30820-Communication Systems

Week 9-11 – Lecture 22-30

(Ref: Chapter 5 of text book)

ANGLE MODULATION AND DEMODULATION

Contents

AM vs. Angle Modulation

- Amplitude Modulation
  - Amplitude modulation is linear.
    - It just moves the signal to a new frequency band.
    - Spectrum shape does not change.
    - No new frequencies are generated.
  - Spectrum: $\varphi(w)$ is a translated version of $M(w)$
  - Bandwidth $\leq\ 2B$

- Angle Modulation
  - They are nonlinear.
    - Spectrum shape does change.
    - New frequencies generated.
  - $\varphi(w)$ is not just a translated version of $M(w)$
  - Bandwidth is usually much larger than $2B$

Instantaneous Frequency

- Consider a generalized sinusoid signal $\varphi(t)$

$$\varphi(t) = A\cos\theta(t)$$

- By definition a sinusoidal signal has a constant frequency and phase

$$A\cos(w_c t + \theta_0)$$

- A hypothetical case general angle of $\theta(t)$ happens to be tangential to the angle $(w_c t + \theta_0)$ at point t.
- Over the interval Δt i.e., $t_1 < t < t_2$

$$\varphi(t) = A\cos(w_c t + \theta_0)$$

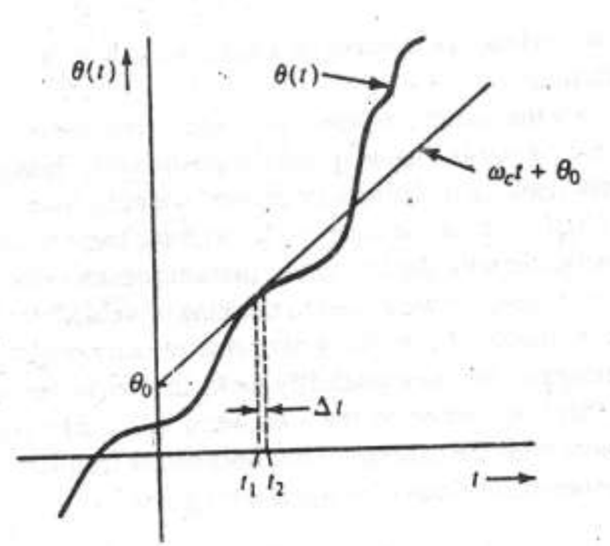
- The angular frequency of $\varphi(t)$ is w_c
- The instantaneous frequency w_i of $\varphi(t)$ is

$$w_i(t) = \frac{d\theta}{dt}$$

i.e., the slope of $\theta(t)$ at t

- The generalized angle can be expressed as

$$\theta(t) = \int_{-\infty}^{t} w_i(\alpha)d\alpha$$



Phase Modulation (PM)

- We can transmit the information of $m(t)$ by varying the angle θ of the carrier.
  - Phase Modulation (PM)
  - Frequency Modulation (FM)
- In phase modulation (PM), the angle $\theta(t)$ is varied linearly with $m(t)$ i.e.,

$$\theta(t) = w_c t + \theta_0 + k_p m(t)$$

  - Where k_p is a constant and w_c is the carrier frequency. Assume $\theta_0 = 0$.
- The resulting PM wave is represented as

$$\varphi_{PM}(t) = A\cos[w_c t + k_p m(t)]$$

- The instantaneous frequency in this case is given by

$$w_i(t) = \frac{d\theta}{dt} = w_c + k_p \dot{m}(t)$$

Frequency Modulation (FM)

- In PM the instantaneous angular frequency w_i varies linearly with the derivative of $m(t)$.
- In frequency modulation (FM), w_i is varied linearly with $m(t)$. Thus

$$w_i(t) = w_c + k_f m(t)$$

  - Where k_f is a constant.
- The angle $\theta(t)$ can now be expressed as

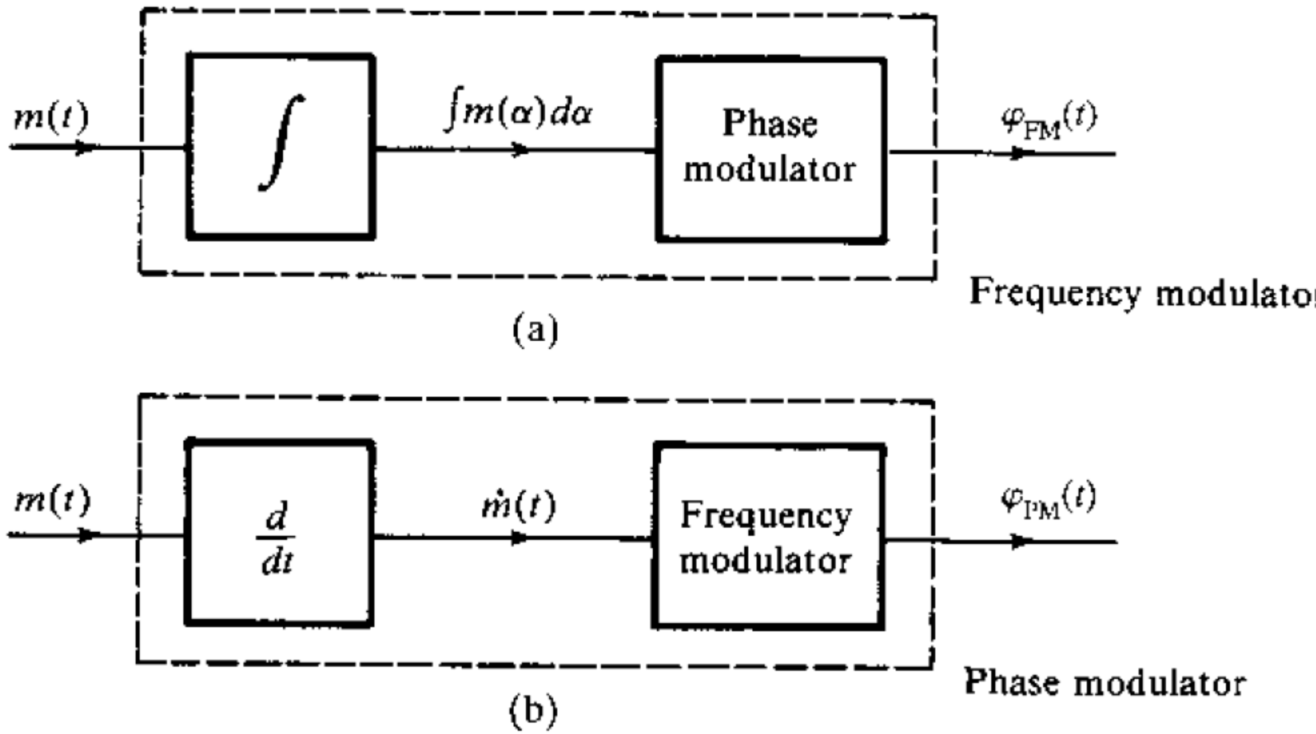
$$\theta(t) = \int_{-\infty}^{t} [w_c + k_f m(\alpha)]\, d\alpha = w_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha$$

- The resulting FM wave is

$$\varphi_{FM}(t) = A\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right]$$

Phase and Frequency Modulator

$$\varphi_{FM}(t) = A\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$



$$\varphi_{PM}(t) = A\cos[w_c t + k_p m(t)]$$

$$w_i(t) = \frac{d\theta}{dt} = w_c + k_p \dot{m}(t)$$

Power of an Angle Modulated wave

- A general angle modulated waveform can be represented as

$$\varphi(t) = A\cos\theta(t)$$

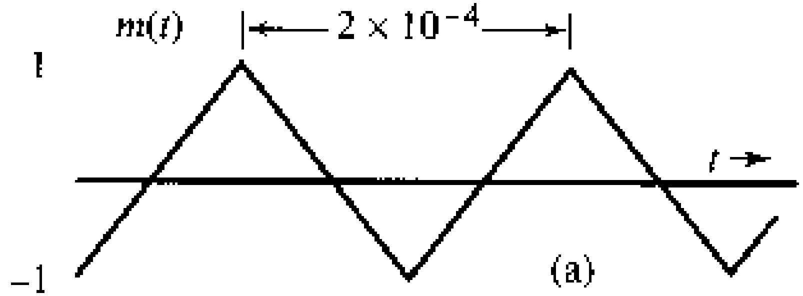
- Instantaneous phase and frequency vary with time, but amplitude remains constant

- Thus, the power of the angle modulated waveform is always

$$P_\varphi = \frac{A^2}{2}$$

Example

- Consider a modulating waveform $m(t)$



$$slope = \pm\frac{2}{0.0001} = \pm 20{,}000$$

- Determine the corresponding PM and FM waveforms for
  - $k_f = 2\pi \times 10^4$
  - $k_p = 10\pi$
  - $\theta_0 = 0$
  - $f_c = 100MHz$

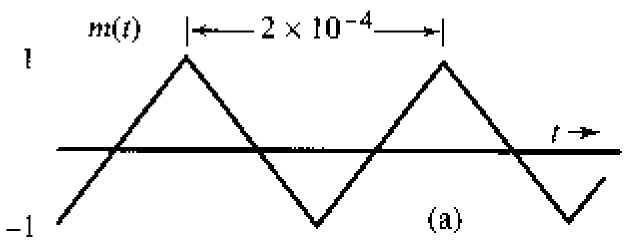
Example

- For PM wave

$$\varphi_{PM}(t) = A\cos[w_c t + k_p m(t)]$$

- The instantaneous frequency in this case is given by

$$w_i(t) = \frac{d\theta}{dt} = w_c + k_p \dot{m}(t)$$



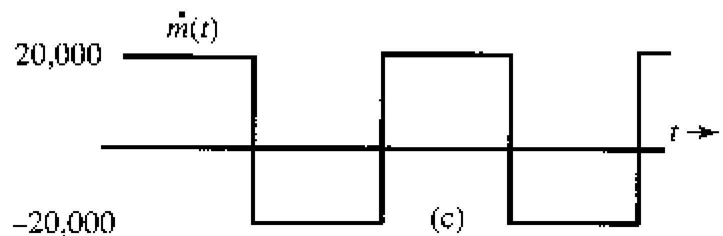
(a)

$$f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t)$$

$$= 10^8 + 5\,\dot{m}(t)$$

$$(f_i)_{\min} = 10^8 + 5\,[\dot{m}(t)]_{\min} = 10^8 - 10^5 = 99.9\text{ MHz}$$

$$(f_i)_{\max} = 10^8 + 5\,[\dot{m}(t)]_{\max} = 100.1\text{ MHz}$$



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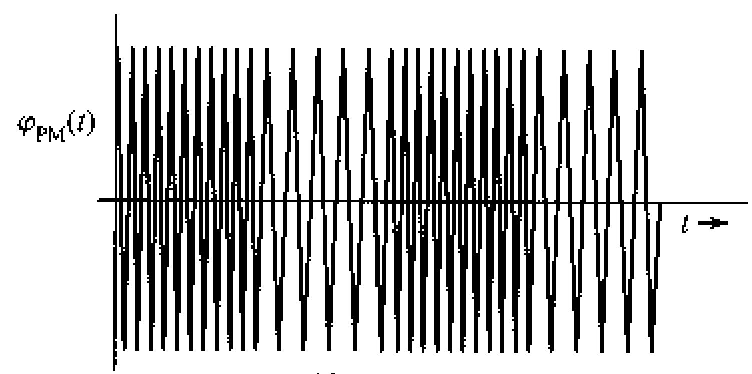


(d)

