Unit 4 (II part)

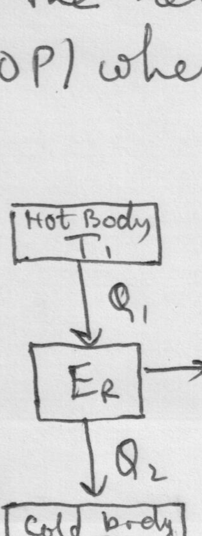
Reversed Heat Engine

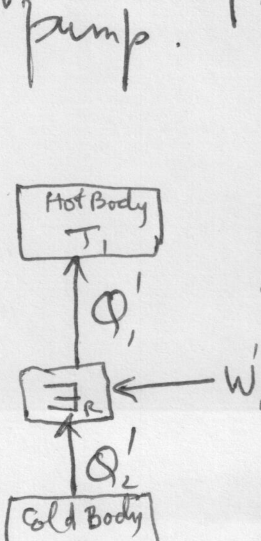
- If reciprocating steam engine is driven backwards, it will act like a reciprocating compressor of refrigerator.
- If a turbine is driven backwards, it will act like rotary compressor.
- But both systems may be inefficient in their backward direction.
- If we imagine a heat engine, i.e. a complete power plant, all components of which would work just as well backwards as forwards. Such a heat engine is called Reversible Heat Engine.

imp – A heat engine which engages in heat transfer with two systems of fixed but different, temperatures, is reversible if its efficiency when operating directly equals to the reciprocal of its coefficient of performance (COP) when operated as heat pump.

$$\eta_{R} = \frac{1}{(COP)_{HP,R}}$$

Reversible Engine operating directly

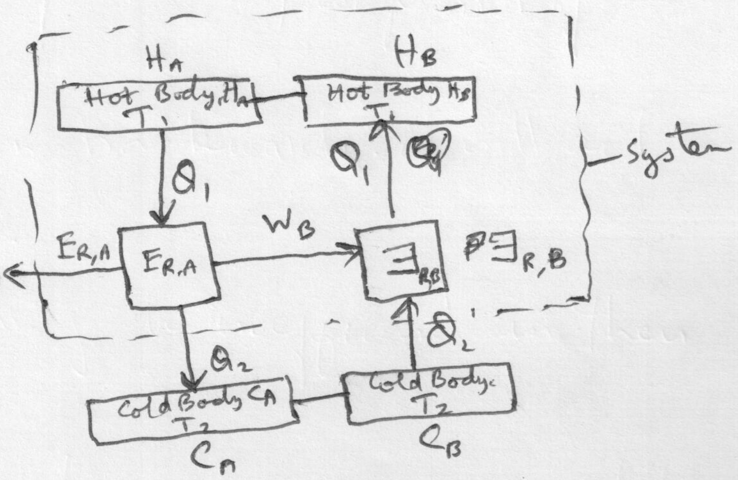




Reversible Engine operating as Heat pump

- Efficiency of a reversible engine is independent of the nature of the engine and of the constant-temperature systems.

Proof:



- H_A & H_B are in thermal contact so no temp. difference exists b/w them (T_1)
- Similarly C_A & C_B are also at same temp. (T_2)
- Let efficiency of $E_{R,A}$ is more than that of $E_{R,B}$. ~~so reverse of $E_{R,B}$ (i.e. $\exists_{R,B}$) is used to drive~~
- $E_{R,A}$ is used to drive reverse of $E_{R,B}$ (i.e. $\exists_{R,B}$) so that same heat Q_1 is returned to the hot system H_B as is withdrawn from H_A.

or $\eta_A > \eta_B$

$$\frac{W_A}{Q_1} > \frac{W_B}{Q_1}$$

$$W_A - W_B > 0$$

- As C_A & C_B are in good thermal contact, so they form a "single cold ~~temperature~~ constant-temperature reservoir."
- Combination of $E_{R,A}$ & $\exists_{R,B}$ together with H_A & H_B form a system operating continuously doing positive work & engaging in heat transfer with a single constant temp. reservoir. As this statement violates II Law so the original assumption would be wrong. or η_A cannot be more than η_B.

