

Monday

23.09.2019

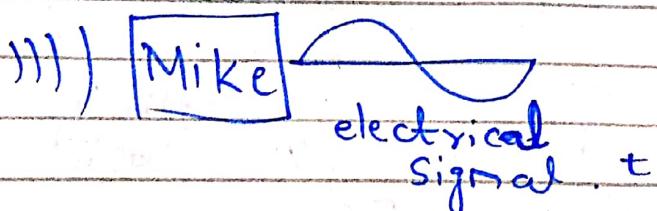
# COMMUNICATION SYSTEM.

## Transducers

## Lecture 1

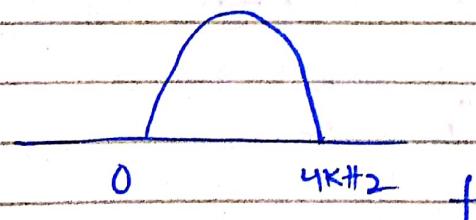
One form into another.  
energy

①



Speed signal frequency:

0.3 to 3.4 kHz.



Problems (for transmitting electro magnetic signal).

i) Frequency translation

[if mostly is range me frequency hogi then no overlap kr jayegi; no voice will be heard.]

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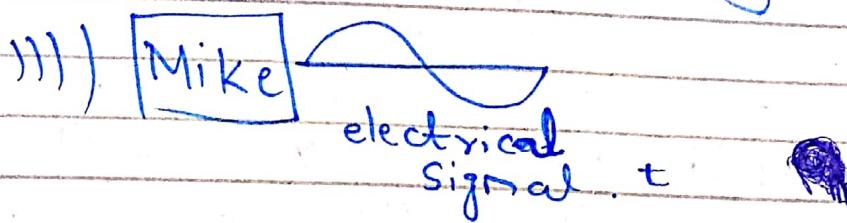
# COMMUNICATION SYSTEM.

## Transducer:

## Lecture 1

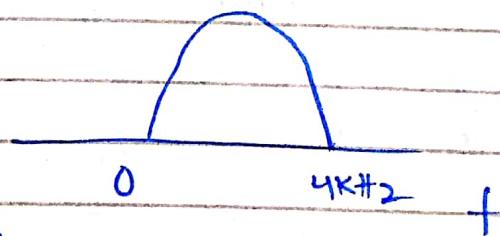
one form into another.  
energy

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Speed signal frequency:

0.3 to 3.4 kHz.

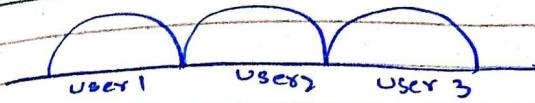


Problems (for transmitting electro magnetic signal).

i) Frequency translation

[if mostly is range me frequency hog; then no overlap kr jayegi;  
no voice will be heard.]

Keep the users in diff frequency space.



so that it will not overlap each other.

→ Time domain Representation of signal.

⇒ for this

(iii) Narrow

(bandwidth ↓  
4KHz)

## ii) Practicality of Antenna.

Antenna Theory: → width & length

The dimensions of antenna should be comparable with the signals.

(1/10n of wavelength.)

2

(first  
one)

$$f\lambda = c$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{4 \times 10^3}$$

$$= \frac{3}{4} \times 10^5$$

(The antenna height for above band should be 75 kmeters.)

75 km

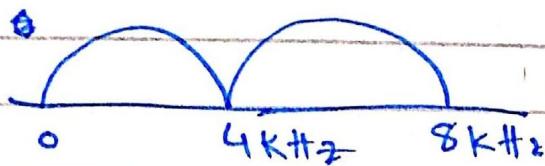
$\Rightarrow$  for this enhance frequency of signal.

(iii) Narrow banding.

(bandwidth)



4 kHz.



3

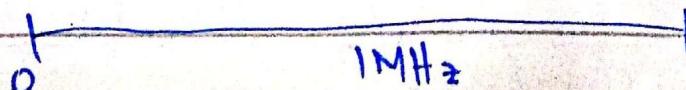
$$\lambda_a = \frac{3 \times 10^8}{8 \times 10^3}$$

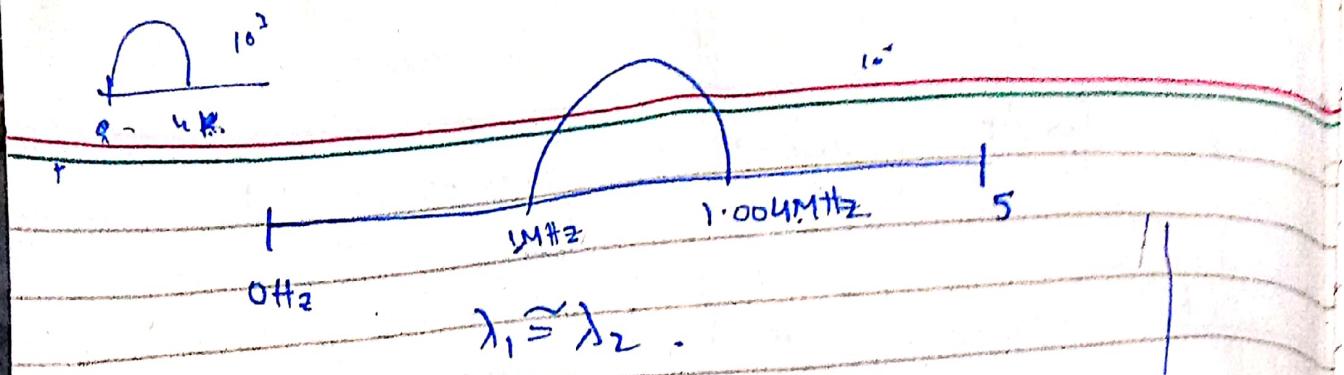
$$= 37.5 \text{ Kmetres.}$$

(first 75 km to  $\Sigma$  then 37.5 km to one antenna is not enough)

(We have to shrink the band)

if we want to shrink, we won't be able to hear proper sound.)





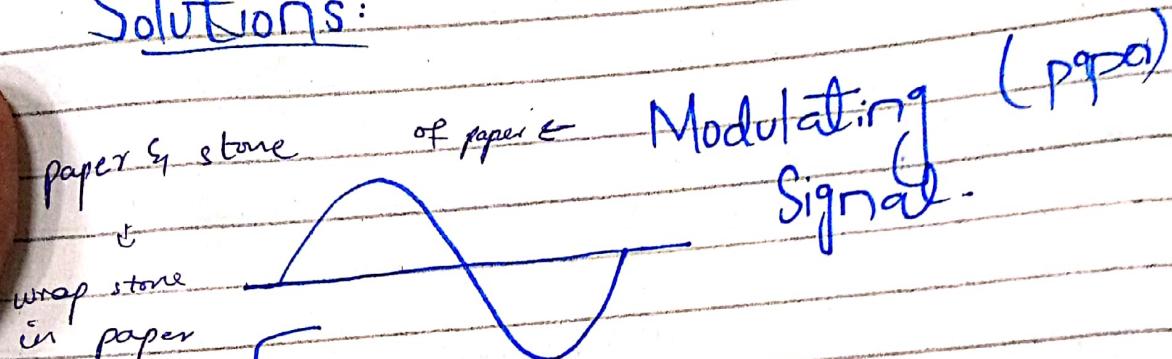
$$1000000 \\ 4000$$

$$\frac{1004000}{1.004 \text{ MHz}}$$

$$\frac{3 \times 10^6}{1 \times 10^6} = 3 \times 10^{-2}$$

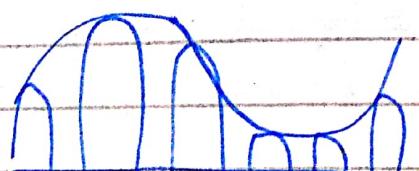
(4)

### Solutions:



Modulated  
(Carrier Signals)

Modulated  
Signal.  
(paper & stone)



→ Transducer : step 1  
→ Transmitter : step 2.

### Attributes of Signals

- amplitude
- frequency
- phase

\* Modulating Signal is same but the amplitude of carrier signal is changed. ] this process is Modulation.

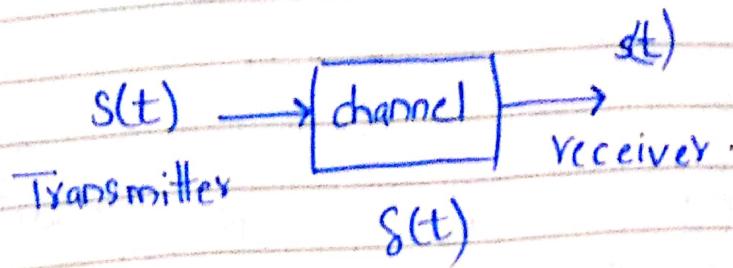
• if we vary any attribute of a carrier signal in accordance with the instantaneous value of the base band modulating signal, the process is said to be modulation.

(5)

## Lecture #2.

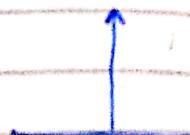
### Channel:

(6)



Transfer function:

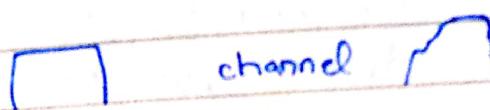
$$S(t)$$



$$s(t) \xrightarrow{*} S(t) = s(t).$$

(Ideal case).

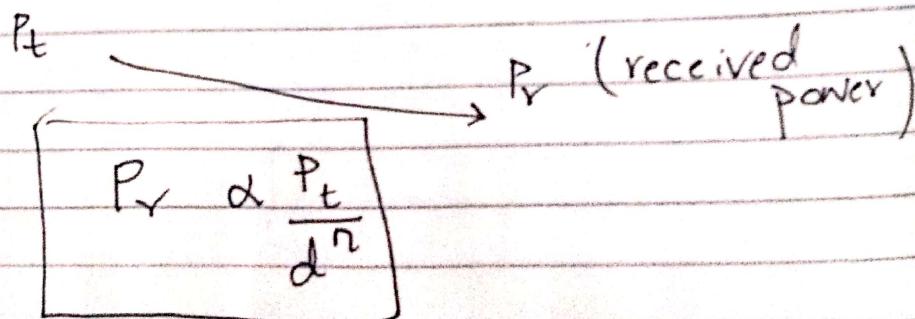
Property  
of  
channel



linear  
distortion.

\* the signal may be attenuated  
(Amplitude change)

non-linear distortion.



$$n = 2 \text{ to } 6.$$

$n$  depends on  
landscape.  
 $\Rightarrow$  streets, hgt<sup>n</sup>.

$n$  is Path loss exponent.

(1)

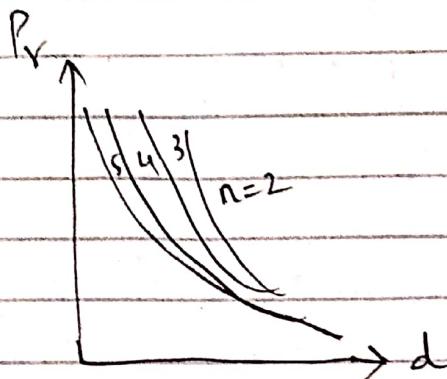
①

channel distortion

② spreading of signal.

little change in  $d$  causes abrupt change in denominator

$$P_r = k \frac{P_t}{d^n}$$



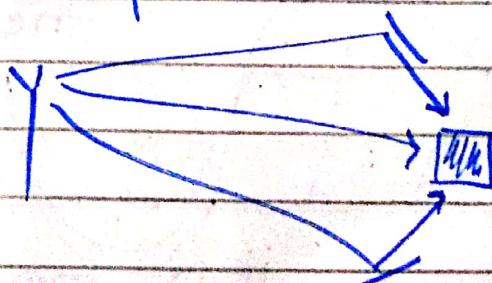
Okumura Hata

## Distortion kinds.

### \* Multipath;

jab signal ko radiate kia jata ha it is in omni directional.

