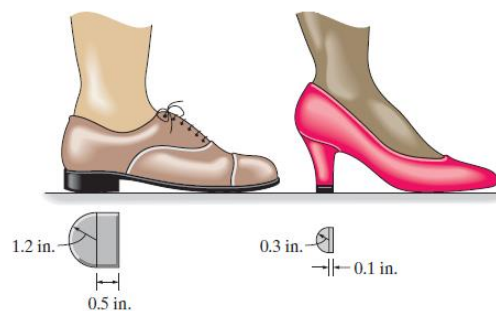


Fig. 6

6. A 175-lb woman stands on a vinyl floor wearing stiletto high-heel shoes. If the heel has the dimensions shown, determine the average normal stress she exerts on the floor and compare it with the average normal stress developed when a man having the same weight is wearing flat-heeled shoes. Assume the load is applied slowly, so that dynamic effects can be ignored. Also, assume the entire weight is supported only by the heel of one shoe. (Fig. 6)

## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 3

#### Normal Strain, Shear Strain & Material Properties

- The slender rod shown in Fig. 1 is subjected to an increase of temperature along its axis, which creates a normal strain in the rod of  $\epsilon_z = 40(10^{-3})z^{1/2}$ , where  $z$  is measured in meters. Determine (a) the displacement of the end B of the rod due to the temperature increase, and (b) the average normal strain in the rod. [ $\delta_B = 2.39\downarrow$ ,  $\epsilon_{avg} = 0.0119\text{mm/mm}$ ]

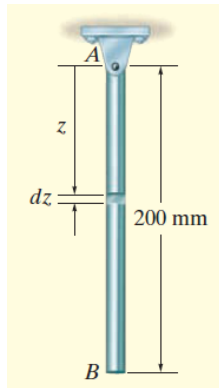


Fig. 1

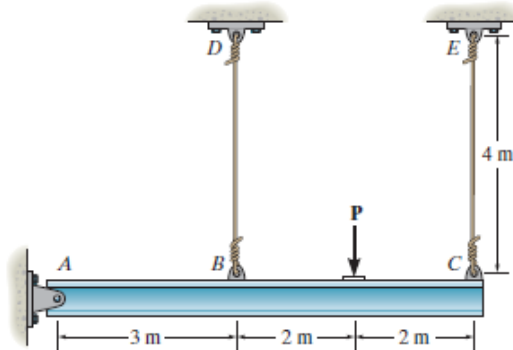


Fig. 2

- The rigid beam shown in Fig. 2 is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD. [ $\epsilon_{CE} = 0.00250\text{mm/mm}$ ;  $\epsilon_{BD} = 0.00107\text{mm/mm}$ ]

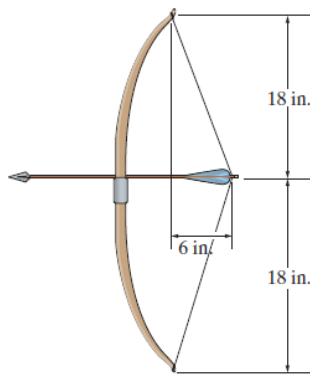


Fig. 3

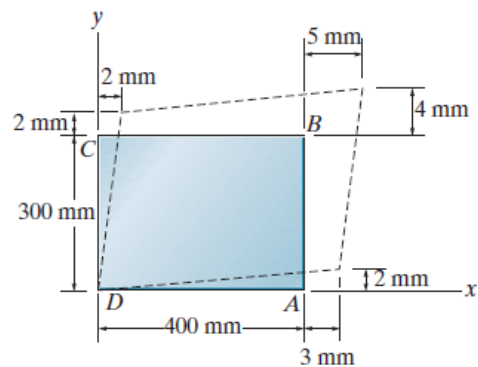


Fig. 4

- If the unstretched length of the bowstring is 35.5 in., determine the average normal strain in the string when it is stretched to the position shown in Fig. 3. [ $\epsilon_{avg} = 0.0689\text{in/in}$ ]
- The piece of plastic shown in Fig. 4 is originally rectangular. Determine the shear strain at corners A, B, C and D if the plastic distorts as shown by the dashed lines. Also determine the average normal strain that occurs along the diagonals AC and DB. [ $(\gamma_B)_{xy} = 11.6(10^{-3})$  rad;  $(\gamma_A)_{xy} = -11.6(10^{-3})$  rad;  $(\gamma_C)_{xy} = -11.6(10^{-3})$  rad;  $(\gamma_D)_{xy} = 11.6(10^{-3})$  rad;  $\epsilon_{AC} = 1.60(10^{-3})$  mm/mm;  $\epsilon_{DB} = 12.8(10^{-3})$  mm/mm]

5. The square plate is deformed into the shape shown by the dashed line in Fig. 5. Determine the average normal strain along diagonal  $AC$  and the shear strain of point  $E$  with respect to the  $x$  and  $y$  axes. [  $(\epsilon_{AC})_{avg} = 0.00347 \text{ mm/mm}$ ;  $(\gamma_E)_{xy} = -0.0332 \text{ rad}$  ]

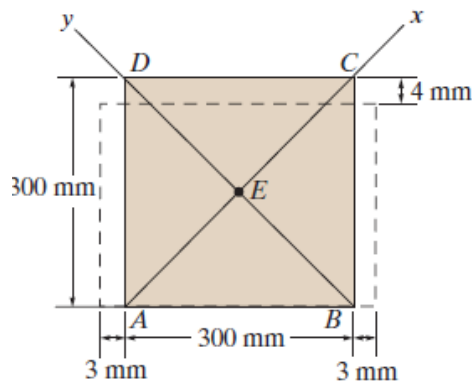


Fig. 5

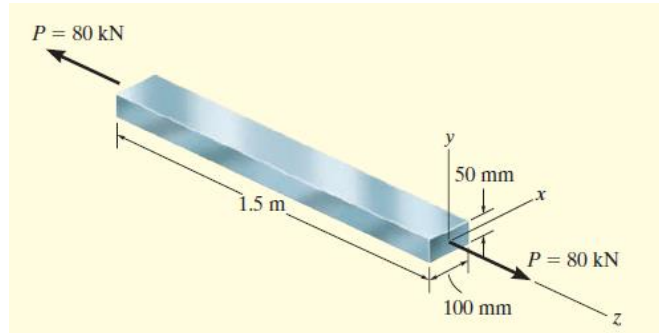


Fig. 6

6. A bar made of A-36 steel has the dimensions shown in Fig. 6. If an axial force of  $P = 80 \text{ kN}$  is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically. [  $\delta_z = 120 \text{ } \mu\text{m}$ ;  $\delta_x = -2.56 \text{ } \mu\text{m}$ ;  $\delta_y = -1.28 \text{ } \mu\text{m}$  ]

