Date 25/01/2018

Strength/Mechanics of Materials

- Strength
- Stability
- Stiffness

- → Fluids
- → Solids/Materials

Statics → Analysis of force in rest

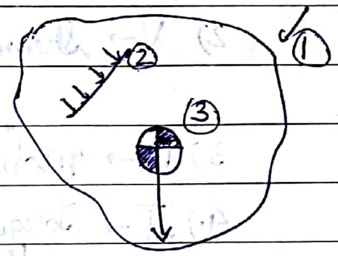
Dynamics → Analysis of force in motion

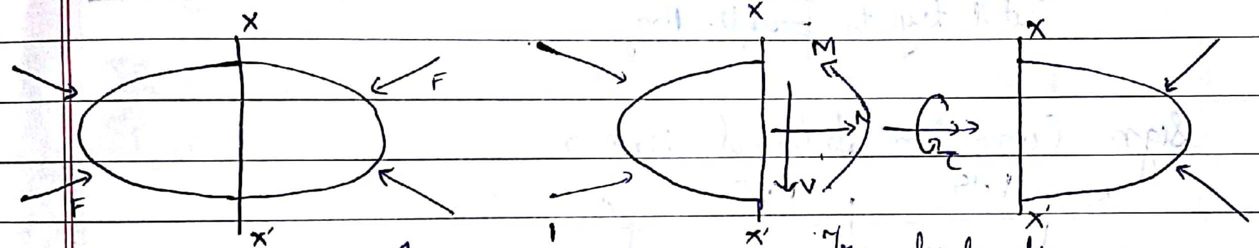
Force → rxn/action due to contact of two bodies

① surface force – forces acting on surface eg. simple pulling/pushing

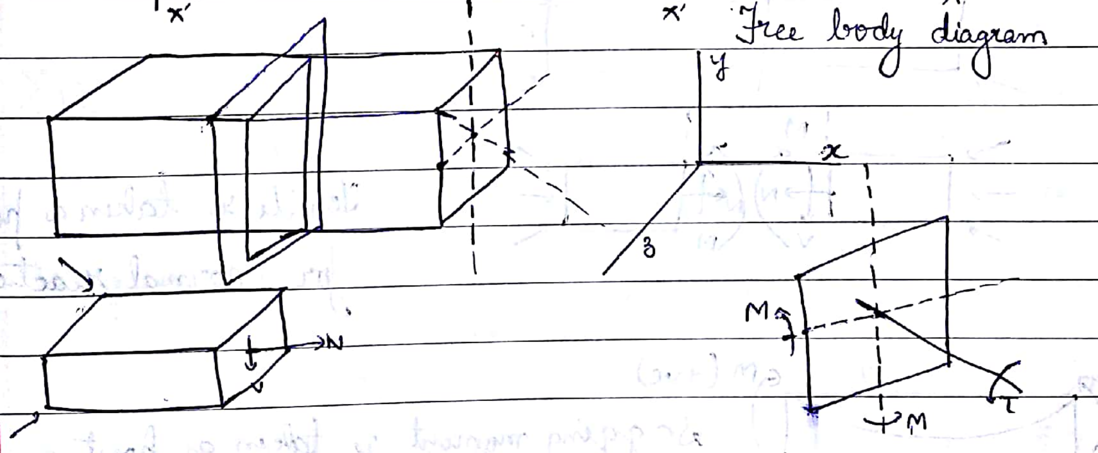
② Body force – Forces acting within the body

Surface forces :- ① Concentrated
② Uniform
③ Pressure



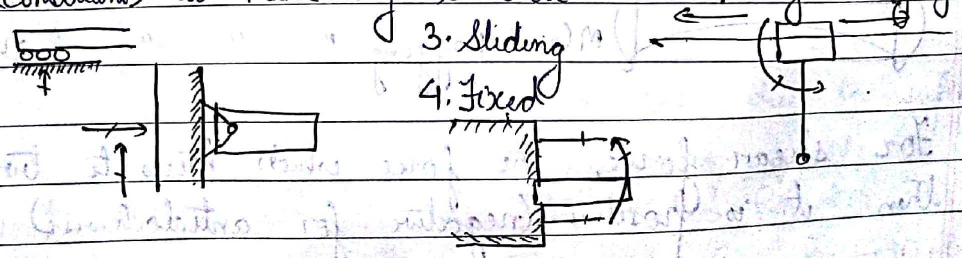


Free body diagram



Supports and Reactions :- Supports are end condition of structural member (constraint) to make any structure stable. They are of following types

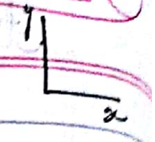
1. Roller
2. Pin
3. Sliding
4. Fixed

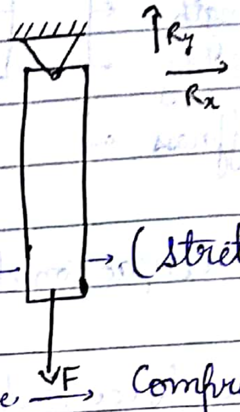


$\Sigma F_x = 0 \Rightarrow R_x = 0$

$\Sigma F_y = 0 \Rightarrow R_y - F = 0$

$R_y = F$

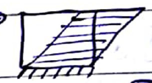




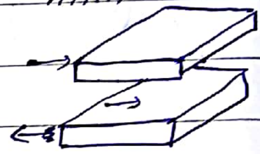
Internal forces/rxns

1.) N → normal reaction force
- Tensile → (stretching or elong
- Compressive → Compression

2.) V → Shear force - distort the body // to plane →

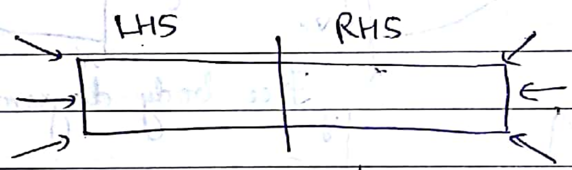


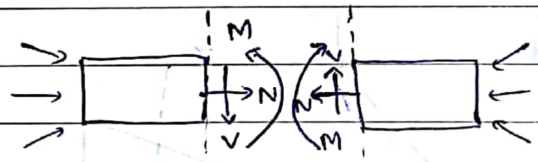
3.) M → moment → it'll try to bend the body



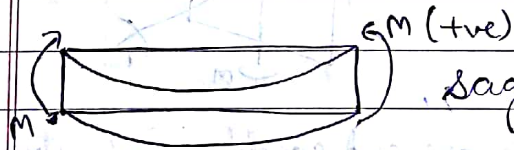
4.) T → Torque → it'll try to ~~push~~ twist the body

Sign Convention Internal rxn ⇒

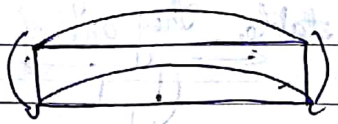




Tensile is taken as positive for normal reaction force



sagging moment is taken as positive

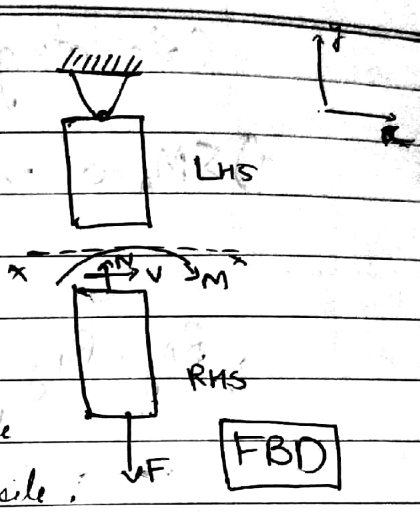


M(-ve) hogging " " " " negative

For shear force, the force which tries to turn it clockwise, then it is positive (negative for anticlockwise)

Always consider sagging as default in internal force

Applying equilibrium eqn for RHS

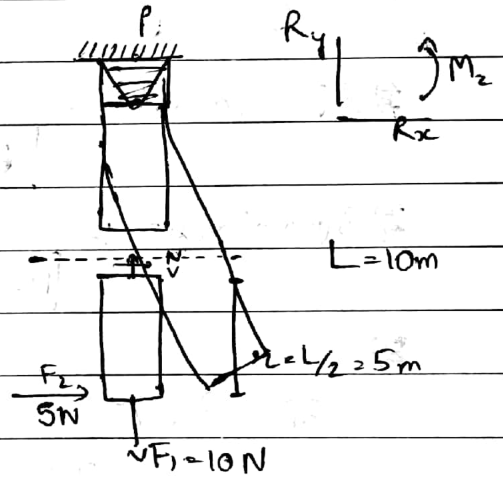


$\xrightarrow{+} \sum F_x = 0; \quad V = 0$

$+\uparrow \sum F_y = 0, \quad N - F = 0; \; N = F$ Tensile action due to force. F is tensile.

$+\circlearrowleft \sum M_z = 0 \Rightarrow M = 0$, as no perpendicular force to act.
(anticlockwise is +ve)

Always go for external reaction first



$\sum F_x = 0 \; ; \; V + 5 = 0 , \; V = +5N \leftarrow$

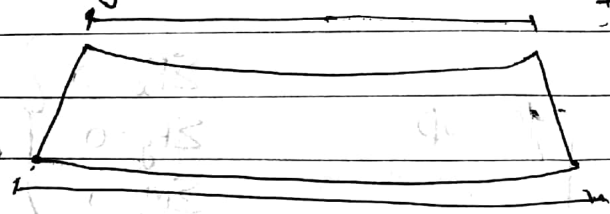
$\sum F_y = 0, \quad N - F_1 = 0$

$N = F_1 = 10N \uparrow$

$+\circlearrowleft \sum M_z = 0, \quad M = F_2 \times r$ (for RHS)

$M = 5 \times 5 = 25 \, Nm$

Sagging body



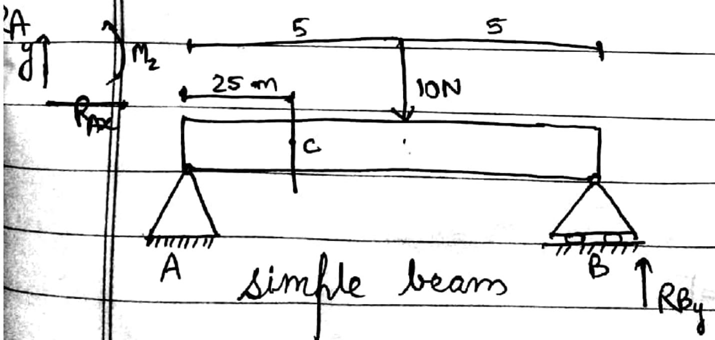
At P.

$+\circlearrowleft \sum M_z = 0$

$M_z + F_2 \times 10 = 0$

$M_z = 50 \, Nm$
(−)

Our assumption was wrong.



External rxn due to support A

Applying eq. eqn for whole segment.

$\sum F_x = 0 \Rightarrow R_{Ax} = 0$

$\sum F_y = 0 \rightarrow R_{Ay} + R_{By} - 10 = 0$

$R_{Ay} + R_{By} = 10N$

Calculating moment about A due to all forces

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow R_{By} \times 10 + (-10 \times 5) = 0$

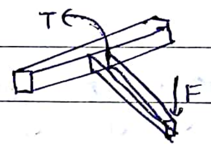
$R_{By} = \underset{(+)}{5\,N}$ anticlockwise

$R_{Ay} + R_{By} = 10$

$R_{Ay} = \underset{(+)}{5\,N}$

Both are equal because load is applied at midpoint.

Degree of Constraints :- (External support)



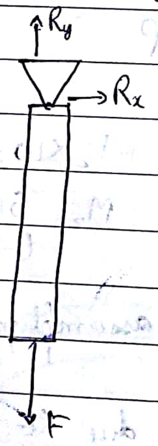
1) 3 Translational :- U_x, U_y, U_z $\boxed{R_x, R_y, R_z}$

2) 3 Rotational :- $\theta_x, \theta_y, \theta_z$ $\boxed{M_x, M_y, M_z}$

3)

3D Bodies

2D $\boxed{R_x, R_y, M_z}$



$$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0 \\ \Sigma M_x &= 0 \\ \Sigma M_y &= 0 \\ \Sigma M_z &= 0 \end{aligned}\right\} \text{for 3D}$$

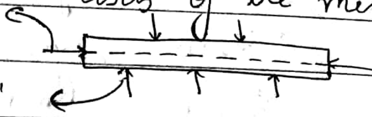
$$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_z &= 0 \end{aligned}\right\} \text{for 2D}$$

$\Sigma F_x = 0$; $R_x = 0$

$\Sigma F_y = 0$; $R_y - F = 0$; $R_y = F$

Loads (External)

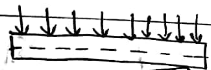
→ Axial- If a force is being applied along the longitudinal (in line) ∠ to the axis of the member



→ Transverse- If " " " " ⊥ to " " "

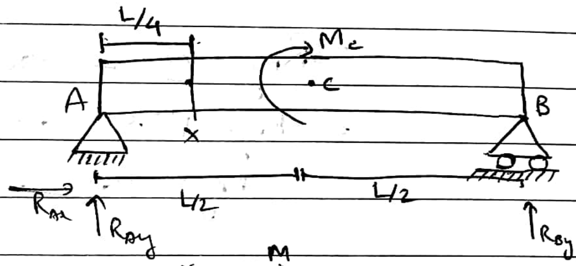
→ Concentrated force/load (kN, N).

