Data Mining

Chapter 5

Association Analysis: Basic Concepts

Support Counting of Candidate Itemsets

- Scan the database of transactions to determine the support of each candidate itemset
  - Must match every candidate itemset against every transaction, which is an expensive operation

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread, Diaper, Milk}
{ Beer, Bread, Milk}

Support Counting of Candidate Itemsets

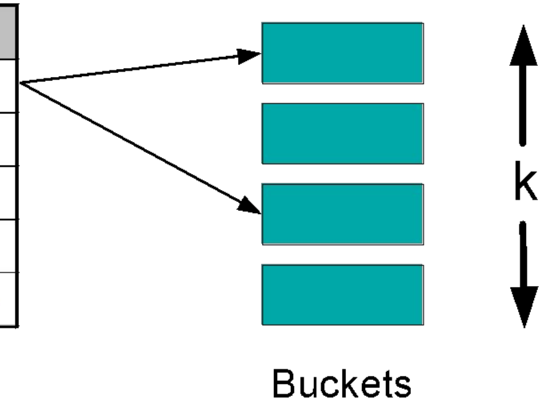
- To reduce number of comparisons, store the candidate itemsets in a hash structure
  - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

N

Hash Structure

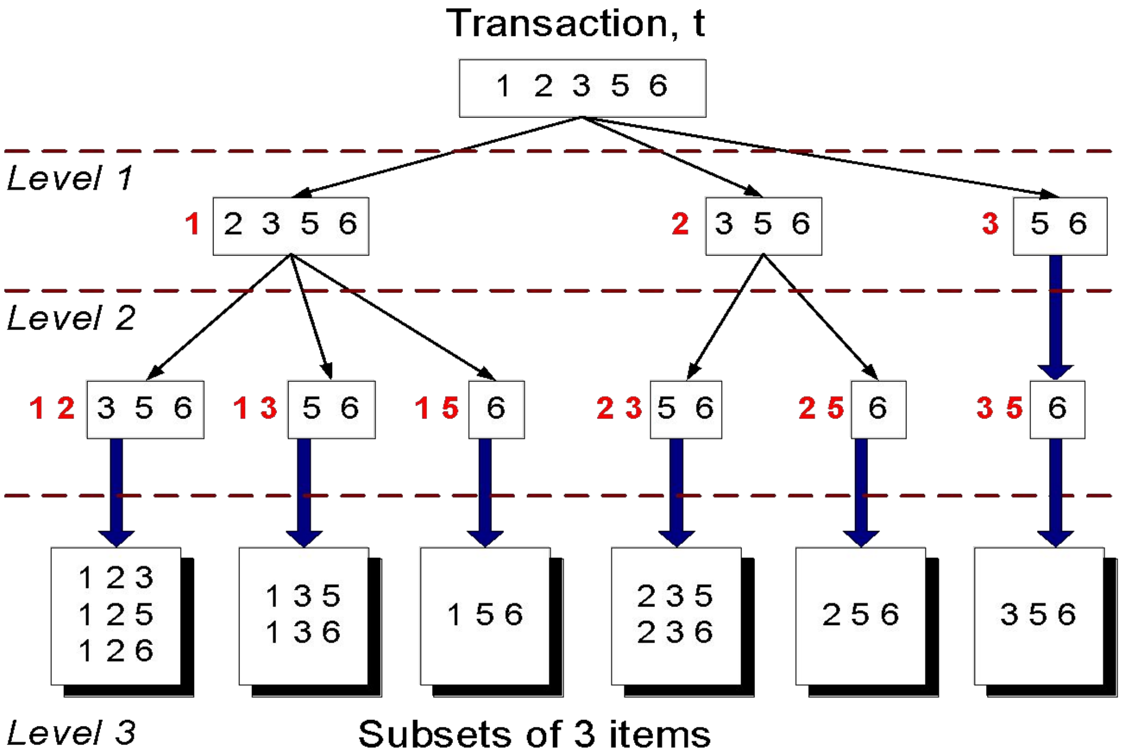


Support Counting: An Example

Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

How many of these itemsets are supported by transaction (1,2,3,5,6)?



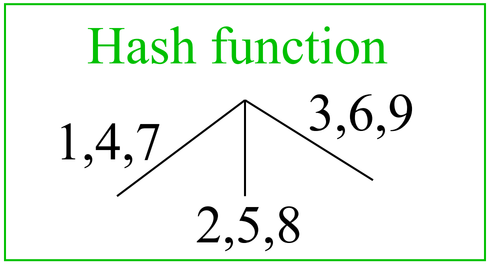
Support Counting Using a Hash Tree

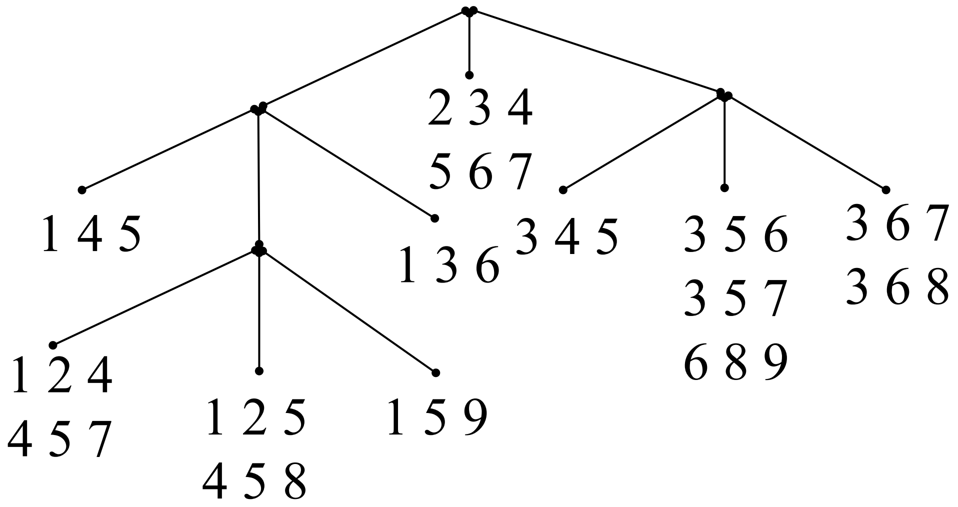
Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

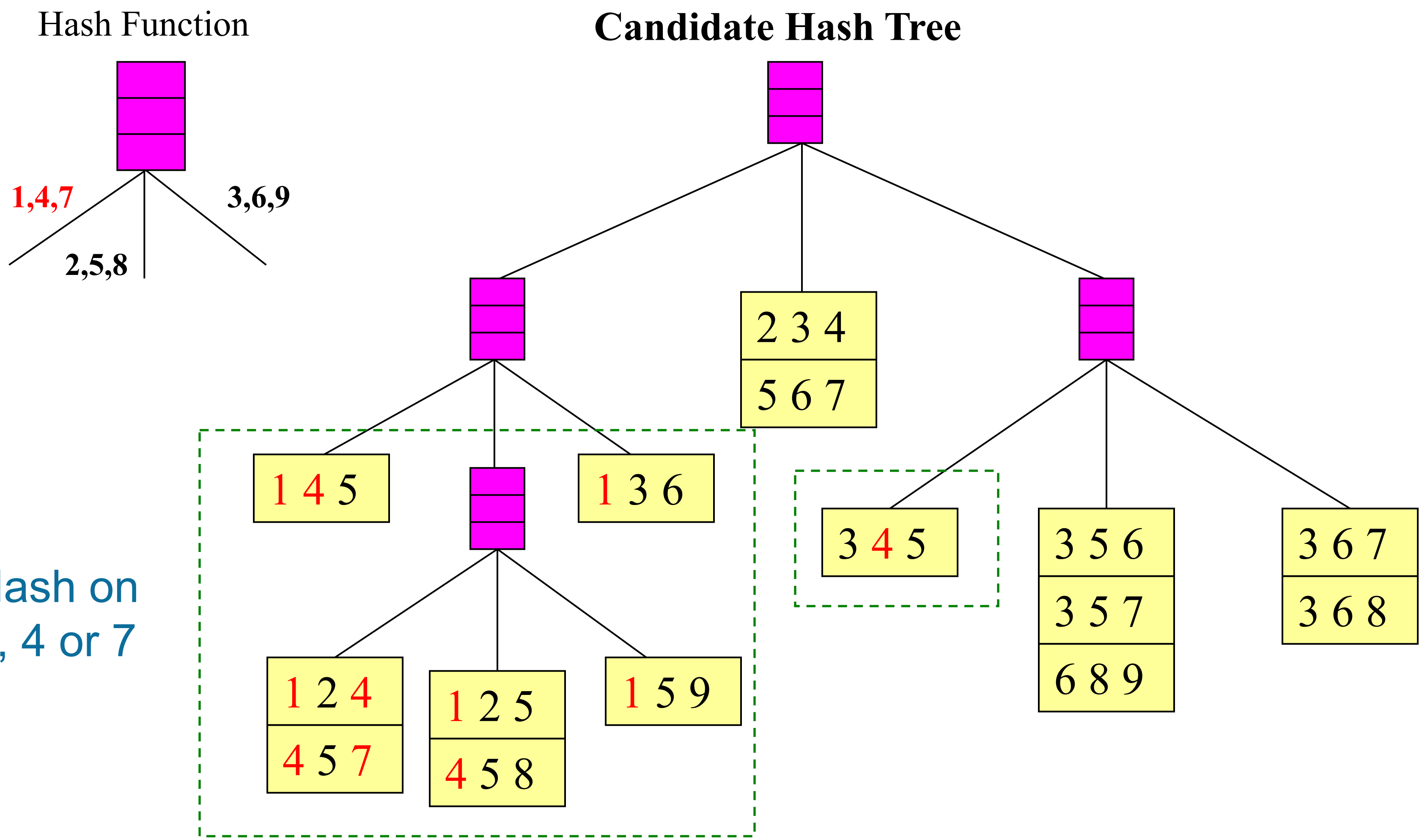
You need:

- Hash function
- Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)



