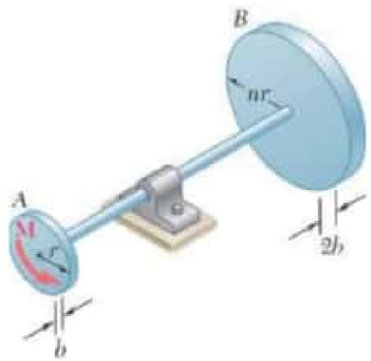


PROBLEM 17.3

Two uniform disks of the same material are attached to a shaft as shown. Disk A has a weight of 10 lb and a radius $r = 6$ in. Disk B is twice as thick as disk A. Knowing that a couple $\mathbf{M}$ of magnitude 22 lb·ft is applied to disk A when the system is at rest, determine the radius nr of disk B if the angular velocity of the system is to be 480 rpm after 5 revolutions.

SOLUTION

Moments of inertia.

Disk A:
$$I_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}\left(\frac{10}{32.2}\right)\left(\frac{6}{12}\right)^2 = 0.03882 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Disk B:
$$m_B = m_A\left(\frac{t_B}{t_A}\right)\left(\frac{r_B}{r_A}\right)^2 = \left(\frac{10}{32.2}\right)(2)(n)^2 = 0.62112n^2 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$I_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}\left(0.62112n^2\right)\left(\frac{6}{12}n\right)^2 = 0.07764n^4 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Total:
$$I = I_A + I_B = \left(0.03882 + 0.07764n^4\right) \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Work.
$$\theta_2 - \theta_1 = 5 \text{ rev} = 10\pi \text{ radians}$$

$$U_{1\to 2} = M(\theta_2 - \theta_1) = (22)(10\pi) = 220\pi \text{ ft}\cdot\text{lb}$$

Kinetic energy.
$$\omega_1 = 0 \qquad T_1 = 0$$

$$\omega_2 = 480 \text{ rpm} = 16\pi \text{ rad/s} \qquad T_2 = \frac{1}{2}I\omega_2^2$$

Principle of work and energy.
$$T_1 + U_{1\to 2} = T_2$$

$$0 + U_{1\to 2} = \frac{1}{2}I\omega_2^2$$

Solving for I,
$$I = \frac{2U_{1\to 2}}{\omega_2^2} = \frac{(2)(220\pi)}{(16\pi)^2} = 0.547095 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

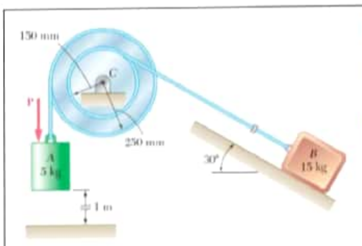
Radius of Disk B.

Equating the two expressions for I,
$$0.03882 + 0.07764n^4 = 0.547095$$

$$n^4 = 6.5466 \qquad n = 1.5996$$

$$r_B = nr_A = (1.5996)(6 \text{ in.}) = 9.597 \text{ in.}$$

$$r_B = 9.60 \text{ in.} \blacktriangleleft$$



PROBLEM 17.14

The double pulley shown has a mass of 15 kg and a centroidal radius of gyration of 160 mm. Cylinder *A* and block *B* are attached to cords that are wrapped on the pulleys as shown. The coefficient of kinetic friction between block *B* and the surface is 0.2. Knowing that the system is at rest in the position shown when a constant force $P = 200$ N is applied to cylinder *A*, determine (*a*) the velocity of cylinder *A* as it strikes the ground, (*b*) the total distance that block *B* moves before coming to rest.