Example

- For FM wave

$$\varphi_{FM}(t) = A\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$

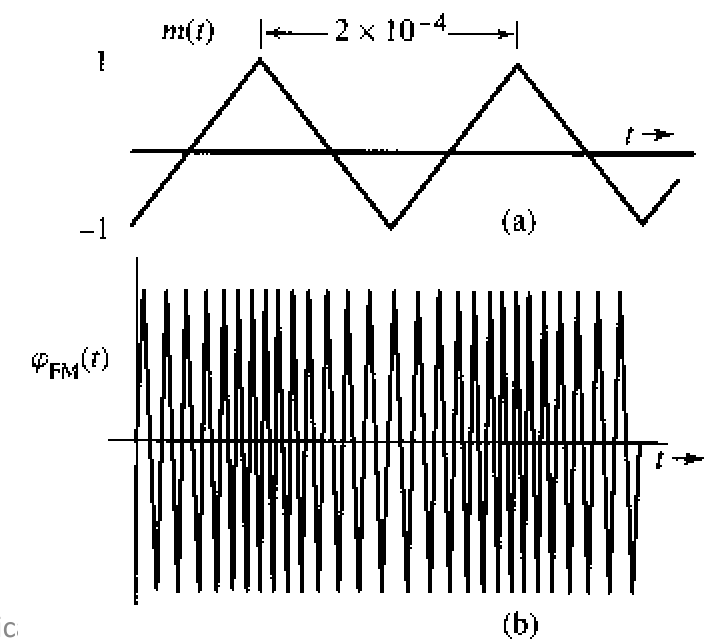
- The instantaneous frequency in this case is given by

$$w_i(t) = w_c + k_f m(t)$$

$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$
$$= 10^8 + 10^5 m(t)$$

$$(f_i)_{\min} = 10^8 + 10^5\,[m(t)]_{\min} = 99.9\ \text{MHz}$$

$$(f_i)_{\max} = 10^8 + 10^5\,[m(t)]_{\max} = 100.1\ \text{MHz}$$



Bandwidth of Angle Modulated Waves

- Angle modulation is nonlinear
  - No properties of Fourier transform can be directly applied
- To determine the bandwidth of FM waves, define

$$a(t) = \int_{-\infty}^{t} m(\alpha)d\alpha$$

and define

$$\hat{\varphi}_{FM} = Ae^{j[w_c t + k_f a(t)]} = Ae^{jw_c t}e^{jk_f a(t)}$$

- Its relation to FM signal is

$$\varphi_{FM} = Re\{\hat{\varphi}_{FM}\}$$

Bandwidth of Angle Modulated Waves

- Expanding $e^{jk_f a(t)}$ in power series

$$\hat{\varphi}_{FM} = A[1 + jk_f a(t) - \frac{k^2{}_f}{2!} a^2(t) + \cdots + j^n \frac{k^n{}_f}{n!} a^n(t) + \cdots]e^{jw_c t}$$

- Since, $\varphi_{FM} = Re\{\hat{\varphi}_{FM}\}$

$$\varphi_{\mathrm{FM}}(t) = A_c\left[\cos\omega_c t - K_f \boxed{a(t)} \sin\omega_c t - \frac{K_f^2}{2!} \boxed{a^2(t)} \cos\omega_c t - \frac{K_f^3}{3!} \boxed{a^3(t)} \sin\omega_c t + \cdots\right]$$

$a(t)$ is bandlimited to B_m Hz

$a^2(t)$ is bandlimited to $2B_m$ Hz

$a^3(t)$ is bandlimited to $3B_m$ Hz

The FM signal $\varphi_{\mathrm{FM}}(t)$ is **not bandlimited** ! ...

$\cdots$

$a^n(t)$ is bandlimited to nB_m Hz

Narrowband FM

- The bandwidth of FM signal is theoretically infinite. However, practical signals are always finite in bandwidth.
  - Reason?

$$\frac{k^n{}_f}{n!} a^n(t) \simeq 0 \text{ for large } n$$

- If $\left|k_f a(t)\right| \ll 1$, then

$$\varphi_{FM} = A[\cos w_c t - k_f a(t) \sin w_c t - \frac{k^2{}_f}{2!} a^2(t) \cos w_c t + \frac{k^3{}_f}{3!} a^3(t) + \cdots]$$

Will reduce to

$$\varphi_{FM} \simeq A[\cos w_c t - k_f a(t) \sin w_c t]$$

- This case is called narrow-band FM.
- Similarly, narrow-band PM is given by

$$\varphi_{PM} \simeq A[\cos w_c t - k_p m(t) \sin w_c t]$$

Comparison of Narrowband FM with AM

- Narrow-band FM

$$\varphi_{FM} \simeq A[\cos w_c t - k_f a(t) \sin w_c t]$$

- Full AM

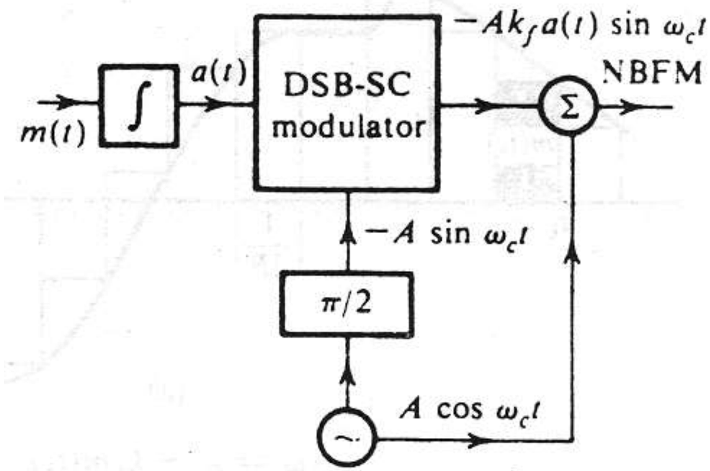
$$\varphi_{AM} = [A + m(t)] \cos w_c t = A \cos w_c t + m(t) \cos w_c t]$$

- Narrow-band FM and full AM require a transmission bandwidth equal to $2B\ Hz$.
- The sideband spectrum for FM has a phase shift of ${}^{\pi}/_{2}$ with respect to the carrier, whereas that of AM is in phase with the carrier.
  - Despite the similarities, AM and FM have different waveforms.
- The above equation suggests a way to generate narrowband FM or PM signals by using DSB-SC modulators.

Narrow Band Angle Modulation Generation

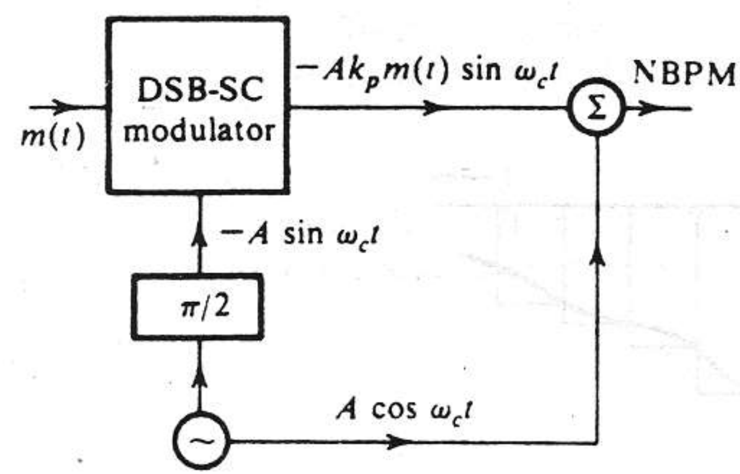
- Narrow-band FM

$\varphi_{FM} \simeq A[\cos w_c t - k_f a(t) \sin w_c t]$



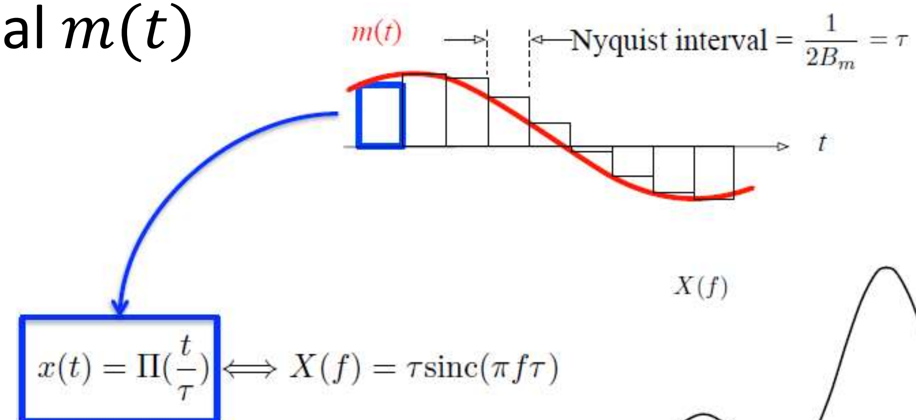
- Narrow-band PM

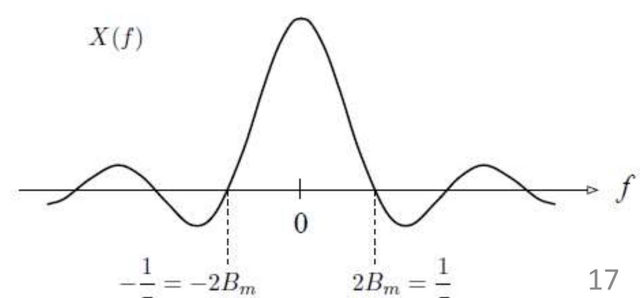
$\varphi_{PM} \simeq A[\cos w_c t - k_p m(t) \sin w_c t]$



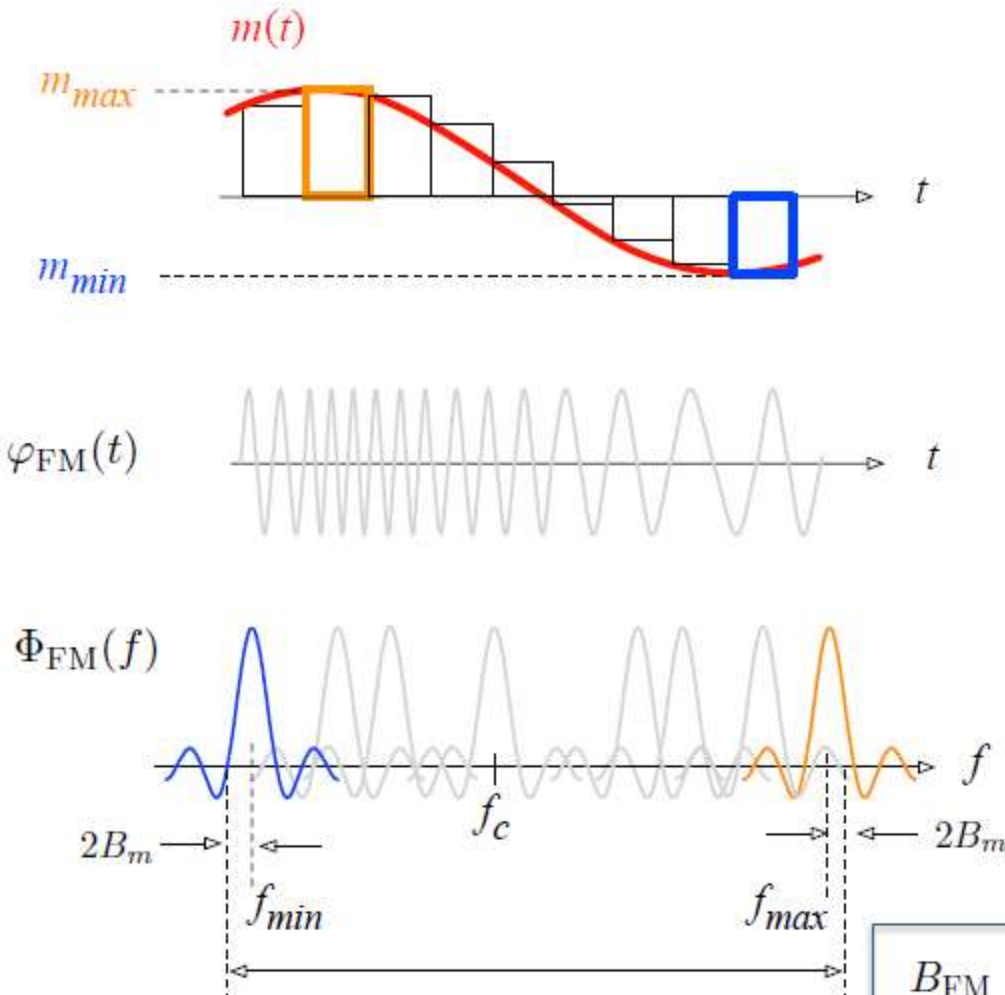
Wideband FM

- FM signal is more meaningful only if the frequency deviation is large enough.
  - We select k_f large enough so that $|k_f a(t)| \ll 1$ is not satisfied.
  - In such case it is called Wideband FM (WBFM).
  - We cannot ignore all the higher order terms.
- To determine the bandwidth of WBFM, consider the modulating signal $m(t)$





