Q#1

$T_A = 4n^2$

$T_B = 32n\log n$

$4n^2 < 32n\log n$

$n < 8\log n$

$n/8 < \log n$

$2^{n/8} < n$

For n = 8

$2^{8/8} < 8$

$2 < 8$

For n = 32

$2^{32/8} < 32$

$2^4 < 32$

$16 < 32$

For n = 44

$2^{44/8} < 44$

$45.2 < 44$

Hence T_A beats T_B

Q#2

$T_B = 100n^2$

$T_A = 2n$

$2^n < 100n^2$

$\log_2 2^n < \log_2 100n^2$

$n\log 2 < \log 100 + \log n^2$

$n < 6.64 + 2\log_2 n$

For n = 1

$1 < 6.64$

For n = 2

$2 < 8.64$

For n = 15

$15 < 14.45$

Hence T_A Beats T_B

Q#3 (a)

```
int s,i,n;                    1
Cin >> n;                     1
s = 0;                        1
for(i=n; i>1; i--){           n
    s++;                      n-1
}
```

$1 + 1 + 1 + n + (n-1)$

$2n + 2$

$O(n)$

(b)

```
int sum, i, j, n ;          1
  Sum = 0   ;               1
  Cin >> n ;                1
for (int i=1; i<n; i*2){    log2n
  for (j=1; j<n; j*2){      log2n
      Sum ++ ;              log2n
```

O(log n).

(c)

```
int x=0, j=n ;         1
while (j>0){           n
    x += j x 3;        log4n
    j /=4 ;            log4n
}
```

O(nlog4 n)

Q#1

```
selectionsort (arr, size) {
   int min=0;
   for (i=0 ; i<size-1; i++) {
      min = i ;
   for(j= i+1; j<size; j++) {
     if (arr[i] < arr[min]) {
        min = arr [i];
     }
   if (i != min) {
    Swap(arr[i], arr[min]);
```

Since the algorithm is comparison based so after each iteration of the outerloop element is automatically placed in it's correct position

Best Case → $\theta(n^2)$

Worst Case → $O(n^2)$

Q#5

$T(n) = \frac{1}{8}n^3 - 5n^2$ is $\theta(n^3)$

$c_1 n^3 \le \frac{1}{8}n^3 - 5n^2 \le c_2 n^3$

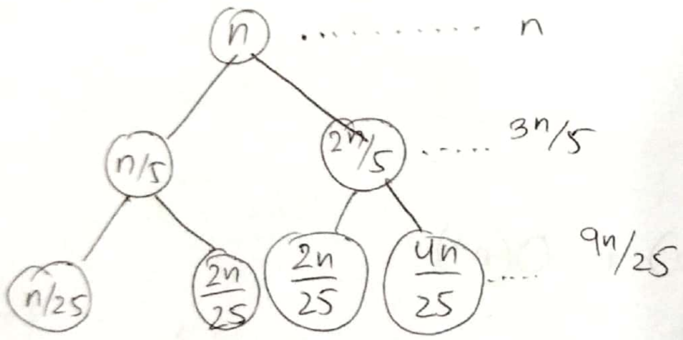
$c_1 \le 1/8 - 5/n \le c_2$

if $n = 1$

$c_1 \le \frac{1}{8} - 5 \le c_2$

Q#6

$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{2n}{5}\right) + O(n)$



$n\left[1 + \frac{3n}{5} + \frac{9n}{25} \cdots\cdots + \left(\frac{3}{5}\right)^{\log_{5/2} n}\right]$