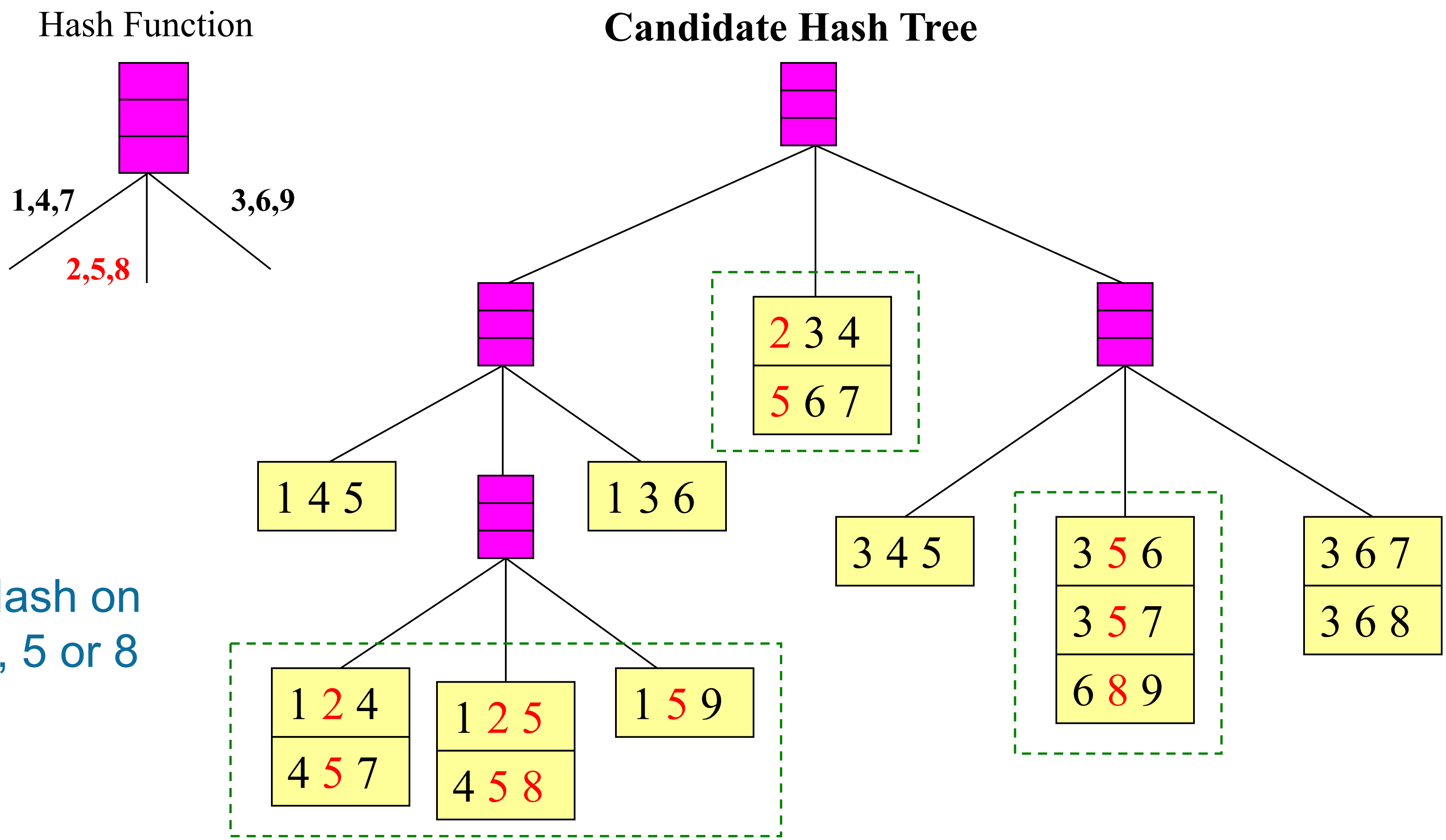


Support Counting Using a Hash Tree



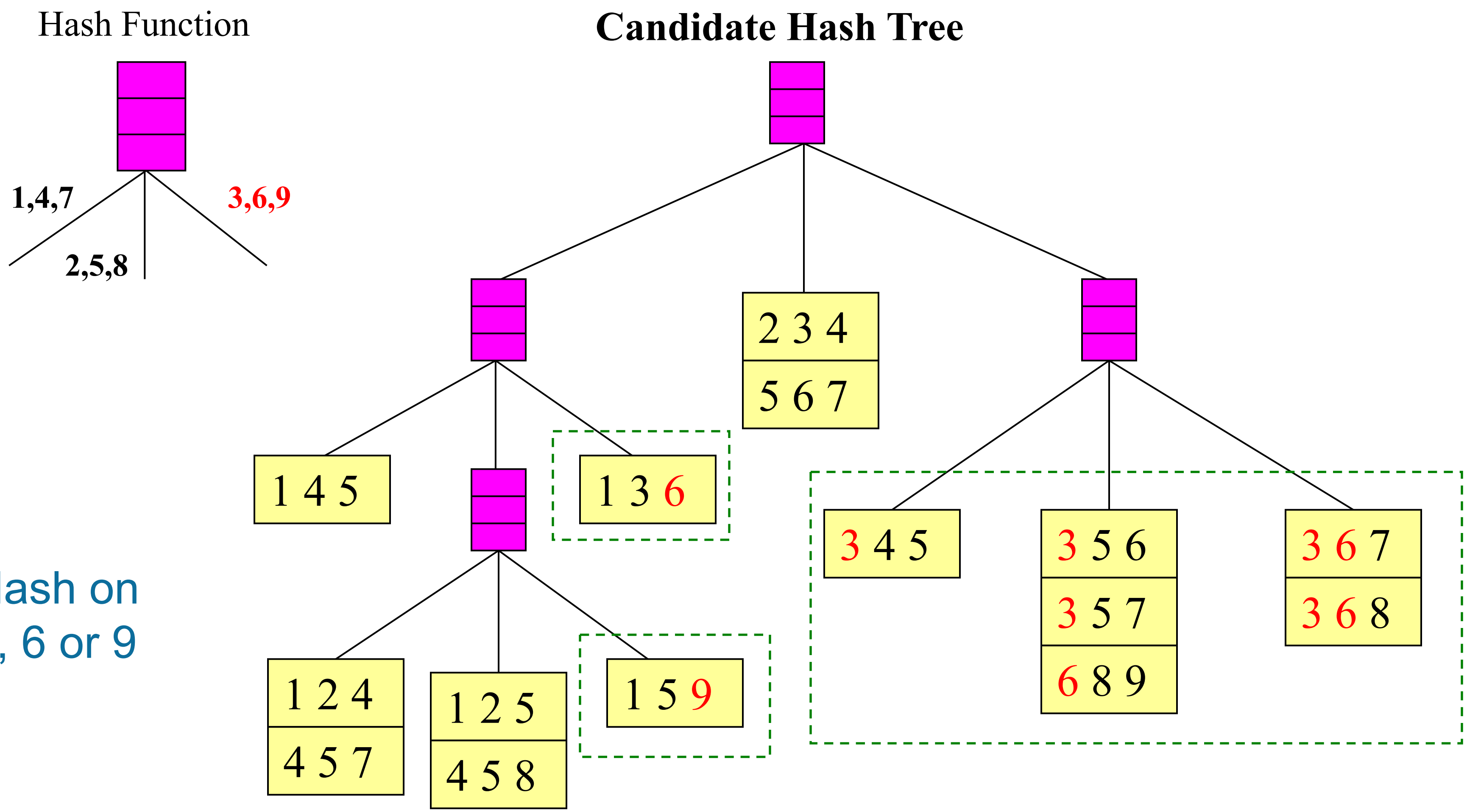
Hash on
1, 4 or 7

Support Counting Using a Hash Tree



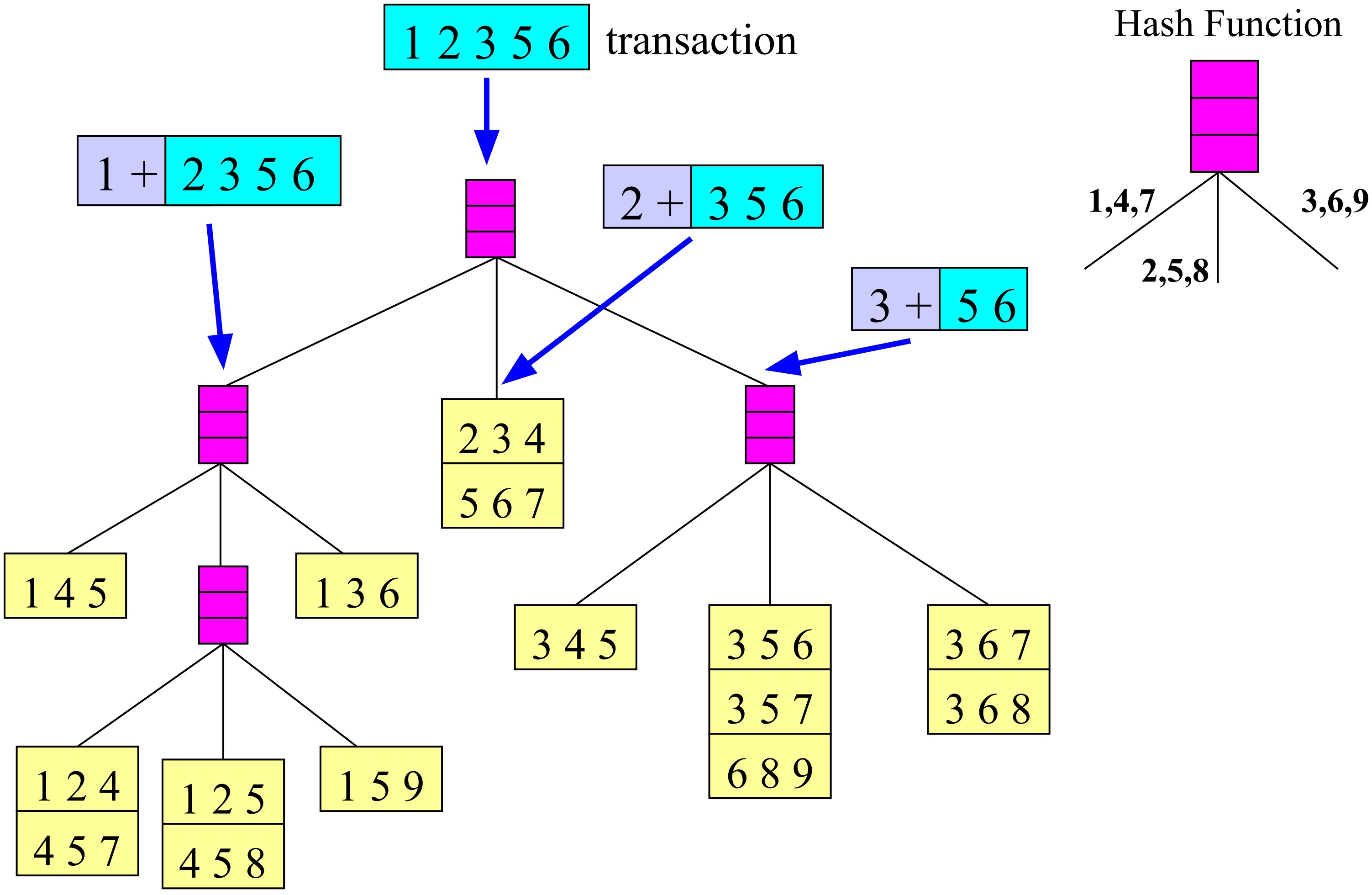
Hash on
2, 5 or 8

Support Counting Using a Hash Tree

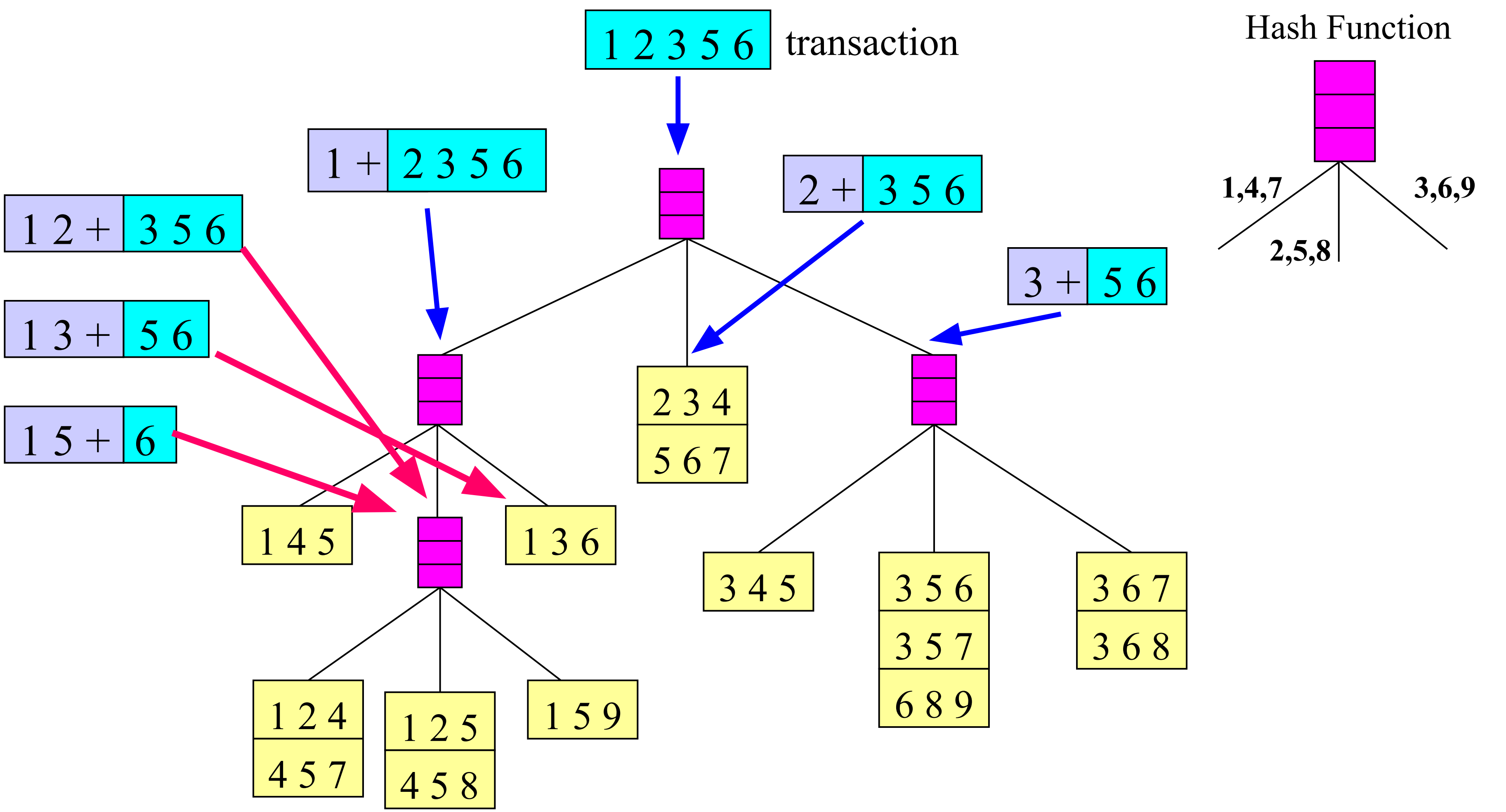


Hash on
3, 6 or 9

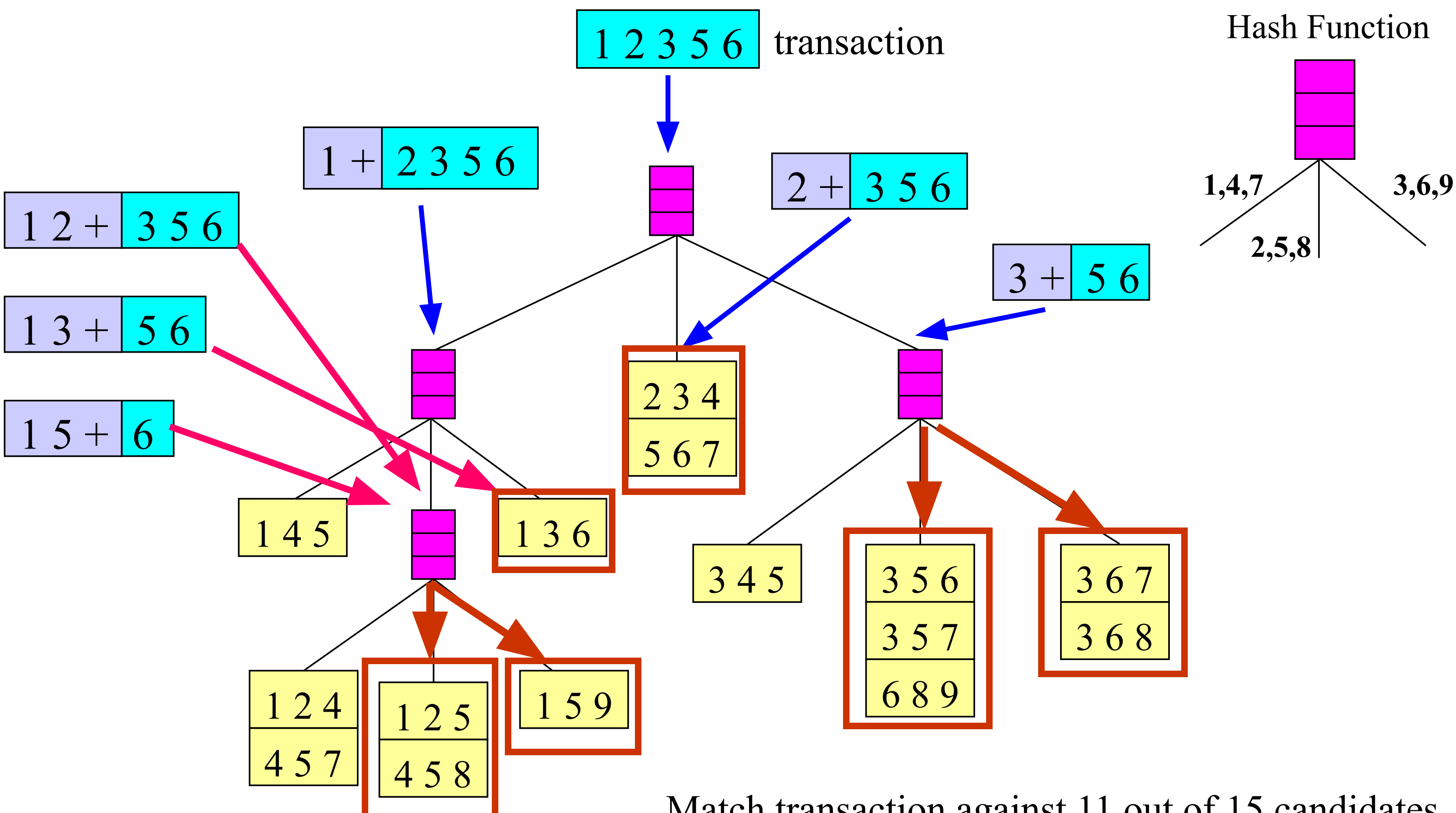
Support Counting Using a Hash Tree



Support Counting Using a Hash Tree



Support Counting Using a Hash Tree



Match transaction against 11 out of 15 candidates

Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

    ABC →D, ABD →C, ACD →B, BCD →A,
    A →BCD, B →ACD, C →ABD, D →ABC
    AB →CD, AC → BD, AD → BC, BC →AD,
    BD →AC, CD →AB,

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

Rule Generation

- In general, confidence does not have an anti-monotone property

  $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

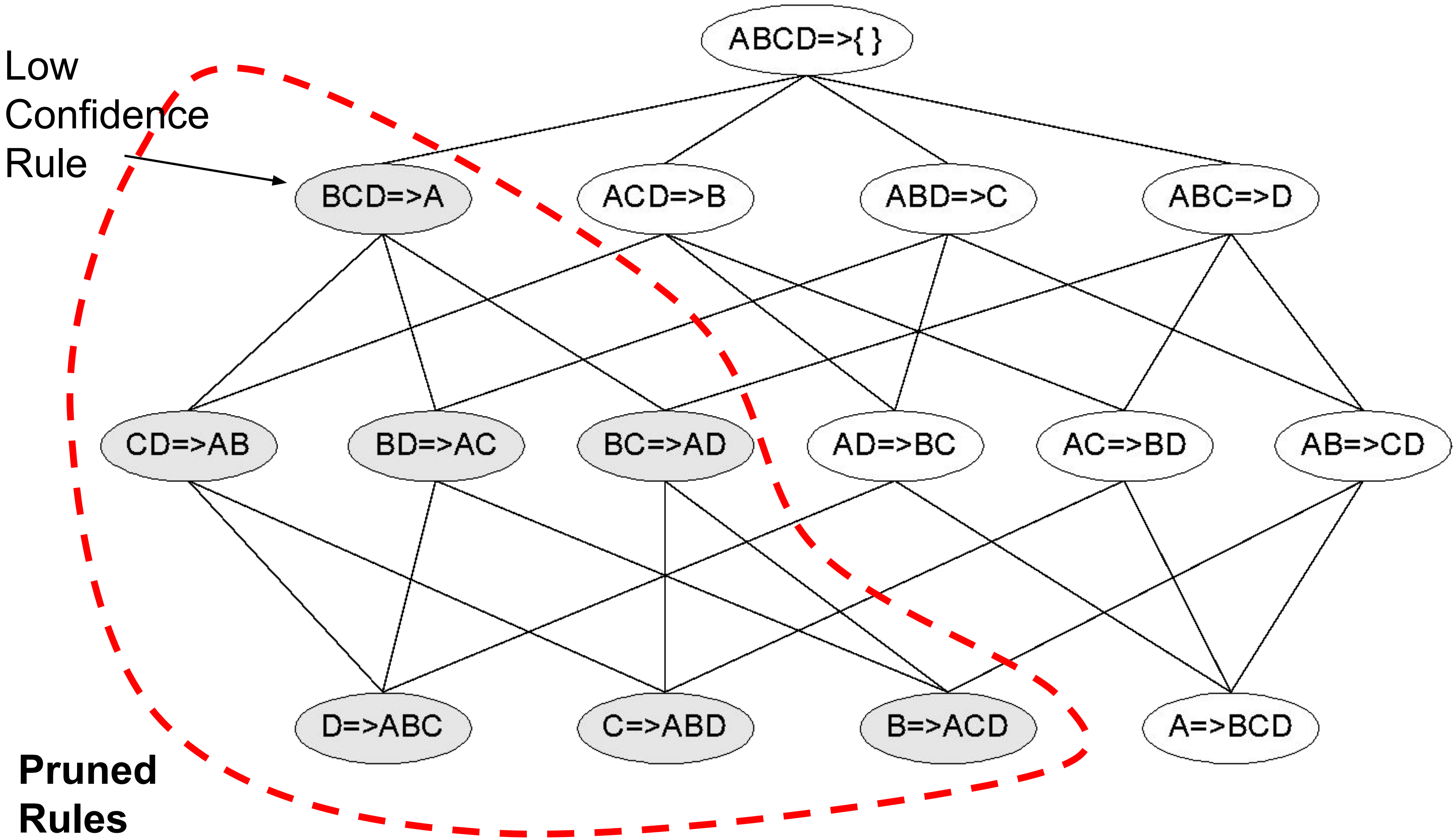
- But confidence of rules generated from the same itemset has an anti-monotone property
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

Lattice of rules



Association Analysis: Basic Concepts and Algorithms

Algorithms and Complexity

Factors Affecting Complexity of Apriori

- Choice of minimum support threshold
- Dimensionality (number of items) of the data set
- Size of database
- Average transaction width
  –

Factors Affecting Complexity of Apriori

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  -
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  -
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  -

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Impact of Support Based Pruning

TID	Items
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

Minimum Support = 2

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4$

$6 + 15 + 20 + 15 = 56$

Factors Affecting Complexity of Apriori

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - More space is needed to store support count of itemsets
  - if number of frequent itemsets also increases, both computation and I/O costs may also increase
- Size of database
- Average transaction width
  -