WBFM: Bandwidth Estimation



$$\boxed{B_{\text{FM}} \approx [f_{max} - f_{min}] + 2(2B_m)}$$

WBFM: Bandwidth Estimation

- As we know

$$f_{min} = f_c + \frac{k_f}{2\pi}\min[m(t)] = f_c - \frac{k_f}{2\pi}m_p \qquad f_{max} = f_c + \frac{k_f}{2\pi}\max[m(t)] = f_c + \frac{k_f}{2\pi}m_p$$

$$B_{FM} \approx [f_{max} - f_{min}] + 2(2B_m)$$

$$= 2\,\frac{k_f}{2\pi}m_p + 2(2B_m)$$

$$B_{FM} = 2(\Delta f + 2B_m)$$

where $\Delta f = \frac{k_f}{2\pi}m_p$

- This approximation is not a very good one,
  - In NBFM case when $\Delta f \approx 0$, $B_{NBFM} = 2B_m$
- A better approximation is given by **Carson's Rule**

$$B_{FM} \approx 2(\Delta f + B_m)$$

- Let $\beta = \frac{\Delta f}{B_m}$ defines the deviation ratio (a.k.a., modulation index)

$$B_{FM} \approx 2B_m(1 + \beta)$$

- For PM signal the bandwidth estimate is the same as in case of FM with $\Delta f = \frac{k_p}{2\pi}\dot{m}_p$

FM: Spectral Analysis of Tone Frequency Modulation

- Determining a closed-form expression for the spectrum of signal $\varphi_{FM}(t)$ for arbitrary modulation/deviation index values is not possible.
- Therefore we will investigate the single-tone modulation case:

$$m(t) = \alpha \cos w_m t$$

- Since we know that, $a(t) = \int_{-\infty}^{t} m(\alpha) d\alpha$

$$a(t) = \frac{\alpha}{w_m} \sin w_m t$$

- As we know that

$$\hat{\varphi}_{FM} = Ae^{j[w_c t + k_f a(t)]}$$

$$= Ae^{j[w_c t + k_f \frac{\alpha}{w_m} \sin w_m t]}$$

FM: Spectral Analysis of Tone Frequency Modulation

- As we have already established that

$$\Delta w = k_f m_p = \alpha k_f$$

and the bandwidth of $m(t)$ is $f_m\ Hz$ or $2\pi f_m$rad/sec

- The deviation ratio will now be

$$\beta = \frac{\Delta w}{2\pi f_m} = \frac{\alpha k_f}{w_m}$$

- Hence, we can write

$$\hat{\varphi}_{FM} = Ae^{j[w_c t + \beta \sin w_m t]}$$
$$= Ae^{j w_c t} e^{j\beta \sin w_m t}$$

- Note that the $e^{j\beta \sin w_m t}$ is a periodic signal with period $\frac{2\pi}{w_m}$
  - It can be expand using exponential Fourier series.

FM: Spectral Analysis of Tone Frequency Modulation

$$e^{j\beta \sin w_m t} = \sum_{n=-\infty}^{\infty} D_n e^{jnw_m t}$$

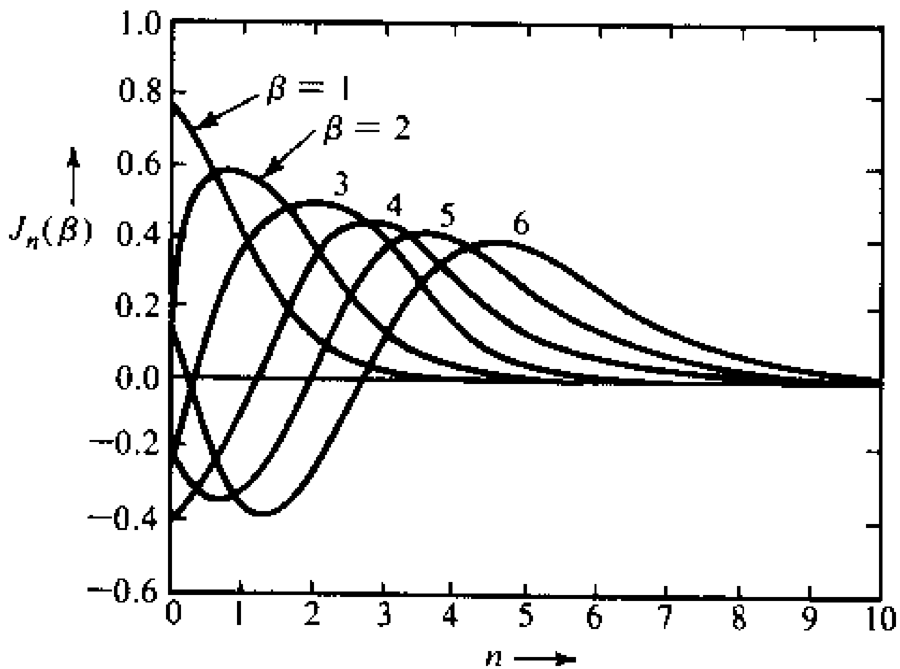
where

$$D_n = \frac{w_m}{2\pi} \int_{-\pi/w_m}^{\pi/w_m} e^{j\beta \sin w_m t}\, e^{-jnw_m t} dt = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

- The integral on the right hand side cannot be evaluated in closed form.
  - It must be integrated by expanding the integrand in infinite series.
- This integral has been extensively tabulated and denoted by $J_n(\beta)$, i.e., the Bessel function of the first kind and the n^{th} order.

FM: Spectral Analysis of Tone Frequency Modulation

- The function is plotted as a function of n for various values of β as follows



$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\, dx$$

FM: Spectral Analysis of Tone Frequency Modulation

$$e^{j\beta \sin w_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jnw_m t}$$

- As $\hat{\varphi}_{FM} = Ae^{jw_c t} e^{j\beta \sin w_m t}$ so

$$\hat{\varphi}_{FM} = A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jnw_m t}\, e^{jw_c t}$$

$$= A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(w_c t + nw_m t)}$$

- Since, $\varphi_{FM} = Re\{\hat{\varphi}_{FM}\}$

$$\varphi_{FM} = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(w_c + nw_m)t$$

FM: Spectral Analysis of Tone Frequency Modulation

- A tone modulated FM signal has a carrier component and infinite number of sidebands of frequencies $w_c \pm w_m$, $w_c \pm 2w_m$, $w_c \pm 3w_m$,…
  - The strength of n^{th} sideband is $J_n(\beta)$
  - $J_n(\beta)$ is negligible for $n > \beta + 1$

- Hence, the number of significant sideband impulses is $\beta + 1$
- The bandwidth of the FM carrier is given by

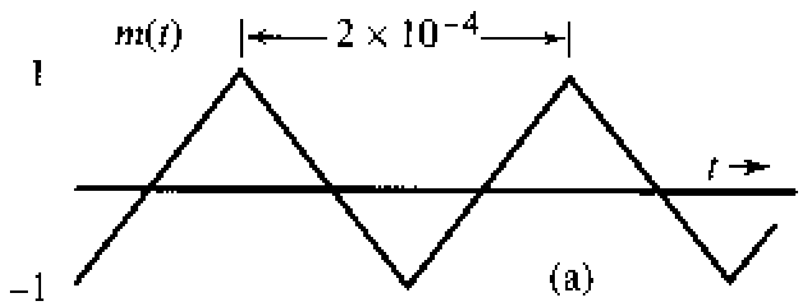
$$B_{FM} = 2(\beta + 1)f_m$$
$$= 2(\Delta f + B)$$

where $\beta = \frac{\Delta f}{f_m}$

- This verifies Carson's formula.

Example (1)

- For the following modulating signal $m(t)$ for $k_f = 2\pi \times 10^5$ and $k_p = 5\pi$



$$slope = \pm\frac{2}{0.0001} = \pm 20{,}000$$

- What will be the essential bandwidth of the signal $m(t)$?
- Estimate B_{FM} and B_{PM}?
- What will happen if the amplitude of $m(t)$ is doubled?

Example (1)

- Fourier series of $m(t)$ can be found out as

$$m(t) = \sum_{n} C_n \cos n w_0 t$$

where $w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\times 10^{-4}} = 10^4\pi$ and $C_n = \begin{cases} \frac{8}{\pi^2 n^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$

- Note that 3rd harmonic is only 11% of the fundamental and 5th harmonic is only 4% of the fundamental.
  - 3rd harmonic power is 1.21% and 5th harmonic is 0.16%
- It is justified to say that essential bandwidth of $m(t)$ is within its 3rd harmonic i.e.,

$$B_m = 3f_m = 15KHz$$

Example (1)

- For FM:

$$\Delta f = \frac{k_f m_p}{2\pi} = 100KHz$$

So

$$B_{FM} = 2(\Delta f + B_m) = 230KHz$$

Alternatively,

$$\beta = \frac{\Delta f}{B_m} = 6.667$$

So

$$B_{FM} = 2B_m(1 + \beta) = 230KHz$$

- Doubling the amplitude of $m(t)$
  - $m_p = 2$
  - Frequency deviation Δf is doubles

Example (1)

- For PM:

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = 50KHz$$

So

$$B_{PM} = 2(\Delta f + B_m) = 130KHz$$

Alternatively,

$$\beta = \frac{\Delta f}{B_m} = 3.334$$

So

$$B_{PM} = 2B_m(1 + \beta) = 130KHz$$

- Doubling the amplitude of $m(t)$
  - $\dot{m}_p$ is doubled
  - Frequency deviation Δf is doubles

Example (2)

An angle modulated signal with carrier frequency $w_c = 2\pi \times 10^5$ is described as

$$\varphi_{EM}(t) = 10\cos(w_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

- Find the power of the modulating signal
- Find the frequency deviation Δf
- Find the deviation ratio β
- Find the phase deviation $\Delta\varphi$
- Estimate the bandwidth of $\varphi_{EM}(t)$

Example (2)

