30820-Communication Systems

Week 6-8 – Lecture 16-21

(Ref: Chapter 4 of text book)

AMPLITUDE MODULATIONS AND DEMODULATIONS

Contents

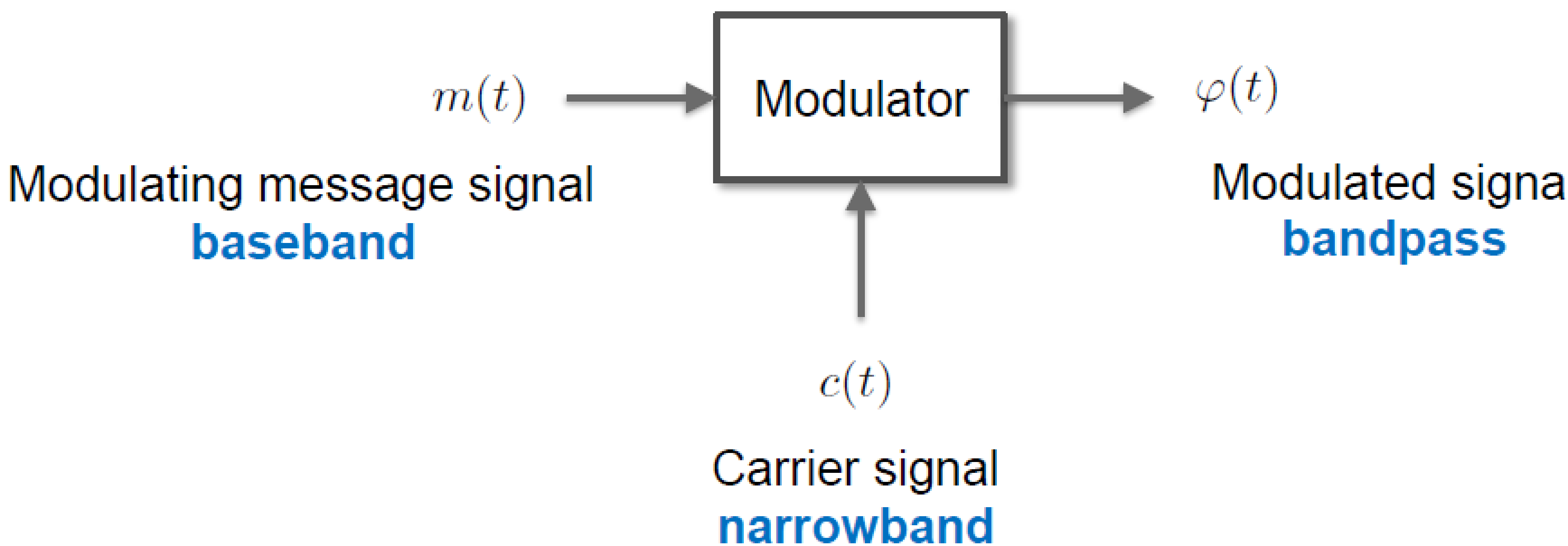
- Baseband versus Carrier Communications
- Double-Sideband Amplitude Modulation
- Amplitude Modulation (AM)
- Bandwidth Efficient Amplitude Modulation
  - Amplitude Modulation: Single Sideband (SSB)
  - Quadrature Amplitude Modulation (QAM)
- Amplitude Modulation: Vestigial Sideband (VSB)
- Phase Locked Loop and Some Applications
- Superheterodyne AM Receiver

Modulation

- Modulation is a process that causes a shift in the range of frequencies in a signal.
- Two types of communication systems
  - Baseband Communication
  - Carrier Modulation
- The baseband is used to designate the band of frequencies of the source signal i.e., audio signal 4KHz, video 4.3MHz
- In analog modulation the basic parameter such as amplitude, frequency or phase of a sinusoidal carrier is varied in proportion to the baseband signal $m(t)$.
- This result in amplitude modulation (AM) or frequency modulation (FM) or phase modulation (PM).

Modulation

- $m(t)$= Modulating Message Signal
- $c(t)$=Carrier Signal
- $\varphi(t)$=Modulated Signal



Why Modulation?

- To use a range of frequencies more suited to the transmission medium.

- To allow a number of signals to be transmitted simultaneously
  - Frequency division multiplexing (FDM)

- To reduce the size of the antennas in wireless links

Modulation

- Un-modulated Carrier

$$a(t) = A_c,$$
$$\theta(t) = 2\pi f_c t + \theta_0$$

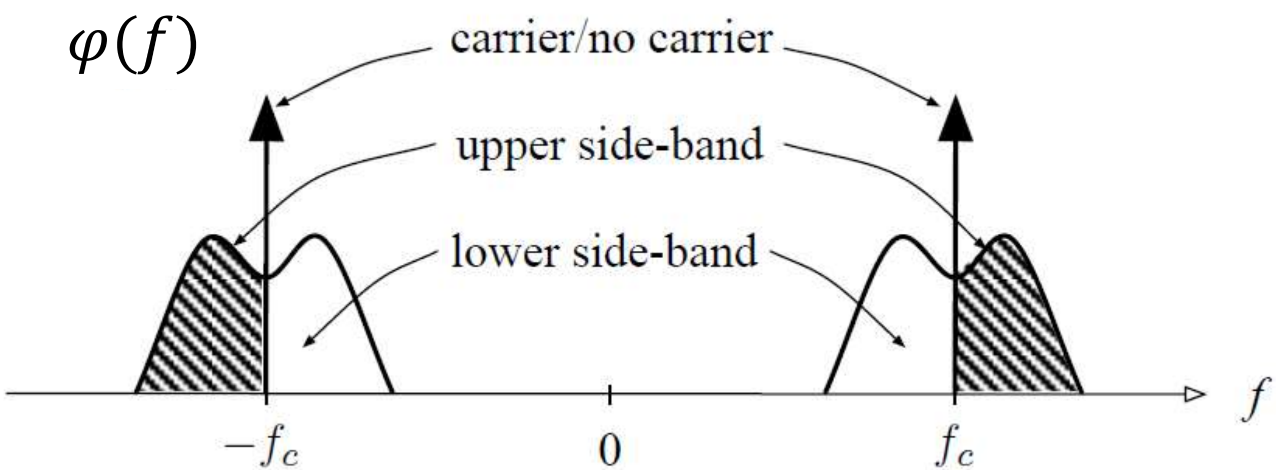
Such that

$$\varphi(t) = A_c \cos(2\pi f_c t + \theta_0)$$

- The unmodulated carrier has constant amplitude and constant instantaneous frequency.
- It carries no information-all components are independent of the message signal $m(t)$.

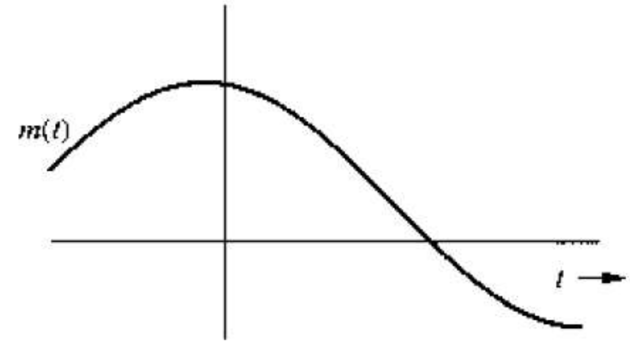
Modulation

- **Carrier:** $\varphi(t)$ may or may not include a separate carrier term. If a carrier is presented, (f) has a line spectrum components at $\pm f_c$
- **Lower Side-Band:** Frequency components in $\varphi(f)$ for $|f| < f_c$
- **Upper Side-Band:** Frequency components in $\varphi(f)$ for $|f| > f_c$



Double Sideband Suppressed-Carrier (DSB-SC) Modulation

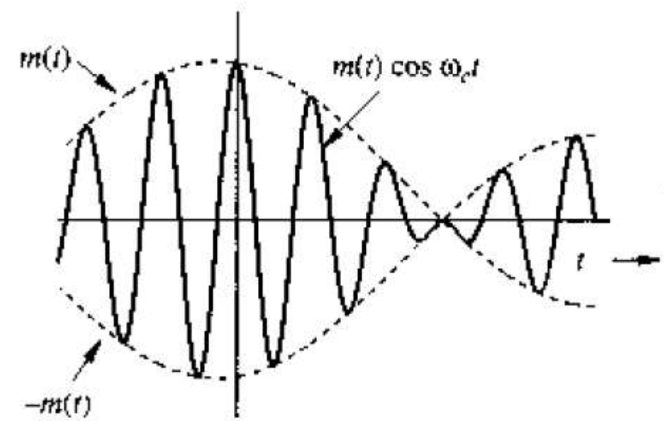
- **DSB-SC generation: Multiplier Modulator**
- Time Domain Description



$$a(t) = m(t)$$
$$\theta(t) = 2\pi f_c t + \theta_c$$
$$f_i(t) = f_c$$

Such that

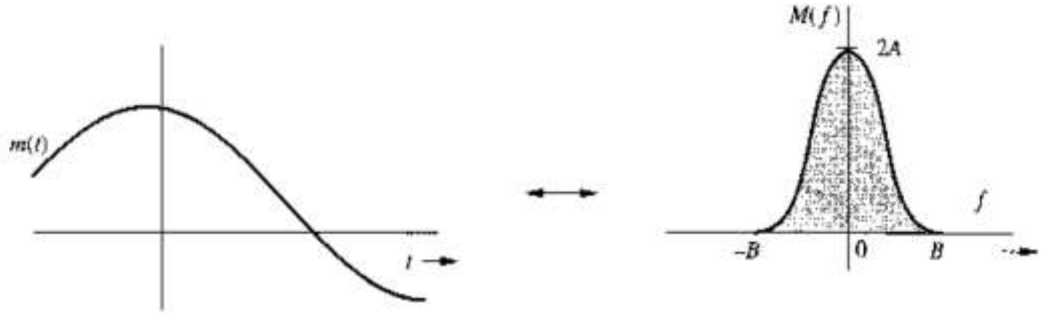
$$\varphi_{AM}(t) = m(t)\cos(2\pi f_c t + \theta_c)$$



- Information contained in $m(t)$ is embedded in the time varying amplitude.

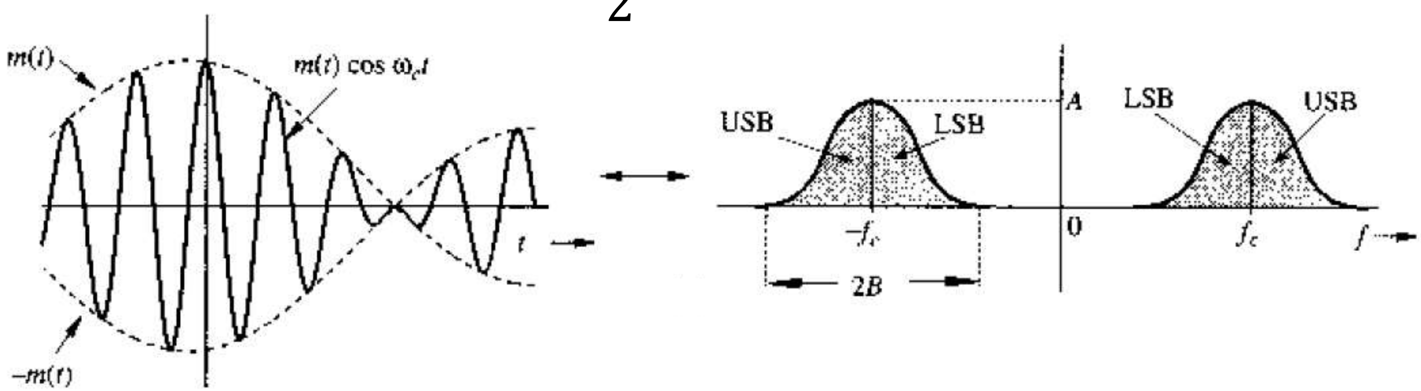
Double Sideband Suppressed-Carrier (DSB-SC) Modulation

- Frequency Domain Description
  - Baseband Spectrum $M(f)$ of message signal $m(t)$



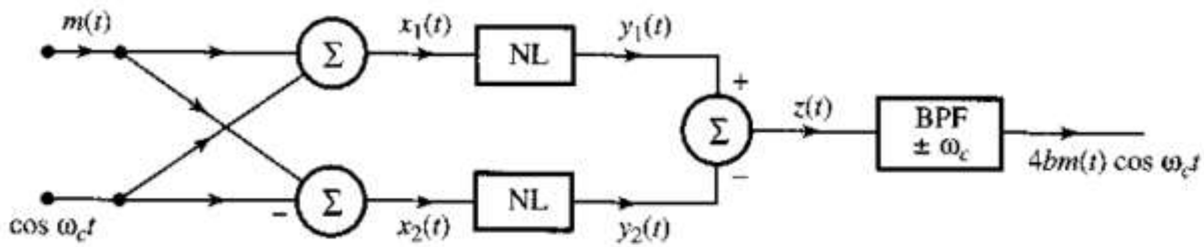
  - $M(f)$ is shifted to $M(f - f_c)$ and $M(f + f_c)$ after modulation

$$m(t)\cos 2\pi f_c t = \frac{1}{2}[M(f - f_c) + M(f + f_c)]$$



Double Sideband Suppressed-Carrier (DSB-SC) Modulation

- **DSB-SC generation: Non-linear Modulator**
  - Modulation can also be achieved using non-linear devices e.g., semiconductor diode or transistor.



