

finals Course

→ Distortion less transmission

$$g(t) \xrightarrow{h(t)} y(t) = g(t) + h(t)$$

$$Y(\omega) = G(\omega) \cdot H(\omega)$$

→ Delay ✓

→ same frequency attenuation.

$$y(t) = k g(t - t_d)$$

$$Y(\omega) = k G(\omega) e^{-j\omega t_d}$$

$$H(\omega) = k e^{-j\omega t_d}$$

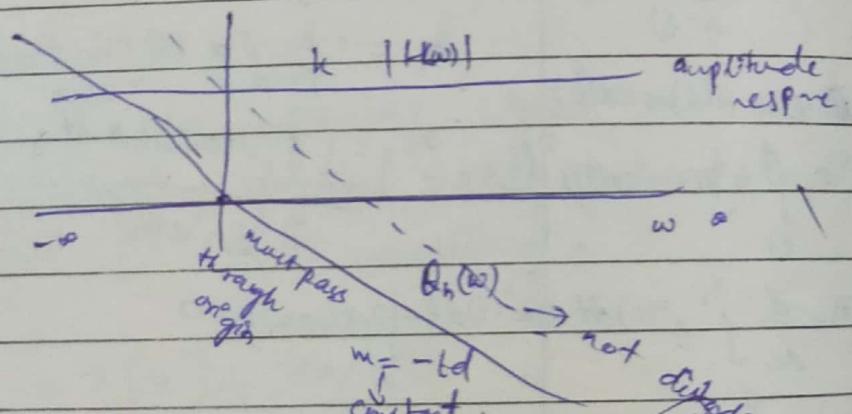
magnitude

$$\Theta_h(\omega) = -\omega t_d$$

$$\frac{d\Theta_h(\omega)}{d\omega} = -t_d$$

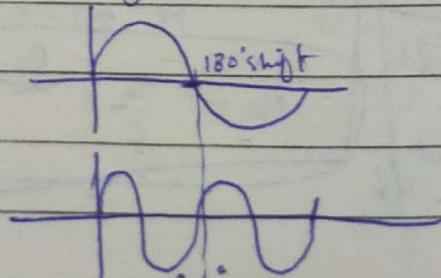
delay possible  
but shape not  
change.

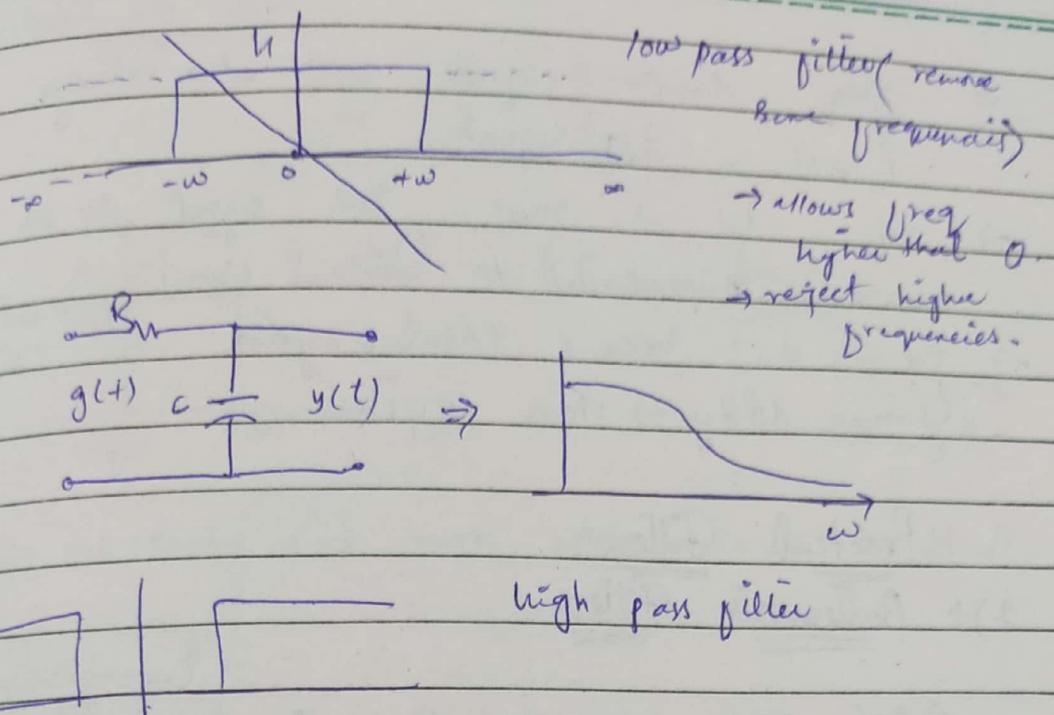
→ every freq.  
component delay  
with same d.



→ amplified response const for all freq components. (attenuate with same amount, shape of sig will not change).

→ all components delay with same (higher freq undergoes more delay).





⇒ Ideal Filters

$-\omega$  to  $+\omega$  or  $-2\pi B$  to  $+2\pi B$ .

Constant gain.

(all higher frequencies will be rejected)

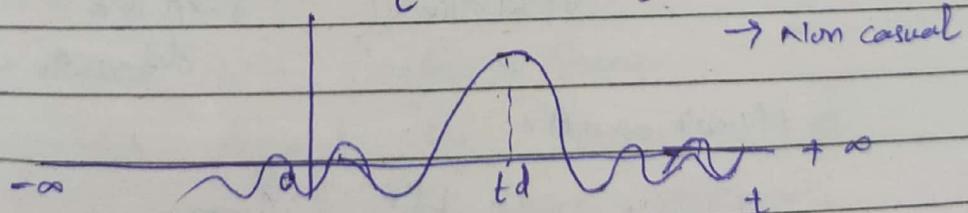
Not exists in reality:

$$H(\omega) = \text{rect} \left( \frac{\omega}{2\omega_0} \right) e^{-j\omega t_d}$$

variation integrat.  $\int \left( \frac{t}{\tau} \right)$

frequency.

$$h(t) = \frac{\omega_0}{\pi} \text{sinc} [\omega_0(t-t_d)]$$

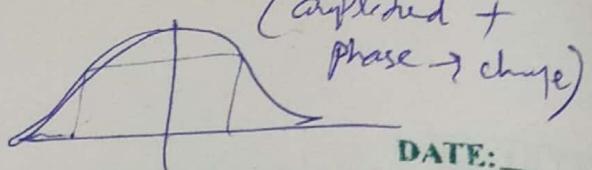


→ non-causal never exist in history.

→ removes components before 0.

$$h^u(t) = h(t) \cdot u(t)$$

$$u(t) = 0 \quad t < 0$$



- freq domain signal will also change.  
(not a rect signal)
- If  $t_d$  is more, the signal will be more approximated to ideal signal.
- $t_d \rightarrow \infty$ , becomes ideal again.  
↳ more delay  $\rightarrow$  slows system.

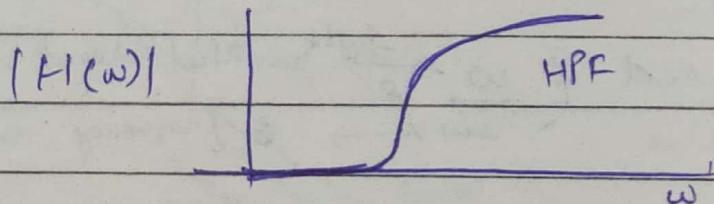
### Practical Filters

#### 1) Butterworth Filters

→ For lower freq, it will attenuate signal

$$|H(\omega)| = \frac{A_0}{\sqrt{1 + (\omega_0/\omega)^{2n}}} \quad B$$

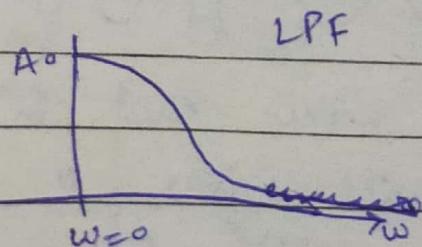
if  $\omega = 0$ ,  $B = \infty$ ,  $\Rightarrow |H(\omega)| = 0$



$$H(\omega) = \frac{A_0}{\sqrt{1 + (\omega/\omega_0)^{2n}}}$$

If  $\omega_0 = 0$   
denominator = 1

$$H(\omega) = A_0$$

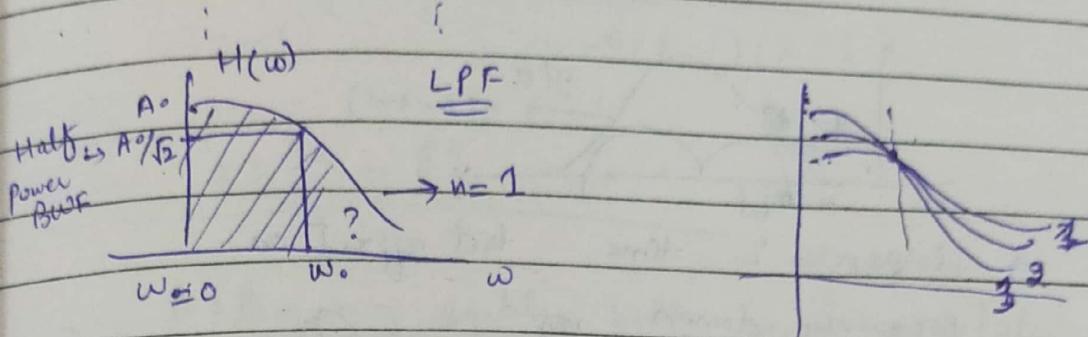


M T W T F S

DATE:

Values of  $n$  :- $n = 1$ 

First order BWF.