- Find the power of the modulating signal?
  - The carrier amplitude is 10, and the power is

$$P = \frac{A^2}{2} = \frac{10^2}{2} = 50$$

- To find frequency deviation Δf,
  - Find the instantaneous frequency w_i i.e.,

$$w_i = \frac{d\theta}{dt}$$

$$w_i = \frac{d\theta}{dt} = \frac{d(w_c t + 5\sin 3000t + 10\sin 2000\pi t)}{dt}$$
$$= w_c + 15000\cos 3000t + 20000\pi\cos 2000\pi t$$

- The carrier deviation is

$$\Delta w = 15000\cos 3000t + 20000\pi\cos 2000\pi t$$

Example (2)

- The two sinusoid will add in phase at some point and the maximum value will be

$$\Delta f = \frac{\Delta w}{2\pi} = \frac{15000 + 20000\pi}{2\pi} = 12{,}387.32 Hz$$

- The deviation ratio β

$$\beta = \frac{\Delta f}{B}$$

  – The signal bandwidth is the highest frequency in $m(t)$

$$\varphi_{EM}(t) = 10\cos(w_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

$$B = \frac{2000\pi}{2\pi} = 1000 Hz$$

So

$$= \frac{12387.32}{1000} = 12.387$$

Example (2)

- Find the phase deviation $\Delta\varphi$?
  - The angle $\theta(t)$ can be expressed as

  $$\theta(t) = w_c t + (5\sin 3000t + 10\sin 2000\pi t)$$

  - The phase deviation is the maximum value of the angle inside the parenthesis

  $$\Delta\varphi = 5 + 10 = 15\ \mathrm{rad}$$

- The estimated bandwidth of $\varphi_{EM}(t)$

$$B_{EM} = 2(\Delta f + B)$$
$$= 2(12{,}387.32 + 1000)$$
$$= 26{,}774.65\ Hz$$

Generating FM Waves

- There are two ways of generating FM waves
  - Indirect Method
  - Direct Method
- We first discuss the NBFM generator that is utilized in the indirect FM generation of WBFM signals.
  - Generate NBFM first, then NBFM is frequency multiplied for targeted Δf.
- Good for the requirement of stable carrier frequency
- Commercial-level FM broadcasting equipment all use indirect FM
- A typical indirect FM implementation: Armstrong FM

Narrow Band Angle Modulation Generation

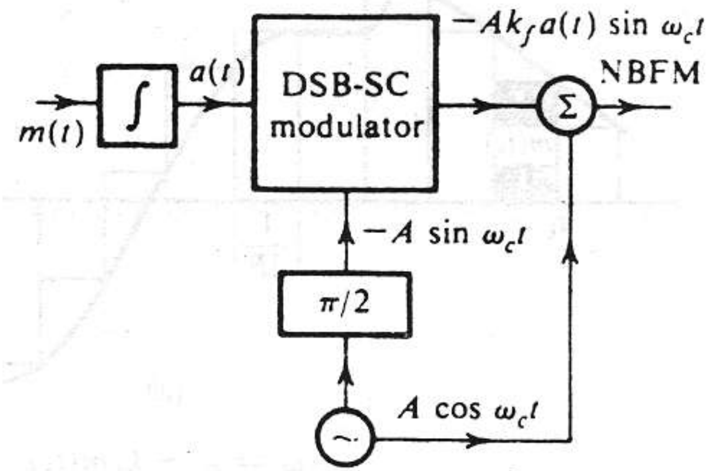
- Narrow-band FM: $\left|k_f a(t)\right| \ll 1$

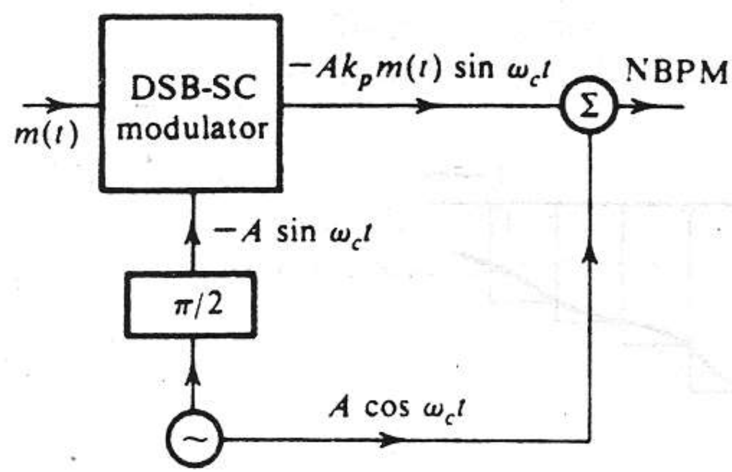
  $\varphi_{FM} \simeq A[\cos w_c t - k_f a(t) \sin w_c t]$

- Narrow-band PM: $\left|k_p m(t)\right| \ll 1$

  $\varphi_{PM} \simeq A[\cos w_c t - k_p m(t) \sin w_c t]$

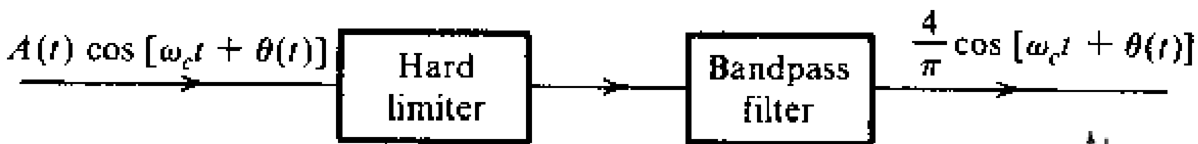
- The NBFM generation in this way will have some distortion because of the approximation.
  - Output of the modulator has some amplitude variations.
- A nonlinear device designed to limit the amplitude of a bandpass signal can remove this distortion.



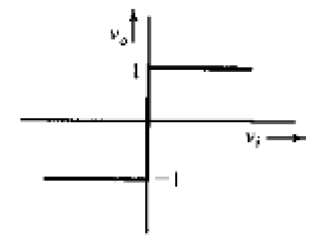


Bandpass Limiter

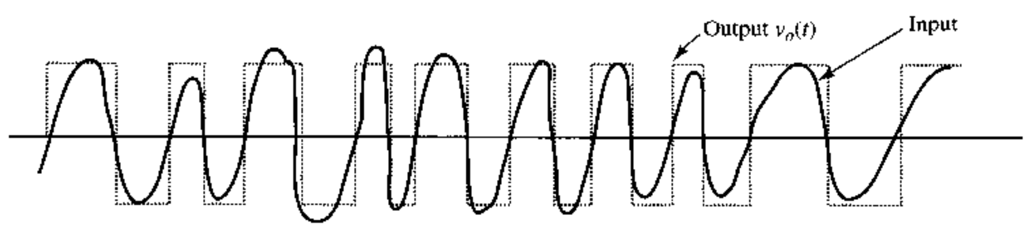
- The amplitude variations of an angle modulated carrier can be eliminated by a bandpass limiter.
  - It consists of a hard limiter and a bandpass filter



- The input-output characteristics of hard limiter is



- The output of the bandpass limiter to a sinusoid will be a square wave of unit amplitude. Moreover, the zero crossings are preserved.



- Thus an angle modulated sinusoid input $v_i(t) = A(t) \cos \theta(t)$ results in a constant amplitude , angle modulated square wave $v_o(t)$

Bandpass Limiter

- When $v_o(t)$ is passed through a bandpass filter centered at w_c, the output is a angle modulated wave of constant amplitude .
- Consider the incoming angle-modulated wave as

$$v_i(t) = A(t)\cos\theta(t)$$

where $\theta(t) = w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha$

- The output of the hard limiter is either +1 or -1, depending on the signal $v_i(t)$ being positive or negative.

$$v_o(t) = \begin{cases} +1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$

- Note that v_o as a function of θ is a periodic square wave function with period 2π

Bandpass Limiter

- We can expand $v_o(\theta)$ using Fourier series as follows

$$v_o(\theta) = \frac{4}{\pi}\left(\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta + \cdots\right)$$

- At any time t, $\theta = w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha$

$$v_o[\theta(t)]$$
$$= \frac{4}{\pi}\left\{\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right] - \frac{1}{3}\cos 3\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]\right.$$

Indirect Method: Armstrong

- The NBFM is converted to WBFM by using additional frequency multipliers.
- A frequency multiplier can be recognized by a nonlinear device followed by a bandpass filter.
- Consider a non-linear device whose output signal $y(t)$ to input signal $x(t)$ is given by

$$y(t) = a_2 x^2(t)$$

- If the FM signal is passed through this device, the output signal will be

$$y(t) = a_2 \cos^2\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$

$$= \frac{a_2}{2} + \frac{a_2}{2}\cos\left[2w_c t + 2k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$

- A bandpass filter centered at $2w_c$ will recover the FM signal with twice the original instantaneous frequency.

Indirect Method: Armstrong

- To generalize this concept, consider a nonlinear device characterized by

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \cdots + a_n x^n(t)$$

- If $x(t) = A\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$, then by using trigonometric identities, we can show that

$$y(t) = c_0 + c_1 \cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right] + c_2 \cos\left[2w_c t + 2k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right] + \cdots + c_n \cos\left[n w_c t + n k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$

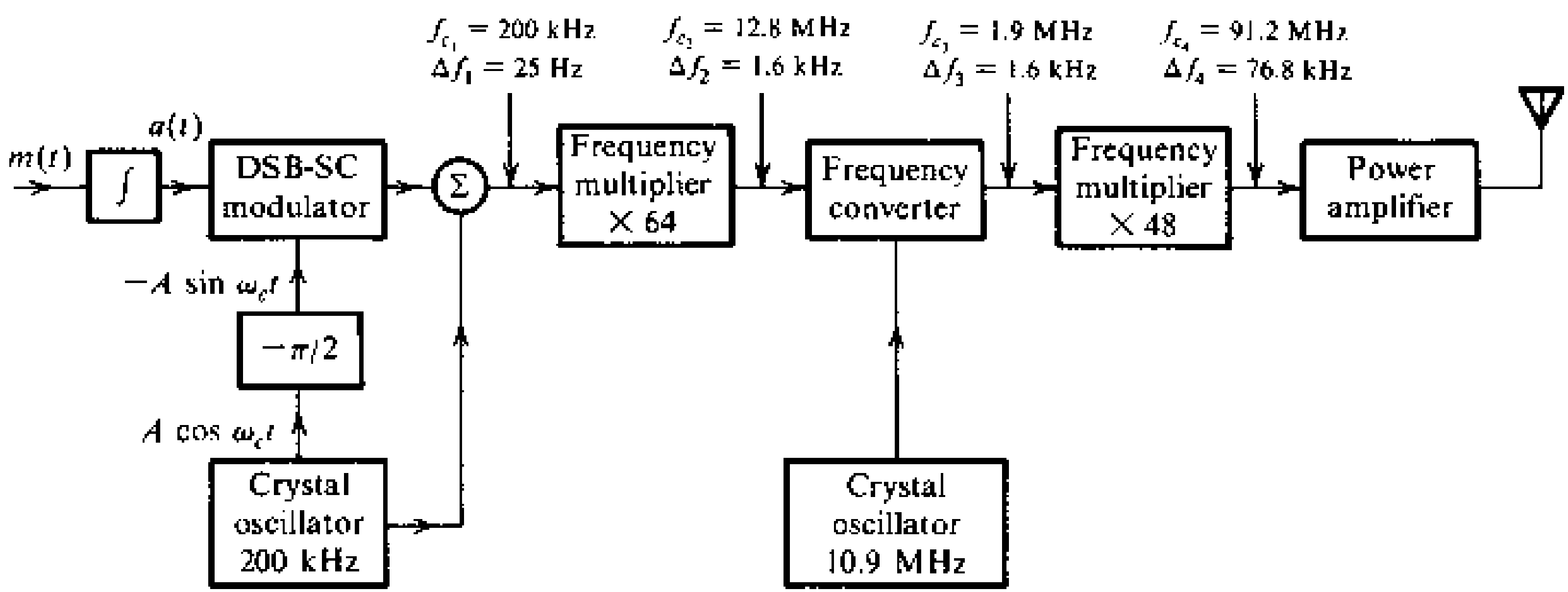
- The output will have spectra at $w_c, 2w_c, \ldots, nw_c$ with frequency deviation $\Delta f, 2\Delta f, \ldots, n\Delta f$ respectively.
- A bandpass filter centered at nw_c can recover an FM signal whose instantaneous frequency has been multiplied by a factor of n.
- These devices (nonlinearity and bandpass filter) are called frequency multipliers.

