FP-growth Algorithm

- Use a compressed representation of the database using an FP-tree

- Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

FP-growth Algorithm

- An FP-tree is a compressed representation of the input data. It is constructed by reading the data set one transaction at a time and mapping each transaction onto a path in the FP-tree.

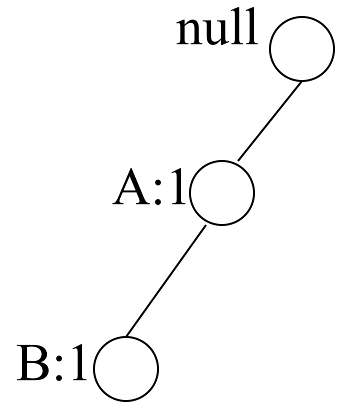
FP growth algorithm

- As different transactions can have several items in common, their paths may overlap. The more the paths overlap with one another, the more compression we can achieve using the FP-tree structure. If the size of the FP-tree is small enough to fit into main memory, this will allow us to extract frequent itemsets directly from the structure in memory instead of making repeated passes over the data stored on disk.

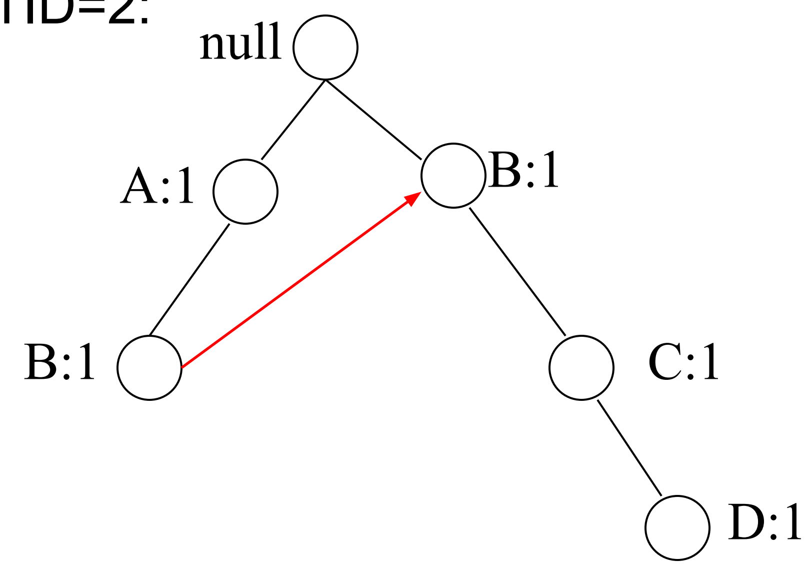
FP-tree construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

After reading TID=1:



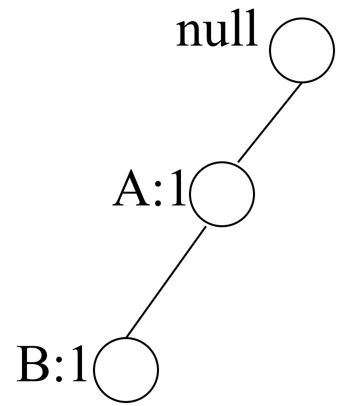
After reading TID=2:



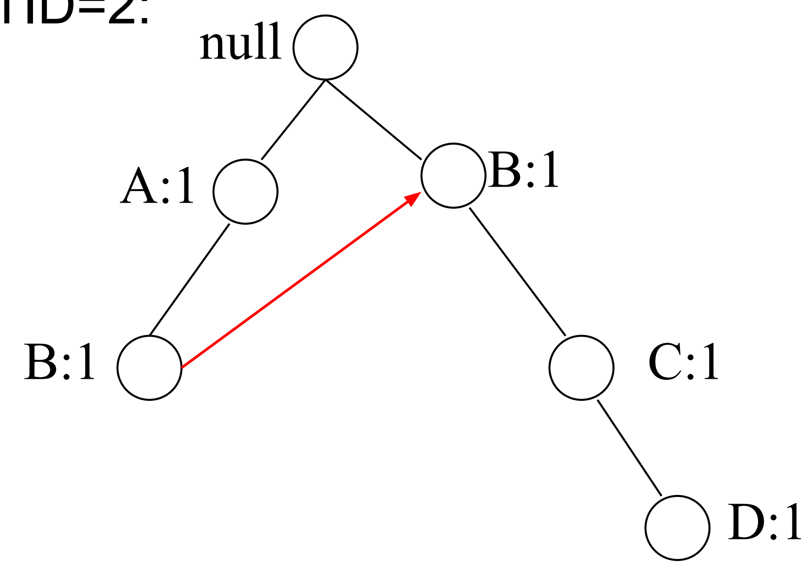
FP-tree construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

After reading TID=1:

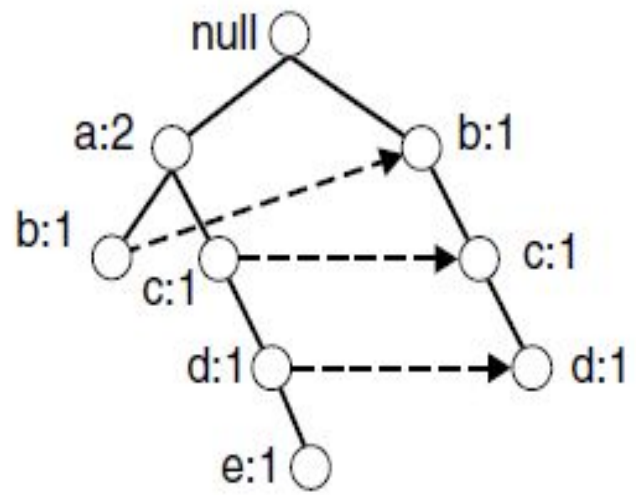


After reading TID=2:



FP-tree construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}



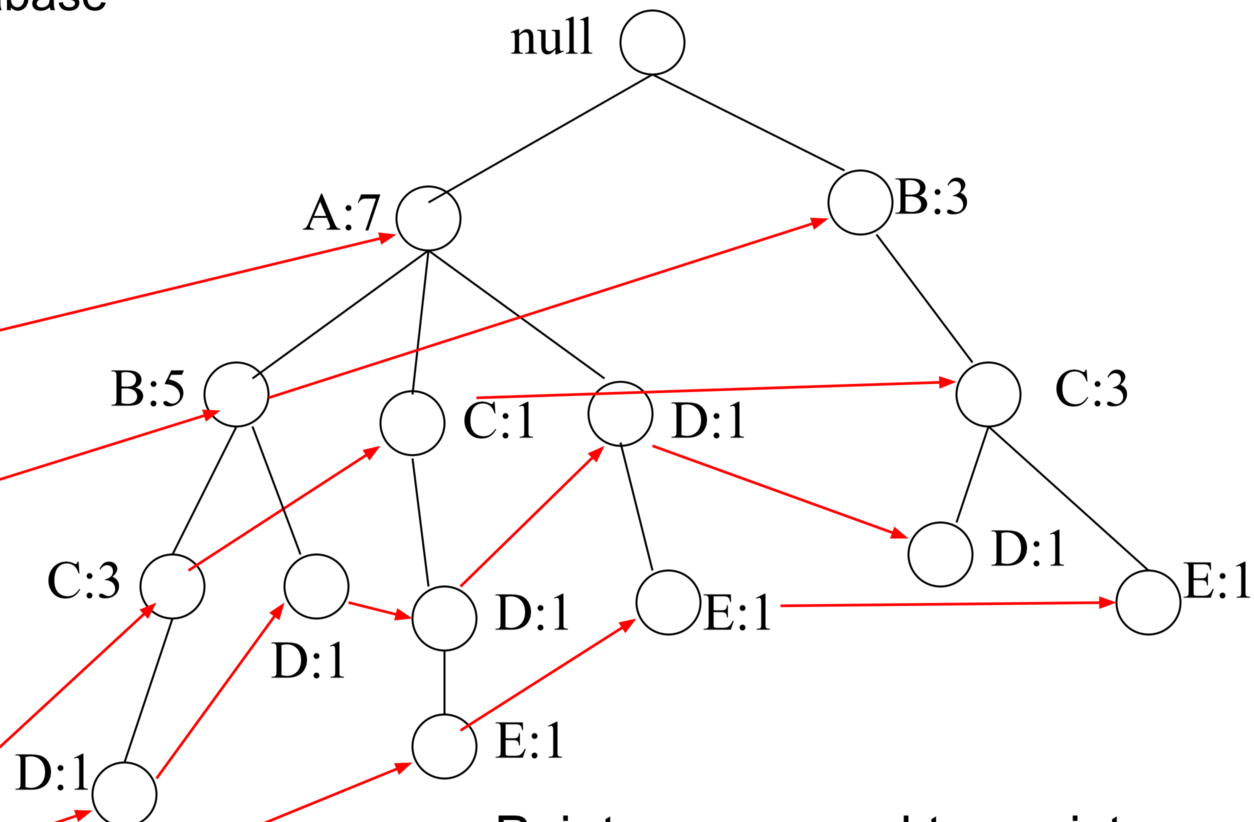
FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Transaction Database