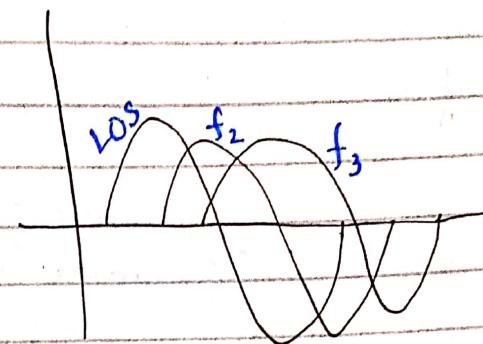
All power will be distributed.



we don't need copies of signal  
due to spreading of signal.

(8)

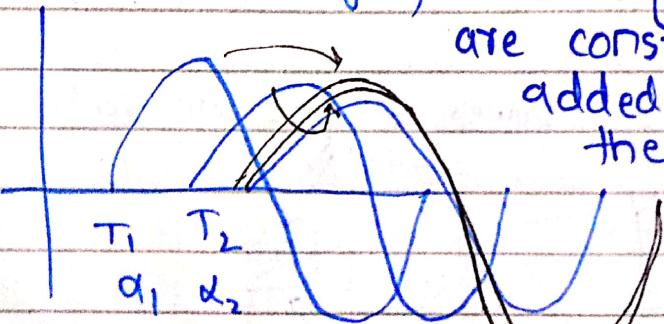
one signal is from LOS (line of sight)



When it is added together,  
all the signals have diff  
time delay.

### RAKE Receiver

→ Delay the first signal to the last signal) When all the signals are constructively added then



it is more beneficial.

Channel →  
contains .

(9)

Noise

PTA

Pak Telecom..

Authority.

→ Internal Noise

→ External Noise

Galactic Noise .

Atmosphere Noise .

⇒ (Sun radiations)

⇒ (Star radiations) (Man made)

28

Distortion due to manmade is  
interference .

External noise act as  
interference .

ye

ex

The noise which is created in the device  
is called internal noise

⇒ Due to temp , initiation of energy  
occurs creates thermal  
noise

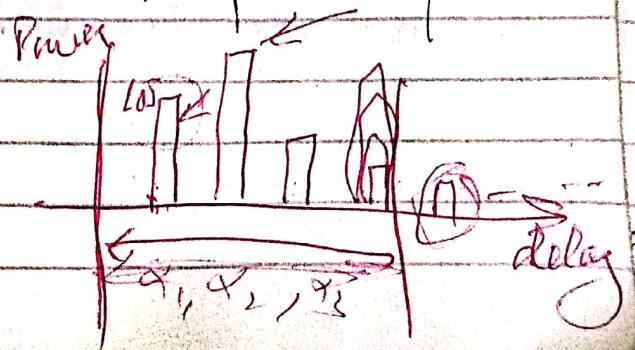
Noises are part of channel .

Signal to Noise  
Ratio .

$$= \frac{S \text{ power}}{N \text{ power}} \quad | \quad \frac{S}{1+N}$$

↓  
internal  
noise

created  
due to  
thermal  
noise



(10)

Ufone      Mobilink

10 MHz

10 MHz



License Band :

( Previous  
freq Band )

Shenon ~~Theory~~  
Capacity

$$C = B \log_2 (1 + SNR)$$

Dependant on Bandwidth .

u can't transmit more than the  
given capacity .

if noise = 0

then S/N will become  
infinite .

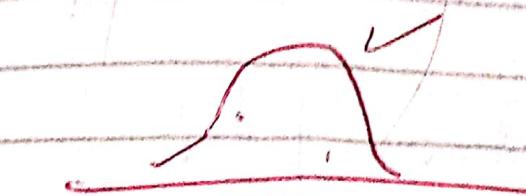
In zero time v infinite data is  
transmitted

↓  
impossible .

Noise is important.



noise



noise



(A) (B)

→ What is Regenerative Repeaters?

Digital

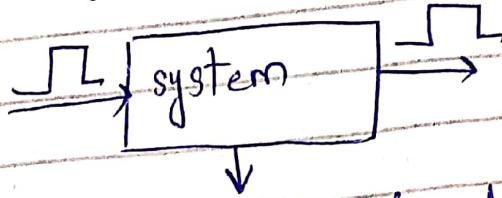


Analog  
Repeater

# Lecture 3:

201

Transfer function:



Transfer function:

↳ Transfer function can be represented by a mathematical expression or logical expression.



$$c = a \oplus b$$

= Transfer function.

System:

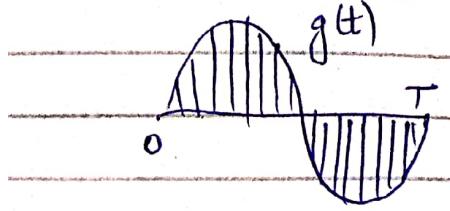
which give you desired output when you give inputs.

Size of Signal:

The area under the curve is size of the signal.

e.g.  $g(t)$  is a signal

$$\text{size} = \int_{-\infty}^{\infty} g(t) dt$$

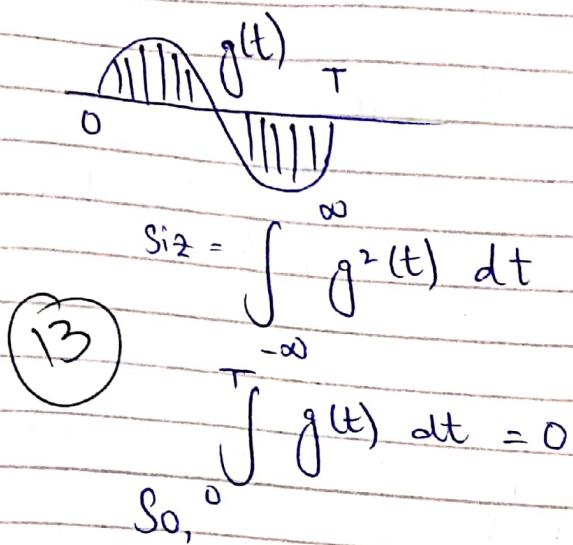
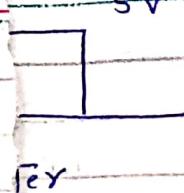


$$\text{size} = \int_{-\infty}^{\infty} g^2(t) dt = \text{Energy}$$

This is called  
T of signal

Size of cylinder

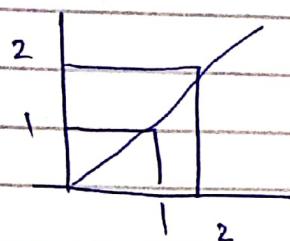
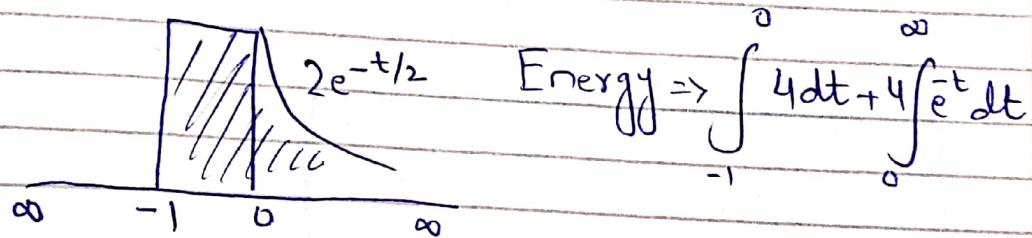
$$\pi \int_0^h r^2 dr.$$



$$\text{Size} = \int_{-\infty}^{\infty} g^2(t) dt$$

$$\int_0^T g(t) dt = 0$$

So,



→ It has infinite energy because it is a continuous

signal.

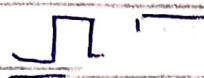
- Every signal is not an energy signal.
- ↳ if a signal has infinite value from  $-\infty$  to  $\infty$  then its energy cannot be calculated

- ↳ Those signals which have statistical symmetry and are continuous cannot be calculated. They have infinite E &

# Lecture

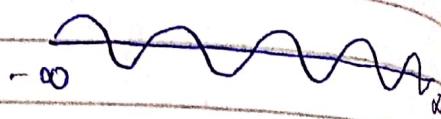
## Power Signals

Transfer



Some signals are not energy signals but have some power

$$P_g = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t) dt$$



$C \cos(\omega t + \phi)$   
⇒ energy cannot be calculated  
but power can be.

## Digital Signal:

↳ Conversion of Analog to Digital Signal

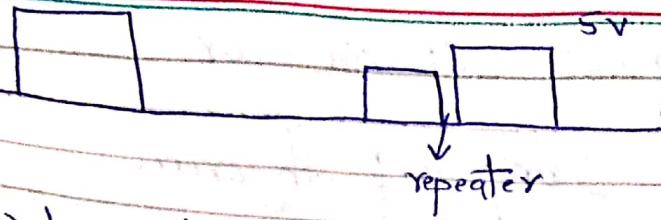


↳ We amplify it

$$\frac{S}{N}$$

↳ SNR increases

(15)



↳ In digital system:

↳ We have some regenerative repeaters at some distances which generate the original signal.

↳ No effect on SNR.

↳ Steps of Conversion.

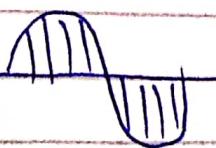
## Sampling:

- Take the samples of Analogue signal
- The samples have to be taken.

According to Sampling theorem

↳ No. of Samples should be taken at least twice of the highest frequency component present in signal.

e.g.



$$\rightarrow f = 4 \text{ kHz}$$

No. of samples taken should be 8k samples/sec

Otherwise the signal cannot be recovered.

# Nyquist Criteria or Sampling Theorem

Step #1:

Simplify



Step #2 : QUANTIZATION

↳ Must give a no. or quantity to the signal.

↳ Give levels

↳ More no. of levels give accurate signal.

Step #3

Encoding

0 0000

1 0001

2

⋮

i

↓

16

⋮

1111

↳ Every sample will be

converted into binary.

Telecom using 128 levels

256 levels

↳ 7 bits

require to represent the signal.

(11)

The entire process is said to be

Pulse Code Modulation

Bandwidth :

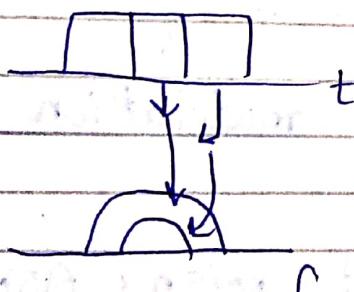
power,  $\hookrightarrow$  Transmitting signal with more less bandwidth will be required.

If signals has greater width, it will acquire less space in time domain. (less bandwidth) and viceversa.

$\Rightarrow$  Signal Trade b/w Signal Power & bandwith.

rect function

$$\frac{t}{T}$$



$\hookrightarrow$  Only analog have the property to travel

$\hookrightarrow$  Analog have one to one correspondance with digital signal

## Pulse Code Modulation

18

- ↳ More no. of levels
- ↳ less quantization error
- but bandwidth increases.
- ↳ If digital signal undergoes some errors during transmission
- ↳ junk of data packets

Steps:

- Detection of error
- Send that packet again  
or (req the sender)

Receiver detects the error  
and corrects it

that is make the receiver smart.