→ Distributed load- along length — [uniform distribution (UDL) (N/m) ; varying " (VDL)]



→ on certain area (intensity of loading). $[N/m^2]$

Q $\Sigma F_x = 0$ (for whole beam)

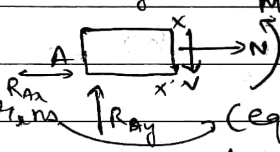


$R_{Ax} = \cancel{0}\ \cancel{N} = 0$; $R_{Ax} = \cancel{N}\,0$

$\Sigma F_y = 0$

$\cancel{A}\ R_{Ay} + R_{By} = 0$; $R_{Ay} = -R_{By} = -M/L$

at A → +) $\Sigma M_A^{+} = 0$; $R_{By} \times L - M_c = 0$, $R_{By} = \dfrac{M_c}{L}$



FBD of segment Ax

Step 1: Always calculate external supports r×ns (eq eqns on whole structure)

Step 2: Calculate internal support r×n (eq eqns only on FBD segment)

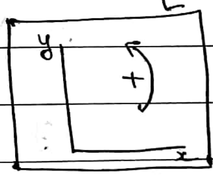
For the segment Ax

$\Sigma F_x = 0$

$\cancel{A_{Ax}}\ R_{Ax} + N = 0$; $N = 0$

$\Sigma F_y = 0$

$R_{Ay} - V = 0$, $R_{Ay} = V$; $V = \dfrac{-M}{L}$



+) $\Sigma M_{\cancel{x}x}^{+} = 0$

$M - \cancel{V} \overset{R_{Ay}}{} \times \dfrac{L}{4} = 0$; ; $M \mp \dfrac{M_c}{4} = 0$, $M = \cancel{0}\ -\dfrac{M_c}{4}$

$M = -\dfrac{M_c}{4}$

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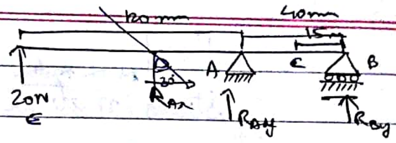
6. $\sum F_x = 0$ $\qquad$ $\sum F_y = 0$

$\boxed{R_{Ax} = 0}$ $\qquad$ $R_{Ay} + R_{By} + 20 = 0$

$R_{Ay} + R_{By} = -20$; $R_{Ay} = -80N$

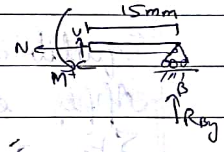


$\sum M_A^+ = 0$

$-20 \times 120 + R_{By} \times 40 = 0$

$R_{By} = \frac{\cancel{2400}^{60}}{\cancel{40}} = 60N$

Section CB



$\sum F_x = 0$ $\qquad$ $\sum F_y = 0$

$N = 0$ $\qquad$ $V + R_{By} = 0$

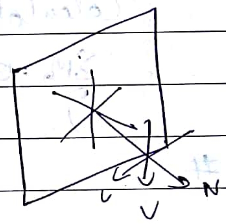
$V = -R_{By} = -60N$

$\sum M_C^+ = 0$

$M + R_{By} \times 15 = 0$; $M = -900\ N$

Stress (mechanics).-

Internal forces



- Normal force (direct)
- Shear (tangential)

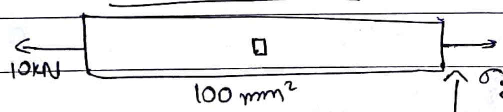
$\boxed{\text{Any Stress} = \frac{\text{Force}}{\text{Area}}}$; It's unit is N/m^2.

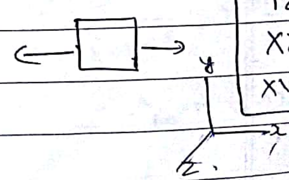
Tensor: Stress holds rank 2:→ Magnitude, direction & plane to signify

This def was given by Cauchy → 3D ⇒ Stress matrix

Plane names
$YZ \to x$
$XZ \to y$
$XY \to z$

Material Element

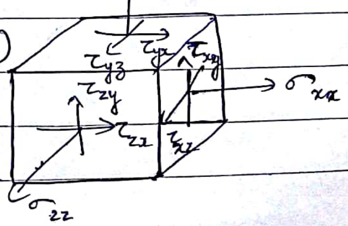




→ Normal stress (direct) (σ) (6)

→ Shear stress (tangential) (τ) (12)

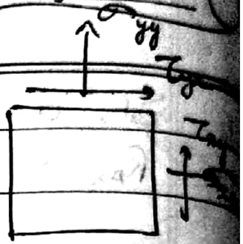
↳ 18 unknowns



Here, σ is taken σ_{xx}

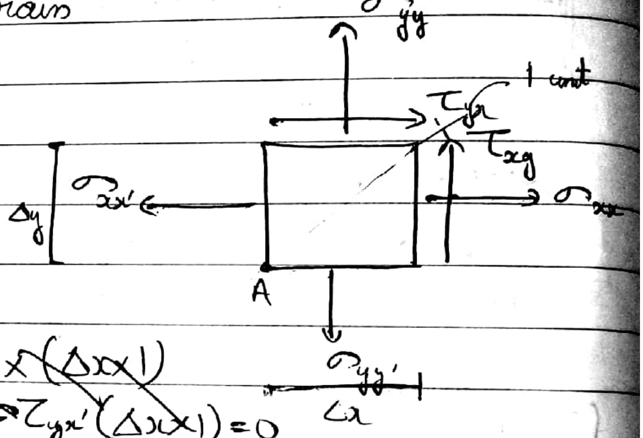
Suffix σ_{ij} — i: Plane, j: direction of axis

$$[\sigma]_{\text{Stress matrix}} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}_{3D}$$



The shortage or elongation of material on applying compressive or tensile stress is called strain

$$\text{Strain} = \frac{\Delta l}{L}$$



Equilibrium of stresses
Apply eq. eqs
$\Sigma F_x = 0$

$$\sigma_{xx}(\Delta y \times 1) - \sigma_{xx'}(\Delta y \times 1) + \tau_{yx} \times (\Delta x \times 1) - \tau_{yx'}(\Delta x \times 1) = 0$$

$\boxed{\sigma_{xx} = \sigma_{xx'}}$; Similarly , $\boxed{\sigma_{yy} = \sigma_{yy'}}$

Calculating moment about A

$\Sigma M_A = 0$

$$\tau_{xy}(\Delta y \times 1)\Delta x - \tau_{yx}(\Delta x \times 1)\Delta y = 0$$

\# $\boxed{\tau_{xy} = \tau_{yx}} = \tau_{xy'} = \tau_{yx'} = \tau$

This all holds if the material is in static equilibrium

\# This is also called complimentary property of shear stress.

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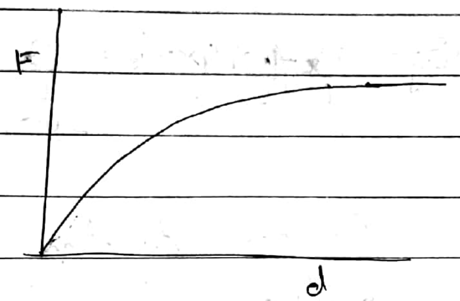
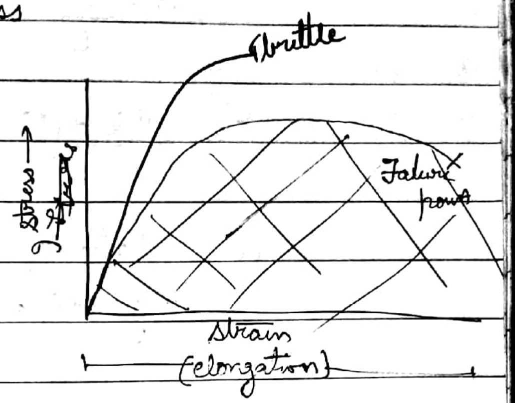
Mechanical properties of Material.

1. Brittleness
2. Hardness
3) Ductility
4) Toughness





$$\text{Strain} = \frac{\overset{L'-L}{\Delta L}}{L}$$

A brittle material fails to reach the level of elongation a ductile material reachs

1) Brittleness is a property of material which'll have less plastic deformation (or negligible).
3) Ductility is a property of material by which it can achieve higher elongation or more plastic deformation.
2) Hardness is a property of material by virtue of which it can resist surface deformation or abrasion. There are two types of hardness.
   a) Scratch Hardness → scratch to surface
   b) Indentation Hardness → damage on surface
4.) Toughness is the ability of material to absorb strain energy due to any work done on the material

Toughness = Strain energy / volume

Relationship between E, G & K

Young's modulus ← Modulus of rigidity (shear) → Bulk modulus $\sigma_x = K(\epsilon_v)$ → volumetric strain (Dilatation)

$\sigma = E(\epsilon)$ longitudinal strain

$\tau = G(\gamma)$

$y = mx$

$\mu = \nu$ → Poisson's ratio

$$\boxed{G = \frac{E}{2(1+\nu)}}$$ → Proof of the relation is in your mediums & your first HA

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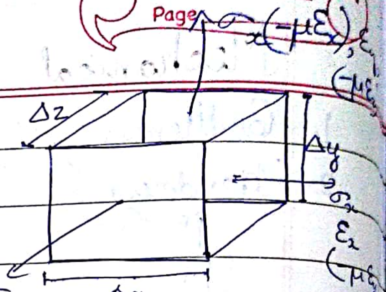
Relationship between K & E



$V = \Delta x \Delta y \Delta z$ $\qquad \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$\varepsilon = \frac{\sigma}{E}$ $\qquad \varepsilon_v = \frac{3\sigma}{E}(1-2\mu)$

$\varepsilon_x = \frac{\sigma_x}{E} - \left(\frac{\mu\sigma}{E}\right) - \left(\frac{\mu\sigma}{E}\right)$ $\quad \varepsilon_y = \frac{\sigma_y}{E} - \left(\frac{\mu\sigma}{E}\right) - \left(\frac{\mu\sigma}{E}\right)$ $\quad \varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma}{E} - \frac{\mu\sigma}{E}$

σ_x $(-\mu\varepsilon_x)$ $(-\mu\varepsilon_y)$ ε_z

$\mu = \frac{\text{lateral}}{\text{longitudinal}}$

In each direction, there's one component of longitudinal strain & two components of lateral st.

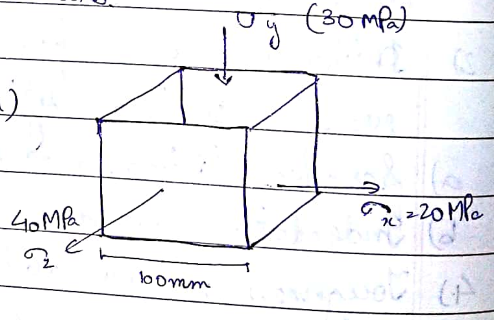
$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$\varepsilon_v = \frac{3\sigma}{E}(1-2\mu)$, $\quad \varepsilon_v = \frac{3\sigma}{E}(1-2\mu)$, $\quad \varepsilon_v = \frac{\sigma}{K}$

$$\boxed{K = \frac{E}{3(1-2\mu)}}$$

or ν

$\frac{\text{Change in vol}}{\text{Original vol}} = \frac{\Delta V}{V} \cdot \boxed{\varepsilon_v = \frac{\sigma}{K}}$ → All stresses are same in all directions



$\varepsilon_x = \frac{\sigma}{E} - \frac{\mu\sigma}{E} - \frac{\mu\sigma}{E} = \frac{\sigma}{E}(1-2\mu)$ (20)

$\varepsilon_y = -\frac{\sigma}{E} + \frac{\mu\sigma}{E} + \frac{\mu\sigma}{E} \quad \frac{\sigma}{E}(2\mu-1)$ (30)

$\varepsilon_z = \frac{\sigma}{E} - \frac{\mu\sigma}{E} - \frac{\mu\sigma}{E} = \frac{\sigma}{E}(1-2\mu)$ (40)

$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \cancel{\frac{\sigma}{E}(1-2\mu)} \quad \frac{60}{E}(1-2\mu) + \frac{30}{E}(2\mu-1)$

$\frac{\Delta V}{100^3} = \frac{30}{E}(1-2\mu) \qquad = \frac{30}{E} - \frac{60\mu}{E} = \frac{30}{E}(1-2\mu)$