TID	*Items*
1	Bread, Milk
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- Size of database
  - run time of algorithm increases with number of transactions
- Average transaction width

TID	*Items*
1	Bread, Milk
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- Choice of minimum support threshold
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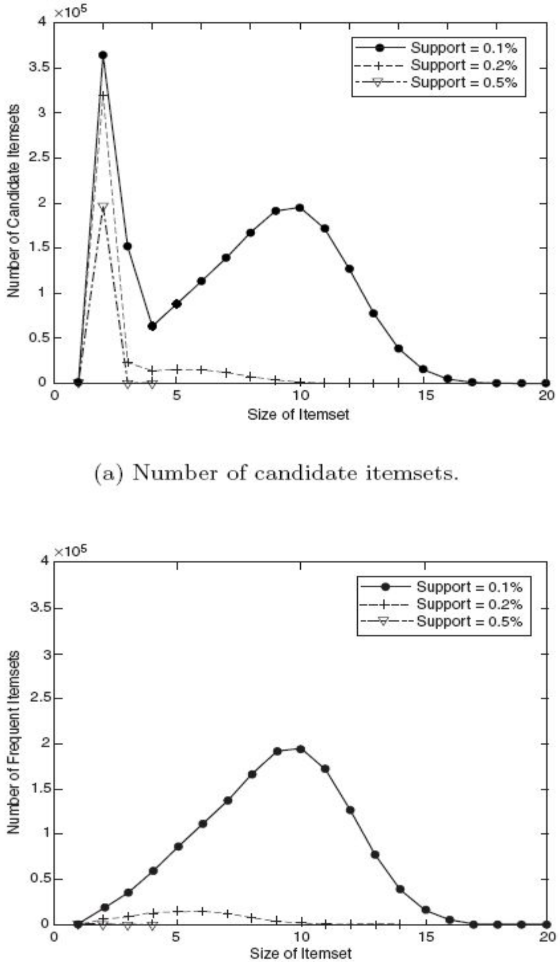
- Dimensionality (number of items) of the data set
  - More space is needed to store support count of itemsets
  - if number of frequent itemsets also increases, both computation and I/O costs may also increase
- Size of database
  - run time of algorithm increases with number of transactions
- Average transaction width
  - transaction width increases the max length of frequent itemsets
  - number of subsets in a transaction increases with its width, increasing computation time for support counting

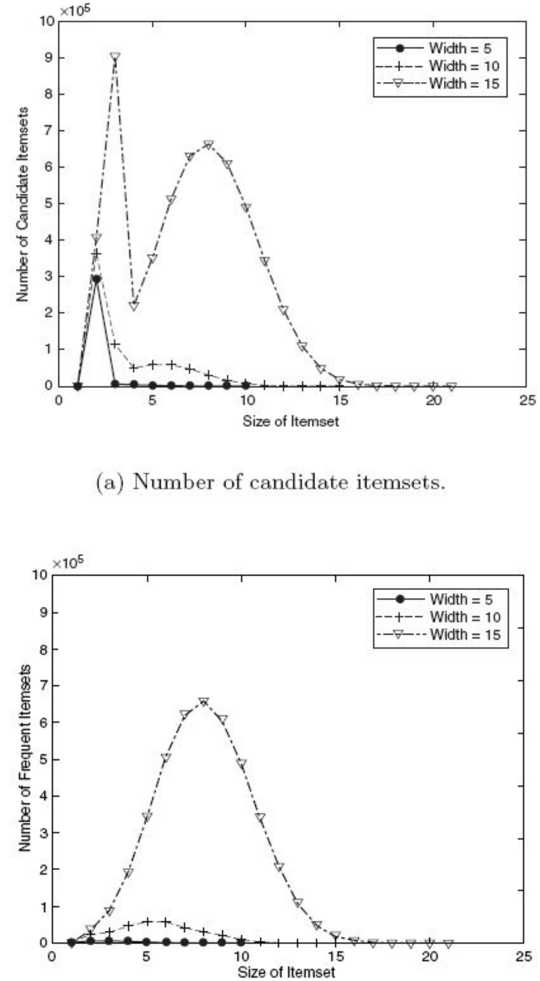
Factors Affecting Complexity of Apriori



(a) Number of candidate itemsets.

(b) Number of frequent itemsets.

Figure 6.13. Effect of support threshold on the number of candidate and frequent itemsets.



(a) Number of candidate itemsets.

(b) Number of Frequent Itemsets.

Figure 6.14. Effect of average transaction width on the number of candidate and frequent itemsets.

Assumptions of Apriori

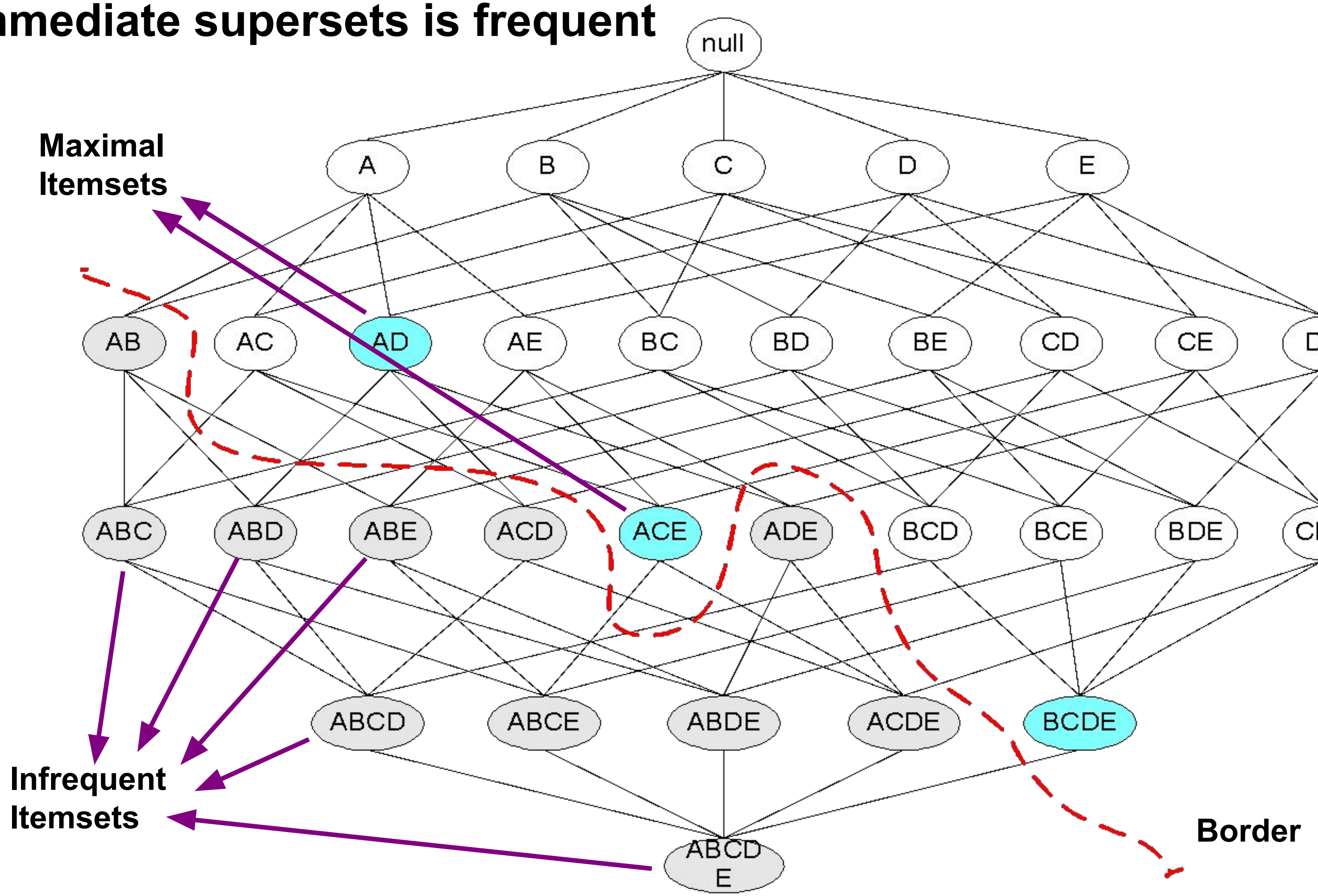
- If an itemset is frequent, then all of its subsets must also be frequent.
- Conversely, if an itemset such as {a, b} is infrequent, then all of its supersets must be infrequent too.

Problem with Apriori

- In practice, the number of frequent itemsets produced from a transaction data set can be very large. It is useful to identify a small representative set of itemsets from which all other frequent itemsets can be derived.
- Two such representations are presented in this section in the form of maximal and closed frequent itemsets.

Maximal Frequent Itemset

An itemset is maximal frequent if it is frequent and none of its immediate supersets is frequent



Maximal Frequent Itemset

- Maximal frequent itemsets effectively provide a compact representation of frequent itemsets. In other words, they form the smallest set of itemsets from which all frequent itemsets can be derived.

- Here there are two groups of frequent itemsets.

- Frequent itemsets that begin with item a and that may contain items c, d, or e. This group includes itemsets such as {a}, {a, c}, {a, d}, {a, e}, and {a, c, e}.

- Frequent itemsets that begin with items b, c, d, or e. This group includes itemsets such as {b}, {b, c}, {c, d},{b, c, d, e}, etc.

- Frequent itemsets that belong in the first group are subsets of either {a, c, e} or {a, d}, while those that belong in the second group are subsets of {b, c, d, e}.

Maximal Frequent Itemset

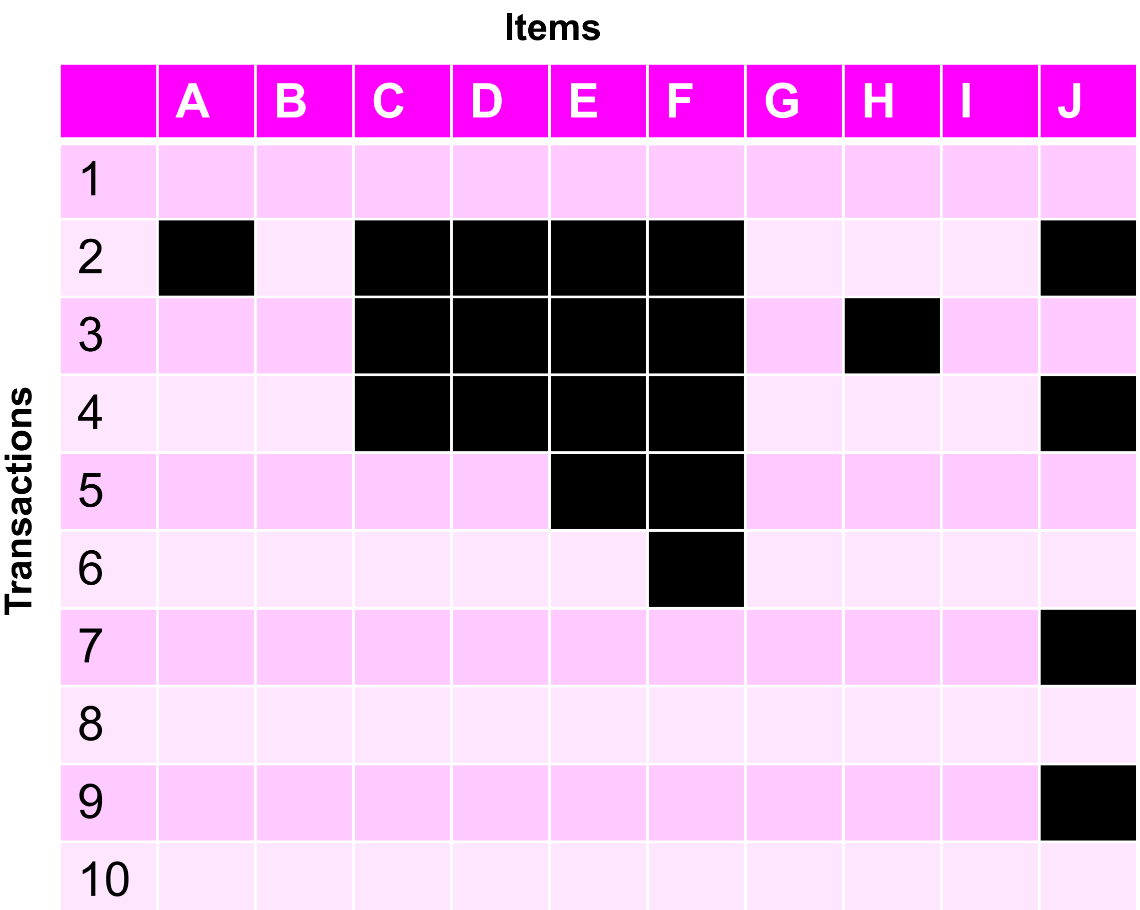
- Maximal frequent item sets provide a valuable representation for data sets that can produce very long, frequent item sets, as there are exponentially frequent item sets in such data. Nevertheless, this approach is practical only if an efficient algorithm exists to explicitly find the maximal frequent item sets without having to enumerate all their subsets

