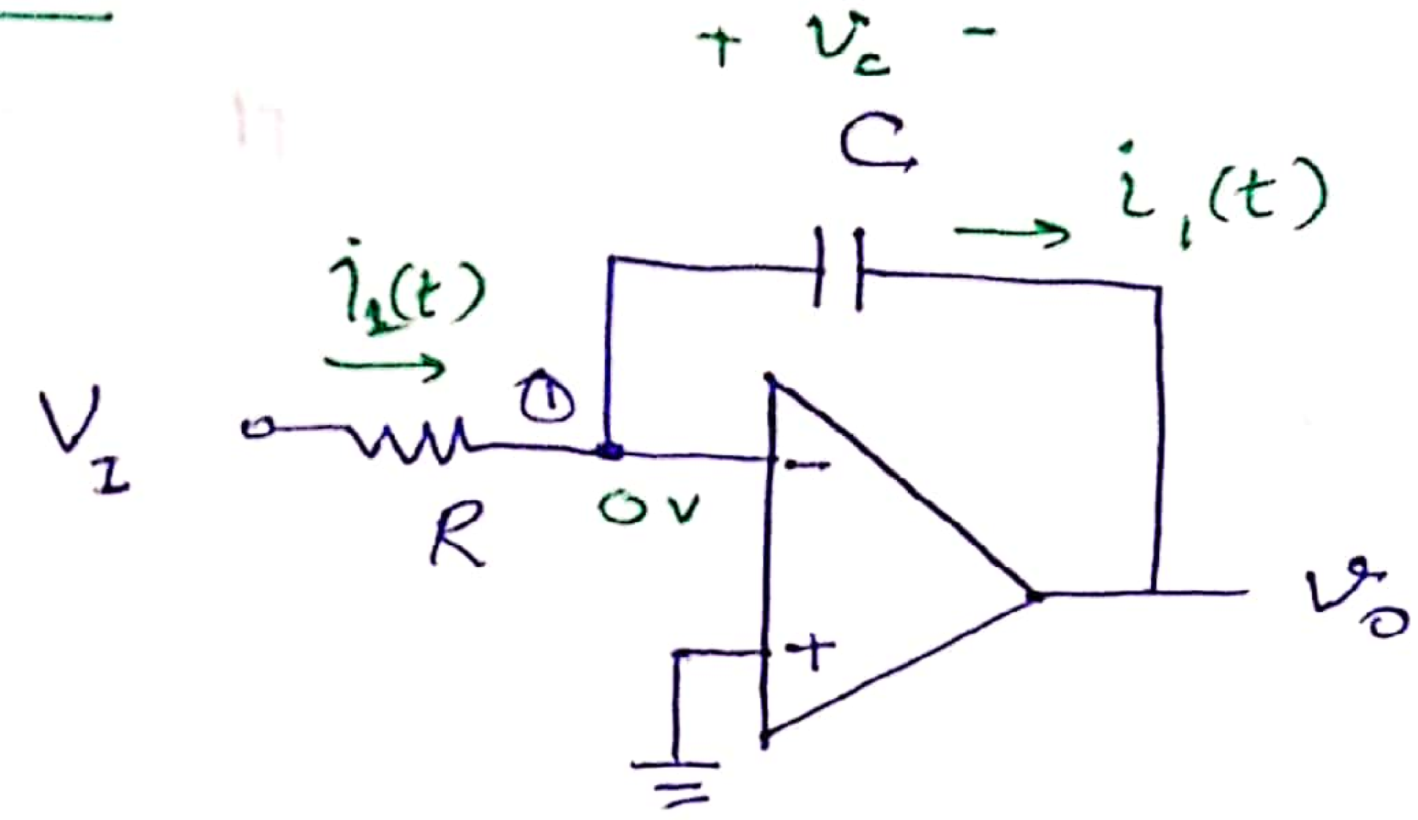
Opamp Integrator



Inverting Integrator Configuration

writing nodal equation at 1

$$0\left(\frac{1}{R}+sC\right) = \frac{V_I}{R} + v_o(sC)$$

capacitor impedance

$$X_c = \frac{1}{j\omega C}$$

$$X_C = \frac{1}{sC}$$

$(s = j\omega)$

Voltage gain

$$\boxed{\frac{V_o}{V_i} = -\frac{1}{sRC}}$$

(freq. domain)

for physical freq., $s = j\omega$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega RC}$$

Magnitude: $\left|\frac{V_o}{V_i}\right| = \frac{1}{\omega RC}$

Phase: $\phi = +90^\circ$

[RC is also known as Integrator time constant]

At $\left|\frac{V_o}{V_i}\right| = 1$, $\boxed{\omega = \frac{1}{RC}}$

Known as integrator frequency

Output voltage in time domain:

if $i_1(t)$ be the current flowing through R, the same will also flow through C.

