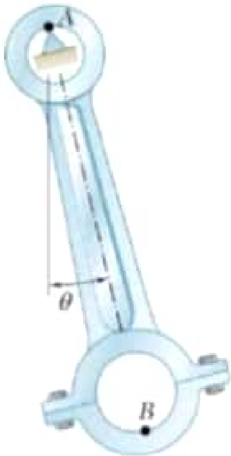


PROBLEM 15.6

A connecting rod is supported by a knife-edge at Point A. For small oscillations the angular acceleration of the connecting rod is governed by the relation $\alpha = -6\theta$ where α is expressed in rad/s^2 and θ in radians. Knowing that the connecting rod is released from rest when $\theta = 20°$, determine (*a*) the maximum angular velocity, (*b*) the angular position when $t = 2$ s.

SOLUTION

Angular motion relations:

$$\alpha = \frac{d\omega}{dt} = \frac{\omega d\omega}{d\theta} = -6\theta \qquad (1)$$

Separation of variables ω and θ gives

$$\omega d\omega = -6\theta d\theta$$

Integrating, using $\omega = 0$ when $\theta = \theta_0$,

$$\int_0^\omega \omega d\omega = -6\int_{\theta_0}^\theta \theta d\theta$$

$$\frac{1}{2}\omega^2 = -3(\theta^2 - \theta_0^2) = 3(\theta_0^2 - \theta^2)$$

$$\omega^2 = 6(\theta_0^2 - \theta^2) \qquad \omega = \sqrt{6(\theta_0^2 - \theta^2)}$$

(*a*) ω is maximum when $\theta = 0$.

Data: $\theta_0 = 20° = 0.34907$ radians

$$\omega_{\max}^2 = 6(0.34907^2 - 0) = 0.73108 \text{ rad}^2/\text{s}$$

$\omega_{\max} = 0.855$ rad/s ◀

(*b*) From $\omega = \dfrac{d\theta}{dt}$ we get $dt = \dfrac{d\theta}{\omega} = \dfrac{1}{\sqrt{6}} \dfrac{d\theta}{\sqrt{\theta_0^2 - \theta^2}}$

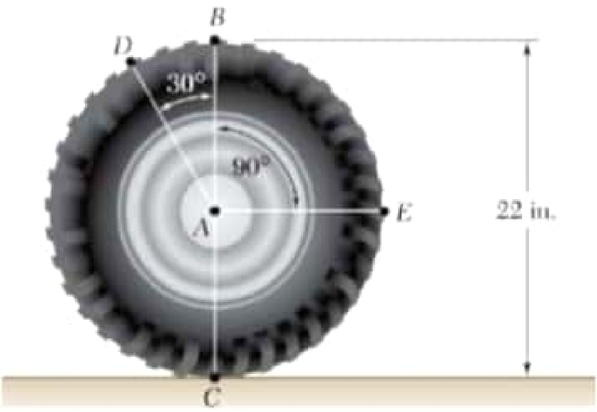
Integrating, using $t = 0$ when $\theta = \theta_0$,

$$\int_0^t dt = \frac{1}{\sqrt{6}}\int_{\theta_0}^\theta \frac{d\theta}{\sqrt{\theta_0^2 - \theta^2}}$$

$$t = -\frac{1}{\sqrt{6}}\cos^{2-1}\frac{\theta}{\theta_0}\bigg|_{\theta_0}^{\theta} = -\frac{1}{\sqrt{6}}\left[0 - \cos^{-1}\frac{\theta}{\theta_0}\right] = -\frac{1}{\sqrt{6}}\cos^{-1}\frac{\theta}{\theta_0}$$

$$\theta = \theta_0 \cos(\sqrt{6}t) = 0.34907\cos[(\sqrt{6})2] = (0.34907)(0.18551) = 0.064756 \text{ radians}$$

$\theta = 3.71°$ ◀



PROBLEM 15.38

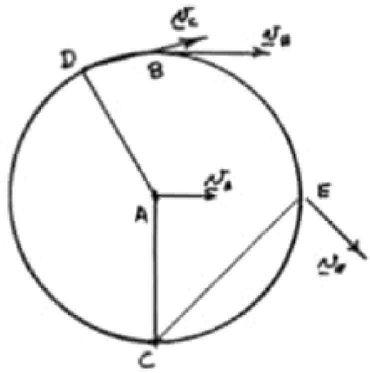
An automobile travels to the right at a constant speed of 48 mi/h. If the diameter of a wheel is 22 in., determine the velocities of Points B, C, D, and E on the rim of the wheel.

