

## ✔ Congratulations! You passed!

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To pass 70% or higher

**Go to next item**

1. Which of the following is the expression for the Fourier Transform?

**1 / 1 point**

- ☐  $F(u) = \int_0^{\infty} f(x)e^{-i2\pi ux} dx$
- ☐  $F(u) = \int_{-\infty}^{\infty} f(x)e^{-\frac{i2\pi u}{x}} dx$
- ☐  $F(u) = \int_{-\infty}^{\infty} f(x+u)e^{-i2\pi ux} dx$
- ☒  $F(u) = \int_{-\infty}^{\infty} f(x)e^{-i2\pi ux} dx$

✔ **Correct**

See definition of the Fourier Transform.

2. Which of the following must be a property of functions for which Fourier's Theorem applies?

**1 / 1 point**

- ☐ Bounded
- ☒ Periodic
- ☐ Differentiable
- ☐ Continuous

✔ **Correct**

Fourier's Theorem applies to periodic functions which are reasonably continuous. This includes discontinuous functions with a finite number of discontinuities and unbounded functions (see Dirichlet Conditions for further reading).

3. What is the expression for the frequency of the following function?

2 / 2 points

$$f(x) = B \sin \left( \frac{x}{T} + \psi \right)$$

☐  $\frac{1}{T}$

☐  $T$

☐  $\frac{2\pi}{T}$

☒  $\frac{1}{2\pi T}$

✓ **Correct**

The expression for the frequency of a sinusoidal expression is given as the coefficient of the angle divided by  $2\pi$ .

4. What is the expression for the phase shift of the following function with respect to  $f(x) = \sin(x)$ ?

3 / 3 points

$$g(x) = A \cos(\pi u + \psi)$$

$$x = \pi u$$

☐  $\psi$

☐  $\frac{\psi}{2}$

☒  $\psi - \frac{\pi}{2}$

☐  $\frac{\psi + \pi}{2}$

✓ **Correct**

$g(x)$  can be rewritten as follows:

$$g(x) = A \cos(\pi u + \psi)$$

$$= A \sin \left( \pi u + \left( \psi - \frac{\pi}{2} \right) \right).$$

From this the phase shift can be read off as  $\psi - \frac{\pi}{2}$ .

5. Recall that the Fourier transformation of a standard Gaussian is  $\sqrt{\pi}e^{-\pi^2 u^2}$ . 2 / 2 points

What would be the Fourier transformation of the second derivative of the Gaussian?

- ☒  $-4\pi^{5/2}u^2e^{-\pi^2 u^2}$
- ☐  $2\pi^{3/2}iu^2e^{-\pi^2 u^2}$
- ☐  $2\pi^{5/2}(2\pi^2 u^2 - 1)e^{-\pi^2 u^2}$
- ☐  $-2\pi^{3/2}iue^{-\pi^2 u^2}$

☒ **Correct**

Recall that the Fourier transform of the  $n$ th derivative of a function  $f$  is given by

$(i2\pi u)^n F$ , where  $F$  is the Fourier transform of  $f$ . If we apply this property we get the above result.

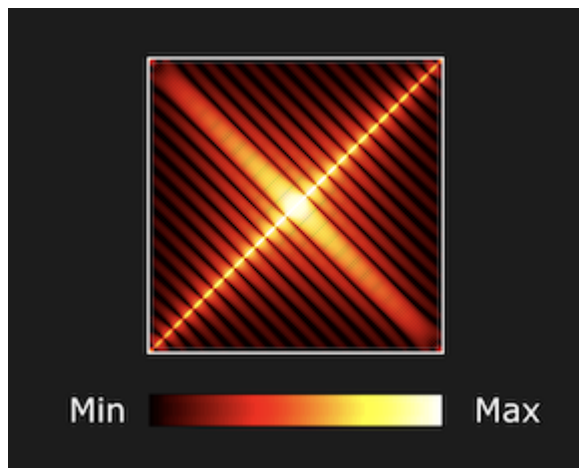
6. Which of the following is not a property of the Fourier transform? 1 / 1 point

- ☐  $\alpha f(x) + \beta g(x) \iff \alpha F(u) + \beta G(u)$
- ☐  $f(ax) \iff \frac{1}{|a|} F\left(\frac{u}{a}\right)$
- ☐  $\frac{d^n}{dx^n} f(x) \iff (i2\pi u)^n F(u)$
- ☒  $h(x) = f(x) \cdot g(x) \iff H(u) = F(u) \cdot G(u)$

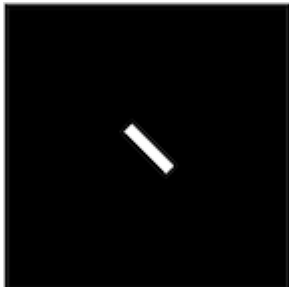
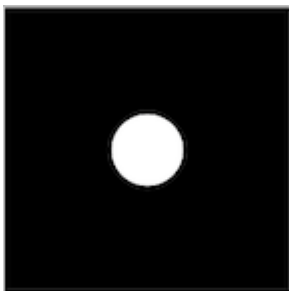
☒ **Correct**

The first answer choice represents the linearity property. The second answer choice is the scaling property. The third answer choice is the differentiation property. The last answer choice is not valid.

7. 2 / 2 points



The magnitude of the Fourier transform of an image is shown above. Which of these images is it the Fourier transform of?



**Correct**

This object has four sets of strong edges (4 sides of the rectangle). That is, all the edges are aligned with two directions, where the directions are perpendicular to each other. This manifests as two lines of non-zero frequencies in the Fourier transform.

8. If the Fourier transformation of a function  $f$  is ever 0 then convolving a signal with  $f$  is irreversible.

**0 / 1 point**

True



False

**Incorrect**

9.

1 / 1 point

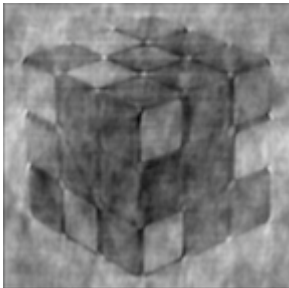
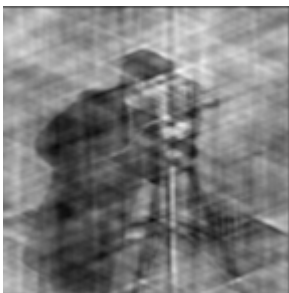


Image 1



Image 2

If the magnitude of the frequencies in image 1 were combined with the phases of the frequencies in image 2, which of these options would it most closely resemble?

☐☒☐

**Correct**

The phases of the frequencies have more of an impact on an object's recognizability than the magnitudes. Changing the magnitudes will, however, affect the saturation of various parts of the image.

10. The maximum frequency in a continuous (optical) image is  $u_{\max}$ . To recover the continuous signal from its discrete samples, the sampling frequency  $f_s$  must satisfy:

**2 / 2 points**

☒  $f_s \geq 2u_{\max}$

☐  $f_s \geq u_{\max}$

☐  $f_s \leq \frac{u_{\max}}{2}$

☐  $f_s \geq u_{\max}^2$

**Correct**

This result follows directly from the Nyquist Theorem which states that to be able to fully recover a continuous signal, it must be sampled at a frequency that is at least twice the maximum frequency of the signal.