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1

Maximal Frequent Itemset

- Despite providing a compact representation, maximal frequent itemsets do not contain the support information of their subsets. For example, the support of the maximal frequent itemsets {a, c, e}, {a, d}, and {b,c,d,e} do not provide any hint about the support of their subsets. An additional pass over the data set is therefore needed to determine the support counts of the non-maximal frequent itemsets

An illustrative example

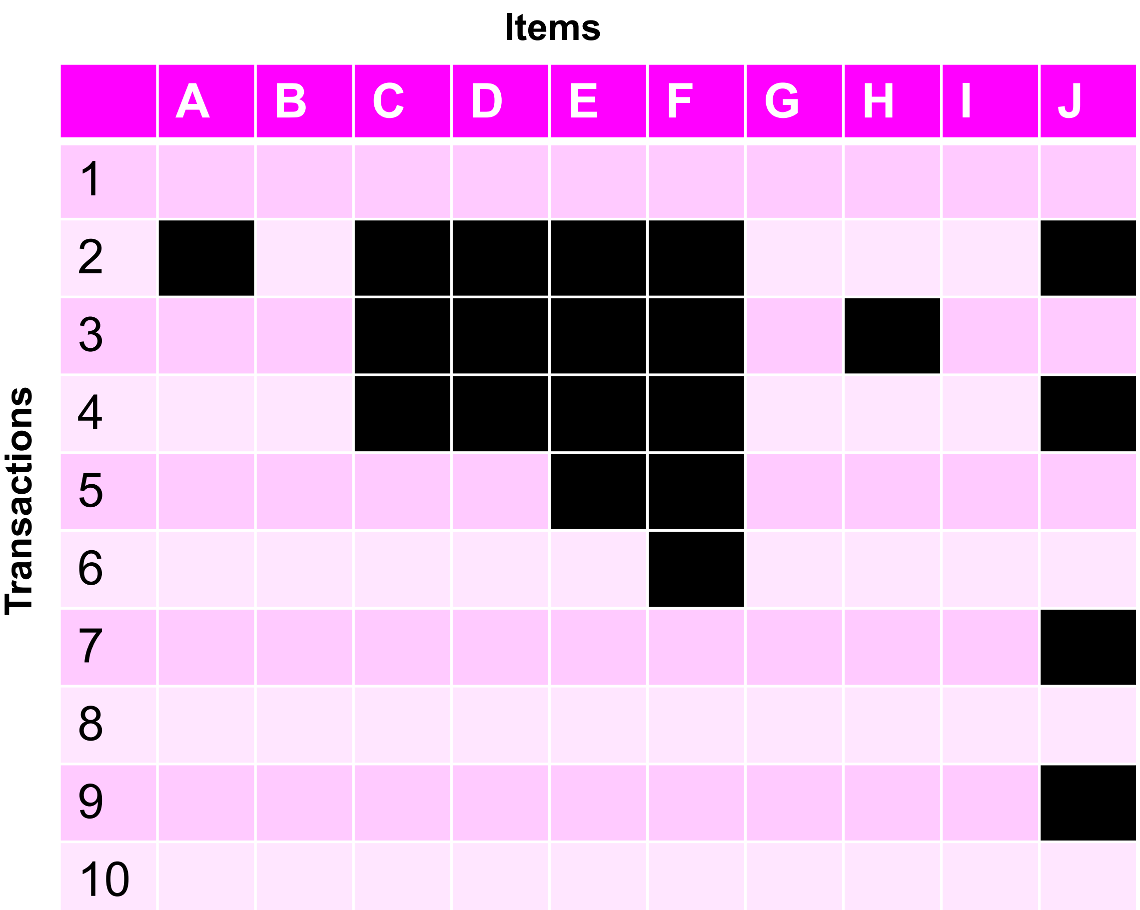


Support threshold (by count) : 5

Frequent itemsets: ?

Maximal itemsets: ?

An illustrative example



Support threshold (by count) : 5

Frequent itemsets: {F}

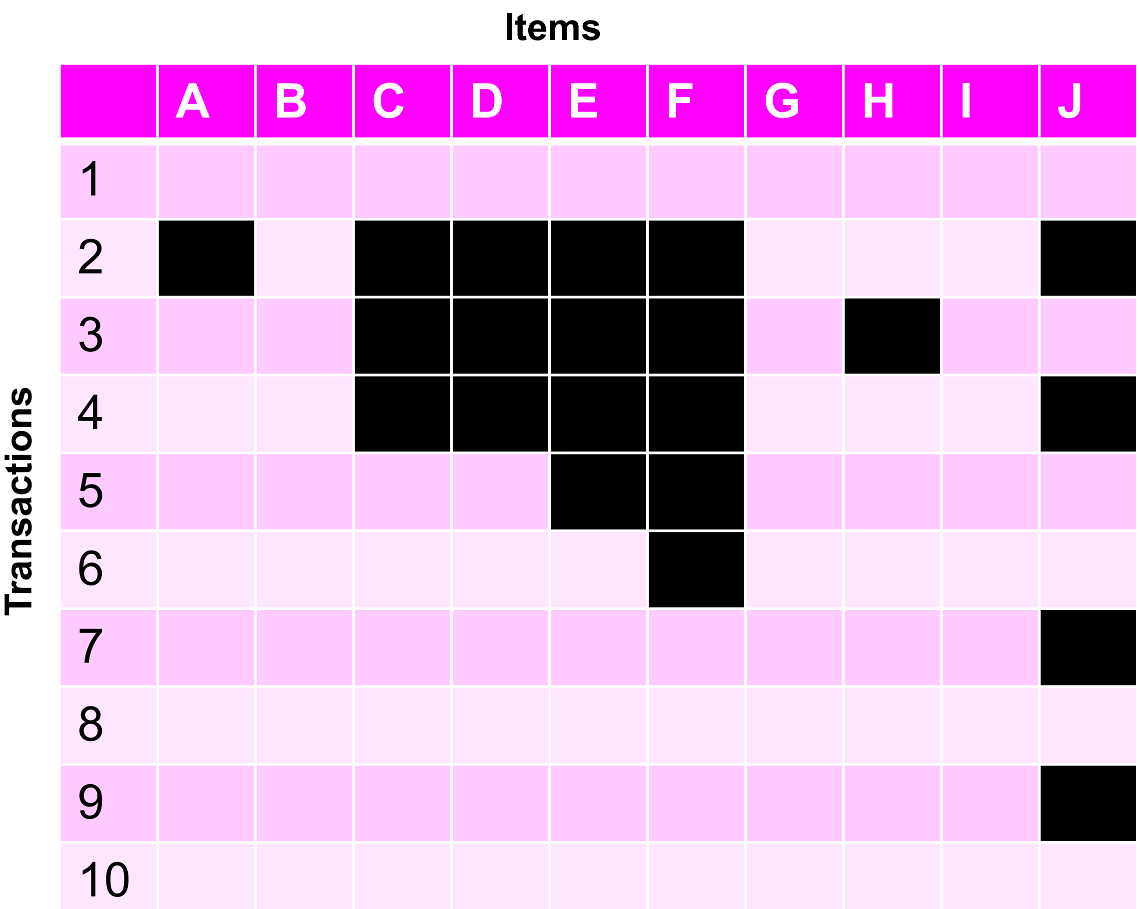
Maximal itemsets: {F}

Support threshold (by count): 4

Frequent itemsets: ?

Maximal itemsets: ?

An illustrative example



Support threshold (by count) : 5

Frequent itemsets: {F}
Maximal itemsets: {F}

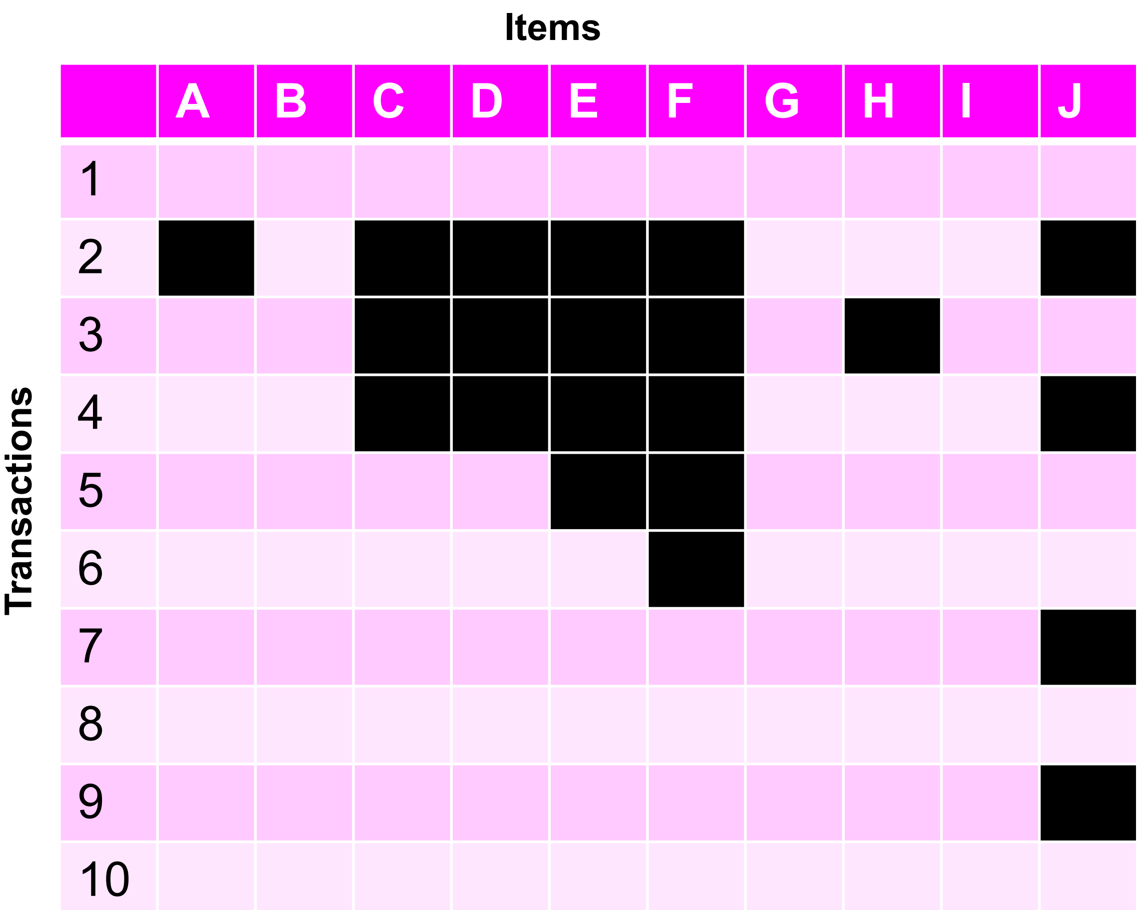
Support threshold (by count): 4

Frequent itemsets: {E}, {F}, {E,F}, {J}
Maximal itemsets: {E,F}, {J}

Support threshold (by count): 3

Frequent itemsets: ?
Maximal itemsets: ?