## Lecture # 4

↳ Three kinds of modulation in digital signals

- ① Amplitude Shift Keying (ASK)
- ②
- ③

(19)

## ⇒ Error Detection & Correction:

↪ Receiver is smart, it detects the error.

### Hamming Error Detection Code:-

① Error detection and error correction would only be possible at the far end if some redundant bits transmitted along with the message.

② These redundant bits are not part of the message but will facilitate the communication.

Let we have 8 bit of communication.

$D_7 \ D_6 \ D_5 \ D_4 \ D_3 \ D_2 \ D_1 \ D_0$

The number of redundant bits should satisfy the below formula.

$$2^n \geq n+m+1$$

$$2^4 \geq 4+7+1$$

$$16 \geq 13$$

$n =$  redundant bits

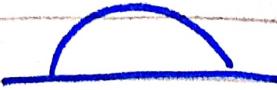
$m =$  message //

12	11	10	9	8	7	6	5	4	3	2	1	0
0101	0100	0010										
0011	0001											

## Lecture #4

CDMA - Code division multiple access.

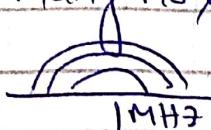
QD



1MHz to 1 user

data rate increased  $\rightarrow$  more no. of bits increased / s

$\rightarrow$  f band limited  $\rightarrow$  many no. of users in same band



$\rightarrow$  all messages mix with one another

but in CDMA they give no. to each user and accommodate them in same f band.

BER Bit Error Rate:

Some bits loss during communication  
 $\downarrow$  information is lost.

- bcz of noise.

$$\text{BER} = 10^{-3} = \frac{1}{1000}$$

E is large  
make packet

$\downarrow$   
Hamming Correction Code:

have some number of bits

$\rightarrow$  we have to append some extra bits with message

$\rightarrow$  These extra bits are called redundant bits.

(21)

→ no part of actual info but it facilitates communication.

- Extra bits should ~~be~~ not be appendant at any side of msg.

$D_7 D_6 D_5 D_4 D_3 D_2 D_1 D_0$

→ Spread within the message.  
How many ~~redundant~~ redundant bits

$$2^n \geq n+m+1$$

$$n = \text{no. of redundant bits}$$

$$m = \text{no. of bits in message.}$$

$$2^4 \geq 4 + 8 + 1$$

$$16 \geq 13$$

$n = 4 \rightarrow$  no. of redundant bits.

4 redundant, 8 message.

1100 12	1011 11	1010 10	1001 9	10000 8	0111 7	0110 6	0101 5	0100 4	0101 3	0111 2	0010 1	1001 0
$D_7$	$D_6$	$D_5$	$D_4$	$R_8$	$D_3$	$D_2$	$D_1$	$R_4$	$D_0$	$R_2$	$R_1$	

In binary → only one time one = 1 → 1<sup>st</sup> redundant bit placed here  
 2 → 2<sup>nd</sup> " "  
 4 → 3<sup>rd</sup> " "  
 8 → 4<sup>th</sup> " "

Rest position is reserved for data bit.

Values of redundant bits

22

Exclusive OR operation

1 on least significant bit included  
is

$$R_1 = D_0 \oplus D_1 \oplus D_3 \oplus D_4 \oplus D_6$$

1 on next significant position.

$$R_2 = D_0 \oplus D_2 \oplus D_3 \oplus D_5 \oplus D_6$$

$$R_4 = D_1 \oplus D_2 \oplus D_3 \oplus D_7$$

$$R_8 = D_4 \oplus D_5 \oplus D_6 \oplus D_7$$



1010

D<sub>7</sub> D<sub>6</sub> D<sub>5</sub> D<sub>4</sub> R<sub>8</sub> D<sub>3</sub> D<sub>2</sub> D<sub>1</sub> R<sub>4</sub> D<sub>0</sub> R<sub>2</sub>, R<sub>1</sub>

1 0 1 0 0 1 0 1 1 0 0 0

$$R_1 = 0 \ 1 \ 1 \ 0 \ 0 = 0$$

$$R_2 = 0 \ 0 \ 1 \ 1 \ 0 = 0$$

$$R_4 = 1 \ 0 \ 1 \ 1 = 1$$

$$R_8 = 0 \ 1 \ 0 \ 1 = 0$$

$$\begin{array}{r} 0 \\ 1 \\ \hline 1 \end{array} = 1$$

$$R_8 R_4 R_2 R_1 = 0100$$

if no. of 1's  
even = 0

if no. of 1's  
odd = 1

(23)

At receiver  $R_5 = 0$

Receiver again calculated  $R_1, R_2, R_4, R_8$

$$R_1 = 0$$
$$R_2 = D_0 \oplus D_2 \oplus D_3 \oplus D_5 \oplus D_6 = 1$$

$$R_4 = 1$$

$$R_8 \oplus D_5 = 1$$

$$R_8' R_4' R_2' R_1' = 1110$$

↓  
Perform  
exclusive  
OR

different than  
previous  
Now receiver  
know that there  
is error.

↓

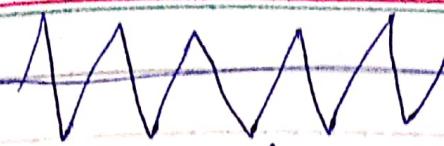
$$R_8 R_4 R_2 R_1 = 0100$$
$$R_8' R_4' R_2' R_1' \underline{1110}$$
$$1010$$

↓

(10)

at position  
10 it  
was an  
error.

on sc



(24)

infinite  
energy = power  
signal

→ Hence receiver have  
ability to correct  
message → no need to transmit msg  
again.

2 → bit error ⇒ then cannot  
fix it

Every system have limitations

$$\text{BER} = 10^{-6} = \frac{1}{1000,000}$$

Redundant bit is lost

0001	}	No need of correction
0010		
0100		
1000		

0001

R<sub>4</sub> lost

received  
→ errors

$$R_8 R_4 R_2 R_1 = 010\underset{1}{1} \quad (1)$$

$$R_8' R_4' R_2' R_1' = 0100 \quad \begin{matrix} \text{originally} \\ \text{data} \\ \text{bit is safe} \end{matrix}$$

Assignment

MATLAB

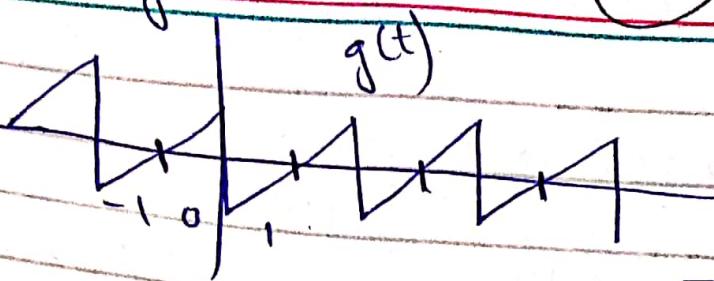
Transmitter  
Receiver

$$c = a \oplus b$$

0001

$$g(t) = 1$$

(25)



$$Pg = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t) dt$$

$$\begin{aligned} Pg &= \frac{1}{2} \int_{-1}^1 t^2 dt \\ &= \frac{1}{2} \cdot \frac{t^3}{3} \Big|_{-1}^{+1} \end{aligned}$$

$$= \frac{1}{6} \left[ (1)^3 - (-1)^3 \right]$$

$$= \frac{1}{6} \times 3 = \frac{1}{2}$$

7 Oct 2019

# Communication Lab

Signal  $\rightarrow$  electromagnetic  
form of  
data .  
  
Mathematical  
expressions .

26

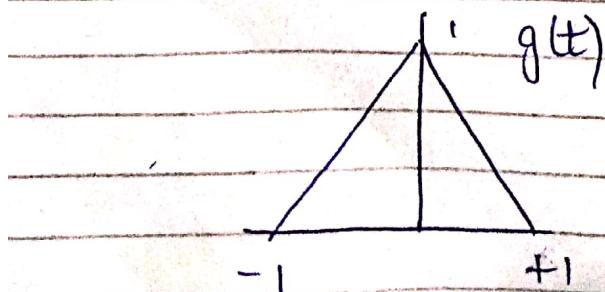
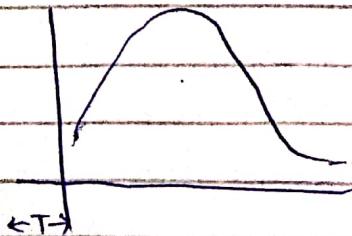
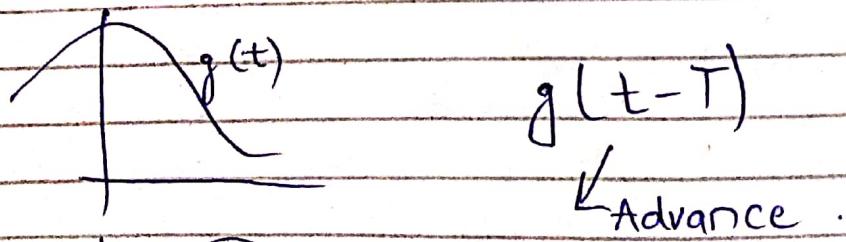
⇒ Here we will assume signal as source of communication.

# Lecture # 5

## Time Shifting Property

$$g(t) = \phi(t + T)$$

$$g(t-\tau) = \phi(t)$$

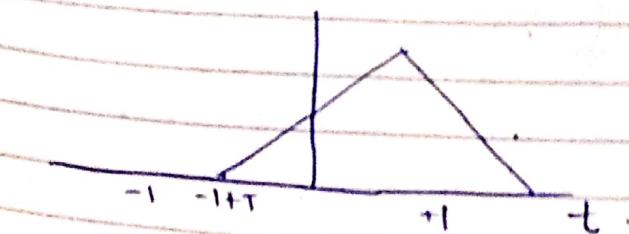


(21)

$$g(t-T)$$

$$t-T = -1$$

$$t = -1+T$$



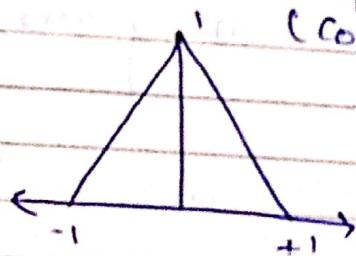
Time Scaling Property

$$g(t) = \phi(t/2)$$

$$g(2t) = \phi(t)$$

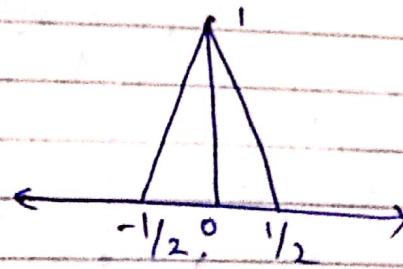
$$\begin{aligned} g(2t) &= ? \\ 2t &= 1 \\ t &= -1/2 \end{aligned}$$

Multiplication  
(Compression)