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Tutorial Sheet #3

1. $\dfrac{\text{original length} - \text{Change length}}{\text{Original length}}$; $\varepsilon_z = 40 \times 10^{-3} z^{1/2}$

$\dfrac{\delta z}{dz} = \dfrac{dz - dz'}{dz} = \delta z\, \varepsilon_z = \int_0^L dz' = \int_0^L dz + \int_0^L dz (40\times10^{-3}) z^{1/2}$

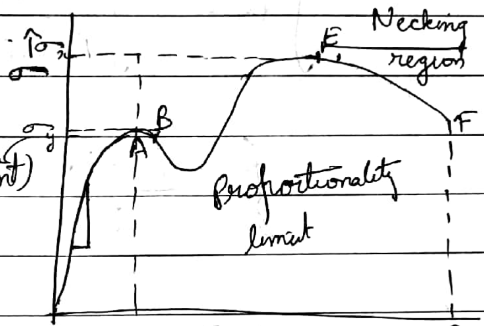
$= L + 40\times10^{-3} \times \dfrac{2}{3} \, z L^{3/2} = 2.585$

Mechanical properties

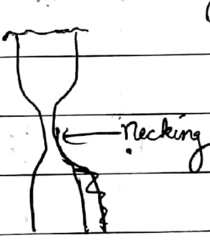
1) Elasticity. The body regains it's original shape & size
According to Hooke's law

$\sigma \propto \varepsilon$
stress $\propto$ strain

$[y = mx]$

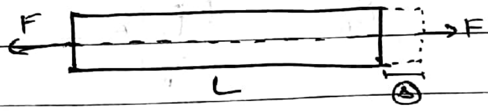


A → proportionality limit
B → yield point → After this point it goes to permanent deformation (Plastic stage)
σ_y → yield stress
E → Young's modulus of rigidity
BD → Perfectly plastic region
E → Ultimate point / strain softening region
F → Failure/rupture point

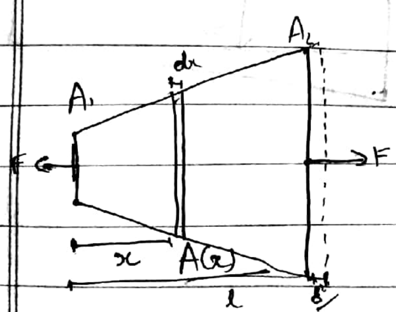


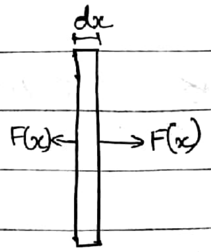
Deformation in Axially loaded Members

$\sigma = \dfrac{F}{A}$; $\sigma = E\varepsilon$



$\varepsilon = \dfrac{\delta l}{l}$ $\boxed{\delta = \dfrac{FL}{AE}}$





$d\delta$ = Elongation strip

$\varepsilon = \dfrac{d\delta}{dx}$

$\sigma = \dfrac{F(x)}{A(x)}$

$\boxed{\int_0^L d\delta = \int_0^L \dfrac{F(x)\,dx}{A(x)\,E}}$

Total elongation

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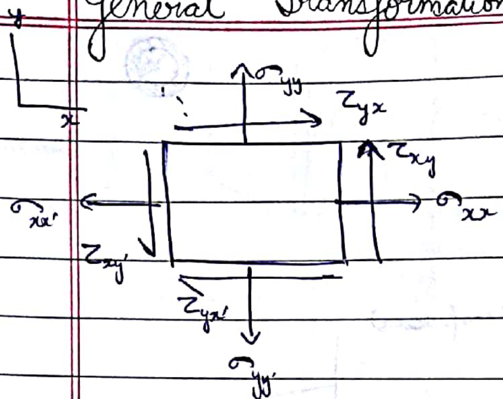
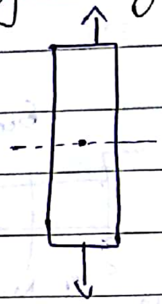
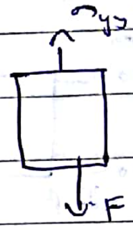
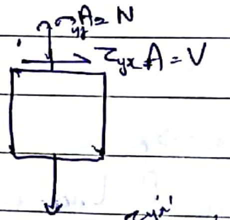
General Transformation of state of stress & strain







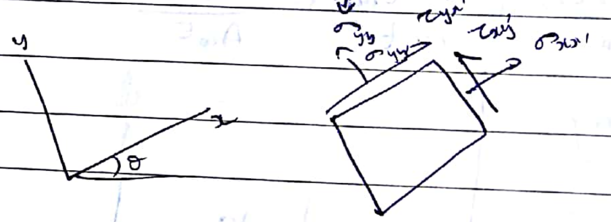
(Due to complementary property of shear stress $\tau_{xy} = \tau_{yx}$)

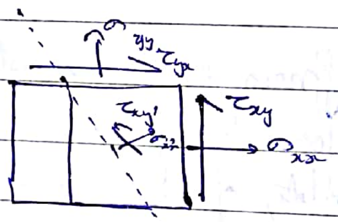


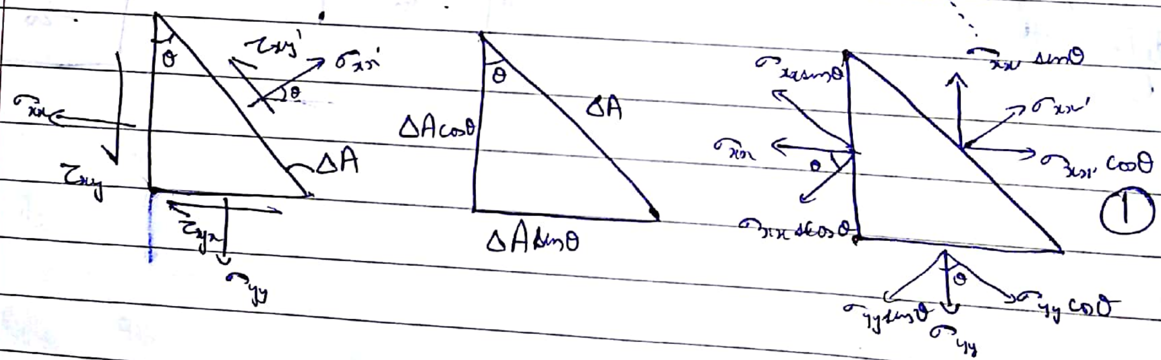
If $\sigma_{yy} > \sigma_{yield}$ body will break

$> \sigma_{ultimate}$

$\sigma_{fracture}$







①

$$\sum F_{x'} = \sigma_{xx'}\Delta A - \sigma_{xx}\Delta A\cos^2\theta - \sigma_{yy}\Delta A\sin^2\theta - \tau_{xy}\Delta A\sin\theta\cos\theta - \tau_{yx}\Delta A\cos\theta\sin\theta = 0$$

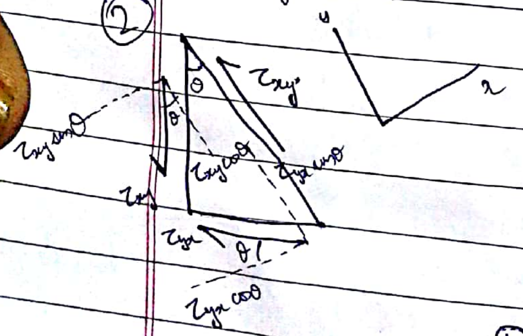
$F_x = (\sigma_{xx}\Delta A\cos\theta)$

Then take forces components $F_x\cos\theta$

$$\sum F_{y'} = \sigma_{xx}\Delta A\cos\theta\sin\theta - \sigma_{yy}\cos\theta\,\Delta A\sin\theta + \tau_{xy'}$$

$$-\tau_{xy}\Delta A\cos^2\theta + \tau_{yx}\sin^2\theta\,\Delta A = 0$$

②



Sum both the components of shear & normal forces

$$\sigma_{xx'} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + \overbrace{\tau_{xy}\cos\theta\sin\theta + \tau_{yx}\sin\theta\cos\theta}^{2\tau_{xy}\sin 2\theta}$$

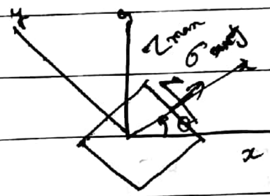
$$= \sigma_{xx}\frac{(1+\cos 2\theta)}{2} + \sigma_{yy}\frac{(1-\cos 2\theta)}{2} + \tau_{xy}\sin 2\theta$$

$$= \cos 2\theta\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right) + \tau_{xy}\sin 2\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)$$

$$\tau_{xy'} = \tau_{xy}\cos 2\theta - \sin\theta\cos\theta(\sigma_{xx}-\sigma_{yy})$$

$$= \tau_{xy}\cos 2\theta - \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta$$

These are eqns of transformed stress

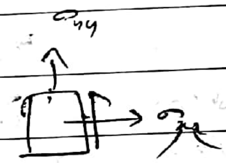


Principal stresses / Maximum Shear stresses

Max. normal stress

$$\sigma_{xx'} = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{yy'} = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) - \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$



$$\tau_{xy'} = -\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal stress

$$\frac{d|\sigma_{xx'}|}{d\theta} = -2\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$= 2\tau_{xy}\cos 2\theta - (\sigma_{xx}-\sigma_{yy})\sin 2\theta = 0$$

$$\tan 2\theta_p = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\tau_{xy}}{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)}, \quad 2\theta_p = ?$$

$$\theta_{p_1} \ \& \ \theta_{p_2} = \frac{\pi}{2} + \theta_{p_1}$$

Max. shear stress

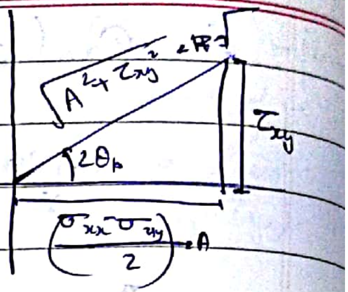
$$\frac{d\tau_{xy'}}{d\theta} = -(\sigma_{xx}-\sigma_{yy})\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\tan 2\theta_s = -\frac{(\sigma_{xx}-\sigma_{yy})}{2\tau_{xy}} \qquad \theta_{s_1} \ \& \ \theta_{s_2} = \frac{\pi}{2} + \theta_{s_1}$$

$\tan 2\theta_p$ & $\tan 2\theta_s$ are −ve reciprocals of each other

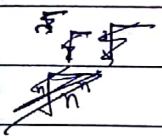
$\boxed{\tan 2\theta_p \times \tan 2\theta_s = -1}$ – Negative reciprocal property

$$\sin 2\theta_p = \frac{\tau_{xy}}{\sqrt{\;}}$$



$$\cos 2\theta_p = \frac{(\sigma_{xx} - \sigma_{yy})/2}{\sqrt{\;}}$$

$$\sigma_{xx'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2}{\sqrt{\;}} + \frac{\tau_{xy}^2}{\sqrt{\;}} \qquad \boxed{\sqrt{\;} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}}$$



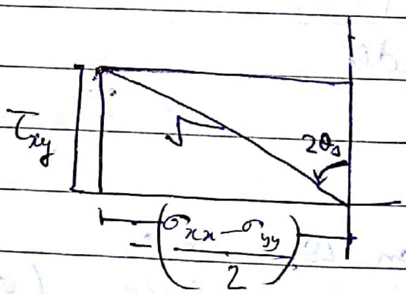
$$\sigma_{1,2} = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 \rightarrow$ Max principal stress
$\sigma_2 \Rightarrow$ Min. " "
$\tau_{xy}' = 0$

$\sigma_{yy}' =$

Shear stress $= 0$ $\longrightarrow$ $$\tau_{xy}' = -\frac{\tau_{xy}\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)}{\sqrt{\;}} + \frac{\tau_{xy}\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)}{\sqrt{\;}} = 0$$

$$\sin 2\theta_s = \frac{-\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)}{\sqrt{\;}}$$



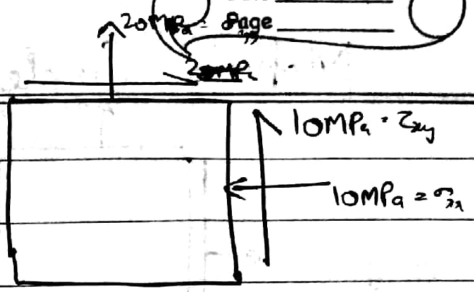
$$\cos 2\theta_s = \frac{\tau_{xy}}{\sqrt{\;}}$$

$$\sigma_{xx'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \cancel{\frac{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)\tau_{xy}}{\sqrt{\;}}} - \cancel{\frac{\tau_{xy}\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)}{\sqrt{\;}}}$$

$$\sigma_{xx'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \sigma_{avg}$$

Max shear stress $$\tau_{xy'} = \frac{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2}{\sqrt{\;}} + \frac{\tau_{xy}^2}{\sqrt{\;}} = \pm\sqrt{\;} = \tau_{max}$$