- Let the input-output characteristics of non-linear (NL) device be defined using power series

$$y(t) = ax(t) + bx^2(t)$$

- The output $z(t)$ can be defined as

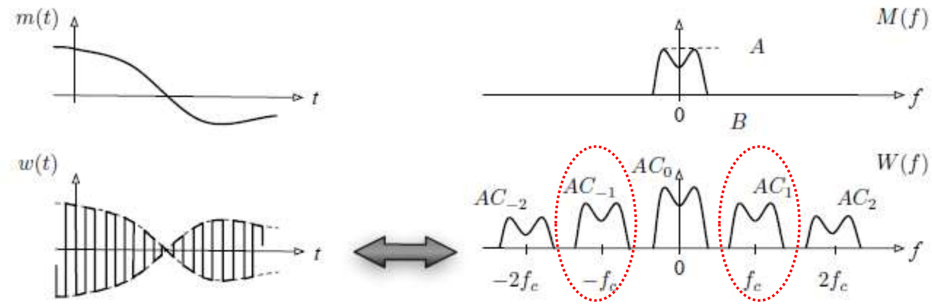
$$z(t) = y_1(t) - y_2(t)$$
$$= [ax_1(t) + b{x_1}^2(t)] - [ax_2(t) + b{x_2}^2(t)]$$

- Substituting $x_1(t) = m(t) + \cos w_c t$ and $x_2(t) = m(t) - \cos w_c t$

$$z(t) = 2am(t) + 4bm(t)\cos w_c t$$

Double Sideband Suppressed-Carrier (DSB-SC) Modulation

- **DSB-SC generation: Switching Modulator**
  - Modulation can also be achieved using a simpler switching operation.
  - If we realize that a modulated signal can be obtained by multiplying $m(t)$ with any periodic signal $\varphi(t)$ with fundamental frequency w_c. Such periodic signal can be represented as



$$\varphi(t) = \sum_{n=0}^{\infty} C_n \cos(nw_c t + \theta_n)$$

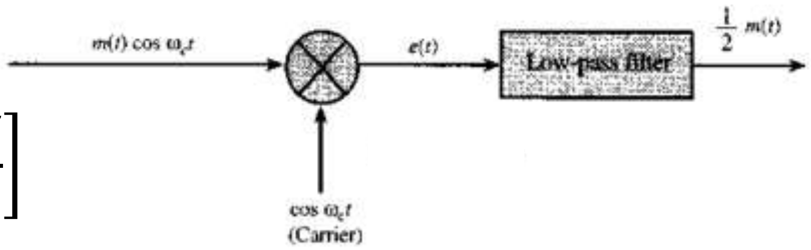
Hence

$$w(t) = m(t)\varphi(t) = \sum_{n=0}^{\infty} C_n m(t) \cos(nw_c t + \theta_n)$$

- This shows that the spectrum of $w(t)$ is the spectrum $M(w)$ shifted to $\pm w_c, \pm 2w_c, \pm 3w_c, \dots, \pm nw_c$.
- If we pass this signal through a bandpass filter (BPF) of bandwidth $2B\ Hz$ and tuned to w_c, then we get the desired modulated signal $c_1 m(t) \cos(w_c t + \theta_1)$.
- Note: Read types of switching modulator e.g., diode bridge modulator, ring modulator (page 147).

Double Sideband Suppressed-Carrier (DSB-SC) Modulation

- Demodulation
  - Process modulated signal $m(t)\cos w_c t$
- Multiply modulated signal with $\cos w_c t$

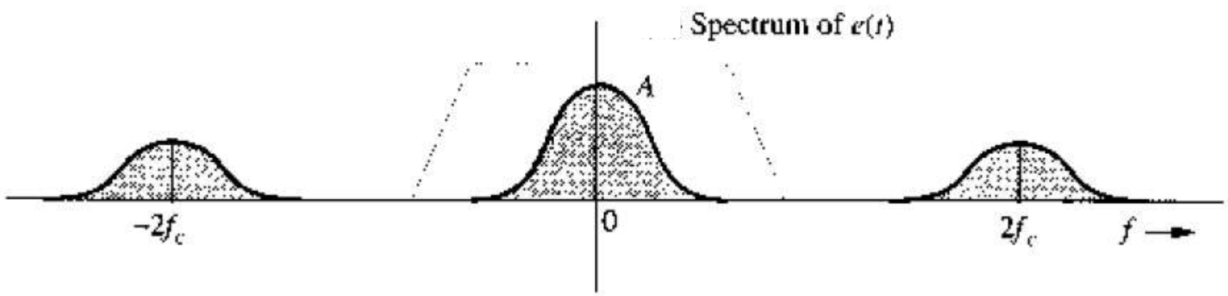


$$e(t) = m(t)\cos^2 w_c t = m(t)\left[\frac{1+\cos 2w_c t}{2}\right]$$

$$e(t) = \frac{1}{2}[m(t) + m(t)\cos 2w_c t]$$

$$\Updownarrow$$

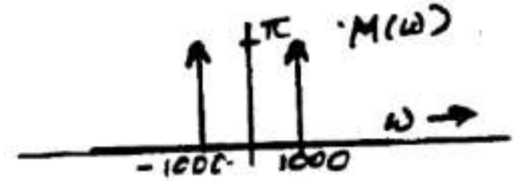
$$E(f) = \frac{1}{2}M(f) + \frac{1}{4}[M(f+2f_c) + M(f-2f_c)]$$



Example

- For the following baseband signal

$$m(t) = \cos 1000t$$



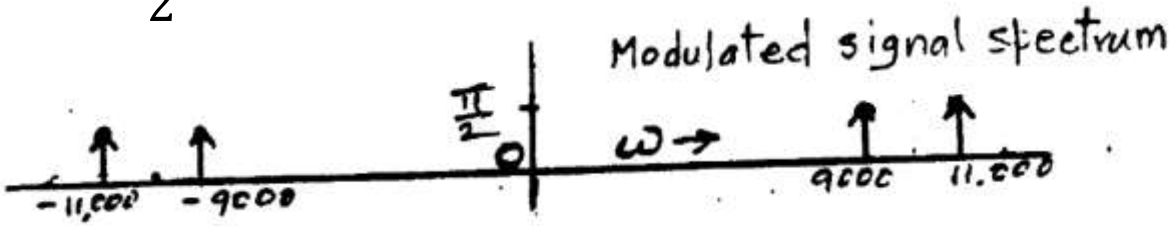
Sketch the spectrum of the DSB-SC signal $m(t)\cos 10000t$

$$\varphi_{DSB-SC}(t) = m(t)\cos 10000t = \cos 1000t \cos 10000t$$

$$\varphi_{DSB-SC}(t) = \frac{1}{2}[\cos 9000t + \cos 11000t]$$

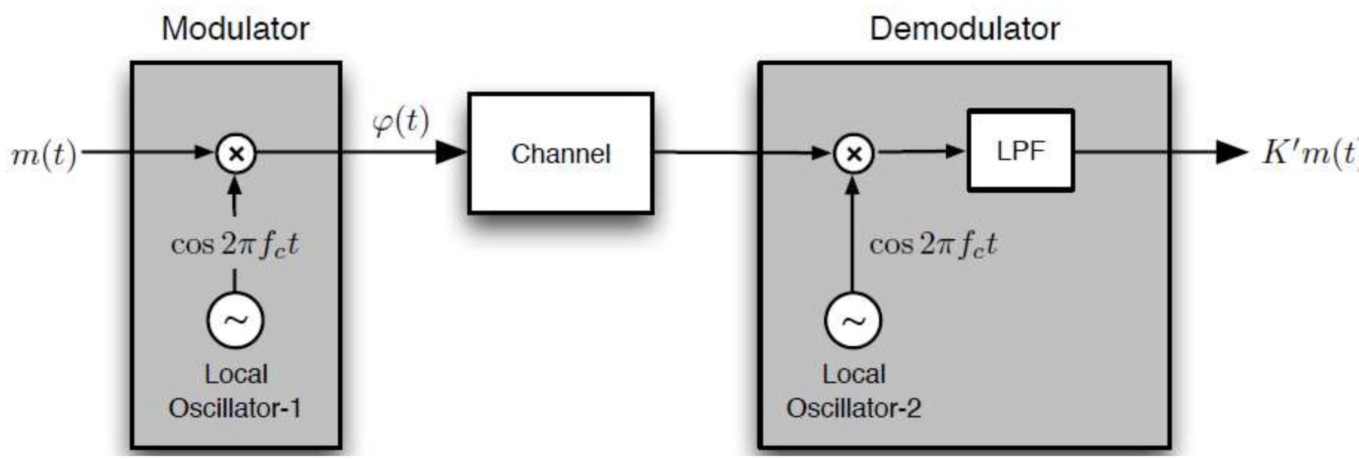
$$\Updownarrow$$

$$\varphi_{DSB-SC}(w)$$
$$= \frac{\pi}{2}[\delta(w-9000) + \delta(w+9000)]$$
$$+ \frac{\pi}{2}[\delta(w-11000) + \delta(w+11000)]$$



Double Sideband Suppressed-Carrier (DSB-SC) Modulation

- The complete **modulation-transmission-demodulation** chain for DSB-SC amplitude modulated signal is given as



- Note: We assume LO-1 in the TX and LO-2 in the RX are synchronized.
- What happens if they are not synchronized…

DSB-SC: Coherent Demodulation

- To investigate effects of lack of synchronization , let's assume
  - LO-1 is defined as $l_m(t) = \cos 2\pi f_c t$
  - LO-2 is defined as $l_d(t) = \cos[2\pi(f_c+\Delta f)t + \theta_0]$
- The DSB-SC signal can be expressed as

$$\varphi_{DSB-SC}(t) = m(t)\cos 2\pi f_c t$$

and demodulated signal is expressed as

$$\varphi_{DSB-SC}(t) l_d(t) = m(t)\cos 2\pi f_c t \cos[2\pi(f_c+\Delta f)t + \theta_0]$$

$$= \frac{1}{2} m(t)\cos[2\pi\Delta f t + \theta_0] + \frac{1}{2} m(t)\cos[2\pi(2f_c + \Delta f)t + \theta_0]$$

- The second term will be centered at $\pm 2f_c$ and it will be filtered out by the lowpass filter in the demodulator.

DSB-SC: Coherent Demodulation

- Lowpass filter output

$$y(t) = \varphi_{DSB-SC}(t) l_d(t) = \frac{1}{2} m(t) \cos[2\pi \Delta f t + \theta_0]$$

- If we assume $\Delta f = 0$, demodulator output becomes

$$y(t) = \varphi_{DSB-SC}(t) l_d(t) = \frac{1}{2} m(t) \cos \theta_0$$

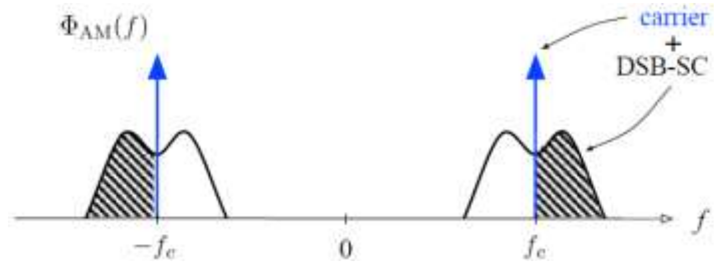
  - If we are lucky and $\theta_0 = 0 \rightarrow y(t) = \frac{1}{2} m(t)$
  - If we are unlucky and $\theta_0 = \frac{\pi}{2} \rightarrow y(t) = 0$
  - Other values of $\theta_0 \rightarrow$ varying degrees of attenuation
- Successful demodulation of DSB-SC amplitude modulated signal requires accurate synchronization of the two local oscillators i.e., synchronous detection/coherent detection.

Amplitude Modulation

- Modulated waveforms with suppressed carrier terms require fairly complex circuitry at the receiver to acquire frequency/phase synchronization i.e., coherent detection, which makes receivers expensive to manufacture.
- In applications where we have one or few transmitters and much larger number of receivers e.g., AM/FM radio broadcasting, it makes economic sense that the receivers are as simple as possible.
- To facilitate simple demodulation we consider the idea of transmitting a separate carrier term in the same frequency band as a DSB-SC AM signal.
  - Double Sideband and Large Carrier

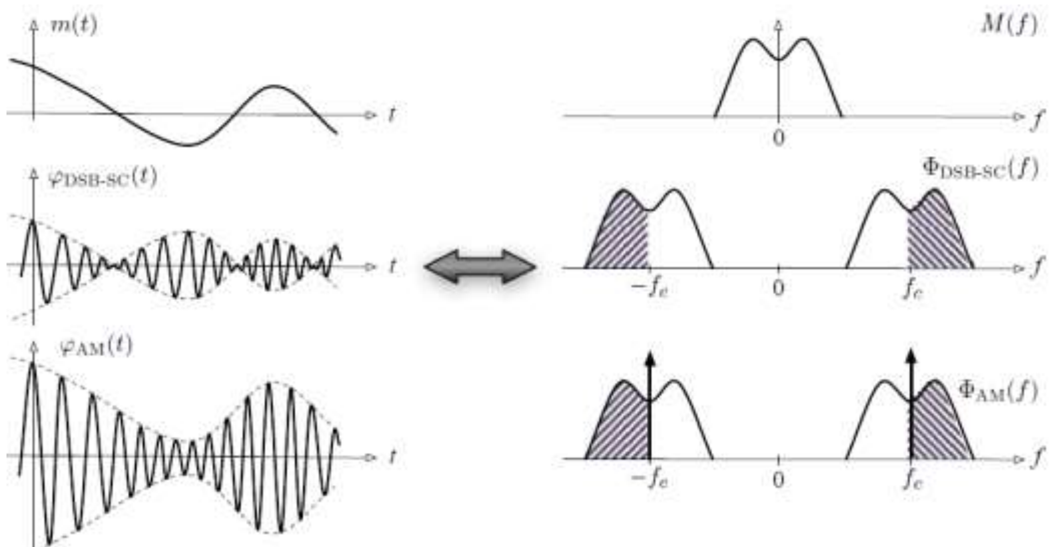
AM: Double Sideband Large Carrier

$$\varphi_{AM}(t) = \varphi_{DSB-SC}(t) + carrier$$
$$= m(t)\cos w_c t + A\cos w_c t$$
$$= [m(t) + A]\cos w_c t$$



Its Fourier spectrum,

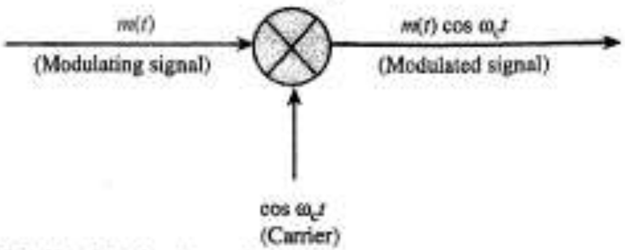
$$\varphi_{AM}(f) = \frac{1}{2}[M(f - f_c) + M(f + f_c)] + \frac{A}{2}[\delta(f - f_c) + \delta(f + f_c)]$$



