2-3 OCTAL NUMBER SYSTEM

The octal number system is often used in digital computer work. The **octal number system** has a base of *eight*, meaning that it has eight possible digits: 0, 1, 2, 3, 4, 5, 6, and 7. Thus, each digit of an octal number can have any value from 0 to 7. The digit positions in an octal number have weights as follows:

8^4	8^3	8^2	8^1	8^0	• 8^{-1}	8^{-2}	8^{-3}	8^{-4}	8^{-5}

Octal point

Octal-to-Decimal Conversion

An octal number, then, can be converted easily to its decimal equivalent by multiplying each octal digit by its positional weight. For example:

$$
\begin{aligned}
372_8 &= 3 \times (8^2) + 7 \times (8^1) + 2 \times (8^0) \\
&= 3 \times 64 + 7 \times 8 + 2 \times 1 \\
&= 250_{10}
\end{aligned}
$$

$$
\begin{aligned}
24.6_8 &= 2 \times (8^1) + 4 \times (8^0) + 6 \times (8^{-1}) \\
&= 20.75_{10}
\end{aligned}
$$

Decimal-to-Octal Conversion

A decimal integer can be converted to octal by using the same repeated-division method that we used in the decimal-to-binary conversion (Figure 2-1), but with a division factor of 8 instead of 2. An example follows.

$$\frac{266}{8} = \boxed{33} + \text{remainder of } 2 \quad \text{LSD}$$

$$\frac{\boxed{33}}{8} = 4 + \text{remainder of } 1$$

$$\frac{4}{8} = 0 + \text{remainder of } 4 \quad \text{MSD}$$

$$266_{10} = \boxed{412_8}$$

Note that the first remainder becomes the least significant digit (LSD) of the octal number, and the last remainder becomes the most significant digit (MSD).

Octal-to-Binary Conversion

The primary advantage of the octal number system is the ease with which conversion can be made between binary and octal numbers. The conversion from octal to binary is performed by converting *each* octal digit to its three-bit binary equivalent. The eight possible digits are converted as indicated in Table 2-1.

TABLE 2-1

Octal Digit	0	1	2	3	4	5	6	7
Binary Equivalent	000	001	010	011	100	101	110	111

Using these conversions, we can convert any octal number to binary by individually converting each digit. For example, we can convert 472_8 to binary as follows:

$$\begin{array}{ccc} 4 & 7 & 2 \\ \downarrow & \downarrow & \downarrow \\ 100 & 111 & 010 \end{array}$$

Thus, octal 472 is equivalent to binary 100111010. As another example, consider converting 5431_8 to binary:

$$\begin{array}{cccc} 5 & 4 & 3 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 101 & 100 & 011 & 001 \end{array}$$

Thus, $5431_8 = 101100011001_2$.

Binary-to-Octal Conversion

Converting from binary integers to octal integers is simply the reverse of the foregoing process. The bits of the binary number are grouped into groups of *three* bits, starting at the LSB. Then each group is converted to its octal equivalent (Table 2-1). To illustrate, consider the conversion of 100111010_2 to octal.

$$\begin{array}{ccc} \underbrace{1\ 0\ 0} & \underbrace{1\ 1\ 1} & \underbrace{0\ 1\ 0} \\ \downarrow & \downarrow & \downarrow \\ 4 & 7 & 2_8 \end{array}$$

Sometimes the binary number will not have even groups of three bits. For those cases, we can add one or two 0s to the left of the MSB of the binary number to fill out the last group. This is illustrated below for the binary number 11010110.

$$\begin{array}{ccc} \underbrace{\boxed{0}\ 1\ 1} & \underbrace{0\ 1\ 0} & \underbrace{1\ 1\ 0} \\ \downarrow & \downarrow & \downarrow \\ 3 & 2 & 6_8 \end{array}$$

Note that a 0 was placed to the left of the MSB to produce even groups of three bits.