Indirect Method: Armstrong

- For example, if we want a twelfth fold increase in the frequency deviation
  - Use a twelfth order nonlinear device.
  - Two second order and one third order device in cascade.
- The output will then be passed through a bandpass filter centered at $12w_c$
  - The output of the bandpass filter will be an FM signal whose carrier frequency as well as the frequency deviation are 12 times the original values.
- Armstrong used this concept to proposed the indirect method of generating WBFM signals.

Indirect Method: Armstrong

- A simplified diagram of a commercial FM transmitter using Armstrong's method is shown as follows



- The final output is required to have a carrier frequency of $91.2MHz$ and $\Delta f = 75KHz$

Indirect Method: Armstrong

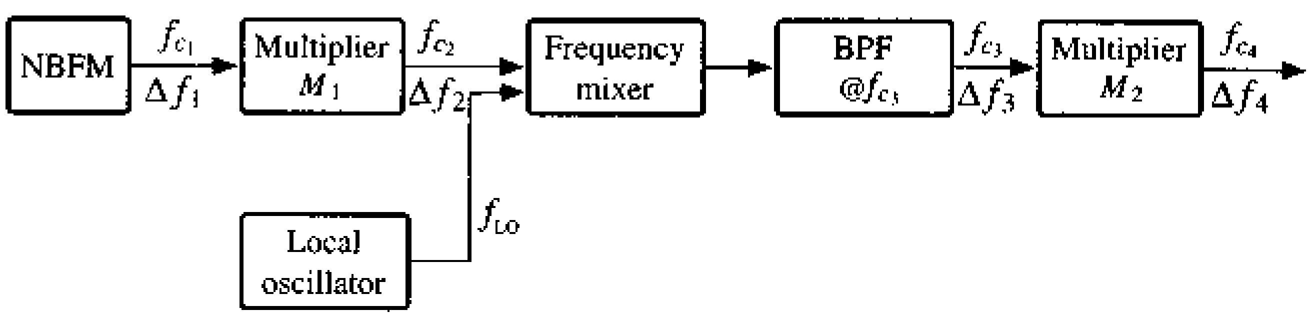
- We first generate a NBFM signal with carrier frequency $f_{c1} = 200KHz$ generated by a crystal oscillator.
- To maintain $\beta \ll 1$, the deviation Δf is chosen to be $25Hz$.
- The baseband spectrum ranges from 50Hz to 15KHz. So with $\Delta f = 25Hz$, $\beta = 0.5$ for the worst possible case i.e., when $f_m = 50Hz$
- To achieve $\Delta f = 75KHz$, we need multiplication of 3000.
  - This can be achieved by two multiplier stages of 64 and 48.
  - This will give us multiplication factor 3072 and $\Delta f = 76.8KHz$
  - Note that 64 multiplier can be obtained with 6 doublers in cascade and 48 multiplier can be obtained by 4 doublers and a tripler in cascade.
- Multiplication of $200KHz$ with 3072 will get $600MHz$. Too high!
  - How to get $91.2MHz$?
  - The problem is solved using frequency translation.

Indirect Method: Armstrong

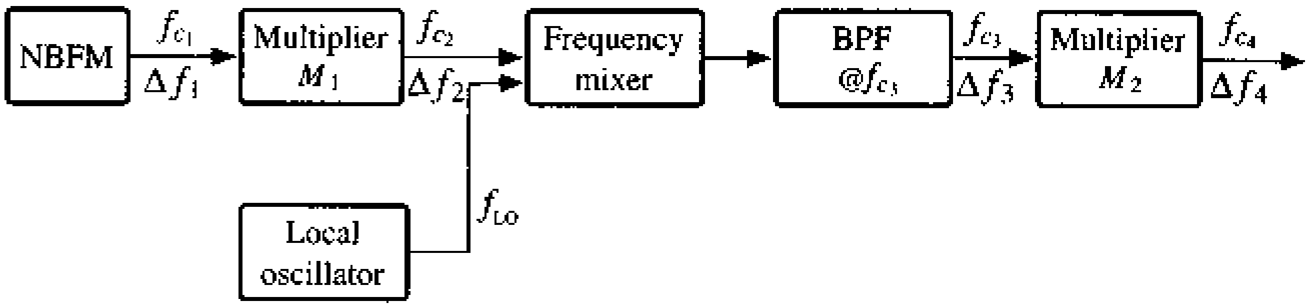
- After first multiplication by 64 result in the carrier frequency $f_{c2} = 200KHz \times 64 = 12.8MHz$ and the carrier deviation $\Delta f_2 = 25Hz \times 64 = 1.6KHz$
- We now use frequency converter (mixer) with carrier frequency $10.9MHz$ to shift the entire spectrum.
  - The new carrier frequency will be $f_{c3} = 12.8 - 10.9 = 1.9MHz$
  - The frequency converter has not effect on Δf so $\Delta f_3 = 1.6KHz$
- Further multiplication by 48, yields
  - $f_{c4} = 1.9 \times 48 = 91.2MHz$
  - $\Delta f_4 = 1.6 \times 48 = 76.8KHz$
- The scheme has the advantage of frequency stability but suffers from inherent noise cause by excessive multiplications and distortion at lower modulating frequencies, where β is not small enough.

Example

- Design an Armstrong indirect FM modulator to generate an FM signal with carrier frequency $97.3MHz$ and $\Delta f = 10.24KHz$.
  - A NBFM generator of $f_{c1} = 20KHz$ and $\Delta f = 5Hz$ is available.
  - Only frequency doublers can be used as multipliers.
  - A LO with adjustable frequency between $400 - 500KHz$ is available for mixing.



Example



- Total Multiplication required

$$M = M_1 M_2 = \frac{\Delta f_4}{\Delta f_1} = \frac{10240}{5} = 2048 = 2^{11}$$

- Since, we can only use frequency doublers we have three conditions

$$M_1 = 2^{n_1}$$
$$M_2 = 2^{n_2}$$
$$n_1 + n_2 = 11$$

- It is clear that

$$f_{c2} = 2^{n_1} f_{c1} \qquad \text{and} \qquad f_{c4} = 2^{n_2} f_{c3}$$

Example

- As we know that available local oscillator support variable frequency i.e.,

$$400{,}000 \leq f_{LO} \leq 500{,}000$$

- To find f_{LO}, we need to test the following possibilities

$$f_{c3=}f_{c2} \pm f_{LO} \quad \text{and} \quad f_{c3=}f_{LO} - f_{c2}$$

- If $f_{c3=}f_{LO} - f_{c2}$

$$f_{LO} = 2^{-n_2}(13.826 \times 10^7)$$

No integer value of n_2 will lead to a realizable f_{LO}

- If $f_{c3=}f_{c2} - f_{LO}$

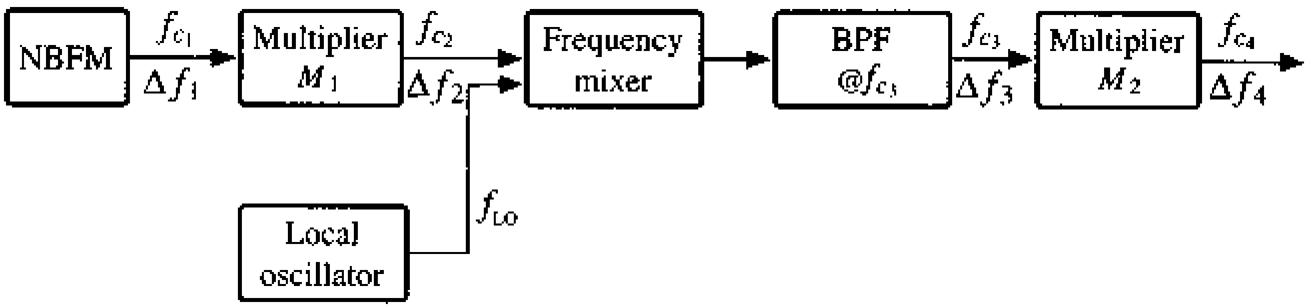
$$f_{LO} = 2^{-n_2}(4.096 \times 10^7 - 9.73 \times 10^7) < 0$$

- If $f_{c3=}f_{c2} + f_{LO}$

$$f_{LO} = 2^{-n_2}(5.634 \times 10^7)$$

If $n_2 = 7$, then $f_{LO} = 440KHZ$. Which is well within the range.

Example



- For $n_2 = 7$,

$$n_1 = 11 - n_2 = 4$$

- The final design will be

$$M_1 = 2^{n_1} = 2^4 = 16$$
$$M_2 = 2^{n_2} = 2^7 = 128$$
$$f_{LO} = 440KHz$$

Distortion in Armstrong indirect FM generator

- Two kinds of distortions arise in this scheme
  - Amplitude distortion
  - Frequency distortion
- The NBFM wave is given by

$$\begin{aligned}\varphi_{FM} &= A[\cos w_c t - k_f a(t) \sin w_c t] \\ &= AE(t) \cos[w_c t + \theta(t)]\end{aligned}$$

where

$$E(t) = \sqrt{1 + k^2{}_f a^2(t)} \quad \text{and} \quad \theta(t) = \tan^{-1}\left[k_f a(t)\right]$$

- Amplitude distortion occurs because $AE(t)$ is not constant.
  - Solution?
  - Bandpass limiter!

Distortion in Armstrong indirect FM generator

- Ideally, $\theta(t)$ should be $k_f a(t)$ but in this case the phase $\theta(t) = \tan^{-1}[k_f a(t)]$.
- The instantaneous frequency $w_i(t)$ is now

$$w_i(t) = \dot{\theta}(t) = \frac{k_f \dot{a}(t)}{1 + k^2{}_f a^2(t)}$$

$$= \frac{k_f m(t)}{1 + k^2{}_f a^2(t)}$$

Maclaurin series expansion

$$= k_f m(t)\left[1 - k^2{}_f a^2(t) + k^4{}_f a^4(t) - \cdots\right]$$

$$= k_f m(t) - k_f m(t)\left[k^2{}_f a^2(t) - k^4{}_f a^4(t) + \cdots\right]$$

- Ideally, $w_i(t)$ should be $k_f m(t)$. The remaining terms are the distortion.