- Similarly η_B cannot be more than η_A.
- So $\eta_A = \eta_B$
- In general, "All reversible engines have the same efficiency when operating b/w two reservoirs having fixed but different temperatures.
- Efficiency is function of temp. only & does not depend on the nature of either the const. temp. system or reversible engine.
- On similar lines, all reversible engines acting as heat pumps or refrigerators, operating b/w at same two temps. have equal coefficient of performance.
- Also efficiency of a reversible engine is greatest that can be achieved by any engine operating b/w same two temps.
- Similarly, COP of HP & Ref. systems is greatest that can be exhibited by any engine operating b/w same two temps.
- Also $\eta_R = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

Also $\eta_R = f(T_1, T_2)$ or $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

so $\eta_R = 1 - \frac{T_2}{T_1}$

Also $(COP)_{ref,R} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1} = \frac{1}{\frac{T_1}{T_2} - 1}$

$(COP)_{ref,R} = \frac{T_2}{T_1 - T_2}$ $\cancel{(COP)_{ref,R} = \frac{T_2}{T_1 - T_2}}$

Similarly $(COP)_{HP,R} = \dfrac{T_1}{T_1 - T_2}$

$$\boxed{(COP)_{HP,R} = (COP)_{ref,R} + 1}$$

~~Also $Q_1 = (COP)_{HP}$~~

Absolute temperature scale (Kelvin scale)

or

Thermodynamic temp. scale

- The previously mentioned scales (in unit I) were dependent on thermometric ~~subs~~ substance.
- Kelvin proposed a temperature scale in terms of efficiency of reversible engine.
- Kelvin's ~~thermo~~ scale is based on thermodynamic properties ~~prop~~ & is independent of thermometric substance.
- Absolute temperature scale is defined as.

$$\eta_R = \frac{T_1 - T_2}{T_1} \quad - (A)$$

η_R = efficiency of reversible engine
T_1 = Absolute temp. of hotter body
T_2 = " " " colder body

also ~~$\eta_R = \frac{Q}{Q_1}$~~ $\eta_R = \dfrac{W_E}{Q_1} = \dfrac{Q_1 - Q_2}{Q_1} = \dfrac{T_1 - T_2}{T_1}$

$$\boxed{\frac{W_E}{T_1 - T_2} = \frac{Q_1}{T_1} = \frac{Q_2}{T_2}} \quad - (B)$$

- Expression (B) implies that given two const. temp. systems, any number can be assigned to the temp. of one of them; then reversible engine will fix the number which must be assigned to the second one.

- The assigning of values to known pts of freezing & boiling of water was done experimentally & choosing 100 divisions b/w them we get

$$T_{freezing} = 273.15 \text{ or } 273 \text{ °C abs or K}$$

$$T_{boiling} = 373.15 \text{ or } 373 \text{ °C abs or K}$$

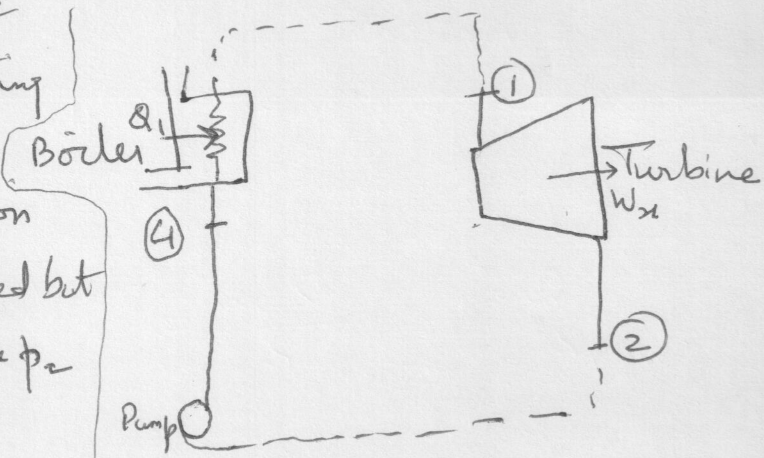
also $T_{in\,K} = t_{in\,°C} + 273$

Note: Although a reversible H.E. cannot be constructed in practicality, it is possible to use the definition of $\eta_R \left(= \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}\right)$ to determine absolute temp. by means of measurements of pressure, volume & heat transfer.

Entropy

- What is the maximum work obtainable from a given process?