$$i_1(t) = \frac{V_i(t)}{R} \qquad -(1)$$

Charge stored on capacitor, if from 0 to t,

$$Q = \int_0^t i_1(t)\,dt$$

(capacitor principle)

Capacitor voltage, $v_c(t) = \frac{1}{C}\int_0^t i_1(t)\,dt$

$$v = \frac{Q}{C}$$

but here $v_o = -v_c$

$$\therefore \quad v_o(t) = -\frac{1}{C}\int_0^t i_1(t)\,dt$$

$$\boxed{v_o(t) = -\frac{1}{RC}\int_0^t v_i(t)\,dt} \qquad \text{Using (i)}$$

DC problem in integrator

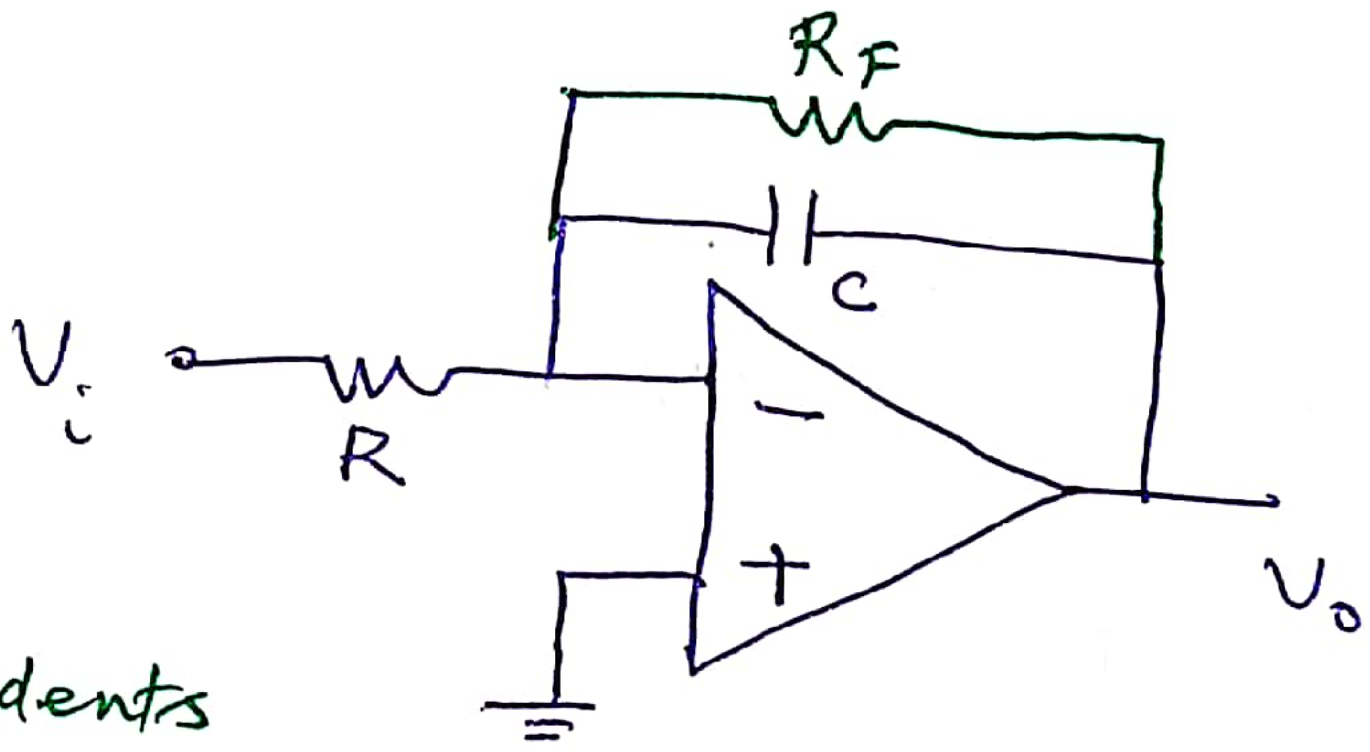
$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\cancel{j}\omega C} \qquad \frac{1}{\omega RC}$$

at dc $(\omega = 0)$, $\frac{V_o}{V_i} \Rightarrow \infty$

The gain becomes infinite and opamp saturates

To avoid this problem, generally, a feedback resistor, R_F is connected in parallel with C