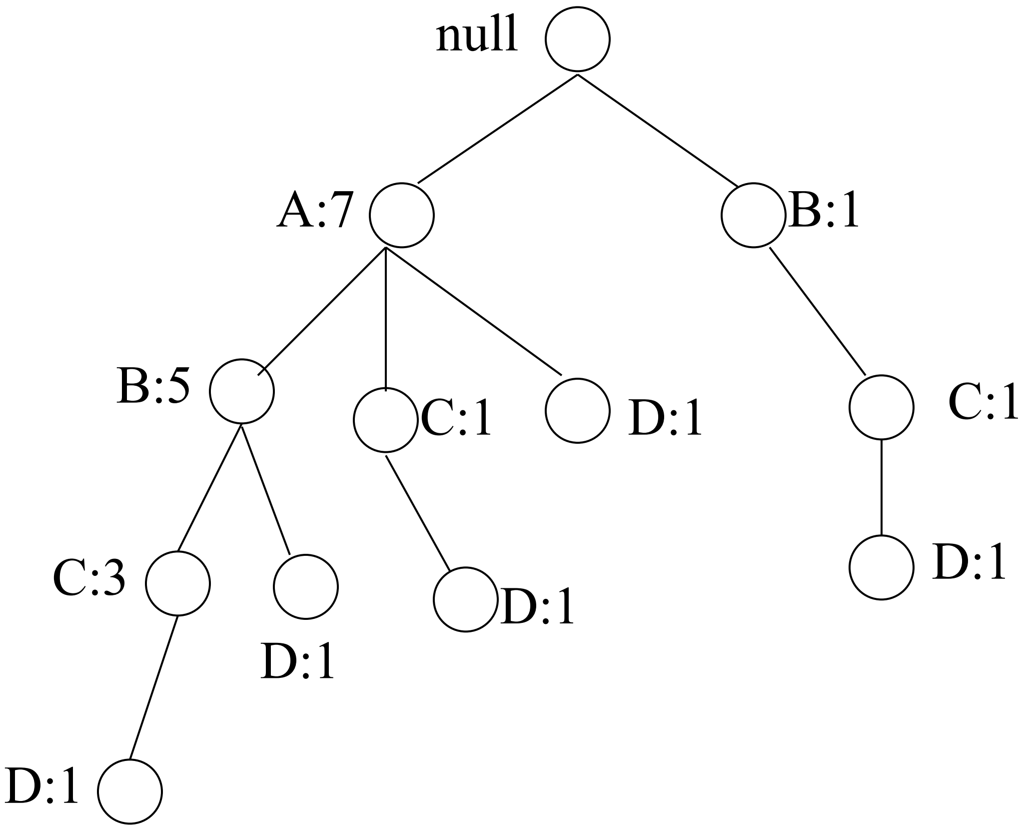
Header table

Item	Pointer
A	
B	
C	
D	
E	



Pointers are used to assist frequent itemset generation

FP-growth



Conditional Pattern base for D:

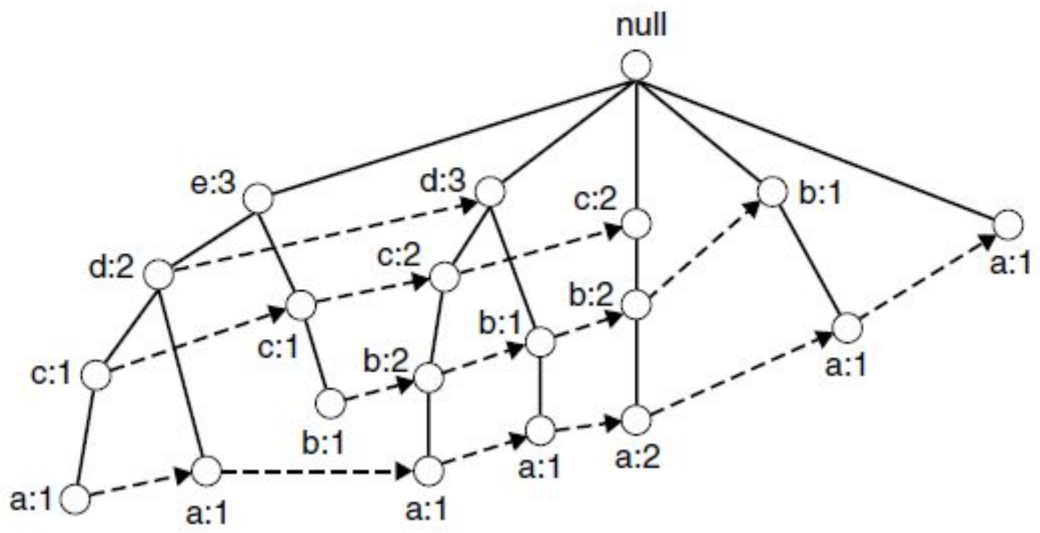
P = {(A:1,B:1,C:1),
(A:1,B:1),
(A:1,C:1),
(A:1),
(B:1,C:1)}