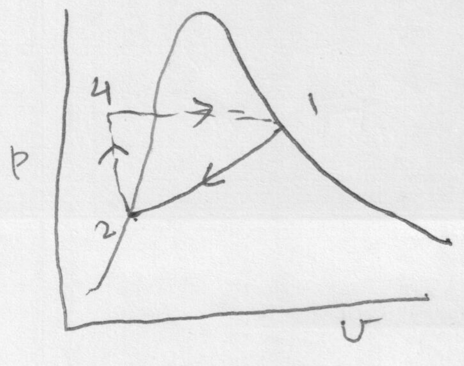
- Example: Turbine



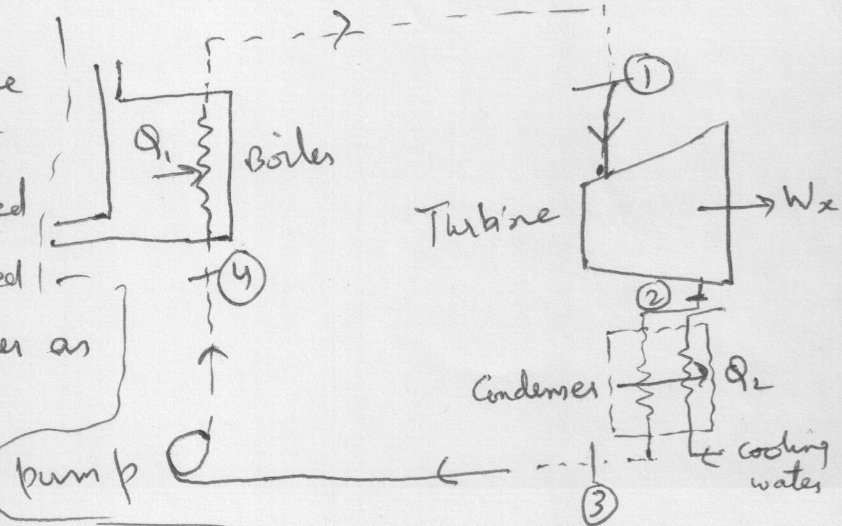
- Consider a turbine operating b/w states ① & ②.
- Sufficient (two) condition at state ① are specified but at state ② only pressure p_2 is known.
- Turbine is adiabatic
- From SFEE

$$W_x = h_1 - h_2 \quad -(a)$$

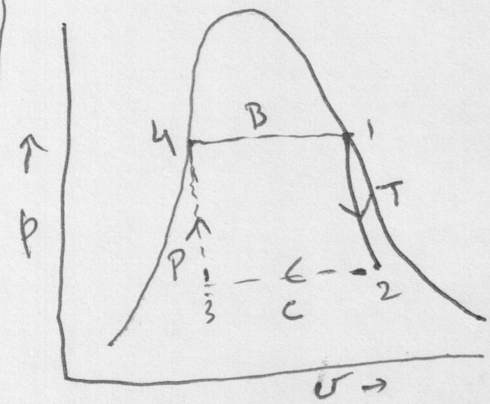
here h_1 is known but h_2 is not known as state ② is not completely defined.



- Let state is saturated water. Attach a feed pump & a boiler to supply heat, so that turbine is fed with constant supply of steam at state 1.
- Examining this system ~~we~~ shows that a 100% efficiency system has been created. (pump work being negligible)
- This system violates the II Law so this system is incapable of realization in practice. In other words, if system is not 100% efficient, the dryness fraction of exhausted steam cannot be zero. So How low it can be?
- To answer above question reconstruct the system mentioned above by adding condenser b/w turbine exhaust & feed pump inlet.



- Steam leaving the turbine is almost completely condensed before being pumped back into the boiler as saturated water.
- Turbine work output is greatest when the turbine & all others components work reversibly.



- From SFEE

$$\boxed{W_{x,R} = h_1 - h_2} \quad -(b)$$

Also for reversible H.E.

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad -(c)$$

Also applying SFEE to condenser plus feed pump.

$$-Q_2 + W_{P,R} = h_4 - h_2 \quad -(d)$$

where; $W_{P,R} = (p_1 - p_2)\, v_{f4}$

For boiler SFEE gives

$$Q_1 = h_1 - h_4 \quad -(e)$$

combining this with (c) & (d) we can obtain h_2 (not known earlier)

$$h_2 = h_4 + \frac{T_2}{T_1}(h_1 - h_4) - W_{P,R} \quad -(f)$$

putting eqn. (f) in (b)

$$\boxed{W_{T,R} = (h_1 - h_4)\left(1 - \frac{T_2}{T_1}\right) + W_{P,R}} \quad -(g)$$

The above expression provides us with a standard of actual performance of the turbine.

