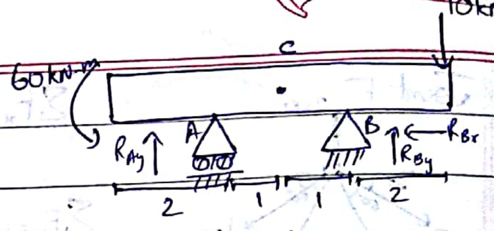
Force, Moments & FBD

Tutorial #1



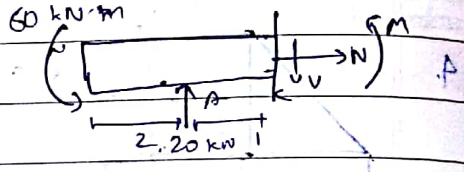
1 a) For the beam, applying eq eqns

$\Sigma F_x = 0 ;\ R_{Bx} = 0$

$\Sigma F_y = 0 ;\ R_{Ay} + R_{By} = 10$

$\circlearrowleft \Sigma M_A = 0 ;\ 60 + 2R_{By} - 40 = 0$

$R_{By} = -10\text{kN} ,\ R_{Ay} = 20\text{ kN}$



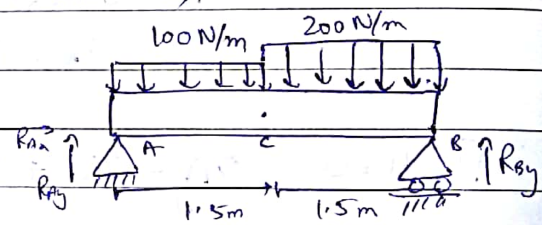
For section AC (LHS)

$\Sigma F_x = 0 ;\ N = 0$

$\Sigma F_y = 0 ;\ R_{Ay} - V = 0 ,\ V = R_{Ay} = 20\text{ kN}$

$\circlearrowleft \Sigma M_c = 0 ;\ M - R_{Ay} + 60 = 0$

$M = -40\text{ kN.m} \Rightarrow 40\text{ kN-m (clockwise)}$



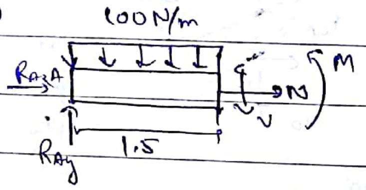
b) For the beam apply eq. eqns

$\Sigma F_x = 0 ,\ F_{Ax} = 0$

$\Sigma F_y = 0 ,\ R_{Ay} + R_{By} = 450$

$\circlearrowleft \Sigma M_B = 0 ;\ -3R_{Ay} + 100 \times 1.5 \times \left(1.5 + \frac{1.5}{2}\right) + 200 \times \frac{1.5^2}{2} = 0$

$R_{Ay} = 187.5\text{ kN} ,\ R_{By} = 262.5\text{ kN}$



For section AC (LHS)

$\Sigma F_x = 0 ;\ N + R_{Ax} = 0 ;\ N = -R_{Ax} = 0$

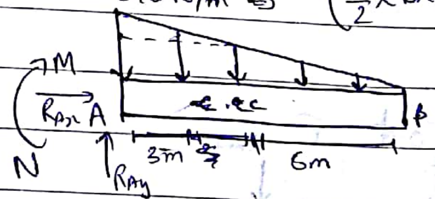
$\Sigma F_y = 0 ;\ R_{Ay} - V - 100 \times 1.5 = 0$

$V = \cancel{262}\ 187.5 - 150 = 37.5\text{ N}$

$\circlearrowleft \Sigma M_c = 0 ;\ M - 1.5R_{Ay} + 100 \times \frac{1.5^2}{2} = 0 ;\ M = 168.75\text{ N-m}$

c) For the beam, apply eq eqn

$270\text{ N/m} \Rightarrow \left(\frac{1}{2} \times b \times h\right) = \frac{1}{2} \times 9 \times 270 = 1215\text{ N}$



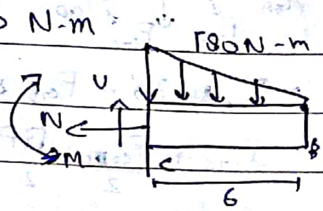
$\Sigma F_x = 0 ;\ R_{Ax} = 0$

$\Sigma F_y = 0 ;\ R_{Ay} - \frac{270 \times 9}{2} = 0 ,\ R_{Ay} = 1215\text{ N}$

$\circlearrowleft \Sigma M_A = 0 ;\ -M - \frac{270 \times 9}{2} \times \frac{9}{3} = M = -3645\text{ N-m}$

By similar triangles

$\frac{\cancel{270}\,90}{\cancel{9}\,3} = \frac{x}{\cancel{6}\,2} ,\ x = 180\text{ N-m}$



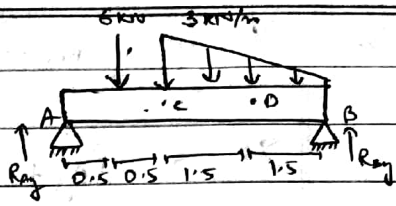
Loading $= \frac{180 \times \cancel{6}\,3}{\cancel{2}} = 540\text{ N}$

For section BC

$\sum F_x = 0;\ N = 0$

$\sum F_y = 0;\ V - 540 = 0;\ V = 540\ N$

$\sum M_c = 0;\ M - \frac{540 \times 6}{3} = 0;\ M = 1080\ N\text{-}m$



d) For the beam, apply eq eqns

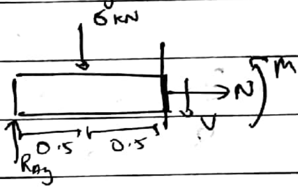
$\sum F_y = 0:\ R_{Ay} + R_{By} - 6 - \frac{3 \times 3}{2} = 0$

$R_{Ay} + R_{By} = \cancel{9+6+15} = 10.5$

~~$\sum M_B = 0,\ 4R_{Ay} - 6\times 3.5 - \frac{3\times 1.5}{3}\ \frac{}{2} = 0;\ R_{Ay} =$~~

$4R_{Ay} - 6\times3.5 - \frac{3\times3}{2} \times 2 = 0;\quad R_{Ay} = 7.5\ kN:\ R_{By} = 3\ kN$

At section AC



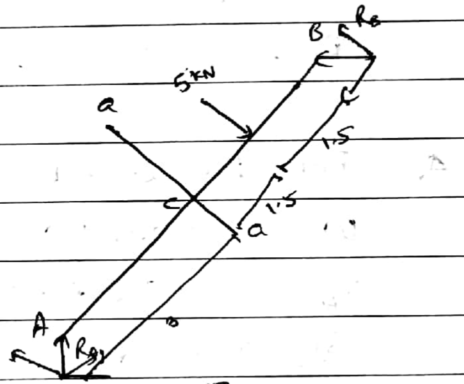
$\sum F_y = 0;\ R_{Ay} - 6 - V = 0;\quad V = \cancel{3}\ 1.5\ kN$

$\sum F_x = 0;\ N = 0$

$\sum M_c = 0;\ -R_{Ay} + 6\times 0.5 + M = 0$

$M = 4.5\ kN\text{-}m$

2. Along section aa

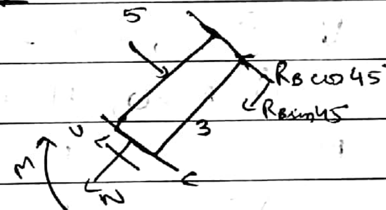


For the beam, apply eq eqn.

$\sum M_A = 0$

$-5\times 4.5 + R_B \sin 45 \times 6 = 0$

$R_B = 5.303\ kN$



$\sum F_x = 0;$

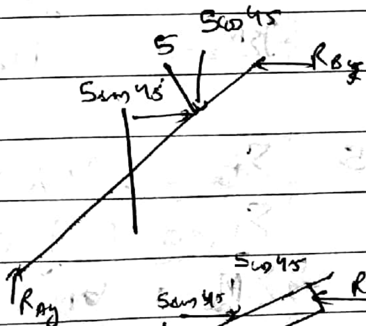
$-N - R_B \sin 45° = 0;\quad N = -3.75\ kN$

$\sum F_y = 0;$

$V + R_B \cos 45 = 0;\quad V = \cancel{5}\,{-5}\ 3.75\ \cancel{kN} = 1.25\ kN$

$\sum M_c = 0;\ -M \mp 5\times 1.5 + R_B \cos 45 \times 3 = 0$

$M = 3.75\ kN\text{-}m$

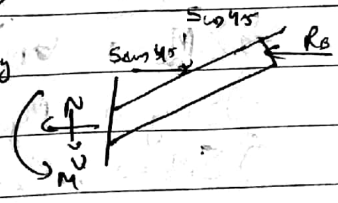


Along section bb

$\sum F_x = 0$

$-N + 5\sin 45 - R_B = 0$

$N = -1.767\ kN$

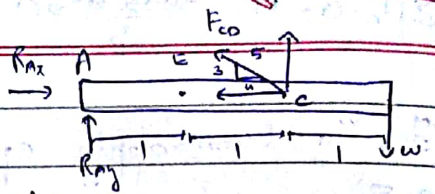


$\sum F_y = 0$

$V - 5\cos 45 = 0,\quad V = 3.535\ kN$

$\sum M_c = 0;\ M_c - 5\cos 45 \cancel{\times 1.5} = \quad M_c = 3.75\ kN\cdot m$

3. $\Sigma F_x = 0$; $R_{Ax} - F_{CD} \times \frac{4}{5} = 0$

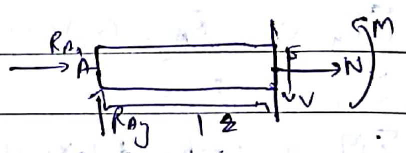


$\Sigma F_y = 0$ $R_{Ay} + \frac{3}{5} F_{CD} = W \Rightarrow R_{Ay} + \frac{3}{5} F_{CD} = 4905$

$\circlearrowleft \Sigma M_A = 0$ $\frac{3}{5} F_{CD} \times 2 - 3 \times 4905 = 0$; $F_{CD} = 12262.5$ N
$= 12.263$ kN

$R_{Ay} = -2452.5$ N ; $R_{Ax} = 9810$ kN
$= -2.453$ kN $= 9.810$ kN

For section AE



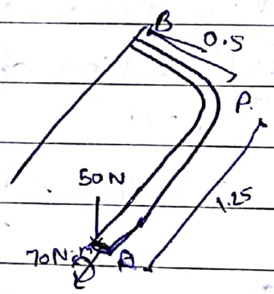
$\Sigma F_x = 0$

$N + R_{Ax} = 0$, $N = -R_{Ax} = \cancel{2.453}$ -9.810 kN

$\Sigma F_y = 0$ $\circlearrowleft \Sigma M_E = 0$, $M - R_{Ay} = 0$

$R_{Ay} - V = 0$, $V = R_{Ay} = -2.453$ kN $M = R_{Ay} = -2.453$ kN-m

4. FBD till section B



$W_{BD} = 2 \times 9.81 \times 0.5 = 9.81$ N

$W_{AD} = 2 \times 9.81 \times 1.25 = 24.525$ N

$\Sigma F_x = 0$; $F_x = 0$; $\Sigma F_y = 0$; $F_y = 0$

V= $\Sigma F_z = 0$; $F_z - 9.81 - 24.525 - 50 = 0$; $F_z = 84.335$ N

$\circlearrowleft \Sigma M_x = 0$; $M_x + 70 - 50 \times 0.5 - 24.525 \times 0.5 - 9.81 \times 0.25 = 0$

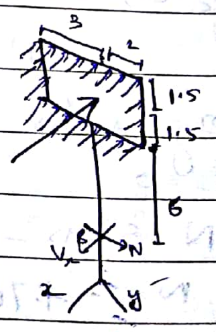
M. $M_x = -30.463$ N-m

$\circlearrowleft \Sigma M_y = 0$; $M_y + 24.625 \times 0.625 + 50 \times 1.25 = 0$

Z= $M_y = -77.89$ N-m

$\circlearrowleft \Sigma M_z = 0$; $M_z = 0$

5. $\Sigma F_x = 0$; $(V_B)_x - 7 \times 3 \times 5 = 0$, $(V_B)_x = 105$ lb



$\Sigma F_y = 0$: $(V_B)_y = 0$; $\Sigma F_z = 0$, $N_z = 0$

$\circlearrowleft \Sigma M_x = 0$ $M_x = 0$ $(6+1.5)$

$\circlearrowleft \Sigma M_y = 0$; $M_y - 105 \times (7.5) = 0$; $M_y = 787.5$ lb ft

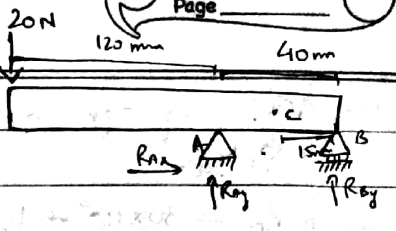
$\circlearrowleft \Sigma M_z = 0$; $M_z - 105 \times (2.5 - 2) = 0$, $M_z = 52.5$ lb ft

6. $\Sigma F_x = 0;\ R_{Ax} = 0$

$\Sigma F_y = 0;\ R_{Ay} + R_{By} - 20 = 0;\quad R_{Ay} + R_{By} = 20$

$\Sigma M_B = 0;\quad 20 \times 160 - 40 R_{Ay} = 0$

$R_{Ay} = 80\ N;\quad R_{By} = -60 N$

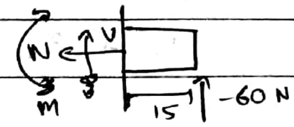


At section BC

$\Sigma F_x = 0 \qquad N = 0$

$\Sigma F_y = 0,\quad V + R_{By} = 0;\quad V = 60N$

$\Sigma M_c = 0\ ;\quad -M + R_{By} \times 15 = 0;\quad M = 900\ N\text{-}mm$ (clockwise)

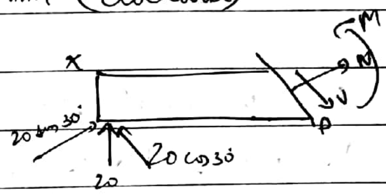


At section XD

$\Sigma F_x = 0\ ;\quad N - 20 \sin 30^\circ = 0,\quad N = 10\ N$

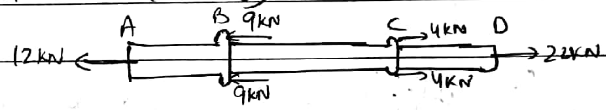
$\Sigma F_y = 0\ ;\ -V + 20 \cos 30^\circ = 0,\quad V = 17.32\ N$

$\Sigma M_D = 0,\quad M - \frac{20 \times 80}{1000} = 0 \cdot M = 1.6 N\text{-}m$



Tutorial #2 (Normal & Shear stress)

1. In section AB, apply eq. eqn $\Sigma F_x = 0$



12kN ⟵[ ]⟶ N ; $N - 12 = 0;\quad N = 12kN$

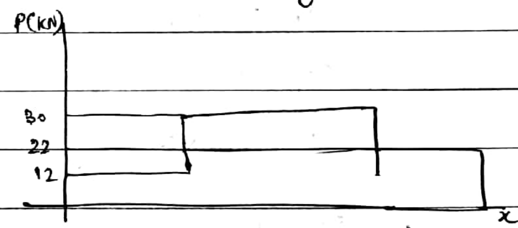
In section AC; $\Sigma F_x = 0$

$N - 18 - 12 = 0,\ N = 30\ kN$

In section AD $\quad \Sigma F_x = 0$

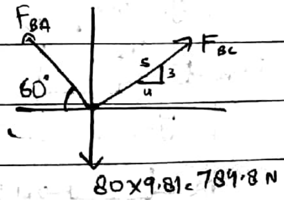
$-N + 22 = 0,\quad N = 22\ kN$

Axial Force diagram



Max avg. normal stress ⟹ $\frac{\text{Force}}{\text{Area}} = \frac{30 \times 10^3}{35 \times 10^{-6} \times 10} = 85.7\ MPa$

2. Apply eq. eqns;



$\Sigma F_x = 0;\quad \frac{4}{5} F_{BC} - F_{BA} \cos 60^\circ = 0\ ;\ \frac{4}{5} F_{BC} - \frac{1}{2} F_{BA} = 0$

$\Sigma F_y = 0;\quad \frac{3}{5} F_{BC} + F_{BA} \sin 60^\circ = 784.8;\ \frac{3}{5} F_{BC} + \frac{\sqrt{3}}{2} F_{BA} = 784.8$

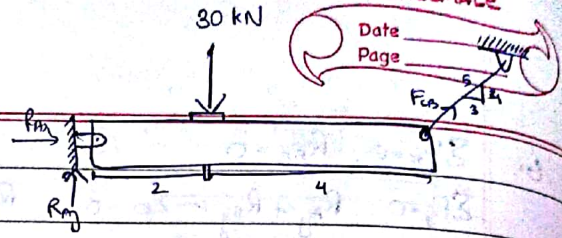
$F_{BC} = 395.24\ N;\quad F_{BA} = 632.38 N$