Direct Generation of FM Waves

- In voltage controlled oscillator (VCO), the frequency is controlled by an external voltage.
  - The oscillation frequency varies linearly with the control voltage.
- We can generate an FM wave by using the modulating signal $m(t)$ as a control signal.

$$w_i = w_c + k_f m(t)$$

- Carrier frequency is directly varied by the message through voltage-controlled oscillator (VCO)
  - Vary one of the reactive parameters i.e., C or L of the VCO
- The frequency of oscillation is given by

$$w_o = \frac{1}{\sqrt{LC}}$$

Direct Generation of FM Waves

- If the capacitance C is varies by the modulating signal $m(t)$, that is

$$C = C_o - km(t)$$

then

$$w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L[C_o - km(t)]}} = \frac{1}{\sqrt{LC_o\left[1 - \frac{km(t)}{C_o}\right]}}$$

$$= \frac{1}{\sqrt{LC_0}\left[1 - \frac{km(t)}{C_o}\right]^{1/2}}$$

- Apply Taylor series expansion i.e.,

$$(1+x)^n \approx 1 + nx \qquad \text{if } |x| \ll 1$$

- So we get,

$$w_o \approx \frac{1}{\sqrt{LC_0}}\left[1 + \frac{km(t)}{2C_o}\right] \qquad \frac{km(t)}{C_o} \ll 1$$

Direct Generation of FM Waves

$$w_o = \frac{1}{\sqrt{LC_0}}\left[1 + \frac{km(t)}{2C_o}\right] = w_c\left[1 + \frac{km(t)}{2C_o}\right]$$

$$w_o = w_c + k_f m(t)$$

where, $w_c = \frac{1}{\sqrt{LC_0}}$ and $k_f = \frac{kw_c}{2C_o}$

- As $C = C_o - km(t)$, the maximum capacitance deviation will be

$$\Delta C = km_p = \frac{2k_f C_o m_p}{w_c}$$

- Hence,

$$\frac{\Delta C}{C_o} = \frac{2k_f m_p}{w_c} = \frac{2\Delta f}{f_c}$$

- The similar frequency deviation can be achieved by varying the inductance of the variable inductor.
- Direct FM generation produces sufficient frequency deviation.
  - It has poor frequency stability and require feedback to stabilize the frequency.

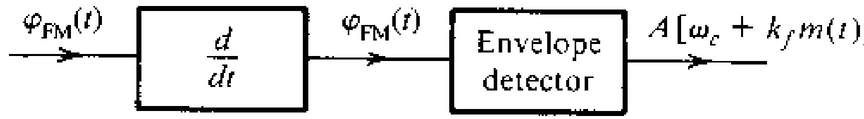
Demodulation of FM Signals

- The information in an FM signal resides in the instantaneous frequency

$$w_i = w_c + k_f m(t)$$

- FM demodulation can be achieved using different techniques
  - Frequency Discriminator (Frequency to Voltage Converter)
    - Differentiator + Envelope Detector
  - Zero Crossing Detector
    - Hard Limiter + Digital Frequency Counter
  - Phase Locked Loop (PLL)
    - Phase Detector + Loop Filter + VCO

Frequency Discriminator



- FM signal is represented as

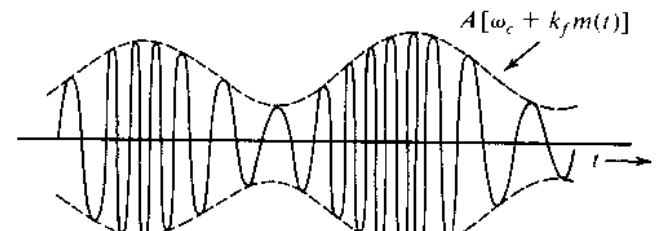
$$\varphi_{FM} = A\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$

- If we apply $\varphi_{FM}(t)$ to a differentiator, the output is

$$\dot{\varphi}_{FM}(t) = \frac{d}{dt}\left[A\cos\{w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\}\right]$$

$$= A\left[w_c + k_f m(t)\right]\left[\sin\left\{w_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha - \pi\right\}\right]$$

- Observe that both amplitude and the frequency of the signal $\dot{\varphi}_{FM}(t)$ are modulated.



(b)

Frequency Discriminator

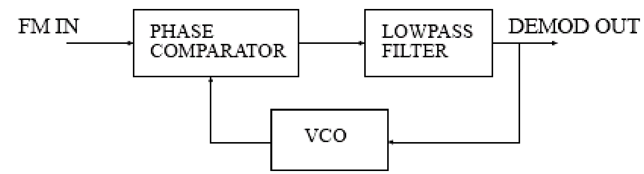
- The envelope being in this case $A\left[w_c + k_f m(t)\right]$
  - As $\Delta w = k_f m_p < w_c$, we have $w_c + k_f m(t) > 0$ for all t.
  - $m(t)$ can be obtained by using envelope detection of $\dot{\varphi}_{FM}(t)$.
- The amplitude A of the incoming FM carrier is assumed constant, otherwise the envelope will have the time varying effects of the amplitude $A(t)$.
- Before applying input to a frequency discriminator, the fluctuation in amplitude (due to say channel fading) should be removed.
  - How?
  - Bandpass Limiter

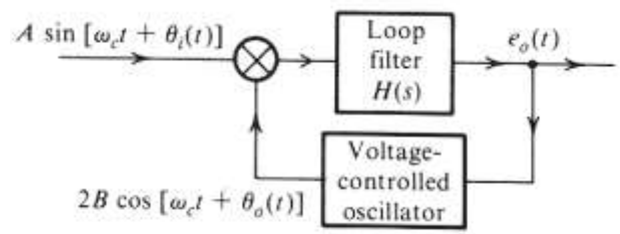
Zero Crossing Detector

- A zero crossing detectors can also be used due to the advances in digital integrated circuits.
- The first step is to use an amplitude limiter to generate rectangular pulse.
- The resulting rectangular pulse train of varying width can then be applied to trigger a digital counter .
- The digital counters are frequency counters designed to measure the instantaneous frequency from the number of zero crossings.
- The rate of zero crossings is equal to the instantaneous frequency of the input signal.

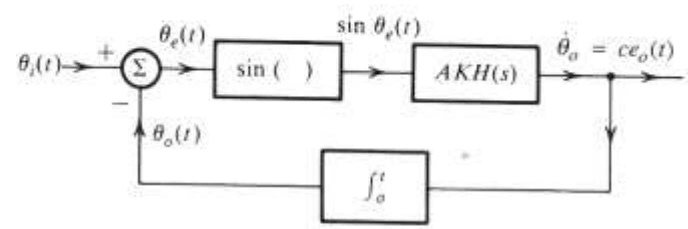
Phase Locked Loop (PLL)

- A PLL is a closed-loop feedback control circuit, where the feedback action will drive the time-varying phase of the VCO output to match the time-varying phase of the input i.e., $\theta_o(t) \rightarrow \theta_i(t)$
- We assume that initially we have adjusted the VCO so that when the control voltage is zero, two conditions are satisfied
  - The frequency of the VCO is precisely set at the unmodulated carrier frequency f_c
  - The VCO output has a 90^o phase shift with respect to the unmodulated carrier wave





(a)



PLL Analysis

- Since, the instantaneous frequency of the VCO is given as

$$w(t) = w_c + ce_o(t)$$

  - Where w_c is the free running frequency
  - c is the constant sensitivity factor of the VCO and is measured in $rad/\sec/volt$
- If VCO output is $B\cos[w_c t + \theta_o(t)]$, the instantaneous frequency is given as

$$w_i = w_c + \frac{d\theta_o(t)}{dt}$$

- Hence, $\dot{\theta}_o(t) = ce_o(t)$
- Since, $\theta_e(t) = \theta_i(t) - \theta_o(t)$

$$\theta_o(t) = \theta_i(t) - \theta_e(t)$$

- If VCO is locked to the input signal frequency and phase then $\theta_e(t) \sim 0$

$$\dot{\theta}_o(t) = \dot{\theta}_i(t)$$

$$e_o(t) = \frac{1}{c}\frac{d}{dt}\left[k_f \int_{-\infty}^{t} m(\alpha)d\alpha + \frac{\pi}{2}\right] = \frac{k_f}{c} m(t)$$

- Thus, PLL acts as a FM demodulator.
- For PM, $e_o(t) = \frac{k_p}{c}\dot{m}(t)$
  - Integrate $e_o(t)$ to obtained the desired signal $m(t)$

Effects of Nonlinear Distortion

- Nonlinear distortion in AM not only cause unwanted modulation with carrier frequencies nw_c but also causes distortion of the desired signal.
- For instance, if a DSB-SC signal i.e., $m(t)\cos w_c t$ pass through a nonlinear system defined as

$$y(t) = ax(t) + bx^3(t)$$

- The output will be defined as

$$y(t) = a[m(t)\cos w_c t] + bm^3(t)\cos^3 w_c t$$
$$= \left[am(t) + \frac{3b}{4}m^3(t)\right]\cos w_c t + \frac{b}{4}m^3(t)\cos 3w_c t$$

- Passing the signal though a bandpass filter still yield

$$\left[am(t) + \frac{3b}{4}m^3(t)\right]\cos w_c t$$

  - The distortion component $\frac{3b}{4}m^3(t)$ is present along the desired message signal.

Effects of Nonlinear Distortion

- A very useful feature of angle modulation is its constant amplitude.
  - Less susceptible to nonlinearities.
- Consider an amplifier with nonlinear distortion and input $x(t)$ and output $y(t)$ related by

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \cdots + a_n x^n(t)$$

  - The first term is the desired signal amplification term
  - The remaining terms are the unwanted nonlinear distortion
- For an angle modulated signal

$$\varphi_{EM}(t) = A\cos[w_c t + \theta(t)]$$

- Using trigonometric identities we can write the output of the amplifier as

$$y(t) = c_0 + c_1\cos[w_c t + \theta(t)] + c_2\cos[2w_c t + 2\theta(t)] + \cdots + c_n\cos[nw_c t + n\theta(t)]$$

Effects of Nonlinear Distortion

- A sufficient large w_c makes each component of $y(t)$ separable in frequency domain.
  - A bandpass filter centered at w_c with bandwidth equal to B_{FM} or B_{PM} can extract the desired signal component i.e., $c_1 \cos[w_c t + \theta(t)]$ without any distortion.
- Immunity from nonlinearity is the primary reason for the use of angle modulation.
- The constant amplitude of FM gives it a kind of immunity to rapid fading .
  - The rapid fading can be eliminated by using AGC and bandpass limiting.
- These advantages made FM an attractive technology for the first generation (1G) cellular phone systems.

Effects of Interference

- Angle modulation is also less vulnerable than AM.
- Let us consider the simple case of the interference of an unmodulated carrier $A\cos w_c t$ with another sinusoid $I\cos(w_c + w)t$
- The received signal $r(t)$ can be written as

$$\begin{aligned} r(t) &= A\cos w_c t + I\cos(w_c + w)t \\ &= (A + I\cos wt)\cos w_c t - I\sin wt \sin w_c t \\ &= E_r(t)\cos\theta(t) \end{aligned}$$

Where

$$E_r(t) = \sqrt{A^2 + I^2 + 2AI\cos wt}$$

$$\theta(t) = w_c t + \theta_d(t)$$

$$\theta_d(t) = \tan^{-1}\frac{I\sin wt}{A + I\cos wt}$$

- If the interfering signal is small compared to the carrier i.e., $I \ll A$

$$\theta_d(t) \approx \frac{I}{A}\sin wt$$

Effects of Interference

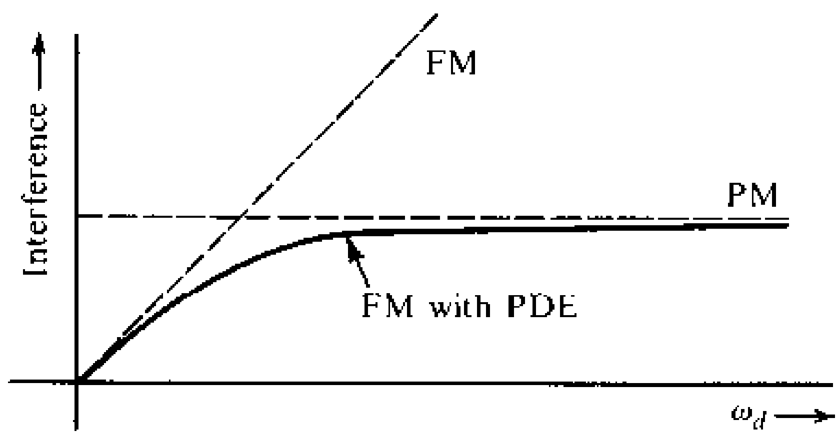
- The instantaneous frequency of $E_r(t)\cos\theta(t)$ is
$$w_i = \dot{\theta} = w_c + \dot{\theta}_d(t)$$
- If the signal $r(t)$ is applied to an ideal phase demodulator, the output would be
$$y_d(t) = \theta_d(t) = \frac{I}{A}\sin wt$$
- If the signal $r(t)$ is applied to an ideal frequency demodulator, the output would be
$$y_d(t) = \dot{\theta}_d(t) = \frac{Iw}{A}\cos wt$$
- In both cases, the interference output is inversely proportional to the carrier amplitude A.
  - The larger the carrier amplitude A, the smaller the interference effect.
- Hence, angle modulated systems are much better than AM systems at suppressing weak interference ($I \ll A$).