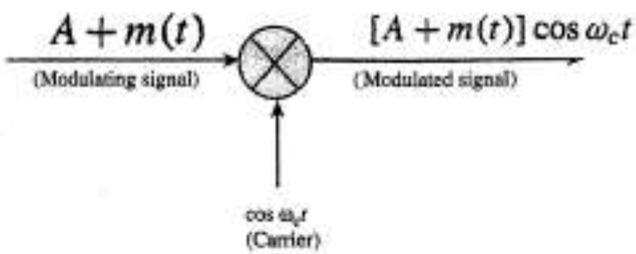
AM: Double Sideband Large Carrier

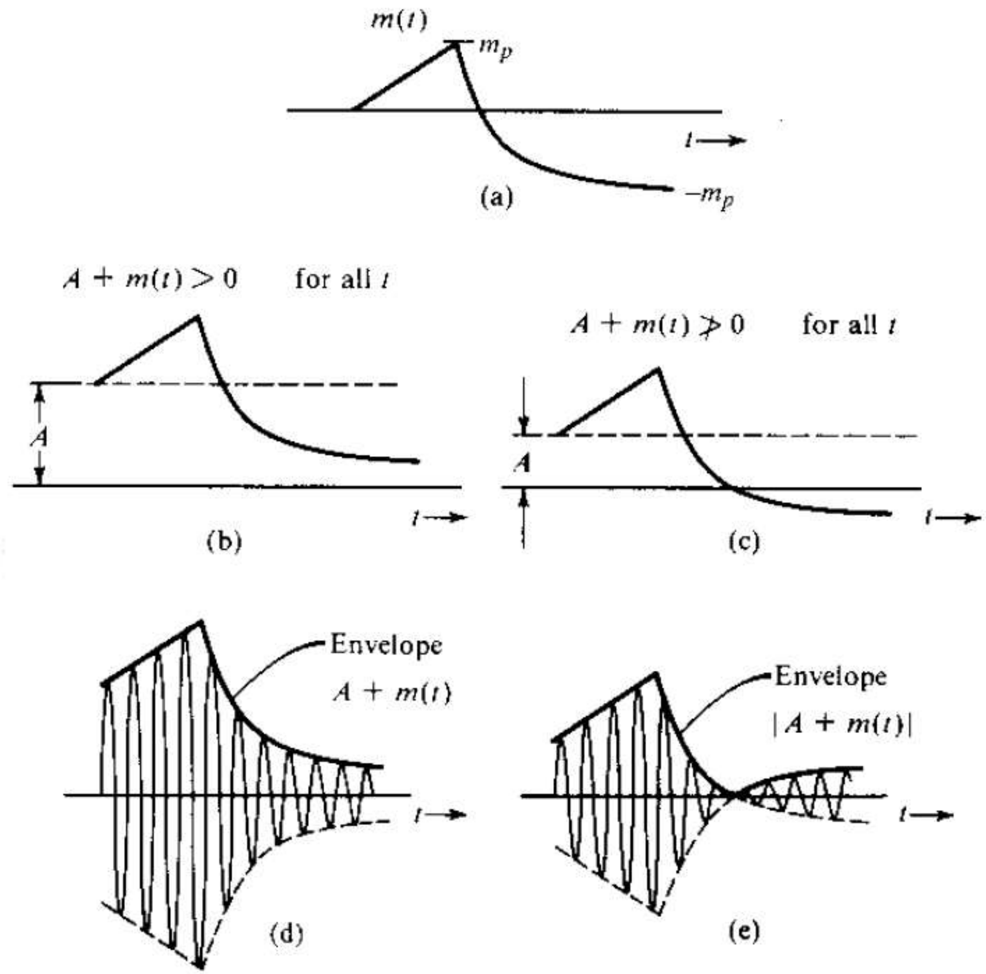
- The size of A affects the time domain envelope of the modulated signal.

- **DSB Modulated signal:**



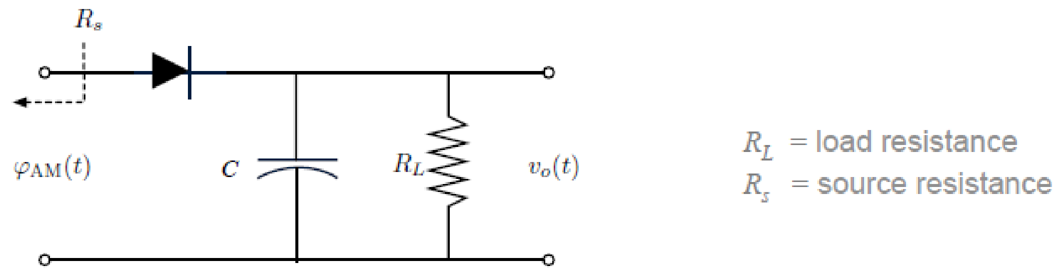
- **Full AM signal**

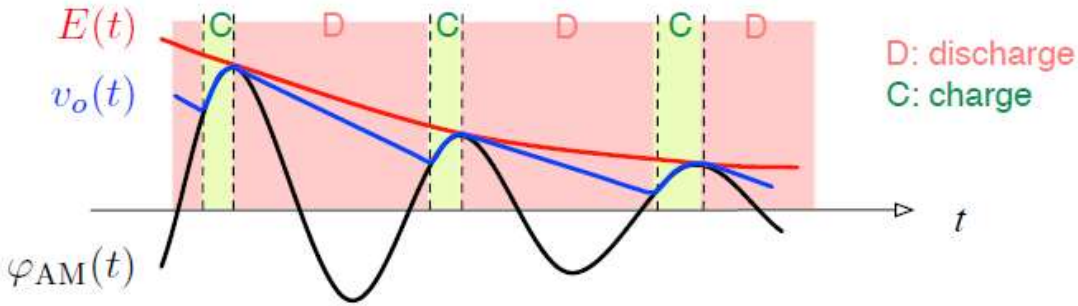




AM: Demodulation

- A simple and inexpensive envelope detector can demodulate AM signal if
  - $f_c \gg \text{BW of } m(t)$
  - $A + m(t) \ \geq 0 \ for\ all\ t$
- AM: Envelope detector

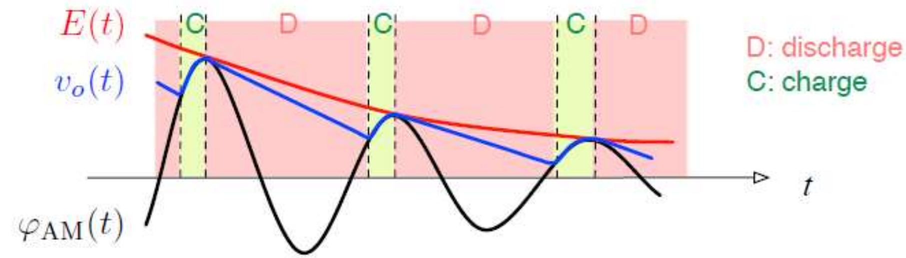


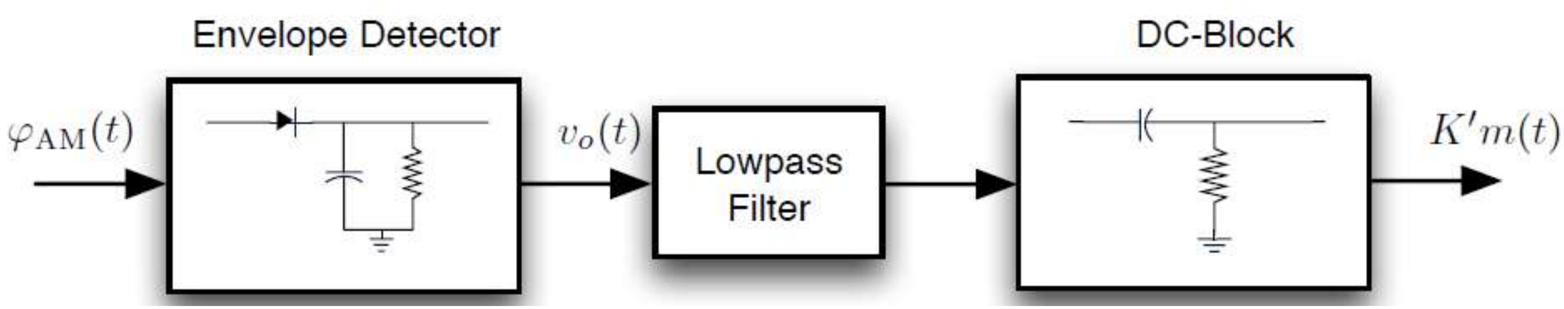


- When $\varphi_{AM}(t) > 0$ and $\varphi_{AM}(t) > v_0(t)$
  - Diode is forward biased and conducts
  - C charges to the maximum value of $\varphi_{AM}(t)$
- When $\varphi_{AM}(t) < 0$ and/or $v_0(t) > \varphi_{AM}(t)$
  - Diode is reversed bias, does not conduct
  - C discharge over R_L

AM: Envelope Detector

- Output of the envelope detector
  - It has ripples
  - Exhibits a DC component
- Solution?
  - Use Low pass filter to smooth ripples
  - Use a DC blocking unit





AM: Modulation Index

- Let $\pm m_p$ be the maximum and minimum values of $m(t)$.
- The AM modulation index is a measure based on the ratio of the peak value of message signal to the level of the unmodulated carrier.

$$\mu = \frac{m_p}{A}$$

- Since condition of envelope detection is $A + m(t) \geq 0$ for all t
  - $A \geq m_p$
- Hence for envelope detection to be distortionless

$$0 \leq \mu \leq 1$$

- When $A < m_p$, it means $\mu > 1$ (over-modulation)
  - Envelope detection is not possible.
  - Need to use synchronous demodulation

AM: Sideband and Carrier Power

- Consider a Full AM signal

$$\varphi_{AM}(t) = m(t)\cos w_c t + A\cos w_c t$$

- The first term on the right hand side are the **sidebands** and the second term represents the **carrier** .
- Power P_c of the carrier $A\cos w_c t$

$$P_c = \frac{A^2}{2}$$

- Power P_s of the sideband signal

$$P_s = \frac{\widetilde{m^2(t)}}{2}$$

- Power efficiency

$$\eta = \frac{useful\ power}{total\ power} = \frac{P_s}{P_c + P_s} = \frac{\widetilde{m^2(t)}}{A^2 + \widetilde{m^2(t)}} 100\%$$

- The useful message information is in the sideband power, whereas carrier power is used for modulation/demodulation.
  - The total power is the sum of carrier power (wasted) and sideband (useful) power.

Example

- Determine power efficiency (η) and the percentage of the total power carried by the sidebands of the AM wave for tone modulation when $\mu = 0.5$

Note that a tone signal can be represented as $A_m \cos w_m t$

$$\eta = \frac{\mu^2}{2+\mu^2} 100\%$$
$$= \frac{(0.5)^2}{2+(0.5)^2} 100\%$$
$$= 11.11\%$$

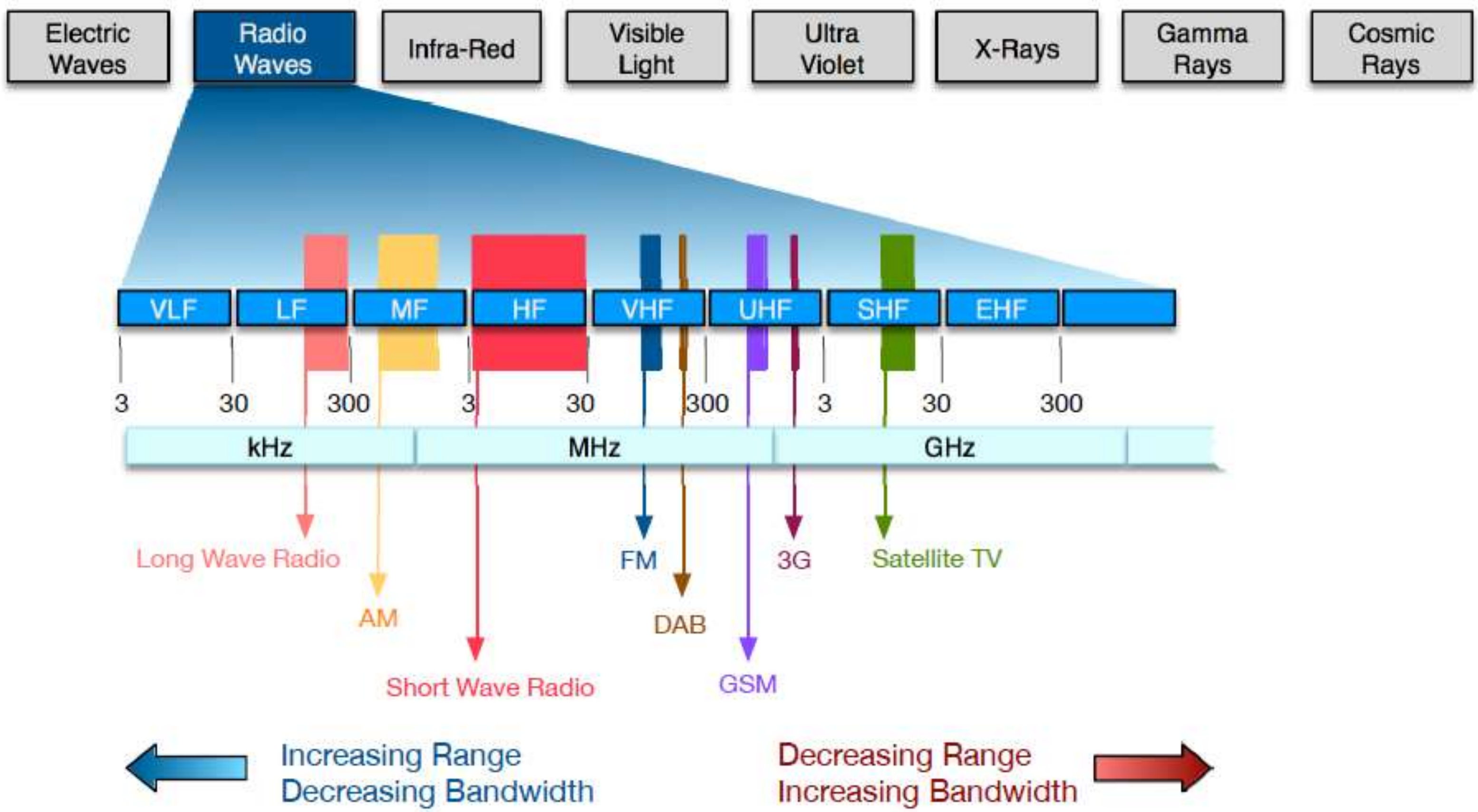
- Only about 11% of the total power is in the sideband.

Bandwidth Efficient Amplitude Modulation

- Since DSB spectrum has two subbands i.e., USB and LSB
  - Required transmission bandwidth is twice the bandwidth of the modulating signal
- Two ways to improve the bandwidth efficiency of AM
  - Quadrature Amplitude Modulation (QAM)
  - Single Sideband (SSB) Modulation
- QAM improves the bandwidth efficiency by transmitting two messages over the same bandwidth of $2B\ Hz$
- SSB modulation removes either LSB or USB and uses only $B\ Hz$ for one message signal $m(t)$
- But before discussing these schemes…

Why are we still discussing AM?

