

## ✔ Congratulations! You passed!

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1. Given the points (1, 1), (2, 1), (4,3) and (5, 3), find the slope  $m$  and y-intercept  $c$  of the line of best fit using the vertical distance least-squares method. The estimated line parameters are: **2 / 2 points**

- ☐  $m = 5, c = 1$   
☐  $m = 6, c = 0$   
☐  $m = 0.1, c = 0.7$   
☒  $m = 0.6, c = 0.2$

✔ **Correct**

Use the closed-form expression for the least squares solution.

2. For the line formulation  $E = \frac{1}{N} \sum_i (x_i \sin \theta - y_i \cos \theta + \rho)^2$ , which is the correct equation for  $\frac{\partial E}{\partial \rho}$ ? **3 / 3 points**

- ☐  $\frac{2\rho}{N} \sum_i (x_i \sin \theta - y_i \cos \theta + \rho)$   
☐  $\frac{-2}{N} \sum_i (x_i \sin \theta - y_i \cos \theta + \rho)$   
☒  $\frac{2}{N} \sum_i (x_i \sin \theta - y_i \cos \theta + \rho)$   
☐  $\frac{-2}{N} \sum_i \rho (x_i \sin \theta - y_i \cos \theta + \rho)$

✔ **Correct**

This is a straightforward application of the chain rule, and is very similar to  $\frac{\partial E}{\partial c}$  seen in the lecture

3. Suppose we want to fit a curve to our edge points with the equation  $y = mx^5 + nx^3 + ox^2 + p$ . Which of the following is the objective function we want to minimize?

2 / 2 points

- ☐  $E = \frac{1}{N} \sum_i (y_i - mx_i^5 - nx_i^3 - ox_i^2 - p)$
- ☐  $E = \frac{1}{N} \sum_i (y_i + mx_i^5 + nx_i^3 + ox_i^2 + p)^2$
- ☒  $E = \frac{1}{N} \sum_i (y_i - mx_i^5 - nx_i^3 - ox_i^2 - p)^2$
- ☐  $E = \frac{1}{N} \sum_i (y_i - mx_i^5 - ax_i^4 - nx_i^3 - ox_i^2 - bx_i - p)^2$

☒ **Correct**

Each of the other answers have one or more errors - The first answer choice does not square the interior function, the second answer choice sums the measured and expected values of y instead of finding their difference, and the last answer choice adds two extraneous terms.

4. Given the coordinates of several points on a polynomial, the parameters of the polynomial can be found using a:

1 / 1 point

- ☐ Non-linear system of equations
- ☒ Linear system of equations
- ☐ Differential equation
- ☐ Quadratic equation

☒ **Correct**

Irrespective of the order of the polynomial, it remains linear with respect to its parameters. This makes it possible to solve for the parameters using a linear system of equations as long as the number of points given is greater than the number of parameters.

5. We want to fit a line of the form  $y = ax^2 + bx + c$  to the points (1,1), (2,2) and (3, 1). Using a linear system, what are the best-fit values of a, b and c?

3 / 3 points

- ☐  $a = 3, b = -2, c = -2$
- ☐  $a = -1, b = -4, c = 5$

☒  $a = -1, b = 4, c = -2$

☐  $a = 1.5, b = 1, c = 0$

☒ **Correct**

We find the values by solving the linear system below. Note that since the matrix is invertible, we do not have to use a pseudo-inverse: we simply left-multiply the vector on the right-hand side of the equation with the matrix inverse.

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.5 & -1 & 0.5 \\ -2.5 & 4 & -1.5 \\ 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix}$$

6.

What is the pseudo-inverse of the following matrix:  $\begin{bmatrix} 2 & 4 \\ 9 & 3 \\ 5 & 1 \end{bmatrix}$

3 / 3 points

☐  $\begin{bmatrix} 110 & 40 \\ 40 & 26 \end{bmatrix}$

☐  $\begin{bmatrix} 13/360 & -2/63 \\ -2/63 & 11/126 \end{bmatrix}$

☒  $\begin{bmatrix} -3/35 & 19/210 & 1/14 \\ 2/7 & -1/42 & -1/14 \end{bmatrix}$

☐  $\begin{bmatrix} -6 & 6 & 5 \\ 20 & -1 & -5 \end{bmatrix}$

☒ **Correct**

The formula for the pseudo-inverse is  $(X^T X)^{-1} X^T$ , thus for the matrix in question:

$$\left( \begin{bmatrix} 2 & 9 & 5 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 9 & 3 \\ 5 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 2 & 9 & 5 \\ 4 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3/35 & 19/210 & 1/14 \\ 2/7 & -1/42 & -1/14 \end{bmatrix}$$

7. Given a continuous curve  $\mathbf{v}(s) = (x(s), y(s))$ , it can be made smooth by minimizing:

1 / 1 point

- ☐  $\left\| \frac{dx}{dy} \right\|^2$
- ☐  $\left\| \frac{d^2x}{dy^2} \right\|^2$
- ☐  $\left\| \frac{d\mathbf{v}}{ds} \right\|^2$
- ☒  $\left\| \frac{d^2\mathbf{v}}{ds^2} \right\|^2$

✓ **Correct**

This term is high when there is a sharp turn in the curve. Minimizing it ensures that the curve remains as smooth as possible.

8. Given a point  $(x_i, y_i)$  on a continuous curve  $\mathbf{v}$ , how many neighboring points (in addition to the point itself) do you need to approximate the elasticity of the curve at  $(x_i, y_i)$  using finite differences?

0 / 1 point

- ☐ 1
- ☐ 2
- ☒ 3
- ☐ 4

✗ **Incorrect**

9. Which of the following statements about accumulator arrays for line detection is false?

0 / 1 point

- ☐ Voting over a patch of cells makes up for inaccurate edge detection.
- ☐ A small cell size may reduce the effect of noise but causes a computational blowup.
- ☐ A large cell size may merge close but distinct lines.

- ☒ In a perfect accumulator array, the number of lines is equivalent to the number of peaks in the array.

☐ **Incorrect**

10. What defines the direction of an edge  $\phi_i$  at point  $i$  in the generalized Hough transform?

1 / 1 point

- ☐ Angle formed by line to the reference point
- ☐ Both the angle formed by line to the reference point and the distance to it
- ☐ Tangent to the edge at point  $i$
- ☒ Normal to the edge at point  $i$

☒ **Correct**

Following the definition laid out in lecture, we use the normal, not the tangent, to define direction. The first answer choice and the second answer choice refer to features used to determine location with respect to the reference point, not direction.