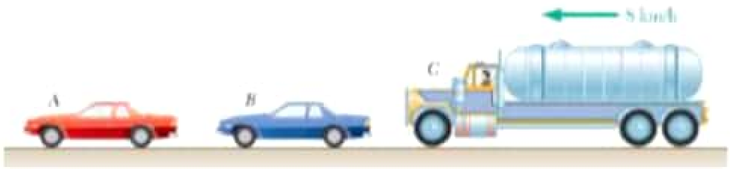


PROBLEM 14.2

Two identical 1350-kg automobiles A and B are at rest with their brakes released when B is struck by a 5400-kg truck C which is moving to the left at 8 km/h. A second collision then occurs when B strikes A. Assuming the first collision is perfectly plastic and the second collision is perfectly elastic, determine the velocities of the three vehicles just after the second collision.

SOLUTION

Given: $m_A = m_B = 1350$ kg and $m_C = 5400$ kg.

Let v_A, v_B, and v_C be the sought after final velocities, positive to the left.

Initial velocities: $(v_A)_0 = (v_B)_0 = 0,\ (v_C)_0 = 8 \text{ km/h} = 2.2222 \text{ m/s}$

First collision. Truck C strikes car B. Plastic impact: $e = 0$

Let $(v_{BC})_0$ be the common velocity of B and C after impact.

Conservation of momentum for B and C:

$$(m_B + m_C)v_{BC} = m_B(v_B)_0 + m_C(v_C)_0$$

$$6750\ v_{BC} = 0 + (5400)(2.2222) \qquad v_{BC} = 1.77778 \text{ m/s}$$

Second collision. Car-truck BC strikes car A.

Elastic impact. $e = 1$

$$v_A - v_{BC} = -e\left[(v_A)_0 - (v_{BC})_0\right] = -1.77778 \text{ m/s} \qquad (1)$$

Conservation of momentum for A, B, and C.

$$m_A v_A + (m_B + m_C)(v_{BC}) = m_A(v_A)_0 + (m_B + m_C)(v_{BC})_0$$

$$1350\ v_A + 6750\ v_{BC} = 0 + (6750)(1.77778) \qquad (2)$$

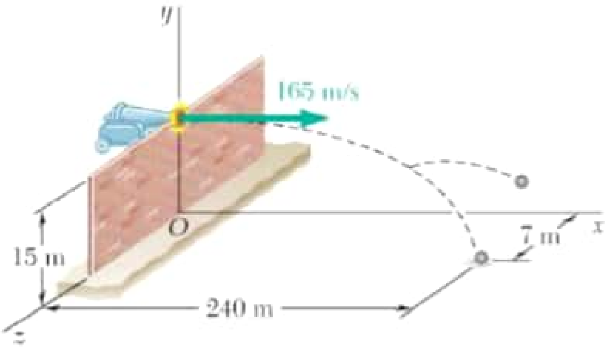
Solving (1) and (2) simultaneously for v_A and v_{BC},

$$v_A = 2.9630 \text{ m/s}, \quad v_{BC} = 1.18519 \text{ m/s} = v_B = v_C$$

$\mathbf{v}_A = 10.67 \text{ km/h} \longleftarrow$ ◀

$\mathbf{v}_B = 4.27 \text{ km/h} \longleftarrow$ ◀

$\mathbf{v}_C = 4.27 \text{ km/h} \longleftarrow$ ◀



PROBLEM 14.18

An 18-kg cannonball and a 12-kg cannonball are chained together and fired horizontally with a velocity of 165 m/s from the top of a 15-m wall. The chain breaks during the flight of the cannonballs and the 12-kg cannonball strikes the ground at $t = 1.5$ s, at a distance of 240 m from the foot of the wall, and 7 m to the right of the line of fire. Determine the position of the other cannonball at that instant. Neglect the resistance of the air.

SOLUTION

Let subscript A refer to the 12-kg cannonball and B to the 18-kg cannonball.