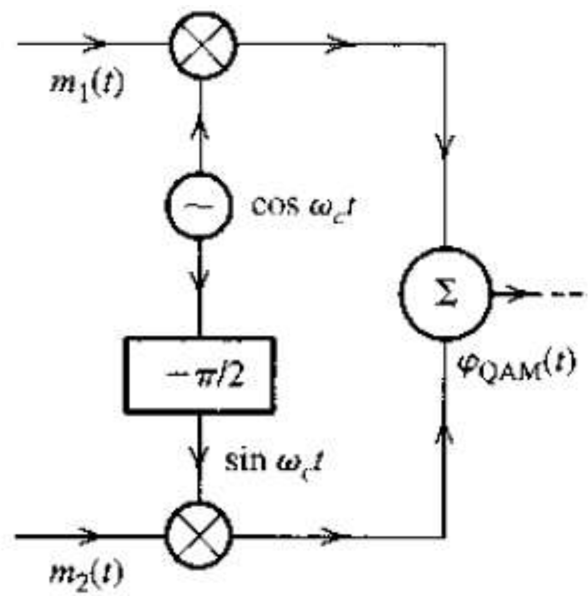
Why AM is Important?



- Most long range radio communication in the LF-HF range is done using AM techniques.

AM: Quadrature Amplitude Modulation

- QAM improves the bandwidth efficiency by transmitting two DSB signals using carriers of same frequency but in phase quadrature.

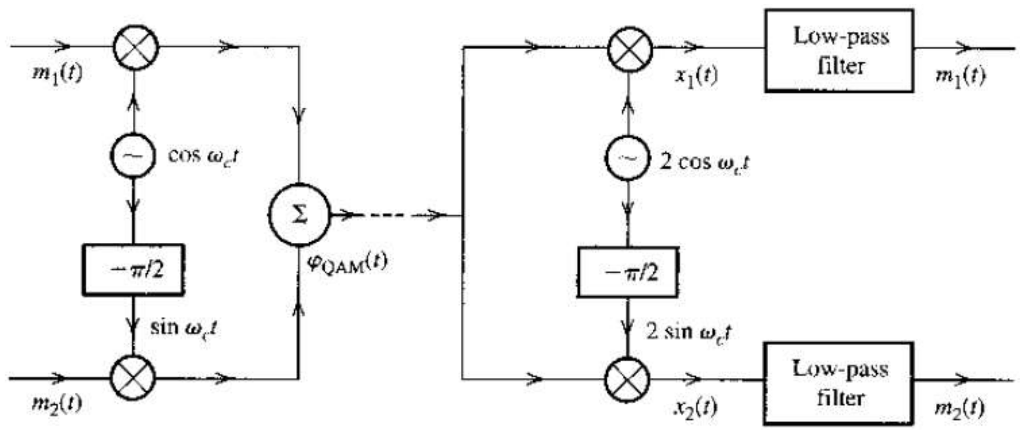


- Both signals occupy the same band. Yet two baseband signals can be separated at the receiver by synchronous detection.

AM: Quadrature Amplitude Modulation

- If the two baseband signals are $m_1(t)$ and $m_2(t)$, the corresponding QAM signal $\varphi_{QAM}(t)$ can be written as

$$\varphi_{QAM}(t) = m_1(t)\cos w_c t + m_2(t)\sin w_c t$$



- At the receiver side, $x_1(t)$ can be defined as

$$x_1(t) = 2\varphi_{QAM}(t)\cos w_c t = 2[m_1(t)\cos w_c t + m_2(t)\sin w_c t]\cos w_c t$$
$$= m_1(t) + m_1(t)\cos 2w_c t + m_2(t)\sin 2w_c t$$

- Applying a LPF will give us

$$x_1(t) = m_1(t)$$

AM: Quadrature Amplitude Modulation

- The upper channel is known as in-phase (I) channel
- The lower channel is known as quadrature (Q) channel
- Note that QAM demodulation is totally synchronous.
  - An error in the phase or the frequency will result in interference between the two signals.
- Suppose the I channel is demodulated using $2\cos(w_c t + \theta)$

$$x_1(t) = 2\varphi_{QAM}(t)\cos(w_c t + \theta)$$
$$= 2[m_1(t)\cos w_c t + m_2(t)\sin w_c t]\cos(w_c t + \theta)$$
$$= m_1(t)\cos\theta - m_2(t)\sin\theta + m_1(t)\cos(2w_c t + \theta)$$
$$+ m_2(t)\sin(2w_c t + \theta)$$

- Applying a LPF will give us

$$x_1(t) = m_1(t)\cos\theta - m_2(t)\sin\theta$$

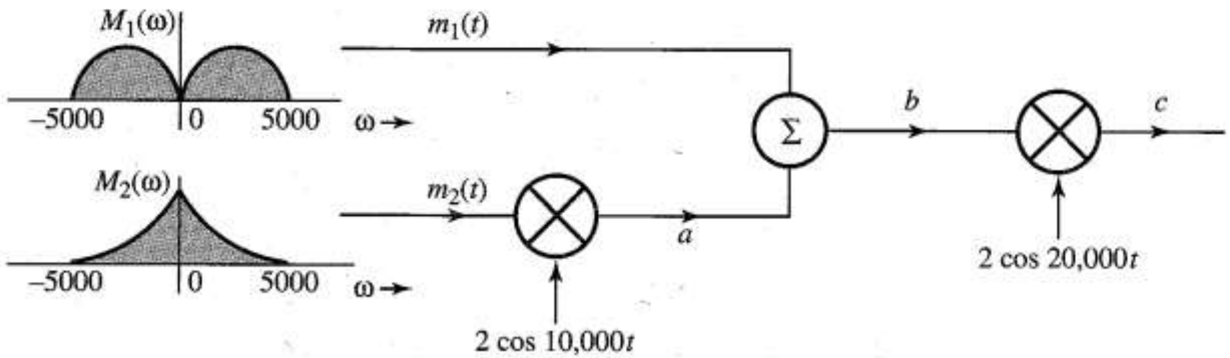
- We observe interference known as “Co-channel Interference” or “Cross Talk”

Properties of QAM

- Bandwidth of QAM signal: $2B\ Hz$
- Transmission power of QAM signal
  - Summation of two DSB-SC signal powers
- QAM is carpool of two message signals in a two-seat car
  - Each message signal has a unique carrier
  - Key point: simple demodulation is required to separate the two signals at the receiver
- Major advantage:
  - Double Bandwidth Efficiency
- Major drawback:
  - Critical requirement of synchronization for demodulation

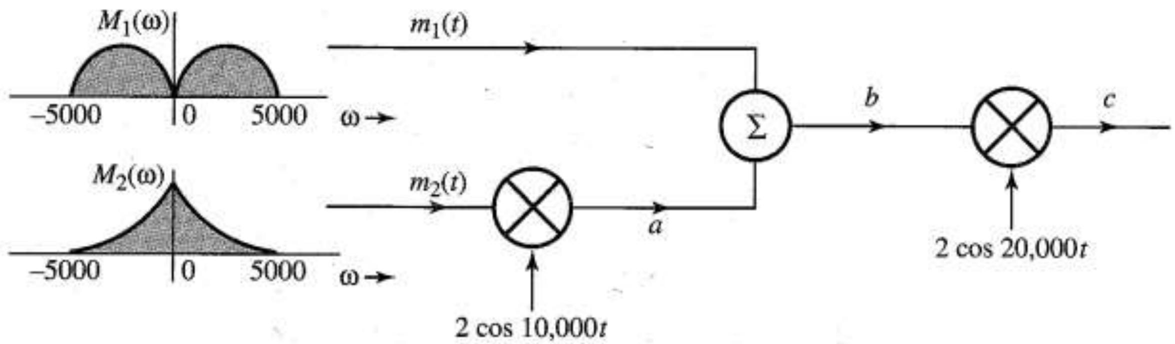
Example

- Two signals $m_1(t)$ and $m_2(t)$, both band limited to 5000 rad/s are to be transmitted simultaneously over a channel as follows



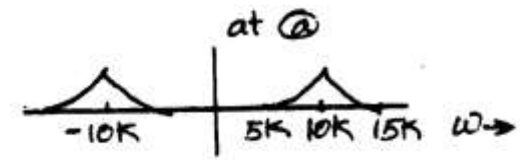
  - Sketch the signal spectra at point a, b and c.
  - What must be the bandwidth of the channel?
  - Design a receiver to recover signals $m_1(t)$ and $m_2(t)$.

Example (cont.)

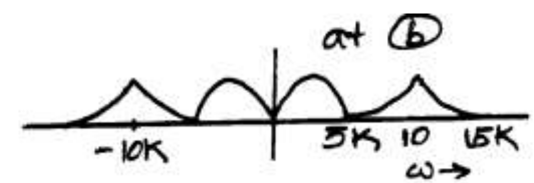


- Derive and Sketch the signal spectra at point a, b and c.

$$\varphi_a(w) = \pi[M_2(w-10000) + M_2(w+10000)]$$

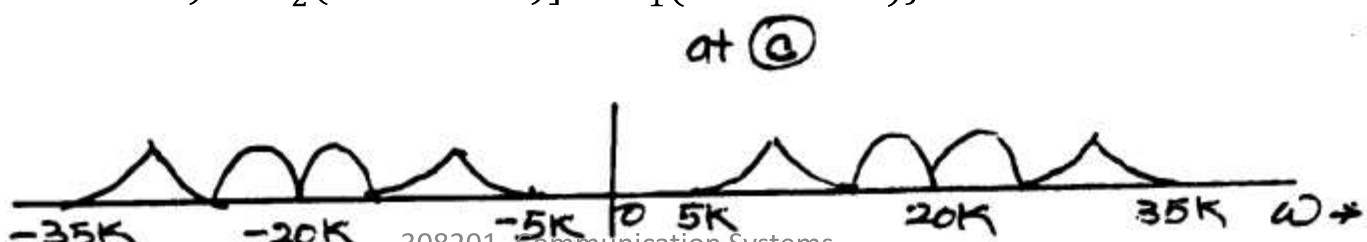


$$\varphi_b(w) = \pi[M_2(w-10000) + M_2(w+10000)] + M_1(w)$$

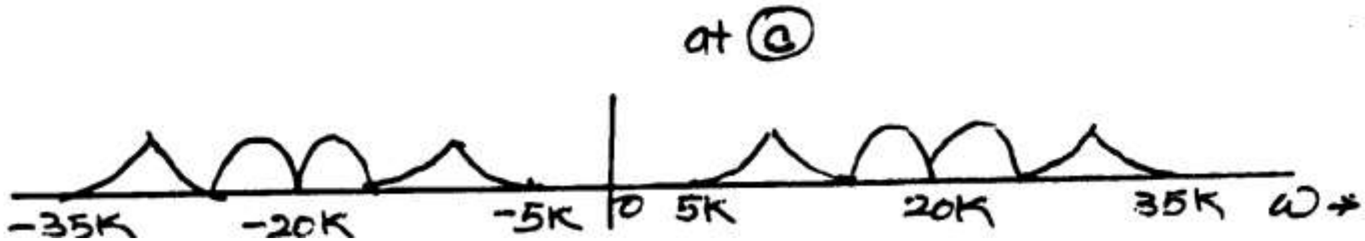


$$\varphi_c(w) = \pi[\varphi_b(w-20000) + \varphi_b(w+20000)]$$

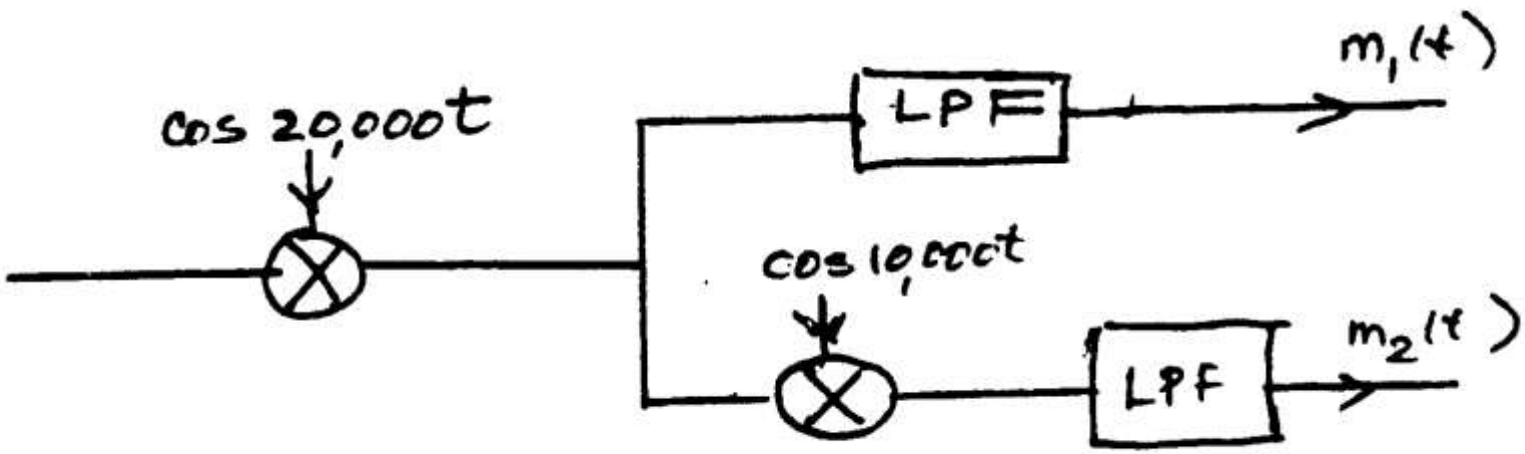
$$\varphi_c(w) = \pi[\{\pi[M_2(w-30000) + M_2(w-10000)] + M_1(w-20000)\}$$



Example (cont.)

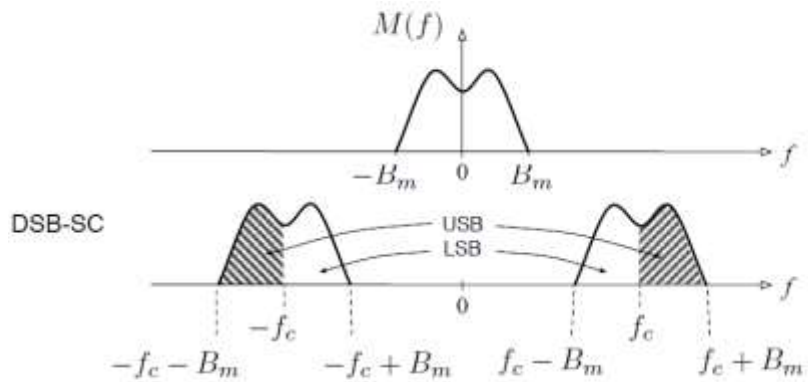


- What must be the bandwidth of the channel?
  - Channel bandwidth must be at least 30K rad/s (5K rad/s to 35K rad/s)
- Design a receiver to recover signals $m_1(t)$ and $m_2(t)$.