An illustrative example



Support threshold (by count) : 5

Frequent itemsets: {F}
Maximal itemsets: {F}

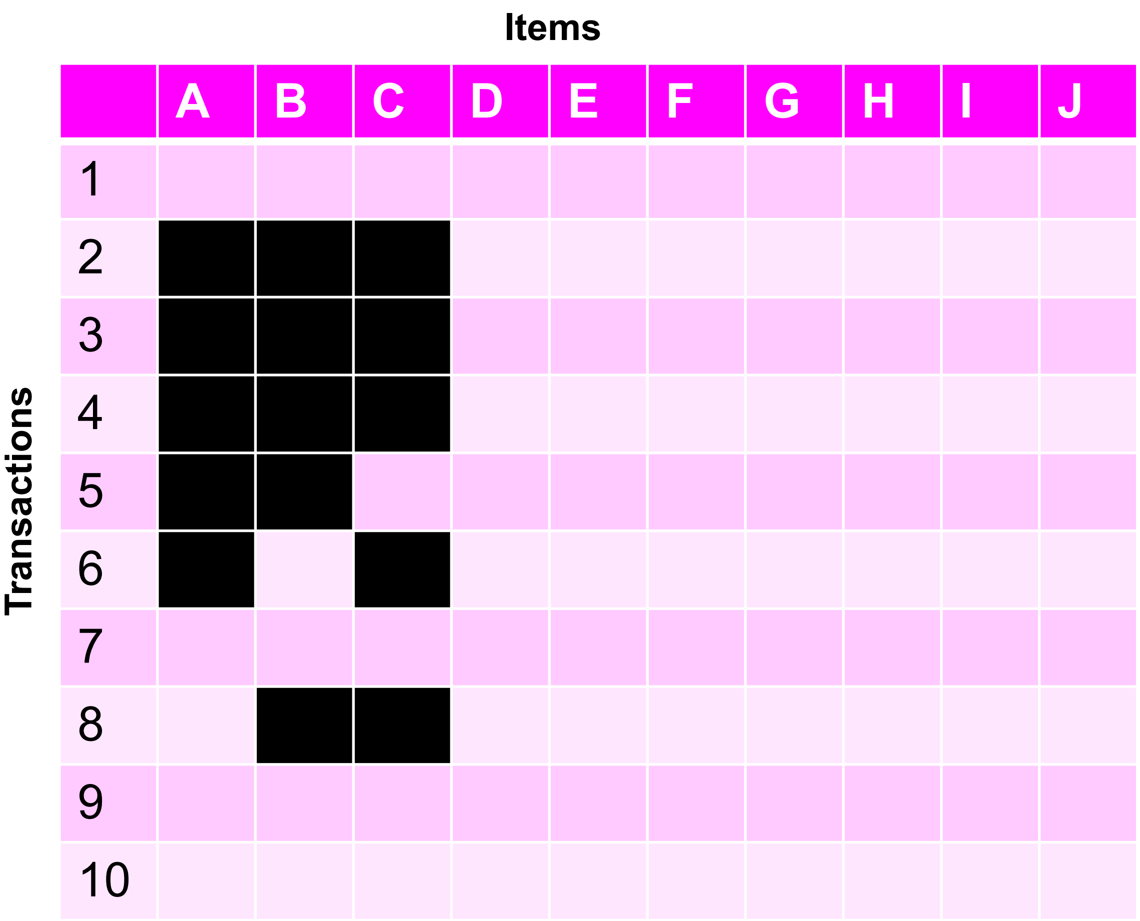
Support threshold (by count): 4

Frequent itemsets: {E}, {F}, {E,F}, {J}
Maximal itemsets: {E,F}, {J}

Support threshold (by count): 3

Frequent itemsets:
All subsets of {C,D,E,F} + {J}
Maximal itemsets:
{C,D,E,F}, {J}

Another illustrative example



Support threshold (by count) : 5
Maximal itemsets: {A}, {B}, {C}

Support threshold (by count): 4
Maximal itemsets: {A,B}, {A,C},{B,C}

Support threshold (by count): 3
Maximal itemsets: {A,B,C}

Closed Itemset

- Closed itemsets provide a minimal representation of itemsets without losing their support information
- An itemset X is closed if none of its immediate supersets has the same support as the itemset X.
- X is not closed if at least one of its immediate supersets has same support count as X.

Closed Itemset

- An itemset X is closed if none of its immediate supersets has the same support as the itemset X.
- X is not closed if at least one of its immediate supersets has support count as X.

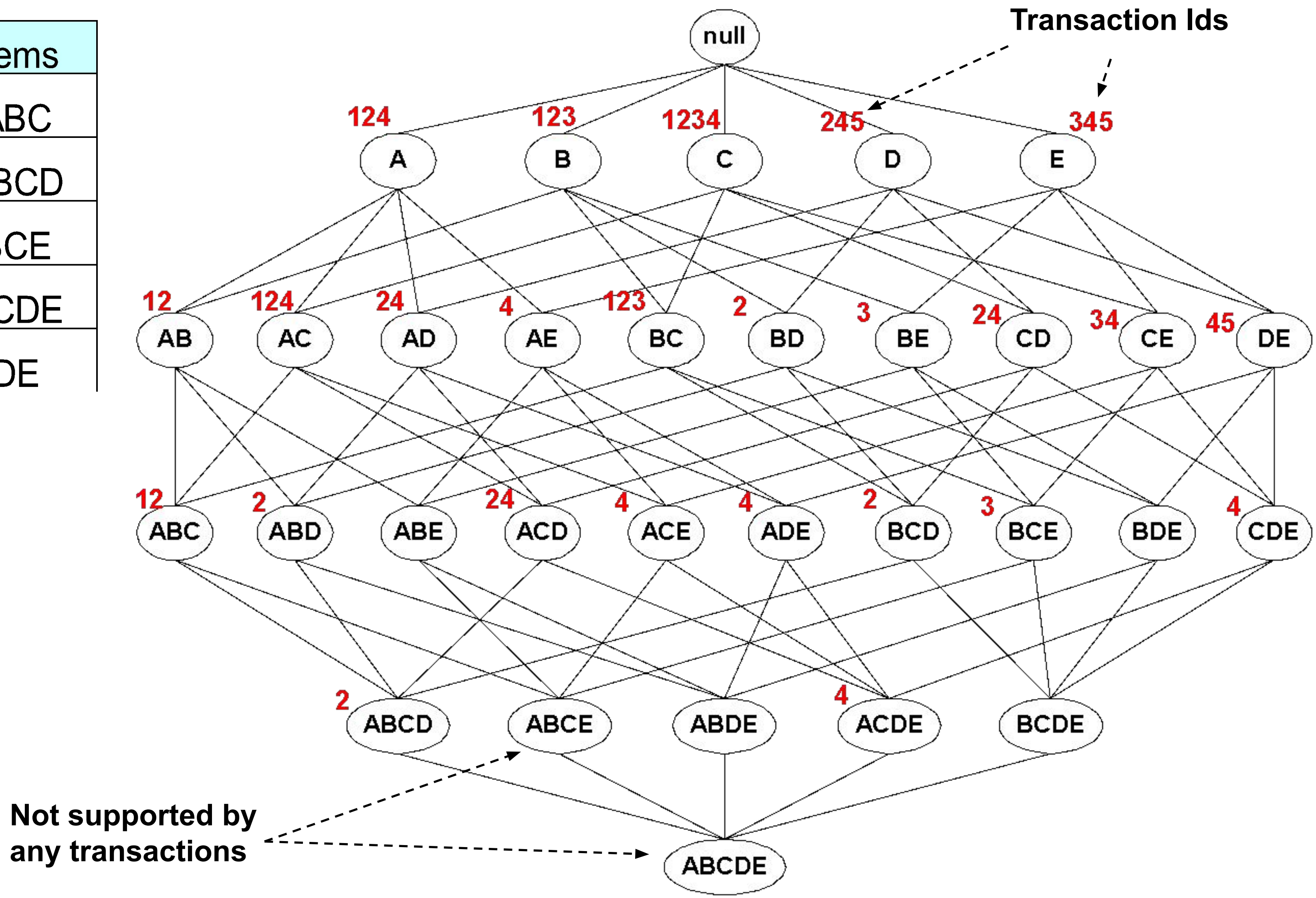
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	2
{A,B,C,D}	2

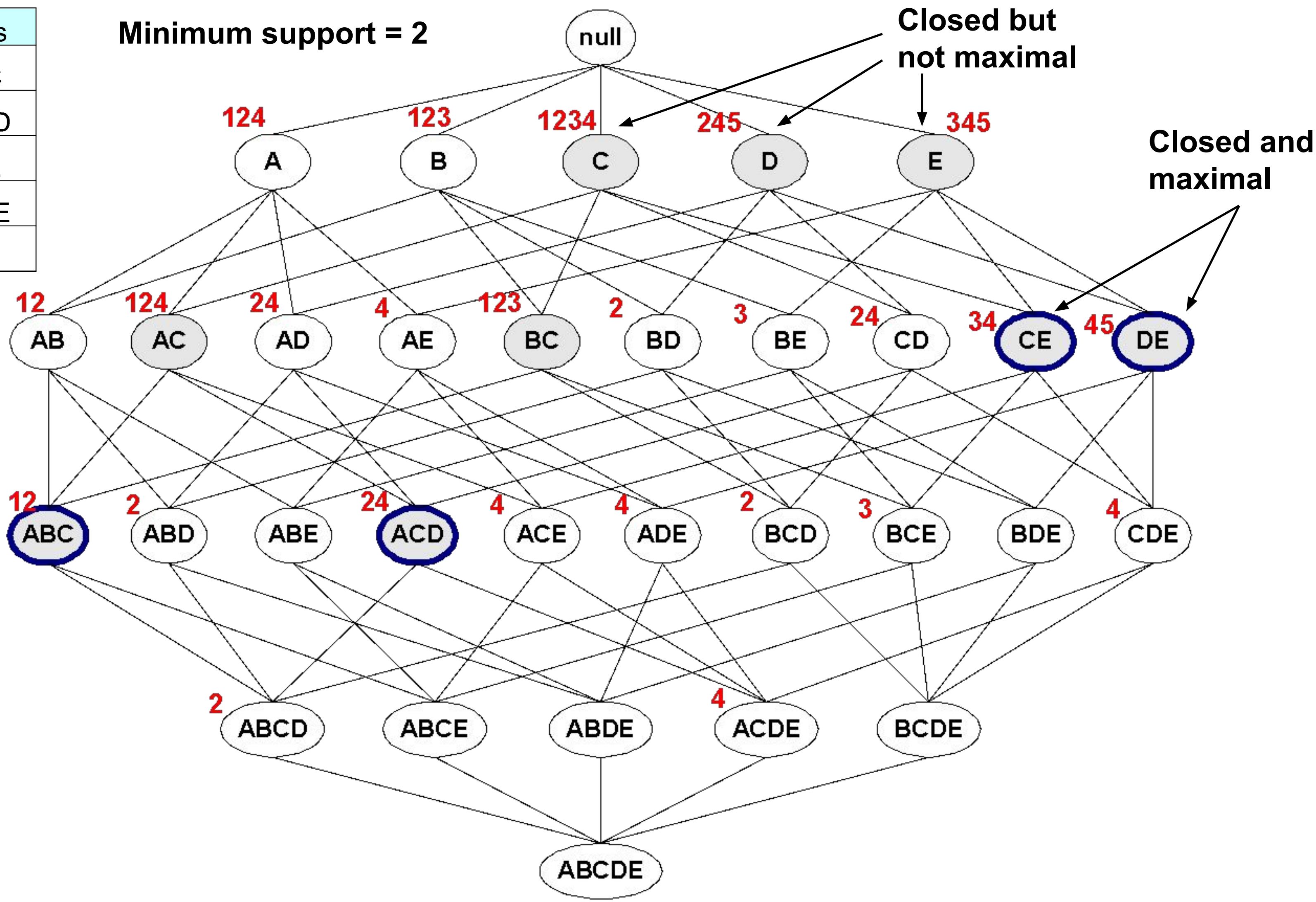
Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal Frequent Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Closed Frequent Itemset