Recursively apply FP-growth on P

Frequent Itemsets found (with sup > 1):
AD, BD, CD, ACD, BCD

Size of FP Tree

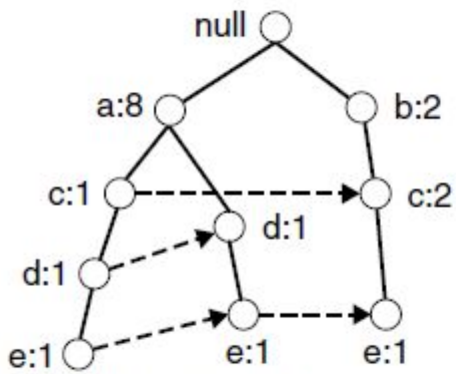
- The size of an FP-tree also depends on how the items are ordered. If the ordering scheme in the preceding example is reversed, i.e., from lowest to highest support item.



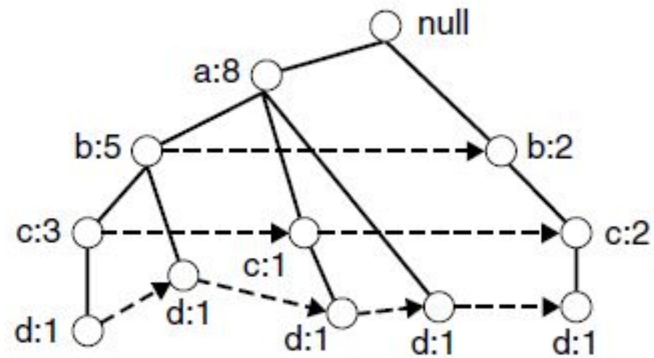
Frequent Itemset generation

- FP-growth is an algorithm that generates frequent itemsets from an FP-tree by exploring the tree in a bottom-up fashion. The algorithm looks for frequent itemsets ending in e first, followed by d, c, b, and finally, a.

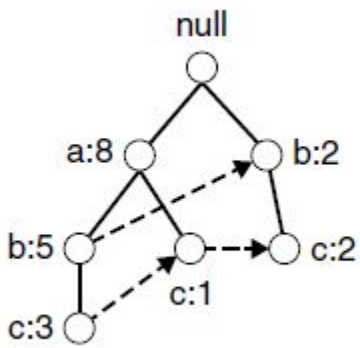
FP growth path



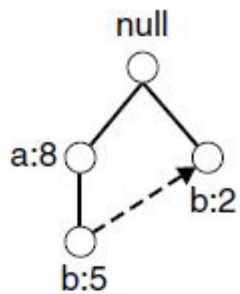
(a) Paths containing node e



(b) Paths containing node d



(c) Paths containing node c



(d) Paths containing node b

(e) Paths containing node a

Example

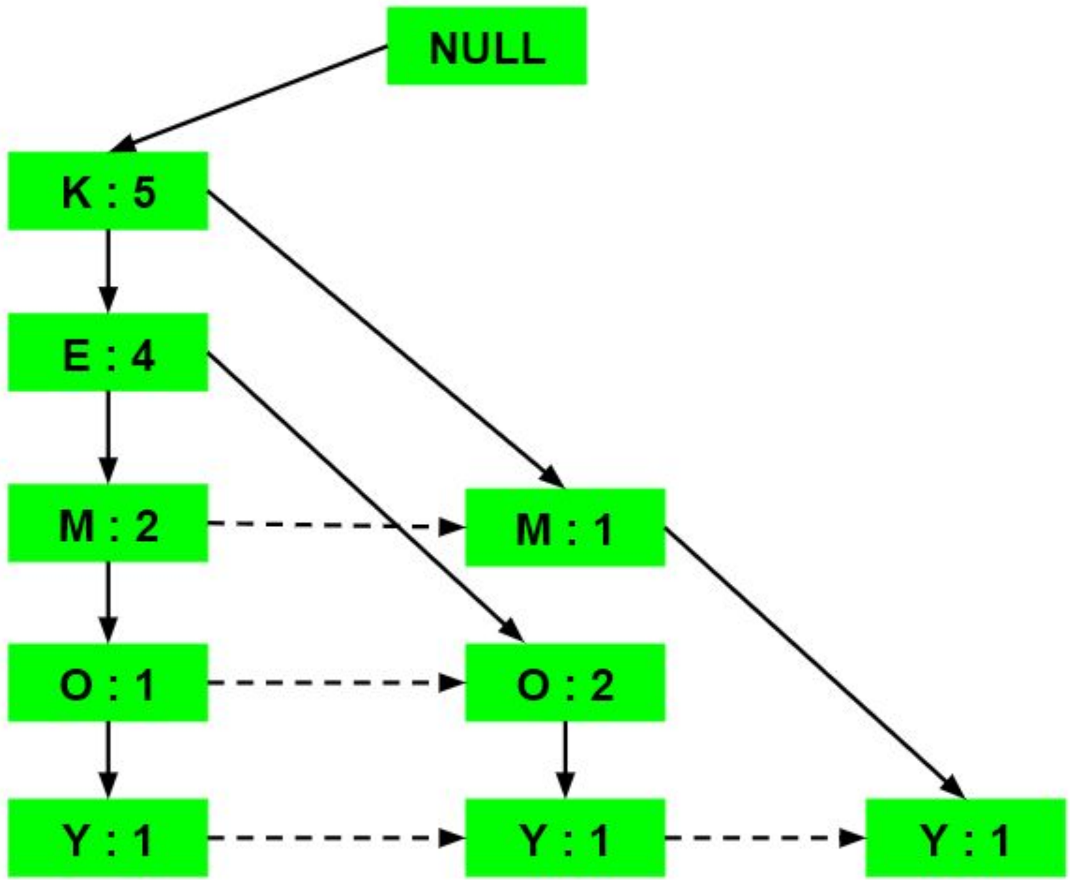
Item	Frequency
A	1
C	2
D	1
E	4
I	1
K	5
M	3
N	2
O	3
U	1
Y	3