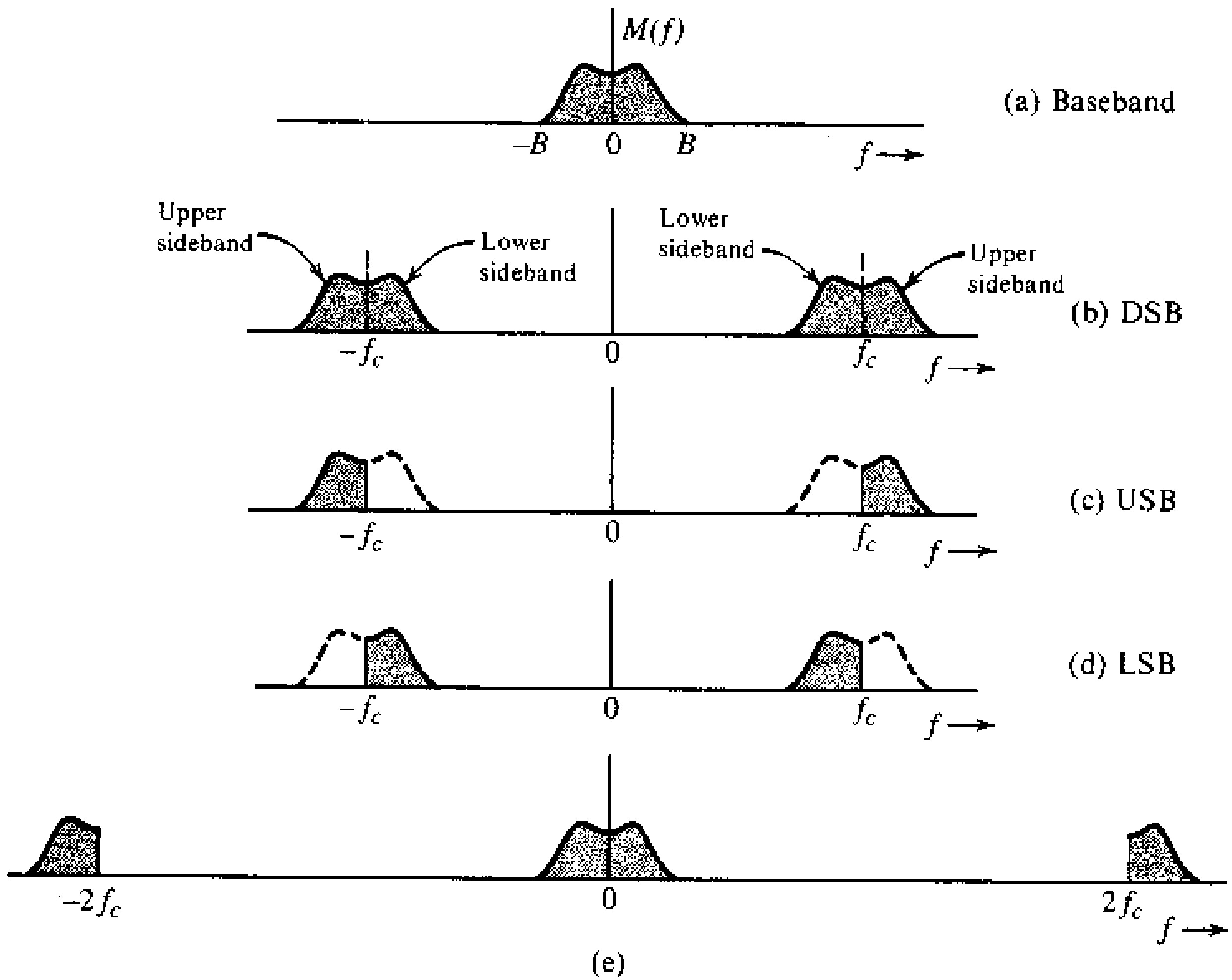
AM: Single Sideband Modulation

- Standard AM and DSB-SC amplitude modulation techniques are wasteful of bandwidth.
  - Both require $2B\ Hz$ transmission bandwidth whereas the modulating signal $m(t)$ bandwidth is just $B\ Hz$
- Doubling the transmission bandwidth is the result of transmitting both upper and lower sidebands.



- Observe that the USB and the LSB signals are uniquely related as they are symmetric.
- Can we transmit only one sideband while retaining $m(t)$?

AM: Single Sideband Modulation



Hilbert Transform

- Let us introduce a tool known as Hilbert Transform for later use.

$$x_h(t) = \mathcal{H}\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\alpha)}{t-\alpha}\,d\alpha$$

- We can observe that the above definition has the form of convolution i.e.,

$$x(t) * \frac{1}{\pi t}$$

- Applying duality property to pair 12 of Table 3.1

$$\frac{1}{\pi t} \Leftrightarrow -j\,\mathrm{sgn}(f)$$

- Therefore

$$X_h(f) = -jX(f)\mathrm{sgn}(f)$$

- This shows that if a message signal $x(t)$ passes through a system with transfer function $H(f) = -j\mathrm{sgn}(f)$, then the output will be the Hilbert Transform of $x(t)$.

Hilbert Transform

- The transfer function of Hilbert Transform can be written as

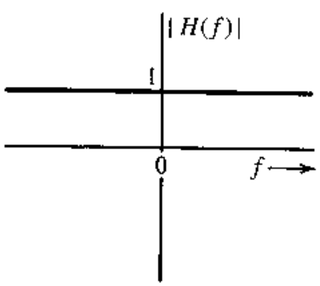
$$H(f) = -j\text{sgn}(f) = \begin{cases} -j = e^{-j\frac{\pi}{2}} & for\ f > 0 \\ j = e^{j\frac{\pi}{2}} & for\ f < 0 \end{cases}$$

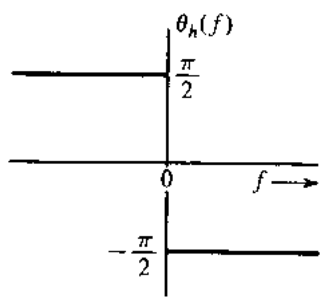
- It shows that

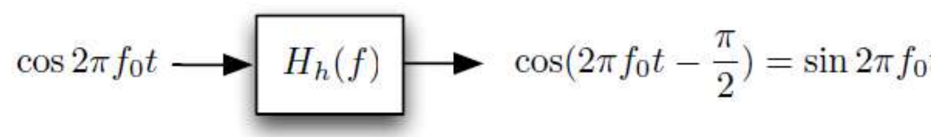
$$|H(f)| = 1$$

$$\theta_h(f) = -{}^{\pi}/_{2} \text{ for } f > 0\ and\ {}^{\pi}/_{2} \text{ for } f < 0$$

- If we change the phase of every component of $x(t)$ by ${}^{\pi}/_{2}$, we get $x_h(t)$.
- Hilbert transformer is an ideal phase shifter that shifts the phase of every spectral component by $-{}^{\pi}/_{2}$







Time Domain Representation of SSB Signal

$$M_+(f) = M(f).u(f)$$
$$M_-(f) = M(f).u(-f)$$

- As we know that

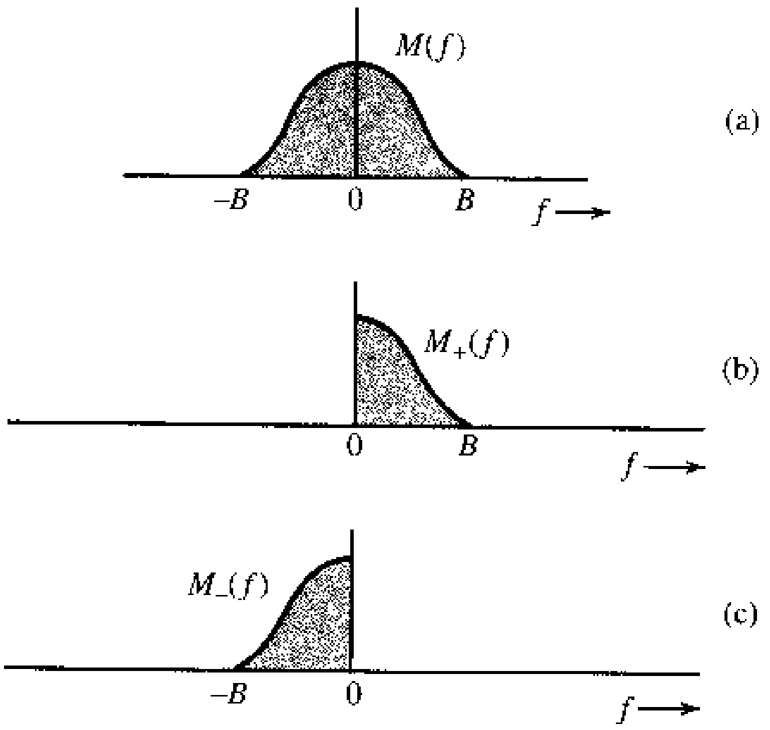
$$u(f) = \frac{1}{2}[1 + \text{sgn}(f)]$$
$$u(-f) = \frac{1}{2}[1 - \text{sgn}(f)]$$

- So

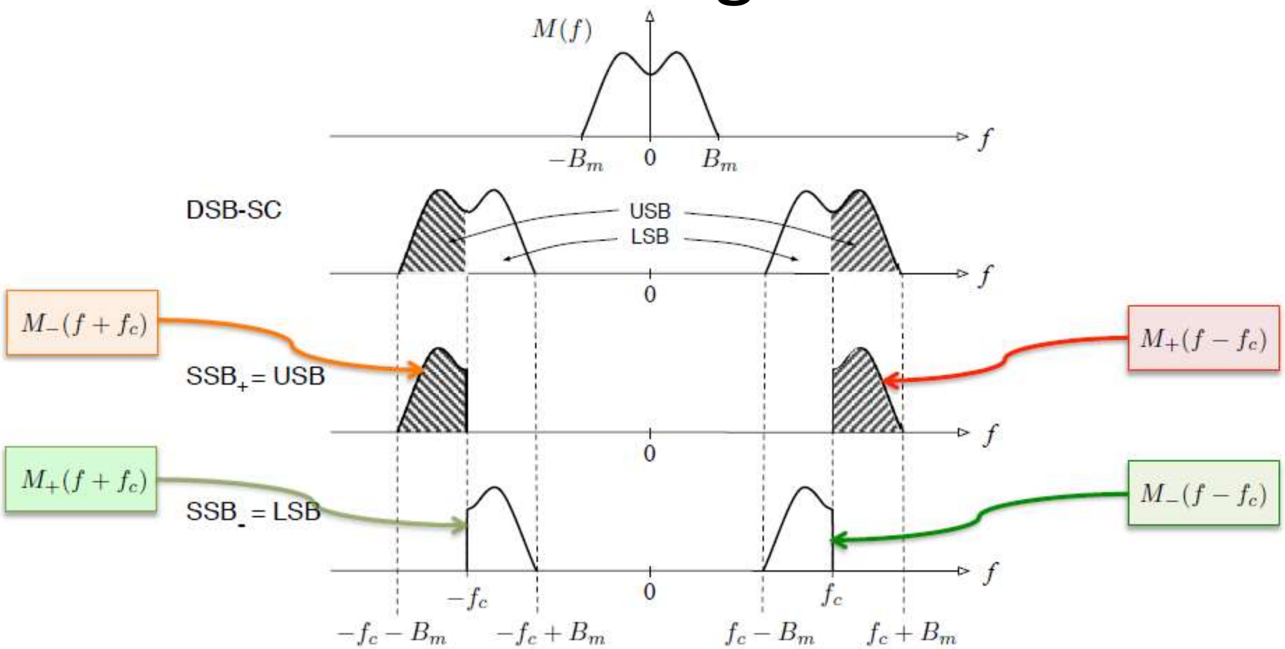
$$M_+(f) = \frac{1}{2}[M(f) + M(f)\text{sgn}(f)]$$
$$M_-(f) = \frac{1}{2}[M(f) - M(f)\text{sgn}(f)]$$

- We can hence write

$$M_+(f) = \frac{1}{2}[M(f) + jM_h(f)]$$
$$M_-(f) = \frac{1}{2}[M(f) - jM_h(f)]$$



Time Domain Representation of SSB Signal



$$\Phi_{\mathrm{SSB}_+}(f) = \boxed{M_+(f-f_c)} + \boxed{M_-(f+f_c)}$$

$$\Phi_{\mathrm{SSB}_-}(f) = \boxed{M_-(f-f_c)} + \boxed{M_+(f+f_c)}$$

$$\varphi_{SSB+}(f) = \frac{1}{2}[M(f-f_c) + M(f+f_c)] - \frac{1}{2j}[M_h(f-f_c) - M_h(f+f_c)]$$

$$\varphi_{SSB+}(t) = m(t)\cos w_c t - m_h(t)\sin w_c t$$

Time Domain Representation of SSB Signal

- Similarly

$$\varphi_{SSB-}(t) = m(t)\cos w_c t + m_h(t)\sin w_c t$$

- General SSB signal can be expressed as

$$\varphi_{SSB}(t) = m(t)\cos w_c t \mp m_h(t)\sin w_c t$$

- The SSB signal can be coherently demodulated as follows

$$2\varphi_{SSB}(t)\cos w_c t = [m(t)\cos w_c t \mp m_h(t)\sin w_c t]2\cos w_c t$$

$$= m(t)[1 + \cos 2w_c t] \mp m_h(t)\sin 2w_c t$$

$$= m(t) + m(t)\cos 2w_c t \mp m_h(t)\sin 2w_c t$$

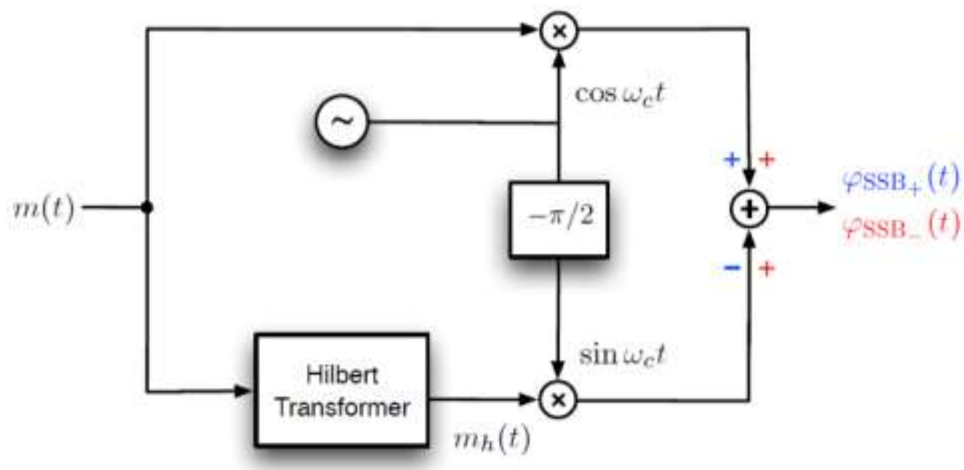
- The product $2\varphi_{SSB}(t)\cos w_c t$ yields the baseband signal and another SSB signal centered at $2w_c$.
- A low pass filter can be used to extract the baseband signal and suppress the unwanted SSB spectrum.

