Investigating the affect of Magnetic island growth on Fusion output

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This project models magnetic island growth in a Tokamak using the Modified Rutherford Equation to assess its impact on fusion output. Simulations show that bootstrap-driven island expansion significantly reduces confinement time and fusion power. Results highlight the need for active island control to maintain efficient fusion conditions.

1 Introduction

This project report consists of the investigation of different parameters and how they affect the growth of the width of magnetic islands within a Tokamak reactor. Additionally, it also investigates how magnetic island size will affect fusion output using the Lawson Criterion. A Tokamak reactor wants to confine hot plasma long enough to satisfy the Lawson criterion. However, no material can withstand the immense temperature of the hot plasma; thus, magnetic fields are used to shape the plasma away from the walls of the reactor. Within this report, interest lies in magnetic island formation and growth. Magnetic islands are localised disruptions in the nested magnetic field configuration [1,2]. Formation comes from magnetic field lines wrapping around plasma in two directions, poloidally and toroidally. At certain surfaces within the plasma, magnetic field lines wrap around the Tokamak geometry in both the toroidal and poloidal directions and eventually return to their exact starting point. These surfaces are known as rational magnetic surfaces. The number of times a field line wraps toroidally for each poloidal circuit is quantified by the safety factor q:

$$q = \frac{m}{n}$$

Figure 1: Represents the safety factor equation. Where m = number of turns in the toroidal direction. n = number of turns in the poloidal direction.

Specifically, the safety factor represents how many times the field line travels around the torus (toroidally) for each complete loop around the cross-section (poloidally). These rational magnetic surfaces are very sensitive to changes. These changes, for example, from pressure gradients or current ripples, break the magnetic surface, forming loops known as magnetic islands. Magnetic islands disrupt the confinement of plasma by allowing heat to escape from the magnetic plasma "cage" that contains it. This heat loss makes it more difficult to achieve the desired triple product, which is necessary for self-sustaining fusion.

$$n \cdot T \cdot \tau_E \geq 3 \times 10^{21} \left\{keV \cdot \frac{s}{m}\right\}^3$$

Figure 2: Represents the fusion triple product, where n is the plasma density, T is the plasma temperature, and τ_E is the energy confinement time. The product must exceed this value to achieve ignition in deuterium-tritium fusion [3].

Additionally, magnetic islands grow over time, modelled by a simplified form of the Rutherford equations, leading to Neoclassical Tearing Modes (NTMs) [4,5]. Instabilities that degrade performance and/or damage the Tokamak reactor physically, potentially leading to serious safety concerns. This is the central motivation behind the investigation and creation of the simulation of magnetic island growth. They represent a significant obstacle for the feasibility of fusion as a practical / efficient energy source in the future. In order to achieve safe and efficient fusion, magnetic islands must be studied, monitored, and actively controlled. Their presence directly reduces magnetic confinement time, one of the three key parameters alongside plasma temperature and fuel density that govern fusion performance. The purpose of this research project is to model the time evolution of magnetic island width within a Tokamak using reduced forms of the Rutherford equations. The simulation provides a framework for understanding how key plasma parameters influence stability. Magnetic islands are known to degrade confinement time and reduce fusion performance, and remain one of the most challenging aspects of magnetic confinement fusion. This simulation aims to provide insight into their growth dynamics, equipping researchers with a computational tool to analyse the effect of changing different parameters such as bootstrap current, saturation, and others. In future applications, such a simulation can be implemented into reactor control systems to focus on island suppression through targeted heating and general optimisation of the parameters that cause the least amount of island growth and formation. There is potential for contribution to the broader challenge of maintaining high performance, disruption-free operation in next-gen reactors such as ITER.