Stress = Force / Area

$\sigma_{BC} = \frac{395.24}{\pi \times 4^2 \times 10^{-6}} = 7.863\ MPa$

$\sigma_{BA} = \frac{632.38}{\pi \times 5^2 \times 10^{-6}} = 8.052\ MPa$

3. Apply eq eqn on beam



$\sum F_y = 0$

$R_{Ay} - 30\times10^3 + F_{CB}\times\frac{4}{5} = 0$

$+\circlearrowright \sum M_A = 0$

$-30\times2 + \frac{4}{5}F_{CB}\times6 = 0$

$F_{CB} = 12.5\,kN$

$\sum F_x = 0$

$R_{Ax} + \frac{3}{5}F_{CB} = 0$; $R_{Ax} = -7.5\,kN$

$R_{Ay} = 20\,kN$; $R_{A} = \sqrt{R_{Ax}^2 + R_{Ay}^2} = 21.36\,kN$

Due to double shear, in pin A ;

$V_A = R_A/2 = 10.68\,kN$

while, for pin B

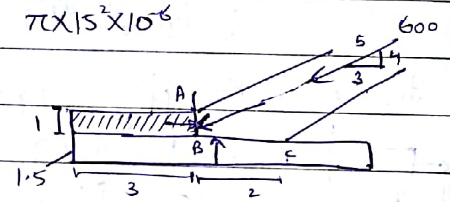
$V_B = R_B = F_{CB} = 12.5\,kN$

Avg shear stress $\Rightarrow \tau_A = \frac{V_A}{A_A}$; $\tau_B = \frac{V_B}{A_B}$

$= \frac{10680}{\pi\times10^2\times10^{-6}} = 34MPa$

$\frac{12500}{\pi\times15^2\times10^{-6}} = 17.7\,MPa$

4. Applying eq eqn on section



$\sum F_x = 0$

$F_{AB} - \frac{3}{5}\times600 = 0$; $F_{AB} = 360$ ~~kip~~ kip

$\sigma_{AB_{avg}} = \frac{\cancel{3600}\,240}{1.5\times1} = 240\,lb/in^2$

$\sum F_y = 0$

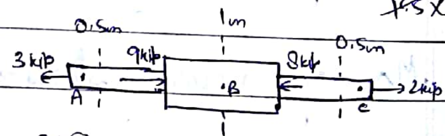
$F_{BC} - \frac{4}{5}\times600 = 0$; $F_{BC} = 480$ ~~kip~~ kip

$\tau = \frac{\cancel{3600}\,\cancel{240}\,80}{\cancel{1.5}\times3} = 80\,lb/in^2$

$\sigma_{B_{y}} = \frac{\cancel{4800}\,\cancel{320}\,160}{\cancel{1.5}\times2} = 160\,lb$

5 For 1st section

3 kip ; $N - 3 = 0$; $N = 3\,kip$

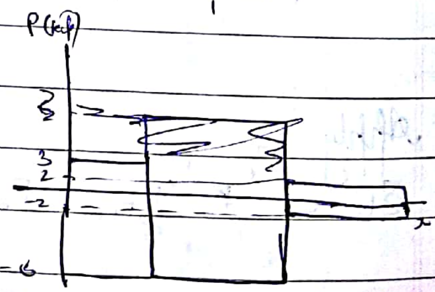


For 2nd section

9 kip, 3 kip ; $N + 9 - 3 = 0$; $N = -6\,kip$

For 3rd section

2 kip ; $-N + 2 = 0$; $N = 2\,kip$



$\sigma_A = \frac{3\times10^3\times4}{\pi\times0.5^2} = 15.276\,psi$ $\sigma_B = \frac{6\times10^3\times4}{\pi\times1^2} = 7.636\,psi$ $\sigma_C = \frac{2\times10^3\times4}{\pi\times0.5^2} = 10.185\,psi$

6. For lady

$\sigma = \frac{P}{A} = \frac{175}{\left(\left(\frac{1}{2}\times0.3^2\times\pi\right) + 0.1\times0.6\right)} = 869.04\,psi$

For man

$\sigma = \frac{P}{A} = \frac{175}{\left(\left(\frac{1}{2}\times1.2^2\times\pi\right) + 0.5\times2.4\right)} = 50.55\,psi$

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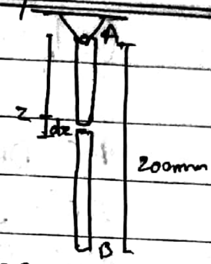
Tutorial #3 - Normal Strain, Shear Strain & Material Properties

1. Deformed length



$$dz' = dz + \varepsilon_z dz$$

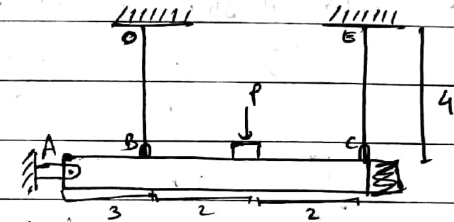
$$= (1 + 40\times10^{-3}\sqrt{z})\,dz$$

$$z' = \int_0^{0.2} (1 + 40\times10^{-3}\sqrt{z})\,dz = 0.2 + \frac{40\times10^{-3}(0.2)^{3/2}\times 2}{3} = 0.20239$$

a) Displacement of end B of rod $= 0.20239 - 0.2 = 0.00239\,m = 2.39\,mm$

b) $\varepsilon_{avg} = \frac{\Delta x' - \Delta x}{\Delta x} = \frac{2.39}{200} = 0.0119$ mm/mm

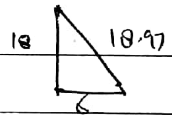
2. Via similar Δs

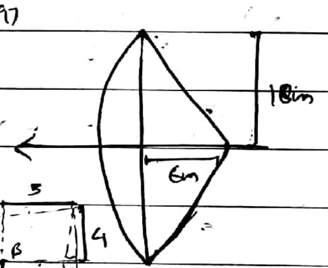


$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}; \quad \Delta L_{BD} = \frac{3\times10}{7} = 4.286\,m$$

$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L_{CE}} = \frac{10}{4000} = 0.00250\,mm/mm \qquad \varepsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{4.286}{4000} = 0.00107\,mm/mm$$

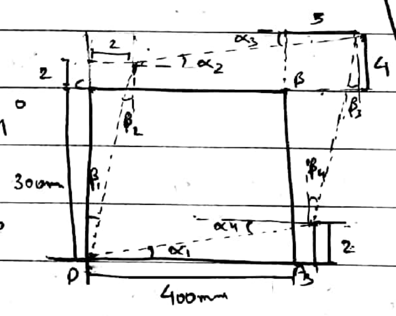
3. $x = 2L' = 2\sqrt{18^2+6^2} = 37.95\,in$





$$\varepsilon_{avg} = \frac{37.95 - 35.5}{35.5} = 0.0689\,in/in$$

4. At D,



$$\tan\alpha_1 = \frac{2}{403}, \quad \alpha_1 = \tan^{-1}\left(\frac{2}{403}\right) = 0.284^\circ$$

$$\tan\beta_1 = \frac{2}{302}; \quad \beta_1 = \tan^{-1}\left(\frac{2}{302}\right) = 0.379^\circ$$

Converting $\angle$s to rads, $\alpha_1 = 0.00496$ rad, $\beta_1 = 0.00662$ rad

At C,

$$\tan\alpha_2 = \frac{4-2}{405-2} \Rightarrow \alpha_2 = \tan^{-1}\left(\frac{2}{403}\right) = 0.284^\circ \Rightarrow 0.00496\,rad$$

$$\tan\beta_2 = \frac{2}{302} \Rightarrow \beta = \tan^{-1}\left(\frac{2}{302}\right) = 0.379^\circ \Rightarrow 0.00662\,rad$$

At B,

$$\tan\alpha_3 = \frac{304-302}{405-2} = \frac{2}{403}; \quad \alpha_3 = \tan^{-1}\left(\frac{2}{403}\right) = 0.284^\circ \Rightarrow 0.00496\,rad$$

$$\tan\beta_3 = \frac{\cancel{304-302}\ 5-3}{304-\cancel{2}} = \frac{2}{302}; \quad \beta_3 = \tan^{-1}\left(\frac{2}{302}\right) = 0.379^\circ \Rightarrow 0.00662\,rad$$

At A,

$$\tan\alpha_4 = \frac{2}{403}; \quad \alpha_4 = \tan^{-1}\left(\frac{2}{403}\right) = 0.284^\circ = 0.00496\,rad$$

$$\tan\beta_4 = \frac{2}{304-2}, \quad \beta = \tan^{-1}\left(\frac{2}{302}\right) = 0.379^\circ = 0.00662\,rad$$

$(\gamma_B)_{xy} = \alpha_3+\beta_3 = 0.01158\,rad$ $\quad (\gamma_C)_{xy} = -(\alpha_2+\beta_2) = -0.01158\,rad$

$(\gamma_A)_{xy} = -(\alpha_4+\beta_4) = -$ " " $\quad (\gamma_D)_{xy} = \alpha_1+\beta_1 =$ " "

$\therefore AC = DB = \sqrt{400^2+300^2} = 500\,mm$

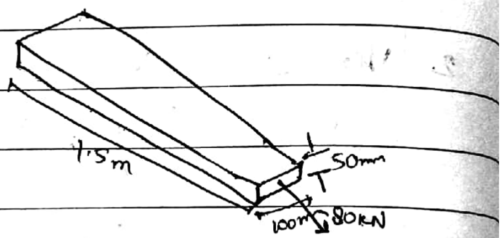
$A'C' = \sqrt{(403-2)^2+(302-2)^2} = 500.8\,mm$

$B'D' = \sqrt{405^2+304^2} = 506.4\,mm$

$\varepsilon_{AC} = \frac{0.8}{500} = 1.6\times10^{-3}\,mm/mm$ $\quad \varepsilon_{BD} = \frac{6.4}{500} = 1.28\times10^{-3}\,mm/mm$

6. Normal stress on the bar

$\sigma_z = \frac{F}{A} = \frac{80\times10^3}{100\times50\times10^{-6}} = 16\,MPa$



$E_{st} = 200\,GPa$

$\boxed{\varepsilon_z = \frac{\sigma_z}{E_{st}}} \Rightarrow \frac{16\times10^6}{200\times10^9} = 80\times10^{-6}\,mm/mm$

Axial elongation of bar

$\delta_z = \varepsilon_z L_z = 80\times10^{-6}\times1.5 = 120\,\mu m$

$\varepsilon_x = \varepsilon_y = -\nu_{st}\,\varepsilon_z$ $\quad$ (ν = Poisson's ratio $= 0.32$)

$= -0.32\times80\times10^{-6} = -25.6\,\mu m/m$

$\delta_x = \varepsilon_x L_x = -25.6\times0.1 = -2.56\,\mu m$ $\quad \delta_y = \varepsilon_y L_y = -\cancel{25.6}\times0.05 = -1.28\,\mu m$

7. a) $\sigma = F/A = \frac{800\times4}{\pi\times0.5^2} = 4074.37\,psi$

$\varepsilon_{longitudinal} = \sigma/E \Rightarrow \frac{-4074.37}{10.6\times10^6} = -0.0003844$

$\delta = \varepsilon_{longitudinal}\cdot L = -0.0003844\times1.5 = -0.577\times10^{-3}\,in$

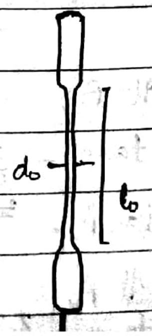
b) $\varepsilon_{lateral} = \nu\,\varepsilon_{longitudinal} = -0.35\times-0.0003844 = 0.0001345$

$\Delta d = \varepsilon_{lateral}\times d = 0.0001345\times0.5 = 0.00006727$

$d' = d+\Delta d = 0.50006727$

8) Avg normal stress $\sigma = F/A = \frac{165\times10^3\times4}{\pi\times(0.025)^2} = 336.1\,MPa$

Avg normal strain $\varepsilon_{long} = \frac{\delta}{l} = \frac{1.20}{250} = 0.00480\,mm/mm$



$E = \frac{\sigma}{\varepsilon_{long}} = \frac{336.1\times10^6}{0.00480} = 70\,GPa$

By relation between young's modulus & shear's modulus

$G = \frac{E}{2(1+\nu)}$; $26\times10^9 = \frac{70\times10^9}{2(1+\nu)}$; $\nu = 0.346$

$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$; $\varepsilon_{lat} = -\nu\times\varepsilon_{long} = -0.00166\,mm/mm$; $\delta'_{zz} = -\varepsilon_{lat}\times d = -0.00166\times25 = 0.0416\,m$

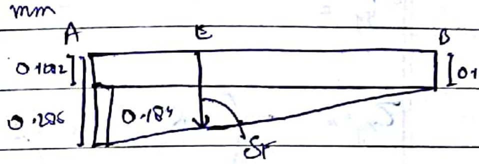
Tutorial#4 (Elastic deformation of an axially loaded member).

1. $F_{AC} = \frac{400}{600}\times90 = 60\,kN$ $\quad F_{BD} = \frac{200}{600}\times90 = 30\,kN$

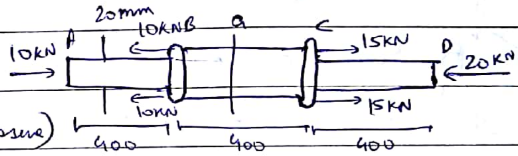
$\delta_{AC} = \frac{F_{AC}L_{AC}}{A_{AC}E_{st}} = -\frac{60\times10^3\times300\times10^{-3}}{\pi\times10^2\times10^{-6}\times200\times10^9} = -286\,\mu m$
$= \downarrow 0.286\,mm$

$\delta_{BD} = \frac{F_{BD}L_{BD}}{A_{BD}E_{Al}} = -\frac{30\times10^3\times300\times10^{-3}}{\pi\times20^2\times10^{-6}\times70\times10^9} = -102\,\mu m$
$= \downarrow 0.102\,mm$

$\delta_F = 0.102 + 0.184\times\frac{400}{600} = 0.225\,mm\downarrow$



2. For section AB, $\Sigma F_x = 0$



10 → N : $N + 10 = 0$, $N = -10\,kN$ (compressive)

For section AC, $\Sigma F_x = 0$

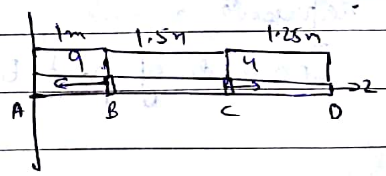
10, 10, 10 → N ; $N + 10 - 20 = 0$, $N = 10\,kN$ (Tensile)

For section CD, $\Sigma F_x = 0$

N ← → 20kN, $-N - 20 = 0$, $N = -20\,kN$ (Compressive)

$\delta_{D/A} = \delta_{AB} + \delta_{BC} + \delta_{CD} = -0.449\,mm$ $\quad \left(\delta = \frac{PL}{AE}\right)$

3. $\delta_{AC} = \delta_{AB} + \delta_{BC}$



$= \frac{9\times10^3\times1}{50\times10^{-6}\times200\times10^9} + \frac{6\times10^3\times1.5}{200\times10^9\times50\times10^{-6}}$

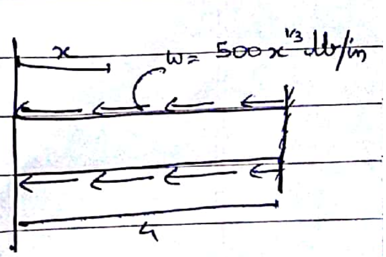
$= 0.600\,mm$

$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$

$= \frac{10^3}{200\times10^9\times50\times10^{-6}}\left(2\times1.25 + 6\times1.5 - 3\times1\right) = 0.850\,mm$

4. $P(x) = \int_0^x w(x)\,dx = \int_0^x 500x^{1/3} = \frac{500\times x^{4/3}\times3}{4}$

$= \frac{1500}{4}x^{4/3}$



$\delta(x) = \int_0^4 \frac{P(x)\,dx}{AE} = \frac{1}{35\times10^3\times3}\times\frac{1500}{4}\int_0^{48} x^{4/3}\,dx$

$\frac{1}{3\times35\times10^3}\times\frac{1500}{4}\times\frac{3}{7}\left[x^{7/3}\right]_0^{48} \Rightarrow \delta_A = 0.0128\,in$

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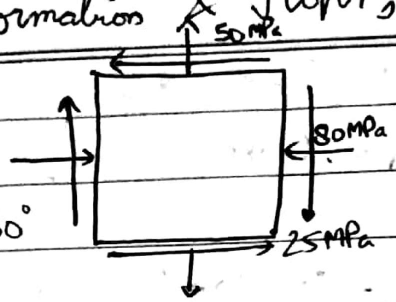
Tutorial#5 (Stress transformation & Mohr's Circle)