 $n = 2$ 2<sup>nd</sup> order BWF.

→ increasing  $n \rightarrow$  more approximated to ideal.

### Assignment :-

→ draw BWF for diff values of  $n$  in MATLAB. (vary  $\omega$  from  $0 \rightarrow \infty$ ).

→ write note on ideal & practical filters.

### ny Bodes Plot.

$$n = 1, -20 \text{ dB/decade}, n = 2, -40 \text{ dB/decade}$$

$\omega = \infty, \infty / \text{decade}$ .

→ Linear Distortion

→ Nonlinear Distortion (will not discuss)

$$y(t) = a_0 + a_1 g(t) + a_2 g^2(t) + \dots + a_n g^n(t)$$

derived from MacLaurins Theorem.

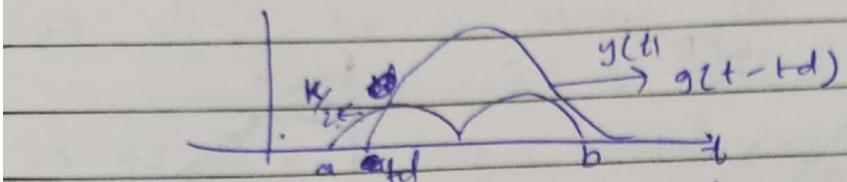
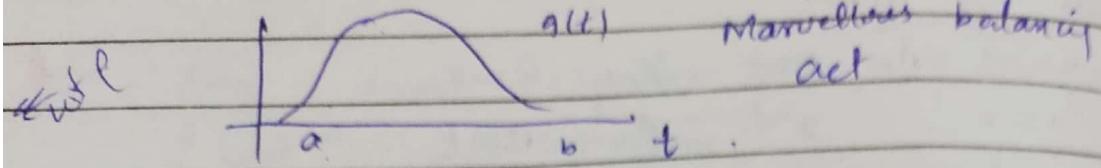
→ distortion caused by imperfection of magnitude or phase response:

→ linear distortion is due to dispersion (time)

\* diff. sign of w  
then they give a sign of b/w a & b and  
nullified other components.

M T W T F S

DATE:

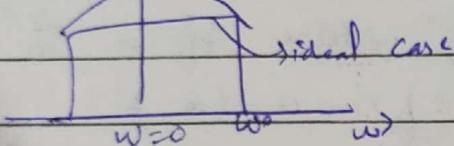


→ dispersion in time but affect on frequency domain - way

$$H(\omega) = (1 + k \cos \tau \omega) e^{j\omega t d} \rightarrow \text{slope}$$

→ amp varies by cosine

$\rightarrow 1 + k$  → linear distortion (response)



$$H(\omega) = (1 + k \cos \tau \omega) e^{j\omega t d}$$

$$= e^{j\omega t d} + k \cos \tau \omega e^{j\omega t d}$$

- (T +

$$Y(\omega) = G(\omega) e^{-j\omega t d} + G(\omega) \frac{k}{2} e^{j\omega (T - td)} + G(\omega) \frac{k}{2} e^{-j\omega (T + td)}$$

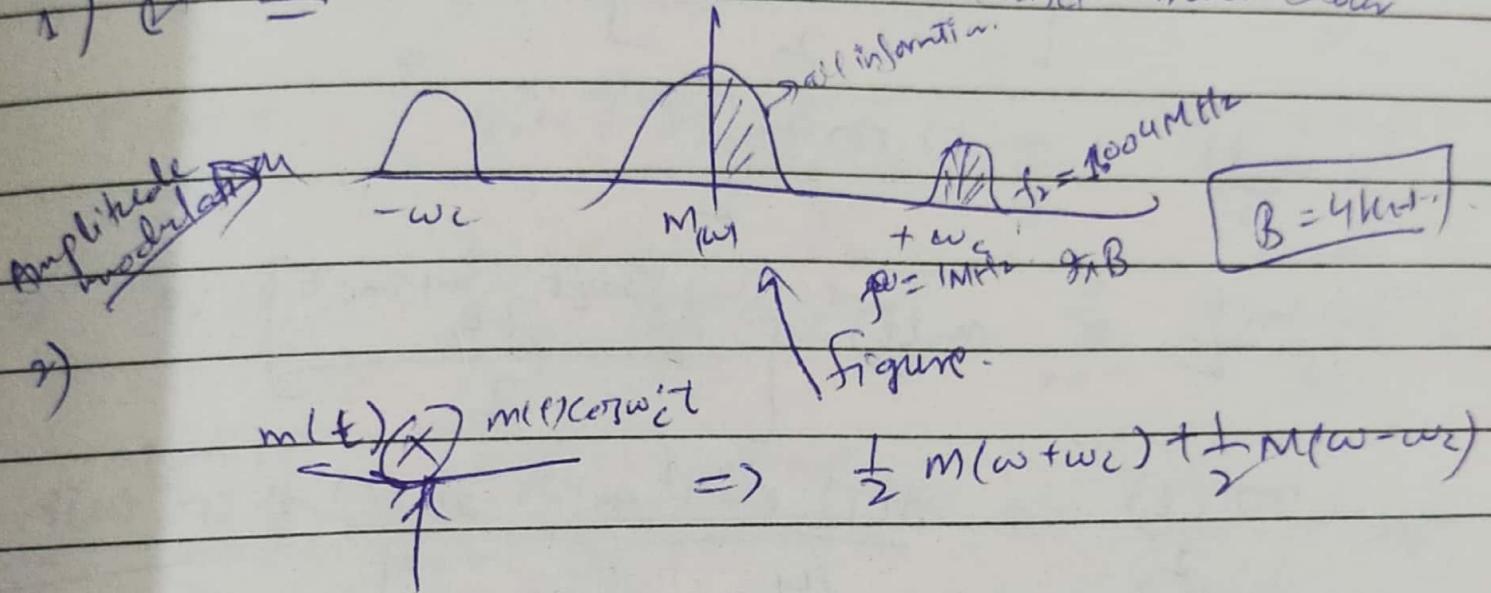
$$Y(t) = g(t - td) + \frac{k}{2} g(t + T - td) + \frac{k}{2} g(t + T + td)$$

→ linear distortion in Time & Frequency

→ in time domain → in form of signal dispersion

In time domain, 3 terms.

1) DSB - desirable side band modulation



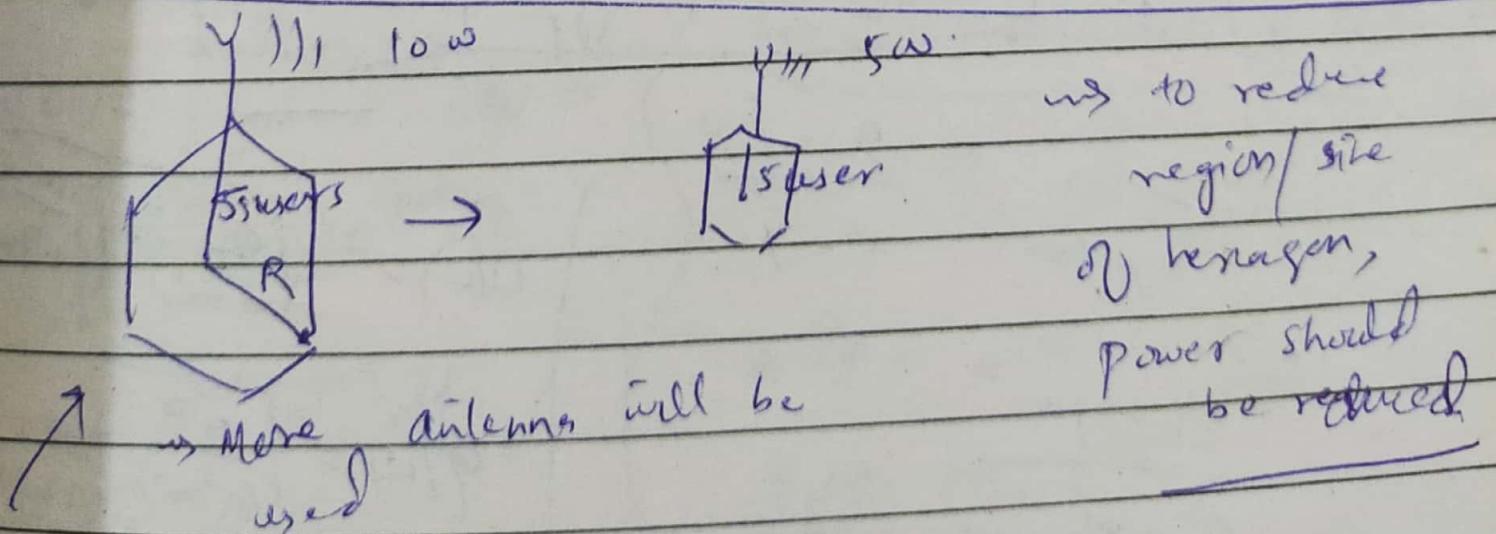
2)

$$m(t) \cos \omega_c t$$

$$\Rightarrow \frac{1}{2} m(\omega + \omega_c) + \frac{1}{2} M(\omega - \omega_c)$$

figure.