Support Count = 3

Ordered Itemset

Transaction ID	Items	Ordered-Item Set
T1	{E, K, M, N, O, Y}	{K, E, M, O, Y}
T2	{D, E, K, N, O, Y}	{K, E, O, Y}
T3	{A, E, K, M}	{K, E, M}
T4	{C, K, M, U, Y}	{K, M, Y}
T5	{C, E, I, K, O, O}	{K, E, O}

FP TREE



Conditional Pattern Base

Items	Conditional Pattern Base
Y	{{K,E,M,O : 1}, {K,E,O : 1}, {K,M : 1}}
O	{{K,E,M : 1}, {K,E : 2}}
M	{{K,E : 2}, {K : 1}}
E	{K : 4}
K	

Conditional Frequent Pattern Tree

Items	Conditional Pattern Base	Conditional Frequent Pattern Tree
Y	{{K,E,M,O : 1}, {K,E,O : 1}, {K,M : 1}}	{K : 3}
O	{{K,E,M : 1}, {K,E : 2}}	{K,E : 3}
M	{{K,E : 2}, {K : 1}}	{K : 3}
E	{K : 4}	{K : 4}
K		

Frequent Pattern Generated

Items	Frequent Pattern Generated
Y	{<K,Y : 3>}
O	{<K,O : 3>, <E,O : 3>, <E,K,O : 3>}
M	{<K,M : 3>}
E	{<E,K : 4>}
K	

Example2

Transaction	List of items
T1	I1,I2,I3
T2	I2,I3,I4
T3	I4,I5
T4	I1,I2,I4
T5	I1,I2,I3,I5
T6	I1,I2,I3,I4

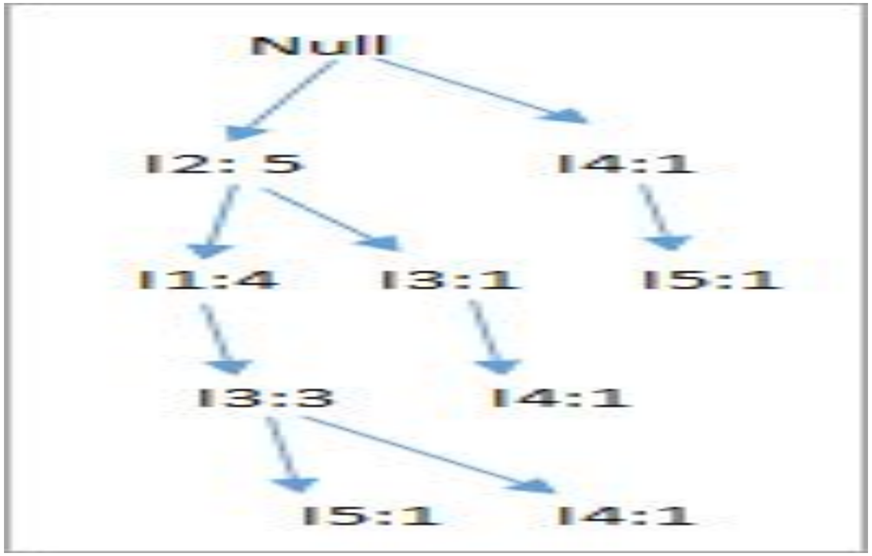
Support Count

Item	Count
I1	4
I2	5
I3	4
I4	4
I5	2

Newly Ordered

Item	Count
I2	5
I1	4
I3	4
I4	4

FP Tree



Frequent Pattern

Item	Conditional Pattern Base	Conditional FP-tree	Frequent Patterns Generated
I4	{I2,I1,I3:1},{I2,I3:1}	{I2:2, I3:2}	{I2,I4:2},{I3,I4:2},{I2,I3,I4:2}
I3	{I2,I1:3},{I2:1}	{I2:4, I1:3}	{I2,I3:4}, {I1:I3:3}, {I2,I1,I3:3}
I1	{I2:4}	{I2:4}	{I2,I1:4}