$|\theta_p - \theta_s| = 45^\circ$ } Verification process



Q Calculate $\sigma_1, \sigma_2, \tau_{max}, \sigma_{avg}, \theta_s$ & θ_p as

$$\sigma_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$= 23.03$ MPa

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} = 18.03 \text{ MPa}$$

$$\sigma_{avg} = \frac{\sigma_{xx}+\sigma_{yy}}{2} = 5 \text{ MPa}$$

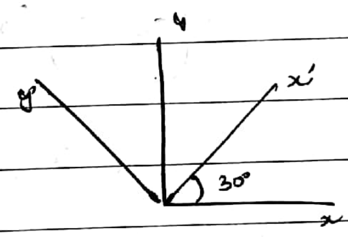
τ_{yy}

$$\theta_s = \cos^{-1}\left(\frac{\tau_{xy}}{18.03}\right) = 56.31^\circ$$

$$\theta_p = \sin^{-1}\left(\frac{\tau_{xy}}{18.03}\right) = 33.685^\circ$$

$$\sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} = -13.03 \text{ MPa}$$

$$\cancel{\theta_p = \tan^{-1}\left(\frac{\tau_{xy}}{\frac{\sigma_{xx}-\sigma_{yy}}{2}}\right)}$$



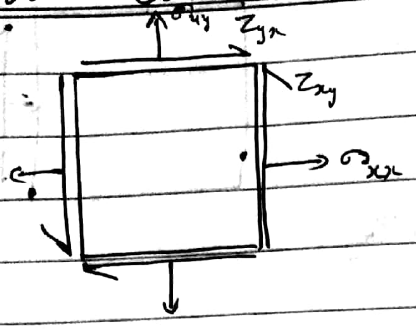
If $\theta = 30^\circ$ what are transformed stresses?

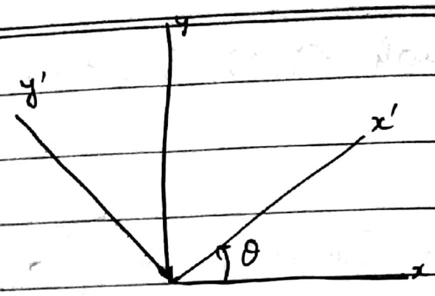
$$\sigma_{xx'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta = 6.16 \text{ MPa}$$

$\sigma_{yy'} =$ " $-$ " $-$ " $=$ 3.84 MPa

$\tau_{xy'} =$

Mohr's Circle





$$\sigma_x' = \underbrace{\frac{\sigma_{xx}+\sigma_{yy}}{2}}_{a} + \underbrace{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}_{b} \quad \text{— ①}$$

$$\tau_{xy}' = -\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{— ②}$$

$①^2 + ②^2$

$$\Rightarrow \left[\sigma_x' - \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)\right]^2 + \tau_{xy}'^2 = \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2\cos^2 2\theta + \tau_{xy}^2\sin^2 2\theta$$

$$+ \cancel{2\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\tau_{xy}\cos 2\theta\sin 2\theta} + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2\sin^2 2\theta + \tau_{xy}^2\cos^2 2\theta$$

$$- \cancel{2\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\tau_{xy}\sin 2\theta\cos 2\theta}$$

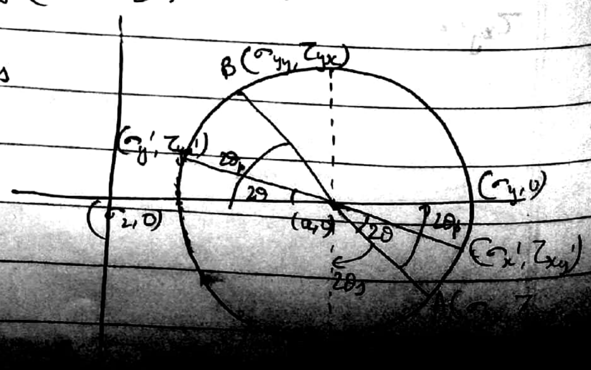
$$= \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2$$

$$\Rightarrow \left[\sigma_x' - \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)\right]^2 + \tau_{xy}'^2 = \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2$$

$$(x-a)^2 + y^2 = R^2$$

$$a = \frac{\sigma_{xx}+\sigma_{yy}}{2} = \sigma_{avg} \qquad R = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

every pt of circle represents a stress



classmate

Date 27-03-18

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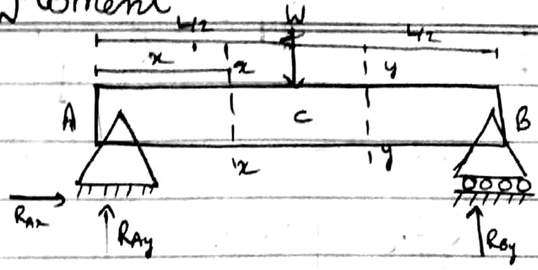
UNIT:2 Shear Force & Bending Moment



$M(x) = ?$; $V(x) = ?$

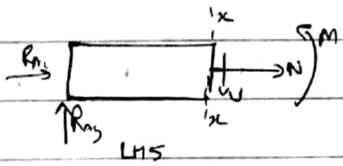
$+\circlearrowleft \sum M_A = 0$

$$R_{By} \times L - W \times \frac{L}{2} = 0 \; ; \quad R_{By} = \frac{W}{2}$$

$\sum F_y = 0$, $\quad R_{Ay} + R_{By} - W = 0$

$$R_{Ay} = W - \frac{W}{2} = \frac{W}{2}$$

$\sum F_x = 0$; $R_{Ax} = 0$



LHS

For segment AC; N=0

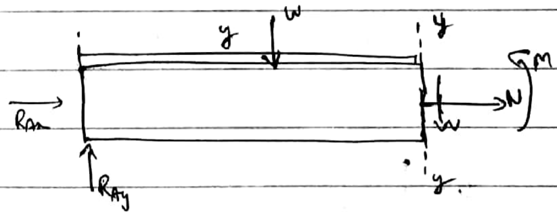
$\sum F_y = 0$, $-V(x) + R_{Ay} = 0$, $\boxed{V(x) = \frac{W}{2}}$

$+\circlearrowleft \sum M_{xx}^{+} = 0$

$M(x) - R_{Ay} \times x = 0$, $\boxed{M(x) = \frac{Wx}{2}}$

$x = 0$ @ A, $M = 0$

$x = L/2$ @ C, $M = WL/4$



For this segment

$\sum F_y = 0$

$$\boxed{V(y) = R_{Ay} - W = -\frac{W}{2}}$$

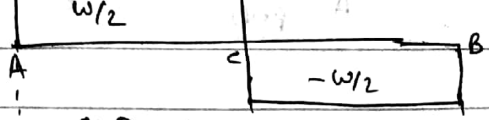
$+\circlearrowleft \sum M_{yy}^{+} = 0$

$$M_y + W \times (y - L/2) - R_{Ay} \times y = 0$$

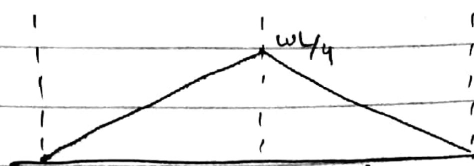
$$M(y) = \frac{Wy}{2} - W\left(y - \frac{L}{2}\right)$$

@ $y = L/2$, @ C, $M = WL/4$

@ $y = L$ @ B, $M = 0$

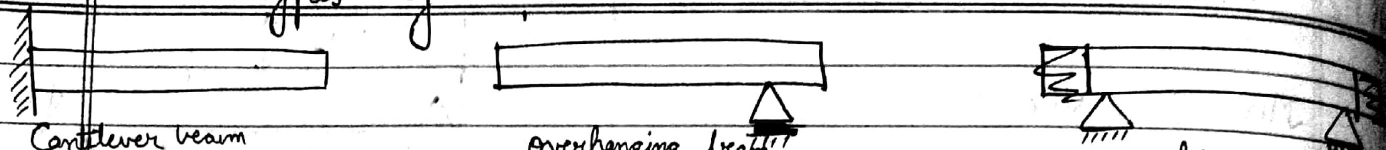


SFD - Shear Force Diagram



BMD - Bending moment diagram

Three types of beams

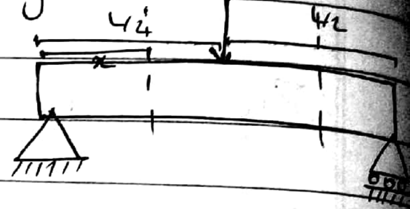


Cantilever beam — Overhanging beam — Simple beam

Types of loads

→ Concentrated

Distributed → Uniform (UDL), Varying (VDL)



Distributed

$V(x) = \frac{wL}{2} - wx$

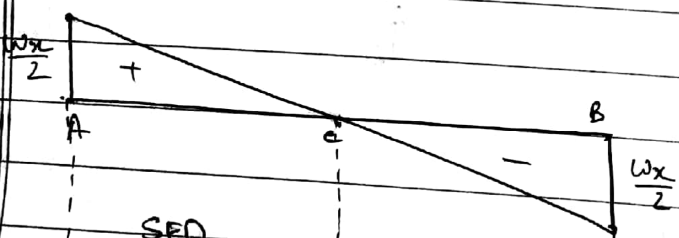
$\circlearrowleft M_{xy} = M(x) - R_{Ay} \times \cancel{L}\,x + wx \times \frac{\cancel{x}L}{4} = 0$

$= M(x) - \frac{wLx}{2} + \frac{wx\cancel{x}L}{} = 0$

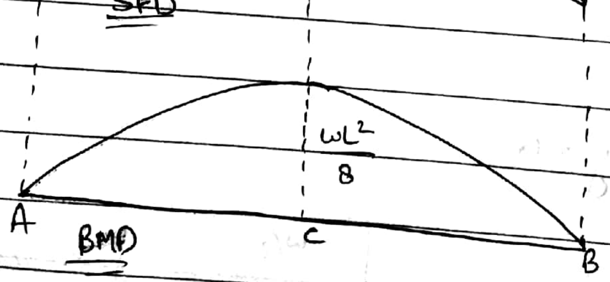
$= M(x) = \frac{wLx}{2} - wxL$

When $x=0$, $V(x) = wL/2$
$x = L/2$, $V(x) = 0$
$x = L$, $V(x) = -wL/2$

When $x=0$, $M(x) = 0$
$x = L/2$, $M(x) = wL^2/8$
$x = L$, $M(x) = 0$

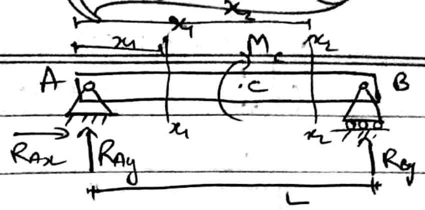


SFD



BMD

Draw SFD & BMD



$+\circlearrowleft \sum M_A^+ = 0 \Rightarrow R_{By} \times L - M_c = 0; \; R_{By} = \frac{M_c}{L}$

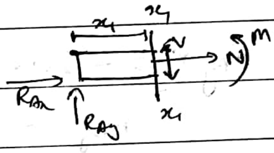
$R_{Ay} + R_{By} = 0$

Supporting rxn.