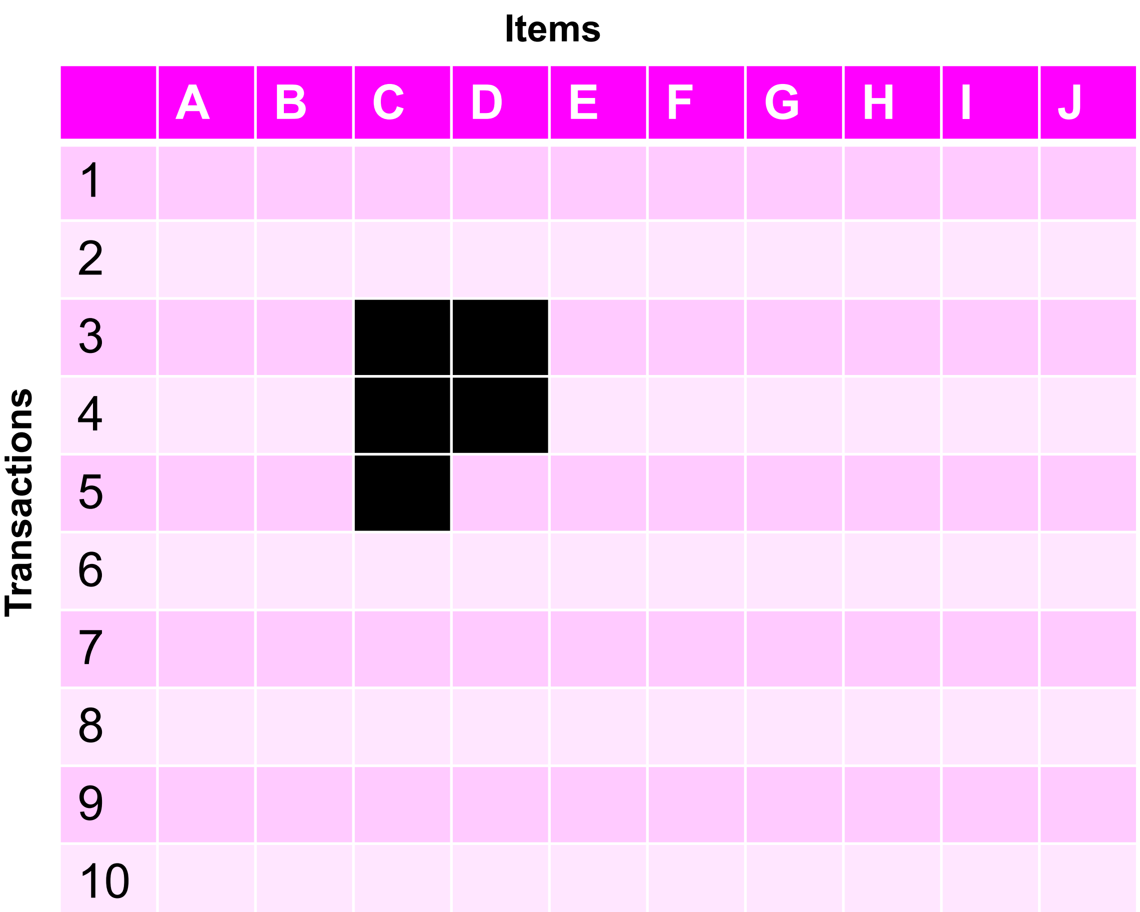
- An itemset is a closed frequent itemset if it is closed and its support is greater than or equal to minsup.
- In the previous example, assuming that the support threshold is 40%, {b,c} is a closed frequent itemset because its support is 60%.
- Algorithms are available to explicitly extract closed frequent itemsets from a given data set.

What are the Closed Frequent Itemsets in this Data?

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

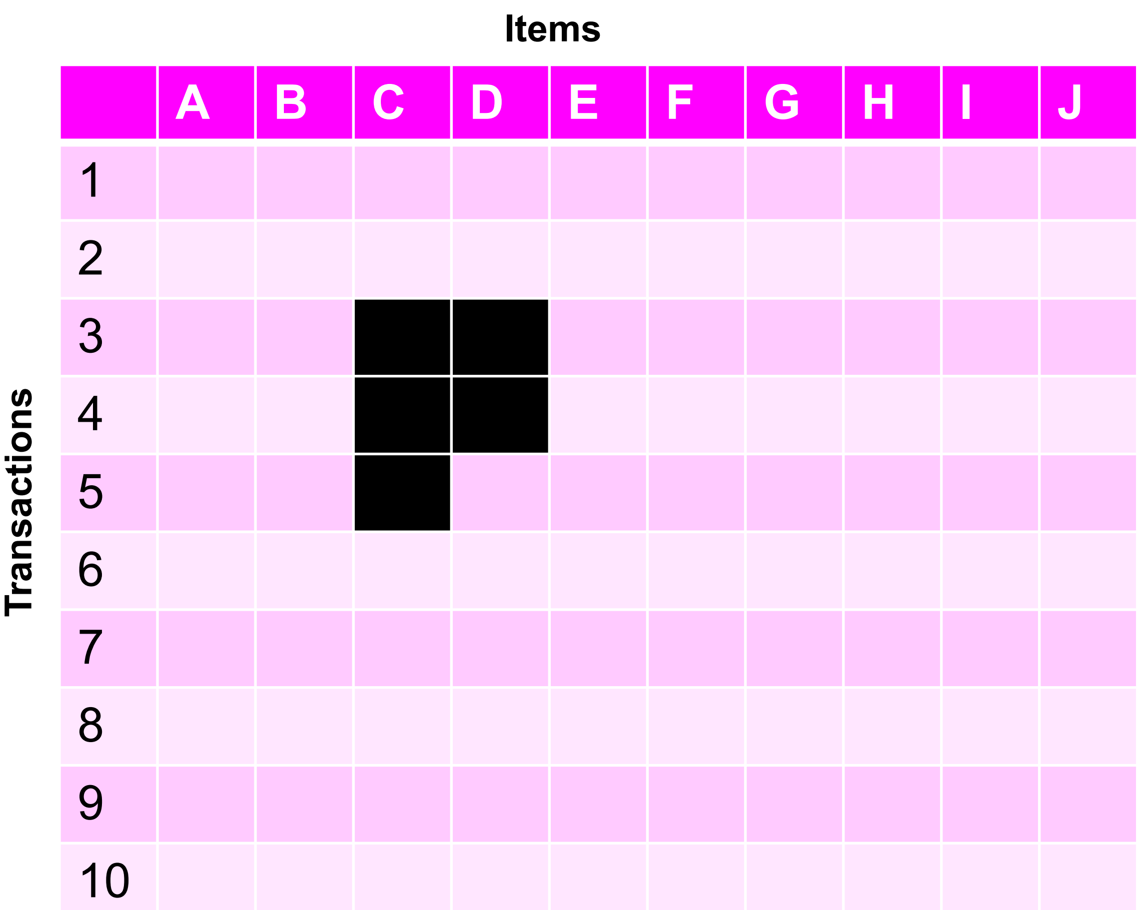
(A1-A10)
(B1-B10)
(C1-C10)

Example 1



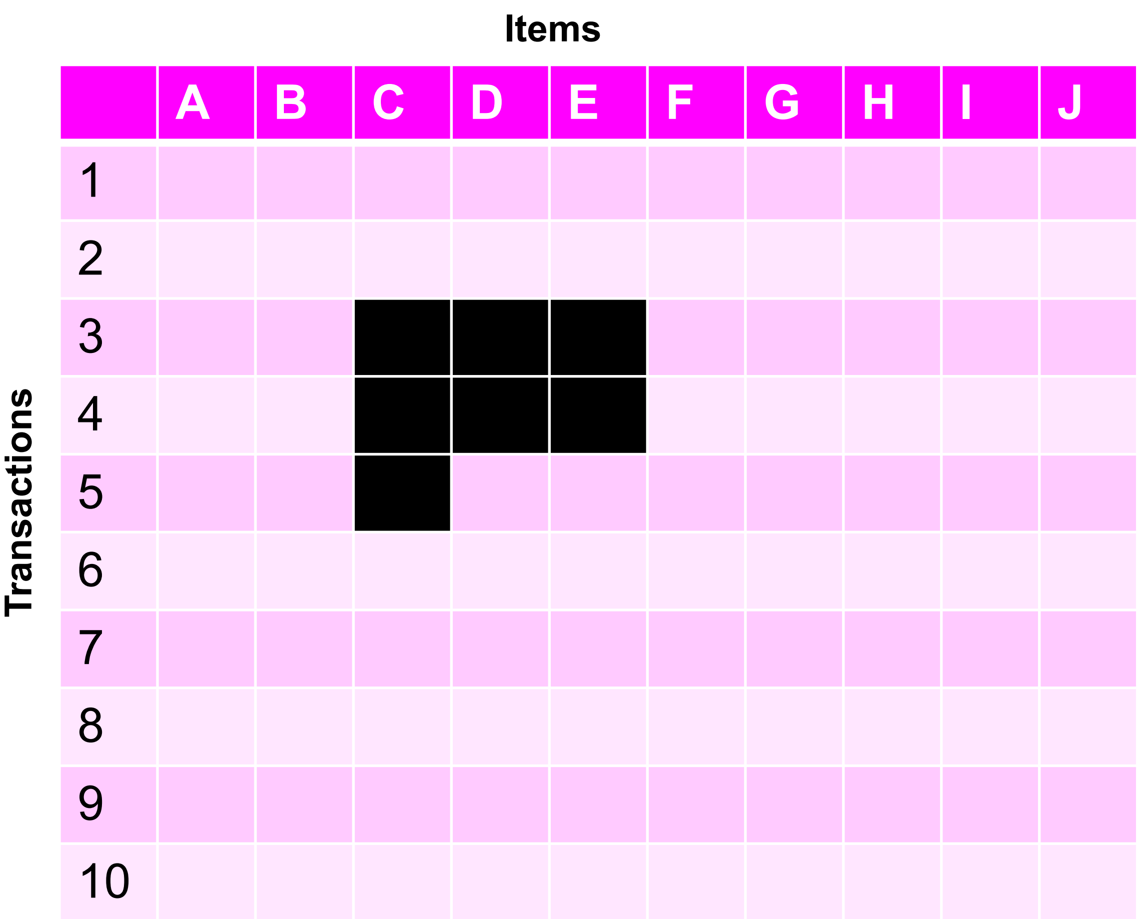
Itemsets	Support (counts)	Closed itemsets
{C}	3	
{D}	2	
{C,D}	2	