→ modulation and demodulation can be done with same signal.



Frequency can be reuse

demodulation example

$$m(t) \cos \omega_c t \xrightarrow{\text{Q}} m(t) \cos^2 \omega_c t$$

$\cos \omega_c t$

$\rightsquigarrow$  open formula.  
 write in exponential.

$$= m(t) \left[ \frac{(1 + \cos 2\omega_c t)}{2} \right]$$

$$\frac{m(t)}{2} + m(t) \cos 2\omega_c t$$

$$\frac{m(t)}{2} + \frac{m(t)}{2} \left[ \frac{e^{j2\omega_c t} + e^{-j2\omega_c t}}{2} \right]$$

$$\frac{m(t)}{2} + \frac{m(t)}{4} e^{j\omega_c t} + \frac{m(t)}{4} e^{-j\omega_c t}$$

$$= \frac{m(\omega)}{2} + \frac{m(\omega + 2\omega_c)}{4} + \frac{m(\omega - 2\omega_c)}{4}$$

If pass through low pass filter  
last two terms will be omitted

$$\frac{1}{a + j\omega} \times \frac{a - j\omega}{a - j\omega}$$

$$G(\omega) = \frac{1}{(a^2 + \omega^2)^{1/2}}$$

$$\frac{a - j\omega}{a^2 + \omega^2}$$

$$|H(\omega)| = \sqrt{\frac{a^2}{(a^2 + \omega^2)^2} + \omega^2}$$

$$\left( \frac{a}{a^2 + \omega^2} - \frac{j\omega}{a^2 + \omega^2} \right)$$

$$G(\omega) = \sqrt{\frac{a^2}{(a^2 + \omega^2)^2}}$$

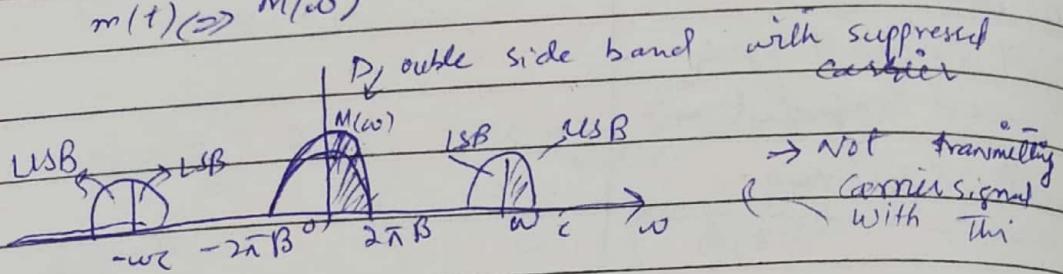
Modulation = a process that cause the range of frequencies in a signal.

M T W T F S Base band  
 (doesn't require) communication Carrier DATE:  
 (wave mod)

### Amplitude Modulation

$$m(t) \cos \omega_c t$$

$$m(t) \Leftrightarrow M(\omega)$$



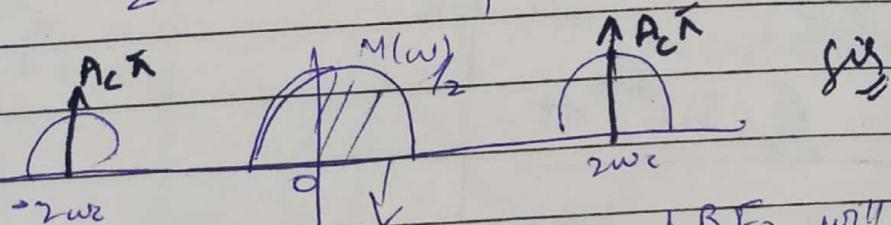
### Demodulation

sig again with carrier.

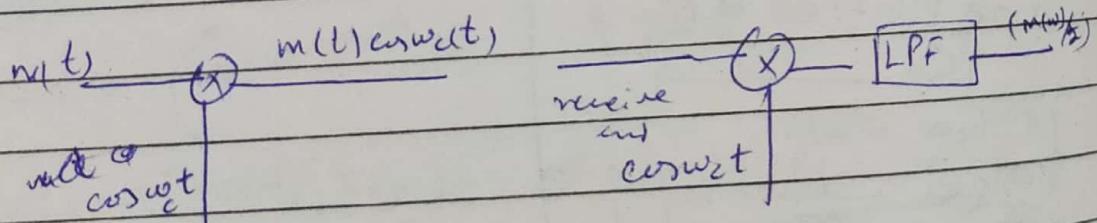
$$m(t) \cos^2 \omega_c t \Rightarrow m(t) \left( \frac{1 + \cos 2\omega_c t}{2} \right)$$

$$\frac{m(t)}{2} + \frac{m(t) \cos 2\omega_c t}{2}$$

$$\frac{m(\omega)}{2} + \frac{m(\omega + 2\omega_c)}{4} + \frac{m(\omega - 2\omega_c)}{4}$$



Pass through LBF, will retrieve original signal.



Coherent detection / synchronize

$$m(t) \cos \omega_c t \cos (\omega_c + \Delta\omega) t$$

$$m(t) \left[ \cos(\omega_c + \omega_c + \Delta\omega)t + \cos(\omega_c - \omega_c - \Delta\omega)t \right]$$

M T W T F S

DATE:

\* But when delay  $\tau$  is there will be  
sure delay  $\tau$ , then will be  
 $g(t) = g(t - \tau) \Leftrightarrow g(\omega) \text{ is shift}$   
 $\partial g(\omega) = -\omega \tau$

$$= \frac{m(t) \cos(2\omega_c t + \phi_0)}{2} + \frac{m(t) \cos \omega_c t}{2} \rightarrow \text{damping}$$

at receiver, initially zero,  
 $= m(t) \cos(\omega_c t + \phi_0)$

w) transmit carrier with signal &  
then demodulate it to overcome the  
failure.

$$\phi(t) = m(t) \cos \omega_c t + A_c \cos \omega_c t$$

↓ transmitted modulated signal

with carrier signal

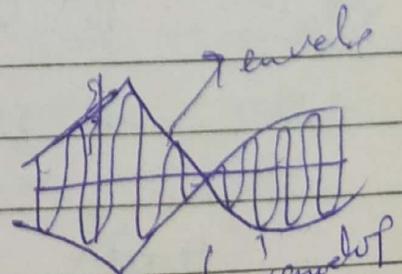
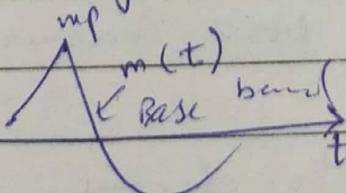
also called Full Amplitude modulation.

F.F. =  $2\pi S(\omega - \omega_c)$

$\boxed{\begin{array}{l} \text{DSB = LC} \\ \text{large carrier amplitude enough that it can have full balance} \end{array}}$

$$\phi_{AM}(t) = (m(t) + A_c) \cos \omega_c t$$

→ without sending carrier with signal  
represented by,



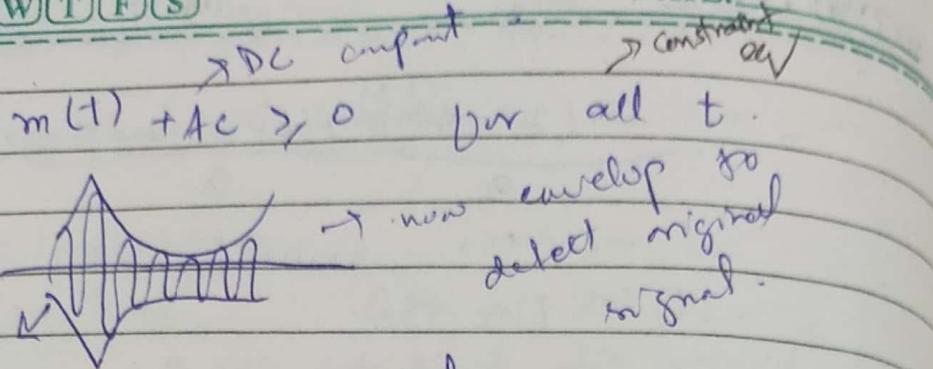
we can't do envelop detection

as signal is not representing  
the original signal

can't do envelop detection

M T W T F S

DATE:



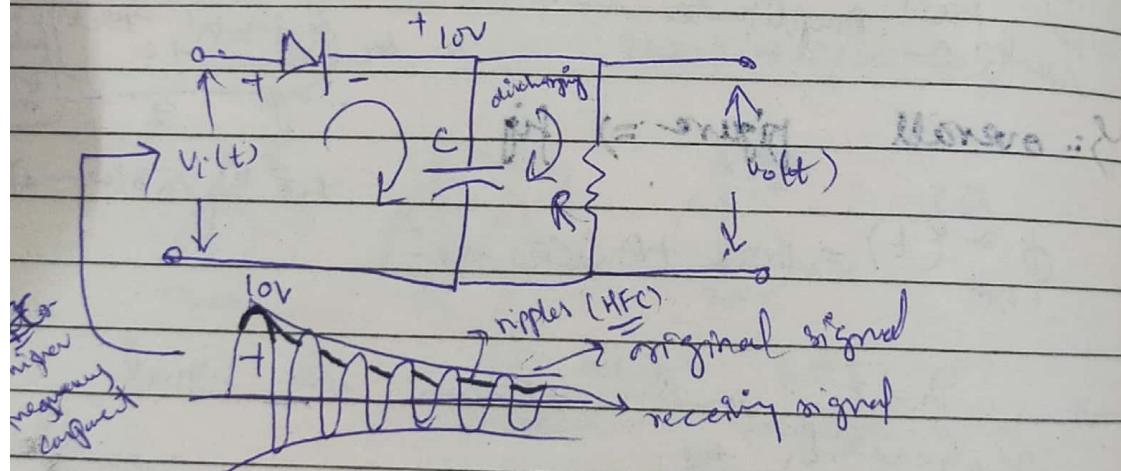
Cotenant detection <sup>but</sup> of envelop detection

some signal  
carrier

vs envelop detection  $\Rightarrow$  to detect original signal  
b/c more receiver than transmitter

advantage: easy, inexpensive

How we do envelop detection?



$\Rightarrow$  More voltage, diode becomes forward biased

$\rightarrow$  Capacitor starts charging.

$\rightarrow$  It does on peak

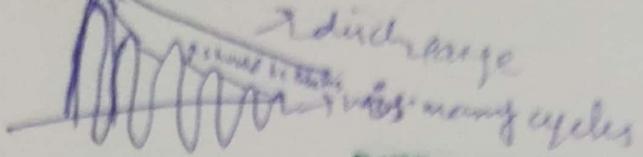
$\rightarrow$  When <sup>input</sup> goes down, diode  $\rightarrow$  reverse and act as open circuit, no current flows.

$\rightarrow$  it starts discharging.

$\rightarrow$  again voltage/amplitude increases, and flows and cycle repeats.

M T W T F S

\* To over  
RC should be  
adjusted



DATE:

- RC time constant  $e^{-t/\text{time constant}}$
- we can trace envelop there.  $\log$  ~~difference~~
- (black signal)
- This envelop represents original signal.

Ques. 1 Can we send base band over wireless?

Ans. NO, coz base band have very low freq and high ( $\lambda$ ), so antenna length will be greater that is impossible

Ques. 2  $m(t) = \cos(1000t)$

$$m_r(t) = \cos(1000\pi t)$$

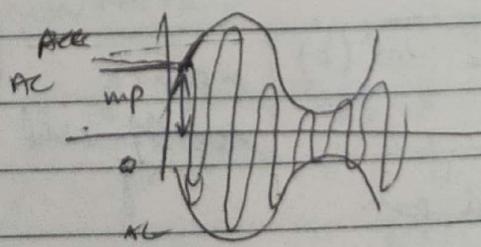
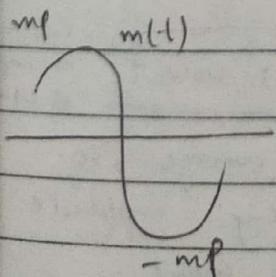
$$\omega_m = 1000$$

$$f_m = \frac{1000}{2\pi}$$

Lecture # ~

Modulation Index ~

$$m(t) + Ac \geq 0 \quad \text{Constraint eqn.}$$



all values should be +, for envelop detection

$M = \frac{m_p}{A_c}$  → Peak amplitude

$A_c$  → P.A of carrier signal

for  $M < 1$ ,  $A_c > m_p$ . ( $m_p$  must be greater)  
if less than, over modulation

$$0 \leq M \leq 1$$

Modulation index

let suppose  $M > 1$ ,

Condition of  $\Rightarrow m_p > A_c$

overmodulation.

$$\text{Power Efficiency} = \frac{\text{DSB - LC}}{\text{use}}$$

We are occupying double band gap

single, , ,

$$\phi_{AM}(t) = m(t) \cos \omega_c t + A_c \cos \omega_c t$$

useful sig      Additional

→ additional sig will facilitate signal  
but not part of original signal.

$$P_c = \frac{A_c^2}{2}$$

Additional power      Power of carrier signal

$$P_s = \frac{m^2(t)}{2}$$

useful power      power of modulated sig

$$m(t) \cdot \cos \omega_c t$$

Find average, so  
Damping amplitude  
of cos  $\omega_c t$ .

$$\text{Efficiency} = \eta = \frac{\text{useful power}}{\text{Total power}} = \frac{P_s}{P_c + P_s}$$

$$\eta = \frac{m^2(t)/2}{A_c^2/2 + m^2(t)/2} = \frac{m^2(t)}{A_c^2 + m^2(t)}$$

power efficiency of signal

In percentage

$$\eta \% = \frac{m^2(t)}{A_c^2 + m^2(t)} \times 100$$

$\rightarrow$  If  $m(t)$  is a tone signal;

impulse signal  $\rightarrow$  1 freq (Signal of same frequency)  
 $(\cos \omega t)$ ,  $f_c \rightarrow$  single frequency.

$m(t) = m_p \cos \omega_0 t$

$$= \frac{U^2 A^2}{2}$$

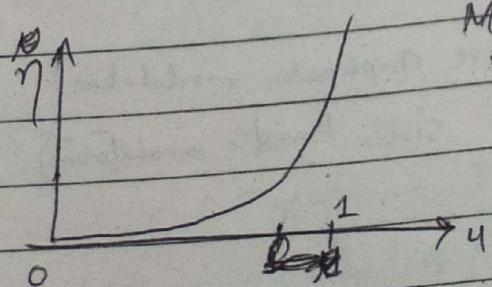
$\Rightarrow$  composite signal  $\rightarrow$   $f_1, f_2$  freq  
 $\Rightarrow m_p = U A$

$$\eta = \frac{U^2 A^2 / 2}{A_c^2 + U^2 A^2 / 2} = \frac{U^2}{2 + U^2}$$

$$\eta \% = \frac{U^2}{2 + U^2} \times 100$$

, percentage of power

Graphically representation:



Mathematically increasing function

MATLAB

$$\mu = 0.5, \eta = 11.1\% \quad \mu = 0.25, n = 4.5\%$$

DATE:

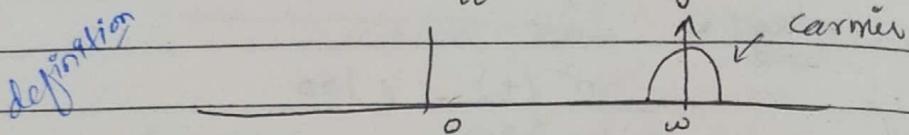
M T W T F S

$$\eta = \frac{f \times 100}{2+2} = 33.3\%$$

$\rightarrow$  66.66% utilize by carrier signal itself.

$$Q_m(t) = m(t) \cos \omega t + A_c \cos \omega t \rightarrow 66.66$$

$\rightarrow$  Not a power efficient system.

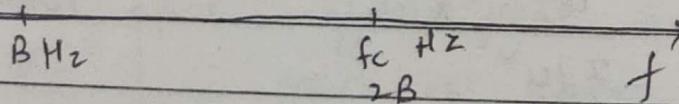
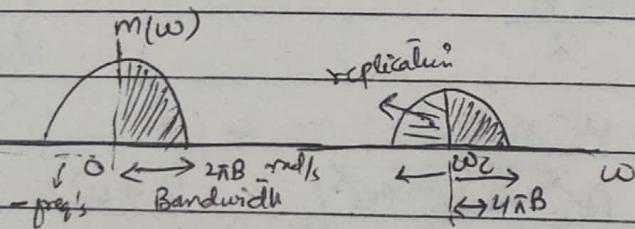


### Assignment:-

Note on Modulation index

and implementation in Matlab:-

### With Bandwidth Efficiency :-



Not Bandwidth efficient coz wasting band.