Capture Effect

- The suppression of weak interference in FM leads to **capture effect** when listening to FM radio.
- For two transmitters with carrier frequency separation less than the audio range , instead of getting interference , we observe that the stronger carrier suppresses (captures) the weak carrier.
- **Capture Effect:** The stronger carrier effectively suppresses (captures) the weaker carrier.
- Subjective tests show that an interference level as low as 35dB in the audio signals can cause objectionable effects (hence the restriction in AM).
  - In AM, the interference level should kept below 35dB.
  - For FM, because of the capture effect, the interference level need only be below 6 dB.

Interference due to channel noise

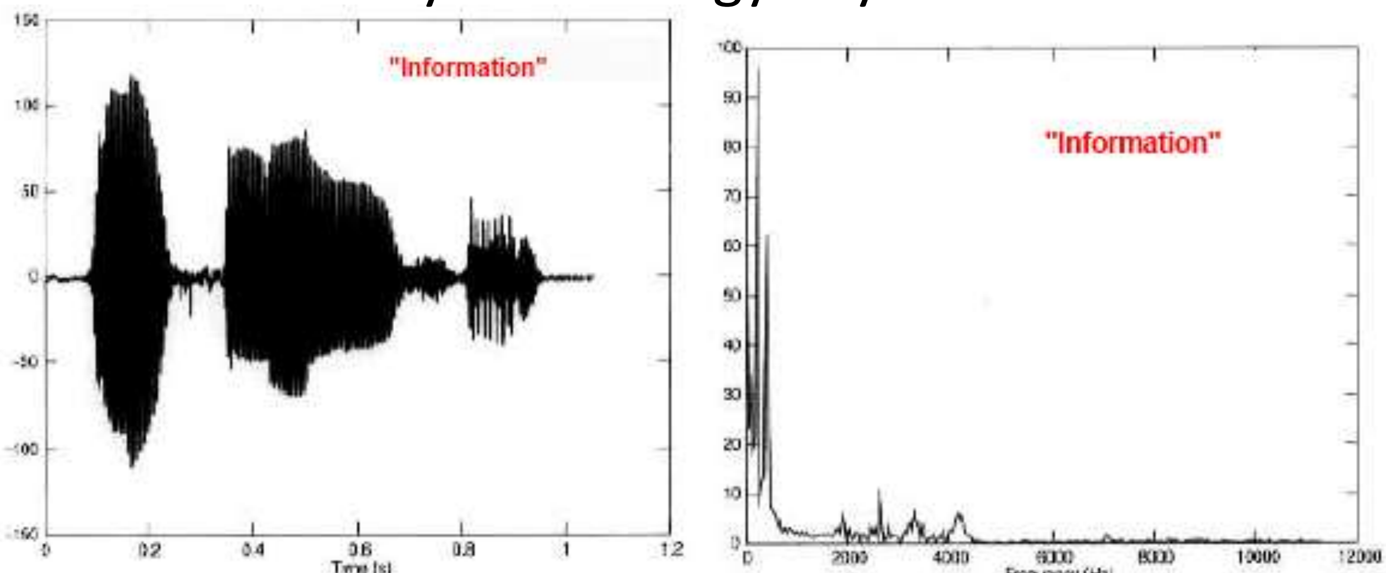
- The channel noise act as interference in angle modulated signals.
- The interference amplitude (I/A for PM and Iw/A for FM) vs. w at the receiver output is shown below.



- The interference amplitude is constant for all w in PM.
- The interference amplitude varies linearly with w in FM.

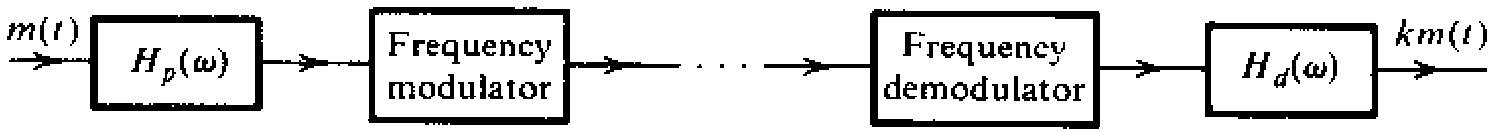
Preemphasis and Deemphasis in FM Broadcasting

- Audio frequency Spectrum used in FM radio transmission does not exhibit uniform levels across the commercial range of 12 KHz.
- The power in human voice is concentrated between 400~2200 Hz and there is very little energy beyond 3.2 KHz.



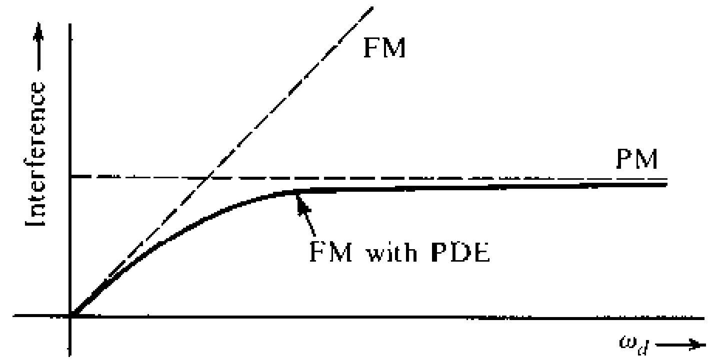
Waveform and spectrum of the word “information” uttered by a female speaker.

Preemphasis and Deemphasis in FM Broadcasting

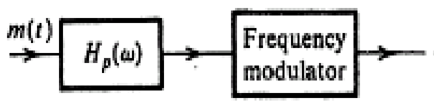


- **Preemphasis:** Weaker signal components (beyond 2.2 KHz) are boosted before transmission by a preemphasis filter of transfer function $H_p(w)$.
- **Deemphasis:** The deemphasis undoes the preemphasis by passing the received signal through a filter having the transfer function $H_d(w) = 1/H_p(w)$
- Since the noise entered the channel and has not been boosted, the deemphasis filter leaves the desired signal untouched but reduces the noise power considerably.

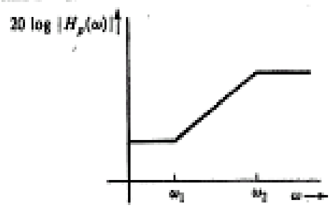
Preemphasis and Deemphasis Filters

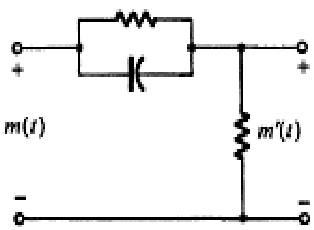


To improve noise immunity of FM signals we use a pre-emphasis circuit at transmitter

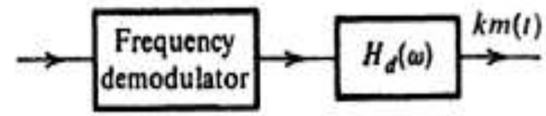


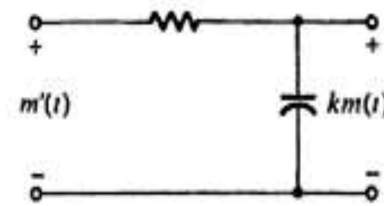
$$H_p(\omega) = K\frac{j\omega + \omega_1}{j\omega + \omega_2}$$

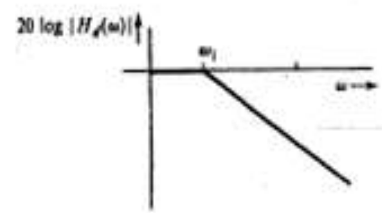




Receiver de-emphasis circuit

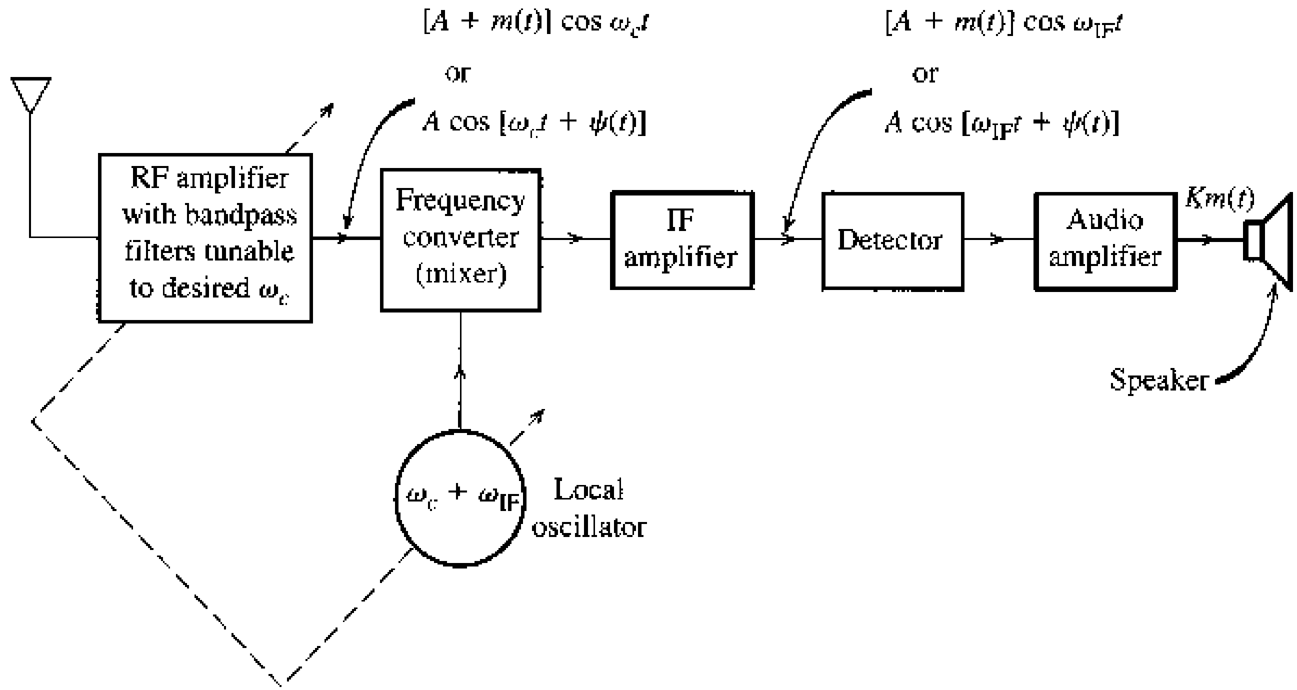






PDE enhances the SNR by 13.27dB

Superheterodyne Analog FM Receivers



- The frequency mixer (convertor), translates the carrier from w_c to a fixed IF frequency of $455\,kHz$ in case of AM while $10.7MHz$ in case of FM.
- A superheterodyne FM receiver is a monophonic FM receiver.