SOLUTION

Kinematics. Let r_A be the radius of the outer pulley and r_B that of the inner pulley.

$$v_A = r_A\omega_C \qquad v_B = r_B\omega_C = \frac{r_B}{r_A}v_A$$

$$s_A = r_A\theta_C \qquad s_B = \frac{r_B}{r_A}s_A$$

Use the principle of work and energy with position 1 being the initial rest position and position 2 being when cylinder *A* strikes the ground.

$$T_1 + U_{1\to 2} = T_2:$$

where $\quad T_1 = 0$

and $\quad T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}\bar{I}_C\omega_C^2$

with $\quad m_A = 5\text{ kg}, \quad m_B = 15\text{ kg}, \quad \bar{I}_C = m_C\bar{k}_C^2 = (15\text{ kg})(0.160\text{ m})^2 = 0.384\text{ kg}\cdot\text{m}^2$

$$T_2 = \frac{1}{2}\left[m_A + \frac{m_B r_B^2}{r_A^2} + \frac{\bar{I}_C}{r_A^2}\right]v_A^2$$

$$= \frac{1}{2}\left[5\text{ kg} + \frac{(15\text{ kg})(0.150\text{ m})^2}{(0.250\text{ m})^2} + \frac{0.384\text{ kg}\cdot\text{m}^2}{(0.250\text{ m})^2}\right]v_A^2$$

$$= (8.272\text{ kg})v_A^2$$

Principle of work and energy applied to the system consisting of blocks *A* and *B* and the double pulley *C*.

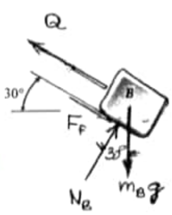
Work. $\quad U_{1\to 2} = Ps_A + m_A g\, s_A - F_f s_B - m_B g\, s_B \sin 30°$

where $\quad s_A = 1\text{ m}$

PROBLEM 17.14 (Continued)

and $\quad s_B = \frac{r_B}{r_A}s_A = \frac{0.150\text{ m}}{0.250\text{ m}}(1\text{ m}) = 0.6\text{ m}$

To find F_f use the free body diagram of block *B*.



$\measuredangle 60°\ \Sigma F = 0$: $\quad N_B - m_B g\cos 30° = 0$

$$N_B = m_B g\cos 30° = (15\text{ kg})(9.81\text{ m/s})\cos 30° = 127.44\text{ N}$$

$$F_f = \mu_k N_B = (0.2)(127.44\text{ N}) = 25.487\text{ N}$$

$$U_{1\to 2} = (200\text{ N})(1\text{ m}) + (5\text{ kg})(9.81\text{ m/s}^2)(1\text{ m})$$
$$-(25.487\text{ N})(0.6\text{ m}) - (15\text{ kg})(9.81\text{ m/s}^2)(0.6\text{ m})\sin 30°$$
$$= 189.613\text{ J}$$

Work-energy: $\quad 0 + 189.613\text{ J} = (8.272\text{ kg})v_A^2$

(*a*) *Velocity of A.* $\quad v_A = 4.7877$ m/s $\qquad \mathbf{v}_A = 4.79\text{ m/s}\uparrow$ ◀

when the cylinder strikes the ground,

$$v_B = \frac{r_B}{r_A}v_A = \frac{0.150\text{ m}}{0.250\text{ m}}(4.7877\text{ m/s}) = 2.8726\text{ m/s}$$

$$\omega_C = \frac{v_A}{r_A} = \frac{4.7877\text{ m/s}}{0.250\text{ m}} = 19.1508\text{ rad/s}$$

After the cylinder strikes the ground use the principle of work and energy applied to a system consisting of block *B* and double pulley *C*.