— Analogy with the I law:

- for cyclic processes

$$\oint W = \oint Q$$

for non-cyclic processes

$$\Delta E = \delta Q - \delta W$$

So energy (a property) was introduced to apply I-law for non-cyclic processes.

- Can on similar lines a property be invented to apply II-law to non-cyclic processes.

from eqn. (c)

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{or} \quad \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

Since Q_1 & Q_2 are Heat Transfers to & from steam in reversible cyclic process therefore

$$\boxed{\oint \frac{dQ_R}{T} = 0} \quad -(h)$$

- Define Entropy: Entropy S, is a property of a system such that its increase, $S_2 - S_1$, as the system changes from state 1 to state 2 is given by

$$\boxed{S_2 - S_1 = \Delta S = \int_1^2 \frac{dQ_R}{T}}$$

<u>Note</u>: As S is property, the value of the integral is independent of the path of the change of state & depends only on the end states 1 & 2

- Differential form of eq^n:

$$dS = \frac{dQ_R}{T}$$

<u>Proof that entropy is a property - The Clausius Inequality</u>

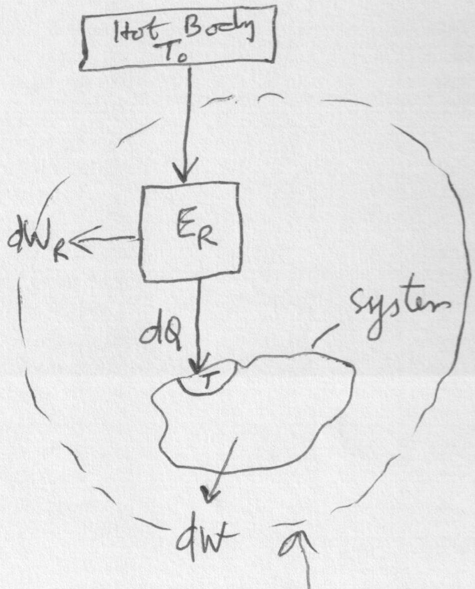
<u>statement</u>: When any system undergoes a cyclic process, the integral around the cycle of $(dQ)/T$ is less than or equal to zero.

$$\oint \frac{dQ}{T} \leq 0$$

$\underline{T}$ is absolute temp of the part of system to which heat transfer $\underline{dQ}$ occurs.

<u>Proof</u>:- Connect the system to const. temp (T_0) hot body & a reversible Heat Engine (E_R) instead of actual surroundings with effecting the actual system.

- Various transfers occuring are shown in fig. →
- Engine performs multiple cycle while the system performs only one.
- Applying I law



$$\oint (\delta Q - \delta W) = 0$$

$$\simeq^a \oint (dQ - dW) = 0 \quad -(i)$$

From II law applied to cyclic process to system in dotted lines

$$\oint (dW + dW_R) \leq 0 \quad -(ii)$$

for absolute temp scale

$$\frac{dW_R}{T_0 - T} = \frac{dQ}{T} \quad -(iii)$$

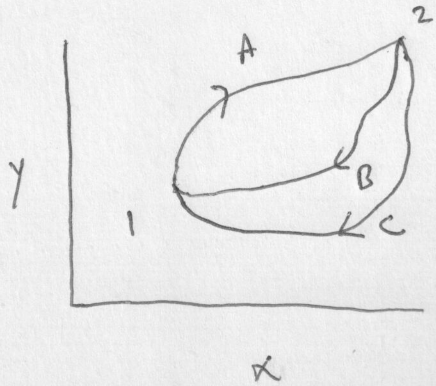
using (i) & (iii) in (ii) yields $\oint \left\{ dQ + \frac{T_0 - T}{T} dQ \right\} \leq 0$

or $T_0 \oint \frac{dQ}{T} \le 0$

So $\boxed{\oint \frac{dQ}{T} \le 0}$ Pd.