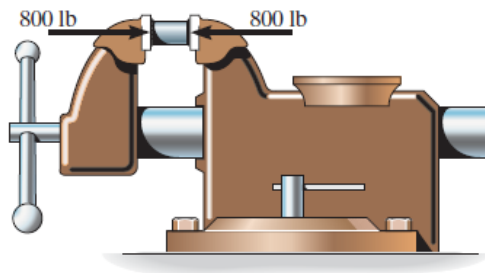


Fig. 7

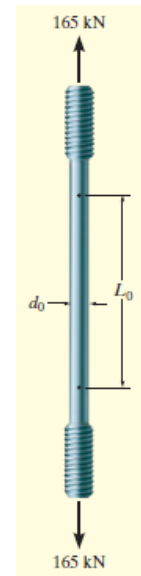


Fig. 8

7. The short cylindrical block of 2014-T6 aluminum, having an original diameter of 0.5 in. and a length of 1.5 in., is placed in the smooth jaws of a vise and squeezed until the axial load applied is 800 lb. Determine (a) the decrease in its length and (b) its new diameter. (Fig. 7) [decrease in length  $\delta = -0.577(10^{-3}) \text{ in.}$ ; new dia.  $d' = 0.50000673$  ]
8. An aluminum specimen shown in Fig. 8 has a diameter of and a gauge length of  $d_0 = 25 \text{ mm}$  and gauge length of  $L_0 = 250 \text{ mm}$ . If a force of 165 kN elongates the gauge length 1.20 mm, determine the modulus of elasticity,  $E$ . Also, determine by how much the force causes the diameter of the specimen to contract.  $G = 26 \text{ GPa}$ , Yield strength,  $\sigma_y = 440 \text{ MPa}$ . [  $E = 70.0 \text{ GPa}$ ; Change in diameter  $\delta' = 0.0416 \text{ mm}$  ]

## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 4

#### Elastic deformation of an axially loaded member

1. Rigid beam  $AB$  rests on the two short posts shown in Fig. 1.  $AC$  is made of steel and has a diameter of 20 mm, and  $BD$  is made of aluminum and has a diameter of 40 mm. Determine the displacement of point  $F$  on  $AB$  if a vertical load of 90 kN is applied over this point. Take  $E_{st} = 200$  GPa,  $E_{al} = 70$  GPa [ $\delta_F = 0.225$  mm]

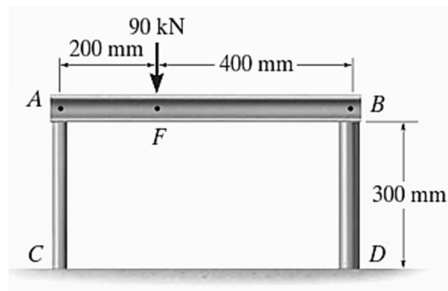


Fig. 1

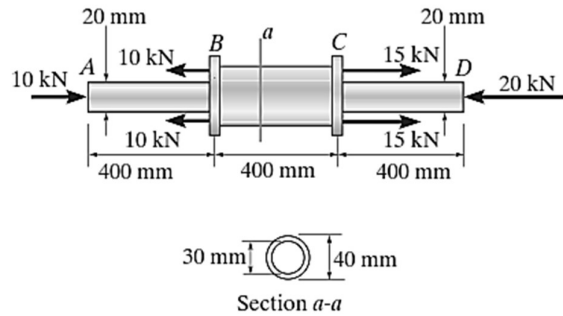


Fig. 2

2. Segments  $AB$  and  $CD$  of the assembly shown in Fig. 2 are solid circular rods, and segment  $BC$  is a tube. If the assembly is made of 6061-T6 aluminum ( $E_{al} = 68.9$  GPa), determine the displacement of end  $D$  with respect to end  $A$ . [ $\delta_{D/A} = -0.449$  mm]

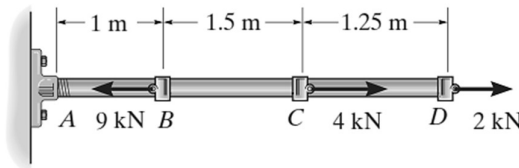


Fig. 3

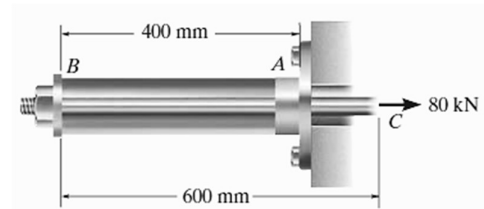


Fig. 4

3. The A-36 steel rod ( $E_{st} = 200$  GPa) is subjected to the loading shown in Fig. 3. If the cross-sectional area of the rod is determine the displacement of  $C$  and  $D$ . Neglect the size of the couplings at  $B$ ,  $C$ , and  $D$ . [ $\delta_C = 0.600$  mm;  $\delta_D = 0.850$  mm]
4. The assembly shown in Fig. 4 consists of an aluminum tube  $AB$  having a cross-sectional area of  $400 \text{ mm}^2$ . A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end  $C$  of the rod. Take  $E_{st} = 200$  GPa,  $E_{al} = 70$  GPa. [4.2mm]
5. If the 20-mm-diameter rod is made of A-36 steel and the stiffness of the spring is,  $k = 50$  MN/m, determine the displacement of end  $A$  when the 60-kN force is applied. [Fig. 5]
6. The ship is pushed through the water using an steel propeller shaft that is 8 m long, measured from the propeller to the thrust bearing  $D$  at the engine. If it has an outer diameter of 400 mm and a wall thickness of 50 mm, determine the amount of axial contraction of the shaft when the propeller exerts a force on the shaft of 5 kN. [Fig. 6]

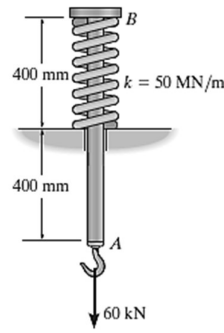


Fig. 5

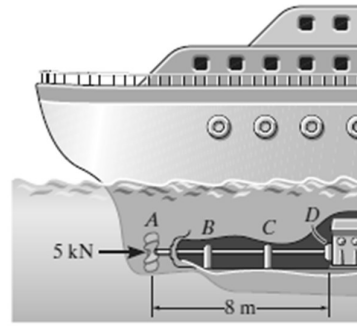


Fig. 6

7. The rod has a slight taper and length  $L$ . It is suspended from the ceiling and supports a load  $P$  at its end. Show that the displacement of its end due to this load is  $\delta = PL/\pi E r_2 r_1$ . Neglect the weight of the material. The modulus of elasticity is  $E$ . [Fig. 7]
8. Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load  $P$ . [Fig. 8]
9. The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the average normal stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm.  $E_s = 200$  GPa,  $E_c = 24$  GPa.
10. The concrete post is reinforced using six steel reinforcing rods, each having a diameter of 20 mm. Determine the stress in the concrete and the steel if the post is subjected to an axial load of 900 kN.