1\. (CPoST) $\sigma_{xx} = -80\,MPa$ $\sigma_{yy} = 50\,MPa$

$\tau_{xy} = \tau_{yx} = -25\,MPa$

As the orientation is 30° ; $\theta = -30°$

$$\sigma_{xx'} = \left(\frac{-80+50}{2}\right) + \left(\frac{-80-50}{2}\right)\cos(-60) + 25\sin(-60) = -25.8\,MPa$$

$$\sigma_{yy'} = \text{"} - \text{"} + \text{"} = -4.15\,MPa$$

$$\tau_{xy'} = -\left(\frac{-80-50}{2}\right)\sin(-60) \mp 25\cos(-60). = -68.8\,MPa$$

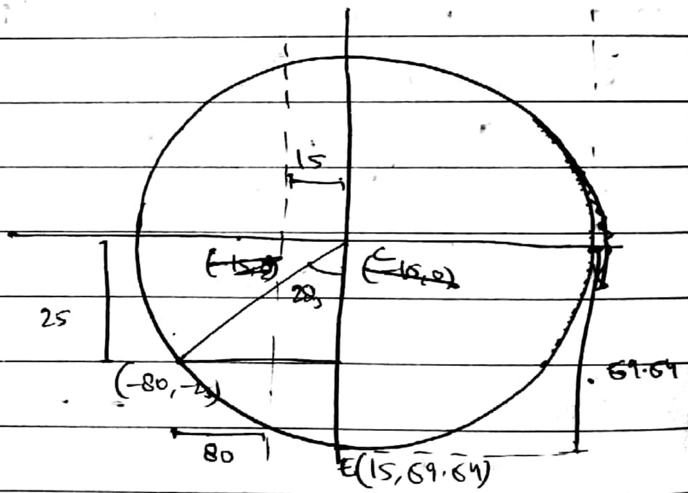
By Mohr's circle

-14.978

$$\sigma_{avg} = \frac{-80+50}{2} = -15\,MPa$$

$$R = \sqrt{\left(\frac{-80-50}{2}\right)^2 + 25^2}$$

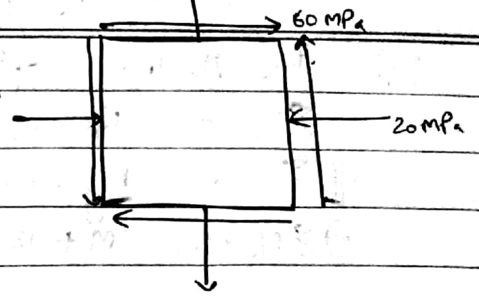
$$= 69.64\,MPa$$



$$(x+15)^2 + y^2 = (69.64)^2$$

represents a circle

Coordinates of point E gives



2. $\sigma_{xx} = -20\,MPa,\quad \sigma_{yy} = 90\,MPa,\quad \tau_{xy} = 60\,MPa$

$$\sigma_{1,2} = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{-20+90}{2}\right) \pm \sqrt{\left(\frac{-20-90}{2}\right)^2 + 60^2}$$

$$= 116\,MPa \quad \& \quad -46.4\,MPa$$

Angle ⇒

$$\sin 2\theta_s = \frac{-\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)}{\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\tau_{xy}^2}} \Rightarrow \theta_s = 21.3^\circ$$

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\tau_{xy}^2} = 81.4\ MPa \quad ; \quad \sigma_{avg} = \frac{\sigma_{xx}+\sigma_{yy}}{2} = 35\,MPa$$

By Mohr's circle

$R = \cancel{\tau_{max}}\ 81.4\,MPa \Rightarrow$ ( )

The coordinates of point E (35, 81.4)

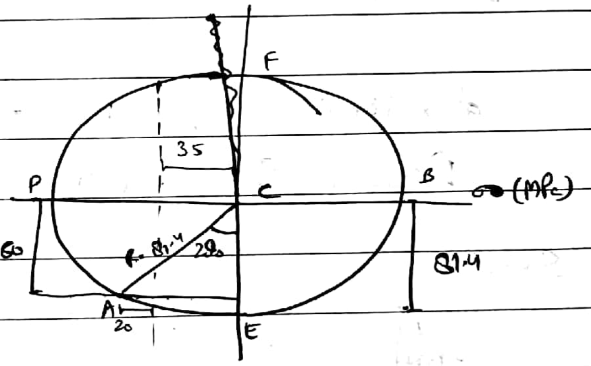
give

$\tau_{max} = 81.4\,MPa$; $\sigma_{avg} = 35\,MPa$

which are max in-plane shear stresses.

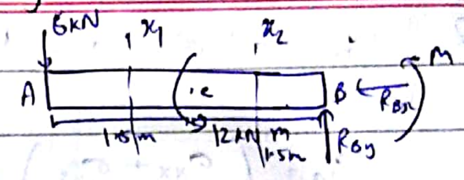
$$2\theta_s = \tan^{-1}\left(\frac{20+35}{60}\right) = 42.5^\circ$$

$$\theta_s = 21.3^\circ$$



Tutorial #6 (Shear Force & Bending moment)

1a) For the beam, apply eq eqn

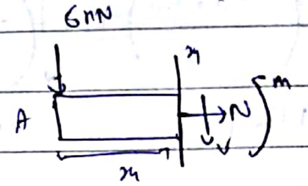


$\Sigma F_x = 0, \quad R_{Bx} = 0$

$\Sigma F_y = 0; \quad R_{By} - 6 = 0, \; R_{By} = 6\ kN$

$\Sigma M_B = 0, \quad M + 12 + 6 \times 3 = 0, \quad M = -30\ kN\text{-}m$ (clockwise)

For section x_1



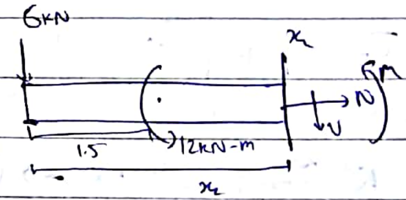
$\Sigma F_x = 0, \; N = 0$

$\Sigma F_y = 0; \; -V - 6\ kN = 0, \quad V = -6\ kN$

$\Sigma M_{x_1} = 0; \quad M_{x_1} + 6x_1 = 0; \quad M_{x_1} = -6x_1\ kNm$

at $x = 0m$; $V_{x_1} = -6kN$, $M_{x_1} = 0\ kNm$ at $x = 1.5m$, $V_{x_1} = -6kN$, $M_{x_1} = -9\ kN\text{-}m$

For section x_2



$\Sigma F_y = 0; \quad -V - 6 = 0, \quad V = -6\ kN$

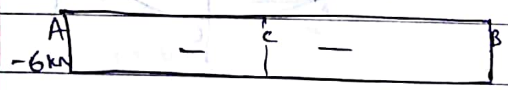
$\Sigma M_{x_2} = 0 \; ; \; M_{x_2} + 12 + 6x_2 = 0$

$M_{x_2} = -12 - 6x_2\ kN\cdot m$

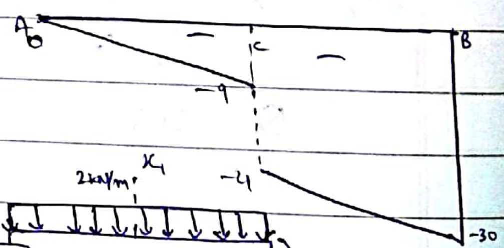
at $x = 1.5m$, $V_{x_2} = -6\ kN$, $M_{x_2} = -21\ kNm$ at $x = 3m$, $V_{x_2} = -6kN$, $M_{x_2} = -30\ kN\text{-}m$

- Drawing SFD & BMD

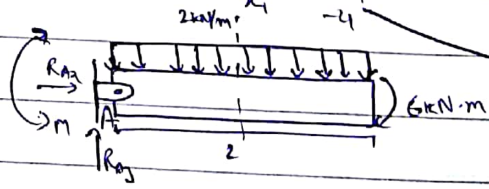
SFD :



BMD.



b) Apply eq eqn on beam

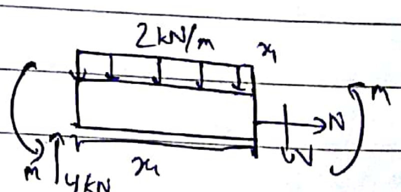


$\Sigma F_x = 0; R_{Ax} = 0$

$\Sigma F_y = 0; \; R_{Ay} - 2 \times 2 = 0; \quad R_{Ay} = 4\ kN$

$\Sigma M_A = 0; \quad M - \frac{2 \times 2^2}{2} - 6 = 0; \quad M = 10\ kNm$

For section x_1



$\Sigma F_x = 0, \; N = 0$

$\Sigma F_y = 0, \; 4 - V - 2x_1 = 0$

$V = 4 - 2x_1\ kN$

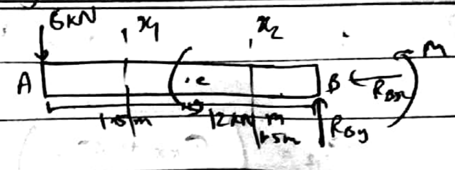
$\Sigma M_{x_1} = 0; \quad M_{x_1} + 10 - 4x_1 + x_1^2 = 0; \quad M_{x_1} = \cancel{10}\, 4x_1 - x_1^2 - 10\ kN.m$

at $x = 0m$. $V_x = 4kN$ $M_{..}$

Scanned by CamScanner

Tutorial #6 (Shear Force & Bending moment)

1a) For the beam, apply eq eqn

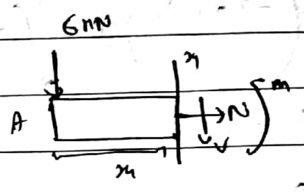


$\Sigma F_x = 0,\quad R_{Bx} = 0$

$\Sigma F_y = 0,\quad R_{By} - 6 = 0,\quad R_{By} = 6\ kN$

$\circlearrowleft \Sigma M_B = 0,\quad M + 12 + 6\times 3 = 0,\quad M = -30\ kN\cdot m$ (clockwise)

For section x_1



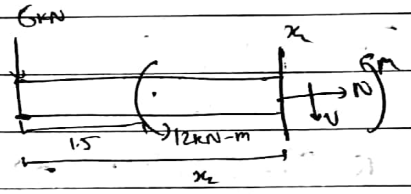
$\Sigma F_x = 0,\quad N = 0$

$\Sigma F_y = 0;\quad -V - 6\ kN = 0,\quad V = -6\ kN$

$\circlearrowleft \Sigma M_{x_1} = 0:\quad M_{x_1} + 6x_1 = 0,\quad M_{x_1} = -6x_1\ kN\cdot m$

at $x = 0m$: $V_{x_1} = -6\ kN$, $M_{x_1} = 0\ kN\cdot m$ at $x = 1.5m$, $V_{x_1} = -6\ kN$, $M_{x_1} = -9\ kN\cdot m$

For section x_2



$\Sigma F_y = 0;\quad -V - 6 = 0,\quad V = -6\ kN$

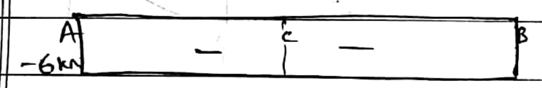
$\circlearrowleft \Sigma M_{x_2} = 0,\quad M_{x_2} + 12 + 6x_2 = 0$

$M_{x_2} = -12 - 6x_2\ kN\cdot m$

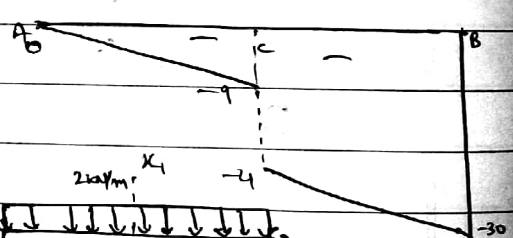
at $x = 1.5m$, $V_{x_2} = -6\ kN$, $M_{x_2} = -21\ kN\,m$ at $x = 3m$, $V_{x_2} = -6\ kN$, $M_{x_2} = -30\ kN\cdot m$

Drawing SFD & BMD

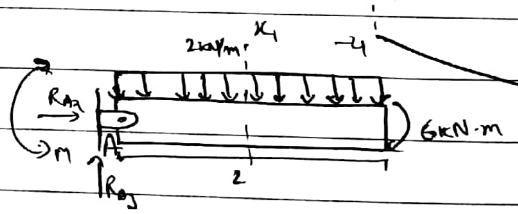
SFD



BMD.



b) Apply eq eqn on beam

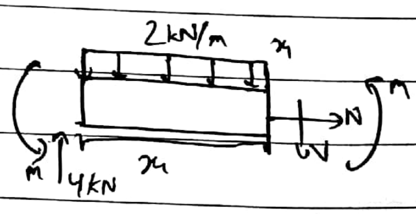


$\Sigma F_x = 0;\ R_{Ax} = 0$

$\Sigma F_y = 0;\ R_{Ay} - 2\times 2 = 0;\quad R_{Ay} = 4\ kN$

$\circlearrowleft \Sigma M_A = 0;\quad M - \frac{2\times 2^2}{2} - 6 = 0;\quad M = 10\ kN\,m$

For section x_1



$\Sigma F_x = 0,\ N = 0$

$\Sigma F_y = 0,\ 4 - V - 2x_1 = 0$

$V = 4 - 2x_1\ kN$

$\circlearrowleft \Sigma M_{x_1} = 0;\quad M_{x_1} + 10 - 4x_1 + x_1^2 = 0;\quad M_{x_1} = 4x_1 - x_1^2 - 10\ kN\cdot m$

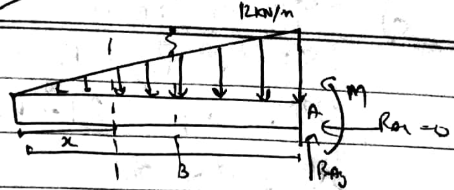
at $x = 0m$: $V_{x_1} = 4\ kN$, $M_{x_1} = -10\ kN\cdot m$

at $x = 2m$ $V_{x_1} = 0\ kN$; $M_{x_1} = -6\ kN\cdot m$

Drawing SFD

c) $\Sigma F_y = 0$

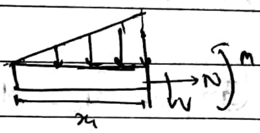
$R_{Ay} - \frac{1}{2} \times \cancel{12}^{6} \times 3 = 0, \quad R_{Ay} = 18\ kN$



$\circlearrowright \Sigma M_A = 0; \quad M + \frac{1}{2} \times \cancel{12}^{6} \times 3 \times \frac{3}{3} = 0, \quad M = \cancel{27}^{18}\ kN.m$ (clockwise) or $-27\ kN\cdot m$

For section x_1 ; ~~lo~~

Via similar Δs; $\frac{\cancel{12}4}{3} = \frac{w}{x_1}, \quad w = 4x_1$



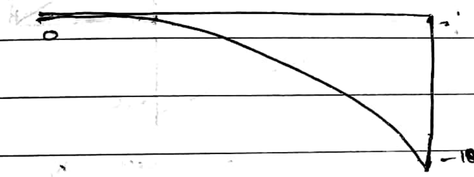
$\Sigma F_x = 0; \quad N = 0$

$\Sigma F_y = 0; \quad -V - \frac{1}{2} \times w \times x_1 = 0, \quad V = -\frac{1}{2} \times 4x_1^2 = -2x_1^2\ kN$

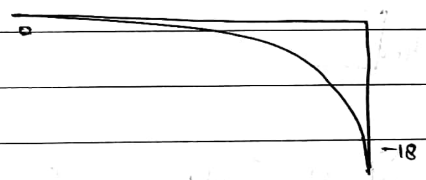
$\circlearrowright \Sigma M_{x_1} = 0; \quad M + \frac{1}{2} \times w \times x_1 \times \frac{x_1}{3} = 0; \quad M = -\frac{1}{2} \times 4x_1^3 \times \frac{1}{3} = \frac{-2x_1^3}{3}\ kN\cdot m$

at $x_1 = 0m$, $V_{x_1} = 0\ kN$, $M_{x_1} = 0\ kN\cdot m$ at $x_1 = 3m$, $V_{x_1} = -18\ kN$, $M_{x_1} = -18\ kN\cdot m$

SFD

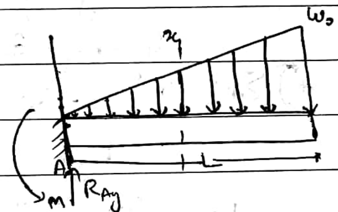


BMD



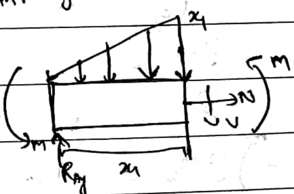
d) $\Sigma F_{y} = 0; \quad R_{Ay} - \frac{1}{2} \times w_0 \times L = 0; \quad R_{Ay} = \frac{w_0 L}{2}$

$\circlearrowright \Sigma M_A = 0 \quad ; \quad M - \frac{1}{2} \times w_0 L \times \frac{L}{3} = 0; \quad M = \frac{w_0 L^2}{6}$