$R_{Ay} = -M_c/L \, (\downarrow)$ $\qquad R_{Ay} = -R_{By} = -\frac{M_c}{L}$

$R_{By} = \frac{M_c}{L} \, (\uparrow)$

Applying eq eqns at ~~x_1x_1~~ section AC

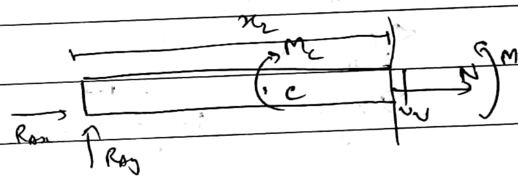


$R_{Ax} + N = 0, \quad N = 0$

$R_{Ay} - V = 0, \quad V = R_{Ay} \Rightarrow \boxed{V = -\frac{M_c}{L}}$ —(1a)

$+\circlearrowleft \sum M_{x_1x_1}^+ = 0, \quad M - R_{Ay}x_1 = 0, \quad \boxed{M = \frac{-M_c x_1}{L} = \frac{-M_c x_1}{L}}$ —(1b)

Applying eq eqn at section CB



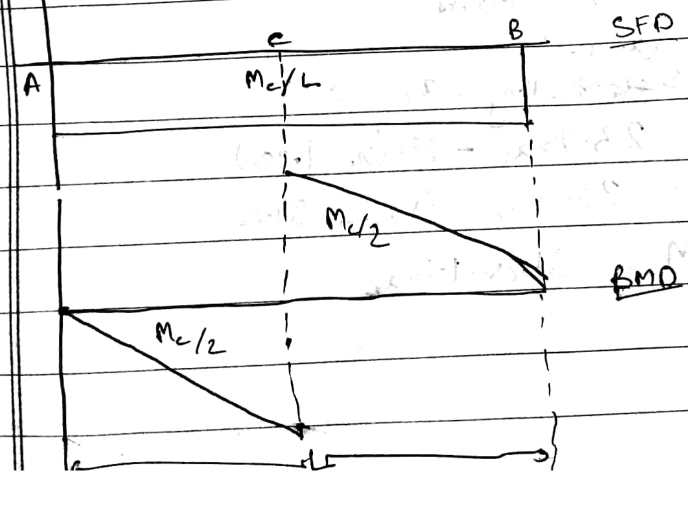
$R_{Ax} + N = 0; \quad N = 0$

$R_{Ay} - V = 0, \quad \boxed{V = R_{Ay} = \frac{-M_c}{L}}$ —2a

$+\circlearrowleft \sum M_{x_2x_2}^+ = 0;$

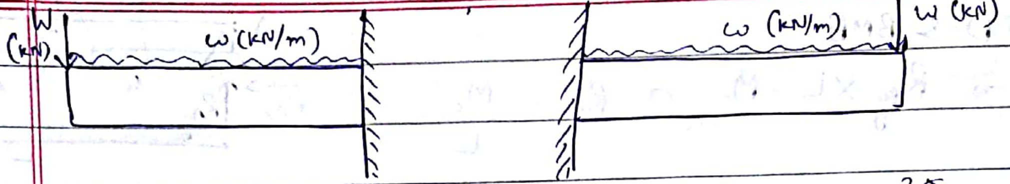
$M - M_c - R_{Ay}x_2 = 0$

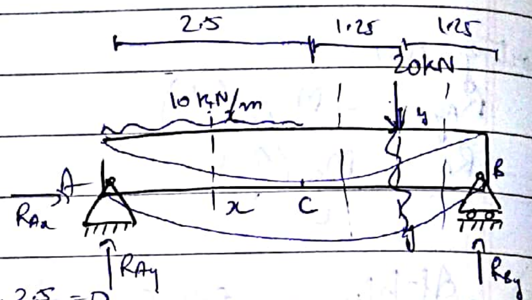
$\boxed{M = M_c - \frac{M_c}{L} x_2}$ —2b



Draw Shear Force Diagram (SFD) & Bending Moment Diagram (BMD)

Cantilever





Q. External support rxn

Moment about A

$\sum M_A = 0$

$R_{By} \times 5 - 20 \times 3.75 - 10 \times 2.5 \times \frac{1}{2} \times 2.5 = 0$

$R_{By} = 21.25$ kN

25.0
12.5
37.5

$\sum F_y = 0$, $R_{Ay} + R_{By} = 20 + 10 \times 2.5$

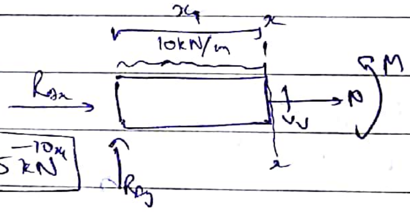
$R_{Ay} + R_{By} = 45$

$R_{Ay} = 23.75$ kN

$\sum F_x = 0$

$R_{Ax} = 0$

For section AC



$\sum F_x = 0$, $R_{Ax} + N = 0$, $N = R_{Ax} = 0$

$\sum F_y = 0$ $R_{Ay} - V \overset{-10x_1}{\wedge} = 0$, $\boxed{V = R_{Ay} \overset{-10x_1}{=} 23.75 \text{ kN} \overset{-10x_1}{}}$

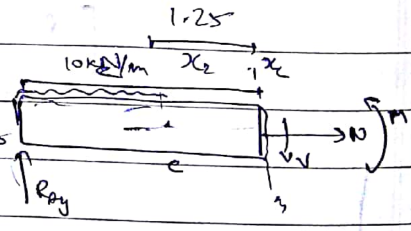
$\sum M_{xx} = 0$, $M + \frac{10 x_1^2}{2} - R_{Ay} \cdot x_1 = 0$

$\cancel{F \times d}$

$10 \times \cancel{x_1 \times x_1}/2$

$\boxed{M = 23.75x_1 - 5x_1^2}$

For other section CD.



$\sum F_y = 0$ $R_{Ay} - \overset{-10x_1}{\cancel{V = 0}}$, $\cancel{V = R_{Ay} = 23.75 - 10x_1}$

$R_{Ay} - V - 10 \times 2.5 = 0$, $V = 23.75 - 25$

$V = -1.25$ kN

$\sum M_{x_2x_2} = 0$

$M + 10 \times 2.5 \overset{(x_2 - 1.25)}{\cancel{\times (x_2 - 1)}} - R_{Ay} \times x_2 = 0$

$M = 23.75x_2 - 25(x_2 - 1.25)$

$M = 23.75x_2 - 25x_2 + 31.25$

$M = 31.25 - 1.25x_2$

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For section DB

$\Sigma F_y = 0$; $R_{By} + V = 0$

$V = -R_{By} = -21.25$ kN

$\curvearrowleft^{+} \Sigma M_{x_3 x_3} = 0$

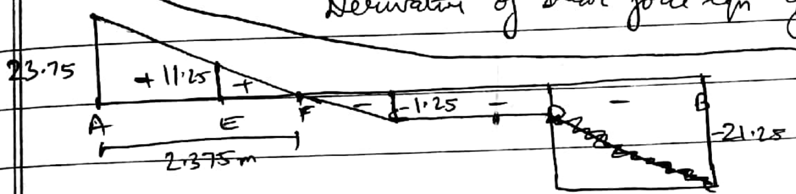
$-M + R_{By} \times x_3 = 0$

$M = +21.25 x_3$

Derivative of moment eqn gives ~~to~~ shear force eqn $= \frac{d(M_{x_4})}{dx_4} = V(x_4)$

Derivative of shear force eqn gives load $= \frac{d V(x)}{dx} = W(x)$

SFD



$M_A = 0$

$M_E = 23.75 \times 1.25 - 5 \times 1.25^2$

$= 21.875$

$M_F = 23.75 \times 2.375 - 5 \times 2.375^2$

$= 28.203125$

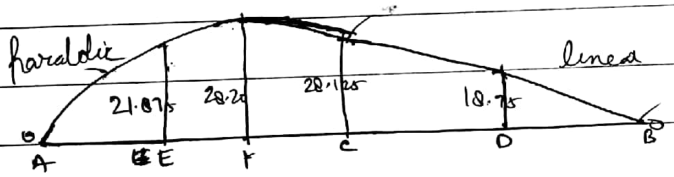
$M_C = 31.25 - 1.25 \times 2.5$

$= 28.125$

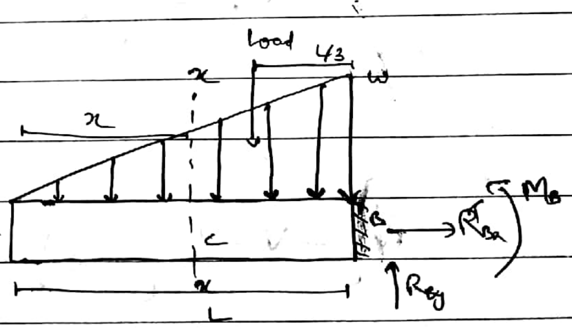
$M_D = 31.25 - 1.25 \times (2.5 + 1.25) = 18.75$

$M_{F} = -106.25$

BMD



Cantilever with varying load



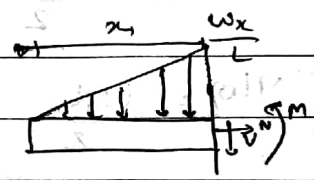
$\Sigma F_y = 0$

$R_{By} = \left(\frac{1}{2} \times W \times L\right)$ kN

$\Sigma M_B = 0 \Rightarrow \quad M_B + \left[\left\{\frac{1}{2} \times W \times L\right\} \times \frac{L}{3}\right] = 0$

$M_B = -\frac{WL^2}{6}$

For section AC



$\Sigma F_x = 0 \quad N = 0$, $\Sigma F_y = 0$

$\cancel{-V - \frac{1}{2} \frac{W_x x}{L} = 0}$, $\cancel{V = -\frac{Wx^2}{2L}}$ $\quad -V - \frac{W_x}{L} \times \frac{1}{2} x_1 = 0$

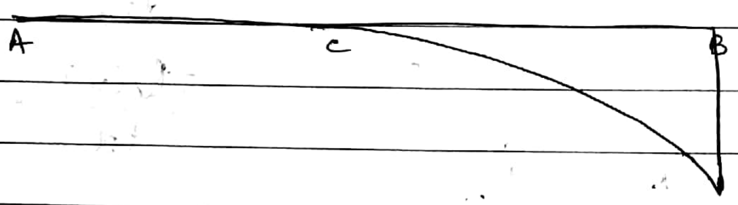
$\cancel{\Sigma M_{aa} \Rightarrow M + \left[\left\{-\frac{Wx}{L}\right\} \times x\right] = 0}$, $\cancel{M = \frac{Wx^2}{L}}$ $\quad V = -\frac{Wx^2}{2L}$

$\circlearrowleft \sum M_{xx} = 0$ F d

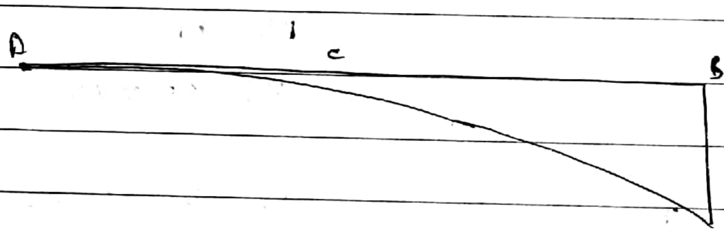
$$\Rightarrow M_x + \left\{\frac{1}{2} \times \frac{W_0 x^2}{L}\right\} \times \frac{x}{3} = 0$$

$$M_x = -\frac{W_0 x^3}{6L}$$

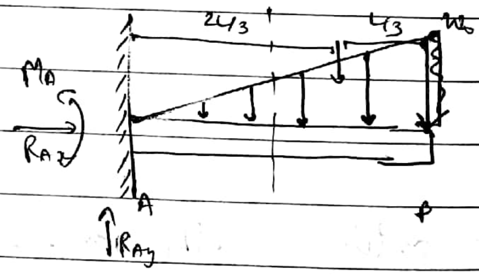
SFD



BMD



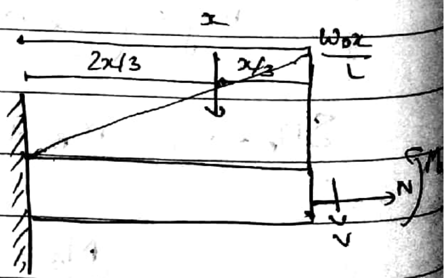
$R_{Ay} - \frac{W_0 L}{2} = 0$; $R_{Ay} = \frac{W_0 L}{2}$



$\sum M_A = 0$

$$M_A - \frac{2L}{3} \times \frac{W_0 L}{2} = 0$$

$$M_A = \frac{W_0 L^2}{3}$$



$$R_{Ay} - V - \frac{W_0 x}{L} \times \frac{1}{2} \times x = 0$$

$$V = \frac{W_0 L}{2} - \frac{W_0 x^2}{2L}$$

$V(0) = \frac{W_0 L}{2}$; $W(L) = 0$

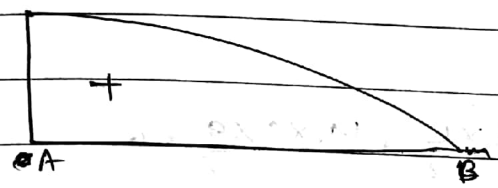
$\sum M_x = 0$

$$M_A + M_x + \frac{x}{3} \times \frac{1}{2} \times \frac{w_0 x^2}{L} - \frac{w_0 L x}{2} = 0$$

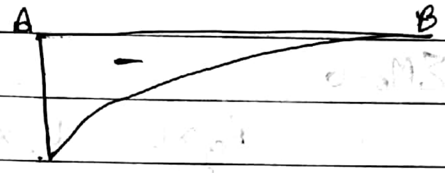
$$M_x = \frac{w_0 L x}{2} - \frac{w_0 x^3}{6L} - \frac{w_0 L^2}{3}$$

$$M_x(0) = -\frac{w_0 L^2}{3} ; \quad M_x(L) = 0$$

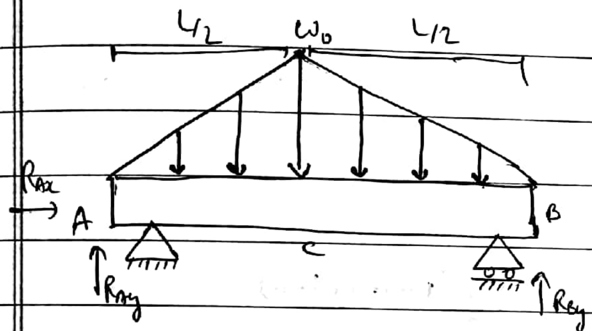
SFD



BMD



Q



$$\text{Total load} = 2 \times \frac{1}{2} \times w_0 \times \frac{L}{2} = \frac{w_0 L}{2}$$