Let T_3 be its kinetic energy when *A* strikes the ground.

$$T_3 = \frac{1}{2}m_B v_B^2 + \frac{1}{2}\bar{I}_C\omega_C^2$$

$$= \frac{1}{2}(15\text{ kg})(2.8726\text{ m/s})^2 + \frac{1}{2}(0.384\text{ kg}\cdot\text{m}^2)(19.1508\text{ rad/s})^2$$

$$= 132.305\text{ J}$$

When the system comes to rest, $\quad T_4 = 0$

$$U_{3\to 4} = -(25.487\text{ N})s_B' - (15\text{ kg})(9.81\text{ m/s}^2)(s_B'\sin 30°)$$
$$= -(99.062\text{ N})s_B'$$

where s_B' is the additional travel of block *B*.

$$T_3 + U_{3\to 4} = T_4: \quad 132.305\text{ J} - (99.062\text{ N})s_B' = 0$$

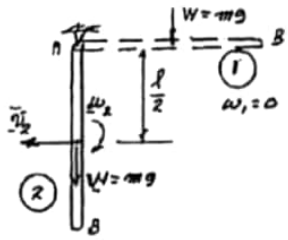
$$s_B' = 1.3356\text{ m}$$

(*b*) *Total distance:* $\qquad s_B + s_B' = 1.936\text{ m}$ ◀

PROBLEM 17.16

A slender rod of length l and weight W is pivoted at one end as shown. It is released from rest in a horizontal position and swings freely. (*a*) Determine the angular velocity of the rod as it passes through a vertical position and determine the corresponding reaction at the pivot, (*b*) Solve part *a* for $W = 1.8$ lb and $l = 3$ ft.

SOLUTION



Position 1:

$$v_1 = 0$$
$$\omega_1 = 0$$
$$T_1 = 0$$
$$\bar{v}_2 = \frac{l}{2}\omega_2$$

Position 2:

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}m\left(\frac{l}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_2^2$$
$$T_2 = \frac{1}{6}ml^2\omega_2^2$$

Work:

$$U_{1\to2} = mg\frac{l}{2}$$

Principle of work and energy:

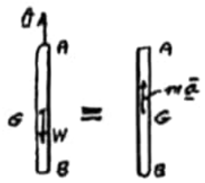
$$T_1 + U_{1\to2} = T_2$$
$$0 + mg\frac{l}{2} = \frac{1}{6}ml^2\omega_2^2$$

(*a*) Expressions for angular velocity and reactions.

$$\omega_2^2 = \frac{3g}{l}$$

$$\boldsymbol{\omega}_2 = \sqrt{\frac{3g}{l}} \;◀$$

$$\bar{a} = \frac{l}{2}\omega_2^2 = \frac{l}{2}\cdot\frac{3g}{l} = \frac{3}{2}g$$



$$+\!\uparrow \Sigma F = \Sigma(F)_{\text{eff}}: \quad A - W = m\bar{a}$$
$$A - mg = m\frac{3}{2}g$$
$$A = \frac{5}{2}mg$$

$$\mathbf{A} = \frac{5}{2}W \uparrow \;◀$$

PROBLEM 17.16 (Continued)

(*b*) Application of data:

$$W = 1.8 \text{ lb}, \quad l = 3 \text{ ft}$$

$$\omega_2^2 = \frac{3g}{l} = \frac{3g}{3} = 32.2 \text{ rad}^2/\text{s}^2$$

$$\boldsymbol{\omega}_2 = 5.67 \text{ rad/s} \;◀$$

$$A = \frac{5}{2}W = \frac{5}{2}(1.8 \text{ lb})$$

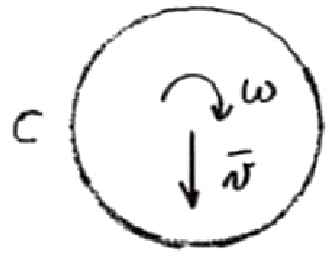
$$\mathbf{A} = 4.5 \text{ lb} \uparrow \;◀$$

PROBLEM 17.25

A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION

Point C is the instantaneous center.