$x = \frac{3}{5}$

$n\left[\frac{1}{1-x}\right]$

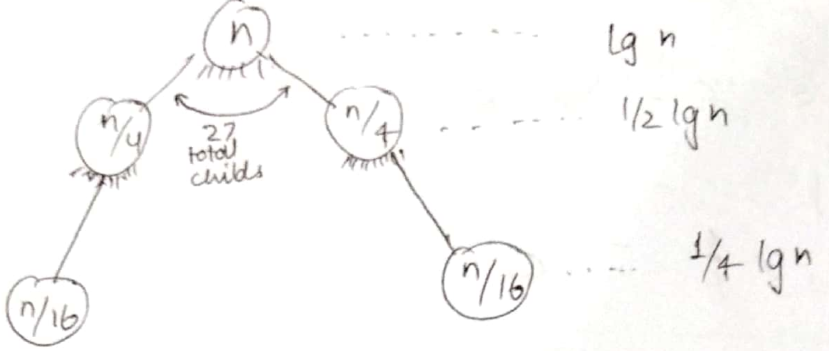
$\frac{1}{1 - 3/5}$

$n\left[\frac{5}{2}\right]$

$O(n)$

Q#7

(a) $T(n) = 27\left(\frac{n}{4}\right) + O(\lg n)$

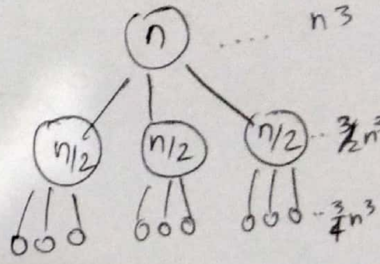


$\left[1 + \frac{1}{2} + \frac{1}{4} + \cdots\cdots \left(\frac{1}{2}\right)^{\lg_4 n}\right]$