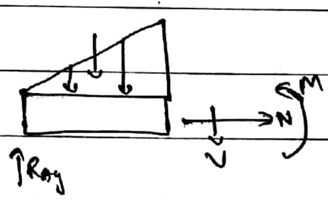
$$R_{Ay} + R_{By} = \frac{w_0 L}{2}$$

$\sum M_A = 0$

$$-\frac{w_0 L}{2} \times \frac{L}{2} + R_{By} \times L = 0 , \quad R_{By} = \frac{w_0 L}{4} ; \quad R_{Ay} = \frac{w_0 L}{4}$$

For section AC

$$\frac{w_0}{L} = \frac{w}{x}, \quad x = \frac{2w x}{L}$$



$$\text{Load} = \frac{1}{2} \times x \times \frac{2wx}{L} = \frac{wx^2}{L}$$

$$R_{Ay} - V - \frac{w_0 x^2}{2} = 0$$

$$; V = \frac{w_0 L}{4} - \frac{w_0 x^2}{4} ; \quad V(0) = \frac{w_0 L}{4} ; \quad V(L/2) = 0$$

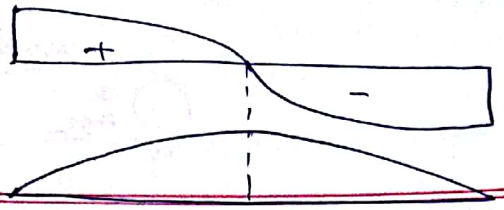
$\sum M_x = 0$

$$M_x + \frac{wx^2}{L} \times \frac{x}{3} - \frac{w_0 L}{4} x = 0 , \quad M_x = \frac{w_0 L x}{4} - \frac{w x^3}{3L}$$

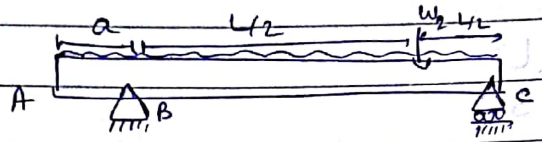
$$M_x(0) = 0, \quad M_x(L/2) = \frac{w_0 L^2}{12}$$

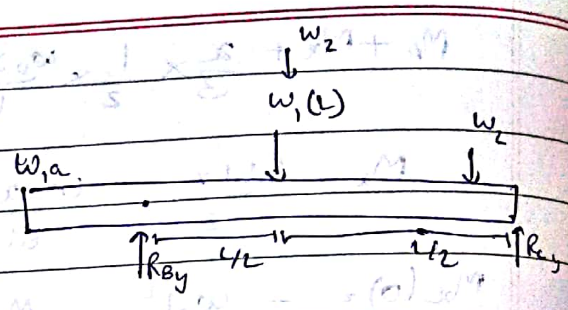
SFD

BMD



Q





$\sum F_y = 0$

$R_{By} + R_{cy} = w_1(L+a) + w_2$

$\sum M_B = 0$

$$R_{cy} \times L - \frac{W_2 \times L}{2} - \frac{w_1 \times L \times L}{2} + \frac{w_1 \times a \times a}{2} = 0$$

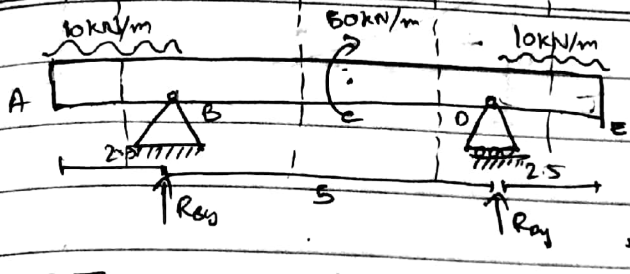
$$R_{cy} = \frac{W_2}{2} + \frac{W_1 L}{2} - \frac{W_1 a^2}{2L}$$

$$R_{cy} = \frac{W_2}{2} + \frac{w_1}{2L}\left(L^2 - a^2\right)$$

$$R_{By} = w_1(L+a) + w_2 - \frac{W_2}{2} - \frac{w_1}{2L}(L+a)(L-a)$$

$$= \frac{W_2}{2} + w_1\left((L+a) + \frac{(L-a)}{2L}\right)$$

Date
Page



$\sum F_y = 0$

$R_{By} + R_{Dy} = ?$

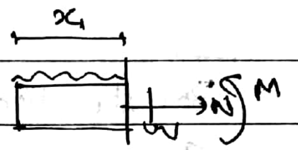
$+\circlearrowright \sum M_B = 0$

$\Rightarrow R_{Dy} \times 5 \mp 50 + 10 \times 2.5 \times \frac{2.5}{2} \mp 10 \times 2.5 \times \frac{\overset{6.25}{(2.5+5)}}{2} = 0$

$R_{Dy} = 35\ kN$

$R_{By} = 15\ kN$

In first segment ~~section~~ AB (limits 0 – 2.5)



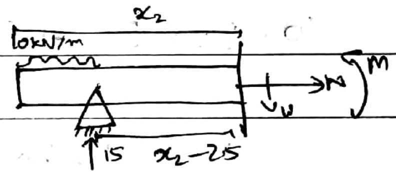
$-V + 10 \times x = 0$

$V = -10x_1$

Moment about x_1, $\sum M_{x_1} = 0$; @ $x_1 = 0$; $V_{x_1} = 0$, $M_{x_1} = 0$

$M_{x_1} + 10x_1 \times \frac{x_1}{2} = 0$; $M_{x_1} = -5x_1^2$ @ $x_1 = 2.5$, $V_{x_1} = -25\ kN$, $M_{x_1} = -31.25\ kN/m$

In second ~~section~~ segment BC (limits 2.5 – 5)



~~$15 - V - 10 \times x_2 \times 2.5 = 0$~~

$15 - V - 10 \times 2.5 = 0$

$V = -10\ kN$

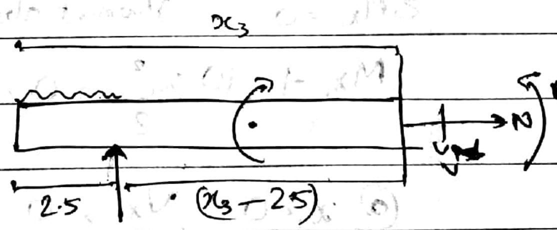
Moment about x_2 $\sum M_{x_2} = 0$

$M_{x_2} - 15 \times (x_2 - 2.5) + 10 \times 2.5 \times \overset{(x_2 - 1.25)}{\cancel{\frac{x_2}{2}}} = 0$ @ $x_2 = 2.5$, $V_{x_2} = -10\ kN$, $M_{x_2} = -31.25\ kN/m$

@ $x_2 = 5$, $V_{x_2} = -10\ kN$, $M_{x_2} = -56.25\ kN/m$

$M_{x_2} = -10x_2 \mp 6.25$

In ~~se~~ third segment ~~AD~~ CD (limits 5 – $\overset{7.5}{\cancel{6.25}}$)



~~$-V + 15 \mp 10 \times 2.5 = 0$~~ $15 - V - 10 \times 2.5 = 0$

~~$V = -10$~~ $V = -10\ kN$

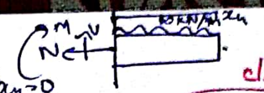
Moment about x_3 $\sum M_{x_3} = 0$

$M_{x_3} - 15(x_3 - 2.5) - 50 - 10 \times 2.5 \times (x_3 - 1.25) = 0$

$M_{x_3} = -10x_3 + \overset{43.75}{\cancel{56.25}}$ @ $x_3 = 5$ $V_{x_3} = -10\ kN$, $M_{x_3} = -6.25\ kN$

@ $x_3 = \overset{7.5}{\cancel{6.25}}$, $V_{x_3} = -10\ kN$, $M_{x_3} = -31.25\ kN$

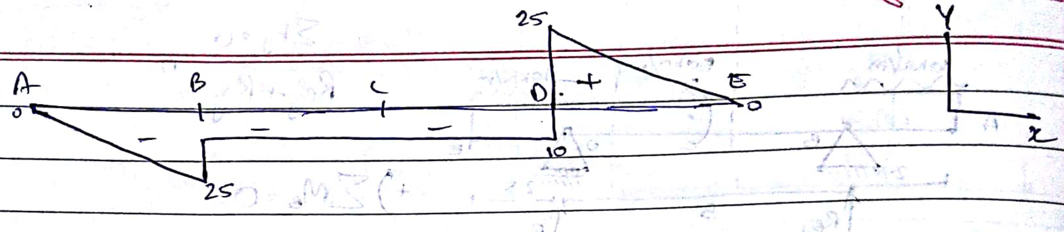
For segment DE (limit 0 to 2.5)



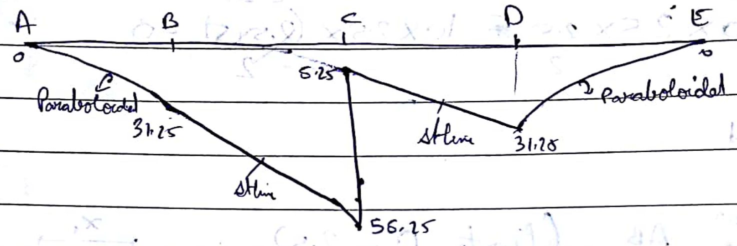
$V_{x_4} = +10x_4$ @ $x_4 = 0$ $V_{x_4} = 0$, $M_{x_4} = 0$

$M = -5x_4^2$ @ $x_4 = 2.5$ $V_{x_4} = 25$ kN, $M_{x_4} = 31.25$ kN

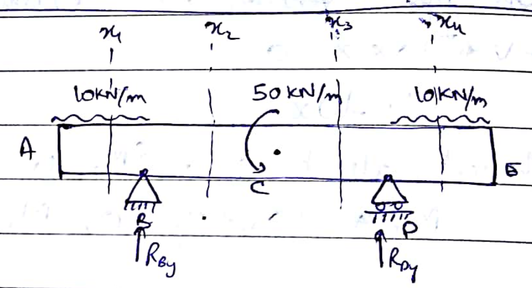
SFD



BMD



For the beam



$\Sigma F_y = 0$

$R_{By} + R_{Dy} = ?$

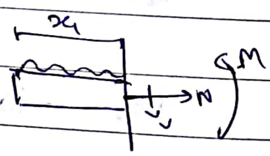
$+\circlearrowleft \Sigma M_B = 0$

$$10 \times 2.5 \times \frac{2.5}{2} + 50 + R_{Dy} \times 5 - 10 \times 2.5 \times 6.25 = 0$$

$R_{Dy} = 15$ kN

$R_{By} = 2(10 \times 2.5) - 15 = 35$ kN

For first segment AB (limit 0 to 2.5)



$\Sigma F_y = 0$

$-V - 10x_1 = 0$

$V = -10x_1$

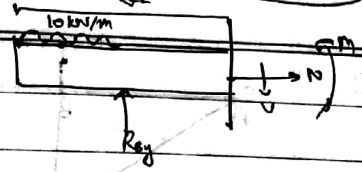
$\Sigma M_{x_1} = 0$ Moment about x_1

$M_{x_1} + 10\frac{x_1^2}{2} = 0$; $M_{x_1} = -5x_1^2$

@ $x_1 = 0$, $V_{x_1} = 0$ kN, $M_{x_1} = 0$ kN/m

@ $x_1 = 2.5$m $V_{x_1} = -25$ kN, $M_{x_1} = -31.25$ kN/m

For second segment BC (limit 2.5 to 5)



$\Sigma F_y = 0$

$-V + R_{By} - 10 \times 2.5 = 0$

$\therefore V = R_{By} - 25 = 35 - 25 = 10\ kN$

$\Sigma M x_2 = 0$, moment about x_2

$M_{x_2} - R_{By}(x_2 - 2.5) + 10 \times 2.5 \times (x_2 - 1.25) = 0 \Rightarrow M_x$

$M_{x_2} = 35(x_2 - 2.5) - 25(x_2 - 1.25)$

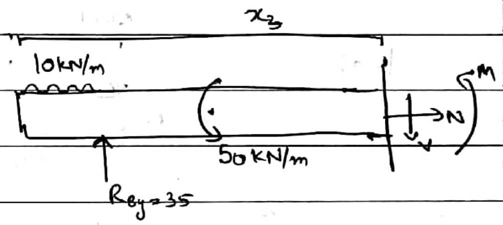
$= 35x_2 - 87.5 - 25x_2 + 31.25$

$M_{x_2} = 10x_2 - 56.25\ kN/m$.