For section x_1

Via similar Δs. $\frac{w_0}{L} = \frac{w}{x_1}, \quad w = \frac{w_0 x_1}{L}$

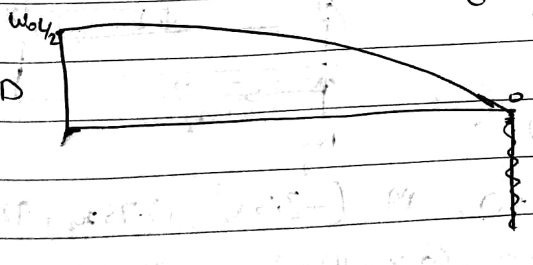


$\Sigma F_y = 0, R_{Ay} - V - \frac{1}{2} \times w \times x_1 = 0, \quad V = \frac{w_0 L}{2} - \frac{1}{2} \times \frac{w_0 x_1^2}{L} = \frac{w_0 L}{2} - \frac{w_0 x_1^2}{2L}$

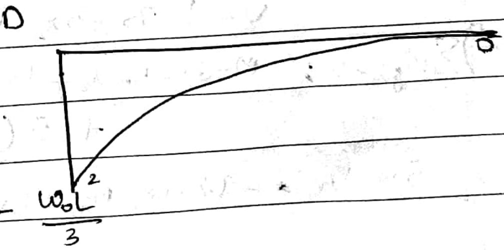
$\circlearrowright \Sigma M_{x_1} = 0, \quad M_{x_1} + M + \frac{1}{2} \times w \times x_1 \times \frac{x_1}{3} - R_{Ay} x_1 = 0; \quad M_{x_1} = -\frac{w_0 L^2}{6} - \frac{w_0 x_1^3}{6L} - \frac{w_0 L x_1}{3}$

at $x = 0$, $V_{x_1} = \frac{w_0 L}{2}\ kN$, $M_{x_1} = \frac{-w_0 L^2}{6}$; at $x = L$; $V_{x_1} = \cancel{\frac{-w_0 L}{2}}\ 0\ kN$, $M_{x_1} = \cancel{\frac{-w_0 L^2}{3}}\ 0$

SFD



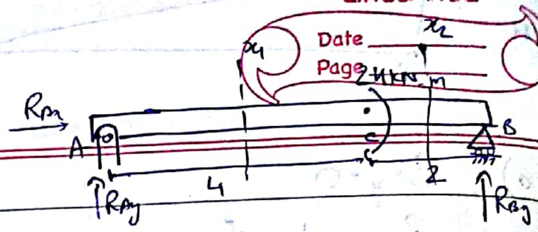
BMD



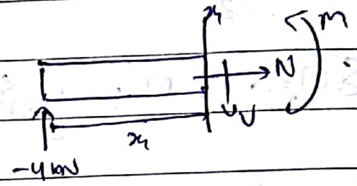
e) $\sum F_x = 0;\ R_{Ax} = 0$

$\sum F_y = 0;\ R_{Ay} + R_{By} = 0$, $R_{Ay} = -4\,kN$

$\circlearrowleft \sum M_A = 0;\ -24 + R_B 6 = 0;\ R_B = 4\,kN$



At section x_1

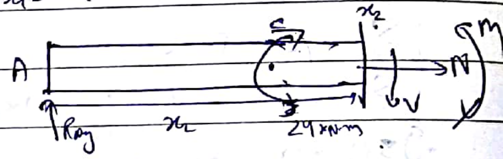


$\sum F_y = 0.\ -V + R_{Ay} = 0,\ V = R_{Ay} = -4\,kN$

$\circlearrowleft \sum M_{x_1} = 0;\ M - R_{Ay} x_1 = 0\ ;\ M = R_{Ay} x_1$
$= -4x_1\ kN{\cdot}m$

$@ x_1 = 0m,\ V_{x_1} = -4kN,\ M_{x_1} = 0\ kN{\cdot}m$, $@ x_1 = 4m,\ V_{x_1} = -4kN,\ M_{x_1} = -16\ kN{\cdot}m$

At section x_2

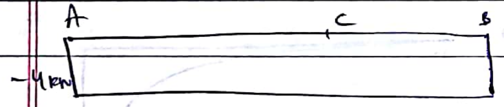


$\sum F_y = 0\ ;\ -V + R_{Ay} = 0;\ V = R_{Ay} = -4\,kN$

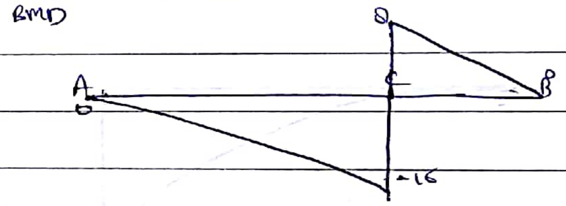
$\circlearrowleft \sum M_{x_2} = 0;\ M \mp 24 - R_{Ay} x_2 = 0\ ;\ M = +24 - 4x_2$

$@ x_2 = 4m, V_{x_2} = -4kN, M_{x_2} = 8kN{\cdot}m;$ $@ x_2 = 6m, V_{x_2} = -4kN, M_{x_2} = 0\,kN{\cdot}m$

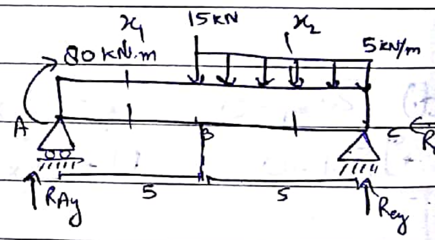
SFD



BMD



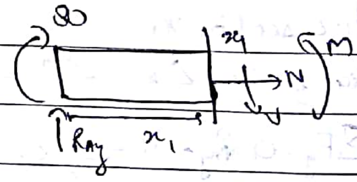
f.) $\sum F_y = 0;\ R_{Ay} + R_{cy} - 15 - 5\times5 = 0$
$R_{Ay} + R_{cy} = \cancel{27.5}\ 40$



$\circlearrowleft \sum M_c = 0;\ -80 - 10R_{Ay} + 15\times5 + \frac{5\times5}{2}\times5 = 0$
$R_{Ay} = 5.75\,kN$

$R_{cy} = 40 - 5.75 = 34.25\,kN$

For section x_1

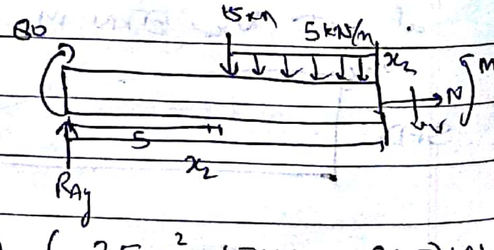


$\sum F_y = 0;\ R_{Ay} - V = 0;\ V = R_{Ay} = 5.75\ kN$

$\circlearrowleft \sum M_{x_1} = 0\ ;\ M - 80 - R_{Ay} x_1 = 0;\ M = 80 + 5.75x_1\ kN{\cdot}m$

$@\ x_1 = 0m,\ V_{x_1} = 5.75kN,\ M_{x_1} = 80\ kNm\ ;\ @ x = 5m,\ V_{x_1} = 5.75kN,\ M_{x_1} = 108.75\ kNm$

For section x_2

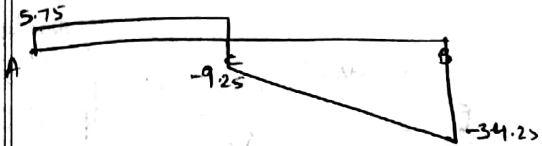


$\sum F_y = 0;\ R_{Ay} - 15 - 5(x_2 - 5) - V = 0$
$V = 15.75 - 5x_2\ kN$

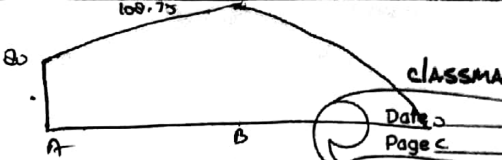
$\circlearrowleft \sum M_{x_2} = 0;\ M - 80 - R_{Ay} x_2 + 15(x_2 - 5) + 5\frac{(x_2-5)^2}{2} = 0;\ M = (-2.5x_2^2 + 15.75x_2 + 92.5)\ kN{\cdot}m$

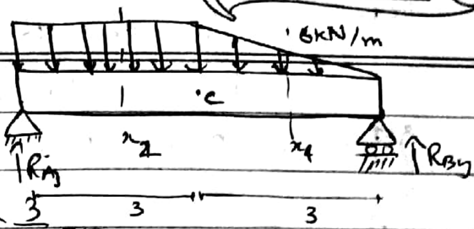
$@\ x_2 = 5m,\ V_{x_2} = -9.25kN,\ M_{x_2} = 108.75\ kN{\cdot}m\ ;\ @ x_2 = 10m,\ V_{x_2} = -34.25kN,\ M_{x_2} = 0\ kN{\cdot}m$

SFD



BMD





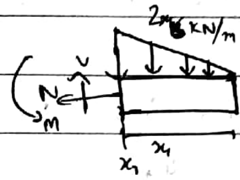
g.) $\Sigma F_y = 0$, $R_{Ay} + R_{By} - 6\times 3 - \frac{1}{2}\times \overset{3}{\cancel{6}}\times 3 = 0$

$R_{Ay} + R_{By} = 27$ kN

$\circlearrowleft + \Sigma M_B = 0$; $-6R_{Ay} + 6\times3\times\left(3+\frac{3}{2}\right) + \left(\frac{1}{2}\times6\times3\right)\times\frac{3}{2} = 0$

$R_{Ay} = 15.75$ kN ; $R_{By} = 11.25$ kN

For section x_1

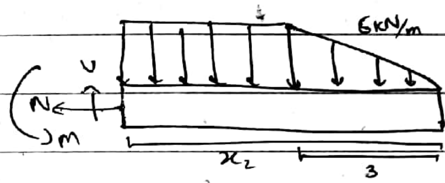


$\Sigma F_y = 0$; $V - 2x_1 = 0$; $V = 2x_1$

$\circlearrowleft + \Sigma M_{x_1} = 0$; $M - 2x_1 \times \frac{x_1}{2} = 0$; $M = x_1^2$

@ $x_1 = 0$ m, $V_{x_1} = 0$ kN, $M_{x_1} = 0$ kNm. @ $x_1 = 3$m ; $V_{x_1} = 6$kN, $M_{x_1} = 9$ kNm

For section x_2



$\Sigma F_y = 0$; $V - \frac{1}{2}\times\overset{3}{\cancel{6}}\times3 - 6(x_2-3) = 0$

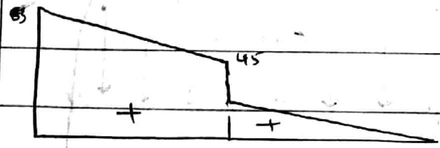
; $V = 27 + 6x_2$ kN

$\circlearrowleft + \Sigma M_{x_2} = 0$; $M - \frac{1}{2}\times\overset{3}{\cancel{6}}\times3\times\left(x_2 - \frac{3}{2}\right) - \frac{\overset{3}{\cancel{6}}(x_2-3)^2}{2} = 0$

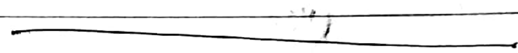
$M = 9(x_2 - 3/2) + 3(x_2^2 + 9 - 6x_2)$

$= 3x_2^2 - 18x_2 + 9x_2 + 27 - \frac{27}{2} = 3x_2^2 - 9x_2 + 27/2$

SFD

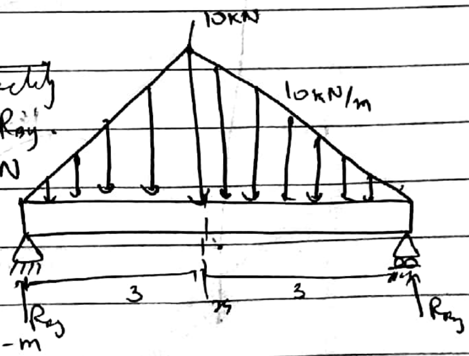


BMD



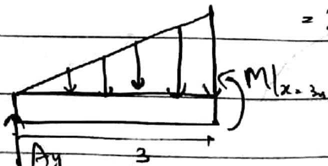
h) $\Sigma F_y = 0$; $R_{Ay} + R_{By} - 10 - 10\times3 = 0$

$R_{Ay} + R_{By} = 40$ kN ; Symmetrically $R_{Ay} = R_{By}$ = 20kN



~~ΣM=0~~

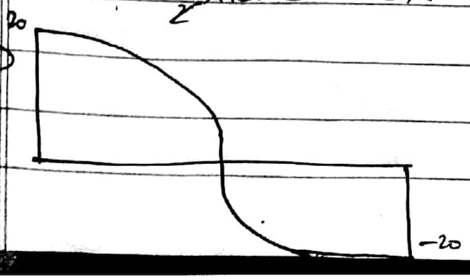
In sections x_1



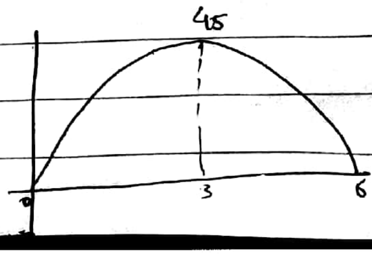
$\circlearrowleft + \Sigma M = 0$

$M|_{x=3m} + \frac{1}{2}\times\overset{5}{\cancel{10}}\times3 - 20\times3 = 0$; $M|_{x=3m} = 45$ kN-m

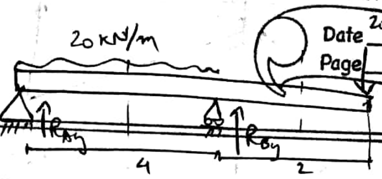
SFD



BMD



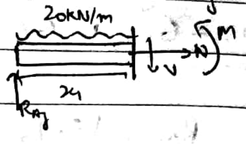
i)



$\sum F_y = 0;\ R_{Ay} + R_{By} - 80 - 20 = 0$

$R_{Ay} + R_{By} = 100$

$\circlearrowright \sum M_A = 0\ \ -\dfrac{20 \times 4 \times 4}{2} + 4R_{By} - 20 \times 6 = 0$; $R_{By} = 70kN$; $R_{Ay} = 30\,kN$



In section x_1

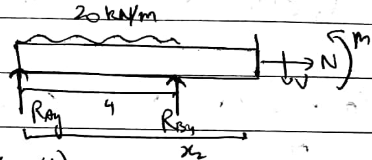
$\sum F_y = 0;\ \ R_{Ay} - V = 0;\ \ V = R_{Ay} = 30\,kN - 20x_1$

$\circlearrowright \sum M_{x_1} = 0;\ \ M_{x_1} - R_{Ay}x_1 + \dfrac{20x_1^2}{2} = 0$

$M_{x_1} = 30x_1 - 10x_1^2$

@ $x = 0\,m$: $V_{x_1} = 30\,kN$, $M_{x_1} = 0\,kNm$

@ $x = 4\,m$ $V_{x_1} = -50\,kN$; $M_{x_1} = -40\,kNm$



In section x_2

$\sum F_y = 0;\ R_{Ay} + R_{By} - V - 20 \times 4 = 0$: $V = 20\,kN$

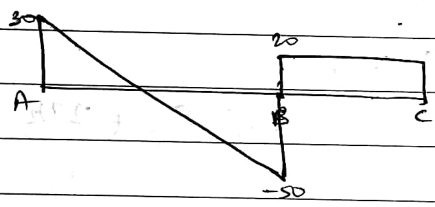
$\circlearrowright \sum M_{x_2} = 0;\ \ M - R_{Ay}x_2 + 20 \times 4 \times (x_2 - 2) - R_{By}(x_2 - 4) = 0$

$M = \cancel{R_{Ay}}\ 100x_2 - 80x_2 + 160 - 280 = 20x_2 - 120$

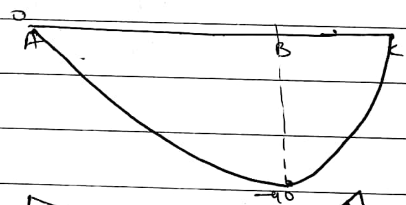
@ $x = 4m$; $V = 20\,kN$ $M = -40\,kN\text{-}m$

@ $x = 6m$ $V = 20\,kN$, $M = 0\,kN\,m$

SFD



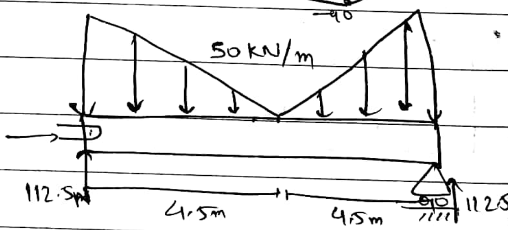
BMD

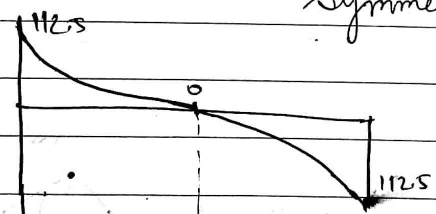


l.)