Generation of SSB Signal

- Phase Shift Method

$$\varphi_{\mathrm{SSB}_+}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

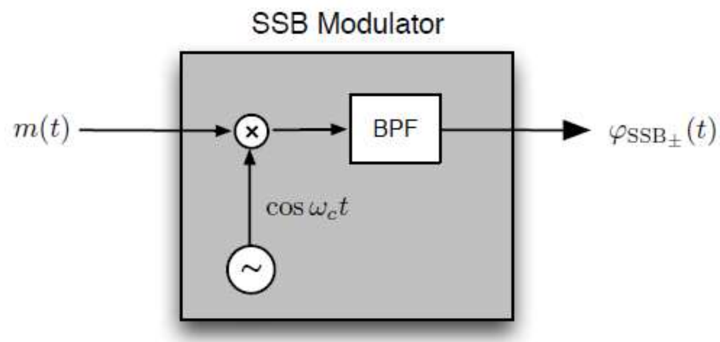
$$\varphi_{\mathrm{SSB}_-}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

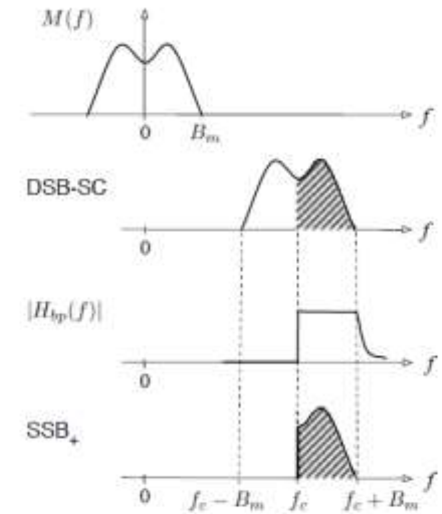


- It is easy to build a circuit to shift the phase of a single frequency component by $\pi/2$ radians, but a device to achieve a phase shift of all the spectral components over a band of frequencies is unrealizable.
- We can, at best, approximate it over a finite band.

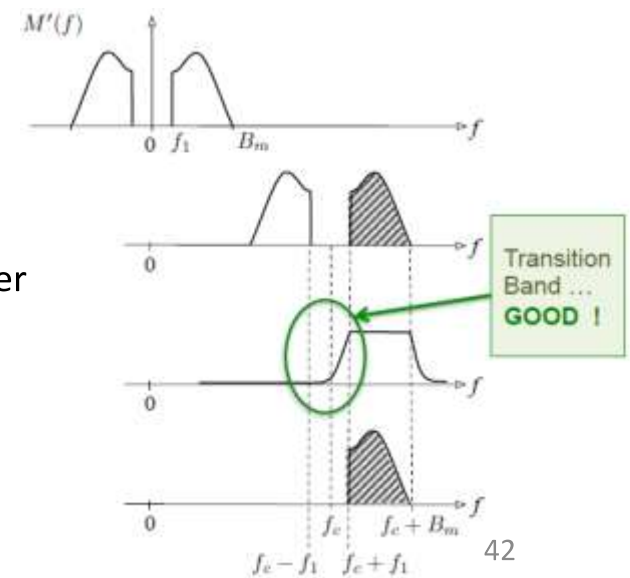
Generation of SSB Signal

- Selective Filtering Method



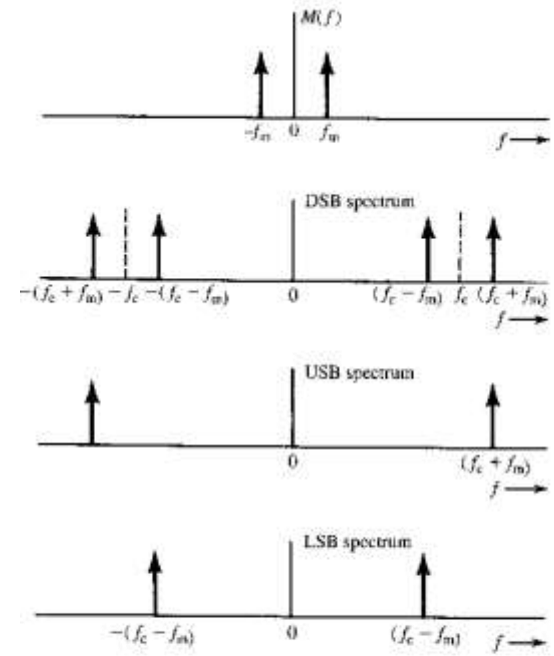


- Most commonly used technique to generate SSB signal.
- Require
  - $B_m \ll f_c$
  - $m(t)$ should have a frequency hole around $f = 0$
    - Why?
    - Without a frequency hole, we need an ideal bandpass filter
- Two step process
  - Generate a DSB-SC signal
  - Filter out one of the sidebands using a BPF.



Example

- Find $\varphi_{SSB}(t)$ for a tone modulation case when the modulating signal is $m(t) = \cos w_m t$. Also demonstrate the coherent demodulation of this SSB signal.



$$m(t) = \cos w_m t$$

$$m_h(t) = \cos\left(\omega_m t - \frac{\pi}{2}\right) = \sin \omega_m t$$

$$\varphi_{\text{SSB}}(t) = m(t)\cos \omega_c t \mp m_h(t)\sin \omega_c t$$

$$\begin{aligned}\varphi_{\text{SSB}}(t) &= \cos \omega_m t \cos \omega_c t \mp \sin \omega_m t \sin \omega_c t \\ &= \cos(\omega_c \pm \omega_m)t\end{aligned}$$

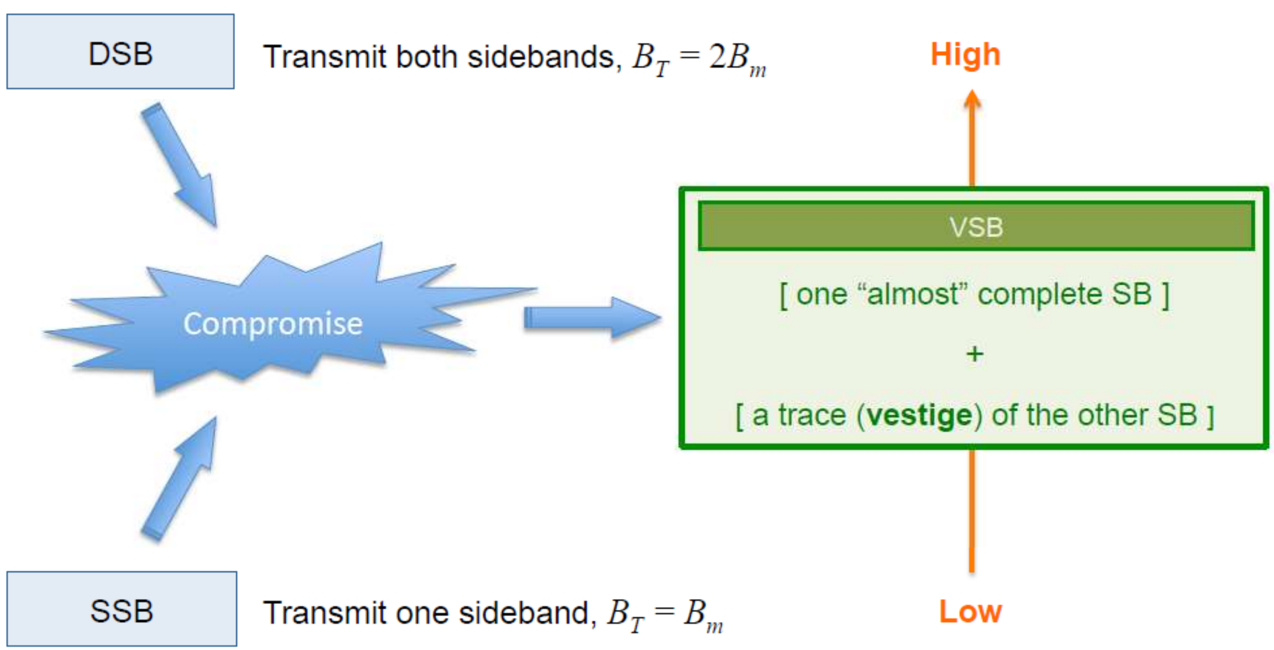
$$\varphi_{\text{USB}}(t) = \cos(\omega_c + \omega_m)t \quad \text{and} \quad \varphi_{\text{LSB}}(t) = \cos(\omega_c - \omega_m)t$$

Finally, the coherent demodulation of the SSB tone modulation is can be achieved by

$$\begin{aligned}\varphi_{\text{SSB}}(t) 2\cos \omega_c t &= 2\cos(\omega_c \pm \omega_m)t \cos \omega_c t \\ &= \cos \omega_m t + \cos(\omega_c + \omega_m)t \xrightarrow{\textbf{LPF}} \cos \omega_m t.\end{aligned}$$

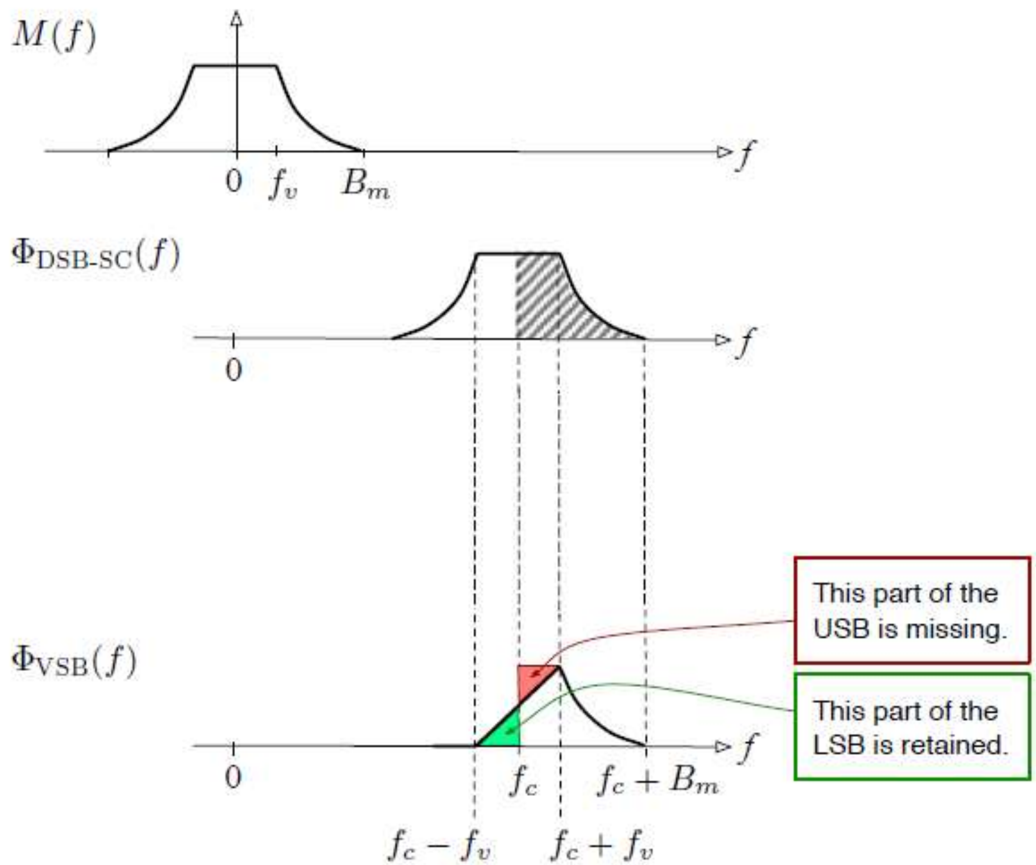
AM: Vestigial Sideband (VSB)

- Bandwidth Efficient Modulation Techniques
  - If we have two signals: **QAM**
  - If $m(t)$ has frequency hole around $f = 0$: **SSB Modulation**
    - But if $m(t)$ has significant low frequency energy: **What to do?**



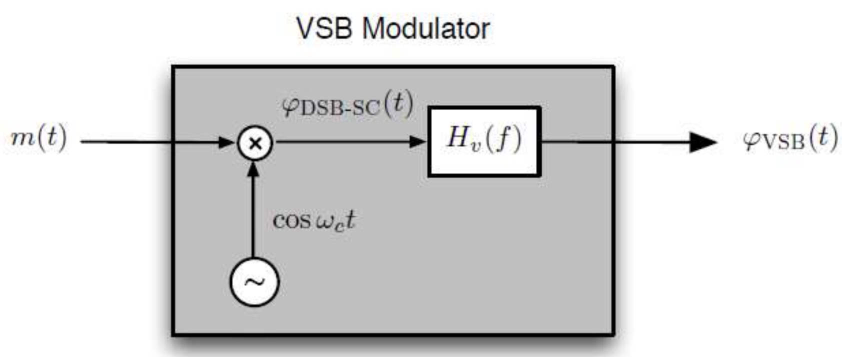
AM: Vestigial Sideband (VSB)

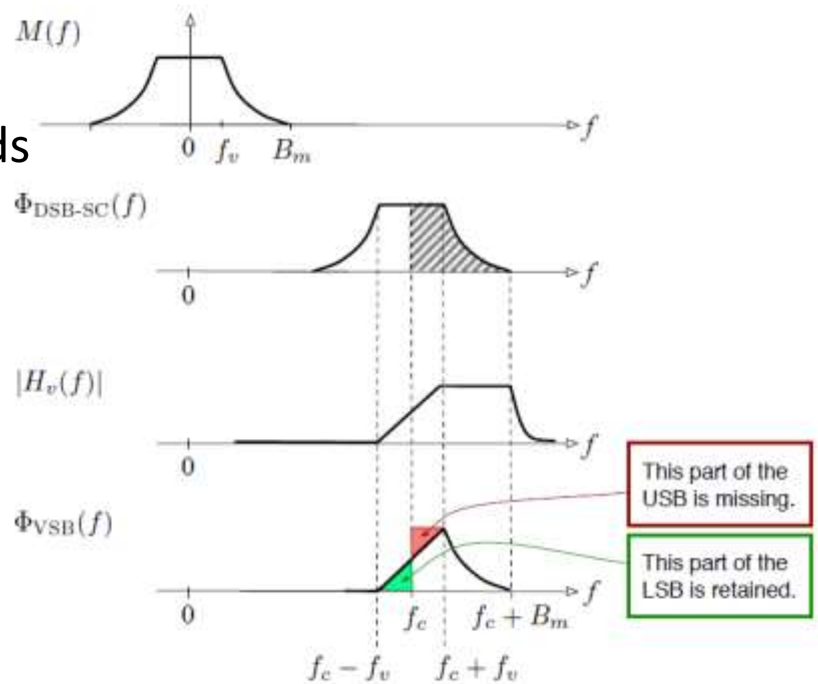
- VSB: One "almost" complete SB + a trace (vestige) of the other SB