The motion of the mass center of A and B is uniform in the x-direction, uniformly accelerated with acceleration $-g = -9.81 \text{ m/s}^2$ in the y-direction, and zero in the z-direction.

$$\bar{x} = (v_0)_x t = (165 \text{ m/s})(1.5 \text{ s}) = 247.5 \text{ m}$$

$$\bar{y} = \bar{y}_0 + (\bar{v}_0)_y t - \frac{1}{2} g t^2$$

$$= 15 \text{ m} + 0 - \frac{1}{2}(9.81 \text{ m/s}^2)(1.55)^2 = 3.964 \text{ m}$$

$$\bar{z} = 0$$

$$\bar{\mathbf{r}} = (247.5 \text{ m})\mathbf{i} + (3.964 \text{ m})\mathbf{j}$$

Definition of mass center:

$$m\bar{\mathbf{r}} = m_A \mathbf{r}_A + m_B \mathbf{r}_B$$

Data: $m_A = 12 \text{ kg}, \quad m_B = 18 \text{ kg}, \quad m = m_A + m_B = 30 \text{ kg}$

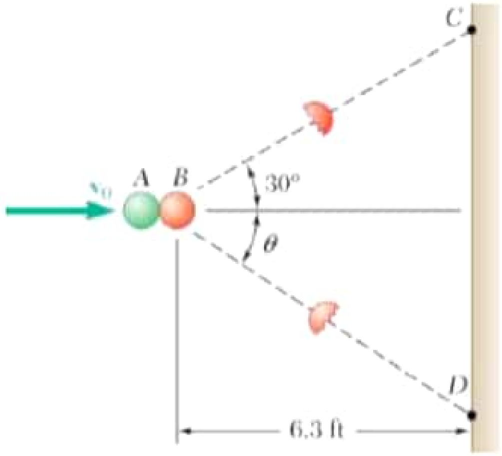
$$t = 1.5 \text{ s}, \quad x_A = 240 \text{ m}, \quad y_A = 0, \quad z_A = 7 \text{ m}$$

$$(30)(247.5\mathbf{i} + 3.964\,\mathbf{j}) = (12)(240\mathbf{i} + 7\mathbf{k}) + (18)(x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k})$$

i: $(30)(247.5) = (12)(240) + 18\,x_B$ $\qquad x_B = 253 \text{ m}$ ◀

j: $(30)(3.964) = (12)(0) + 18\,y_B$ $\qquad y_B = 6.61 \text{ m}$ ◀

k: $(30)(0) = (12)(7) + 18\,z_B$ $\qquad z_B = -4.67 \text{ m}$ ◀



PROBLEM 14.22

Two spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Sphere A is moving at a speed $v_0 = 16$ ft/s when it strikes sphere B which is at rest and the impact causes sphere B to break into two pieces, each of mass $m/2$. Knowing that 0.7 s after the collision one piece reaches Point C and 0.9 s after the collision the other piece reaches Point D, determine (a) the velocity of sphere A after the collision, (b) the angle θ and the speeds of the two pieces after the collision.

SOLUTION

Velocities of pieces C and D after impact and fracture.

$$(v_C')_x = \frac{x_C}{t_C} = \frac{6.3}{0.7} = 9 \text{ ft/s}, \quad (v_C')_y = 9\tan 30° \text{ ft/s}$$

$$(v_D')_x = \frac{x_D}{t_D} = \frac{6.3}{0.9} = 7 \text{ ft/s}, \quad (v_D')_y = -7\tan\theta \text{ ft/s}$$

Assume that during the impact the impulse between spheres A and B is directed along the x-axis. Then, the y component of momentum of sphere A is conserved.

$$0 = m(v_A')_y$$

Conservation of momentum of system:

$$\xrightarrow{+}: \quad m_A v_0 + m_B(0) = m_A v_A' + m_C(v_C')_x + m_D(v_D')_x$$

$$m(16) + 0 = mv_A' + \frac{m}{2}(9) + \frac{m}{2}(7)$$

(a) $\mathbf{v}_A' = 8.00 \text{ ft/s} \longrightarrow$ ◀

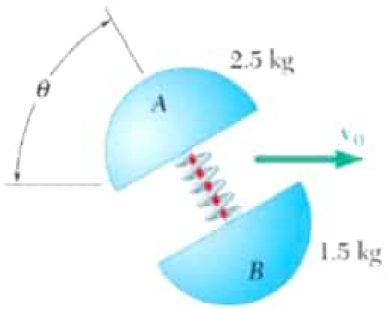
$$+\uparrow: \quad m_A(0) + m_B(0) = m_A(v_A')_y + m_C(v_C')_y + m_D(v_D')_y$$

$$0 + 0 = 0 + \frac{m}{2}(9\tan 30°) - \frac{m}{2}(7\tan\theta)$$

(b) $$\tan\theta = \frac{9}{7}\tan 30° = 0.7423$$ $\theta = 36.6°$ ◀

$$v_C = \sqrt{(v_C)_x^2 + (v_C)_y^2} = \sqrt{(9)^2 + (9\tan 30°)^2}$$ $v_C = 10.39$ ft/s ◀

$$v_D = \sqrt{(v_D)_x^2 + (v_D)_y^2} = \sqrt{(7)^2 + (7\tan 36.6°)^2}$$ $v_D = 8.72$ ft/s ◀