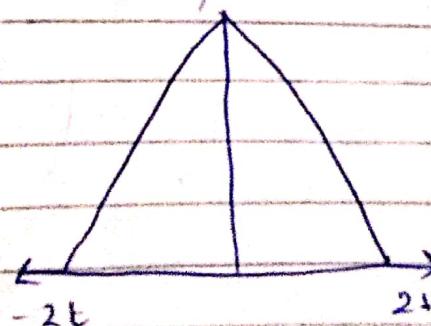


Division

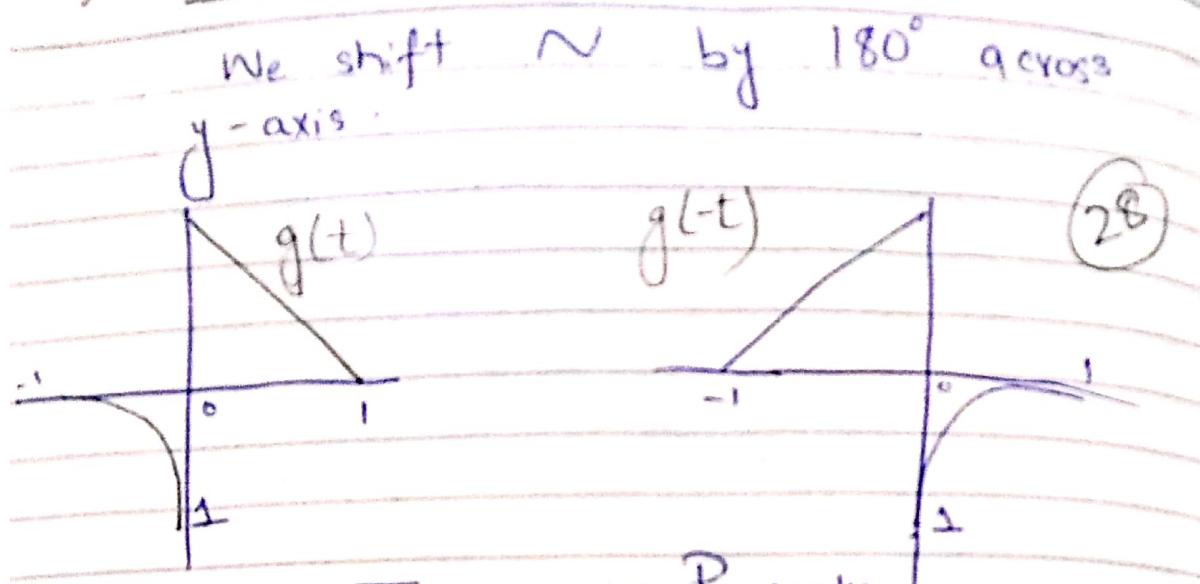
Expansion



$$g(t/2)$$



### 3) Time Inversion:



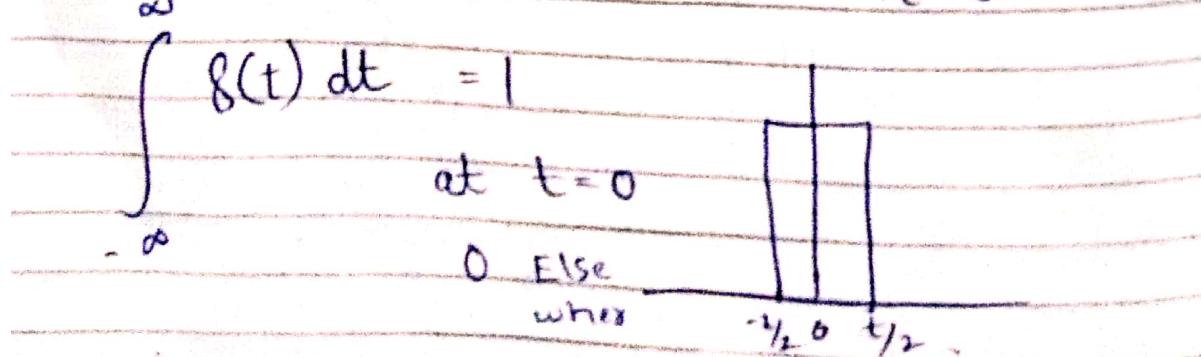
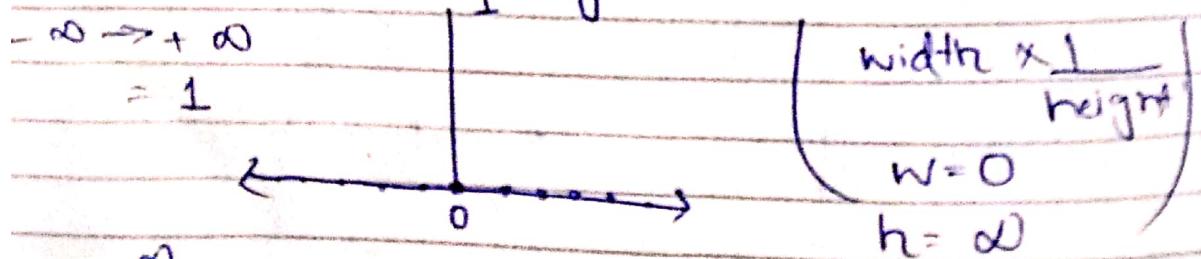
Time inversion Property.

$$\begin{array}{ll} -t = 1 & t = 0 \\ t = -1 & -t = 0 \\ & t = 0 \end{array}$$

$\rightarrow$  invert on y-axis.

### 4) Unit Impulse ftn.

$\delta = 0$  (only defined here)



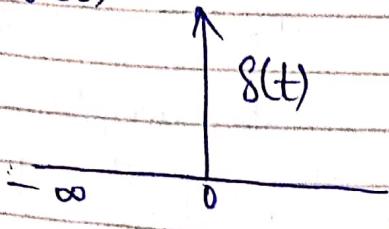
Height: Area under curve = 1 :  $\frac{1}{e}$ .

(29)



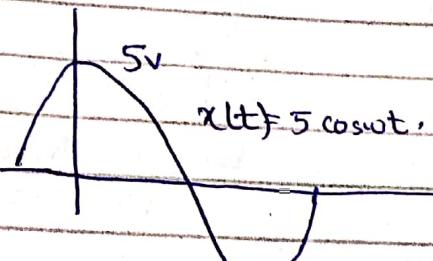
$$e + \frac{1}{e} = \frac{1}{e}$$

$\delta(t)$



$$\int_{-\infty}^{\infty} x(t) \delta(t) dt$$

↓  
derived data function.



$$x(t) = 5\cos\omega t$$

$\delta(t)$



$$= x(0) \int_{-\infty}^{t_0} \delta(t) dt$$

$$= x(0) \Rightarrow 5 \rightarrow$$

We use

Samples kaise le skta  
→ derived data func  $x(t)$ , Sampling

$\Rightarrow$  2 times of the highest frequency component present in signal.

$0 - 4\text{ KHz}$  freq.  $\xrightarrow{\text{Highest}}$

Lapiss  $\rightarrow A \rightarrow D \rightarrow$  take sample  $\rightarrow$  sample rate  
 $\rightarrow$  (2 times the highest frequency component present in signal)

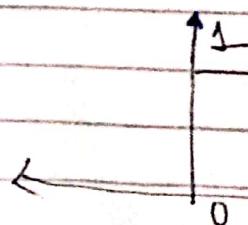
Sampling Rate  $\geq 2B$ .

30

$$2 \times 4\text{ K} = 8\text{ Ksamples}$$

$\downarrow$   
Signal will be recovered with this rate.

Unit Step Function:



$$g(t) = 0 \quad t < 0$$

$$g(t) = 1 \quad t \geq 0$$

This system is known as causal ftr.

started at 0, remain 1 till infinity.

~~Two variable~~

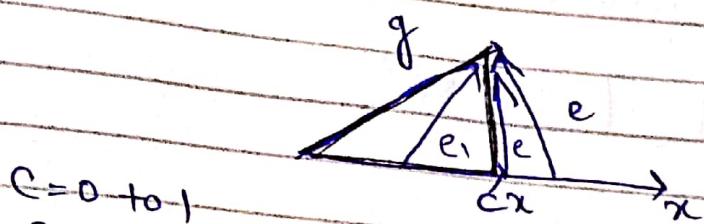
(31)

$$\int_0^\infty g(\tau) d\tau$$

$$\frac{d u(t)}{dt} = s(t).$$

Signals and vector :-

Signals are vectors.



$$c = 0 \text{ to } 1$$

$$0 \leq c \leq 1$$

Tilt  $\rightarrow$  Similarity  
Expand  $\rightarrow$  Difference.

x-vector per projections by

when e  
is perpendicular  
then error will be less.

$$g \approx cx$$

$$g = e + cx.$$

e  $\rightarrow$  vector

$$g = e + c_1 x$$

This is not added  
calculated addition  
this is resultant of  
two vectors

$$g = e + c_2 x$$

$\Rightarrow$  Cos of addition of these two forces

Signals  
are  
similar  
to  
vectors?

NO  
Signal  
are like  
vectors?

NO

Signal  
are  
vector ✓

For right angle  $\Delta$

$\Rightarrow$  Only valid for right angle triangle.

$$|g|^2 = |e|^2 + |c_x|^2$$

$$\cos \theta = \frac{c_x}{|g|} \left( \frac{b}{h} \right)$$

only right angle  $c_x = |g| \cos \theta$

$$c_x(x) = |g| |x| \cos \theta$$

$$c_x^2 = g \cdot x$$

$$c = \frac{g \cdot x}{|x|^2} \rightarrow \text{Minimum error in this case.}$$

$$g(t), x(t)$$

(How much they are related)

(less error)

$$g(t) \approx c_x(t)$$

$$g(t) = e(t) + c_x(t)$$

error of  
fit  
time

$$e(t) = c_x(t) - g(t)$$

formula

What is error energy?

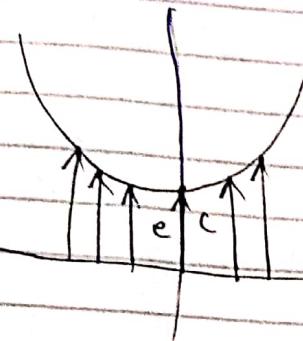
$$\int_{-\infty}^{\infty} e^2(t) dt \quad \int_{-\infty}^{\infty} [x(t) - g(t)]^2 dt$$

$$E_e = \int_{-\infty}^{\infty} c^2 x^2 dt + \int_{-\infty}^{\infty} g^2(t) dt$$

(32)

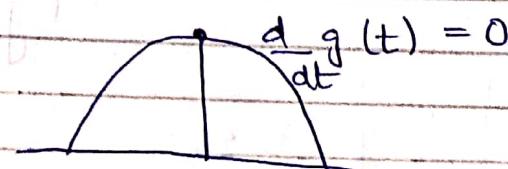
$$= \int_{-\infty}^{\infty} 2c x(t) g(t) dt.$$

$c$  is the  
main contributor  
of error



$E$  of error  
is  
dependant  
on  $c$ .

On one value  
of  $c$ ,  $c$  is minimum otherwise  
increases.



⇒ When Rate of change  $g(t)$  becomes  
Zero.

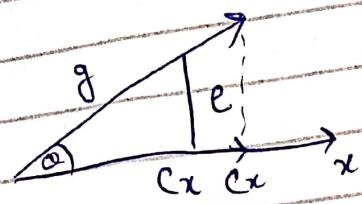
$$\frac{dE_e}{dc} = 0$$

$$\Rightarrow 2c \int_{-\infty}^{\infty} x^2(t) dt + 0 - 0 = 2 \int_{-\infty}^{\infty} x(t) g(t) dt = 0$$

$$c = \frac{\int_{-\infty}^{\infty} x(t) g(t) dt}{\int_{-\infty}^{\infty} x^2(t) dt}$$

This eqtn  
causes  
minimum  
error.