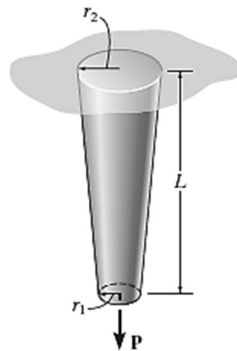


Fig. 7

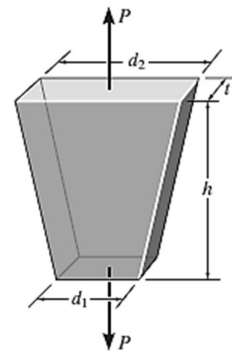


Fig. 8

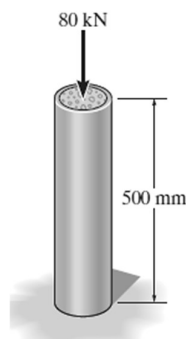


Fig. 9

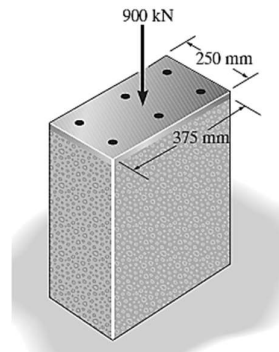


Fig. 10



## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 5

#### Stress Transformation and Mohr's circle

1. The state of plane stress at a point is represented by the element shown in Fig. 1. Determine the state of stress at the point on another element oriented  $30^\circ$  clockwise from the position shown.  $[-25.8, -68.8$  and  $-4.15$  MPa]

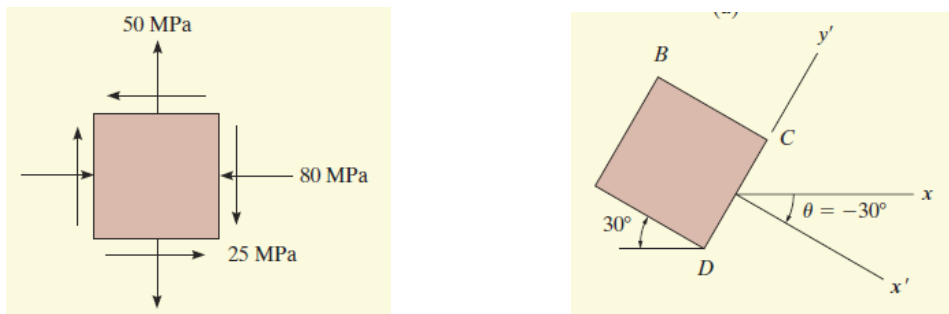


Fig. 1

2. The state of plane stress at a failure point on the shaft is shown on the element in Fig. 2. Represent this stress state in terms of the principal stresses. Also, represent this stress state in terms of the maximum in-plane shear stress and associated average normal stress.  $[116, -46.4$  MPa,  $66.3^\circ$  and  $81.4$  MPa,  $35$  MPa and  $21.3^\circ]$

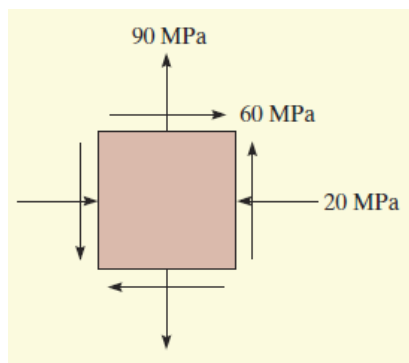


Fig. 2

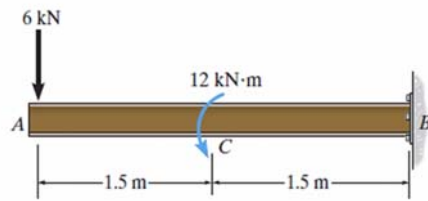
3. Recalculate the stresses in Fig. 1 and Fig. 2 by using Mohr's Circle Method.

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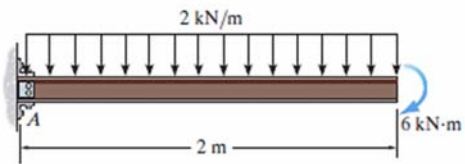
CEA 1120 – Tutorial Sheet # 6

Shear Force and Bending Moment

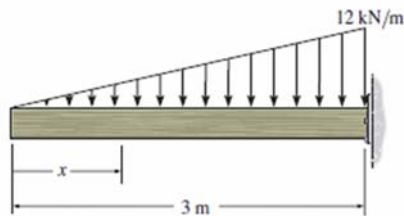
1. Draw the shear force and bending moment diagrams for the beams given below. (Fig. 1)



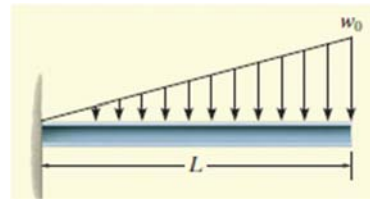
(a)



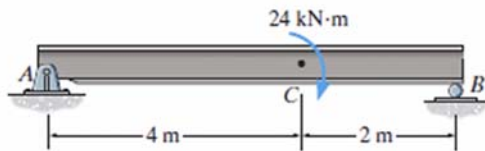
(b)



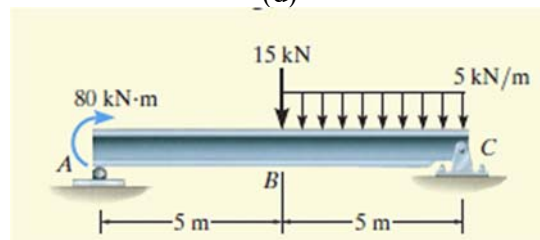
(c)



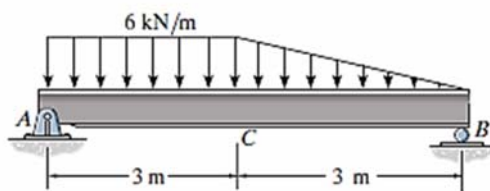
(d)



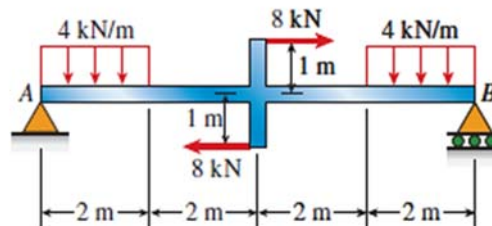
(e)



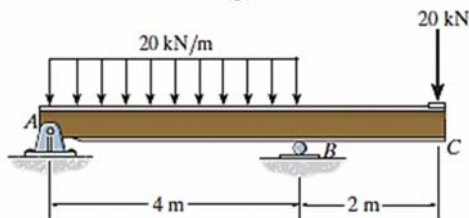
(f)



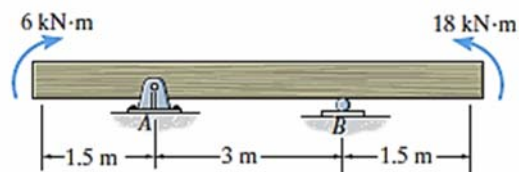
(g)



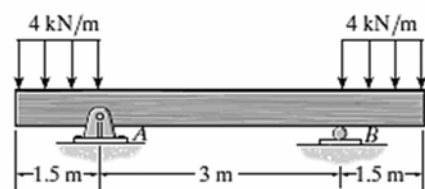
(h)



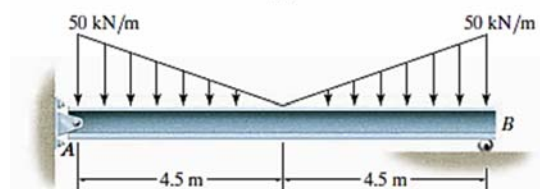
(i)