$$\bar{v} = r\omega \qquad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\bar{v}}{r}\right)^2$$

$$= \frac{3}{4}m\bar{v}^2$$

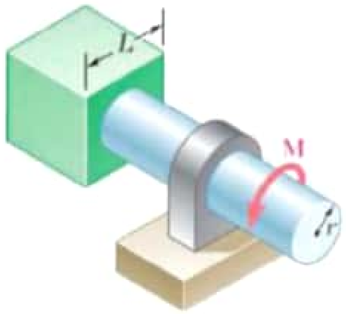
Work. $U_{1\to 2} = mgs$

Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgs = \frac{3}{4}m\bar{v}^2$$

$$\bar{v}^2 = \frac{4gs}{3}$$

$$\bar{\mathbf{v}} = \sqrt{\frac{4gs}{3}} \downarrow \blacktriangleleft$$



PROBLEM 17.55

A uniform 144-lb cube is attached to a uniform 136-lb circular shaft as shown and a couple **M** of constant magnitude is applied to the shaft when the system is at rest. Knowing that $r = 4$ in., $L = 12$ in., and the angular velocity of the system is 960 rpm after 4 s, determine the magnitude of the couple **M**.

SOLUTION

Moments of inertia.

$$\text{Cube: } \frac{1}{12}m\left(L^2 + L^2\right) = \frac{1}{12}\left(\frac{144}{32.2}\right)\left[(1)^2 + (1)^2\right] = 0.74534 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\text{Cylinder: } \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{136}{32.2}\right)\left(\frac{4}{12}\right)^2 = 0.23464 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\text{Total: } \bar{I} = 0.97999 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Final angular velocity.

$$\omega_2 = 960 \text{ rpm} = 32\pi \text{ rad/s}$$

$$\textbf{Syst Momenta}_1 + \textbf{Syst Ext Imp}_{1\to 2} = \textbf{Syst Momenta}_2$$

Moments about cylinder axis:

$$0 + Mt = \bar{I}\omega_2$$

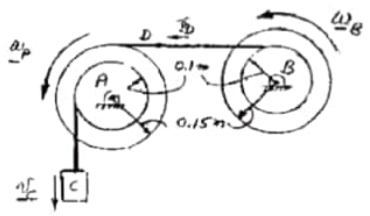
$$M = \frac{\bar{I}\omega_2}{t} = \frac{(0.97999)(32\pi)}{4}$$

$$M = 24.6 \text{ ft}\cdot\text{lb} \blacktriangleleft$$

PROBLEM 17.60

Each of the double pulleys shown has a centroidal mass moment of inertia of 0.25 $\text{kg}\cdot\text{m}^2$, an inner radius of 100 mm, and an outer radius of 150 mm. Neglecting bearing friction, determine *(a)* the velocity of the cylinder 3 s after the system is released from rest, *(b)* the tension in the cord connecting the pulleys.

SOLUTION



Kinematics.

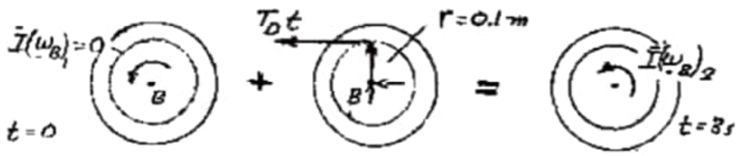
Cylinder *C*. $\mathbf{v}_C = v_C \downarrow$

Pulley *A*. $\omega_A = \dfrac{v_C}{0.100} = 10v_C \circlearrowright$

$\mathbf{v}_D = 0.150\,\omega_A = 1.5v_C \longleftarrow$

Pulley *B*. $\omega_B = \dfrac{v_D}{0.100} = 15v_C \circlearrowright$

Principle of impulse and momentum for pulley B.

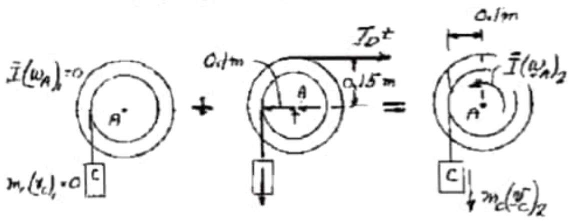


Syst momenta$_1$ + **Syst Ext Imp$_{1\to2}$** = **Syst momenta$_2$**

Moments about *B*: $0 + (T_D t)r = \bar{I}(\omega_B)_2$

$(T_D t)(0.100) = (0.25)(15v_C)$ $\qquad T_D t = 37.5v_C \qquad (1)$

Principle of impulse and momentum for pulley A and cylinder.