SOLUTION

$$\mathbf{v}_A = 48 \text{ mi/h} = 70.4 \text{ ft/s}$$

$$\mathbf{v}_C = 0 \blacktriangleleft$$

$$d = 22 \text{ in.} \quad r = \frac{d}{2} = 11 \text{ in.} = 0.91667 \text{ ft}$$



$$\omega = \frac{v_A}{r} = \frac{70.4}{0.91667} = 76.8 \text{ rad/s} \circlearrowright$$

$$v_{B/A} = v_{D/A} = v_{E/A} = r\omega$$

$$= (0.91667)(76.8) = 70.4 \text{ ft/s}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \longrightarrow]$$

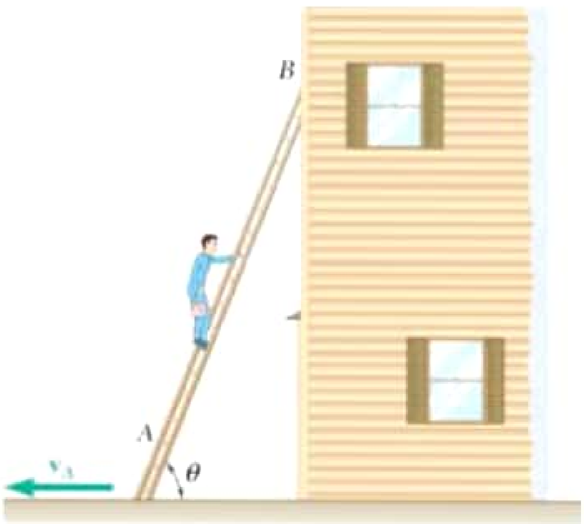
$$\mathbf{v}_B = 140.8 \text{ ft/s} \longrightarrow \blacktriangleleft$$

$$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \measuredangle 30°]$$

$$\mathbf{v}_D = 136.0 \text{ ft/s} \measuredangle 15.0° \blacktriangleleft$$

$$\mathbf{v}_E = \mathbf{v}_A + \mathbf{v}_{E/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \downarrow]$$

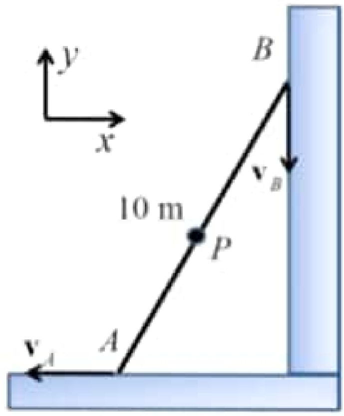
$$\mathbf{v}_E = 99.6 \text{ ft/s} \searrow 45.0° \blacktriangleleft$$



PROBLEM 15.40

A painter is halfway up a 10-m ladder when the bottom starts sliding out from under him. Knowing that point A has a velocity $\mathbf{v}_A = 2$ m/s directed to the left when $\theta = 60°$, determine (*a*) the angular velocity of the ladder, (*b*) the velocity of the painter.

SOLUTION



Given: $\mathbf{v}_A = -2\mathbf{i}$ m/s, $\theta = 60°$

Geometry:

$$\mathbf{r}_{B/A} = 10\cos 60°\mathbf{i} + 10\sin 60°\mathbf{j}$$
$$\mathbf{r}_{P/A} = 5\cos 60°\mathbf{i} + 5\sin 60°\mathbf{j}$$

Relative Velocity:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/A}$$
$$v_B\mathbf{j} = -2\mathbf{i} + \omega_{AB}\mathbf{k} \times (5\mathbf{i} + 8.660\mathbf{j})$$
$$= -(2 + 8.660\omega_{AB})\mathbf{i} + 5\omega_{AB}\mathbf{j}$$