C was  
defective  
error b/w  
 $g$  &  $x$ .



$$g \cdot x = |g||x| \cos \theta$$

$$c_x = \cos \theta = \frac{g \cdot x}{|g||x|}$$

(relation correlation  
b/w co-efficient  
signal)

$$\theta = 0, g = x$$

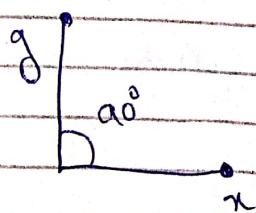
$$\rightarrow g \rightarrow x$$

$g \parallel x$   
similar to  
one another.

( $g$  will be like  
 $x$  and  
is independant  
upon magnitude  
as well)

$$\text{if } \theta = 0 \quad c_n = \cos \theta \Rightarrow \cos 0 = 1$$

$$\theta = 90 \Rightarrow c_n = \cos 90 = 0$$



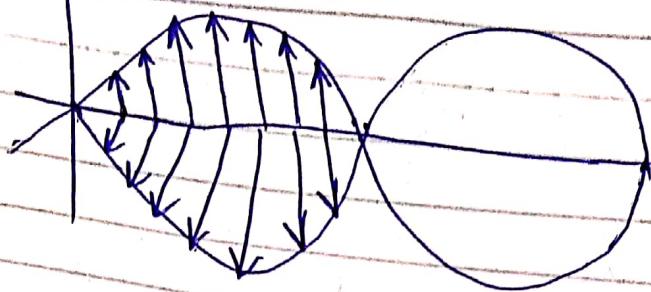
$\Rightarrow$  projection

(orthogonal to  
one  
another)

Same frequency  
Signal  $\rightarrow$   
is orthogonal

to each other  $\rightarrow$   
they won't mix.

$\rightarrow$  such  
signals  
can be  
isolated.



$$Q = 180^\circ, C_x = \cos 180 = -1$$

$$-1 \leq C \leq 1$$

$$C_x = \frac{\int g(t)x(t) dt}{\sqrt{E_g} \sqrt{E_x}}$$

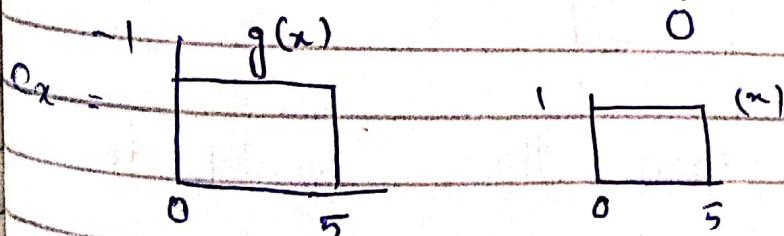
$C_x = 0$   
 have no effect  
 on one another  
 (strangers)  
 $C_x = -1$  (enemies)  
 $C_x = 1$  (friends)

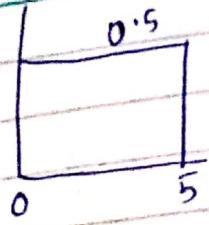
$$C_x = \frac{1}{\sqrt{E_g} \sqrt{E_x}} \int_{t_1}^{t_2} g(t)x(t) dt$$

$$\|g\| = \sqrt{\int_{t_1}^{t_2} g^2(t) dt}$$

if  $g \perp x$   
 $\downarrow$   
 orthogonal

$g^2$  is energy  
 in  
 signals  
 $\sqrt{E}$





$$C_x = \frac{1}{\sqrt{5} \sqrt{1.25}} \left\{ 1 - x \cdot 0.5 dt \right\}$$

$$= \frac{1}{\sqrt{5 \times 1.25}} \quad 0.5 + \Big|_0^5$$

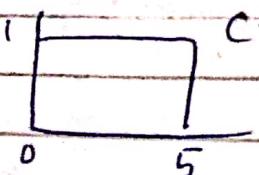
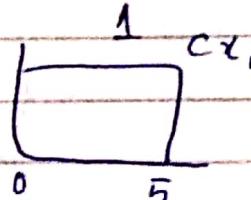
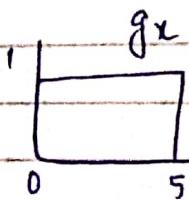
$$= \frac{1}{\sqrt{5 \times 1.25}} \quad 2.5$$

(35)

$$= \frac{2.5}{\sqrt{6.25}}$$

$$= \frac{2.5}{2.5}$$

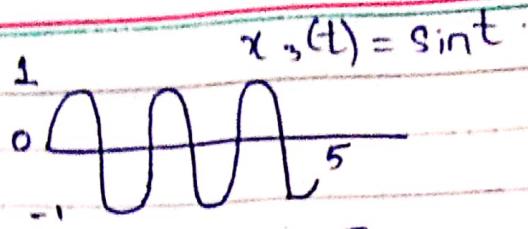
$$= 1$$



Independant upon the amplitude.

$C_{x_1}, \Sigma_1$

$C_{x_2}$  are  
bffriens of  
 $g(x)$



$$E_{x_3} = \int_0^5 \sin^2(t) dt$$

$$= \int_0^5 \left[ 1 - \frac{\cos 2t}{2} \right] dt$$

$$= \int_0^5 \frac{1}{2} + - \int_0^5 \cos 2t dt$$

$$= \frac{1}{2} t$$

$$= \frac{5}{2} = 2.5$$

$$C_x = \frac{1}{\sqrt{5} \sqrt{2.5}} \int_0^5 1.5 \sin t dt = 0.$$

## Lecture #

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

we want to  
see signal  
in

$$g(t) = \sum_{n=-\infty}^{\infty} c_n \cos(n\omega_0 t + \phi_n)$$

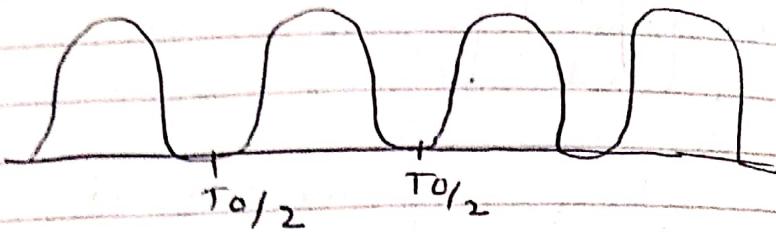
we want to  
see system

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

in frequency

$$D_n = \frac{1}{T_0 - T_0/2} \int_{T_0/2}^{T_0} g(t) e^{-jn\omega_0 t} dt$$

Response



if  $T_0$  increases, loops will be more far away from each other.

(32)

$$T_0 \rightarrow \infty$$

Right & left loops are eliminated.

$$\lim_{T_0 \rightarrow \infty} g_{(T_0)}(t) = g(t)$$

↓  
Periodic  
Signal

↓  
when limit  
→ infinity

↓  
Aperiodic

integrate + limits → expression  
will not be

dependant on

$$(j\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

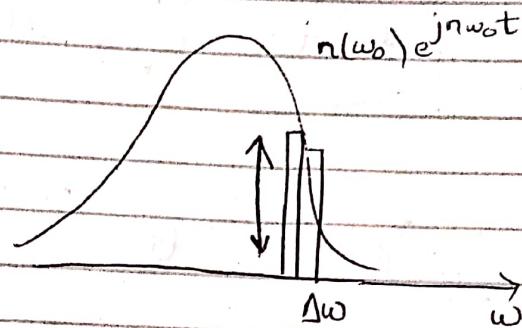
$$D_n = \frac{G(n\omega_0)}{T_0}$$

$$\omega_0 = \frac{2\pi}{T_0} \Rightarrow T_0 = \frac{2\pi}{\omega_0}$$

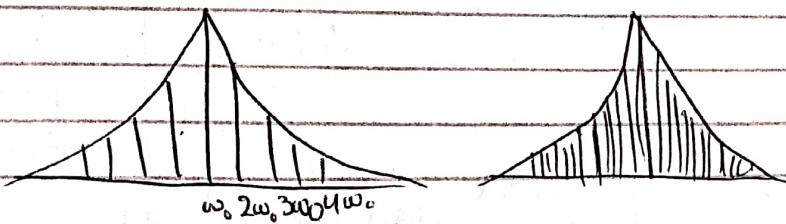
$$g(t) = \sum_{n=-\infty}^{\infty} \frac{G(n\omega_0)}{T_0} e^{jn\omega_0 t}$$

(3)

$$\begin{aligned} \omega &= 0 \\ D_n &= \infty \end{aligned}$$



$$= \sum_{n=-\infty}^{\infty} G(n\Delta\omega) \frac{\Delta\omega}{2\pi} e^{jn\omega_0 t}$$



if  $T_0$  doubles  
 $\omega_0$  becomes  
 half

if  $T_0 = \infty$

$$D_n = 0$$

The graph  
 become

denser {

Compressed

$$g(t) = \sum_{n=-\infty}^{\infty} G(n\Delta\omega) \frac{\Delta\omega}{2\pi} e^{jn\omega_0 t}$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(n\omega) e^{jn\omega_0 t} d\omega$$

$\hookrightarrow$  small change  
 in  $\omega$ .

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} dt$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt \quad -\text{imp}$$

if signal is in  $\omega$ -domain we  
can find its equivalent in  
time Domain.

1

$$g(t) = \frac{1}{2\pi} G(\omega) e^{j\omega t} dt. \quad -\text{imp}$$

$$g(t) \Leftrightarrow G(\omega)$$

(3a)

$$g(t) = e^{-at} u(t)$$

$G(\omega)$

$0 \rightarrow \infty$   
 $e^{at}$   
unit step  
function

$$G(\omega) = \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt.$$

$$G(\omega) = \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt + \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$\int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt$$

~~When base is same powers added~~

$$G(w) = \frac{e^{at + j\omega t}}{e^{at + j\omega t}} \Big|_0^\infty$$

$$= \frac{1}{a + j\omega_0} (e^{-\infty} - e^0)$$

$$= \frac{1}{(a + j\omega_0)} (0 - 1)$$

(40)

$$= \frac{1}{a + j\omega_0}$$

That single jo zero par converge  
nahi hota jo uska Fourier signal  
nahi hota and is known as.

$$\int_{-\infty}^{\infty} |g(t)| dt < \infty$$

$$G(w) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$G(\omega) = |G(\omega)| e^{j\angle G(\omega)}$$

↓  
magnitude

amplitude

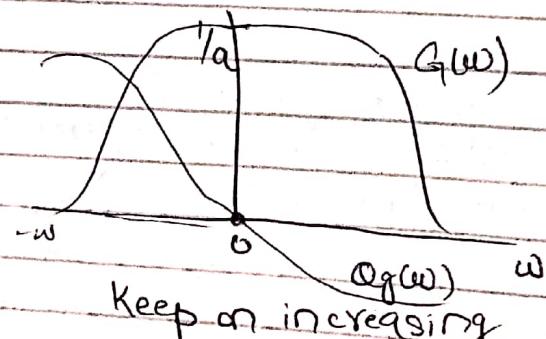
Find Amplitude:

$$G(\omega) = \frac{1}{\sqrt{a^2 + \omega^2}} e^{j \tan^{-1} \omega/a}$$

$$G(\omega) = \frac{1}{\sqrt{a^2 + \omega^2}} e^{-j \tan^{-1} \omega/a}$$

$$G(\omega) = \frac{1}{\sqrt{a^2 + \omega^2}}$$

(U)



when  
 $\omega=0$   
 $G(\omega)=\frac{1}{a}$

Keep on increasing  $\omega$   
 $G(\omega)$  decreases.

$$\angle G(\omega) = \tan^{-1} (\omega/a)$$

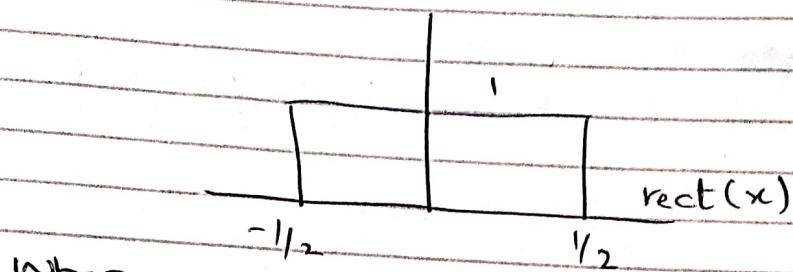
$$\angle G(\omega) = 0 \quad \omega=0$$

All the discussion is valid

the  $e^{-at}$   $a > 0$  otherwise.  
it will never positive & converge.