(j)



(k)#



(l)

Fig. 1

2. A reinforced concrete pier is used to support the stringers for a bridge deck. Draw the shear and moment diagrams for the pier when it is subjected to the stringer loads shown. Assume the columns at  $A$  and  $B$  exert only vertical reactions on the pier. (Fig. 2)

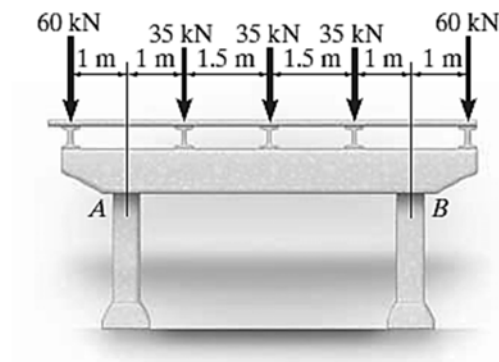


Fig. 2

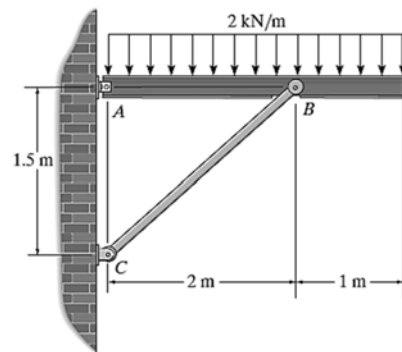


Fig. 3

3. The beam is subjected to the uniform distributed load shown. Draw the shear and moment diagrams for the beam. (Fig. 3)

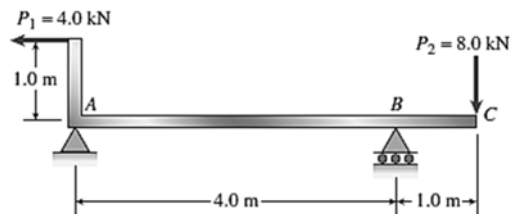


Fig. 4

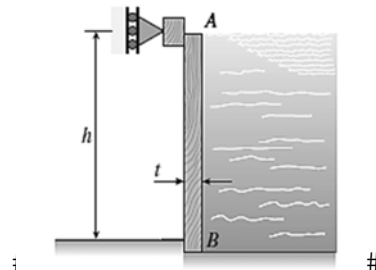


Fig. 5

4. Draw the shear and moment diagrams for the beam shown in Fig. 4.
5. A small dam of height  $h = 2.0$  m is constructed of vertical wood beams  $AB$ . Consider the beams to be simply supported at the top and bottom. Draw the shear and moment diagrams for the beam. (Fig. 5)
6. Under cruising conditions, the distributed load acting on the wing of a small airplane has the idealized variation shown in the figure. Calculate the shear force  $V$  and bending moment  $M$  at the inboard end of the wing (at junction of wing and fuselage).

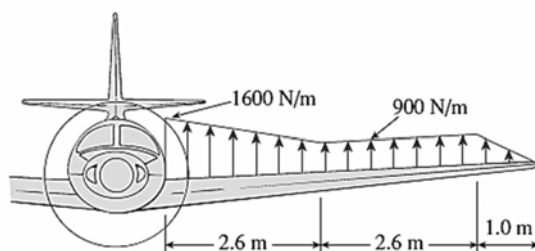


Fig. 6



Wings of small airplane have distributed uplift loads.

## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 7

#### Bending Stress Distribution

1. The simply supported beam has the cross-sectional area shown in Fig. 1. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.

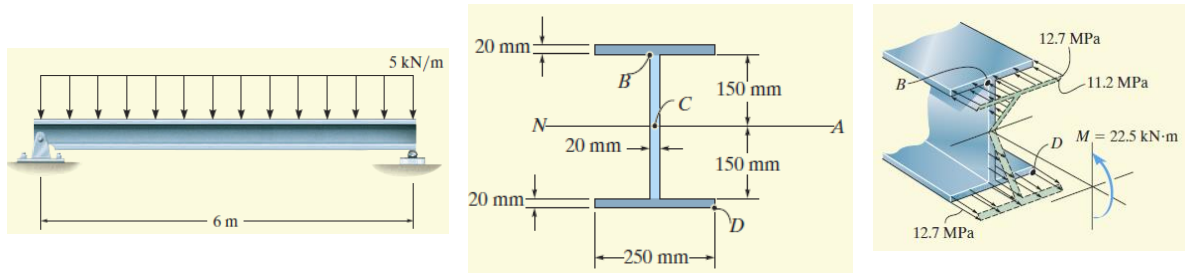


Fig. 1

2. If the beam is subjected to a bending moment of  $M = 20 \text{ kNm}$ , determine the maximum bending stress in the beam. Fig. 2.

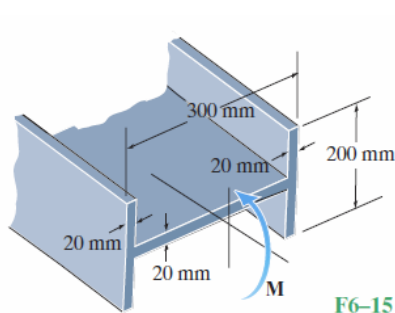


Fig. 2

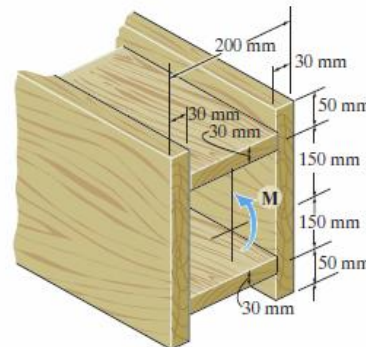


Fig. 3

3. If the beam is subjected to a bending moment of  $M = 10 \text{ kNm}$ , determine the maximum bending stress in the beam. [Fig. 3]
4. A member having the dimensions shown in Fig. 4, is used to resist an internal bending moment of  $M = 90 \text{ kNm}$ . Determine the maximum stress in the member if the moment is applied (a) about the  $z$  axis (b) about the  $y$  axis. Sketch the stress distribution for each case.
5. Determine the moment  $M$  that should be applied to the beam in order to create a compressive stress at point D of  $\sigma_D = 30 \text{ MPa}$ . Also sketch the stress distribution acting over the cross section and compute the maximum stress developed in the beam. (Fig. 5)

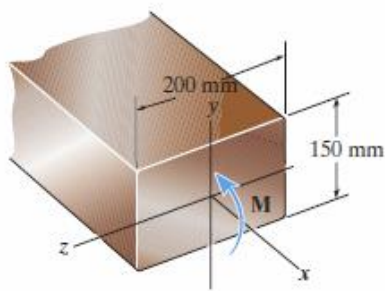


Fig. 4

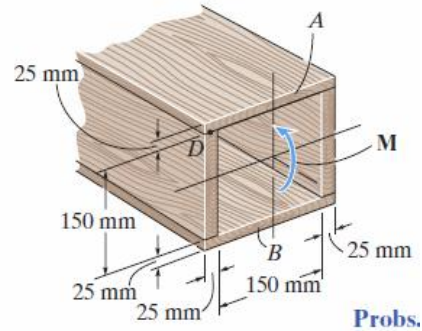


Fig. 5

6. The beam is made from three boards nailed together as shown in Fig. 6. If the moment acting on the cross section is  $M = 600 \text{ N}\cdot\text{m}$ , determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.
7. The aluminum strut has a cross-sectional area in the form of a cross. If it is subjected to the moment  $M = 8 \text{ kNm}$ , determine the maximum bending stress in the beam, and sketch a three-dimensional view of the stress distribution acting over the entire cross-sectional area. Fig. 7.