Equate Components:

(*a*) $\mathbf{i}$:

$$0 = -(2 + 8.660\omega_{AB})$$
$$\omega_{AB} = -0.231 \text{ rad/s}$$

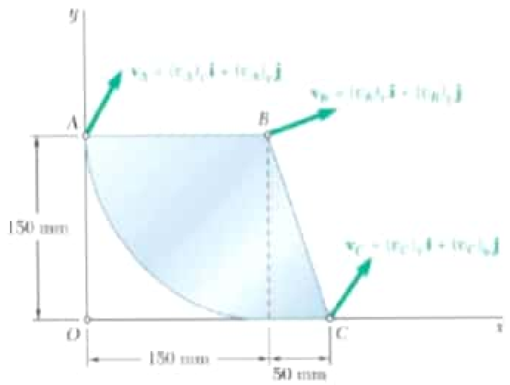
$$\boldsymbol{\omega}_{AB} = 0.231 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Velocity of Painter (point P):

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A} = \mathbf{v}_A + \omega_{AB}\mathbf{k} \times \mathbf{r}_{P/A}$$
$$= -2\mathbf{i} - 0.231\mathbf{k} \times (2.5\mathbf{i} + 4.330\mathbf{j})$$

(*b*)

$$\mathbf{v}_P = -(1.00 \text{ m/s})\mathbf{i} - (0.577 \text{ m/s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.46

The plate shown moves in the xy plane. Knowing that $(v_A)_x = 250$ mm/s, $(v_B)_x = -450$ mm/s, and $(v_C)_x = -500$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point A.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega\mathbf{k}$

Relative position vectors:

$$\mathbf{r}_{B/A} = (150 \text{ mm})\mathbf{i}$$

$$\mathbf{r}_{C/A} = (200 \text{ mm})\mathbf{i} + (150 \text{ mm})\mathbf{j}$$

Velocity vectors:

$$\mathbf{v}_A = (250 \text{ mm/s})\mathbf{i} + (v_A)_y\mathbf{j}$$

$$\mathbf{v}_B = (v_B)_x\mathbf{i} - (450 \text{ mm/s})\mathbf{j}$$

$$\mathbf{v}_C = -(500 \text{ mm/s})\mathbf{i} + (v_C)_y\mathbf{j}$$

Unknowns are ω, $(v_A)_y$, $(v_B)_x$, and $(v_C)_y$.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \omega\mathbf{k} \times \mathbf{r}_{B/A}$$

$$(v_B)_x\mathbf{i} - 450\mathbf{j} = 250\mathbf{i} + (v_A)_y\mathbf{j} + \omega\mathbf{k} \times 150\mathbf{i}$$

$$= 250\mathbf{i} + (v_A)_y\mathbf{j} + 150\omega\mathbf{j}$$

$\mathbf{i}$: $$(v_B)_x = 250 \qquad (1)$$

$\mathbf{j}$: $$-450 = (v_A)_y + 150\omega \qquad (2)$$

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_A + \omega\mathbf{k} \times \mathbf{r}_{C/A}$$

$$-500\mathbf{i} + (v_C)_y\mathbf{j} = 250\mathbf{i} + (v_A)_y\mathbf{j} + \omega\mathbf{k} \times (200\mathbf{i} - 150\mathbf{j})$$

$$= 250\mathbf{i} + (v_A)_y\mathbf{j} + 200\omega\mathbf{j} + 150\omega\mathbf{i}$$

$\mathbf{i}$: $$-500 = 250 + 150\omega \qquad (3)$$

$\mathbf{j}$: $$(v_C)_y = (v_A)_y + 150\omega \qquad (4)$$

(*a*) *Angular velocity of the plate*.

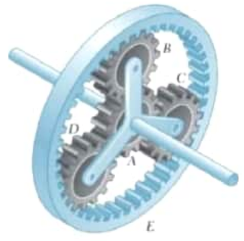
From Eq. (3), $$\omega = -\frac{750}{150} = -5$$

$$\boldsymbol{\omega} = -(5.00 \text{ rad/s})\mathbf{k} = 5.00 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) *Velocity of Point A*.