$\epsilon$  no fourier integral.

## Unit Gate Function :



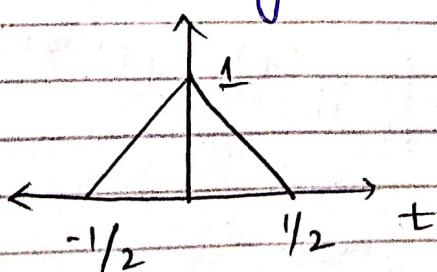
When width & height is  $1$   
then it is known as  $\text{rect}$ .

$$\text{rect}(x) = 0, |x| > \frac{1}{2}.$$

$$\text{rect}(x) = 1, |x| < \frac{1}{2}.$$

$$\Rightarrow \text{rect}\left(\frac{x}{\text{width of signal}}\right)$$

## Unit Triangle Function .



if  $-T/2 \leq t \leq T/2$

$$\begin{aligned} \Delta x &= 0, |x| > \frac{T}{2} \\ &= 1 - 2|x|, |x| < \frac{T}{2} \end{aligned}$$

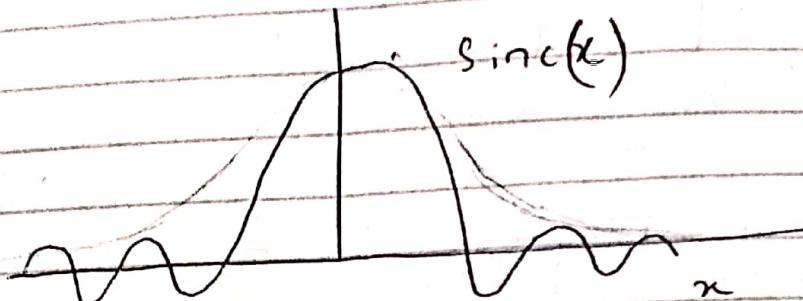
$$\Delta x = 0, |x| > \frac{1}{2}$$

$$= 1 - 2|x|, |x| < \frac{1}{2}$$

# Interpolation Function

$$\text{Sinc}(x) = \frac{\sin x}{x}$$

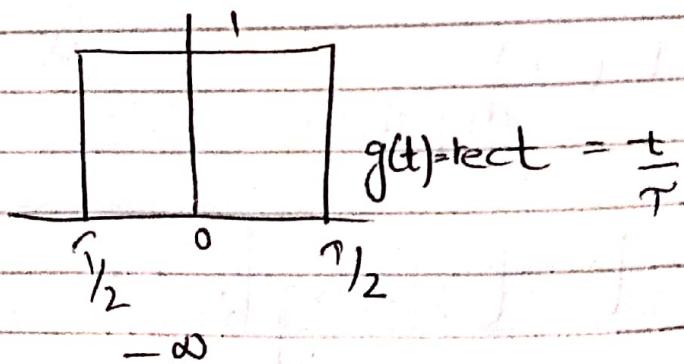
~~x~~ Sinc x will be 0  
at all values except for  $x=0$ .



for

$$x = \pm n\pi, \quad \text{sinc } x = 0$$

(VM)



$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega_0 t} dt$$

$$= \int_0^{\infty} 1 e^{-j\omega t} dt$$

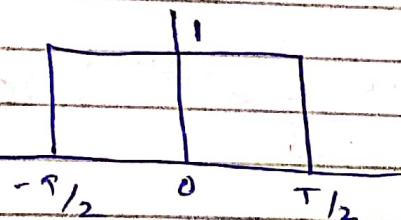
$$\Rightarrow \frac{e^{-j\omega t}}{-j\omega} \Big|_{-T/2}^{T/2}$$

$$= \frac{1}{-j\omega}$$

$\text{uu}$

## Lecture #

$$g(t) = \text{rect}$$



component  
of F.S  
are

orthogonal  
to one  
another.

$$g(t) = \text{rect}\left(\frac{t}{T}\right)$$

$$G(\omega) = \int_{-T/2}^{T/2} \text{rect}\left(\frac{t}{T}\right) e^{-j\omega t} dt$$

frequency

Response

$$* = \frac{e^{-j\omega t}}{-j\omega} \Big|_{-T/2}^{T/2}$$

Exponential F.S :-

$$g(t) = \sum_{n=-\infty}^{\infty} [D_n e^{jn\omega_0 t}]$$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t) e^{-jn\omega_0 t} dt$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega.$$

$\star$

$$G(\omega) = \frac{e^{-j\omega T/2} - e^{j\omega T/2}}{-j\omega}$$

1

$M_{\text{mag}}(t)$

$$= \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\omega}$$

(1)

$$= 2 \left( \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{\omega} \right) / 2j$$

$$\sin Q = \frac{e^{jQ} - e^{-jQ}}{2j}$$

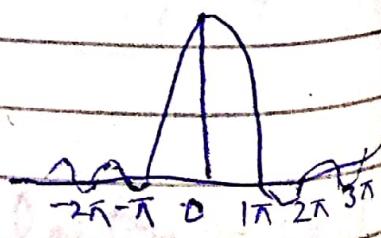
$$2 \frac{\sin \omega T/2}{\omega}$$

$$= \frac{T \sin \frac{\omega T/2}{2}}{\omega T}$$

$$\Rightarrow \text{Sinc } Q = \frac{\sin Q}{Q}$$

$Q = \frac{1}{2} \pi$  it will give one

$$= \frac{T \text{Sinc } \omega T}{2}$$



In time domain  
 $BW \propto \omega$ .

$BW$  is diff b/w two frequencies

$$\textcircled{Q} = T \omega \pi$$

$$\frac{\omega T}{2} = \pi$$

$$\boxed{\omega = \frac{2\pi}{T}} \quad \textcircled{=1}$$

$$\frac{\omega T}{2} = 2\pi$$

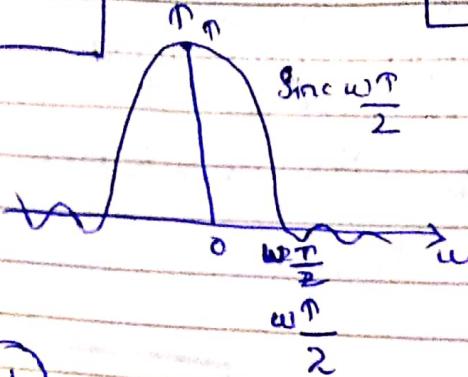
$$\boxed{\omega = \frac{4\pi}{T}} \quad \textcircled{=2}$$

$$\frac{\omega T}{2} = 3\pi$$

$$\boxed{\omega = \frac{6\pi}{T}} \quad \textcircled{=3}$$

time duration when reduced  
 $BW$  increases

freq  
no. of revolutions per sec



46

$\Rightarrow$  The width b/w  $\textcircled{=2\pi}$  is dependant upon  $T$ .

freq is a time term

$0-2\pi$  is the bandwidth of rect.

$T$  (two) is the width of the signal

if width is decreased, the sys will become fast & will occupy

if  $T$  (width) decreases the bandwidth increases

$\Rightarrow$  if  $10 \text{ kHz} = \text{total}$   
 $1 \text{ kHz}$  to 10 user  
if bits per second is increased  
 $2 \text{ kHz} \Rightarrow 5 \text{ user}$ .  
Users decreased.

AMP  
Advanced 1G  
Mobile  
Phone  
System.

PkTEL  $\rightarrow 30 \text{ kHz}$

Instaphone need  $4 \text{ kHz}$

they wanted to  
provide us with

GSM

$\downarrow$  FDMA/TDMA  
 $200 \text{ kHz}$

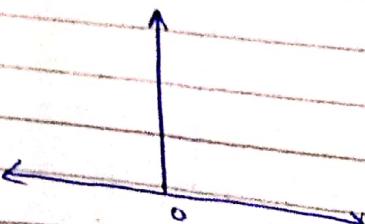
CDMA - 2000  
 $5 \text{ MHz}$



$\downarrow$   
Frequency  
Division  
Multiplexer

$$g(t) = \delta(t)$$

$\delta(t)$



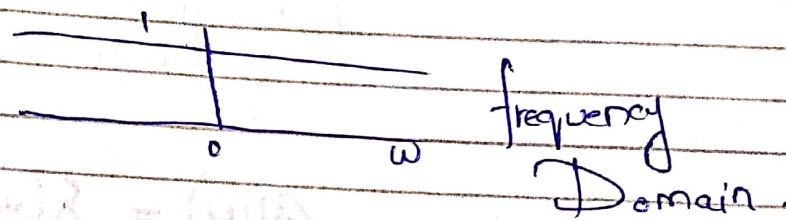
The function  
which don't  
any width  
is  
unit  
impulse  
fn

$$G(w) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$= 1$$

$$f(t) \Leftrightarrow 1$$

T.D



$$G(w) = \delta(w)$$

$$\underset{US}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} 1 e^{j\omega_0 t} d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} 1 e^{j\omega_0 t} d\omega$$

$$= \frac{1}{2\pi} \times 1$$

$\leftarrow$  T.D

$$g(t) \xrightarrow{T.D} = \frac{1}{2\pi} \Leftrightarrow \delta(w)$$

$\curvearrowright$  F.D.

$$\frac{2\pi \times 1}{2\pi} \Leftrightarrow 2\pi f(\omega)$$

$$1 \Leftrightarrow 2\pi g(\omega)$$

$$2\pi g(\omega)$$

frequency Domain  
function.

In  
Time  
domain  
length is  
very  
 $\infty \rightarrow -\infty$

in freq  
bit point  
decrease  
to zero

$$G(\omega) = \delta(\omega - \omega_0)$$

$$g(t) = ?$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega - \omega_0) e^{j\omega_0 t} d\omega$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega_0 - \omega_0) e^{j\omega_0 t} d\omega$$

$$\omega - \omega_0 = 0$$

$$\omega = \omega_0$$

$$\omega = \omega_0$$

$$= \frac{e^{j\omega_0 t}}{2\pi} \Leftrightarrow \delta(\omega - \omega_0)$$

$$e^{j\omega_0 t}$$

T.D

FD

$$e^{-j\omega_0 t} \Leftrightarrow 2\pi(\omega + \omega_0)$$

(5)

$$e^{-j\omega_0 t} \Leftrightarrow 2\pi(\omega + \omega_0)$$

28/10/19

## INVERSE FOURIER

Transform.