### Methods:-

\* QAM (Quadrature Amplitude modulation)

\* SSB (single side Band modulating)

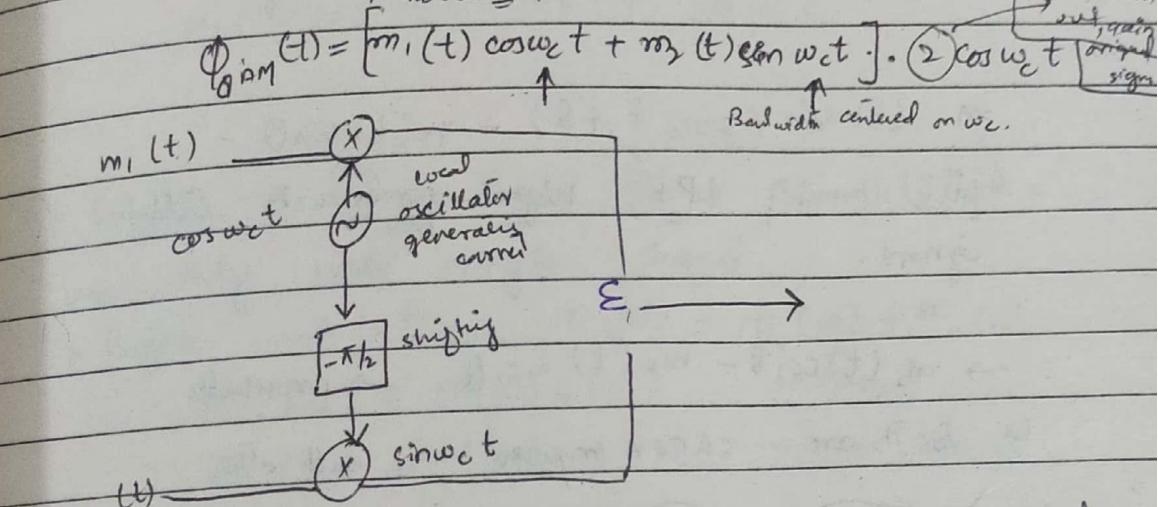
my QAM :-

my orthogonal signals, correlation zero.

we can be separated

for example,  $\cos \omega_c t$ ,  $\sin \omega_c t$ , lie on single frequency but can be separated as orthogonal. (not mix with each other)

Demodulation.



$$\Rightarrow \phi_{QAM}(t) = 2m_1(t)\cos^2 \omega_c t + m_2(t) \cdot \underline{\sin 2\omega_c t}.$$

$$= m_1(t) + m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t.$$

Passing all through L.P.F,

$m_1(\omega)$   $\rightarrow$  I-Phase

$$\Rightarrow \phi_{QAM}(t) = [m_1(t)\cos \omega_c t + m_2(t) \sin \omega_c t] \cdot 2 \sin \omega_c t$$

L.P.F

$= m_2(\omega) \rightarrow Q$ -Phase

w) in phase modulation / demodulating.  
 I-phase  $(M_1(\omega))$

M T W T F S

$$\cos(w_c t + \phi)$$

in Demodulation.

$$Q_{DM} = [m_1(t) \cos w_c t + m_2(t) \sin w_c t] 2 \cos(w_c t + \phi)$$

$$= m_1(t) 2 \cos w_c t \cdot \cos(w_c t + \phi) + 2 m_2(t) \sin w_c t \cdot \cos(w_c t + \phi)$$

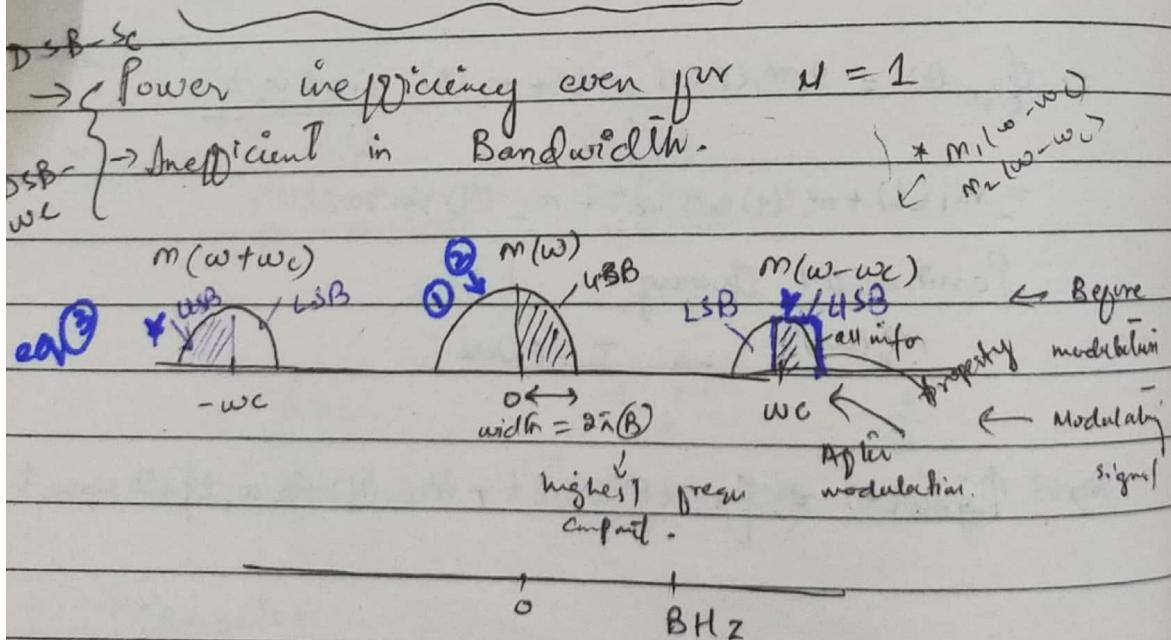
$$= m_1(t) \cdot 2 \cos(2w_c t + \phi) + m_2(t) \cos \phi$$

$$m_2(t) \sin(2w_c t + \phi) + m_2(t) \sin \phi$$

After passing LPF, higher frequencies will ignore.

$$\rightarrow m_1(t) \cos \phi - m_2(t) \sin \phi \rightarrow \text{crosstalk}$$

↳ Both are superimposed on each other.



→ half band is going to lost.

Solutions:

↳ Quadrature Amplitude Modulation

↳ dual signal in same band.

→ not losing any band.

M T W T F S

General

\* QAM  
\* Phase shift method  
\* Frequency Selective Filling

DATE:

$$m_1(t) \Leftrightarrow M_1(\omega) \quad \text{one signal}$$

$$m_2(t) \Leftrightarrow M_2(\omega) \quad \rightarrow \text{second signal}$$

Both signals are in same band  
but orthogonal to each other and can  
be distinguished and at receiver  
end, original signal can be recovered.  
 $m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t$

2) Solution  $\rightarrow$  Single Side Band Modulation -

uses only ~~one~~ single band.

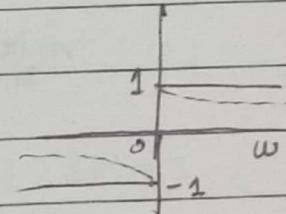
\* Before modulation:  $M(\omega) = M_+(\omega) + M_-(\omega)$

$$M_+(\omega) = M(\omega) u(\omega) \quad \rightarrow \text{causal}$$

$$M_-(\omega) = M(\omega) u(-\omega) \quad \rightarrow \text{non-causal}$$

\*  $M_+(\omega) = M(\omega) \cdot \left[ \frac{1 + \text{sgn}(\omega)}{2} \right]$

$\Rightarrow M_+(\omega) = \frac{M(\omega) + M(\omega) \text{sgn}(\omega)}{2}$  to put  
H. Transf. on



Hilbert Transform -

① Const Amplitude response (unit region)

②  $\pi/2$  phase shifter. (shift every input by  $90^\circ$ )

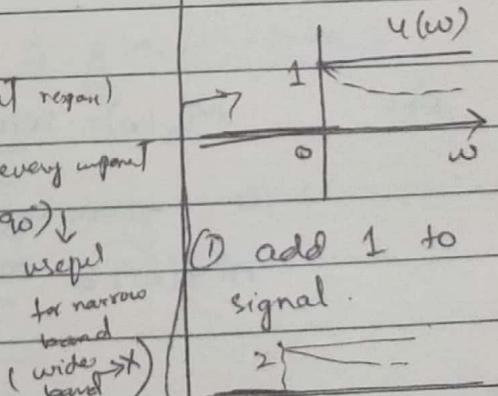
$$H_h(\omega) = -j \text{sgn}(\omega)$$

Hilbert  $H_h$

$$M(\omega) \rightarrow \boxed{-j \text{sgn}(\omega) \quad M(\omega) \text{sgn}(\omega)}$$

$$X(\omega) \rightarrow Y(\omega)$$

$$Y(\omega) = X(\omega) \cdot H_h(\omega)$$



M T W T F S

$\times \lim_{T \rightarrow \infty}$  (convolution in time) DATE:

$$M_b(\omega) = M(\omega) [-\int sgn(\omega)]$$

$$M_h(\omega) = -jM(\omega) sgn(\omega)$$

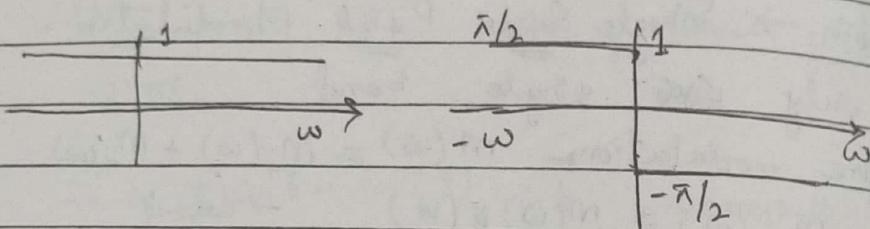
$$\left. \begin{aligned} H_h(\omega) &= (-j) sgn(\omega) = (De^{-j\pi/2}) & \omega > 0 \\ &= 1e^{j\pi/2} & \omega < 0 \end{aligned} \right\}$$