@ $x_2 = 2.5\ m$, $V_{x_2} = 10\ kN$, $M_{x_2} = -31.25\ kN/m$

@ $x_2 = 5\ m$, $V_{x_2} = 10\ kN$, $M_{x_2} = -6.25\ kN/m$

For third segment CD (limit 5 to 7.5)



$\Sigma F_y = 0$

$-V + R_{By} - 10 \times 2.5 = 0$

$V = R_{By} - 25 = 35 - 25 = 10\ kN$

$\Sigma M x_3 = 0$, moment about x_3

$M_{x_3} + 50 - R_{By}(x_3 - 2.5) + 10 \times 2.5 \times (x_3 - 1.25) = 0$

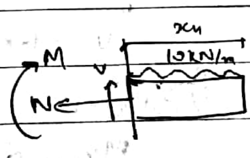
$M_{x_3} = 35(x_3 - 2.5) - 50 - 25(x_3 - 1.25) = 0$

$M_{x_3} = 10x_3 - 106.25\ kN/m$

@ $x_3 = 5\ m$, $V_{x_3} = 10\ kN$, $M_{x_3} = -56.25\ kN/m$

@ $x_3 = 7.5\ m$, $V_{x_3} = 10\ kN$, $M_{x_3} = -31.25\ kN/m$

For last segment DE (limit 2.5 to 0)



$\Sigma F_y = 0$

$V - 10 \times x_4 = 0$; $V = 10x_4$

$\Sigma M x_4 = 0$ moment about x_4

$-M_{x_4} - 10\dfrac{x_4^2}{2} = 0$; $M_{x_4} = -5x_4^2$

@ $x_4 = 2.5\ m$, $V_{x_4} = 25\ kN$, $M_{x_4} = -31.25\ kN/m$

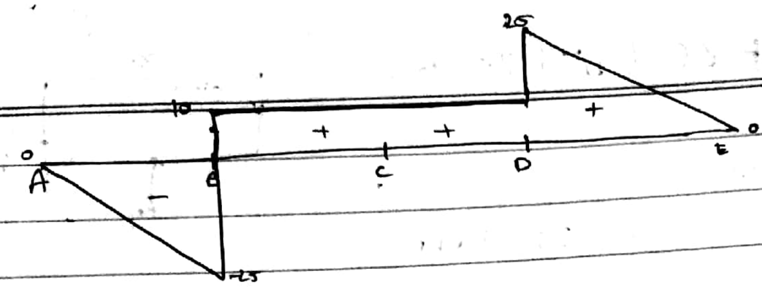
@ $x_4 = 0\ m$, $V_{x_4} = 0\ kN$, $M_{x_4} = 0\ kN/m$

classmate

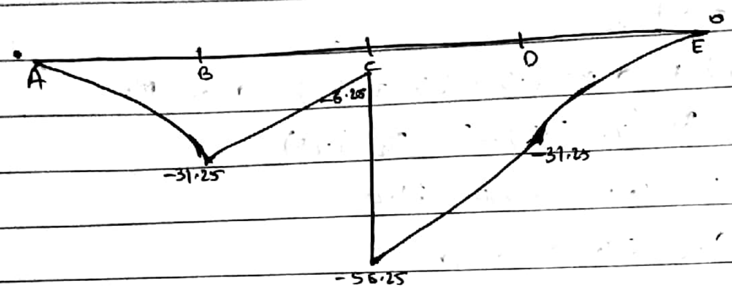
Date ______

Page ______

SFD

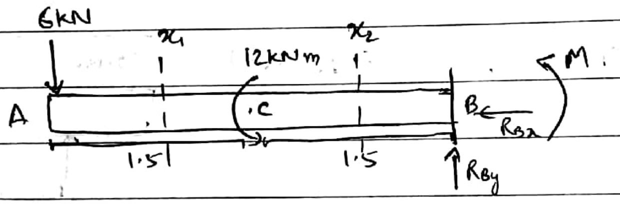


BMD



Tutorial #6

1. a)



For overall beam

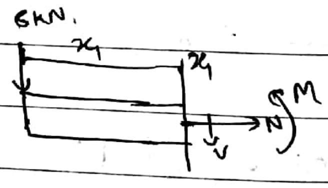
$\Sigma F_y = 0$, $R_{By} = 6\ kN$ $\quad \Sigma F_x = 0$; $R_{Bx} = 0$

$\circlearrowleft^+ \Sigma M_B = 0$

$M + 6 \times 3 + 12 = 0$; $M = -12 - 18 = -\cancel{6}\ 30\ kNm$

$= \cancel{6}\ 30\ kN.m$ (clockwise)

For first segment ABC



$\Sigma F_y = 0$

$V = -6\ kN$

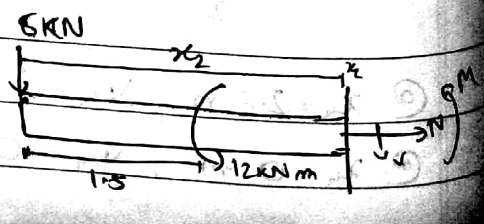
$\circlearrowleft^+ \Sigma M_{x_1} = 0$

$M_{x_1} + 6 \times x_1 = 0$, $M_{x_1} = -6x_1$

@ $x_1 = 0\ m$, $V_{x_1} = -6\ kN$, $M_{x_1} = 0\ kNm$

@ $x_1 = 1.5\ m$, $V_{x_1} = -6\ kN$, $M_{x_1} = -9\ kN.m$

For other segment BC

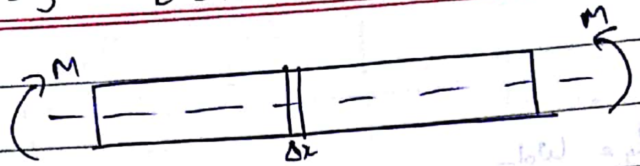


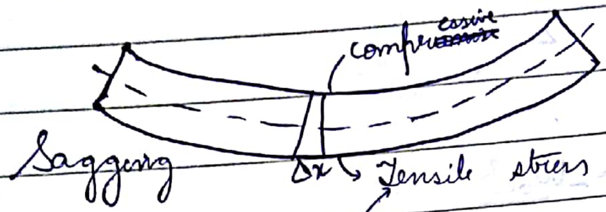
$\Sigma F_y = 0$; $V = -6\ kN$

$\Sigma M_{x_2} = 0$

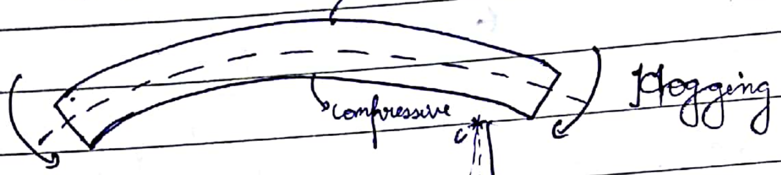
$M_{x_2} + 6(x_2) + 12 = 0$; $M_{x_2} = -12 - 6$

UNIT-3 BENDING STRESS

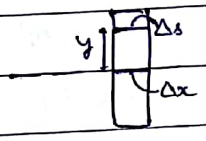


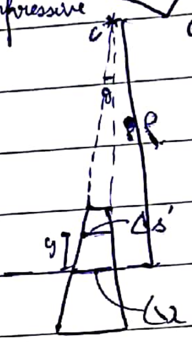


* Neutral surface
* Neutral axis



$$\boxed{f_b \simeq M}$$





$$\varepsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\varepsilon = \lim_{\theta \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta}$$

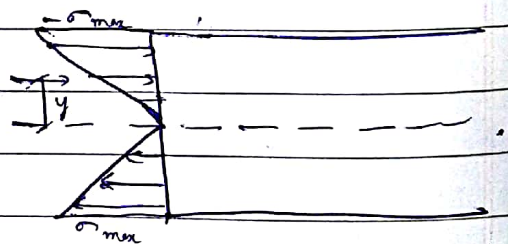
$$\boxed{\varepsilon_{max} = \frac{y_{max}}{\rho}} \Longleftarrow \varepsilon = \frac{-y}{\rho}$$ (−ve shows compression)

$$\sigma = E\varepsilon \quad \text{or} \quad \varepsilon = \frac{\sigma}{E} \text{ —— (I)}$$

$$\varepsilon_{max} = \frac{\sigma_{max}}{E} \text{ —— (II)}, \quad \text{(I)/(II)} \quad \boxed{\frac{\varepsilon}{\varepsilon_{max}} = \frac{\sigma}{\sigma_{max}}}$$

$$\frac{\sigma}{\sigma_{max}} = \frac{y}{y_{max}} \Rightarrow \boxed{\sigma = \left(\frac{y}{y_{max}}\right)\sigma_{max}}$$

$$M = \int dF.y$$



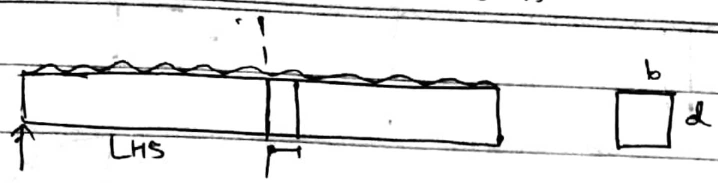
$$M = \int \sigma\, dA.y$$

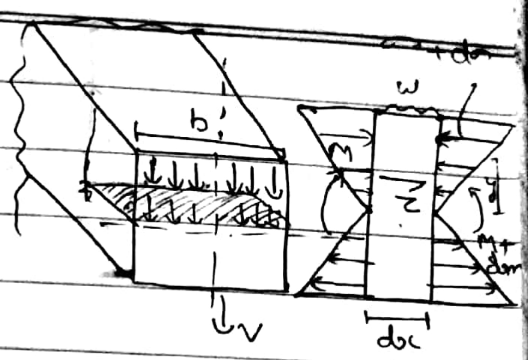
$$M = \frac{\sigma_{max}}{y_{max}} \underbrace{\boxed{\int y^2 dA}}_{I}$$

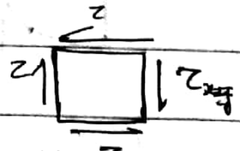
Second moment of area or Moment of Inertia $= \frac{1}{12}bd^3$ (for rectangular

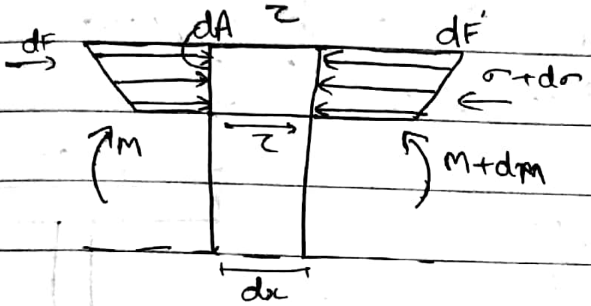
$$\boxed{\frac{M}{I} = \frac{\sigma_{max}}{y_{max}} = \frac{\sigma}{y}}$$

Shear Stress Distribution









$$\frac{M}{I} = \frac{\sigma}{y} \quad , \quad \sigma = \frac{M}{I} \cdot y$$

$$\sigma' = \frac{M'}{I} y$$

$$\sigma + d\sigma = \frac{(M+dM)}{I} y$$

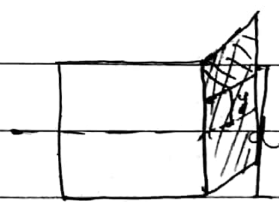
$\Sigma F_x = 0 \quad ; \quad dF - dF' + \tau b\, dx$

$$\Rightarrow \int \sigma\, dA - \int \sigma' dA + \tau b\, dx \quad \text{—— (1)}$$

$$(1) \Rightarrow \int \frac{M}{I} y\, dA - \int \frac{(M+dM)}{I} y\, dA + \tau\, b\, dx = 0$$

$$-\frac{dM}{I} y\, dA + \tau b dx = 0 \quad ; \quad \tau = \left(\frac{dM}{dx}\right) \cdot \frac{y\, dA}{I b}$$

$$\boxed{\tau = \frac{VQ}{Ib}}$$ (for transverse shear)



At top and bottom of X section

$Q = \Sigma A \times D \quad ; \quad Q = 0, \tau = 0$

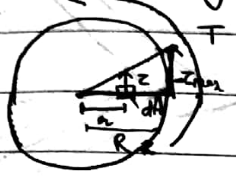
At neutral axis

$Q = \frac{d}{2} \times b \frac{d}{4} \;;$

At a point y from NA, $Q = \left(\frac{d}{2} - y\right) \times b \times \left\{ y + \left(\frac{d/2 - y}{2}\right)\right\} = \frac{1}{2}\left(\frac{d^2}{4} - y^2\right) b$