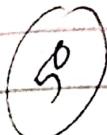
$$\delta(\omega - \omega_0)$$

$$F^{-1}[\delta(\omega - \omega_0)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_0) e^{j\omega t} d\omega$$

\*

$$= \frac{1}{2\pi} e^{j\omega_0 t}$$

$\omega = \omega_0 \rightarrow \text{unit impulse.}$

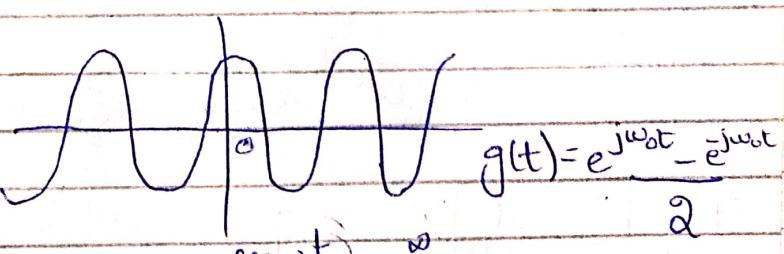


$$\frac{1}{2\pi} e^{j\omega_0 t} \Leftrightarrow \delta(\omega - \omega_0)$$

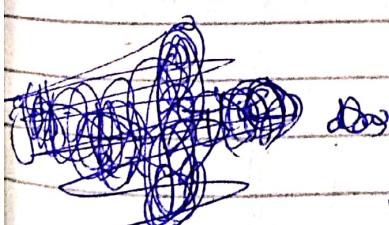
$$e^{j\omega_0 t} \Leftrightarrow 2\pi \delta(\omega - \omega_0)$$

$$e^{-j\omega_0 t} \Leftrightarrow 2\pi \delta(\omega + \omega_0)$$

$$g(t) = \cos \omega_0 t$$



$$g(t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2}$$



$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

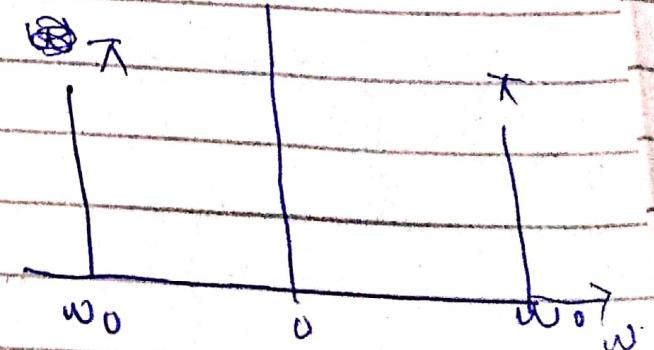
$$G(\omega) = \int_{-\infty}^{\infty} \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2} e^{-j\omega t} dt$$

$$\begin{aligned}
 G(w) &= \frac{1}{2} \int_{-\infty}^{\infty} e^{j\omega_0 t} e^{-j\omega t} dt + \frac{1}{2} \int_{-\infty}^{\infty} e^{-j\omega_0 t} e^{-j\omega t} dt \\
 &= \frac{1}{2} [2\pi \delta(\omega - \omega_0) + 2\pi \delta(\omega + \omega_0)] \\
 &= \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]
 \end{aligned}$$

$$G(w) = \int_{-\infty}^{\infty} \cos \omega_0 t e^{j\omega t} dt$$

$$\begin{aligned}
 &= \frac{1}{2} \int_{-\infty}^{\infty} e^{j\omega_0 t} e^{-j\omega t} dt + \frac{1}{2} \int_{-\infty}^{\infty} e^{-j\omega_0 t} e^{-j\omega t} dt \\
 &= \frac{1}{2} \left[ e^{-j(\omega - \omega_0)t} dt \right] + \frac{1}{2} \int_{-\infty}^{\infty} e^{-j(\omega + \omega_0)t} dt \\
 &= \frac{1}{2} [2\pi \delta(\omega - \omega_0) + 2\pi \delta(\omega + \omega_0)] \\
 &= \pi (\delta(\omega - \omega_0) + \delta(\omega + \omega_0))
 \end{aligned}$$

$$F = [\cos \omega_0 t] = \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$



$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

## Duality Property

If you having shift in time  
so, we change the corresponding  
signal in T.D to F.D.

$$g(t - t_0) \leftrightarrow G(\omega) e^{-j\omega t_0}$$

$$g(t) e^{j\omega_0 t} \leftrightarrow G(\omega - \omega_0).$$

if there is shift so in f  
time

## Scaling Property:

$$g(t) \leftrightarrow G(\omega)$$

$$g(at) \leftrightarrow \frac{1}{|a|} G(\omega/a)$$

$$F[g(at)] = \int_{-\infty}^{\infty} g(at) e^{-j\omega t} dt \quad \text{if } a > 1 \text{ compress}$$

if  $a < 1$  expand

$$x = at$$

$$t = \frac{x}{a}$$

$$dt = dx/a$$

$$\rightarrow \int_{-\infty}^{\infty} g(x) e^{-j\frac{\omega}{a} x} dx$$

53

$$= \frac{1}{a} \int_{-\infty}^{\infty} g(x) e^{-j\frac{\omega}{a} x} dx$$

$$\int_0^{\infty} g(u) e^{-j\omega u} du$$

$$= \frac{1}{a} G_1(\omega/a)$$

$$G_1(\omega)$$

$$\boxed{g(t) = e^{-at} u(t) + e^{at} u(-t)}$$



$$g(t) = e^{-at}$$

bit per sec

$$G(\omega) = \int_{-\infty}^{\infty} (e^{-at} + e^{at}) dt$$

More space covered

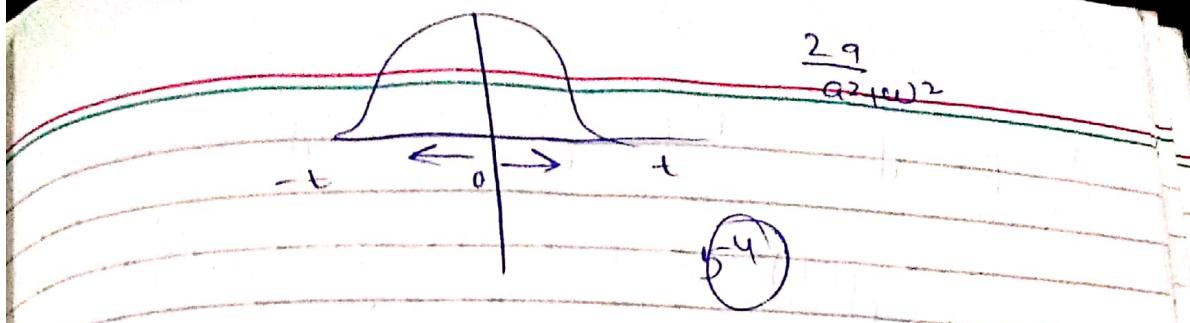
in Fourier Domain

$$G(\omega) = \int_0^{\infty} e^{-at} \frac{e^{-j\omega t}}{dt} + \int_0^{\infty} e^{-at} \frac{e^{-j\omega t}}{dt}$$

$$\Rightarrow e^{-\frac{(a+j\omega)t}{-(a+j\omega)}} \Big|_0^\infty + e^{\frac{(a-j\omega)t}{a-j\omega}} \Big|_0^\infty$$

$$= \frac{a-1}{-(a+j\omega)} + \frac{1}{a-j\omega} \cdot 0 + (1)$$

$$= \frac{a+j\omega + a-\omega}{a^2+\omega^2} = \frac{3}{a^2+\omega^2}$$



$\Rightarrow \omega \uparrow$  Amplitude decrease

## Time Shifting Property.

$$g(t) = G(\omega)$$

$$g(t-t_0) \Leftrightarrow G(\omega) e^{-j\omega t_0}$$

$$F[g(t-t_0)] = \int_{-\infty}^{\infty} g(t-t_0) e^{-j\omega t} dt$$

~~$t = t_0$~~   $2et$

$$t - t_0 = x$$

$$t = x + t_0$$

$$dt = dx$$

$$= \int_{-\infty}^{\infty} g(x) e^{-j\omega(x+t_0)} dx$$

$$e^{-j\omega t_0} \int_{-\infty}^{\infty} g(x) e^{-j\omega x} dx$$

$$\Rightarrow e^{-j\omega t_0} G(\omega) \quad \checkmark$$

## Frequency Shifting Property

$$g(t) e^{j\omega_0 t} \Leftrightarrow G(\omega - \omega_0)$$

③ 6)  $\mathcal{F}[g(t) e^{j\omega_0 t}] =$

$$\Rightarrow \int_{-\infty}^{\infty} \mathcal{F}[g(t) e^{j\omega_0 t}] e^{-j\omega t} dt.$$

$$= \int_{-\infty}^{\infty} g(t) e^{j\omega_0 t} e^{-j(\omega - \omega_0)t} dt.$$

$$= G(\omega - \omega_0).$$

$$[g(t) \cos \omega_0 t]$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cos \omega_0 t$$

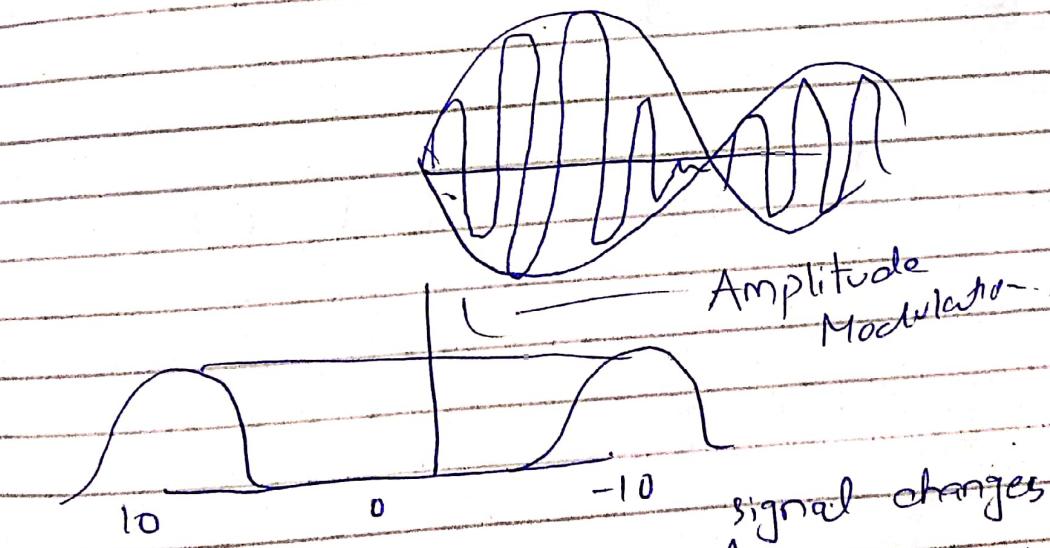
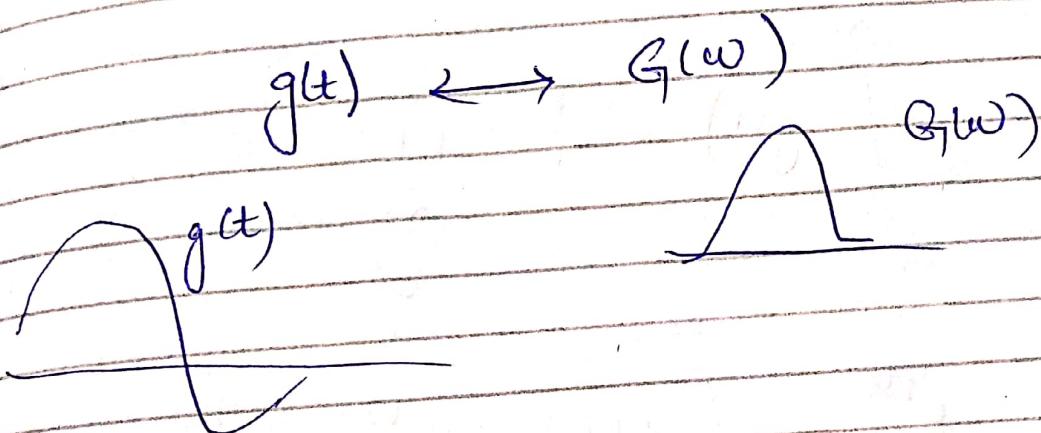
$$= \int_{-\infty}^{\infty} g(t) e^{j\omega_0 t} + \underbrace{g(t) e^{-j\omega_0 t}}_{e^{-j\omega t}} dt$$