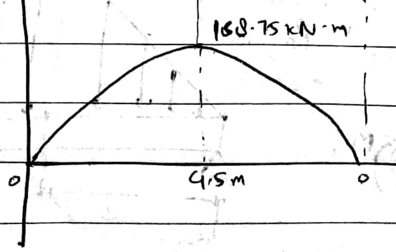
SFD

Symmetry

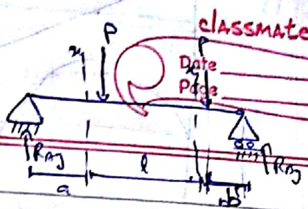




BMD



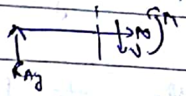
2 $\sum F_y = 0$; $R_{Ay} + R_{By} - 2P = 0$; $R_{Ay} + R_{By} = 2P$



$\sum M_B = 0$; $-R_{Ay}(a+l+b) + P(l+b) + Pb = 0$

$$R_{Ay} = \frac{Pl + 2Pb}{(a+l+b)} \;;\; R_{By} = 2P - \frac{P(l+2b)}{a+l+b} = \frac{2aP + Pl}{a+l+b} \text{ kN}$$

For sections S_1



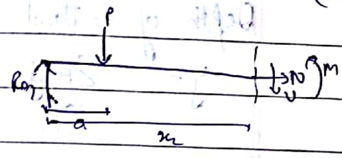
$\sum F_y = 0$; $R_{Ay} - V = 0$, $V = R_{Ay} = \frac{P(l+2b)}{(a+l+b)}$

at $x_1 = 0$; $V = \frac{P(l+2b)}{a+l+b}$, $M = 0$

$\sum M = 0$ $\quad M_{x_1} - R_{Ay} x_1 = 0$; $M_{x_1} = \frac{aP(l+2b)}{(a+l+b)}$

at $x_1 = a$; $V = $ " , $M = \frac{aP(l+2b)}{(a+l+b)}$

For section S_2



$\sum F_y = 0$, $R_{Ay} - P - V = 0$, $V = R_{Ay} - P = \frac{(b-a)P}{(a+l+b)}$

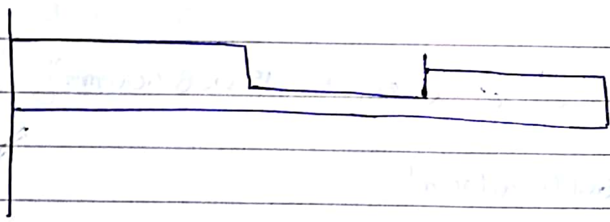
$\sum M = 0$; $M_{x_2} - R_{Ay} x_2 + P(x_2 - a) = 0$; $M_{x_2} = \frac{(l+2b)Px_2}{a+l+b} - Px_2 + Pa$

For $x_2 = a$; $V = \frac{b-a}{a+l+b} P$, $M = \frac{a(l+2b)}{(a+l+b)}$

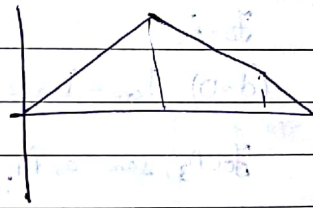
$l^2 + 2bl - Pl(a+l+b)$

$x_2 = a+l$, $V =$ " , $M = \frac{(b-a)l}{(a+l+b)}$

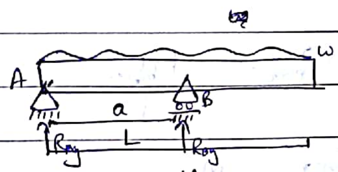
SFD =



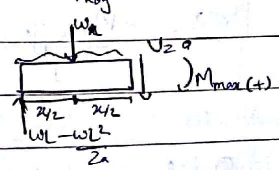
BMD.



3. $\sum F_y = 0$; $wL - \frac{wL^2}{2a} - wx = 0$



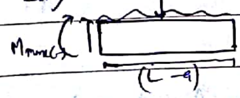
$$x = L - \frac{L^2}{2a} \quad \text{--- (1)}$$



$\sum M = 0$; $M_{max(+)} + wx\left(\frac{x}{2}\right) - \left(wL - \frac{wL^2}{2a}\right)x = 0$

from (1)

$$M_{max(+)} = \left(wL - \frac{wL^2}{2a}\right)\left(L - \frac{L^2}{2a}\right) - \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2 = \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

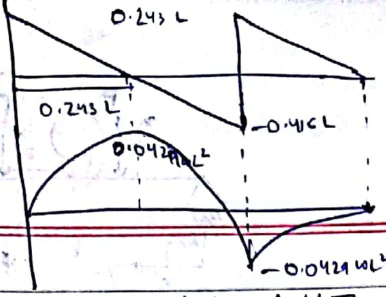


$\sum M = 0$; $M_{max(-)} - w(L-a)\frac{(L-a)}{2} = 0$; $M_{max(-)} = \frac{w(L-a)^2}{2}$

To get abs. min. moment

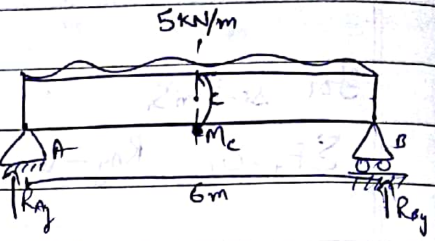
$M_{max(+)} = M_{max(-)}$; $\frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2 = \frac{w}{2}(L-a)^2$; $a = \frac{L}{\sqrt{2}}$

SFD:



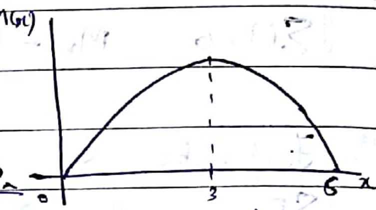
Tutorial #7 (Bending Stress Distribution)

1. $M_x = R_{Ay}x - \frac{5x^2}{2}$; $R_{Ay} = R_{By} = 15$ kN



Max internal moment is at center

$M_c = 15\times3 - \frac{5\times9}{2} = \frac{45}{2} = 22.5$ kN-m

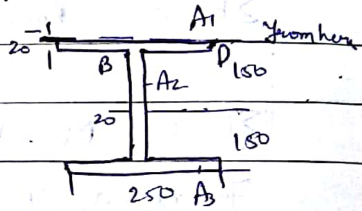


Depth of neutral axis

$y = \frac{\Sigma A\bar{d}}{\Sigma A} = \frac{(20\times250)\times10 + (20\times300)\times170 + \cancel{(20\times250)}\times330}{(20\times250)+(20\times300)+(20\times250)}$

$= \cancel{97.27 \text{ mm}}$

$= 170$ mm from top of section



For A_1

$I_{cg} = \frac{1}{12}bd^3 = \frac{1}{12}\times250\times20^3 = 166666.67 \text{ mm}^4$

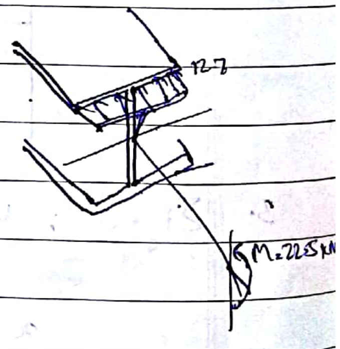
$I_{NA} = I_{cg} + Ad^2 = \cancel{966666.67 \text{ mm}^4}$ $128166666.67 \text{ mm}^4$ (d = 160)

For A_2

(d=0) $I_{cg} = I_{NA} = \frac{1}{12}bd^3 = \frac{1}{12}\times20\times300^3 = 45000000 \text{ mm}^4$

For A_3, same as A_1

$\therefore I_{NA} = \cancel{966666.67}$ 128166666.67 mm^4



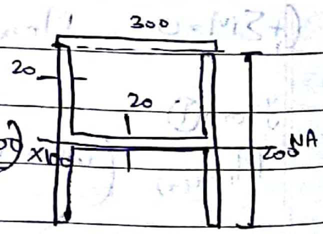
$\sigma_{max} = \frac{M y_{max}}{I} = \frac{22.5\times0.17\times10^3}{301333333.3\times10^{-12}} = 12.7$ MPa

$\sigma_B = \frac{M_{max}\, y_B}{I} = \frac{22.5\times0.15\times10^3}{} = -11.2$ MPa

2. Depth of neutral axis

$y = \frac{\Sigma A\bar{d}}{\Sigma A} = \frac{(20\times200) + (20\times260) + (20\times200)\times100}{}$

$= 100$ mm from top of section



For A_1 & A_3 ⇒ $I_{NA} = \frac{1}{12}bd^3 = \frac{1}{12}\times20\times200^3 = 13333333.33 \text{ m}^4$

For A_2 ⇒ $I_{NA} = \frac{1}{12}bd^3 = \frac{1}{12}\times260\times20^3 = 173333.33 \text{ m}^4$

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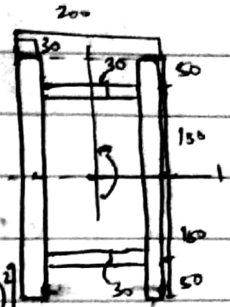
$$\sigma_{max} = \frac{M_{max} y_{max}}{I} = \frac{20 \times 10^3 \times 100 \times 10^{-3}}{26839999.99 \times 10^{-12}} = 74.5 \text{ MPa}$$

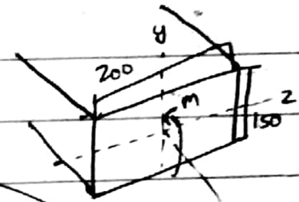


3 Depth of neutral axis $y = \frac{\Sigma Ad}{\Sigma A} =$

$$I = 2\left[\frac{1}{12} \times 0.03 \times (0.4)^3\right] + 2\left[\frac{1}{12} \times 0.14 \times 0.03^3 + 0.14 \times 0.03 \times (0.15)^2\right]$$

$$= 0.50963 \times 10^{-3} \text{ m}^4$$

$$\sigma_{max} = \frac{My}{I} = \frac{10 \times 10^3 \times 0.2}{0.50963 \times 10^{-3}} = 3.92 \text{ MPa}$$



4 Depth of neutral axis $y = \frac{\Sigma Ad}{\Sigma A}$

$I_z = \frac{1}{12} \times 0.2 \times (0.15)^3$, $I_{yy} = \frac{1}{12} \times 0.15 \times 0.2^3$

$= 56.25 \times 10^{-6} \text{ m}^4$, $= 0.1 \times 10^{-3} \text{ m}^4$

For bending about z-axis

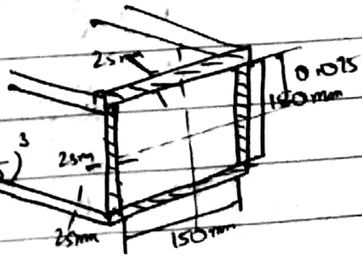
$y = 0.075 \text{ m}$

$$\sigma_{max} = \frac{My}{I_z} = \frac{90 \times 10^3 \times 0.075}{56.25 \times 10^{-6}} = 120 \text{ MPa}$$

For bending about y-axis

$y = 0.1 \text{ m}$

$$\sigma_{max} = \frac{My}{I_y} = \frac{90 \times 10^3 \times \cancel{0.1}}{\cancel{0.1} \times 10^{-3}} = 90 \text{ MPa}$$



5. Section property

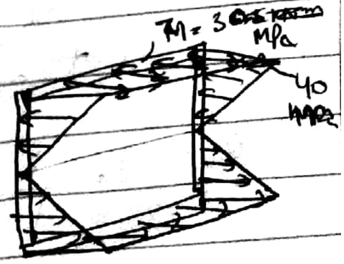
$$I = \cancel{\frac{1}{12} \times 0.2 \times (0.15)}\ \frac{1}{12} \times 0.2 \times 0.2^3 - \frac{1}{12} \times 0.15 \times (0.15)^3$$

$$= 9.11 \times 10^{-5} \text{ m}^4$$

By flexure formula, $\sigma = \frac{My}{I}$; $30 \times 10^6 = \frac{M \times 0.075}{9.11 \times 10^{-5}}$; $M = 36.4 \text{ kN.m}$

$\simeq 36.5 \text{ kN.m}$

$$\sigma_{max} = \frac{M y_{max}}{I} = \frac{36.5 \times 10^3 \times 0.1}{9.11 \times 10^{-5}} = 40 \text{ MPa}$$



6 Depth of neutral axis

$$y = \frac{\Sigma A\bar{d}}{\Sigma A} = \frac{0.025\times0.24\times0.0125 + 2\times 0.02\times0.15\times0.1}{0.025\times0.24 + 2\times0.02\times0.15} = \cancel{0.0795\,m} = 0.0562\,m$$

from top side

$$I_{top} = \frac{1}{12}\times0.24\times(0.025)^3 + 0.24\times0.025\times|0.05625-0.0125|^2$$

$$= 1.18\times10^{-5}\,m^4$$

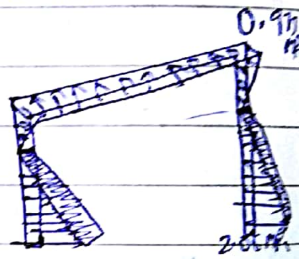
$$I_{side} = \frac{1}{12}\times0.02\times(0.15)^3 + 0.02\times0.15\times|0.1-0.05625|^2$$

$$= 1.14\times10^{-5}\,m^4$$

$$I = 3.46\times10^{-5}\,m^4$$

$$\sigma_{max} = \sigma_b = \frac{My_{max}}{I} = \frac{600\,(0.175-0.05625)}{3.46\times10^{-5}} = 2.06\,MPa$$

$$\sigma_c = \frac{My}{I} = \frac{600\times0.05625}{3.46\times10^{-5}} = 0.977\,MPa$$



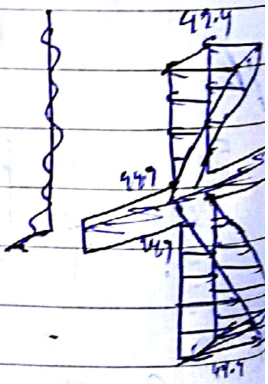
7 By section formula

$$I = \frac{1}{12}\times0.02\times(0.22)^3 + \frac{1}{12}\times(0.1)\times(0.02)^3 = 1.781\times10^{-5}\,m^4$$

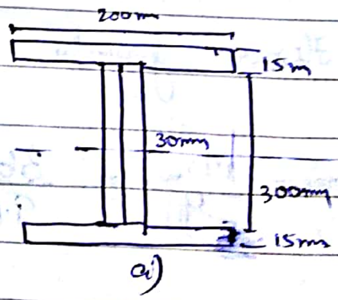
Applying flexure formula

$$\sigma_{max} = \frac{My_{max}}{I} = \frac{8\times10^3\times0.11}{1.781\times10^{-5}} = 49.4\,MPa$$

$$\sigma_c = \frac{My_c}{I} = \frac{8\times10^3\times0.01}{1.781\times10^{-5}} = 44.9\,MPa$$



8 $\cancel{I = 2\times\left[\frac{1}{12}\times0.2\times(0.33)^3 + 0.2\times0.015\times0.1575\right] + \frac{1}{12}\times0.03\times0.3^3 =}$



8 $$I_a = \frac{1}{12}\times0.2\times0.33^3 - \frac{1}{12}\times0.17\times(0.3)^3$$

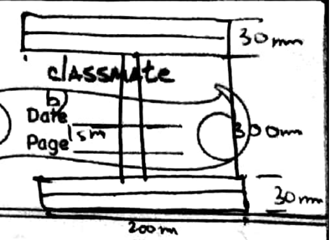
$$= 0.21645\times10^{-3}\,m^4$$

$$\sigma_{max} = \frac{150\times10^3\times0.165}{0.21645\times10^{-3}} = 114.3\,MPa$$

$I_b = \frac{1}{12} \times 0.2 \times (0.36)^3 - \frac{1}{12} \times 0.185 \times (0.3)^3$

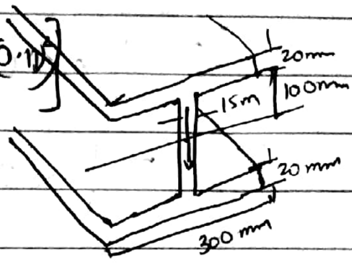
$= 0.36135 \times 10^{-3}\ m^4$

$\sigma_b = \frac{150 \times 10^3 \times 0.18}{0.36135 \times 10^{-3}} = 74.7\ MPa$



Tutorial # 8 (Shear Stress Distribution)

1) $I = \frac{1}{12} \times 0.015 \times 0.1^3 + 2\left[\frac{1}{12} \times 0.3 \times (0.02)^3 + 0.3 \times 0.02 \times (0.11)^2\right]$