Generation of VSB Signal

- Same selective filtering approach as used in generation of SSB signals
- Two step process
  - Generate a DSB-SC signal
  - Filter out and shape the sidebands using an appropriately designed BPF

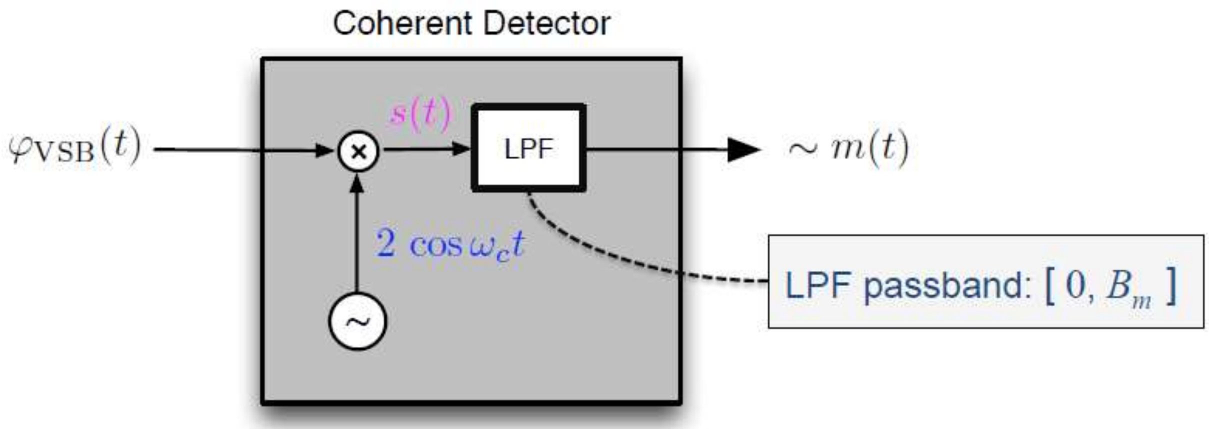




- What conditions do we need on $H_v(f)$?
  - Lets figure out how to demodulate VSB signal first!

VSB Demodulation

- Use coherent detection as before



- Lets analyze the demodulator by first evaluating the signal at the input of the LPF

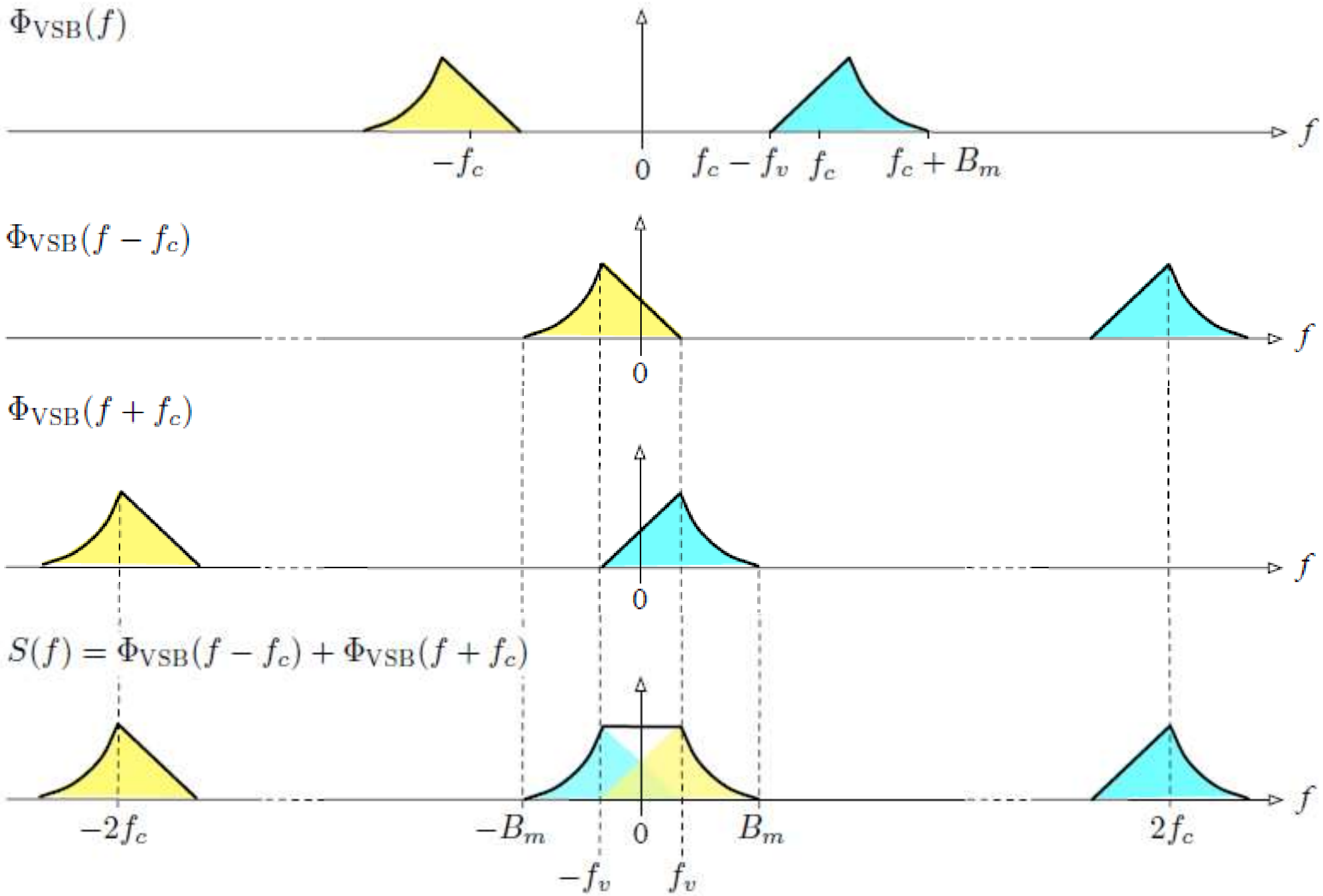
$$s(t) = \varphi_{VSB}(t)[2\cos w_c t]$$

- Its spectrum will be

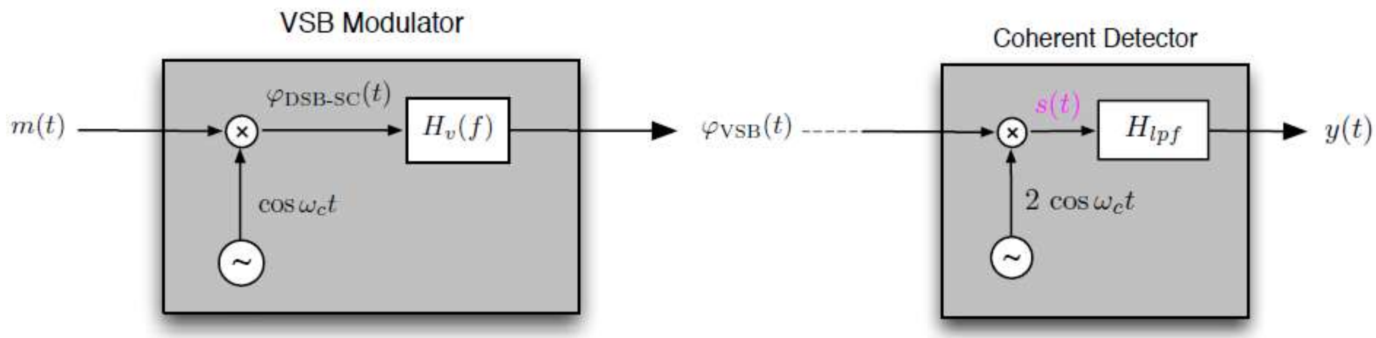
$$S(f) = \varphi_{VSB}(f - f_c) + \varphi_{VSB}(f + f_c)$$

VSB Demodulation

$s(t) = \varphi_{VSB}(t)[2\cos w_c t]$ $\quad$ $S(f) = \varphi_{VSB}(f - f_c) + \varphi_{VSB}(f + f_c)$



How to choose $H_v(f)$?



$$\varphi_{DSB-SC}(f) = M(f - f_c) + M(f + f_c)$$

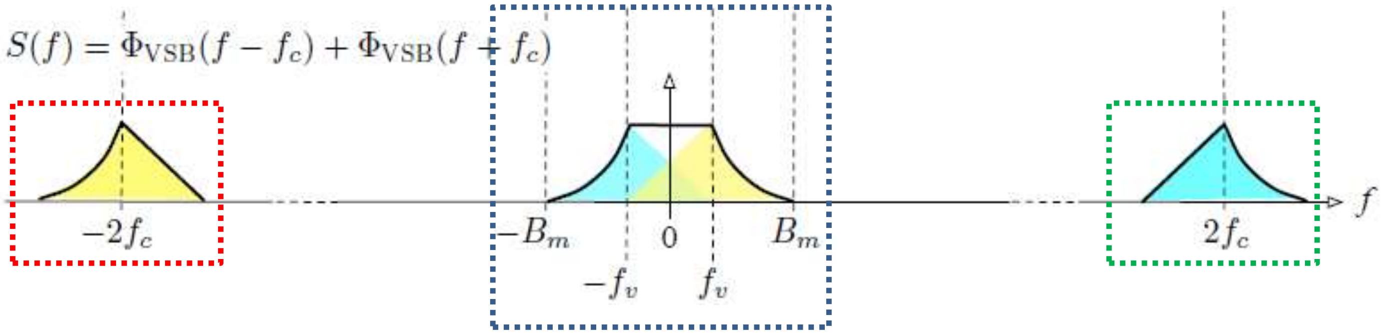
$$\varphi_{VSB}(f) = \varphi_{DSB-SC}(f)H_v(f) = [M(f - f_c) + M(f + f_c)]H_v(f)$$

$$S(f) = \varphi_{VSB}(f - f_c) + \varphi_{VSB}(f + f_c)$$

- The output of the lowpass filter can be written as

$$Y(f) = S(f)H_{lpf}(f)$$

How to choose $H_v(f)$?



$$Y(f) = S(f)H_{lpf}(f) = M(f)$$

$$M(f) = [\varphi_{VSB}(f - f_c) + \varphi_{VSB}(f + f_c)]H_{lpf}(f)$$

$$= [\{[M(f - 2f_c) + M(f)]H_v(f - f_c)\} + \{[M(f) + M(f + 2f_c)]H_v(f + f_c)\}]H_{lpf}(f)$$

$$= [\{H_v(f - f_c) + H_v(f + f_c)\}M(f) + M(f - 2f_c)H_v(f - f_c)$$

Further Comments

- VSB modulation is a compromise between DSB and SSB modulation

$$\lim_{f_v \to 0} \varphi_{VSB}(t) = \varphi_{SSB}(t) \text{ and } \lim_{f_v \to B_m} \varphi_{VSB}(t) = \varphi_{DSB}(t)$$

- Transmission bandwidth B_T

$$B_m = B_T[\varphi_{SSB}(t)] < B_T[\varphi_{VSB}(t)] < B_T[\varphi_{DSB}(t)] = 2B_m$$

- In practice, VSB modulation introduces approximately 25% increase in B_T

$$B_T[\varphi_{VSB}(t)] \approx 1.25 B_T[\varphi_{SSB}(t)] = 1.25 B_m$$

Carrier Acquisition

- All the modulators can be used as demodulators, provided the bandpass filters at the output are replaced by lowpass filters of bandwidth $B\ Hz$.
- The receiver require the same carrier frequency and phase as those used in the received signal.
- However,
  - Electronic device parameter drifting can make the frequency of LO-1 and LO-2 different
  - Propagation delay, different local timing can make the two carriers different in phase
- We therefore need carrier recovery at the receiver for coherent detection.
  - **Pilot Method:** Transmitters send a pilot signal, receiver estimate the correct carrier frequency and phase according to the received pilot signal
  - **Costas Loop:** No pilot signals are sent. The receiver determine the carrier frequency and phase from the received signal.
  - **Signal Squaring Method:** The incoming signal is squared, and then passed through a narrow bandpass filter tuned at $2w_c$

Carrier Acquisition : Costas Phase Lock Loop

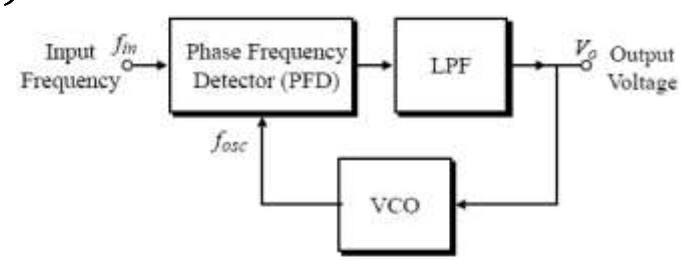
- Costas loop is a phase-lock loop (PLL) used for carrier phase recovery from suppressed-carrier signals such as DSB-SC.
- Frequency is the time-varying rate of phase, so carrier recovery problem can be reduced to phase tracking only

$$f = d\phi(t)/dt$$

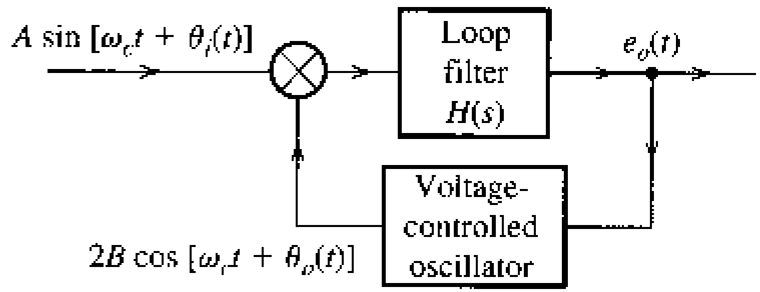
- **Basic principle:** If the local carrier is synchronized, then we have perfect demodulation, otherwise, the demodulation is deviated. So we can use the demodulation error to feedback control the carrier estimation.
  - So just a feedback control loop.