This is known as Clausius Inequality.

<u>For reversible cycle</u>



A reversible process

Consider a reversible cycle shown →

along 1A2B1 ; $\oint \frac{dQ_R}{T} \le 0$ —(iv)

along 1B2A1 ; $\oint \frac{dQ'_R}{T} \le 0$ —(v)

Since the two cycles are same but reverse of the other for reversible cycle $dQ_R' = -dQ_R$

So from eqn. (v) $-\oint \frac{dQ_R}{T} \le 0$

or $\oint \frac{dQ_R}{T} \ge 0$ —(vi)

from (iv) & (vi)

$\oint \frac{dQ}{T} = 0$ (for reversible cycle).

Consider another process C from 2-1

As $\oint \frac{dQ}{T} = 0$

$\Rightarrow \int_{1A}^{2} \frac{dQ_R}{T} + \int_{2B}^{1} \frac{dQ_R}{T} = 0$ —(vii)

Also $\int_{1A}^{2} \frac{dQ_R}{T} + \int_{2C}^{1} \frac{dQ_R}{T} = 0$ (viii)

from (vii) & (viii)

$\int_{2B}^{1} \frac{dQ_R}{T} = \int_{2C}^{1} \frac{dQ_R}{T}$

From def. of entropy ; $(S_2 - S_1)$ will have the same value for any reversible path b/w 1 & 2. It shows that <u>entropy is a property</u>.

Note : About entropy

- Entropy is assigned value zero ~~at~~ for saturated water at triple point
- Units of 'S' is J/K & that of specific entropy 's' (entropy per unit mass) is J/Kg.K
- specific entropy follows the relation

$$s = (1-x)s_f + x s_g$$

~~or~~ $s = s_f + x s_{fg}$

- Many application involve reversible adiabatic processes for such process $\Delta Q_R = 0$ $\Rightarrow$ $\Delta S = 0$

Therefore all reversible adiabatic processes are isentropic.

- Isentropic efficiency: ratio of actual work output to the work output of a reversible adiabatic machine working at same inlet & outlet conditions as ~~original~~ actual m/c.

$$\boxed{\eta_{isen} = \frac{W_x}{W_{x,R}}}$$

when work done is on the fluid.

$$\eta_{isen} = \frac{W_{x,R}}{W_x}$$

- Entropy increase in an irreversible ~~system~~ process is always greater than the integral of (dQ/T)

$$S_2 - S_1 > \int_1^2 \frac{dQ}{T} \quad \text{for any irreversible process}$$

- The entropy of an adiabatic system can only increase

$S_2 - S_1 > 0$ for adiabatic irreversible process

- Entropy increase for pure substance

$$dQ = du + dW \quad (\text{I law})$$

for reversible process $dQ_R = du + p\,dv$

from entropy def. $dQ_R = T\,ds$

so $\boxed{T\,ds = du + p\,dv}$

$h = u + pv$

$dh = du + p\,dv + v\,dp$

so $\boxed{T\,ds = dh - v\,dp}$

- Understanding physical meaning of entropy.
- Mathematically, $\Delta S = \int_1^2 \frac{dQ_R}{T}$
- ~~Quali~~ Qualitatively it is said that, "Entropy is degree of randomness or measure of disorder."

Both representations are not very satisfactory.

- Better def:

The increase in entropy in the system as it changes from state 1 to state 2 is equal to the heat transfer to a reversible engine (from a reservoir at unit temp.) which by interacting with the system, brings about the change in state.

Air Standard Cycles (A-S-C)

- Although working fluid used in I C (Internal Combustion) engines ~~a~~ does not undergo a complete thermodynamic cycle, Air-Standard cycles are conceived in order to simplify the analysis of I.C. engines
- In A-S-C a fixed mass of air ~~o~~ undergoes a complete thermodynamic cycle.
- Otto Cycle [for petrol or SI (spark Ignition) Engines]

Basic processes for a SI engine are:

1-2; Intake of Fuel-Air mixture at const pressure

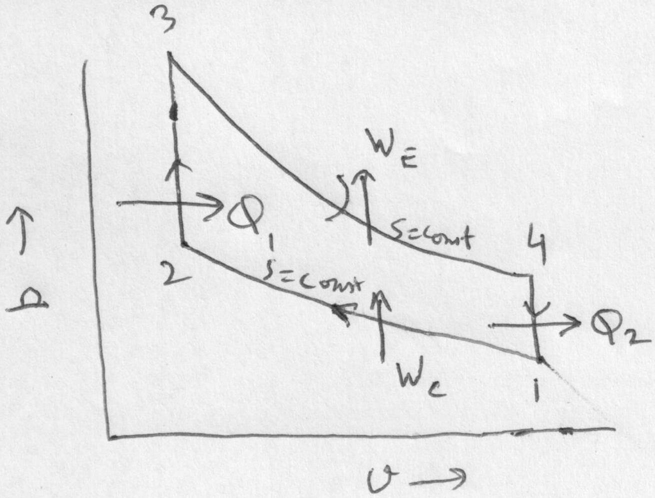
2-3; Compression of $\underbrace{\text{Fuel Air mixture}}_{F-A-M}$ to min. vol.

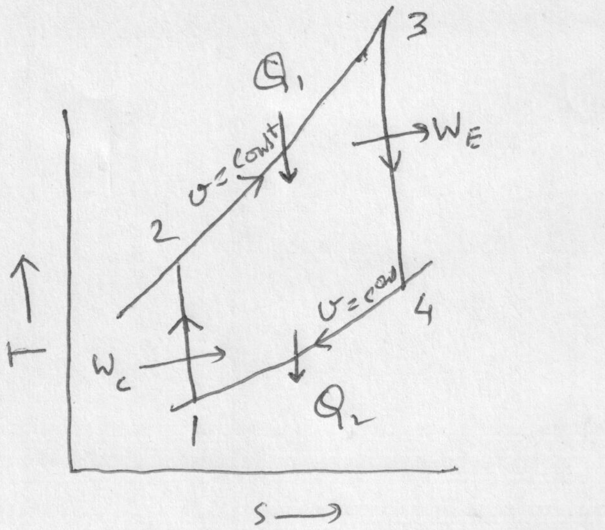
3-4; Combustion of F-A-M using spark plug leading to increase of temp & press.

4-5; Expansion of products of combustion & doing work on piston.

5-6; Blow-down exhaust gases are thrown out leading to initial pressure in ~~chamber~~ cylinder

- ~~6-1~~; The equivalent thermodynamic cycle for above mentioned mechanical cycle are:
  - Reversible ~~ad~~ adiabatic (isentropic) compression & expansion
  - " isochoric heat addition & rejection.





- Diesel or for CI engines

Mech cycle for Diesel engines

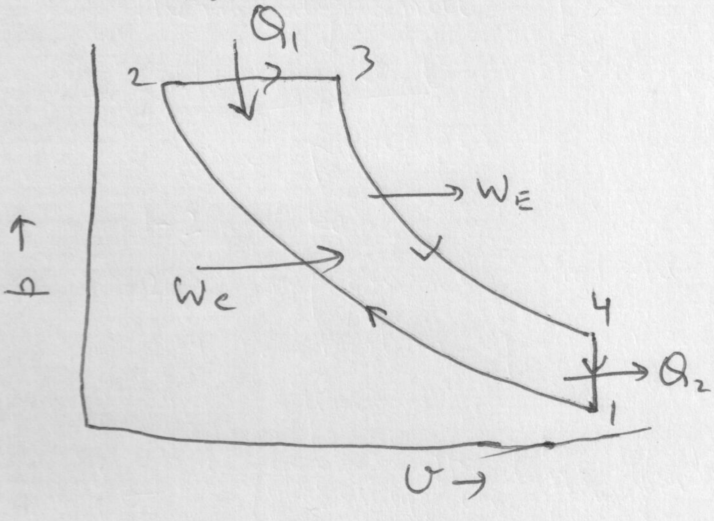
- 1-2; Intake ~~air in~~ of air at const. pressure
- 2-3; Compression of air to min. volume.
- 3-4; Fuel injection & ~~comp~~ combustion at const. pressure