FM Broadcasting System

- Key FM broadcasting parameters
  - Maximum/Peak frequency deviation: $\Delta f_{max} = 75\text{KHz}$
  - Message signal bandwidth: $B_m = 15\text{KHz}$
  - Transmission bandwidth (Carson's Rule): $B_T \approx 2(\Delta f + B_m)$

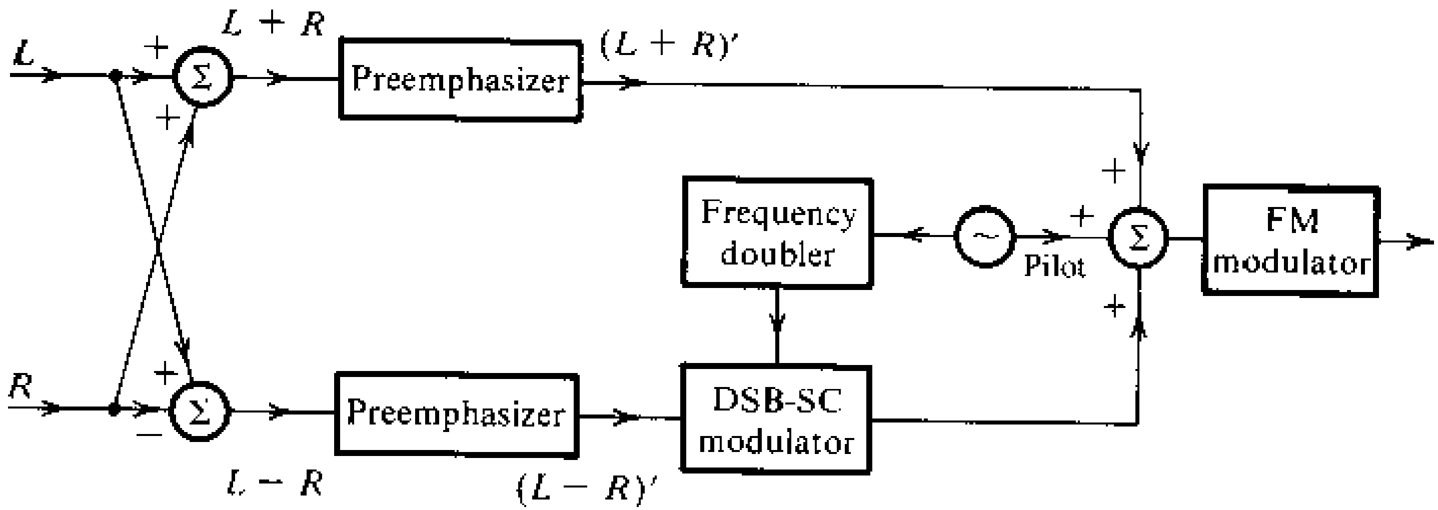
    $B_T \approx 180\text{KHz}$
  - FM broadcasting regulations: $B_T = 200\text{KHz}$

- The FCC has assigned frequency range of 88 to $108MHz$ for FM broadcasting with a separation of $200KHz$ between adjacent stations and peak frequency deviation of $\Delta f = 75KHz$.

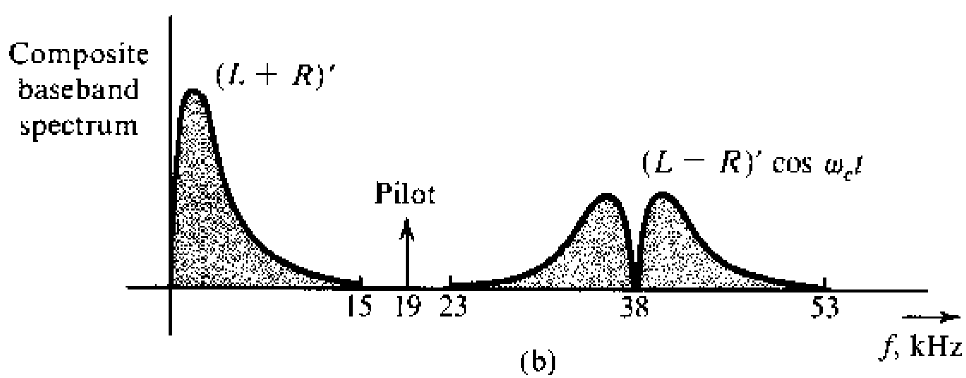
FM Broadcasting System

- Design Considerations
  - Earlier FM broadcasts were monophonic.
  - Stereophonic FM broadcasting
    - Two audio signals , L (left microphone) and R (right microphone), are used for more natural effect.
  - FCC required that stereophonic FM broadcasting should be compatible with monophonic FM broadcasting.
    - Older monophonic receivers should be able to receive $L + R$ signal
    - The total transmission bandwidth for the two signals should still be $200KHz$ with $\Delta f = 75KHz$

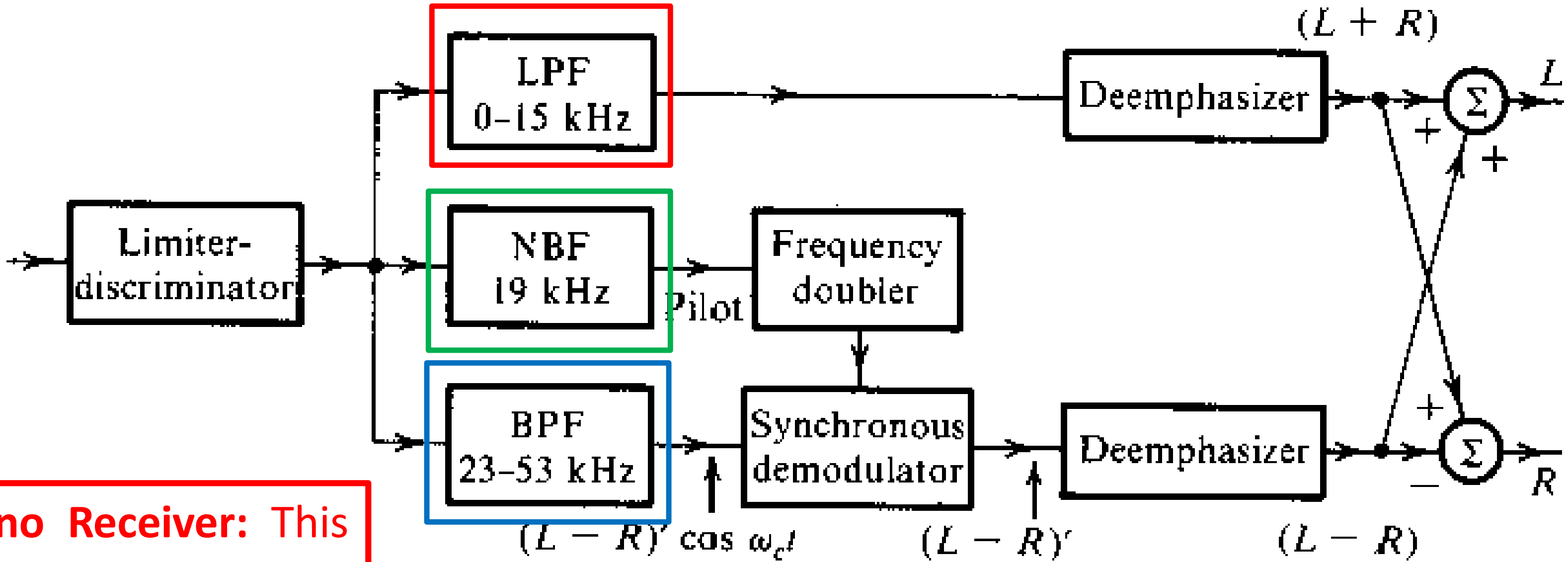
FM Broadcasting System



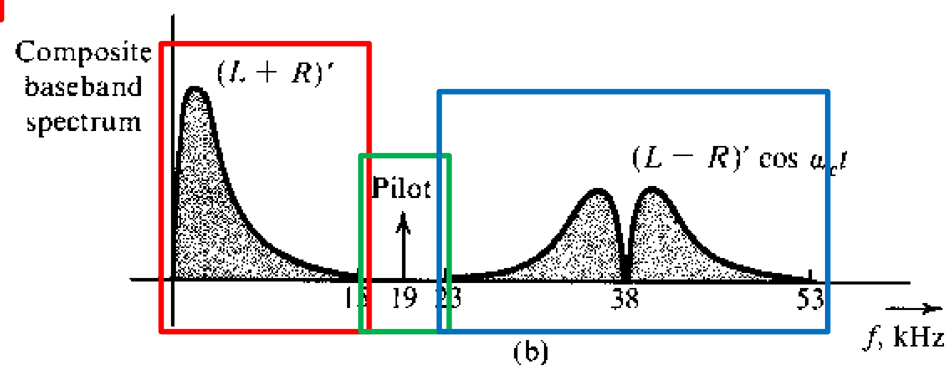
$$m(t) = (L+R)' + (L-R)' \cos w_c t + \alpha \cos \frac{w_c t}{2}$$



FM Broadcasting System

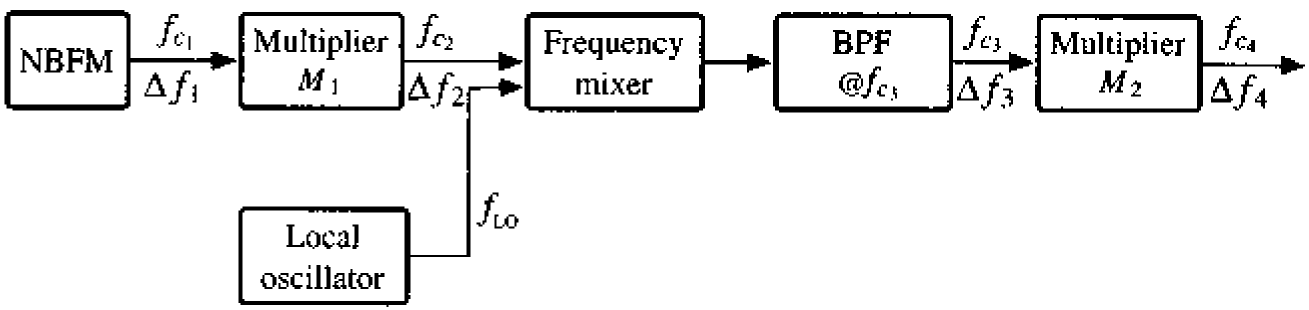


Mono Receiver: This is all what is needed



Example

Design (only the block diagram) an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 98.1 MHz and $\Delta f = 75$ kHz. A narrowband FM generator is available at a carrier frequency of 100 kHz and a frequency deviation $\Delta f = 10$ Hz. The stock room also has an oscillator with an adjustable frequency in the range of 10 to 11 MHz. There are also plenty of frequency doublers, triplers, and quintuplers.



- Total Multiplication: ${}^{\Delta f_4}/_{\Delta f_1} = {}^{75\,KHz}/_{10Hz} = 7500$
- We know that: $7500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 5 = 2^2 \times 3^1 \times 5^4$

$$M_1 = 5 \times 5 \times 5 = 125$$
$$M_2 = 2^2 \times 3 \times 5 = 60$$
$$f_{LO} = 10.865 MHz$$