Amplitude  $e \rightarrow \text{const } (1)$

phase shift  $\rightarrow \pi/2$

$$\omega > 0 \rightarrow \text{sgn} = -1$$

Phase shift



$$\begin{aligned} \text{Time Domain} &\xrightarrow{\text{off sign}} \\ \frac{1}{\pi t} &\Leftrightarrow -j sgn(\omega) \end{aligned}$$

$$\begin{aligned} m_h(t) &= m(t) * \frac{1}{\pi t} \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} m(\alpha) \frac{d\alpha}{(t-\alpha)} \end{aligned}$$

introduce dummy var 'a'  
scroll from  $-\infty$  to  $+\infty$ .

$$H_h(\omega) = -jM(\omega) sgn(\omega)$$

$$*** M_+(\omega) = \underline{m(\omega)} + \underline{m(\omega) sgn(\omega)}$$

To form  $\Rightarrow$  (Hilbert form)  $\xrightarrow{\text{if } j \rightarrow -j}$

$$M_+(\omega) = \frac{m(\omega)}{2} + \frac{-jM(\omega) sgn(\omega)}{2(-j)}$$

$$M_+(\omega) = \frac{m(\omega)}{2} + j M_h(\omega) \rightarrow \textcircled{1}$$

$$m_+(t) = \frac{m(t)}{2} + \frac{\int m_h(\tau) d\tau}{2} \rightarrow \textcircled{1}$$

M T W T F S

DATE:

FOR  $M_-(\omega)$  :-

$$M_-(\omega) = m(\omega) \left[ 1 - \frac{\text{sgn}(\omega)}{2} \right]$$

$$m_-(\omega) = M(\omega) - jM_n(\omega) \rightarrow ①$$

small  $m_-(t) = \frac{m(t)}{2} - j\frac{m_n(t)}{2} \rightarrow \theta^2$

We are going to describe modulated signal.

$$\phi_{\text{USB}}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c)$$

In time domain :-

$$\phi_{\text{USB}}(t) = \bar{m}_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t} \rightarrow ③$$

put ① and ② in ③

$$= m(t) \left( \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} \right) - m_n(t) \left( \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j} \right)$$

$$\phi_{\text{USB}}(t) = m(t) \cos \omega_c t - m_n(t) \sin \omega_c t.$$

For  $\phi_{\text{LSB}}(t) = m(t) \cos \omega_c t + m_n(t) \sin \omega_c t$ .

$$\phi_{\text{CSB}}(t) = m(t) \cos \omega_c t \mp m_n(t) \sin \omega_c t.$$

For do & DSB.

$$\phi_{\text{DSB}}(t) = m(t) \cos \omega_c t.$$

M T W T F S

Example:-let  $m(t) = \cos \omega_m t \rightarrow$  tone signal

Want to do SSB Modulation.

$$m_n(t) = \cos(\omega_m t - \pi/2)$$

(only 1  
freq)  $\omega_m$

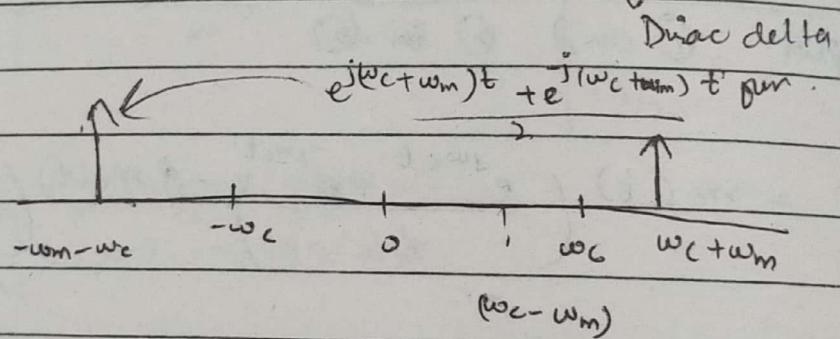
$$\phi_{USB}(t) = \cos \omega_c t \cos \omega_m t - \frac{1}{2} \sin \omega_m t \cdot \sin \omega_c t$$

$$= \cos(\omega_c + \omega_m)t$$

$$\phi_{LSB}(t) = \cos \omega_m t \cos \omega_c t + \frac{1}{2} \sin \omega_m t \cdot \sin \omega_c t$$

$$\phi_{USB}(t) = e^{j(\omega_c + \omega_m)t} + e^{-j(\omega_c - \omega_m)t}$$

2

DEMODULATION :-

$$\phi_{SSB}(t) = [m(t) \cos \omega_c t \mp m_n(t) \sin \omega_c t] 2 \cos \omega_c t$$

↑  
coherent

$$= m(t) + m(t) \cos 2\omega_c t \pm m_n(t) \sin 2\omega_c t$$

$$\phi_{SSB}(\omega) = m(\omega) + m(\omega + 2\omega_c) + m(\omega - 2\omega_c)$$

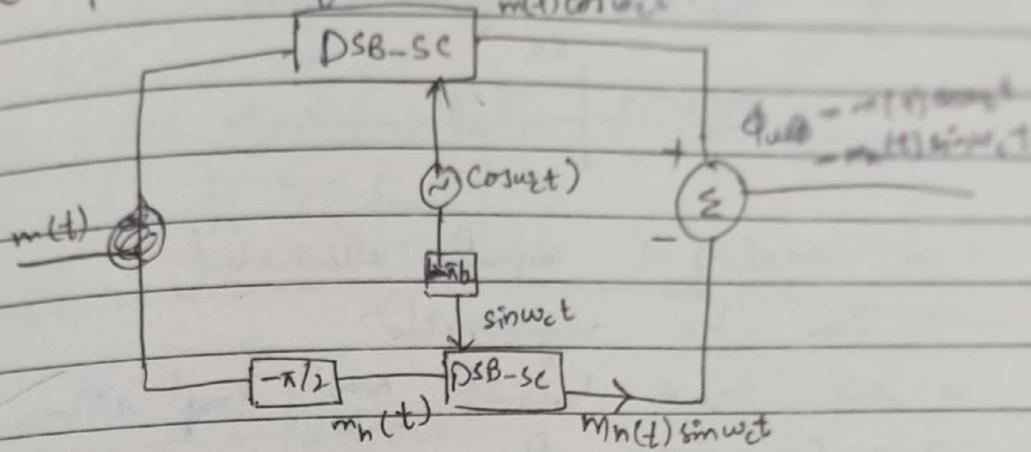
Pass through Low Pass Filter

$\phi_{SSB}(\omega) = m(\omega)$

→ Frequency selective filtering  
↳ unrealizable.  
can't cut sharply all  
Components



### ④ phase shift method $m(t) \cos \omega t$



### ⑤ Vestigial Side Band:

B/w DSB and SSB.

Can implement through phase

$$\phi_{SSB}(t) = m(t) \cos \omega t + m_h(t) \sin \omega t$$

$$m(t) = \cos \omega_m t$$

$$\phi_{SSB}(t) = m(t) \cos \omega t - m_h(t) \sin \omega t$$

$$\{ m_h(t) = \cos(\omega_m t - \pi/2)$$

$$= \sin \omega_m t$$

M T W T F S

Lecture

DATE:

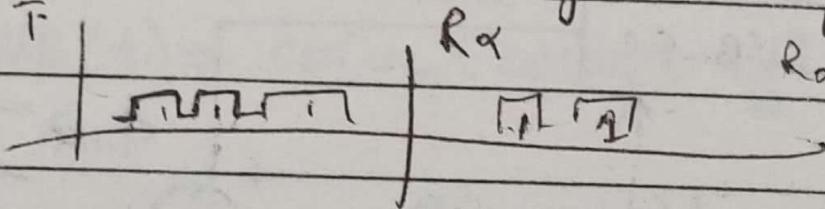
## Analog Modulations

↳ AM, FM, PM (Analog)

↳ PSK, FSK, ASK (digital)

regenerative type (advantage)

→ if transmit digital info,



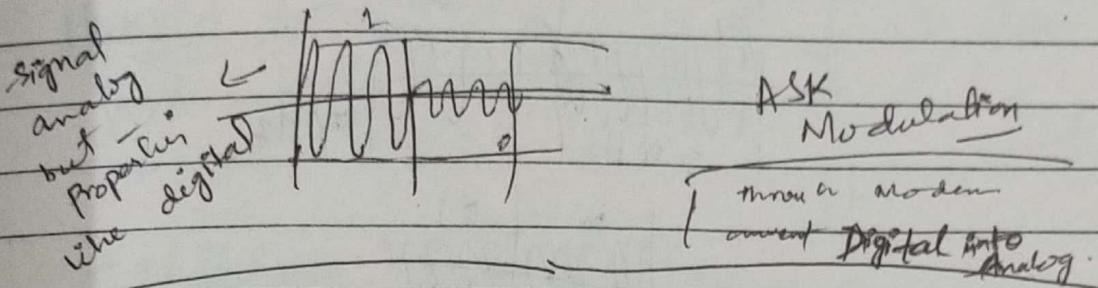
→ in analog signal attenuated,  
can be regenerated

→ dig can't transmit over long dist.