Syst momenta$_1$ + **Syst Ext Imp$_{1\to2}$** = **Syst Momenta$_2$**

PROBLEM 17.60 (Continued)

Moments about *A*: $0 + (W_C t)r_i - (T_D t)r_o = \bar{I}\omega_A + (m_C v_C)r_i$

$(10)(9.81)(3)(0.100) - (T_D t)(0.150) = (0.25)(10v_C) + (10v_C)(0.100)$

$29.43 - 0.15\,T_D t = 3.5v_C$

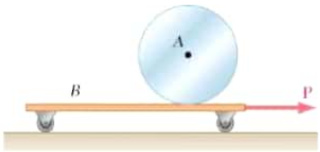
$29.43 - (0.15)(37.5\,v_C) = 3.5v_C$

(a) $v_C = 3.2252$ m/s $\qquad \mathbf{v}_C = 3.23 \text{ m/s}\downarrow$ ◄

(b) From (1), $T_D t = (37.5)(3.2252) = 120.945\ \text{N}\cdot\text{s}$

$T_D = \dfrac{T_D t}{t} = \dfrac{120.945}{3}$ $\qquad T_D = 40.3$ N ◄

PROBLEM 17.72



A 9-in.-radius cylinder of weight 18 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 2.5 lb is applied as shown for 1.2 s. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine the resulting velocity of (*a*) the carriage, (*b*) the center of the cylinder.

SOLUTION

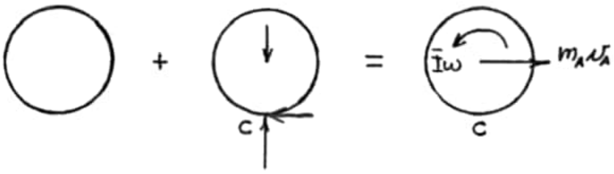
Moment of inertia.

$$\bar{I} = \frac{1}{2} m_A r^2$$

$$= \frac{1}{2}\left(\frac{18 \text{ lb}}{32.2}\right)\left(\frac{9 \text{ in.}}{12}\right)^2$$

$$= 0.15722 \text{ slug} \cdot \text{ft}^2$$

Cylinder alone:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

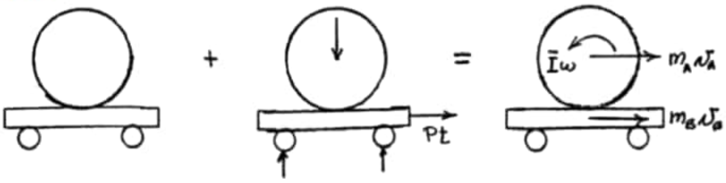
↺ Moments about *C*:

$$0 + 0 = \bar{I}\omega - m_A v_A r$$

or

$$0 = 0.15722\omega - \left(\frac{18}{32.2}\right)\left(\frac{9}{12}\right)v_A \qquad (1)$$

Cylinder and carriage:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Horizontal components:

$$0 + Pt = m_A v_A + m_B v_B$$

or

$$0 + (2.5)(1.2) = \left(\frac{18}{32.2}\right)v_A + \left(\frac{6}{32.2}\right)v_B \qquad (2)$$

PROBLEM 17.72 (Continued)

Kinematics.