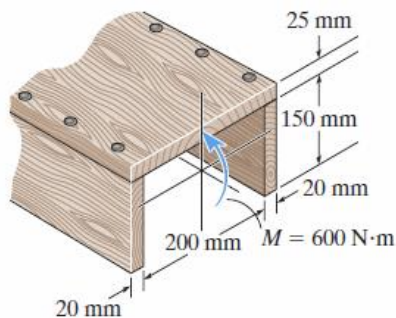


Fig. 6

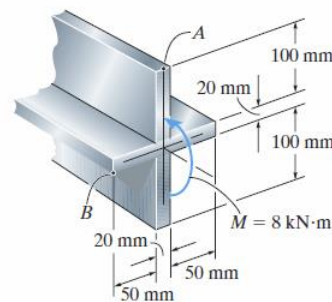


Fig. 7

8. Two designs for a beam are to be considered. Determine which one will support a moment of  $M = 150 \text{ kNm}$  with the least amount of bending stress. What is that stress?

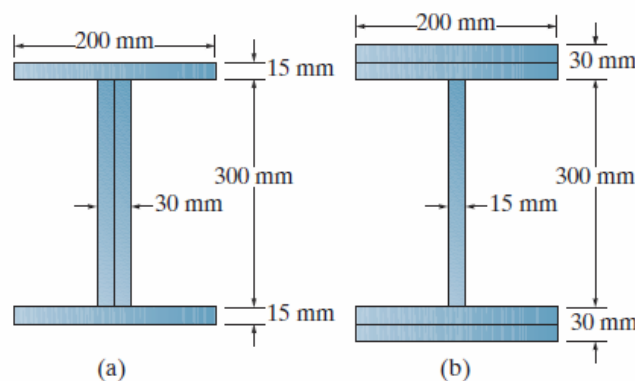


Fig. 8

## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 8

#### Shear Stress Distribution

1. A steel wide-flange beam has the dimensions shown in Fig. 1. If it is subjected to a shear of plot the shear-stress distribution acting over the beam's cross-sectional area.

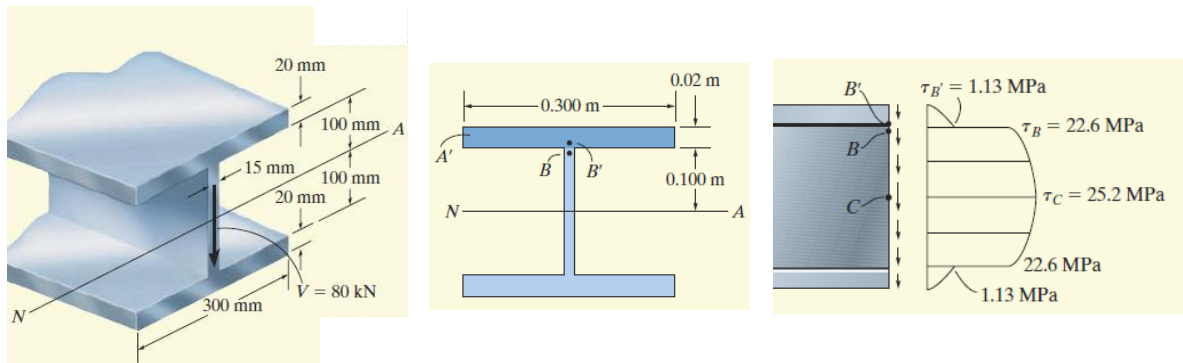


Fig. 1

2. If the beam is subjected to a shear force of  $V = 100 \text{ kN}$ , determine the shear stress developed at point A. Represent the state of stress at A on a volume element. Fig. 2.

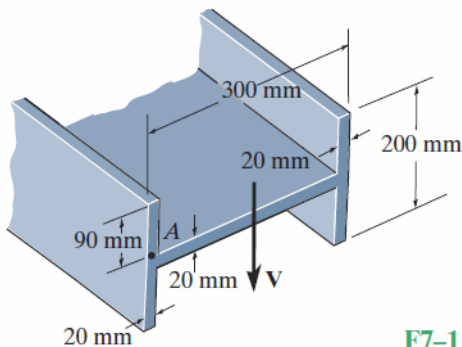


Fig. 2

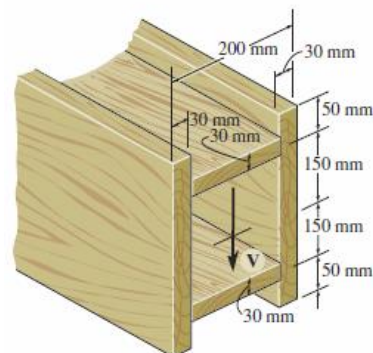


Fig. 3

3. If the beam is subjected to a shear force of  $V = 20 \text{ kN}$ , determine the maximum shear stress developed in the beam. [Fig. 3]
4. Determine the shear stress at point B on the web of the cantilevered strut at section a–a. Determine the maximum shear stress acting at section a–a of the cantilevered strut. (Fig. 4)
5. Determine the maximum shear stress in the T-beam at the critical section where the internal shear force is maximum. Determine the maximum shear stress in the T-beam at point C. Show the result on a volume element at this point. (Fig. 5)

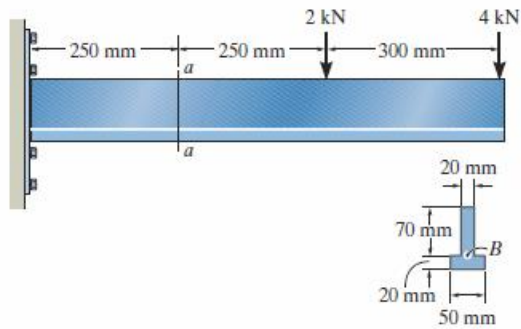


Fig. 4

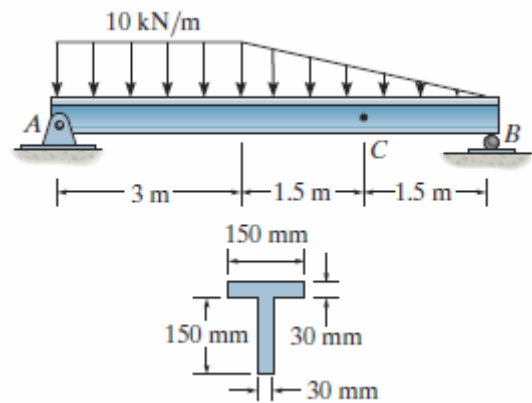


Fig. 5

6. If the beam is subjected to a shear of  $V = 15 \text{ kN}$ , determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Show that the neutral axis is located at from  $y = 0.1747 \text{ m}$  the bottom and  $I_{NA} = 0.21821 \times 10^{-3} \text{ m}^4$ . (Fig. 6)
7. The wood beam has an allowable shear stress of  $\tau_{\max} = 7 \text{ MPa}$ . Determine the maximum shear force  $V$  that can be applied to the cross section. Fig. 7.