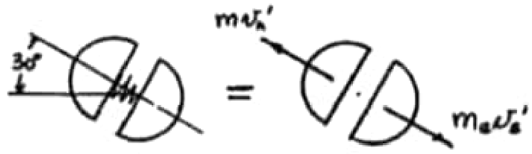


PROBLEM 14.38

Two hemispheres are held together by a cord which maintains a spring under compression (the spring is not attached to the hemispheres). The potential energy of the compressed spring is 120 J and the assembly has an initial velocity $\mathbf{v}_0$ of magnitude $v_0 = 8$ m/s. Knowing that the cord is severed when $\theta = 30°$, causing the hemispheres to fly apart, determine the resulting velocity of each hemisphere.

SOLUTION

Use a frame of reference moving with the mass center.



Conservation of momentum:

$$0 = -m_A v_A' + m_B v_B'$$

$$v_A' = \frac{m_B}{m_A} v_B'$$

Conservation of energy:

$$V = \frac{1}{2} m_A (v_A')^2 + \frac{1}{2} m_B (v_B')^2$$

$$= \frac{1}{2} m_A \left(\frac{m_B}{m_A} v_B' \right)^2 + \frac{1}{2} m_B (v_B')^2$$

$$= \frac{m_B (m_A + m_B)}{2m_A} (v_B')^2$$

$$v_B' = \sqrt{\frac{2m_A V}{m_B (m_A + m_B)}}$$

Data: $m_A = 2.5$ kg $\quad m_B = 1.5$ kg

$$V = 120 \text{ J}$$

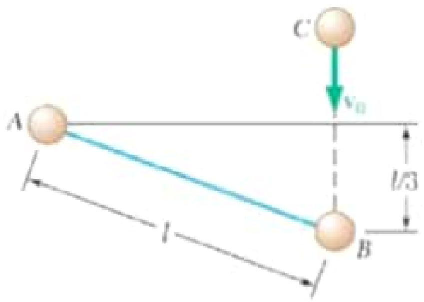
$$v_B' = \sqrt{\frac{(2)(2.5)(120)}{(1.5)(4.0)}} = 10 \qquad v_B' = 10 \text{ m/s} \nwarrow 30°$$

$$v_A' = \frac{1.5}{2.5}(10) = 6 \qquad v_A' = 6 \text{ m/s} \searrow 30°$$

Velocities of A and B.

$$\mathbf{v}_A = [8 \text{ m/s} \longrightarrow] + [6 \text{ m/s} \searrow 30°] \qquad \mathbf{v}_A = 4.11 \text{ m/s} \measuredangle 46.9° \blacktriangleleft$$

$$\mathbf{v}_B = [8 \text{ m/s} \longrightarrow] + [10 \text{ m/s} \nwarrow 30°] \qquad \mathbf{v}_B = 17.39 \text{ m/s} \nwarrow 16.7° \blacktriangleleft$$



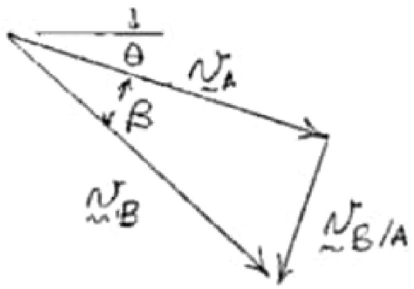
PROBLEM 14.43

Three spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Spheres A and B are attached to an inextensible, inelastic cord of length l and are at rest in the position shown when sphere B is struck squarely by sphere C which is moving with a velocity $\mathbf{v}_0$. Knowing that the cord is taut when sphere B is struck by sphere C and assuming perfectly elastic impact between B and C, and thus conservation of energy for the entire system, determine the velocity of each sphere immediately after impact.