$e^{-j\omega t}$

$$\int_{-\infty}^{\infty} g(t) e^{-j(\omega - \omega_0)t} dt + \int_{-\infty}^{\infty} g(t) e^{-j(\omega + \omega_0)t} dt$$

$$\Rightarrow \frac{G(\omega - \omega_0)}{2} + \frac{G(\omega + \omega_0)}{2}$$



$\Rightarrow$  Coz AM of carries in according to value of base band sig. rate.

signal changes.

$$g(t) \cos(\omega_0 t + \varphi_0)$$

$$G(w) = \int_{-\infty}^{\infty} g(t) \cos(\omega_0 t + \varphi) dt$$

(6A)  $\Rightarrow \int_{-\infty}^{\infty} g(t) e^{j\omega_0 t + \varphi} dt = \frac{e^{j\omega_0 t + \varphi}}{2} \left[ e^{-j\omega_0 t - \varphi} \right]_{-\infty}^{\infty}$

$$\frac{1}{2} \int_{-\infty}^{\infty} g(t) e^{j\omega_0 t} dt + \frac{1}{2} \int_{-\infty}^{\infty} g(t) e^{-j\omega_0 t} dt$$

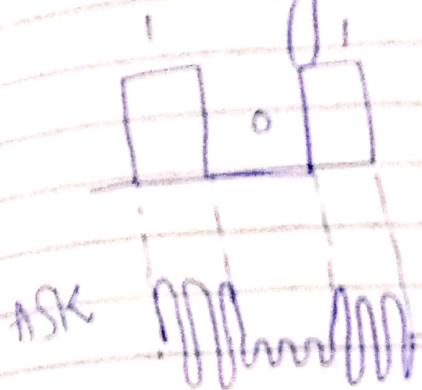
$$\frac{1}{2} e^{j\varphi_0} \int_{-\infty}^{\infty} g(t) e^{(w-w_0)t} dt + \frac{1}{2} e^{-j\varphi_0} \int_{-\infty}^{\infty} g(t) e^{-(w-w_0)t} dt$$

$$\Rightarrow \frac{1}{2} e^{j\varphi_0} G(w-w_0) + \frac{1}{2} e^{-j\varphi_0} G(w-w_0)$$

# Lecture :

## Signal transmission

(a) through linear System.



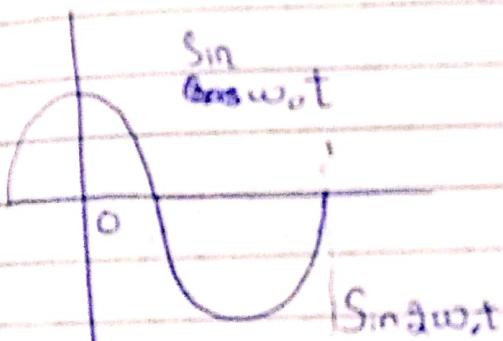
Modulation  
Techniques

→ PCM

→ Amplitude Shift Keying

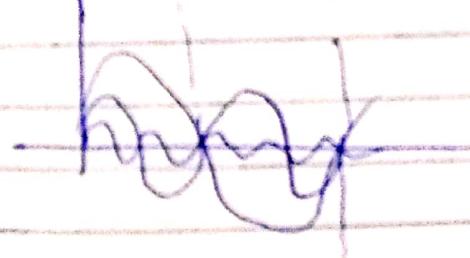
→ Frequency Shifting

## Properties :-



Cosine, cosine & Sine

↓  
all will be  
orthogonal to  
each other.



→ if every one is  
delayed same time

then, there will be

Fundamental

component → Harmonics

no

change  
in  
signal at  
other side.

$\theta_{\text{out}}$

(5)

Phase angle

if a  $f(t)$  has high freq., then

Phase angle increases in proportion

$$\theta = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

$f \propto p$

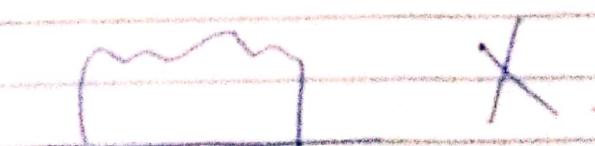
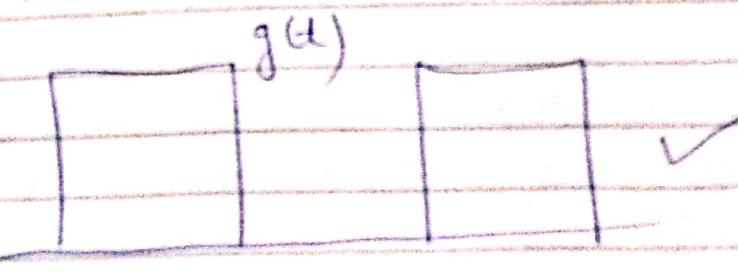
$$= -\left(\frac{\omega}{a}\right) \rightarrow \text{Q angle}$$

for distortion  
180°

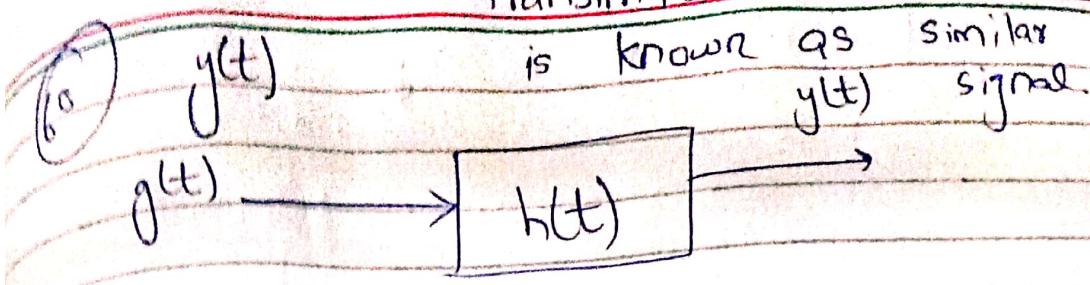
Linear  
freq

→ Every particular comp. of F.S is delayed around and there is no distortion.

→ Phase should be linear fn. of  $\omega$ .



if we get distortion less transmission then it



$$y(t) = g(t) * h(t) \text{ (convolve)}$$

$$\textcircled{1} \quad Y(\omega) = G(\omega)H(\omega) \quad (\text{multiplication})$$

$$G(\omega) = |G(\omega)| e^{j\alpha_g(\omega)}$$

$\downarrow$                      $\downarrow$                      $\downarrow$   
Basic signal      amplitude      base angle.

$$|Y(\omega)| e^{j\alpha_g(\omega)}$$

$$= |G(\omega)| e^{j\alpha_g(\omega)} |H(\omega)| e^{j\alpha_h(\omega)}$$

$$|Y(\omega)| = |G(\omega)| |H(\omega)|$$

$$Q_y(\omega) = Q_g(\omega) + Q_h(\omega)$$

$$g(t) = k g(t - t_d)$$

$$\textcircled{2} \quad Y(\omega) = K G(\omega) e^{-j\omega t_d}$$

$$H(\omega) = K \frac{G(\omega) e^{-j\omega t_d}}{G(\omega)}$$

$$|H(\omega)| e^{j\alpha_h(\omega)} / H(\omega) = K e^{-j\omega t_d}$$

$$|H(\omega)| = K$$

(6)

condition  
for  
distortion  
less

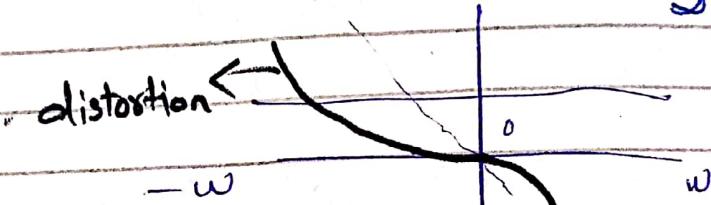
Transmission.

$$\Theta_h(\omega) = -\omega t_d$$

$$\frac{d\Theta_h(\omega)}{d\omega} = -t_d$$

keep on  
increasing  
 $\omega$

~~to decrease~~, }  
①. \* ③



Properties of Distortion ~

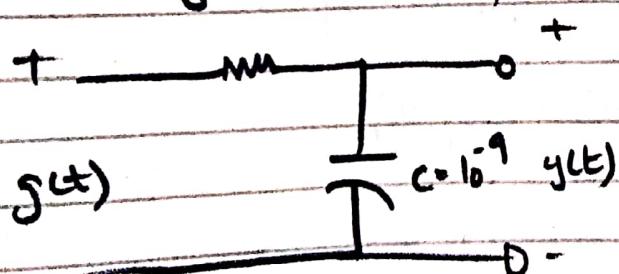
Amplitude constant

phase angle

(linear fn of  $\omega$ )

distortion  
less

## Voltage Division Rule:



$$Y(\omega) = \frac{Y_{j\omega C}}{R + \frac{1}{j\omega C}}$$

shape (signal)

that take

change

ratio nota

jab delay

same comp to  
same amount se  
no.

$$\Rightarrow \frac{R_2}{R_1 + R_2} \quad \textcircled{62}$$

$$= \frac{1}{1 + j\omega RC}$$

$$= \frac{1/R_C}{1/R_C + j\omega} \quad \textcircled{63}$$

$$= \frac{a}{a + j\omega} \quad \boxed{a = 1/R_C}$$

$$|Y(\omega)| e^{j\phi_y(\omega)} =$$

Magnitude

$$|Y(\omega)| = \frac{\sqrt{a^2}}{\sqrt{a^2 + \omega^2}} = \frac{a^2}{\sqrt{a^2 + \omega^2}}$$

$$\phi_y(\omega) = \tan^{-1} \frac{\omega}{a} \quad \begin{matrix} a \\ b \end{matrix} \Rightarrow \frac{a}{\sqrt{a^2 + \omega^2}} e^{j \tan^{-1} \frac{\omega}{a}}$$

$$= \boxed{e^{j \tan^{-1} \frac{\omega}{a}}}$$

$$|Y(\omega)| e^{j\phi_y(\omega)} = a e^{-j \tan^{-1} \frac{\omega}{a}}$$

$$\sqrt{a^2 + \omega^2}$$

$$|Y(\omega)| = \frac{a}{\sqrt{a^2 + \omega^2}}$$

$$Q_y(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

(3)  $a = \frac{1}{R_e} = \frac{1}{10 \times 10^9}$

$$a = 10^9$$

$$\omega \ll a$$

$$Q_y(\omega) = 1$$

$\omega \ll$   
very small

as compared

to  $a$

$$|Y(\omega)| = \frac{a}{\sqrt{a^2 + \omega^2}} \approx 1$$

$$\sqrt{a^2 + \omega^2}$$

$$|Y(\omega)| = \frac{a}{\sqrt{a^2 + \omega^2}}$$

$B_y \uparrow$

omega  
approximation  
ratio  
value  $\gg 0$

distortion

$$\text{if } Q_y(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right) \text{ if } \omega \neq 0$$