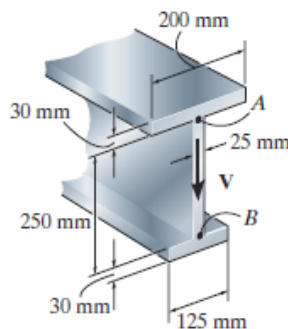


Fig. 6

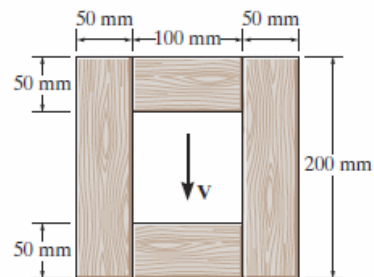
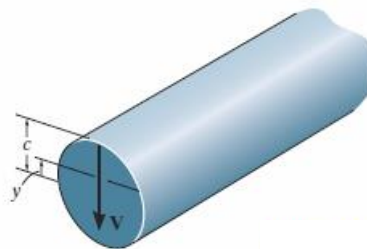


Fig. 7

8. Plot the shear-stress distribution over the cross section of a rod that has a radius  $c$ . By what factor is the maximum shear stress greater than the average shear stress acting over the cross section?



## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 9

#### Torsion

1. The pipe shown in Fig. 1 has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at A using a torque wrench at B, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.

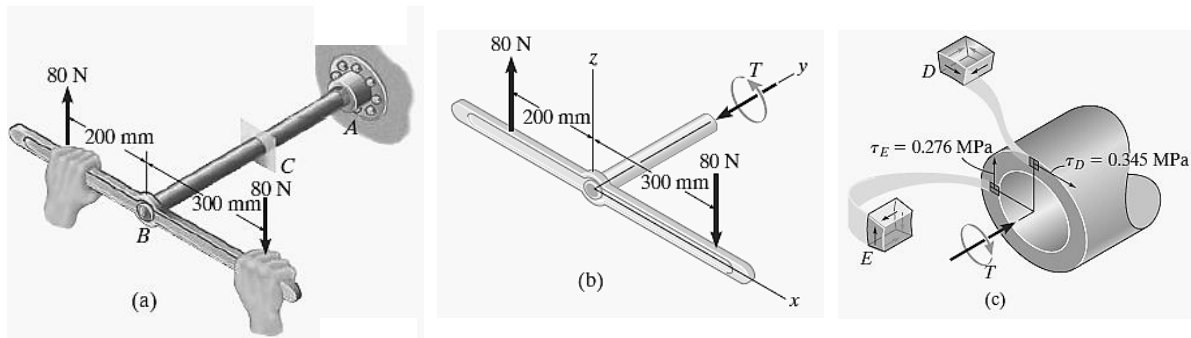


Fig. 1

2. A hollow cylindrical steel shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm (Fig. 2). (a) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120 MPa? (b) What is the corresponding minimum value of the shearing stress in the shaft?

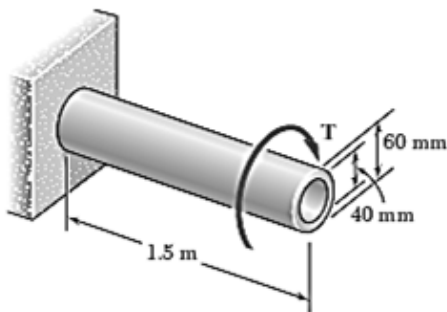


Fig. 2

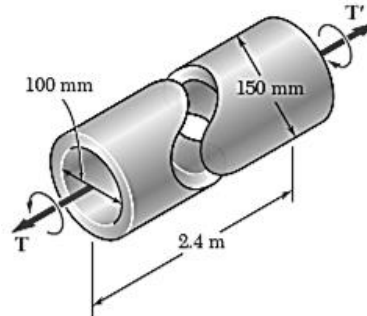


Fig. 3

3. The preliminary design of a large shaft connecting a motor to a generator calls for the use of a hollow shaft with inner and outer diameters of mm and 150 mm, respectively. Knowing that the allowable shearing stress is 84 MPa, determine the maximum torque that can be transmitted (a) by the shaft as designed, (b) by a solid shaft of the same weight, (c) by a hollow shaft of the same weight and of 200-mm outer diameter. (Fig. 3)
4. For the shaft subjected to the applied torques as shown in Fig. 4, determine the angle of twist of end D relative to fixed end A.
5. The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which  $G = 77 \text{ GPa}$ , determine the angle of twist at end A. (Fig. 5)



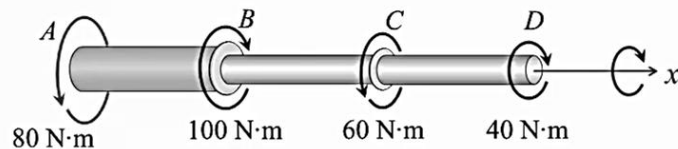


Fig. 4

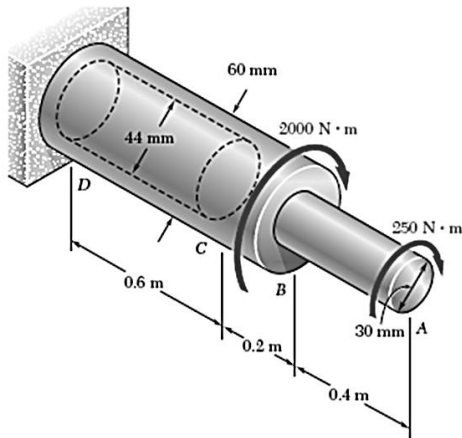


Fig. 5

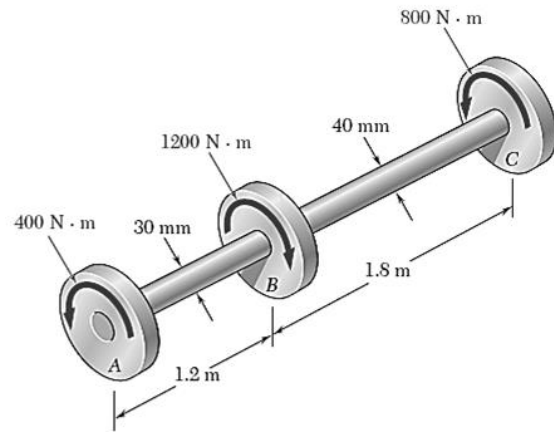


Fig. 6

6. The torques shown are exerted on pulleys A, B and C. Knowing the both shafts are solid and made of brass ( $G = 39 \text{ GPa}$ ), determine the angle of twist between (a) A and B, (b) A and C. (Fig. 6)
7. The aluminium rod AB ( $G = 27 \text{ GPa}$ ) is bonded to the brass rod BD ( $G = 39 \text{ GPa}$ ). Knowing that the portion CD of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at A. (Fig. 7).