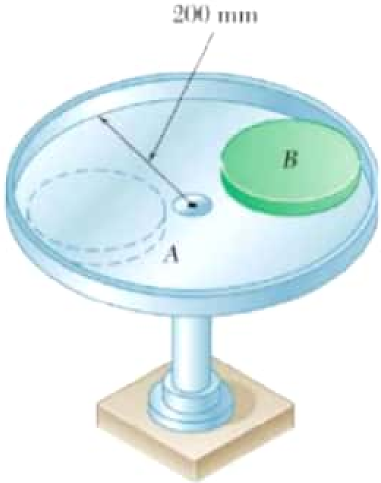
$$v_A = v_B - r\omega$$

$$v_A = v_B - \left(\frac{9}{12}\right)\omega \qquad (3)$$

Solving Equations (1), (2) and (3) simultaneously gives $\omega = 7.16$ rad/s ↻

(*a*) Velocity of the carriage. $\mathbf{v}_B = 8.05$ ft/s ⟶ ◀

(*b*) Velocity of the center of the cylinder. $\mathbf{v}_A = 2.68$ ft/s ⟶ ◀



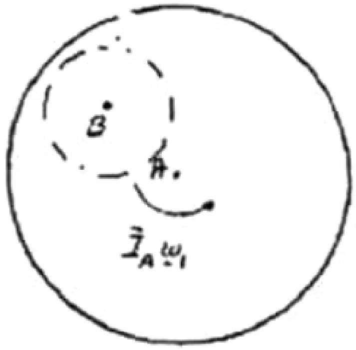
PROBLEM 17.86

The circular platform A is fitted with a rim of 200-mm inner radius and can rotate freely about the vertical shaft. It is known that the platform-rim unit has a mass of 5 kg and a radius of gyration of 175 mm with respect to the shaft. At a time when the platform is rotating with an angular velocity of 50 rpm, a 3-kg disk B of radius 80 mm is placed on the platform with no velocity. Knowing that disk B then slides until it comes to rest relative to the platform against the rim, determine the final angular velocity of the platform.

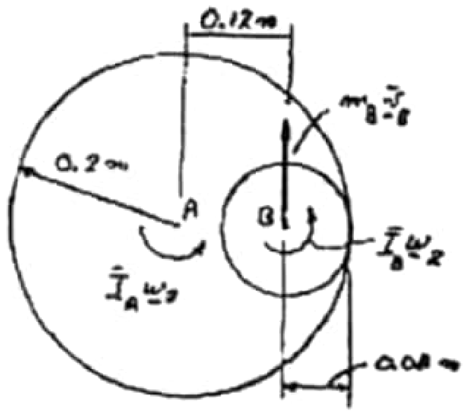
SOLUTION

Moments of inertia.

$$\bar{I}_A = m_A k^2$$
$$= (5\text{ kg})(0.175\text{ m})^2$$
$$= 0.153125\text{ kg}\cdot\text{m}^2$$
$$\bar{I}_B = \frac{1}{2}m_B r_B^2$$
$$= \frac{1}{2}(3\text{ kg})(0.08\text{ m})^2$$
$$= 9.6\times 10^{-3}\text{ kg}\cdot\text{m}^2$$



Syst. Momenta$_1$



Syst. Momenta$_2$

<u>State 1</u> Disk B is at rest.

<u>State 2</u> Disk B moves with platform A.

Kinematics. In <u>State 2</u>, $\bar{v}_B = (0.12\text{ m})\omega_2$

Principle of conservation of angular momentum.

$+\circlearrowleft$ Moments about D: $\bar{I}_A\omega_1 = \bar{I}_A\omega_2 + \bar{I}_B\omega_2 + m_B\bar{v}_B(0.12\text{ m})$

$$(0.153125\text{ kg}\cdot\text{m}^2)\omega_1 = (0.153125\text{ kg}\cdot\text{m}^2)\omega_2 + (9.6\times 10^{-3}\text{ kg}\cdot\text{m}^2)\omega_2 + (3\text{ kg})(0.12\text{ m})^2\omega_2$$
$$0.153125\omega_2 = 0.20593\omega_1$$
$$\omega_2 = 0.7436\omega_1$$
$$= 0.7436(50\text{ rpm})$$

Final angular velocity $\omega_2 = 37.2\text{ rpm}$ ◀