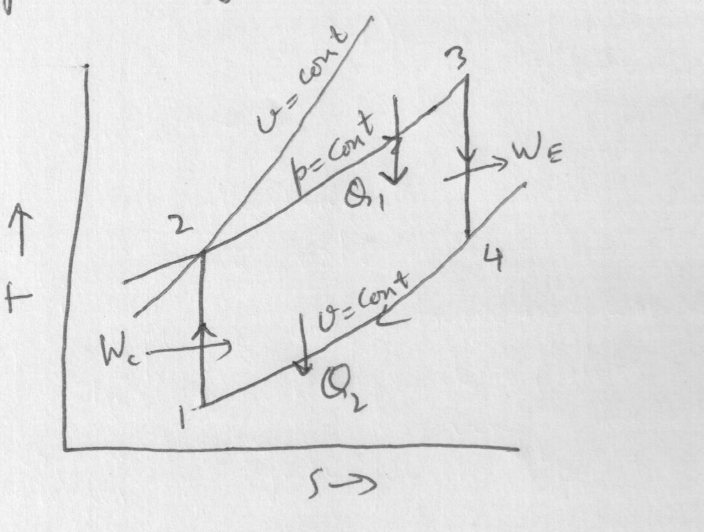
4-5; Expansion by exhaust gases ~~on~~ & work done on piston

5-6; Blow-down & exhaust of gases of combustion to restore initial pressure

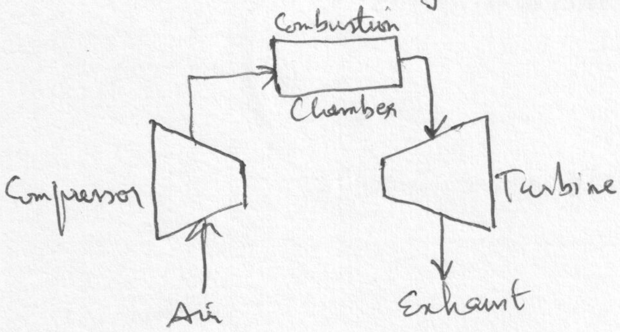
Equivalent thermodynamic cycle

- Reversible adiabatic ~~expansion &~~ compression & expansion of air
- Heat added at const. pressure & heat rejected at const volume
- Both heat addition & rejection being ~~as~~ reversible.

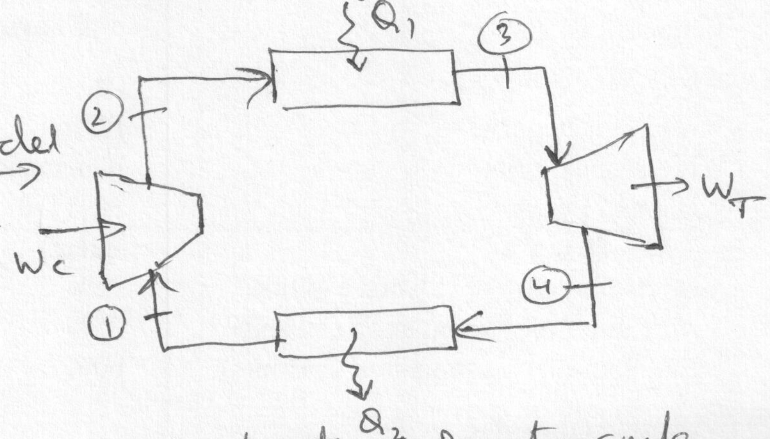




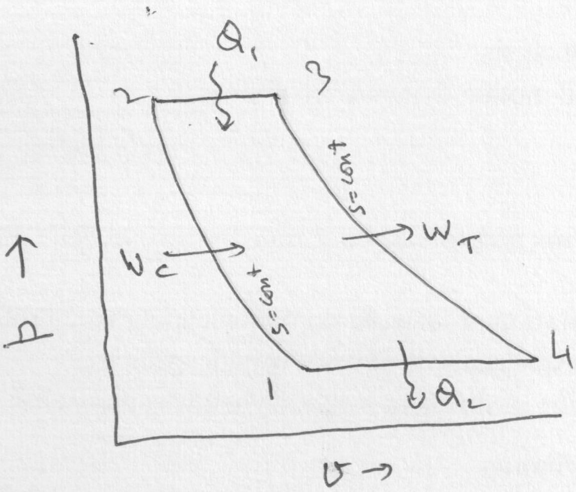
- To model gas turbine power plants Brayton cycle is used