From Eq. (2), $(v_A)_y = -450 - 150\omega = -450 - (150)(-5) = 300$ mm/s

$$\mathbf{v}_A = (250 \text{ mm/s})\mathbf{i} + (300 \text{ mm/s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.49

In the planetary gear system shown, the radius of gears A, B, C, and D is 30 mm and the radius of the outer gear E is 90 mm. Knowing that gear E has an angular velocity of 180 rpm clockwise and that the central gear A has an angular velocity of 240 rpm clockwise, determine (*a*) the angular velocity of each planetary gear, (*b*) the angular velocity of the spider connecting the planetary gears.

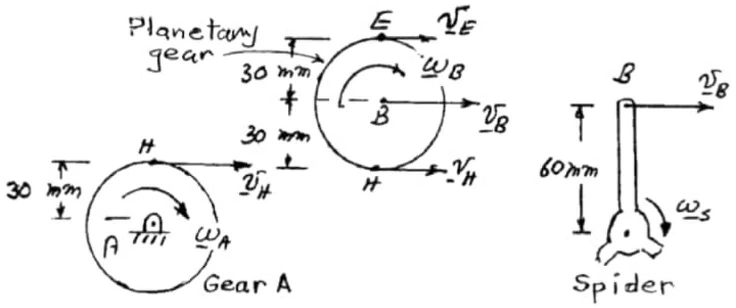
SOLUTION

Since the motions of the planetary gears B, C, and D are similar, only gear B is considered. Let Point H be the effect contact point between gears A and B and let Point E be the effective contact point between gears B and E.

Given angular velocities:

$$\boldsymbol{\omega}_E = 180 \text{ rpm} \circlearrowright = 6\pi \text{ rad/s} \circlearrowright$$

$$\boldsymbol{\omega}_A = 240 \text{ rpm} \circlearrowright = 8\pi \text{ rad/s} \circlearrowright$$



Outer gear E: radius $= r_E = 90$ mm

$$v_E = r_E\omega_E = (90 \text{ mm})(6\pi \text{ rad/s}) = 540\pi \text{ mm/s}$$

$$\mathbf{v}_E = 540\pi \text{ mm/s} \longrightarrow$$

Gear A: radius $= r_A = 30$ mm

$$v_H = r_A\omega_A = (30 \text{ mm})(8\pi \text{ rad/s}) = 240\pi \text{ mm/s}$$

$$\mathbf{v}_H = 240\pi \text{ mm/s} \longrightarrow$$

PROBLEM 15.49 (Continued)

Planetary gear B: radius $= r_B = 30$ mm, $\boldsymbol{\omega}_B = \omega_B \circlearrowright$

$$\mathbf{v}_H = \mathbf{v}_E + \mathbf{v}_{H/E}$$

$$[(30 \text{ mm})\omega_A \longrightarrow] = [540\pi \text{ mm/s} \longrightarrow] + [(60 \text{ mm})\omega_B \longleftarrow]$$

$$30\omega_A = 540\pi - 60\omega_B$$

$$\omega_B = \frac{540\pi + 30\omega_A}{60} = 9\pi + \frac{1}{2}\omega_A = 9\pi - \frac{1}{2}(8\pi) = 5\pi \text{ rad/s} \circlearrowright$$

(*a*) *Angular velocity of planetary gears:*

$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_C = \boldsymbol{\omega}_D = 5\pi \text{ rad/s} \circlearrowright = 150 \text{ rpm} \circlearrowright \blacktriangleleft$$

$$\mathbf{v}_B = \mathbf{v}_H + \mathbf{v}_{B/H} = [(30 \text{ mm})\omega_A \longrightarrow] + [30 \text{ mm } \omega_B \longrightarrow]$$