Phase Locked Loop

- The phase locked loop is a device typically used to track the phase and the frequency of the carrier component of the incoming signal.
- It is useful for synchronous demodulation of AM signal with suppressed carrier (DSB-SC).
- A PLL has three basic components
  - A voltage controlled oscillator (VCO)
  - A multiplier, serving as a phase detector (PD)
  - A loop filter $H(s)$



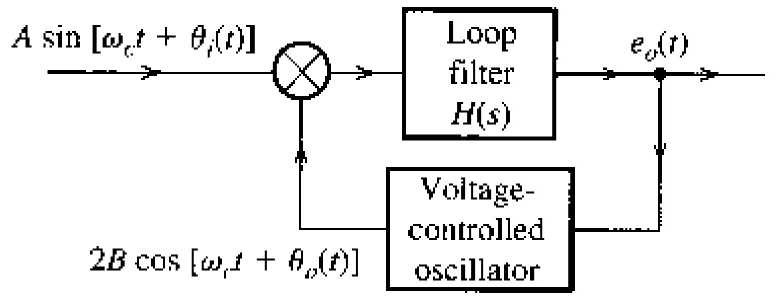
Basic PLL operation



- The operation of PLL is similar to that of a feedback system.
- If the feedback signal is not equal to the input signal, the difference will change the feedback signal until it is close to the input signal.
- A PLL compares the phase of the feedback signal with the input signal and the difference adjusts the VCO frequency and phase so that the feedback can track the phase and frequency of the input signal.

Voltage Controlled Oscillator (VCO)

- A VCO is an oscillator whose frequency can be linearly controlled by an input voltage.

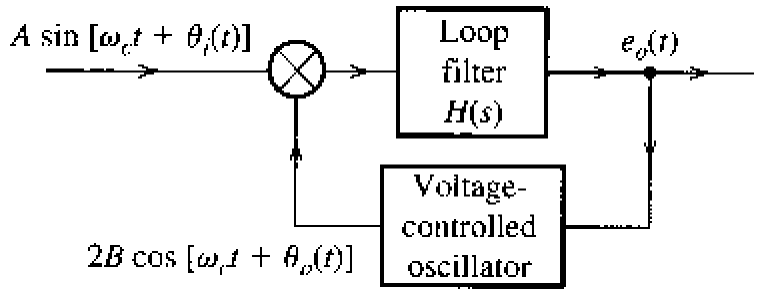


- If input voltage is $e_0(t)$, its output is a sinusoid with instantaneous frequency given by

$$w(t) = w_c + ce_0(t)$$

  - c is a constant of VCO
  - w_c is the free running frequency of the VCO when $e_0(t) = 0$
- If VCO output is $B\cos[w_c t + \theta_0(t)]$, then its instantaneous frequency is $w_c + \dot{\theta}_0(t)$
  - $\dot{\theta}_0(t) = ce_0(t)$

PLL Operation



- The multiplier output is

$$AB \sin[w_c t + \theta_i] \cos[w_c t + \theta_0] = \frac{AB}{2}[\sin(\theta_i - \theta_0) + \sin(2w_c t + \theta_i + \theta_0)]$$

- The loop filter will suppress the $2w_c$ component
- The effective input to the loop filter will be $\frac{AB}{2}[\sin\{\theta_i(t) - \theta_0(t)\}]$
- If $h(t)$ is the unit impulse response of the loop filter

$$e_0(t) = h(t) * \frac{1}{2}AB[\sin\{\theta_i(t) - \theta_0(t)\}]$$

$$= \frac{1}{2}AB \int_0^t h(t-x) \sin\{\theta_i(x) - \theta_0(x)\}\, dx$$

PLL Operation

- As we know that

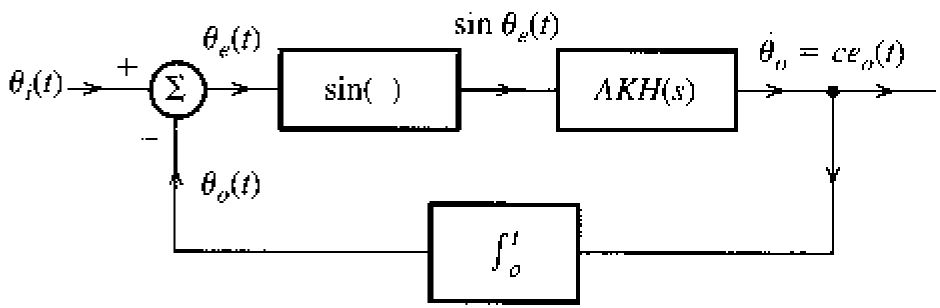
$$\dot{\theta}_0(t) = ce_0(t)$$

- So

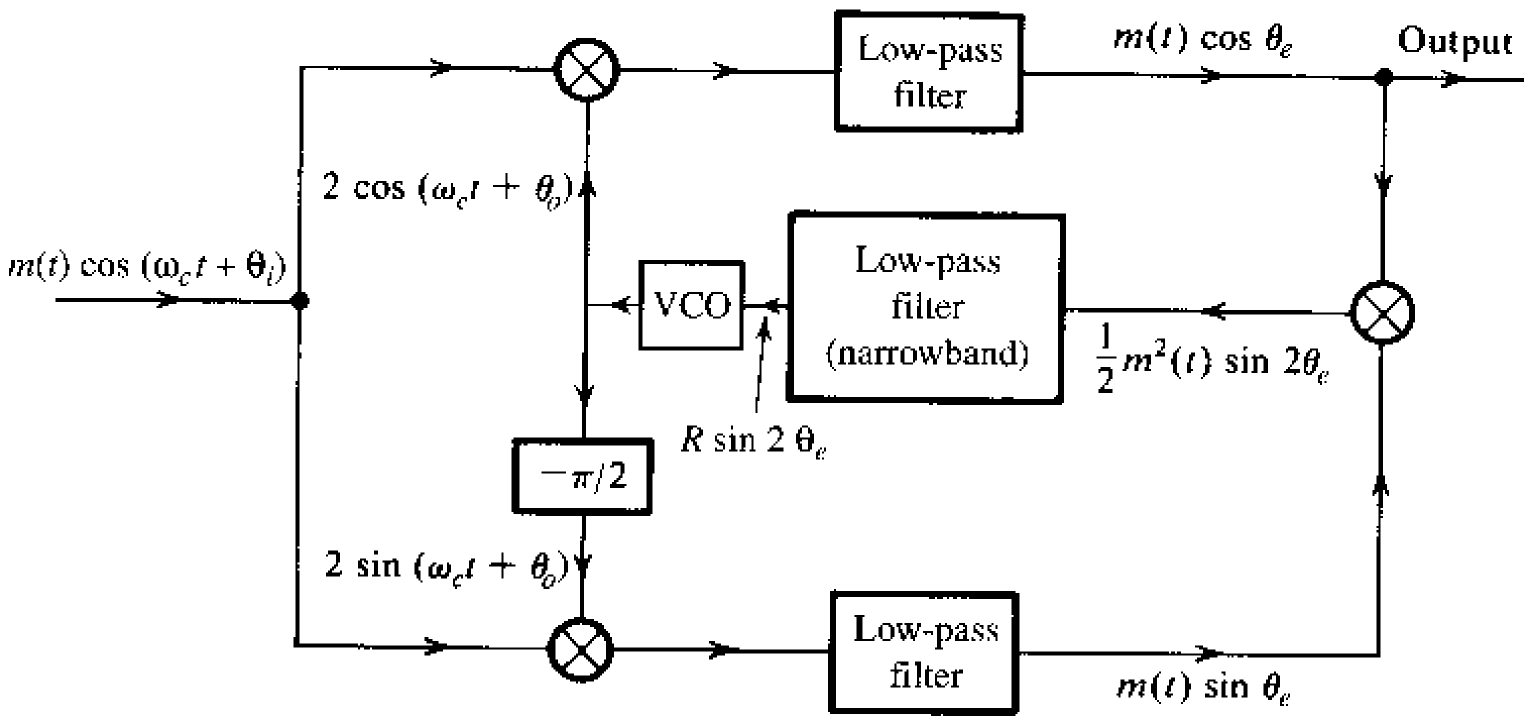
$$\dot{\theta}_0(t) = c\frac{1}{2}AB\int_0^t h(t-x)\sin\{\theta_i(x) - \theta_0(x)\}\,dx$$

$$\dot{\theta}_0(t) = AK\int_0^t h(t-x)\sin\theta_e(x)\,dx$$

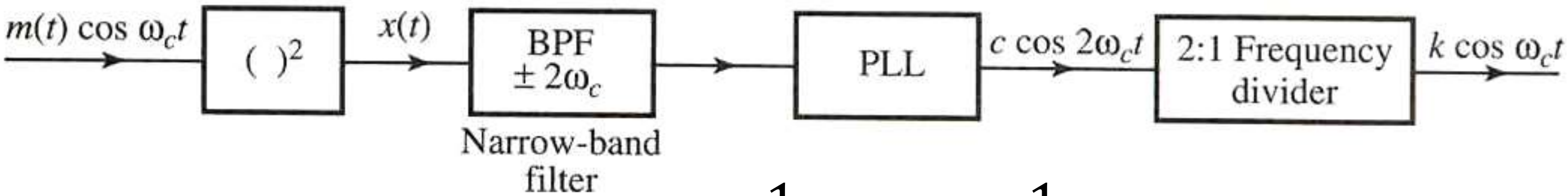
  - Where $K = \frac{1}{2}cB$ and $\theta_e(t) = \theta_i(t) - \theta_0(t)$



Costas Loop Model



Carrier Acquisition : Signal Squaring



$$x(t) = [m(t)\cos w_c t]^2 = \frac{1}{2}m^2(t) + \frac{1}{2}m^2(t)\cos 2w_c t$$

- Let us define

$$\frac{1}{2}m^2(t) = k + \phi(t)$$

- Note that the $\frac{1}{2}m^2(t)$ is a non-negative signal and has a non-negative average value e.g., k and $\phi(t)$ is a zero mean baseband signal i.e., $\frac{1}{2}m^2(t)$ minus its DC component.

$$x(t) = \frac{1}{2}m^2(t) + k\cos 2w_c t + \phi(t)\cos 2w_c t$$

Carrier Acquisition in SSB/VSB Transmission

- The carrier acquisition techniques i.e., Costas loop and squaring method are useful in DSB-SC but not in SSB or VSB transmissions.

$$\varphi_{SSB}(t) = m(t)\cos w_c t \mp m_h(t)\sin w_c t$$
$$= E(t)\cos[w_c t + \theta(t)]$$

- Where

$$E(t) = \sqrt{m^2(t) + m_h^2(t)}$$
$$\theta(t) = \tan^{-1}\left[\frac{\pm m_h(t)}{m(t)}\right]$$

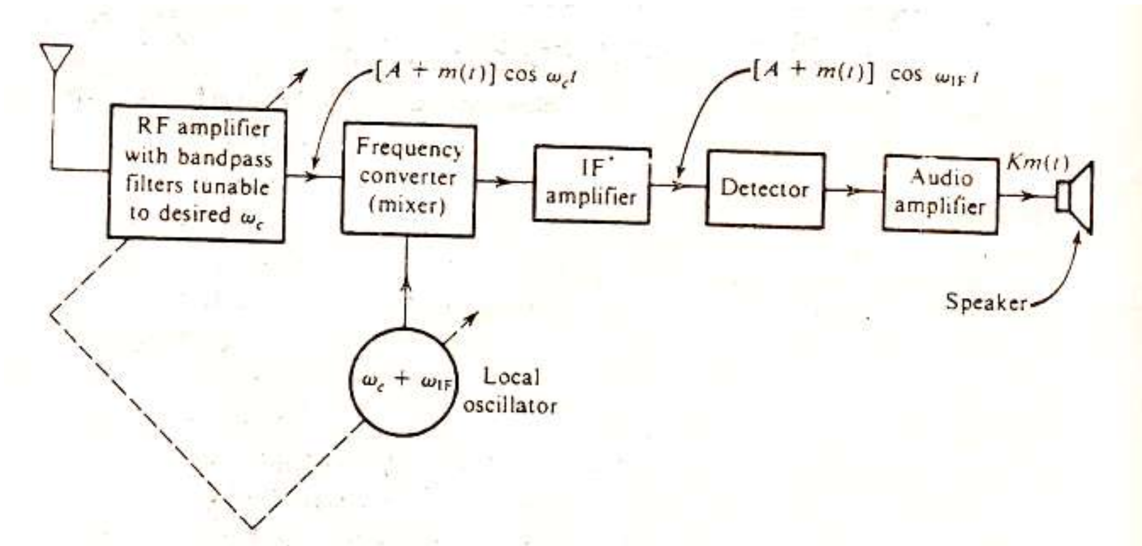
- Squaring this signal yields

$$\varphi_{SSB}^2(t) = E^2(t)[1 + \cos\{2w_c t + 2\theta(t)\}]$$

- $E^2(t)$ is eliminated by the bandpass filter and the second term is not a pure sinusoid of frequency $2w_c$.
- SSB/VSB signals require either highly stable oscillators or pilot based carrier recovery technique.

Superhetrodyne AM Receiver

- The radio receiver used in AM system is called the superheterodyne AM receiver and is illustrated below



- It consists of an RF section, a frequency converter, an intermediate-frequency (IF) amplifier, an envelope detector, and an audio amplifier.

Superhetrodyne AM Receiver

- The RF section is basically a tunable filter and an amplifier whose resonant frequency response curve can be tuned from $540\ kHz$ to $1650\ kHz$ (the standard broadcast band).
- It picks up the desired station by tuning the filter to the right frequency band.
- The frequency mixer (convertor), translates the carrier from w_c to a fixed IF frequency of $455\ kHz$. The mixer is a circuit capable of producing the sum and difference frequencies of the signals receiving from two sources (the RF amplifier and the local oscillator).
- For this purpose, it uses a local oscillator whose frequency f_{LO} is exactly $455 kHz$ above the incoming carrier frequency f_c i.e.,

$$f_{LO} = f_c + f_{IF}.$$

Superhetrodyne AM Receiver

- The reasons for translating all carriers to a fixed frequency of $455\ kHz$ are
  - The components are cheaper at low frequencies compare to high frequencies.
  - It is easy to filter IF signal compare to RF signal
- The output from the mixer circuit is connected to the intermediate frequency amplifier (IF amp), which amplifies a narrow band of select frequencies.
- This new IF frequency contains the same modulated information as that transmitted from the source but at a frequency range lower then the standard broadcast band.
- This signal is rectified and filtered to eliminate one sideband and the carrier (conversion from RF to IF) and is finally amplified for listening.