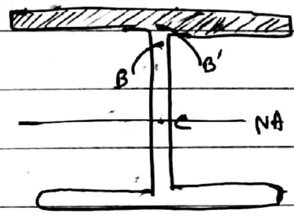


$= 155.6 \times 10^{-6}\ m^4$

For B'; $t_{B'} = 0.3\ m$

$Q_{B'} = \bar{y}'A' = 0.11 \times 0.3 \times 0.02$

$= 0.66 \times 10^{-3}\ m^3$

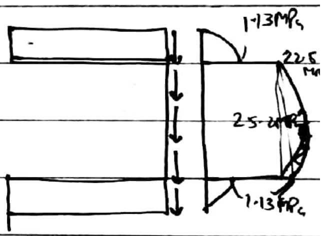


$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80 \times 10^3 \times 0.66 \times 10^{-3}}{155.6 \times 10^{-6} \times 0.3} = 1.13\ MPa$

For B: $t = 0.015\ m$

~~$Q_B = \bar{y}A = 0.015 \times 0.2 \times 0$~~ $Q_B = Q_{B'}$

$\tau_B = \frac{VQ_{B'}}{It_{B'}} = \frac{80 \times 10^3 \times 0.66 \times 10^{-3}}{1.55 \times 10^{-6} \times 0.015} = 22.6\ MPa$



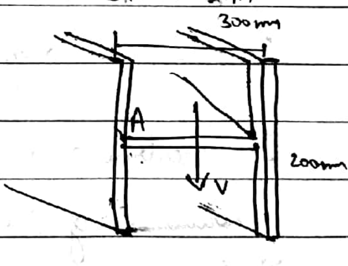
For C, $t = 0.015\ m$

$Q_C = \Sigma \bar{y}A = 0.11 \times 0.3 \times 0.02 + 0.05 \times 0.015 \times 0.1$

$= 0.735 \times 10^{-3}\ m^3$

Thus; $\tau_C = \frac{VQ_C}{It_C} = \frac{80 \times 10^3 \times 0.735 \times 10^{-3}}{155.6 \times 10^{-6} \times 0.015} = 25.2\ MPa$

2. $I = 2\left[\frac{1}{12} \times 0.02 \times 0.2^3\right] + \frac{1}{12} \times 0.02^3 \times 0.26 = 26.84 \times 10^{-6}\ m^4$



$Q_A = y_A A$

$= 0.055 \times 0.02 \times 0.09 = 99 \times 10^{-6}\ m^3$

$\tau_A = \frac{VQ_A}{It_A} = \frac{100 \times 10^3 \times 99 \times 10^{-6}}{26.84 \times 10^{-6} \times 0.02} = 18.4\ MPa$

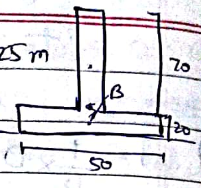
3. $I = \frac{1}{12}\ 2\left[\frac{1}{12} \times 0.03 \times 0.4^3\right] + 2\left[\frac{1}{12} \times 0.14 \times (0.03)^3 + 0.14 \times 0.03 \times 0.15^2\right]$

$= 0.50693 \times 10^{-3}\ m^4$

$Q_{max} = 2y_1'A_1' + y_2'A_2' = 2 \times 0.1 \times 0.2 \times 0.03 + 0.15 \times 0.14 \times 0.03 = 1.83 \times 10^{-3}\ m^3$

$\frac{VQ}{It} = \frac{[illegible] \times 10^3 \times 1.83 \times 10^{-3}}{[illegible]} = 1.2\ MPa$

4. $\bar{y} = \dfrac{0.05\times0.02\times0.01 + 0.02\times0.07\times0.055}{0.05\times0.02 + 0.02\times0.07} = 0.03625\,m$



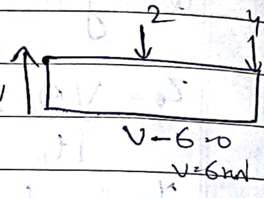
$I = \dfrac{1}{12}\times0.05\times(0.02)^3 + 0.05\times0.02\times(0.03625-0.01)^2$

$+\dfrac{1}{12}\times0.02\times(0.07)^3 + 0.07\times0.02\times(0.055-0.03625)^2$

$= 1.78625\times10^{-6}\,m^4$

$\bar{y}_B = 0.03625 - 0.01 = 0.02625\,m$

$Q_B = 0.02625\times0.05\times0.02 = 26.25\times10^{-6}\,m^3$

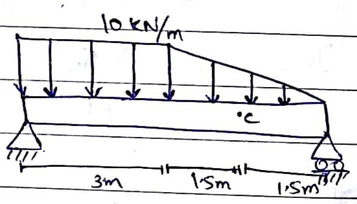


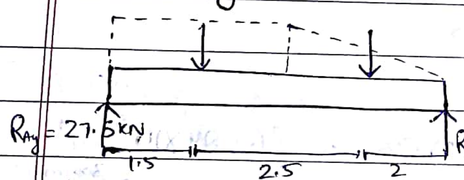
$\tau_B = \dfrac{VQ_B}{It_B} = \dfrac{6\times26.25\times10^{-6}\times10^3}{1.78625\times10^{-6}\times0.02} = 4.41\,MPa$

$Q_{max} = (0.070-0.03625)\times(0.09-0.03625)\times0.02$
$= 28.8906\times10^{-6}\,m^3$

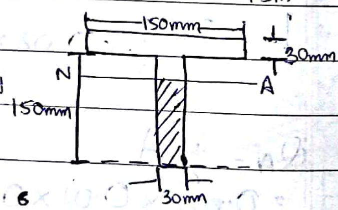
$\tau_{max} = \dfrac{VQ_{max}}{It_{max}} = \dfrac{6\times10^3\times28.8906\times10^{-6}}{1.78625\times10^{-6}\times0.02} = 4.85\,MPa$

5. The FBD of the beam is

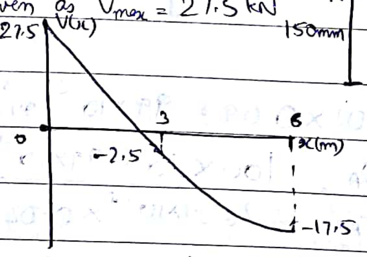




The shear diagram is also given as $V_{max} = 27.5$ kN



The neutral axis passes through centroid C of the cross-section



$\bar{y} = \dfrac{\Sigma\bar{y}A}{\Sigma A} = \dfrac{0.075\times(0.15\times0.03) + 0.165\times(0.03\times0.15)}{0.15\times0.03 + 0.03\times0.15} = 0.12\,m$

$I = \dfrac{1}{12}\times0.03\times(0.15)^3 + 0.03\times0.15\times(0.12-0.075)^2 + \dfrac{1}{12}\times0.15\times(0.03)^3 + 0.15\times0.03\times(0.165-0.$

$= 27\times10^{-6}\,m^4$

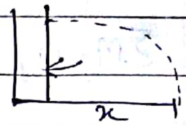
$Q_{max} = \bar{y}A' = 0.06 \times 0.12 \times 0.03 = 0.216 \times 10^{-3}\ m^3$

$\tau_{max} = \frac{VQ_{max}}{It} = \frac{27.5\times10^3\times0.216\times10^{-3}}{27\times10^{-6}\times0.03} = 7.33$ MPa

Now, at point C

$\Sigma F_y = 0; \quad V_c + 17.5 - \frac{1}{2}\times5\times1.5 = 0; \quad V_c = -13.75$ kN

Depth of neutral axis: $\bar{y} = 0.12$ m



$I = 27\times10^{-6}\ m^4$

$Q_c = Q_{max} = 0.216\times10^{-3}\ m^3$

$\tau_{max_c} = \frac{VQ_c}{It} = \frac{-13.75\times10^3\times0.216\times10^{-3}}{27\times10^{-6}\times0.03} = 3.67$ MPa

6. Depth of neutral axis

$$\bar{y} = \frac{(0.03\times0.125)\times0.015+(0.25\times0.025)\times0.155+(0.03\times0.2)\times(0.295)}{0.03\times0.125+0.25\times0.025+0.03\times0.2}$$

$$= 0.1747\ m$$

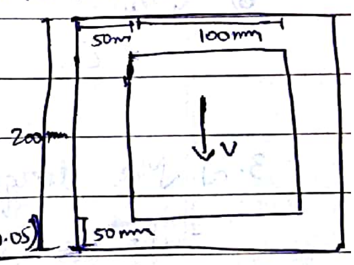
$$I = \frac{1}{12}(0.125)(0.03)^3 + 0.125\times0.03(0.1747-0.015)^2 + \frac{1}{12}(0.025)(0.25)^3 + 0.25\times0.025(0.1747-0.1$$
$$+ \frac{1}{12}(0.2)(0.03)^3 + 0.2\times0.03(0.295-0.1747)^2 = 0.218182\times10^{-3}\ m^4$$

$Q_A = \bar{y}A_A = (0.310-0.015-0.1747)(0.2)(0.03) = 0.7219\times10^{-3}\ m^3$

$Q_B\ \bar{y}A_B = (0.1747-0.015)(0.125)(0.03) = 0.59883\times10^{-3}\ m^3$

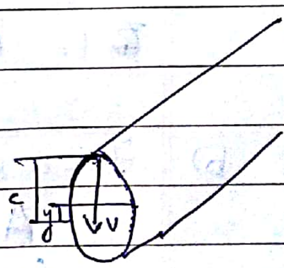
$\tau_A = \frac{VQ_A}{It} = \frac{15\times10^3\times0.7219\times10^{-3}}{0.218182\times10^{-3}\times0.025} = 1.99$ MPa $\quad \tau_B = \frac{VQ_B}{It} = \frac{15\times10^3\times0.59883\times10^{-3}}{0.218182\times10^{-3}\times0.025} = 1.65$ MPa

7. $I = \frac{1}{12}\times0.2\times(0.2)^3 - \frac{1}{12}\times(0.1)\times(0.1)^3 = 125\times10^{-6}\ m^4$



$\tau_{allow} = \frac{VQ}{It}; \quad 7\times10^6 = \frac{V[(0.075)(0.1)(0.05)+2(0.05)(0.1)(0.05)]}{125\times10^{-6}\times0.1}; \quad V = 100$ kN

8. $x = \sqrt{c^2-y^2}; \quad I = \frac{\pi}{4}c^4; \quad t = 2x = 2\sqrt{c^2-y^2}$



$dA = 2x\,dy = 2\sqrt{c^2-y^2}\,dy; \quad dQ = \bar{y}\,dA = 2y\sqrt{c^2-y^2}\,dy$

$Q = \int_y^x 2y\sqrt{c^2-y^2}\,dy = -\frac{2}{3}\left|(c^2-y^2)^{3/2}\right|_y^x = \frac{2}{3}(c^2-y^2)^{3/2}$

$\tau = \frac{VQ}{It} = \frac{V\left|\frac{2}{3}(c^2-y^2)^{3/2}\right|}{\left(\frac{\pi}{4}c^4\right)2\sqrt{c^2-y^2}} = \frac{4V}{3\pi c^4}[c^2-y^2]^{3/2}$

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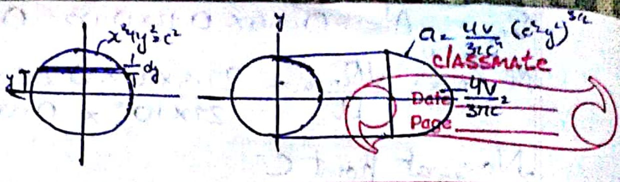
The max shear stress occurs at $y = 0$.

$\tau_{max} = \frac{4V}{3\pi c^2}$; $\tau_{avg} = \frac{V}{A} = \frac{V}{\pi c^2}$;

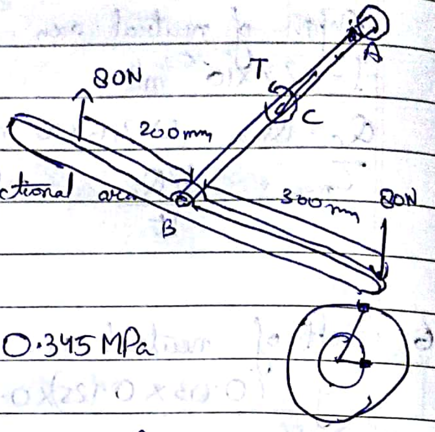
Factor $\Rightarrow \frac{\tau_{max}}{\tau_{avg}} = \frac{4}{3}$



Tutorial #9 (Torsion)

1. $\sum M_y = 0$; $80 \times 0.3 + 80 \times 0.2 - T = 0$

$T = 40\ N.m$



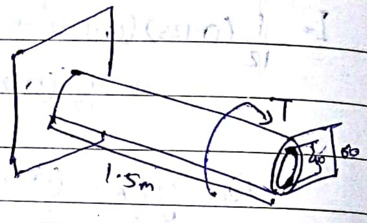
Polar moment of inertia for pipe's cross-sectional area

$J = \frac{\pi}{2}\left[(0.05)^4 - (0.04)^4\right] = 5.796 \times 10^{-6}\ m^4$

Shear stress (For point outside the pipe) $\tau_o = \frac{Tc_o}{J} = \frac{40 \times 0.05}{5.796 \times 10^{-6}} = 0.345\ MPa$

Shear stress (" " inside the pipe) $\tau_i = \frac{Tc_i}{J} = \frac{40 \times 0.04}{5.796 \times 10^{-6}} = 0.276\ MPa$

2. a) Largest permissible torque, $T = \frac{J\tau_{max}}{c}$



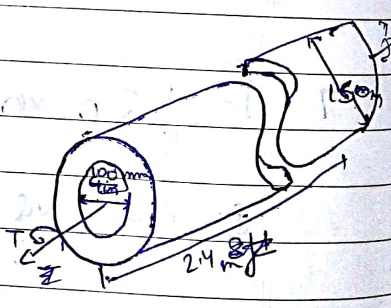
$J = \frac{\pi}{2}\left((0.03)^4 - (0.02)^4\right) = 1.021 \times 10^{-6}\ m^4$

$T = \frac{J\tau_{max}}{c} = \frac{1.021 \times 10^{-6} \times 120 \times 10^6}{0.03} = 4.08\ kN.m$

b) Minimum shearing stress

$\tau_{min} = \frac{c_1}{c_2}\tau_{max} = \frac{0.02}{0.03} \times 120 \times 10^6 = 80\ MPa$

3. a) Max torque in this shaft $= T = \frac{J\tau_{max}}{c}$



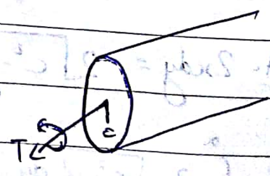
$J = \frac{\pi}{2}(0.15^4 - 0.1^4) = 6.381 \times 10^{-4}\ m^4$

$T = \frac{6.381 \times 10^{-4} \times 84 \times 10^6}{0.15} = 357.356\ kN.m$

b) For a solid shaft of equal weight

$A_a = A_b$

$\pi(0.15^2 - 0.1^2) = \pi r^2$; $r = 0.112\ m$



$J = \frac{\pi}{2} \times 0.112^4 = 2.454 \times 10^{-4}\ m^4$

$T = \frac{2.454 \times 10^{-4} \times 84 \times 10^6}{0.112} = 184.078\ kN.m$

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c) $A_a = A_c$

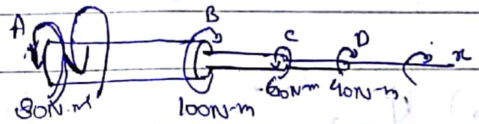
$\pi(0.15^2 - 0.1^2) = \pi(0.2^2 - r^2)$

$r = 0.166\,m$

$J = \frac{\pi}{2}((0.2)^4 - (0.166)^4) = 1.33\times10^{-3}\ m^4$

$T = \frac{1.33\times10^{-3} \times 84\times10^{6}}{0.2} = 558.6\ kN\cdot m$

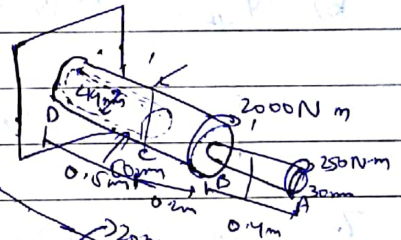
4 Angle of twist



$\phi_A = \frac{\Sigma TL}{JG}$

$= \frac{L}{G}\left(\frac{-80}{\frac{\pi}{2}r_1^4} \frac{\overset{20}{\cancel{+100}}}{\frac{\pi}{2}r_2^4} \cancel{-\frac{60}{\frac{\pi}{2}r_3}} + \frac{40}{\frac{\pi}{2}r_3^4}\right)$