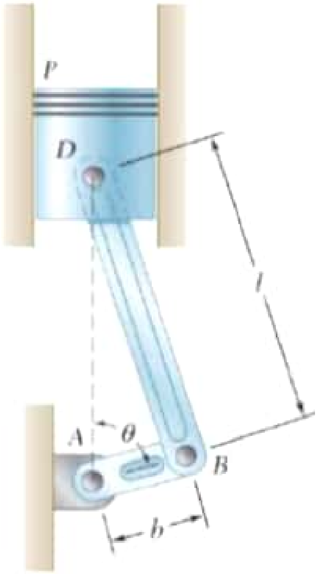
$$v_B = (30 \text{ mm})(8\pi \text{ rad/s}) + (30 \text{ mm})(5\pi \text{ rad/s}) = 390\pi \text{ mm/s}$$

(*b*) *Spider:* arm $= r_s = 60$ mm, $\boldsymbol{\omega}_s = \omega_s \circlearrowright$

$$v_B = r_s\omega_s$$

$$\omega_s = \frac{v_B}{r_s} = \frac{390\pi \text{ mm/s}}{60 \text{ mm}} = 6.5\pi \text{ rad/s}$$

$$\boldsymbol{\omega}_s = 6.5\pi \text{ rad/s} \circlearrowright = 195 \text{ rpm} \circlearrowright \blacktriangleleft$$



PROBLEM 15.61

In the engine system shown, $l = 160$ mm and $b = 60$ mm. Knowing that the crank AB rotates with a constant angular velocity of 1000 rpm clockwise, determine the velocity of the piston P and the angular velocity of the connecting rod when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

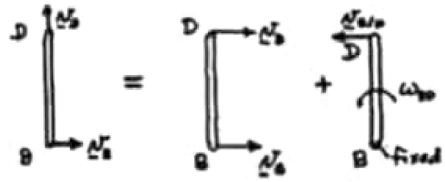
SOLUTION

$$\omega_{AB} = 1000 \text{ rpm} \circlearrowright = \frac{(1000)(2\pi)}{60} = 104.72 \text{ rad/s} \circlearrowright$$

(*a*) $\theta = 0°$. *Crank AB.* (Rotation about A) $\qquad \mathbf{r}_{B/A} = 0.06 \text{ m} \uparrow$

$$\mathbf{v}_B = v_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \longrightarrow$$

Rod BD. (Plane motion = Translation with B + Rotation about B)



$$\mathbf{v}_D = \mathbf{v}_B + v_{D/B}$$

$$v_D \uparrow = [6.2832 \longrightarrow] + [v_{D/B} \longleftarrow]$$

$$v_D = 0$$

$$v_{D/B} = 6.2832 \text{ m/s}$$

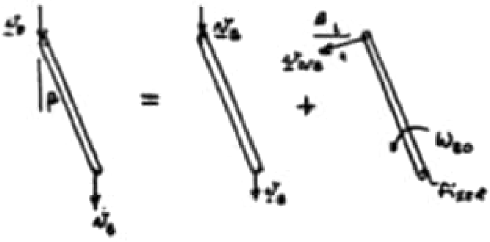
$$\mathbf{v}_P = \mathbf{v}_D \qquad\qquad \mathbf{v}_P = 0 \blacktriangleleft$$

$$\omega_{BD} = \frac{v_B}{l} = \frac{6.2832}{0.16} \qquad\qquad \boldsymbol{\omega}_{BD} = 39.3 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) $\theta = 90°$. *Crank AB.* (Rotation about A) $\qquad \mathbf{r}_{B/A} = 0.06 \text{ m} \longrightarrow$

$$\mathbf{v}_B = r_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \downarrow$$

Rod BD. (Plane motion = Translation with B + Rotation about B.)



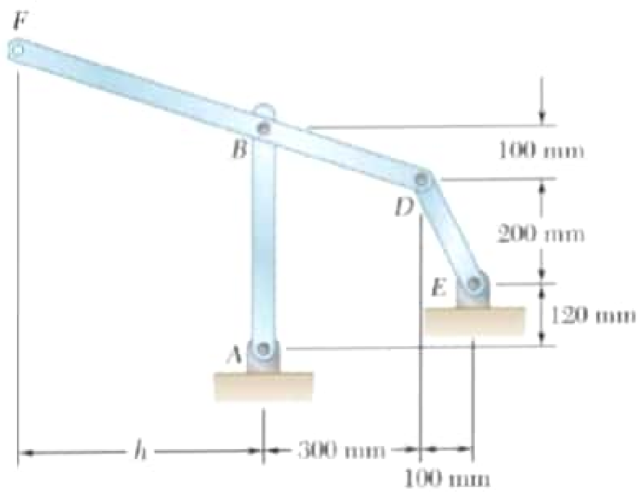
$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [6.2832 \downarrow] + [v_{D/B} \measuredangle \beta]$$