Torsion in beams

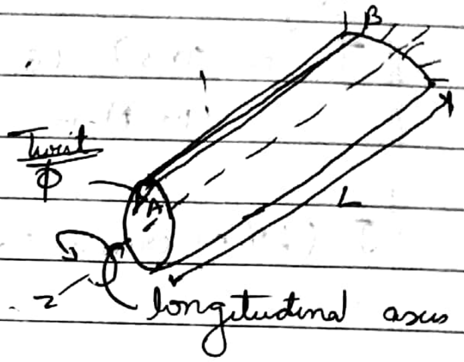
Phenomena of twisting is called torsion



similar Δ

$$\frac{\tau_{max}}{\tau} = \frac{R}{r}$$

$$\tau = \frac{r\tau_{max}}{R}$$



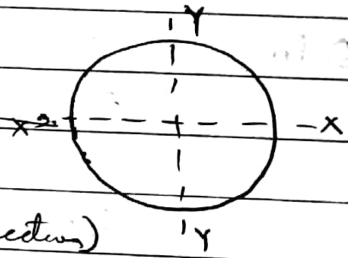
$$\int dT = \int_0^R (\tau dA) \times r$$

$$T = \frac{\tau_{max}}{R} \boxed{\int r^2 dA} \rightarrow J$$

→ Polar moment of inertia = sum of moment of inertia

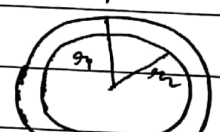
$$\boxed{\frac{T}{J} = \frac{\tau_{max}}{R} = \frac{\tau}{r}} = \frac{G\phi}{L}$$

$$(I_{zz} = I_{xx} + I_{yy})$$



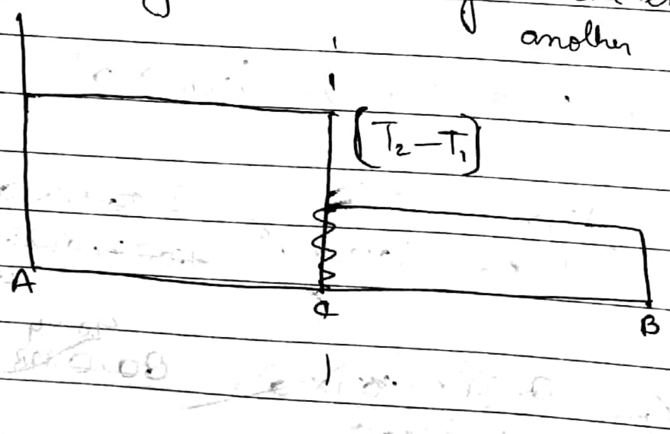
$$J \text{ (for circle)} = \frac{\pi r^4}{2}$$

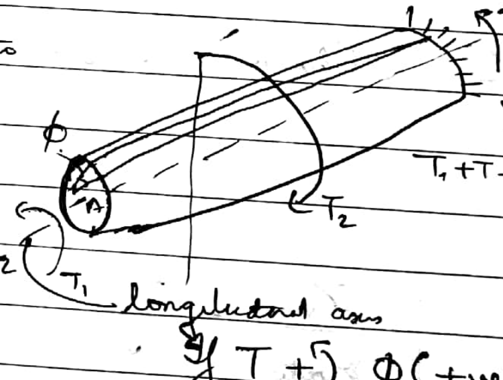
$$J \text{ (for circular cross section)} = \frac{\pi (r_1^4 - r_2^4)}{2}$$



where G is shear modulus, ϕ is twist created by torque, L is length of rod

Torque diagram: move from one end to another



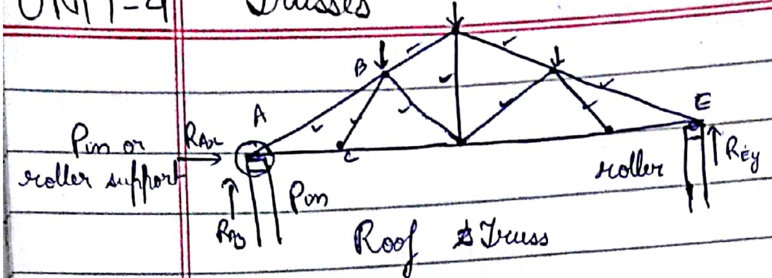


$$\Sigma M_z = 0$$

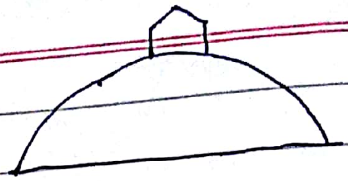
$$T_1 + T - T_2 = 0$$

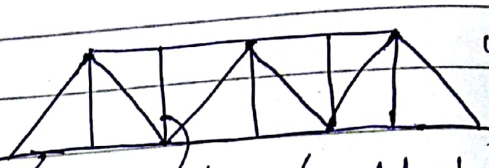
If T +ve, ϕ (+ve)

UNIT-4 Trusses



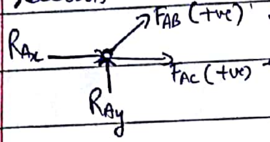
Roof Truss





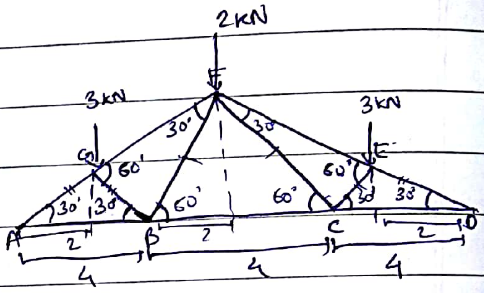
1 → Joint Method $\boxed{+ve \rightarrow Tensile}$

2 → Section method

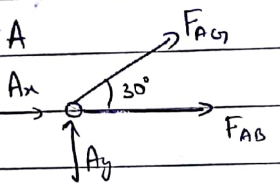


Joint names isolated

Tutorial #10



2 Joint A



$\circlearrowleft \sum M_A = 0$ $\quad - -3\times2 - 2\times6 - 3\times10 + D_y \times 12 = 0$

$D_y = 4\ kN$

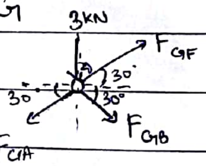
$A_y = 4\ kN$ (loading is symmetric)

$\sum F_y = 0$; $A_y + F_{AG}\sin 30' = 0$ $\qquad \sum F_x = 0$

$F_{AG} = -8\ kN$ (compressive) $\qquad A_x + F_{AB} + F_{AG}\cos 30' = 0$

$F_{AB} = 4\sqrt{3} - \cancel{A_x}^{0} = 4\sqrt{3}\ kN$ (Tensile)

Joint G



$\sum F_x = 0$

$F_{GF}\cos 30' + F_{GB}\cos 30' - F_{GA}\cos 30' = 0$

$F_{GF} + F_{GB} = F_{GA} = -8$

$\sum F_y = 0$ $\quad F_{GF}\sin 30' - 3 - F_{GB}\sin 30' - F_{GA}\sin 30' = 0$

$F_{GF} - F_{GB} = 2(3 \mp 4) = \cancel{14}\ -2$

$F_{GF} = \cancel{11}\ {-5}\ kN$ (~~Tensile~~ Compression) ; $F_{GB} = -3\ kN$ (compressive)

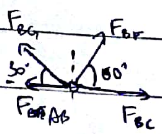
$F_{GF} + F_{GB} = -8$

$F_{GF} - F_{GB} = \cancel{14}\ 2$

$\cancel{F_{GF} = 11 kN}$

$F_{GB} = -5kN$

Joint B



$\sum F_x = 0$

$F_{BC} + F_{BF}\cos 60' - F_{BG}\sin 30' - F_{BA} = 0$

$F_{BC} + F_{BF}\cos 60' = -3 + \frac{3}{2} = -1.5$

$F_{BC} = -2.366\ kN$ (Compressive)

$\Sigma F_y = 0$, $F_{BF} \sin 60 + F_{BG} \sin 30' = 0$; $F_{BF} = \frac{-F_{BG} \sin 30'}{\sin 60'} = \frac{+3}{\sqrt{3}} = \sqrt{3}$ kN (Tensile)

; $F_{BC} = -2.366$ kN (Compressive)

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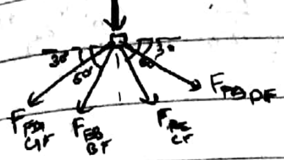
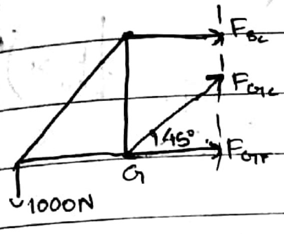
$\cancel{-F_{GF}\cos 30 - F_{BF}\cos 60' + F_{CF}\cos 60 + F_{DF}\cos 30 = 0}$

Joint F 2kN $\Sigma F_x = 0$ $\cancel{2(F_{GF}\cos}$



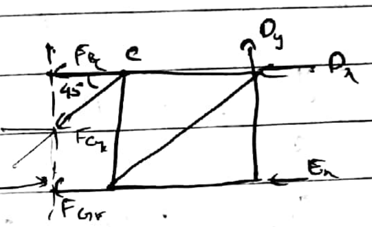
$\Sigma F_y = 0$ $-2 - 2(F_{BF} \sin 60') + F_{GF} \sin 30') = 0$

4.



Calculate moment at G

Calculate moment at C



$\circlearrowleft \Sigma M_G = 0$ $1000 \times 2 - F_{BC} \times 2 - \cancel{F_{GC} \cos 45 \times 1} + \cancel{F_{GC} \sin 45 \times 1} = 0$

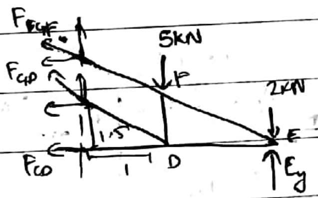
$2F_{BC} + \cancel{\sqrt{2} F_{GC}} = 2000$

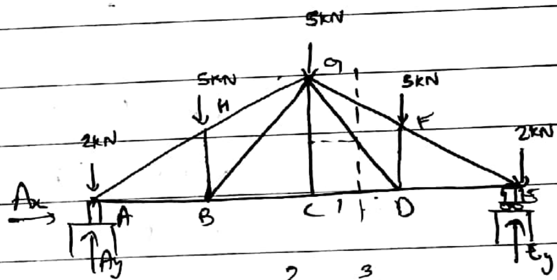
$F_{BC} = 1000\text{ N} = 1\text{ kN}$ (Tensile)

$\circlearrowleft \Sigma M_C = 0$ $2D_y - 2E_x + 2F_{GF} - \cancel{2\frac{1}{\sqrt{2}}F_{GC}} + \cancel{\frac{1}{\sqrt{2}}F_{GC}} = 0$

$D_y - E_x = F_{GF}$

5





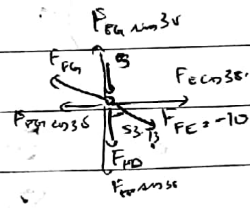
$\frac{2}{\cancel{3}1} = \frac{3}{y}$; $y = 1.5$

$\circlearrowleft \Sigma M_E = 0$

$10 - F_{GD} \sin 60' \times 3 + F_{GD} \cos 60' \times 1.5 - F_{GF} \sin 30 \times 3 + F_{GF} \cos 30 \times 3 = 0$

$\cancel{0.75 - 1.85 F_{GD} + 1.098 F_{GF} = 10}$

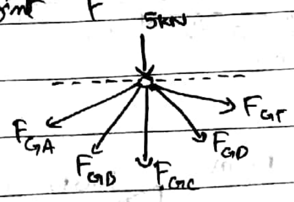
$0.6 F_{GF} + 1.3 F_{GD} = 10$



$F_{FG} = 10$

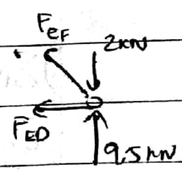
$F_{FD} = 17$

Joint F



$F_{GA} = F_{GF}$ & $F_{GB} = F_{GD}$ (by symmetry)

$\Sigma F_y = 0$



$-F_{GA} \sin 30' - F_{GB} \sin 60 - 5 - F_{GC} - F_{GD} \sin 60' - F_{GF} \sin 30 = 0$

$F_{GC} + \frac{F_{GF}}{2} + \frac{\sqrt{3} F_{GD}}{2} = -5$; $F_{GC} + F_{GF} + \sqrt{3} F_{GD} = 10$

$F_{EF} = 12.5$ kN

$F_{GC} = -10 - F_{GF} = \sqrt{3} F_{GD}$

$F_{ED} = 10$ kN

=

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