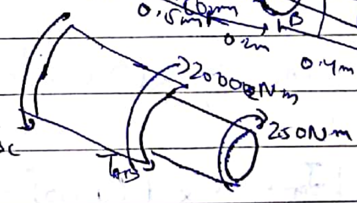
5. For section AB



$\Sigma M_x = 0;\quad 250 - T_{AB} = 0$

$T_{AB} = 250\,N\cdot m$

For section BC



$\Sigma M_x = 0 \quad 2250 - T_{BC} = 0$

$T_{BC} = 2250\,N\cdot m$

$T_{CD} = T_{BC} = 2250\,N\cdot m$

Polar moments of Inertia

$J_{AB} = \frac{\pi}{2}(0.015)^4 = 0.0795\times10^{-6}\ m^4$

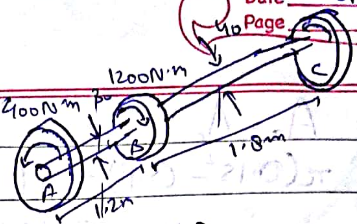
$J_{CD} = \frac{\pi}{2}((0.03)^4 - (0.022)^4) = 0.904\times10^{-6}\ m^4$

$J_{BC} = \frac{\pi}{2}(0.03)^4 = 1.272\times10^{-6}\ m^4$

$\phi_A = \frac{\Sigma TL}{JG} = \frac{1}{G}\left(\frac{T_{AB}L_{AB}}{J_{AB}} + \frac{T_{BC}L_{BC}}{J_{BC}} + \frac{T_{CD}L_{CD}}{J_{CD}}\right)$

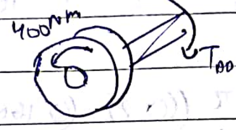
$= \frac{1}{77\times10^9}\left(\frac{250\times0.4}{0.0795\times10^{-6}} + \frac{2250\times0.2}{1.272\times10^{-6}} + \frac{2250\times0.6}{0.904\times10^{-6}}\right) = 0.0403\ rad$

$\phi_A = \frac{0.0403\times180}{\pi} = 2.31^\circ$



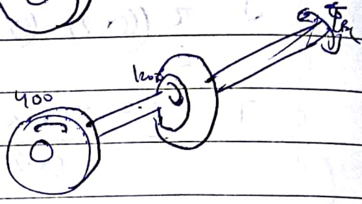
6. $J_{AB} = \frac{\pi}{2}\left(\frac{0.03}{2}\right)^4 = \cancel{1.27}\ 7.95 \times 10^{-8}\ m^4$

$J_{BC} = \frac{\pi}{2}\left(\frac{0.04}{2}\right)^4 = 2.513 \times 10^{-7}\ m^4$



In section AB

$400 - T_{AB} = 0, \quad T_{AB} = 400\ N{\cdot}m$



In section BC

$400 - 1200 - T_{BC} = 0, \quad T_{BC} = -800\ N{\cdot}m$

$= 800\ N{\cdot}m$ (clockwise)

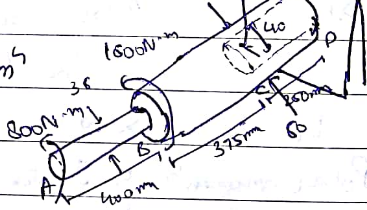
a) $\phi_{B/A} = \frac{T_{AB} \times L_{AB}}{G \times J_{AB}} = \frac{400 \times 1.2}{39 \times 10^9 \times 7.95 \times 10^{-8}} = 0.155\ rad$

$\phi_{B/A} = \frac{0.155 \times 180}{\pi} = 8.87^\circ$

b) $\phi_{C/A} = 0.155 + \frac{(-800) \times 1.8}{2.513 \times 10^{-7} \times 39 \times 10^9} = 8.07 \times 10^{-3}\ rad$

$\phi_{C/A} = \frac{8.07 \times 10^{-3} \times 180}{\pi} = 0.463^\circ$

7. $J_{AB} = \frac{\pi}{2} \times (0.018)^4 = 1.649 \times 10^{-7}\ m^4$ $\quad J_{BC} = \frac{\pi}{2} \times (0.03)^4 = 1.27 \times 10^{-6}\ m^4$

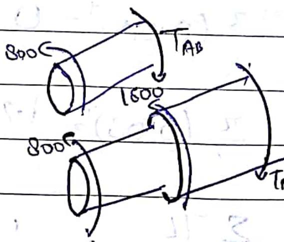


$J_{CD} = \frac{\pi}{2} \times \left((0.03)^4 - (0.02)^4\right) = 1.021 \times 10^{-6}\ m^4$

In section AB

$\Sigma M_x = 0$

$800 + T_{AB} = 0, \quad T_{AB} = 800\ N{\cdot}m$



In section BC

$\Sigma M_x = 0$

$800 + 1600 - T_{BC} = 0; \quad T_{BC} = 2400\ N{\cdot}m$

$T_{CD} = T_{BC} = 2400\ N{\cdot}m$

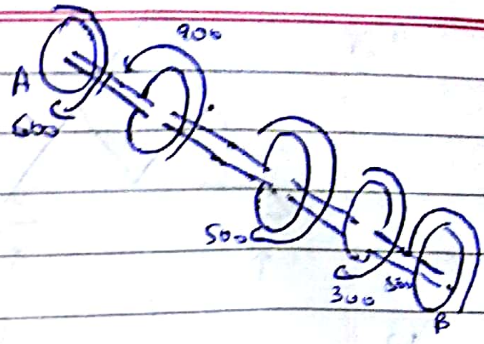
$\phi_A = \frac{T_{AB} \times L_{AB}}{G_{Al} \times J_{AB}} + \frac{T_{BC} \times L_{BC}}{G_{Br} \times J_{BC}} + \frac{T_{CD} \times L_{CD}}{G_{Br} \times J_{CD}}$

$= \frac{800 \times 0.4}{27 \times 10^9 \times 1.65 \times 10^{-7}} + \frac{2400 \times 0.375}{39 \times 10^9 \times 1.27 \times 10^{-6}} + \frac{2400 \times 0.25}{39 \times 10^9 \times 1.021 \times 10^{-6}} = 0.1051\ rad$

$\phi_A = \frac{0.1051 \times 180}{\pi} = 6.02^\circ$

$J = \frac{\pi}{2} \times (0.02)^4 = 2.513 \times 10^{-7}\ m^4$

8 $\phi_A = \frac{\Sigma TL}{JG}$



$= \frac{0.2}{2.513 \times 10^{-7} \times 75 \times 10^9} (600 - 900 + \overset{2}{300} + 500)$

$= 0.01061\ rad \approx 0.608'$

9. $\omega = \frac{2\pi n}{60} = \frac{2\pi \times 1200}{60} = 125.66\ rad/s$

$P = T\omega$

$600 \times \overset{746}{\cancel{550}} = T \times 125.66$

$T = 3562\ N.m$

Shear-stress failure

$\tau_{allow} = \frac{Tc}{J} = \frac{3562 \times c}{\frac{\pi}{2} c^{4}} = 60 \times 10^6$; $c = 0.0335\ m$

Angle of twist limitation

$\phi = \frac{TL}{JG}$

$0.05 = \frac{3562 \times 0.6}{11 \times 10^6 \times \frac{\pi}{2} \times c^4}$; $c = 0.42\ m$

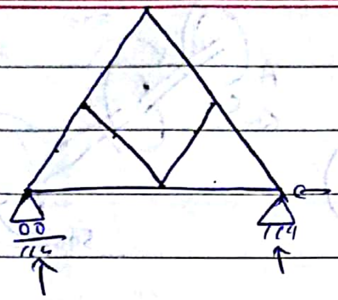
Shear Stress failure controls design

$d = 2c = 2 \times 0.0335 = 0.067\ m$

$d \approx 0.07\ m$

Tutorial #10 (Trusses)

1.a)



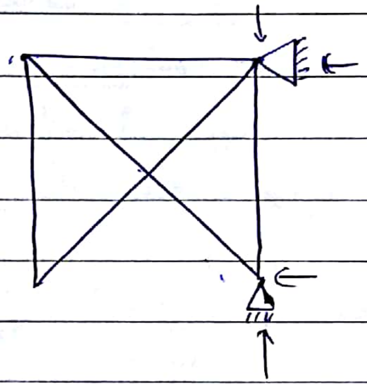
b = no. of bars = 8

r = no. of external rxns = 3

j = no. of joints = 6

$b + r = 11 < 2j$; unstable
$\quad\quad 12$

b)



$b = 7$

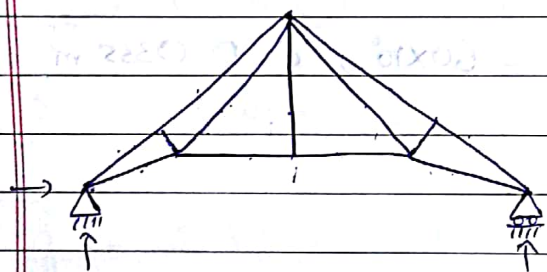
$r = 4$

$j = 5$

$b+r = 11 > 2j$ statically indeterminate to 1^{o}
$\quad\quad 10$

Degree of indeterminacy $= b + r - 2j = 11 - 10 = 1^{o}$

c)



$b = 13$

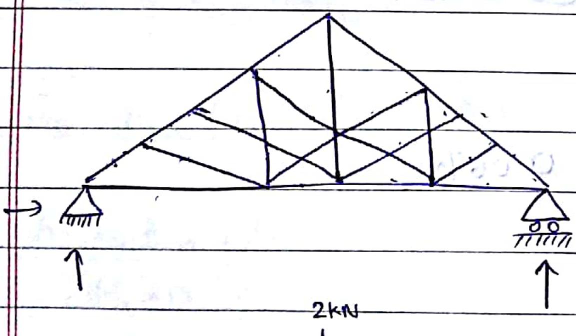
$r = 3$

$j = 8$

$b + r =$ ~~16 < 2j~~ $16 = 2j$
$\quad\quad 16$

Statically determinate

d)



$b = 21$

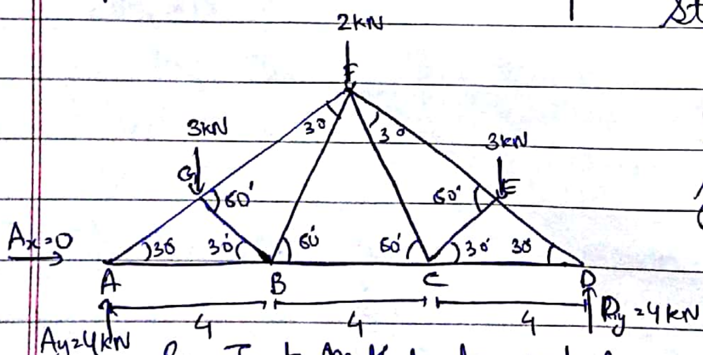
$r = 3$

$j = 12$

$b + r = 24 = 2j$
$\quad\quad 24$

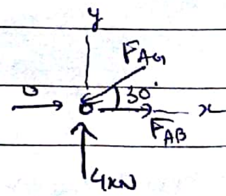
Statically determinate

2



Due to symmetry, only determine forces on half of members

By Joint Method, at joint A



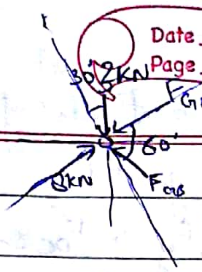
$+\uparrow \sum F_y = 0$; $4 - F_{AG} \sin 30^{\circ}$; $F_{AG} = 8$ kN (C)

$\xrightarrow{+} \sum F_x = 0$, $F_{AB} - F_{AG} \cos 30^{\circ}$; $F_{AB} = 6.928$ kN (T)

At joint G

$+\uparrow \sum F_y = 0$; $F_{GB} \sin 60' - 3 \cos 30' = 0$

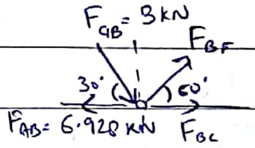
$F_{GB} = 3$ kN (C)



$+\nearrow \sum F_x = 0$; $- F_{GF} - F_{GB} \cos 60' + \cancel{8}\ 4\ 3 \sin 30' = 0$

$F_{GF} = 5$ kN (C)

At joint B



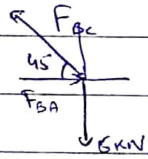
$+\uparrow \sum F_y = 0 \Rightarrow -F_{GB} \sin 30' + F_{BF} \sin 60' = 0$

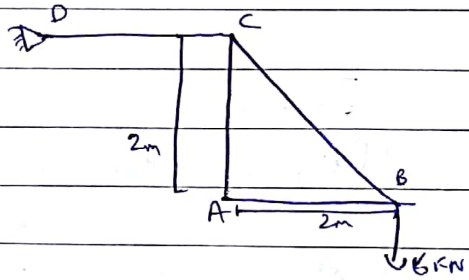
$F_{BF} = 1.732$ kN (T)

$\xrightarrow{+} \sum F_x = 0 \Rightarrow F_{BC} + F_{BF} \cos 60' + F_{GB} \cos 30' - F_{AB} = 0$

$F_{BC} = \cancel{6.928 - 0.866} - 3.464$ kN (T)

3a) At joint B





$+\uparrow \sum F_y = 0$

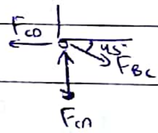
$F_{BC} \sin 45 - 6 = 0$

$F_{BC} = 8.485$ kN (T)

$\xrightarrow{+} \sum F_x = 0$, $F_{BA} - F_{BC} \cos 45 = 0$

$F_{BA} = 6$ kN (C)

At joint C



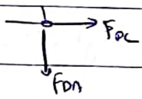
$+\uparrow \sum F_y = 0$; $F_{CA} - F_{BC} \sin 45 = 0$

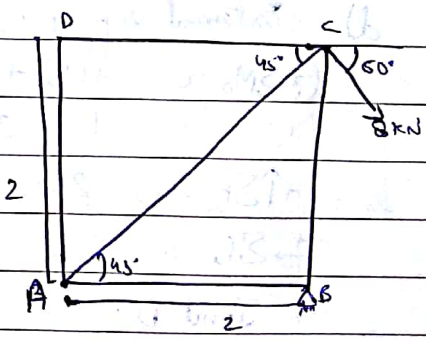
$F_{CA} = F_{BC} \sin 45 = 6$ kN (C)

$\xrightarrow{+} \sum F_x = 0$ $F_{BC} \cos 45 - F_{CD} = 0$

$F_{CD} = 6$ kN (T)

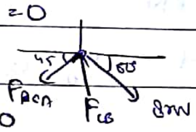
b) For joint D





$+\uparrow \sum F_y = 0$, $F_{DA} = 0$ & $\xrightarrow{+} \sum F_x = 0$, $F_{DC} = 0$

For joint C

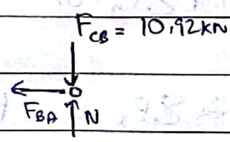


$+\uparrow \sum F_y = 0$; $F_{CB} - 8 \sin 60' - F_{CA} \sin 45 = 0$

$F_{CB} = 10.92$ kN (C)

$\xrightarrow{+} \sum F_x = 0$ $8 \cos 60' - F_{CA} \cos 45 = 0$, $F_{CA} = 5.66$ kN (T)

For joint B

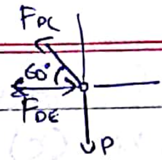


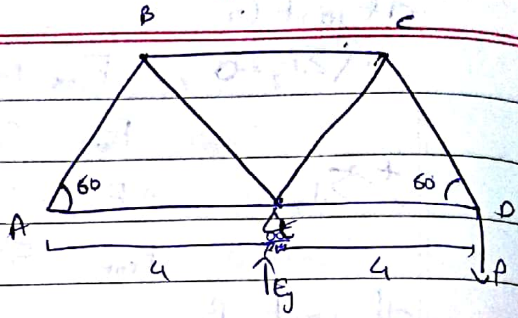
$\xrightarrow{+} \sum F_x = 0$, $F_{BA} = 0$

$+\uparrow \sum F_y = 0$, $N = 10.92$ kN

By methods of joints

c) For joint D





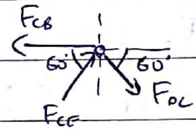
$+\uparrow \sum F_y = 0$; $F_{DE} \sin 60^\circ - P = 0$

$F_{DC} = 1.1547P$ (T)

$\xrightarrow{+} \sum F_x = 0$; $F_{DE} - 1.1547P \cos 60^\circ = 0$

$F_{DE} = 0.57735P$ (C)

Joint C



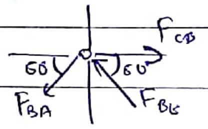
$+\uparrow \sum F_y = 0$; $F_{CE} \sin 60^\circ - F_{DC} \sin 60^\circ = 0$

$F_{CE} = 1.1547P$ (C)

$\xrightarrow{+} \sum F_x = 0$; $F_{DC} \cos 60^\circ + F_{CE} \cos 60^\circ - F_{CB} = 0$

$F_{CB} = 1.1547P$ (C)

Joint B



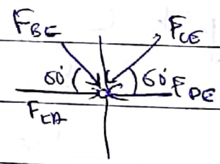
$+\uparrow \sum F_y = 0$; $-F_{BA} \sin 60^\circ + F_{BE} \sin 60^\circ = 0$

$F_{BA} = F_{BE} = F$

$\xrightarrow{+} \sum F_x = 0$ $F_{CB} - 2F \cos 60^\circ = 0$

$F = 1.1547P$ & ; $F_{BA} = 1.1547P$ (T) & $F_{BC} = 1.1547P$ (C)

Joint E

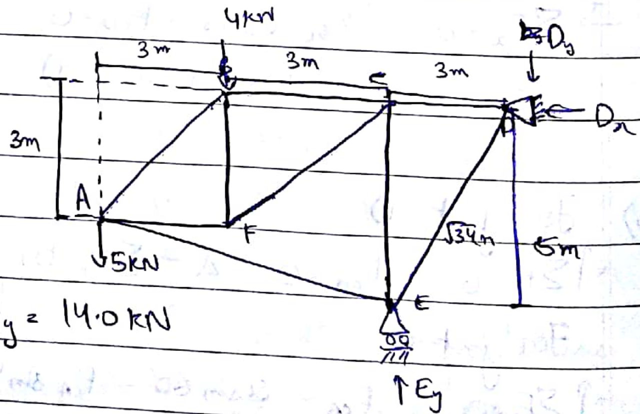


$\xrightarrow{+} \sum F_x = 0$: $F_{EA} - F_{DE} + F_{BE} \cos 60^\circ - F_{CE} \cos 60^\circ = 0$

$F_{EA} = 0.57735P$ (C)

$1.1547P = 6$; $P = 5.20$ kN

d) External support rxn



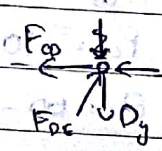
$\circlearrowleft + \sum M_D = 0$; $4(6) + 5(9) - E_y(3) = 0$

$E_y = 23$ kN

$+\uparrow \sum F_y = 0$; $23.0 - 4 - 5 - D_y = 0$; $D_y = 14.0$ kN

$\xrightarrow{+} \sum F_x = 0$; $D_x = 0$

Joint D.