$$v_{D/B} = 0, \qquad v_D = 6.2832 \text{ m/s}$$

$$\omega_{BD} = \frac{v_{D/B}}{l} \qquad\qquad \omega_{BD} = 0 \blacktriangleleft$$

$$\mathbf{v}_P = \mathbf{v}_D = 6.2832 \text{ m/s} \downarrow \qquad\qquad \mathbf{v}_P = 6.28 \text{ m/s} \downarrow \blacktriangleleft$$



PROBLEM 15.68

In the position shown, bar DE has a constant angular velocity of 10 rad/s clockwise. Knowing that $h = 500$ mm, determine (*a*) the angular velocity of bar FBD, (*b*) the velocity of Point F.

SOLUTION

<u>Bar DE</u>: (Rotation about E)

$$\boldsymbol{\omega}_{DE} = 10 \text{ rad/s} \circlearrowright = -(10 \text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{D/E} = -(0.1 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (-10\mathbf{k}) \times (-0.1\mathbf{i} + 0.2\mathbf{j})$$

$$= (1 \text{ m/s})\mathbf{j} + (2 \text{ m/s})\mathbf{i}$$

<u>Bar FBD</u>: (Plane motion = Translation with D + Rotation about D.)

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \quad \mathbf{r}_{B/D} = -(0.3 \text{ m})\mathbf{i} + (0.1 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$

$$= \mathbf{j} + 2\mathbf{i} + (\omega_{BD}\mathbf{k}) \times (-0.3\mathbf{i} + 0.1\mathbf{j})$$

$$= \mathbf{j} + 2\mathbf{i} - 0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i}$$

<u>Bar AB</u>: (Rotation about A)

$$\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} \quad \mathbf{r}_{B/A} = (0.42 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (\omega_{AB}\mathbf{k}) \times (0.42\mathbf{j}) = -0.42\omega_{AB}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_B$,

(*a*) $\mathbf{j}$: $\quad 1 - 0.3\omega_{BD} = 0 \qquad \omega_{BD} = 3.3333 \text{ rad/s} \qquad \boldsymbol{\omega}_{BD} = 3.33 \text{ rad/s} \circlearrowleft$ ◀

$\mathbf{i}$: $\quad 2 - 0.1\omega_{BD} = -0.42\omega_{AB} \qquad 2 - (0.1)(3.3333) = -0.42\omega_{AB}$

$$\omega_{AB} = -3.9683 \text{ rad/s} \qquad \omega_{AB} = 3.97 \text{ rad/s} \circlearrowright$$

<u>Bar FBD</u>: $\quad \mathbf{r}_{F/D} = C\mathbf{r}_{B/D} \quad \text{where} \quad C = \dfrac{h + 0.3}{0.3}$

$$\mathbf{v}_F = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{F/D}$$

$$= \mathbf{j} + 2\mathbf{i} + C(-0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i})$$

$$= \mathbf{j} + 2\mathbf{i} + C(-\mathbf{j} - 0.33333\mathbf{i})$$

With $\quad h = 500 \text{ mm} = 0.5 \text{ m}, \quad C = \dfrac{0.8}{0.3} = 2.6667$

$$\mathbf{v}_F = \mathbf{j} + 2\mathbf{i} - 2.6667\mathbf{j} - 0.88889\mathbf{i}$$

(*b*) $\quad \mathbf{v}_F = (1.11111 \text{ m/s})\mathbf{i} - (1.66667 \text{ m/s})\mathbf{j} \qquad \mathbf{v}_F = 2.00 \text{ m/s} \measuredangle 56.3°$ ◀