Example 1



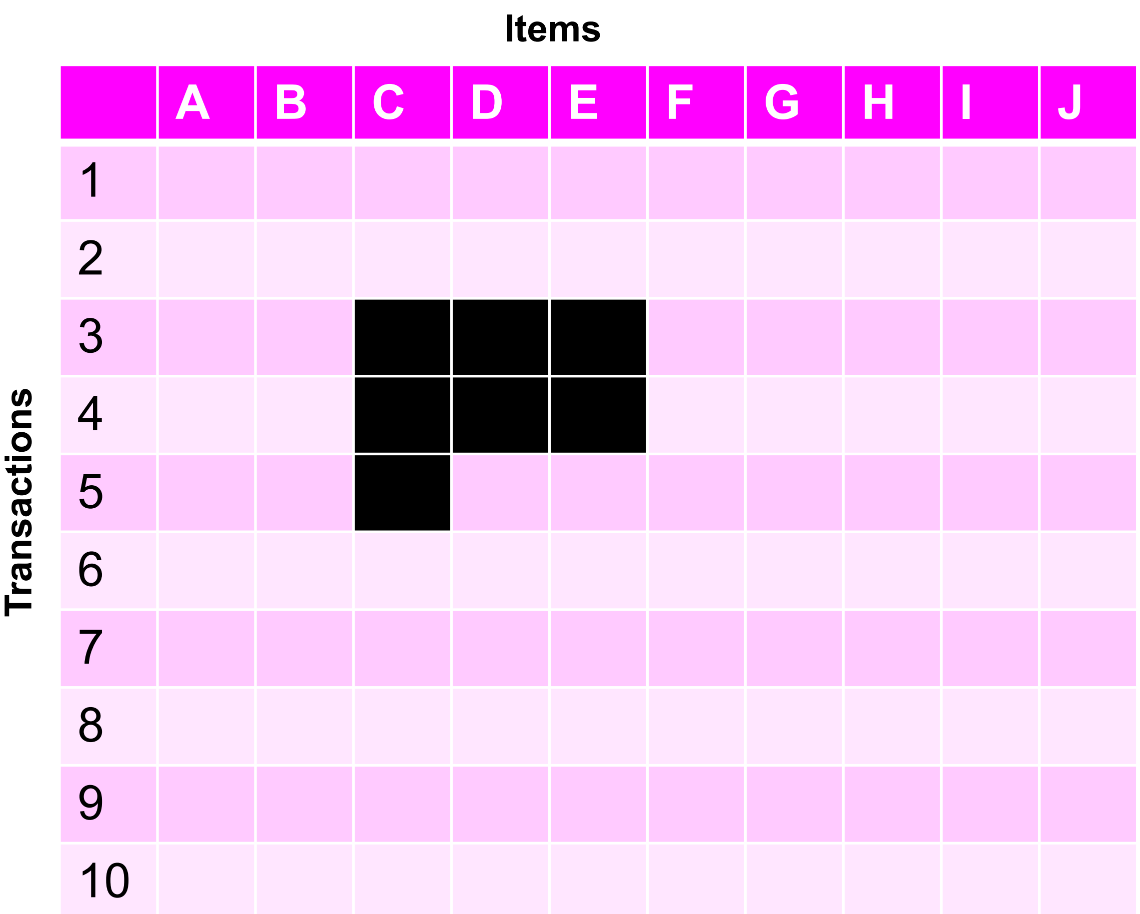
Itemsets	Support (counts)	Closed itemsets
{C}	**3**	✔
{D}	2	
{C,D}	**2**	✔

Example 2



Itemsets	Support (counts)	Closed itemsets
{C}	3	
{D}	2	
{E}	2	
{C,D}	2	
{C,E}	2	
{D,E}	2	
{C,D,E}	2	

Example 2

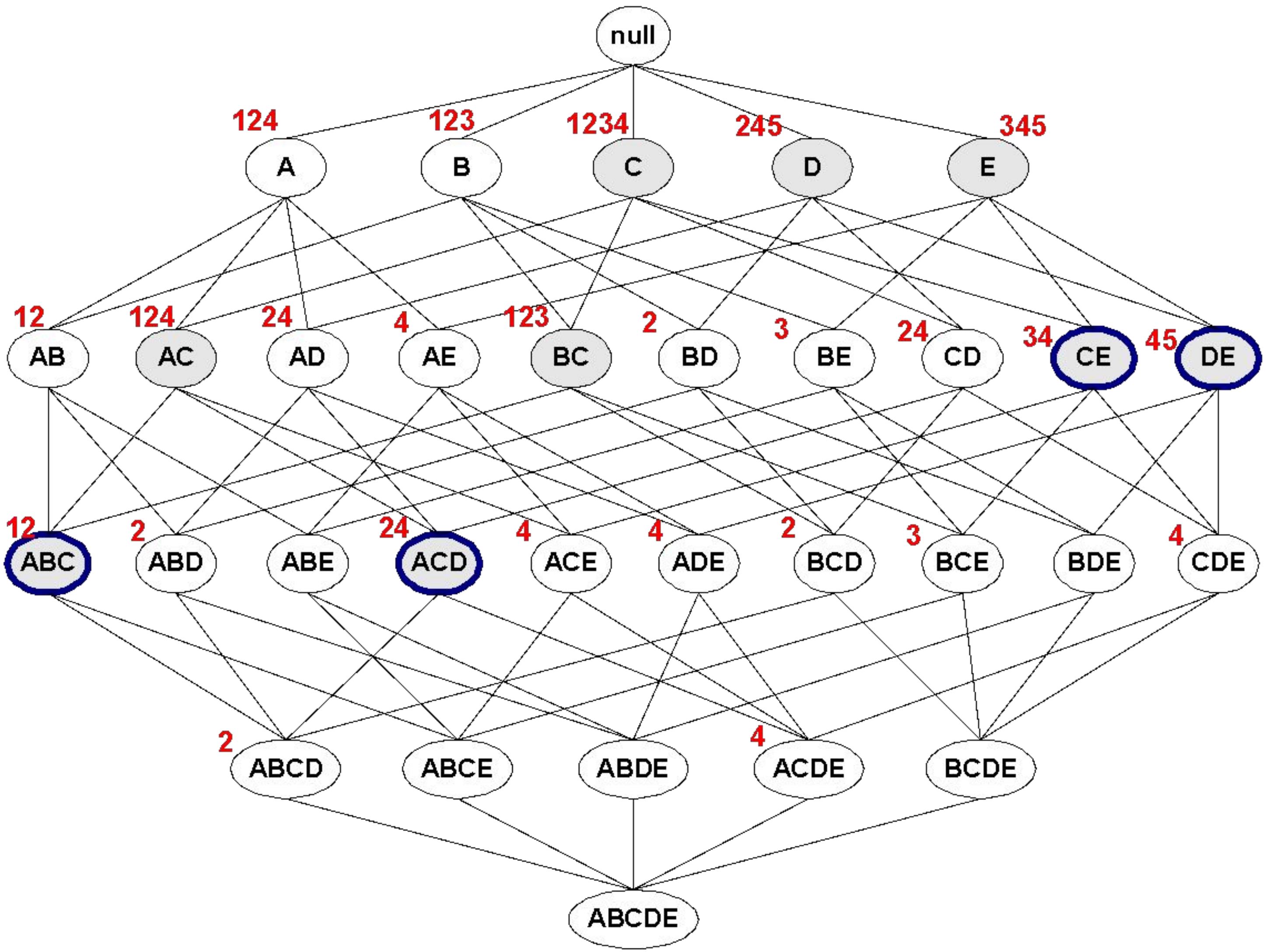


Itemsets	Support (counts)	Closed itemsets
{C}	**3**	✔
{D}	2	
{E}	2	
{C,D}	2	
{C,E}	2	
{D,E}	2	
{C,D,E}	**2**	✔

Why to study closed frequent itemset

- Closed frequent itemsets are useful for removing some of the redundant association rules. An association rule $X \rightarrow Y$ is redundant if there exists another rule $X' \rightarrow Y'$, where X is a subset of X' and Y is a subset of Y', such that the support and confidence for both rules are identical.

Why to study closed frequent itemset



Why to study closed frequent itemset

- {b} is not a closed frequent itemset while {b, c} is closed.
- The association rule {b} → {d, e} is therefore redundant because it has the same support and confidence as {b, c} → {d, e}. Such redundant rules are not generated if closed frequent itemsets are used for rule generation.

Maximal vs Closed Itemsets

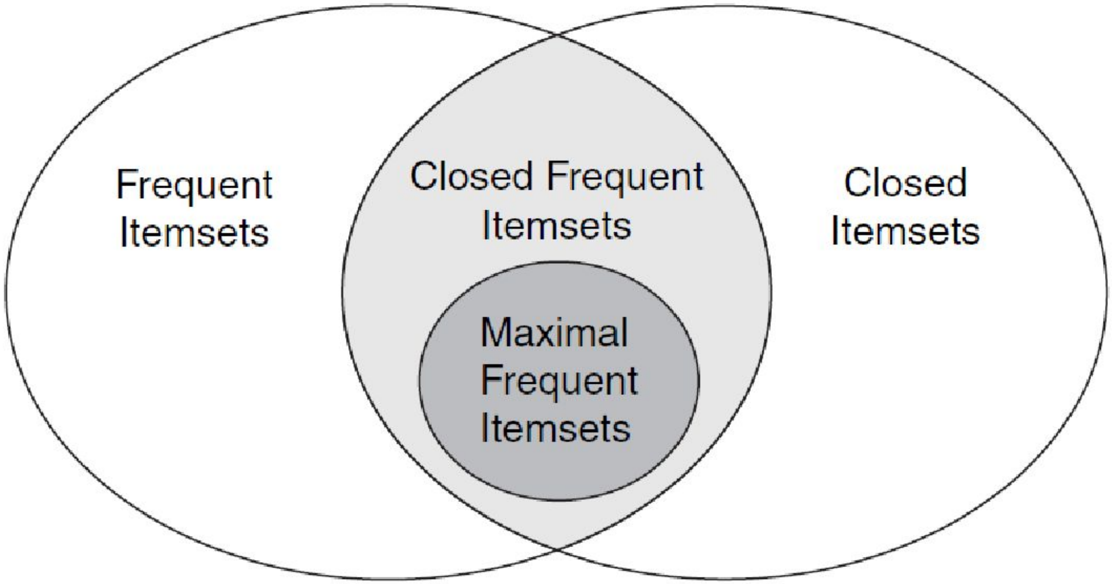


Figure 5.18. Relationships among frequent, closed, closed frequent, and maximal frequent itemsets.

Point to remember

- Finally, note that all maximal frequent itemsets are closed because none of the maximal frequent itemsets can have the same support count as their immediate supersets.