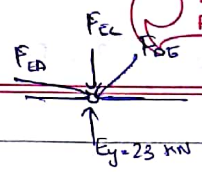
$+\uparrow \sum F_y = 0$; $F_{DE} \times \frac{5}{\sqrt{34}} - D_y = 0$

$F_{DE} = 16.33$ kN (C)

$\xrightarrow{+} \sum F_x = 0$ $F_{DE} \frac{3}{\sqrt{34}} - F_{CD} = 0$

$F_{CD} = 8.40$ kN (T)

Joint E



$\xrightarrow{+} \sum F_x = 0$

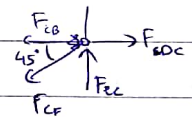
$$F_{EA}\left(\frac{6}{2\sqrt{10}}\right) - F_{DE}\frac{3}{\sqrt{34}} = 0$$

$F_{EA} = 8.85$ kN (C)

$+\uparrow \sum F_y = 0$; $E_y - F_{EA}\left(\frac{2}{2\sqrt{10}}\right) - F_{EC} - F_{DE}\frac{5}{\sqrt{34}} = 0$

$F_{EC} = 6.20$ kN (C)

Joint C



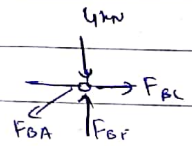
$+\uparrow \sum F_y = 0$; $6.20 - F_{CF}\sin 45° = 0$

$F_{CF} = 8.77$ kN (T)

$\xrightarrow{+} \sum F_x = 0$; $F_{DC} - F_{CB} - F_{CF}\cos 45 = 0$

$F_{CB} = 2.20$ kN (T)

Joint B



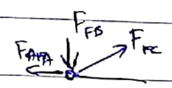
$\xrightarrow{+} \sum F_x = 0$; $F_{BC} - F_{BA}\cos 45 = 0$

$F_{BA} = 3.11$ kN (T)

$+\uparrow \sum F_y = 0$; $F_{BF} - 4 - F_{BA}\sin 45° = 0$

$F_{BF} = 6.20$ kN (C)

Joint F



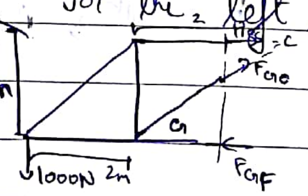
$+\uparrow \sum F_y = 0$; $8.768 \sin 45 - 6.20 = 0$ (Verified)

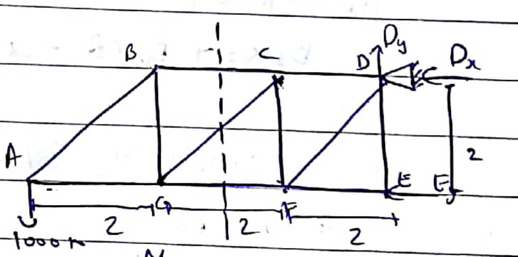
$\xrightarrow{+} \sum F_x = 0$; $8.768 \cos 45 - F_{FA} = 0$

$F_{FA} = 6.20$ kN (T)

4. For the entire truss $\curvearrowleft + \sum M_A = 0$; $-6D_y$ ...

For the left section





$+\curvearrowleft \sum M_G = 0$

$-1000 \times 2 - F_{BC} \times 2 = 0$, $F_{BC} = 1$ kN.

For entire truss

$\curvearrowleft \sum M_G = 0$

$1000 \times 2 + D_y \times 4 + D_x \times 2 = 0$

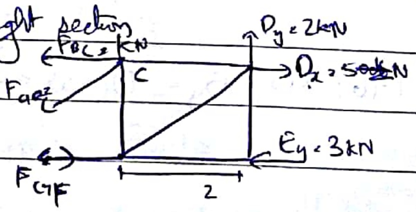
$\curvearrowleft \sum M_E = 0$

$1000 \times 6 + D_x \times 2 + D_y \times 2 = 0$

$D_y = 2000$ kN,

$D_x = -5000$ kN

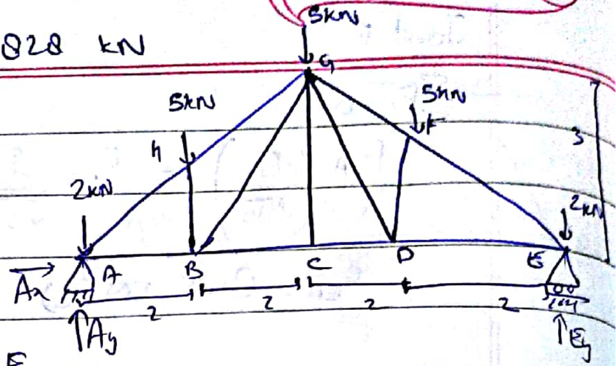
For right section



$+\curvearrowleft \sum M_C = 0$, $-2D_y - 2E_y + 2F_{GF} = 0$, $F_{GF} = 2 - 3 = 1$ kN (C) $\curvearrowleft \sum M_D = 0$

$K = 3000$ kN

$\sum F_y = 0$; $2 - F_{GC}\cos 45 = 0$, $F_{GC} = 2.828$ kN



5. $+\circlearrowleft \sum M_G = 0$

$F_{CD} \times 3 - (9.5) \times 4 + 5 \times 2 + 2 \times 4 = 0$

$F_{CD} = 6.67$ kN (T)

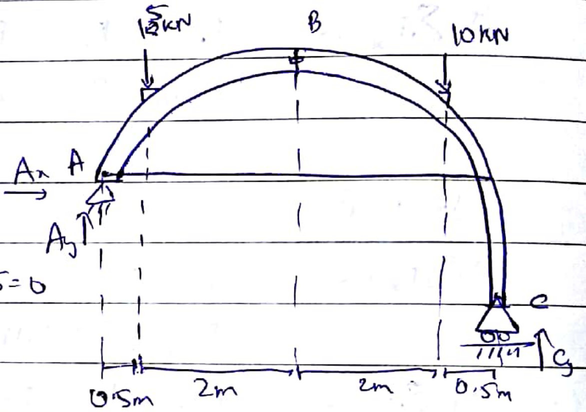
$+\circlearrowleft \sum M_D = 0$; $(-9.5)2 + 2 \times 2 + \frac{4}{5}(1.5) F_{GF} = 0$

$F_{GF} = 12.5$ kN (C)

Joint C

$F_{GC} = 0$

Tutorial #11 (Arches)



1. For entire arch

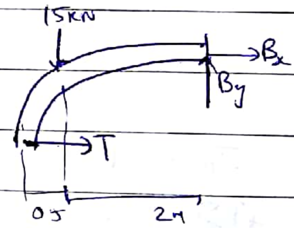
$\sum F_x = 0$; $A_x = 0$

$\circlearrowleft + \sum M_A = 0$, $-15 \times 0.5 - 10 \times 4.5 + C_y \times 5 = 0$

$C_y = 9.55$ kN

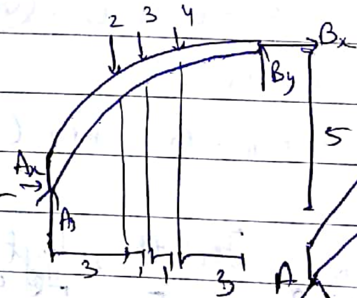
$+\uparrow \sum F_y = 0$; $9.55 - 10 - 15 + A_y = 0$

$A_y = 15.5$ kN



$\circlearrowleft + \sum M_B = 0$; $-15.45 \times 2.5 + T \times 2 + 15 \times 2$

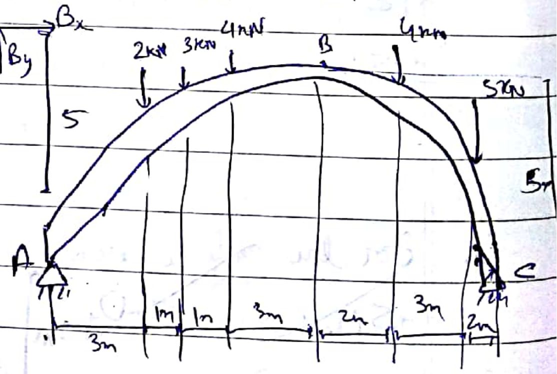
$T = 4.32$ kN



2 Member AB

$\circlearrowleft + \sum M_A = 0$;

$B_x \times 5 + B_y \times 8 - 2 \times 3 - 3 \times 4 - 4 \times 5 = 0$



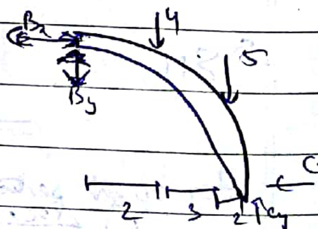
$5B_x + 8B_y = 38$ —— (I)

Member BC

$+\circlearrowleft \sum M_C = 0$

$-B_x \times 5 + B_y \times 7 + 5 \times 2 + 4 \times 5 = 0$

$-5B_x + 7B_y = -30$ —— (II)



Solving, we get, $B_x = 6.7467$ kN, $B_y = 0.533$ kN

Member AB

$\xrightarrow{+} \Sigma F_x = 0$; $A_x = 6.7467$ kN

$+\uparrow \Sigma F_y = 0$, $A_y - 9 + 0.533 = 0$; $A_y = 8.467$ kN

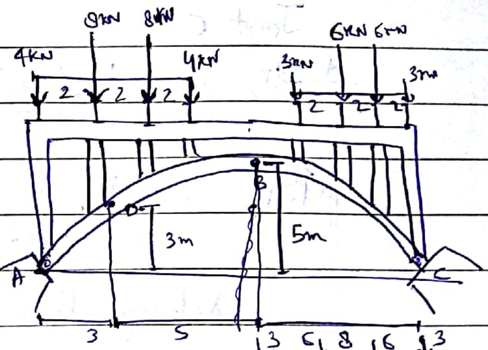
Member BC

$\xrightarrow{+} \Sigma F_x = 0$; $C_x = 6.7467$ kN

$+\uparrow \Sigma F_y = 0$; $C_y - 9 \mp 0.533 = 0$; $C_y = \cancel{8.467}$ 9.533 kN

$F_B = \sqrt{(0.533)^2 + (6.7467)^2} = 6.77$ kN $\quad F_C = \sqrt{(6.7467)^2 + (9.533)^2} = 11.7$ kN

$F_A = \sqrt{(6.7467)^2 + (8.467)^2} = 10.8$ kN



3 $\circlearrowleft + \Sigma M_A = 0$ (For member AB)

$B_x \times 5 + B_y \times 8 - 8\times2 - 8\times4 - 4\times6 = 0$

$B_x + 1.6 B_y = 14.4$ — (I)

Member CB

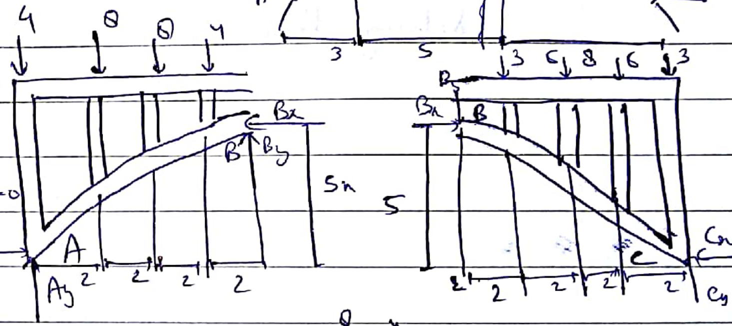
$\circlearrowleft + \Sigma M_c = 0$

$-B_x \times 5 + B_y \times 8 + 6\times2 + 6\times4 + 3\times6 = 0$

$-B_x + 1.6 B_y = 10.8$ — (II)

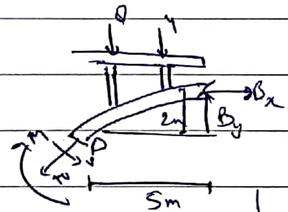
From (I) & (II), we get

$B_x = 12.6$ kN, $\quad B_y = 1.125$ kN



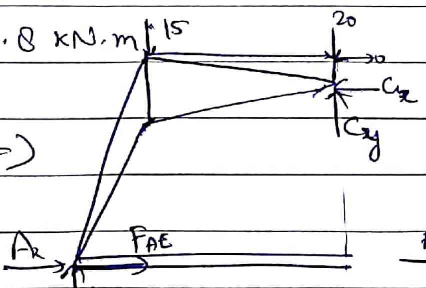
Segment BD

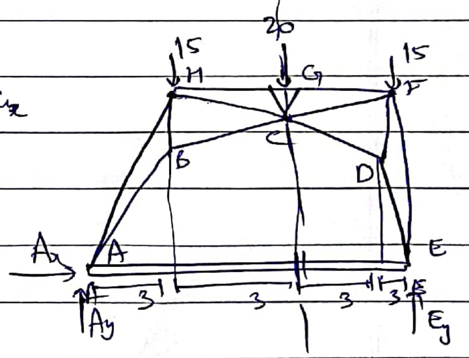
$\circlearrowleft + \Sigma M_D = 0$; $-M_D + 12.6\times2 + 1.125\times5 - 8\times1 - 4\times3 = 0$



$M_D = 10.8$ kN.m

4 Left section taken ⟹





For whole arch

$\circlearrowleft + \Sigma M_A = 0$

$12E_y - 15\times3 - 20\times6 - 15\times9 = 0$; $E_y = 25$ kN

$\xrightarrow{+} \Sigma F_x = 0$; $A_x = 0$ $\quad +\uparrow \Sigma F_y = 0$: $A_y - 15 - 20 - 15 + 25 = 0$; $A_y = 25$ kN

Determining force F_{AE}

$\circlearrowleft + \Sigma M_c = 0$; $F_{AE} \times 5 - 25\times6 + 15\times3 = 0$

$F_{AE} = 21$ kN

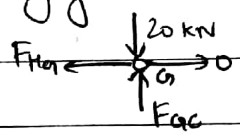
Then

$\xrightarrow{+} \sum F_x = 0; \quad -C_x + 21 = 0, \quad C_x = 21 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad C_y - 15 - 20 + 25 = 0, \quad C_y = 10 \text{ kN}$

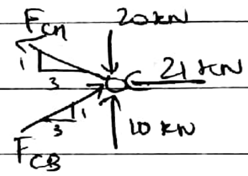
To obtain forces in CH & CG use methods of joints

Joint G



$+\uparrow \sum F_y = 0; \quad F_{GC} - 20 = 0, \quad F_{GC} = 20 \text{ kN (C)}$

Joint C



$\xrightarrow{+} \sum F_x = 0; \quad F_{CB}\left(\frac{3}{\sqrt{10}}\right) \mp F_{CH} \frac{3}{\sqrt{10}} - 21 = 0$

$+\uparrow \sum F_y = 0; \quad F_{CB} \frac{1}{\sqrt{10}} - 10 - F_{CH} \frac{1}{\sqrt{10}} = 0$

Solving these eqns ; $F_{CB} = 26.9 \text{ kN (C)}$

$F_{CH} = 4.74 \text{ kN (T)}$