Assignment-1

Linear Algebra

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1. a. Prove that the matrix $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is involuntary matrix.

b. Express the matrix A as a sum of symmetric and skew-symmetric matrix where

$A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$.

2.. Reduce the matrix into echelon form and find its rank $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$

3. Solve the system of equations completely

$3x + 4y - z - 6w = 0, 2x + 3y + 2z - 3w = 0, 2x + y - 14z - 9w = 0$

$x + 3y + 13z + 3w = 0$

4. Determine whether the following equations have a non-trivial solution, if so solve them:

$4x + 2y + z + 3w = 0,\ 6x + 3y + 4z + 7w = 0, 2x + y + w = 0.$

5. Test for consistency and solve

$x - 2y + 3t = 2\ , 2x + y + z + t = -4, 4x - 3y + z + 7t = 8$

6. Reduce the quadratic form to a canonical form by an orthogonal reduction and discuss its nature $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$

7. Find the Characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

8. Find inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$,by using elementary row operations (Gauss Jordan method).

9. Investigate the values of λ and μ so that the equations

$2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$,

have(i) no solution (ii) unique solution(iii) an infinite number of solutions

10. A) For what values of x the matrix $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular

B) If $A+B=\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A-B=\begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$, Calculate the product AB

11.If the following system has a non-trivial solution, prove that $a+b+c=0\ or\ a=b=c$

$ax+by+cz=0, bx+cy+az=0,\ cx+ay+bz=0$

12.) If $A=\begin{bmatrix} k & 1 \\ 0 & k \end{bmatrix}$, Prove that $A^n=\begin{bmatrix} k^n & n\,k^{n-1} \\ 0 & k^n \end{bmatrix}$

13. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

14. Using Gauss-seidal method solve the system of equations

$3x+4y-z=8, -2x+y+z=3, x+2y-z=2$

15. Show that the vectors $X_1=(1,2,4), X_2=(2,-1,3),\ X_3=(0,1,2)$ and $X_4=(-3,7,2)$ are linearly dependent and find the relation between them