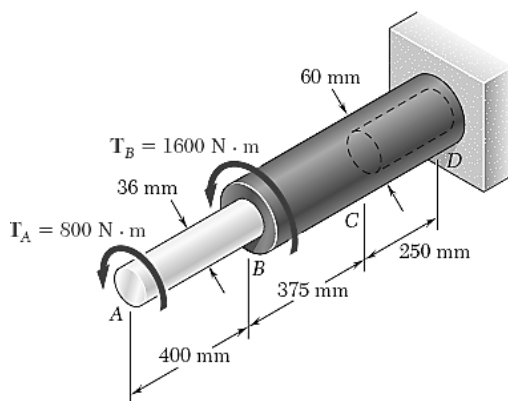


Fig. 7

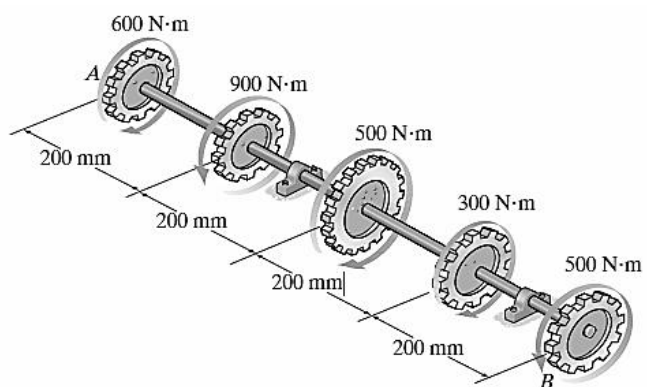
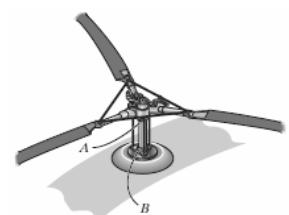


Fig. 7

8. A series of gears are mounted on the 40-mm diameter mild steel shaft. Determine the angle of twist of gear B relative to gear A.
9. The engine of the helicopter is delivering 600 HP to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine the diameter of the shaft AB, if the allowable shear stress is 60 MPa and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 600 mm long and made from steel. (1HP: 746Nm/s)



## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 10

#### Trusses

1. Classify each of the following trusses given in Fig. 1 as statically determinate, statically indeterminate, or unstable. If indeterminate, state its degree.

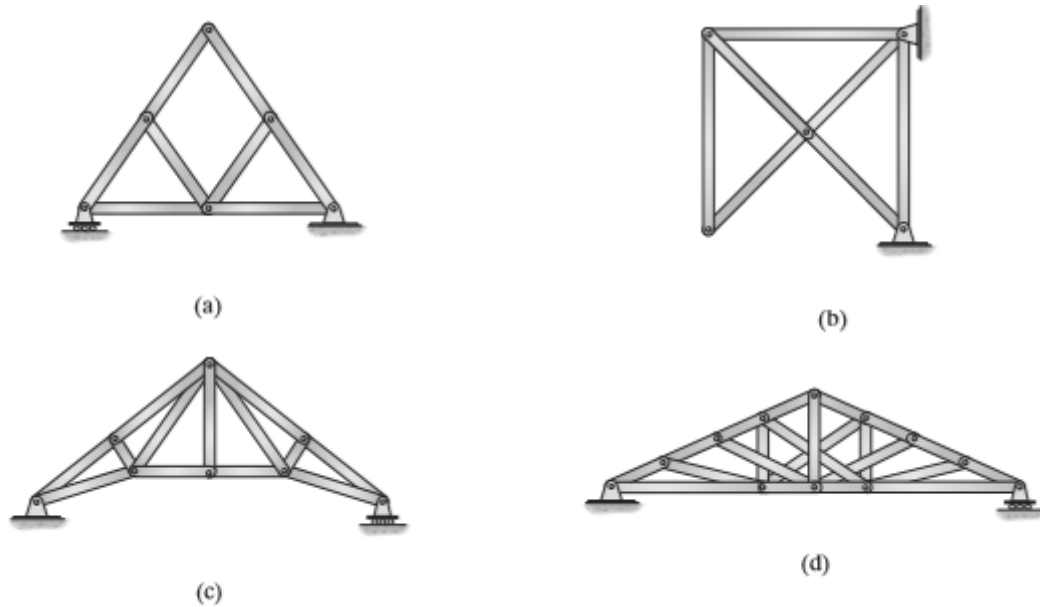


Fig. 1

2. Determine the force in each member of the roof truss shown in Fig. 2. State whether the members are in tension or compression.

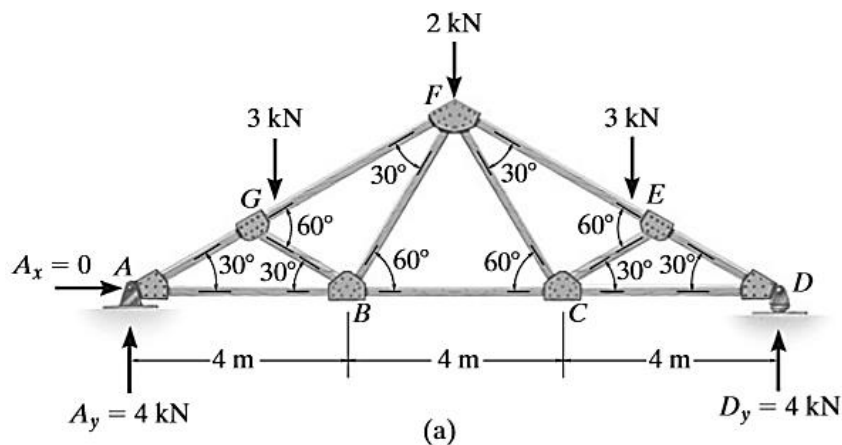


Fig. 2

3. Determine the force in each member of the truss using *Method of Joints* and state whether it is in tension or compression. (Fig. 3)
4. Determine the internal forces in truss members **BC**, **GC** and **GF**, using *Methods of Sections* and state whether it is in tension or compression. (Fig. 4)