SOLUTION

$$\sin\theta = \frac{1}{3}, \qquad \cos\theta = \frac{\sqrt{8}}{3}, \qquad \theta = 19.471^\circ$$

Velocity vectors



$$\mathbf{v}_0 = -v_0\mathbf{j}$$

$$\mathbf{v}_A = v_A(\cos\theta\,\mathbf{i} - \sin\theta\,\mathbf{j})$$

$$\mathbf{v}_{B/A} = u_B(-\sin\theta\,\mathbf{i} - \cos\theta\,\mathbf{j})$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\mathbf{v}_C = v_C\mathbf{j}$$

Conservation of momentum: $\quad m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C = 2m\mathbf{v}_A + m\mathbf{v}_{B/A} + m\mathbf{v}_C$

Divide by m and resolve into components.

$$\mathbf{i}: \quad 0 = 2v_A\cos\theta - u_B\sin\theta$$

$$-\mathbf{j}: \quad v_0 = +2v_A\sin\theta + u_B\cos\theta - v_C$$

Solving for v_A and u_B, $\quad v_A = \frac{1}{6}(v_0 + v_C) \qquad u_B = 0.94281(v_0 + v_C)$

Conservation of energy:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$$= \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(v_A^2 + u_B^2\right) + \frac{1}{2}mv_C^2$$

Divide by $\frac{1}{2}m$ and substitute for v_A and u_B.

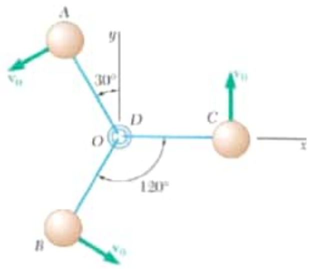
$$v_0^2 = 2\left(\frac{1}{6}\right)^2(v_0 + v_C)^2 + (0.94281)^2(v_0 + v_C)^2 + v_C^2$$

$$v_0^2 - v_C^2 = 0.94445(v_0 + v_C)^2 \qquad v_C = 0.02857v_0 \qquad \mathbf{v}_C = 0.0286v_0 \uparrow \blacktriangleleft$$

$$v_A = 0.17143v_0 \qquad \mathbf{v}_A = [0.17143v_0 \searrow 19.471^\circ]. \qquad \mathbf{v}_A = 0.1714v_0 \searrow 19.5^\circ \blacktriangleleft$$

$$u_B = 0.96975v_0 \qquad \mathbf{v}_{B/A}[0.96975v_0 \swarrow 19.471^\circ]$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{v}_B = 0.985 \searrow 80.1^\circ \blacktriangleleft$$



PROBLEM 14.50

Three small spheres A, B, and C, each of mass m, are connected to a small ring D of negligible mass by means of three inextensible, inelastic cords of length l. The spheres can slide freely on a frictionless horizontal surface and are rotating initially at a speed v_0 about ring D which is at rest. Suddenly the cord CD breaks. After the other two cords have again become taut, determine (*a*) the speed of ring D, (*b*) the relative speed at which spheres A and B rotate about D, (*c*) the fraction of the original energy of spheres A and B that is dissipated when cords AD and BD again became taut.

SOLUTION

Let the system consist of spheres A and B.

State 1: Instant cord DC breaks.

$$m(\mathbf{v}_A)_1 = mv_0\left(-\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

$$m(\mathbf{v}_B)_1 = mv_0\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

$$\mathbf{L}_1 = m(\mathbf{v}_A)_1 + m(\mathbf{v}_B)_1 = -mv_0\mathbf{j}$$

$$\overline{\mathbf{v}} = \frac{\mathbf{L}_1}{2m} = -\frac{1}{2}v_0\mathbf{j}$$

Mass center lies at Point G midway between balls A and B.

$$(\mathbf{H}_G)_1 = \frac{\sqrt{3}}{2}l\mathbf{j} \times (m\mathbf{v}_A)_1 + -\frac{\sqrt{3}}{2}l\mathbf{j} \times (m\mathbf{v}_B)_1$$

$$= \frac{3}{2}lmv_0\mathbf{k}$$

$$T_1 = \frac{1}{2}mv_0^2 + \frac{1}{2}mv_0^2 = mv_0^2$$

State 2: The cord is taut. Conservation of linear momentum:

(*a*)
$$\mathbf{v}_D = \overline{\mathbf{v}} = -\frac{1}{2}v_0\mathbf{j}$$

$v_D = 0.500v_0$ ◀

Let $(\mathbf{v}_A)_2 = \overline{\mathbf{v}} + \mathbf{u}_A$ and $\mathbf{v}_B = \overline{\mathbf{v}} + \mathbf{u}_B$

$$\mathbf{L}_2 = 2m\overline{\mathbf{v}} + m\mathbf{u}_A + m\mathbf{u}_B = \mathbf{L}_1$$

$$\mathbf{u}_B = -\mathbf{u}_A \qquad u_B = u_A$$

$$(\mathbf{H}_G)_2 = lmu_A\mathbf{k} + lmu_B\mathbf{k} = 2lmu_A\mathbf{k}$$

PROBLEM 14.50 (Continued)