ECLAT

- The ECLAT algorithm stands for Equivalence Class Clustering and bottom-up Lattice Traversal. It is one of the popular methods of Association Rule mining

ECLAT

- The basic idea is to use Transaction Id Sets(tidsets) intersections to compute the support value of a candidate and avoiding the generation of subsets which do not exist in the prefix tree. In the first call of the function, all single items are used along with their tidsets. Then the function is called recursively and in each recursive call, each item-tidset pair is verified and combined with other item-tidset pairs.

ECLAT Example

Transaction Id	Bread	Butter	Milk	Coke	Jam
T1	1	1	0	0	1
T2	0	1	0	1	0
T3	0	1	1	0	0
T4	1	1	0	1	0
T5	1	0	1	0	0
T6	0	1	1	0	0
T7	1	0	1	0	0
T8	1	1	1	0	1
T9	1	1	1	0	0

ECLAT Example Continued

Item	Tidset
Bread	{T1, T4, T5, T7, T8, T9}
Butter	{T1, T2, T3, T4, T6, T8, T9}
Milk	{T3, T5, T6, T7, T8, T9}
Coke	{T2, T4}
Jam	{T1, T8}

ECLAT Example Continued

Item	Tidset
{Bread, Butter}	{T1, T4, T8, T9}
{Bread, Milk}	{T5, T7, T8, T9}
{Bread, Coke}	{T4}
{Bread, Jam}	{T1, T8}
{Butter, Milk}	{T3, T6, T8, T9}
{Butter, Coke}	{T2, T4}
{Butter, Jam}	{T1, T8}
{Milk, Jam}	{T8}

ECLAT Example Continued

k = 3

Item	Tidset
{Bread, Butter, Milk}	{T8, T9}
{Bread, Butter, Jam}	{T1, T8}

k = 4

Item	Tidset
{Bread, Butter, Milk, Jam}	{T8}

ECLAT Example Continued

Items Bought	Recommended Products
Bread	Butter
Bread	Milk
Bread	Jam
Butter	Milk
Butter	Coke
Butter	Jam
Bread and Butter	Milk
Bread and Butter	Jam

Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

    $ABC \rightarrow D$, $ABD \rightarrow C$, $ACD \rightarrow B$, $BCD \rightarrow A$,
    $A \rightarrow BCD$, $B \rightarrow ACD$, $C \rightarrow ABD$, $D \rightarrow ABC$
    $AB \rightarrow CD$, $AC \rightarrow BD$, $AD \rightarrow BC$, $BC \rightarrow AD$,
    $BD \rightarrow AC$, $CD \rightarrow AB$,

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \varnothing$ and $\varnothing \rightarrow L$)