$$\boxed{\frac{V_o}{V_i} = -\frac{R_F/R}{1+sR_FC}}$$



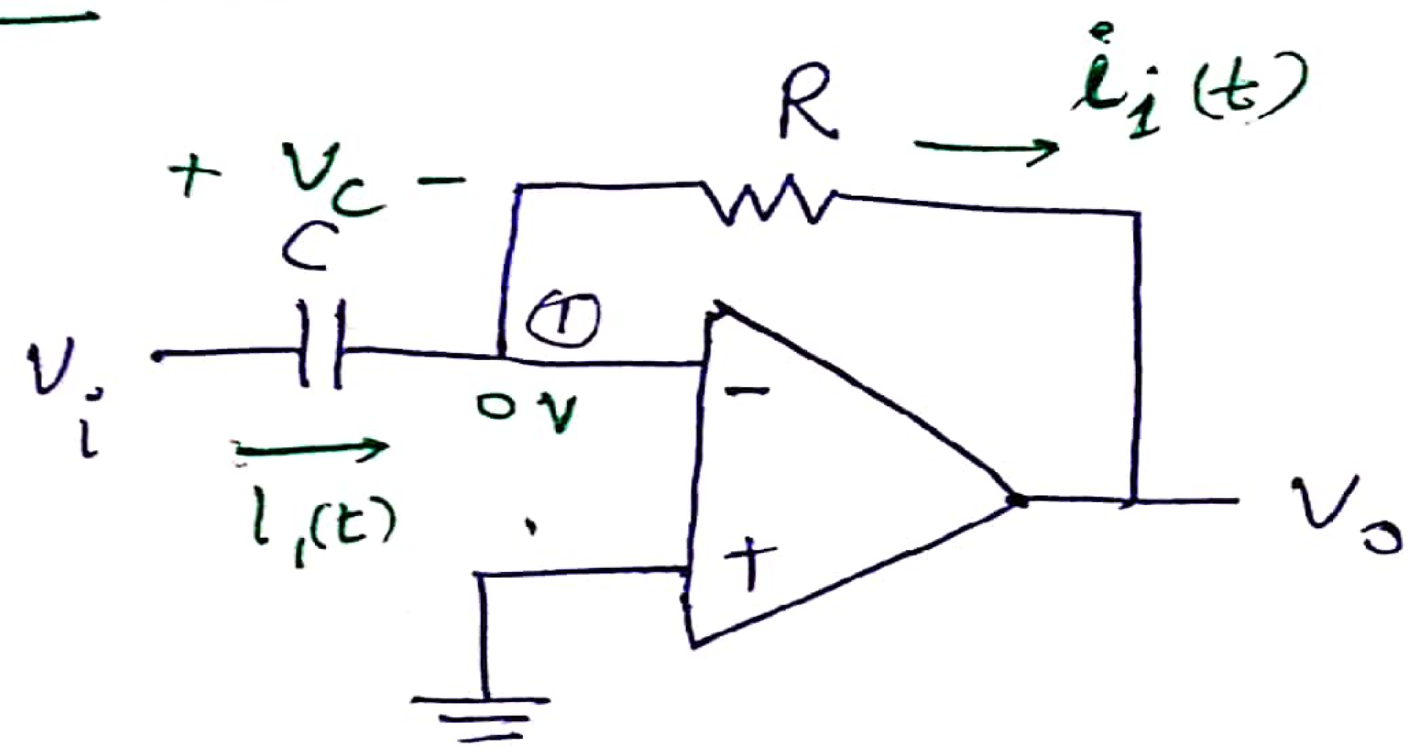
⇒ Derivation is left for students

In this case,

at dc $(\omega = 0)$, capacitor behaves as open circuit and structure simply becomes ~~and~~ an inverting amplifier with voltage gain

$$\frac{V_o}{V_i} = -R_F/R$$

Opamp Differentiator :-



Interchanging the position of R & C results in an opamp differentiator

Writing nodal equation at 1,

$$0\left(\frac{1}{R} + sC\right) = V_i(sC) + V_o \times \frac{1}{R}$$

$$\Rightarrow \quad \frac{V_o}{R} = -V_i sC$$

$$\boxed{\frac{V_o}{V_i} = -sRC}$$

for physical frequency $s = j\omega$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega RC$$

magnitude, $\left|\frac{V_o}{V_i}\right| = \omega RC$

At $|V_o/V_i| = 1$

$$\boxed{\omega = \frac{1}{RC}}$$

Known as differentiator freq.

Phase, $\phi = -90^\circ$

Output voltage in time domain :

for capacitor, ~~$V_c(t)$~~ with voltage $V_c(t)$

$$i_1(t) = C\frac{dV_c(t)}{dt}$$

(Capacitor principle)

but here $V_c(t) = V_i(t)$

$$i_1(t) = C \frac{dV_i(t)}{dt}$$

Same $i_1(t)$ will also flow through R,

$$V_o(t) = -R\, i_1(t)$$

$$\boxed{V_o(t) = -RC \frac{dV_i(t)}{dt}}$$