(*b*) Conservation of angular momentum:

$$(\mathbf{H}_G)_2 = (\mathbf{H}_G)_1$$

$$2lmu_A\mathbf{k} = \frac{3}{2}lmv_0\mathbf{k} \qquad u_A = u_B = \frac{3}{4}v_0$$

$u = 0.750v_0$ ◀

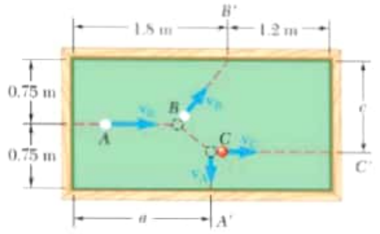
$$T_2 = \frac{1}{2}(2m)\overline{v}^2 + \frac{1}{2}mu_A^2 + \frac{1}{2}mu_B^2$$

$$= \frac{1}{2}mv_0^2\left(\frac{1}{2} + \frac{9}{16} + \frac{9}{16}\right) = \frac{13}{16}mv_0^2$$

(*c*) Fraction of energy lost:
$$\frac{T_1 - T_2}{T_1} = \frac{1 - \frac{13}{16}}{1} = \frac{3}{16}$$

$\frac{T_1 - T_2}{T_1} = 0.1875$ ◀

PROBLEM 14.51



In a game of billiards, ball A is given an initial velocity $\mathbf{v}_0$ along the longitudinal axis of the table. It hits ball B and then ball C, which are both at rest. Balls A and C are observed to hit the sides of the table squarely at A' and C', respectively, and ball B is observed to hit the side obliquely at B'. Knowing that $v_0 = 4$ m/s, $v_A = 1.92$ m/s, and $a = 1.65$ m, determine (*a*) the velocities $\mathbf{v}_B$ and $\mathbf{v}_C$ of balls B and C, (*b*) the Point C' where ball C hits the side of the table. Assume frictionless surfaces and perfectly elastic impacts (that is, conservation of energy).

SOLUTION

Velocities in m/s. Lengths in meters. Assume masses are 1.0 for each ball.

Before impacts: $\quad (\mathbf{v}_A)_0 = v_0\mathbf{i} = 4\mathbf{i}, \quad (\mathbf{v}_B)_0 = (\mathbf{v}_C)_0 = 0$

After impacts: $\quad \mathbf{v}_A = -1.92\mathbf{j}, \quad \mathbf{v}_B = (v_B)_x\mathbf{i} + (v_B)_y\mathbf{j}, \quad \mathbf{v}_C = v_C\mathbf{i}$

Conservation of linear momentum: $\quad \mathbf{v}_0 = \mathbf{v}_A + \mathbf{v}_B + \mathbf{v}_C$

$$\mathbf{i}:\ 4 = 0 + (v_B)_x + v_C \qquad (v_B)_x = 4 - v_C$$

$$\mathbf{j}:\ 0 = -1.92 + (v_B)_y + 0 \qquad (v_B)_y = 1.92$$

Conservation of energy: $\quad \frac{1}{2}v_0^2 = \frac{1}{2}v_A^2 + \frac{1}{2}v_B^2 + \frac{1}{2}v_C^2$

$$\frac{1}{2}(4)^2 = \frac{1}{2}(1.92)^2 + \frac{1}{2}(1.92)^2 + \frac{1}{2}(4 - v_C)^2 + \frac{1}{2}v_C^2$$

$$v_C^2 - 4v_C + 3.6864 = 0$$

$$v_C = \frac{4 \pm \sqrt{(4)^2 - (4)(3.6864)}}{2} = 2 \pm 0.56 = 2.56 \quad \text{or} \quad 1.44$$

Conservation of angular momentum about B':

$$0.75v_0 = (1.8 - a)v_A + cv_C$$

$$cv_C = (0.75)(4) - (1.8 - 1.65)(1.92) = 2.712$$

$$c = \frac{2.712}{v_C}$$

If $v_C = 1.44$, $\quad c = 1.8833$ off the table. Reject.

If $v_C = 2.56$, $\quad c = 1.059$

PROBLEM 14.51 (Continued)

Then, $\quad (v_B)_x = 4 - 2.56 = 1.44, \quad \mathbf{v}_B = 1.44\mathbf{i} + 1.92\mathbf{j}$

Summary.

(*a*) $\quad \mathbf{v}_B = 2.40$ m/s $\angle$ 53.1° ◀

$\mathbf{v}_C = 2.56$ m/s ⟶ ◀

(*b*) $\quad c = 1.059$ m ◀