Rule Generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

    $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

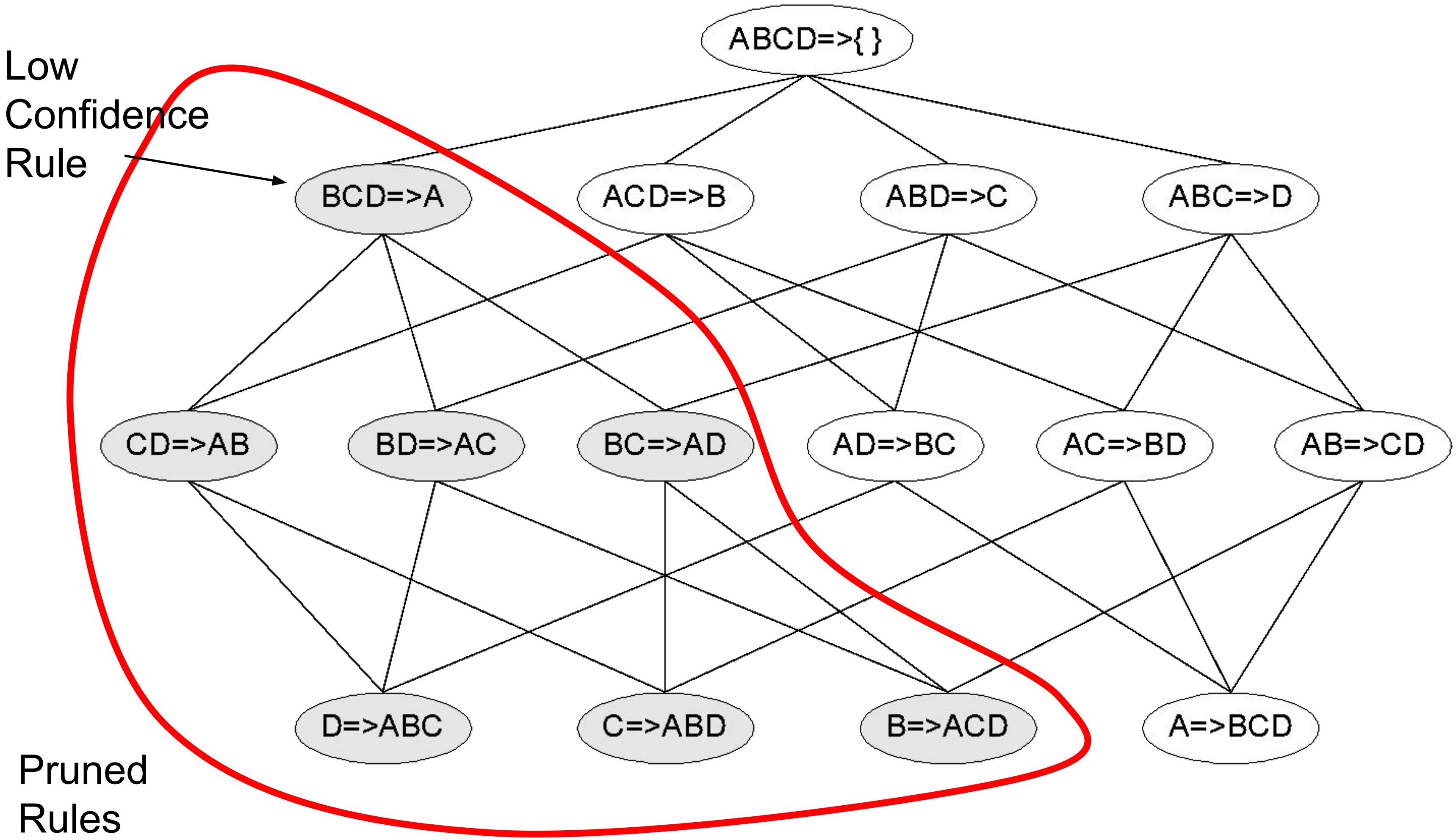
  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., L = {A,B,C,D}:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

    - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

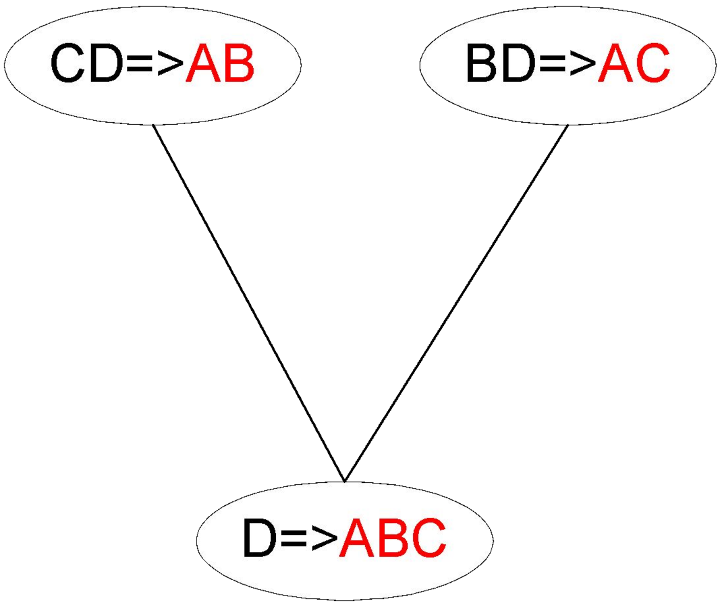
Rule Generation for Apriori Algorithm

Lattice of rules



Rule Generation for Apriori Algorithm

- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent



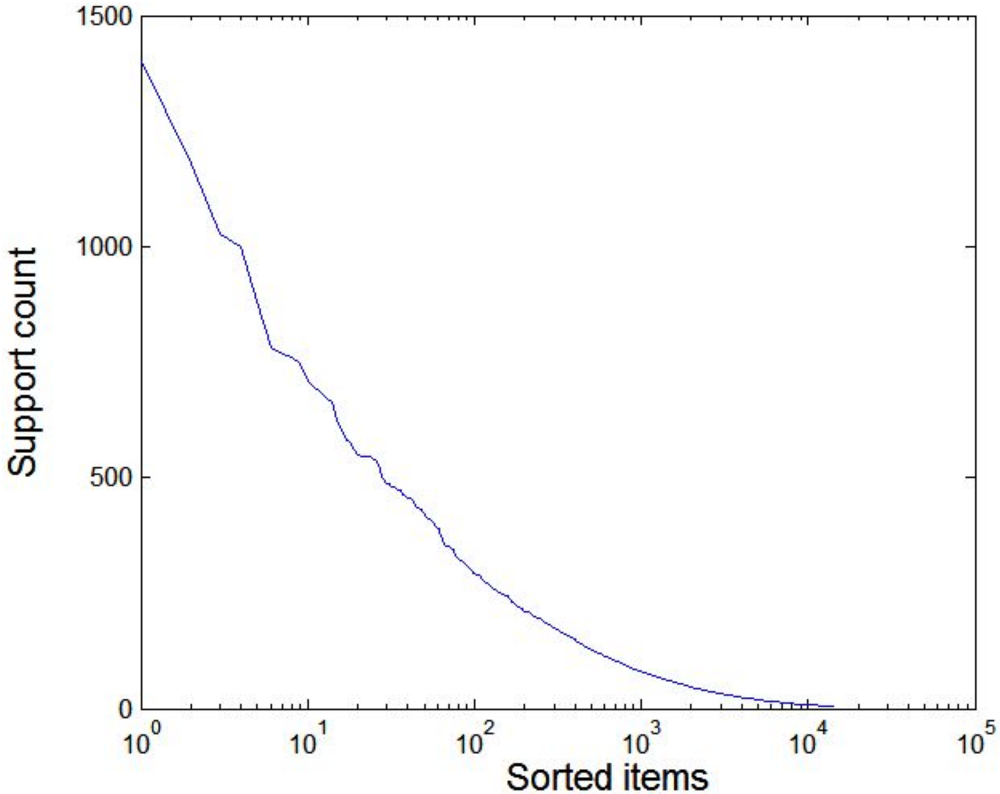
- join(CD=>AB,BD=>AC) would produce the candidate rule D => ABC

- Prune rule D=>ABC if its subset AD=>BC does not have high confidence

Effect of Support Distribution

- Many real data sets have skewed support distribution

Support distribution of a retail data set



Effect of Support Distribution

- How to set the appropriate *minsup* threshold?
  - If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
  - If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large
- Using a single minimum support threshold may not be effective

Computing Interestingness Measure

- The first set of criteria can be established through statistical arguments. Patterns that involve a set of mutually independent items or cover very few transactions are considered uninteresting because they may capture spurious relationships in the data. Such patterns can be eliminated by applying an objective interestingness measure that uses statistics derived from data to determine whether a pattern is interesting. Examples of objective interestingness measures include support, confidence, and correlation.

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Computing Interestingness Measure

- The second set of criteria can be established through subjective arguments.
- A pattern is considered subjectively uninteresting unless it reveals unexpected information about the data or provides useful knowledge that can lead to profitable actions. For example, the rule {Butter} → {Bread} may not be interesting, despite having high support and confidence values, because the relationship represented by the rule may seem rather obvious. On the other hand, the rule {Diapers} −→ {Beer} is interesting because the relationship is quite unexpected and may suggest a new cross-selling opportunity for retailers.
  –

Computing Interestingness Measure

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

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$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

Statistical Independence

- Population of 1000 students
  - 600 students know how to swim (S)
  - 700 students know how to bike (B)
  - 420 students know how to swim and bike (S,B)

  - $P(S \wedge B) = 420/1000 = 0.42$
  - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

- I(S,B) = 1, if *A* and *B* are independent;
- I(S,B) > 1, if *A* and *B* are positively correlated;
- I(S,B) < 1, if *A* and *B* are negatively correlated.

Statistical-based Measures

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y \mid X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

Drawback of Lift & Interest

	Y	$\overline{Y}$	
X	10	0	10
$\overline{X}$	0	90	90
	10	90	100

	Y	$\overline{Y}$	
X	90	0	90
$\overline{X}$	0	10	10
	90	10	100

$$Lift = \frac{0.1}{(0.1)(0.1)} = 10$$

$$Lift = \frac{0.9}{(0.9)(0.9)} = 1.11$$

Statistical independence:

If P(X,Y)=P(X)P(Y) => Lift = 1

Example: ϕ-Coefficient

- φ-coefficient is analogous to correlation coefficient for continuous variables

	Y	$\overline{Y}$	
X	60	10	70
$\overline{X}$	10	20	30
	70	30	100

	Y	$\overline{Y}$	
X	20	10	30
$\overline{X}$	10	60	70
	30	70	100

$$\phi = \frac{0.6 - 0.7 \times 0.7}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

$$\phi = \frac{0.2 - 0.3 \times 0.3}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

φ Coefficient is the same for both tables