→ analog always transmit

in computer      | 1      0 |  
 must convert this onto analog (big + low trans.)  
 over a long distance

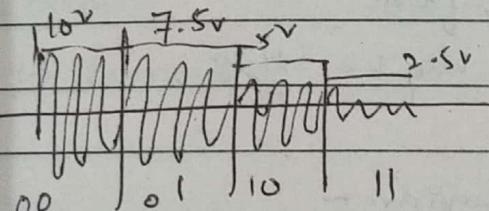
- analog bears properties of digital signal.
- duration of carrier equals to digital.



→ PCM

→ analog signal → samples (Sampling Theorem)

$$f_s \geq 2 f_{max}$$



→ 1 symbol → transmits two bits.

→ speed double.

→ duration same.

→ no of bits increases (more bits transmitted)

(transmitted)

$$\text{Next no of levels} = 2^n = 8$$

disadvantages:-

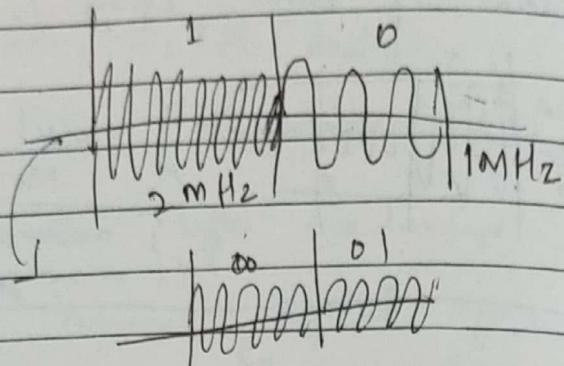
→ difficult to recognize each BER

→ error increases → (bit error rate will be increased)

→ need more sophisticated and expensive receivers.

as # of symbols increases.

→ Frequency Shift Keying -

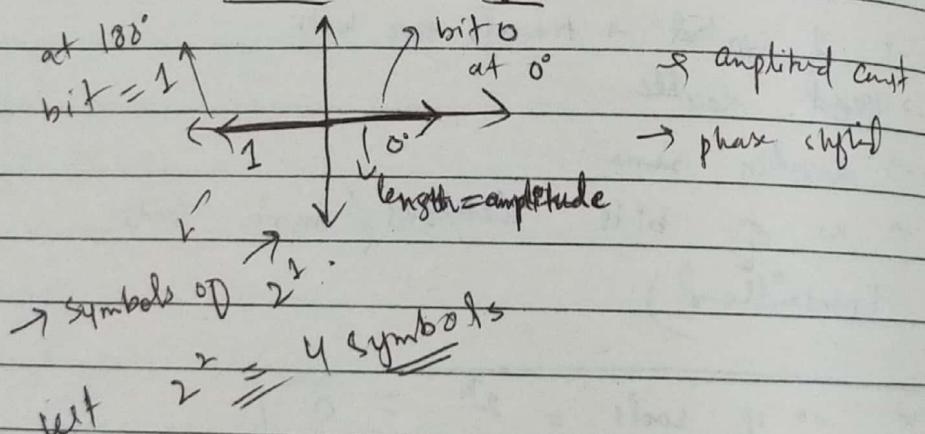


→ Phase Shift Keying -

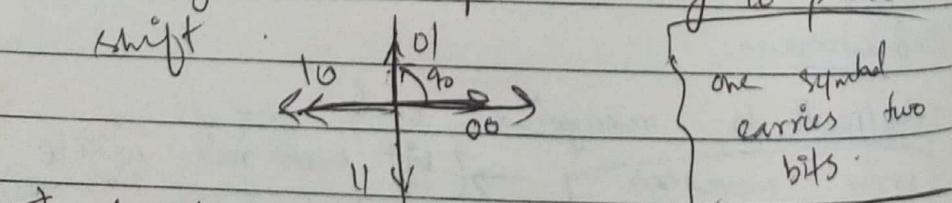
to represent detail, may be symbols are shifted.

→ 1 → 0, other may be  $180^\circ$ .

→ Constellation Diagram



every symbol represented by 90° phase shift



→ duration same

→ signal will be analog.

(proportional  $\sim 1$  correspondence  
in digital)

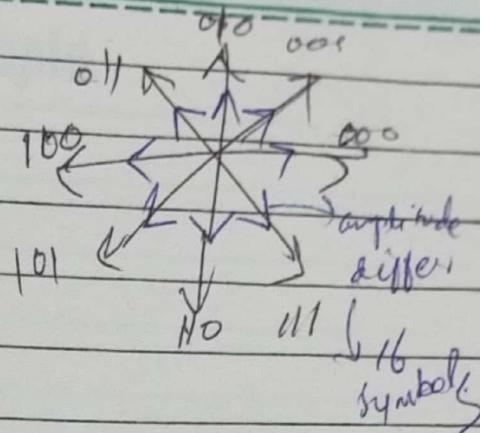
→ bits per sec!  
increase,  
→ bandwidth  
increased

M T W T F S

DATE: \_\_\_\_\_

$\Rightarrow 2^4 = 16 \rightarrow 4 \text{ symbols}$

Hybrid Modulation  
ASK + PSK



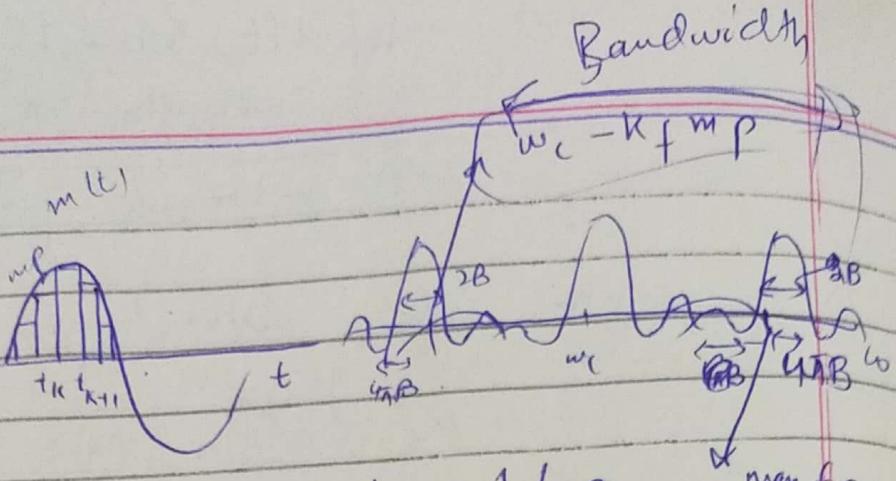
~~→ Euclidean distance~~  
at receiver end (match closest point)

Finds short distance (with nearer points)  
↓

error bit rate increases as points  
on increases on constellation diagram.

by Retransmit signal → error correction.

FM:



$$\text{duration } t_{k+1} - t_k = 1/2B$$

$$B \leq 2B, \quad T = 1/2B$$

$$f_s = 2B$$

$$\text{rect}\left(\frac{t}{T}\right) = \sin\left(\frac{\omega T}{2}\right)$$

$$\frac{\omega T}{2} = \pm n\pi$$

$$\omega_0 = \pm \frac{2n\pi}{T} \Rightarrow \omega_0 = 2n\pi \times 2B = 4\pi B$$

For FM S.

$$\boxed{\omega_c + K_f m(t)} \quad \begin{array}{l} \text{Frequency} \\ \downarrow \\ \text{carrier signal} \leftarrow A \cos(\omega_c t + \phi_0) \end{array} \quad \begin{array}{l} \text{modulation} \\ d\phi(t) = \omega_c + K_f m(t) \cdot t \\ \phi(t) = \omega_c t + \frac{1}{2} K_f m(t) t^2 \end{array}$$

For PM :-

$$\omega_c t + \phi_0 + K_p m(t)$$

$$\phi(t) = \omega_c t + K_p m(t)$$

change in phase

$$\frac{d\phi(t)}{dt} = \omega_c + K_p m(t)$$

$$\boxed{\omega_p = \omega_c + K_p m(t)} \quad (2)$$

$$\cos \omega_c t - k_f m(t) \sin \omega_c t, ] \quad \text{Bandwidth} = 2B \quad \text{Narrow Band}$$

increasing  $m(t)$  increases frequency

→ negative side, subtract from  $\omega_c$

$$\omega_i^o = \omega_c + k_f m(t_k)$$

$$2\pi B_{FM} = \omega_c + k_f m_p - \omega_c + k_f m_p$$

$$B_{FM} = 2 \left( \frac{k_f m_p}{2\pi} \right) \xrightarrow{\Delta f}$$

Adding Bandwidth of remain signal.