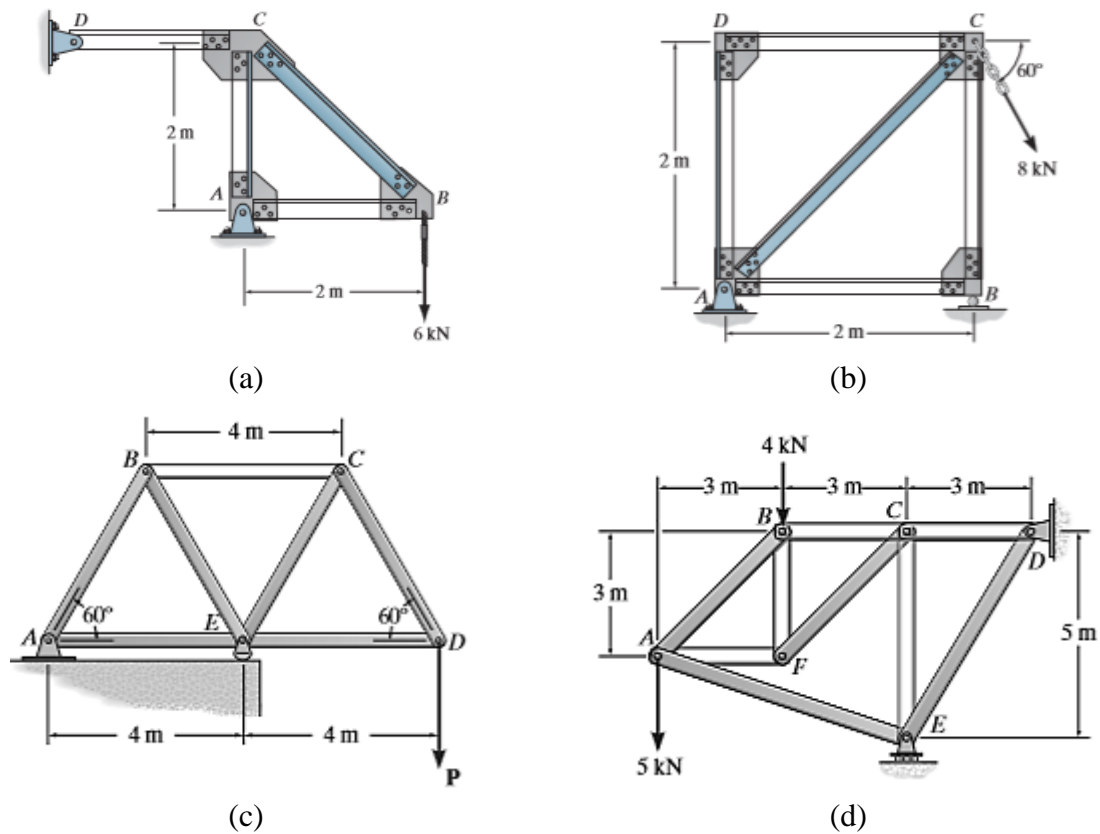


Fig. 3

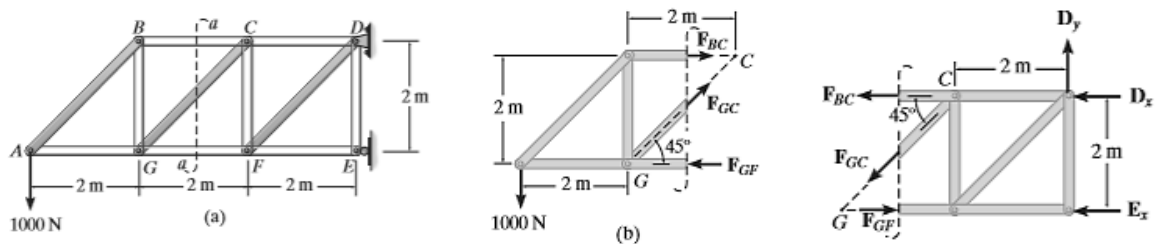


Fig. 4

5. The Howe truss is subjected to the loading shown. Determine the forces in members **GF**, **CD**, and **GC**. State if the members are in tension or compression. Assume all members are pin connected. (Fig. 5)

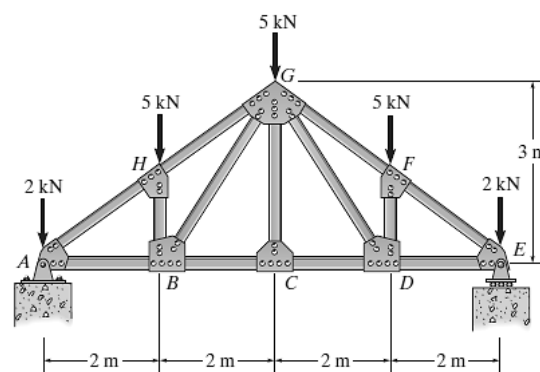


Fig. 5

## Department of Civil Engineering

### CEA 1120 – Tutorial Sheet # 11

#### Arches

1. The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at A and C and the tension in the cable.

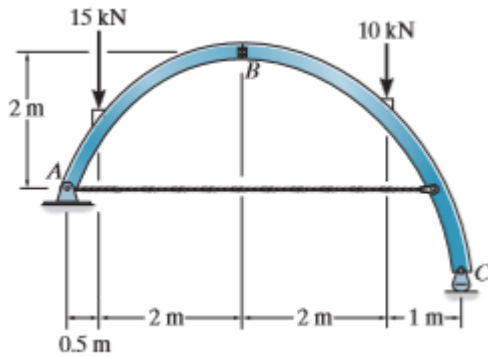


Fig. 1

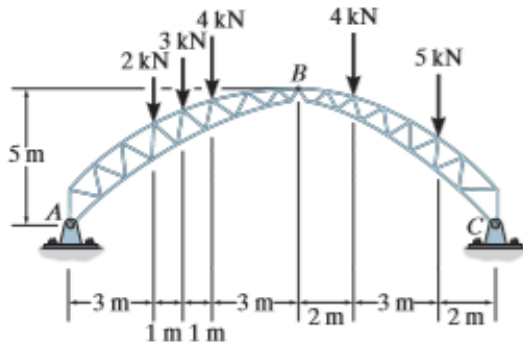


Fig. 2

2. Determine the resultant forces at the pins A, B, and C of the three-hinged arched roof truss. (Fig.2)
3. The three-hinged spandrel arch is subjected to the loading shown. Determine the internal moment in the arch at point D. (Fig. 3)

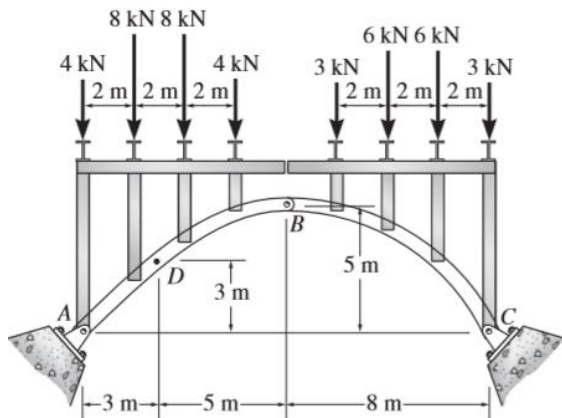


Fig. 3

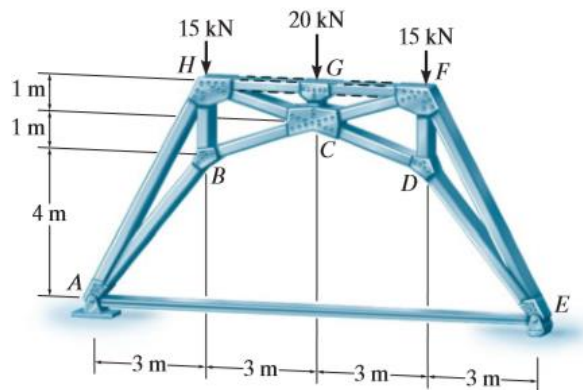


Fig. 4

4. The three-hinged tied arch is subjected to the loading shown in Fig. 4. Determine the force in members **CH** and **CB**. The dashed member **GF** of the truss is intended to carry no force.