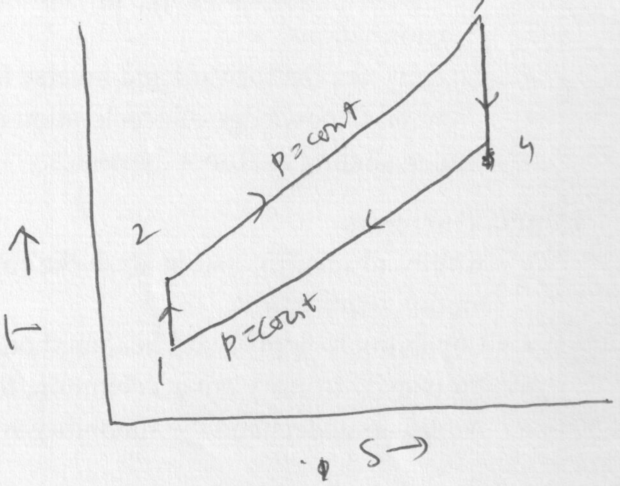


Simple gas turbine



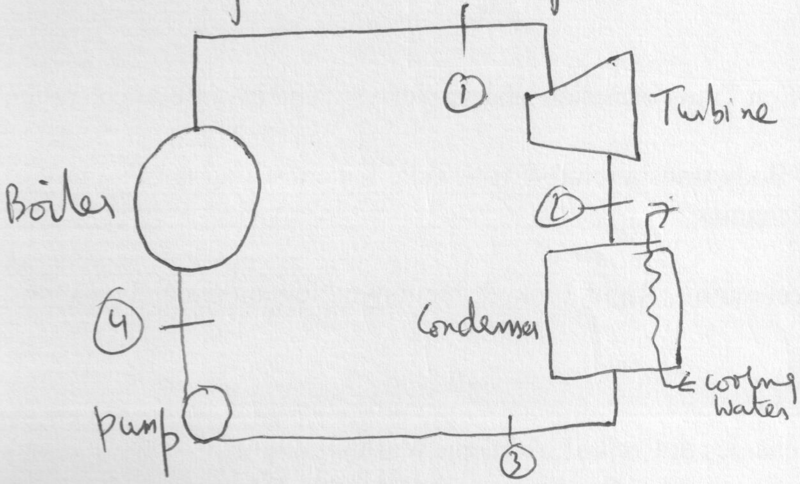
Air standard Brayton cycle

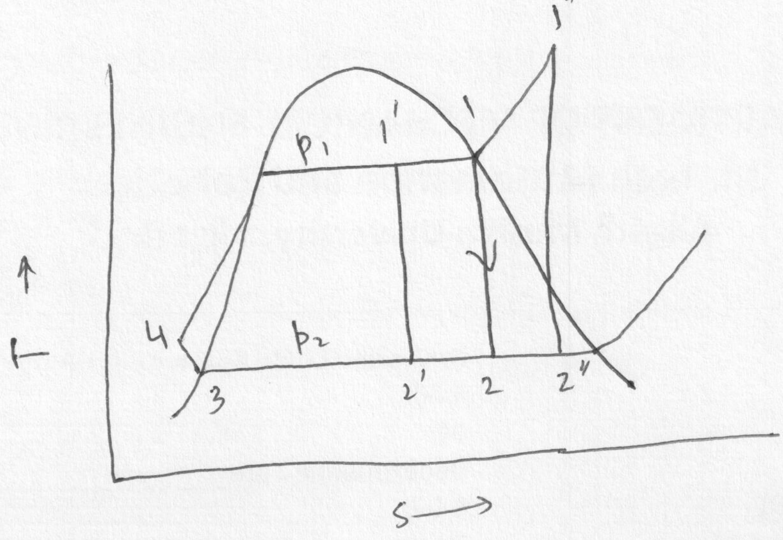




Brayton cycle consists of
- Isentropic work done by compressor & turbine
- Const pressure (isobaric) heat addition & rejection.

- Vapour power systems (Steam turbine powerplants) are modelled using Rankine cycle.





1-2 Isentropic expansion in turbine

2-3 Const. pressure heat rejection in condenser to form saturated liquid

3-4 Isentropic compression by pump.

4-1 Heat transfer to fluid at constant pressure in boiler.

1''-2''-3-4-1'' → shows the possibility of superheated vapor

1'-2'-3-4-1' → " " " " subcooled liquid.