$$B_{FM} = 2\Delta f + 4B$$

$$= 2[\Delta f + 2B]$$

For narrow band, more appropriate

$$B_{FM} = 2 [\Delta f + \frac{2B}{2B}] \xrightarrow{*} \text{Carson's Rule}$$

Same values {  $B_{FM} = 2 [\Delta f + B]$

But difference is,

$$\Delta f_{FM} = \frac{k_f m(t_p)}{2\pi} \xrightarrow{\Delta f} = m(p)$$

$m(p)$   
peak value

$$\Delta f_{PM} = \frac{k_p m^o(t)}{2\pi}$$

For narrow band

$\Delta f$  is much lesser than  $B$

$$B_{FM} \approx 4B$$

{ for  $B$  is much lesser than  $\Delta f$   
 $B_{FM} \approx 2\Delta f$

Theoretical

Bandwidth

$\Delta f$  sinc

sinus sum or

or infinite

$$\beta = \Delta f / B \rightarrow \text{Modulation index}$$

$$\Delta f = \beta B$$

put  $\Delta f$  in eq \*

$$B_{FM} = 2 [\beta B + B]$$

$$B_{FM} = 2B [\beta + 1]$$

Same for  $B_{PM}$ .

$$B_{PM} = 2 [\beta B + B]$$

$$B_{PM} = 2B [\beta + 1]$$

Example

P4PQ  
272

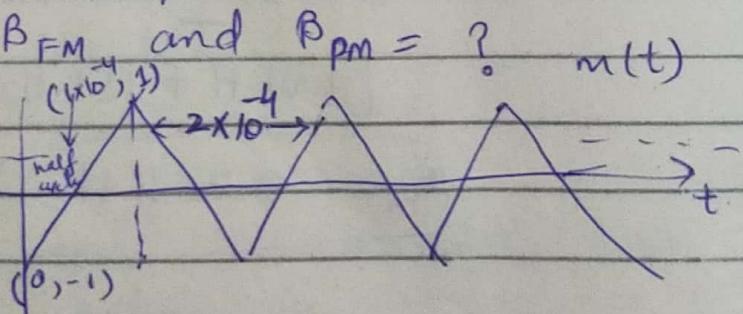
$$k_f = 2\pi \times 10^5$$

$$k_p = 5\pi$$

vertical  $\rightarrow m(t)$   $B_{FM}$  and  $B_{PM} = ?$   $m(t)$

hor  $\rightarrow t$  1

Periodic signal



$$B_{FM} = 2 [\Delta f + B]$$

$$\Delta f = \frac{k_f m_p}{2\pi} = \frac{2\pi \times 10^5 \times 1}{2\pi}$$

$$\Delta f = 0.1 \text{ MHz}$$

For finding Bandwidth,  
Fourier Series

$$\sum_n C_n \cos(n\omega_0 t + \phi_n)$$

amplitude  
↓ signed  
↓ expand  
↓ of F. Series

$$C_n = \frac{8}{n^2 \pi^2} \quad n=odd$$

$$0 \quad n=even$$

$C_n$   
↓ will  
be given  
in exam

Power negligible  
 $\checkmark$  small

$$- \frac{8}{\pi^2} \cos 2\pi(5\omega_0 t) + \frac{8}{9\pi^2} \cos 3\omega_0 t$$

$$f_0 = \frac{1}{T_0}$$

$$+ \frac{8}{25\pi^2} \cos 5\omega_0 t + \dots$$

$$= \frac{1}{2 \times 10^{-4}}$$

4th. major signal

$$f_0 = 5\text{kHz}$$

$\omega_0 = 5\text{kHz}$   
100 kHz weight reduces as freq components increases, so ignore higher comp.

$$B_{FM} = 2 [10\text{kHz} + 15\text{kHz}]$$

$$= 2 [10\text{kHz} + 15\text{kHz}]$$

$$B_{FM} = 230\text{kHz}$$

$$\omega = 2\pi f$$

$$\omega = \text{rad/sec}$$

$$f = \text{cycles/sec}$$

$$\Rightarrow B_{FM} = 2 [\Delta f + B]$$

$$\Delta f = \frac{k_p m_p}{2\pi} \rightarrow \text{derivative}$$

taking derivative of  $m(t)$

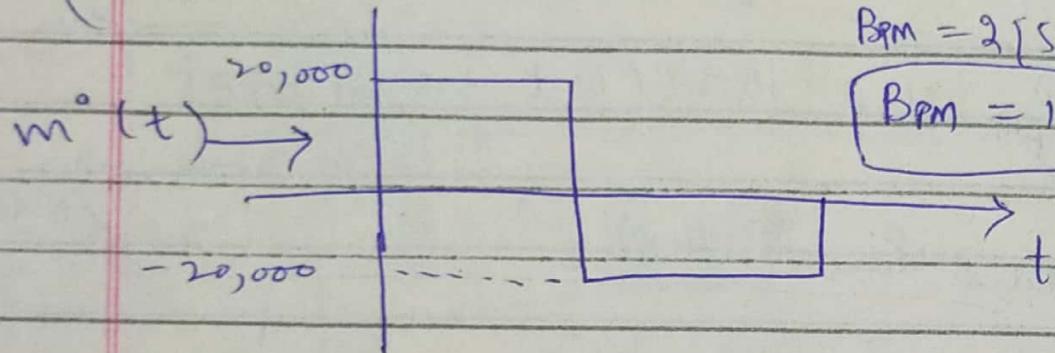
$$\frac{m(t) + 1}{1+1} = \frac{t - 0}{1 \times 10^4 - 0}$$

if

$$\Rightarrow m(t) = 2 \times 10^4 t - 1$$

derivative of signal.

$$\text{of } f(t) \quad m'(t) = 2 \times 10^4 = 20,000$$



$$BPM = 2(50\text{kHz} + 15\text{kHz})$$

$$BPM = 130\text{ kHz}$$

new  
problem

if double  $m(t)$

No effect on bandwidth.

cos time period is not changed

as fundamental frequency remains same

if changes cos depend  
on  $m(t)$

$$\Delta f \rightarrow 2\Delta f \quad \checkmark$$

Note, need to calculate again  
just explain in words.

Example #5-4

expand time by 2

$$t = 4 \times 10^4$$

5.3, 5.4 + similar questions.

Bandwidth changes

cos signal is expanding in time, so compressed in freq during time → double → Bandwidth → half

Example # S.5 +

i) D<sup>ind</sup> Power of modulated signal.

PM  
PM  
carrier angle

$$\phi_{EM.}(t) = 10 \cos(w_c t + 5 \sin 3000t + 10 \sin 2000\pi t)$$

when max, cos will max, freq will increase  
give peak

modulated Power =  $(Amp)^2 / 2$

$$Power = \frac{10^2}{2} = 50$$

2) frequency deviation  $\Delta f \Rightarrow ?$

depend  $\leftarrow \Delta f = k_f m_p$

on max peak  $\Delta f = \frac{\Delta \omega}{2\pi}$

$$\phi(t) = w_c(t) + 5 \sin 3000t + 10 \sin 2000\pi t$$

↓ derivative

$$\omega_i = w_c + 5 \times 3000 \cos(3000t) + 10 \times (2000\pi) \cos(2000\pi t)$$

↑ max dev from  $w_c$   
carrier freq + some deviation.

$$\omega_i = w_c + 15000 \cos(3000t) + 20,000 \pi \cos(2000\pi t)$$

representing bandwidth +  $f$  ↓ deviation in freq

now how much deviation depends on cycles of cosine and man 1 when both are in same and will add

$$\omega_i = \omega_c + \Delta f \Rightarrow \Delta f = \frac{\Delta \omega}{2\pi}$$

will now written both are ( $\cos=1$ ) at same phase angle.

when in phase with each other gives

$$\Delta f = \frac{15000 + 20,000\pi}{2\pi}$$

$$\Delta f = 12887 \text{ Hz}$$

max free deviation is

$$B = \Delta f / \beta$$

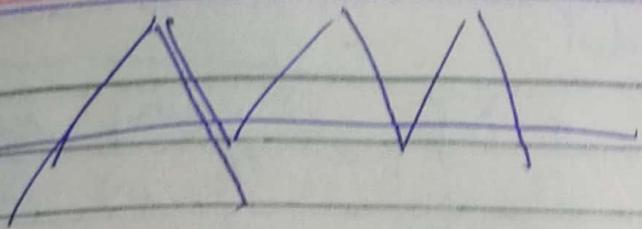
~~Ratio~~ find bandwidth of both, then man will be bandwidth.

$$\omega = i) \quad \omega_1 = 3000, f_1 = 3000 / 2\pi \cdot \frac{4\pi}{1000}$$

$$ii) \quad \omega_2 = 2000\pi, f_2 = 2000\pi / 2\pi = 1000$$

So bandwidth = 1000

$$B = \frac{12887}{1000} =$$



$$B_{FM} = 2 [\Delta f + B]$$

$$\Delta f = \frac{b_{fmp}}{2\pi} = \frac{2 \times 10^5 \times 2}{2\pi} \quad (\Delta f = 2 \times 10^5)$$

$$B_{FM} = 2 [2 \times 10^5 + 15 \text{ kHz}]$$

$$B_{FM} = 430 \text{ kHz}$$

$$B_{FM} = 2 [\Delta f + B]$$

$$B_{FM} = 2 [ \quad + 15 \text{ kHz} ]$$

$$\Delta f = 2 \Delta f$$

$$\Delta f =$$