$\lg n \left[\frac{1}{1 - \frac{1}{2}}\right]$

$O(\lg n)$

(b) $T(n) = 3(n/2) + O(n^3)$

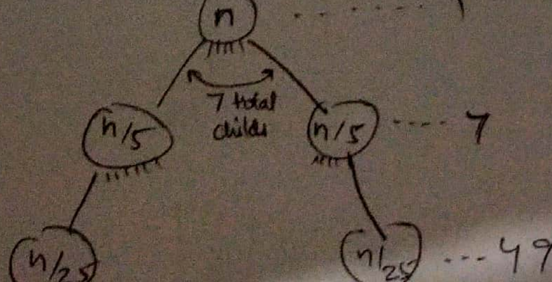


$n^3\left[1 + 3/2 + 3/4 \cdots\cdots (3/2)^{\log_2 n}\right]$

$n^3\left[\frac{(3/2)^{\log_2 n+1} - 1}{3/2 - 1}\right]$

$\theta(n^{0.3})$

(c) $7T(n/5) + \theta(1)$

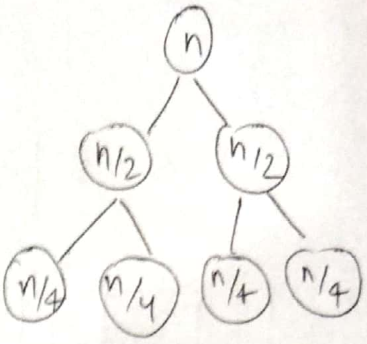


$\frac{7^{\log_5 n+1} - 1}{6}$

$\frac{n^{\log_5 7} \times 7 - 1}{6}$

$O(n^{1.18})$

(d) $T(n) = 2T(n/2) + \cancel{n\log n}\ n/\log n$

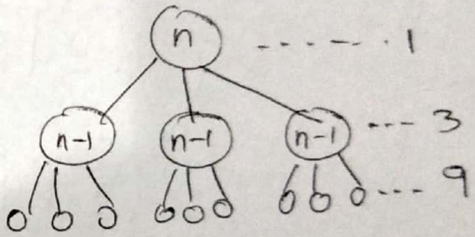


$$\frac{n}{\lg n}\left(1+2+4+\cdots\cdots 2^{\lg_2 n}\right)$$

$$\frac{n}{\lg n}\left(\frac{2^{\lg_2 n+1}}{1}\right)$$

$$O(n/\lg n)$$

(e) $T(n) = 3T(n-1) + \Theta(1)$

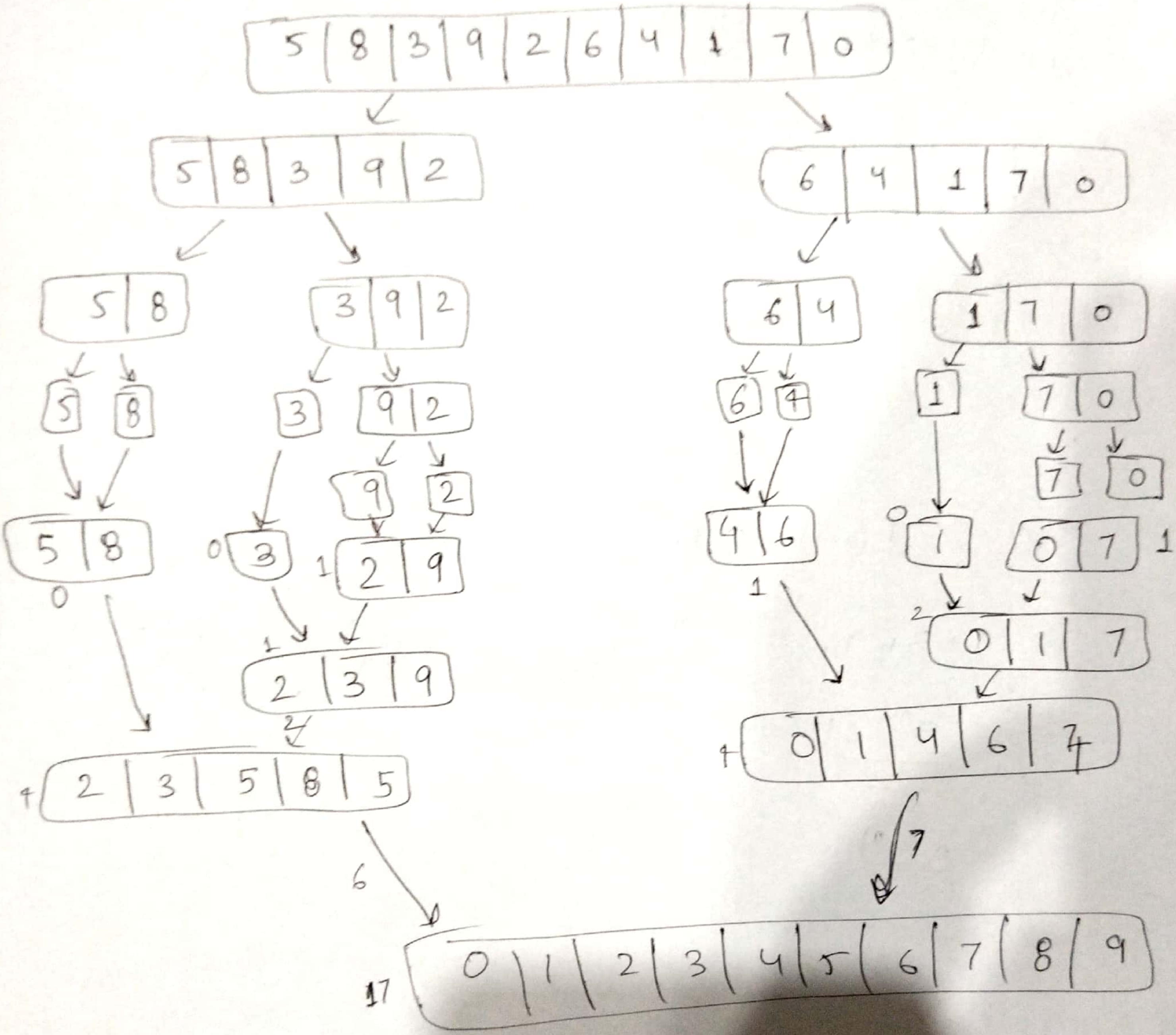


$$1+3+9+\cdots\cdots 3^n$$

$$= \frac{3^{n+1}-1}{2}$$

$$= O(3^n)$$

Q#8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5 8 3 9 2
6 4 1 7 0
5 8
3 9 2
6 4
1 7 0
2 3 9
0 1 7
2 3 5 8 5
0 1 4 6 7
0 1 2 3 4 5 6 7 8 9