2 System Architecture

All code used to build the simulation framework can be found in the GitHub repository at the end of this report. This section provides a brief yet comprehensive analysis of the system's architecture and logic, explaining how magnetic islands grow and their effects on confinement are modelled computationally. The project is structured into modular python files, each responsible for a distinct duty in the modelling process. The modules collectively simulate the growth of magnetic islands within

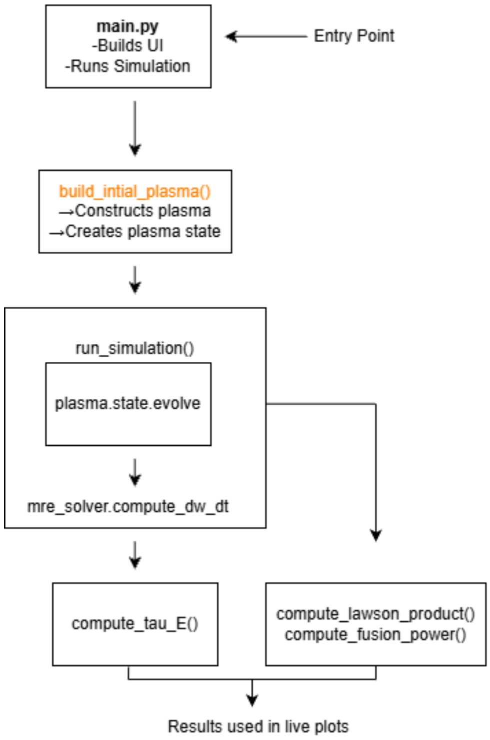

Figure 3: Simulation flow diagram showing how magnetic island growth is modeled and linked to confinement and fusion output calculations.

a Tokamak plasma and quantify their impact on fusion performance. The main.py file initialises and runs the actual simulation. It constructs the initial plasma profile, which is composed of several flux surfaces. If any flux surface has a safety factor close to a rational value (e.g. q = 2/1), a magnetic island is formed at that location. The simulation puts the island through a time-stepping loop, where the width of the island is updated at each step using the simplified MRE (Modified Rutherford Equations). After every step, the script then calls the computational functions which calculate the confinement time, Lawson criterion and triple product.

The flux-surface.py file defines the structure and characteristics of magnetic flux surfaces and magnetic islands. The magnetic island module allows us to model a magnetic island with key attributes: initial width, mode numbers (m,n), and bootstrap current. Additionally, the magnetic island module contains an evolve method which updates the island width over time using the MRE. Lastly, the FluxSurface class represents a magnetic island, defined by its radius and safety factor (q), these two classes are the foundation for defining and modelling a plasma configuration. The plasma-state.py module organises multiple flux surfaces into a complete plasma configuration via the PlasmaState class. Plasmas in fusion are modelled as a collection of nested magnetic surfaces, some of which may contain a magnetic island depending on the safety factor (q). This plasma object is passed through the simulation loop, where it evolves over time. The mre-solver.py file implements the core physics model for the magnetic island growth using a simplified version of the MRE. It calculates the rate of change of island width based on the defined parameters the user sets at the beginning of the simulation, such as bootstrap current or saturation, for example. These parameters are adjustable via the user interface, allowing users to analyse the impact of different physical parameters on magnetic island growth and subsequently on fusion output. Lastly, the two modules confinement.py and fusion-output.py are calculation modules which calculate key values which are shown in the graphs discussed in the coming section. These values are the energy confinement time, which represent how long energy remains confined within the plasma, and the Lawson criterion, which determines whether the plasma is capable of ignition.

3 Results & Discussion

This section discusses the results of the built framework and simulation based on the four key parameters: initial island width, bootstrap drive, delta scale, and saturation. These parameters are all inputs into the MRE, governing the rate of change in width for a magnetic island in Tokamak plasma. Island width is the starting width of the magnetic island. Physically, it represents the seed size of the perturbation on the magnetic surface. A large width implies an already disrupted confinement region, increasing the risk of further instability. Bootstrap Drive models the self-driven current within the island, caused by pressure gradients within the plasma. This parameter amplifies island growth; the larger the bootstrap current, the faster and more persistent the island grows. Delta scale represents the local magnetic shear and geometry near rational surfaces. It determines whether small perturbations are stable or unstable. Higher the delta scale; increases the growth-driving component in the MRE, making the island more sensitive to small perturbations between mode numbers, hence the general safety factor (q). The saturation coefficient is the only parameter which opposes island growth, representing a stabilising effect. It essentially represents the limit to how large a magnetic island can get through non-linear physics such as magnetic tension and current redistribution.

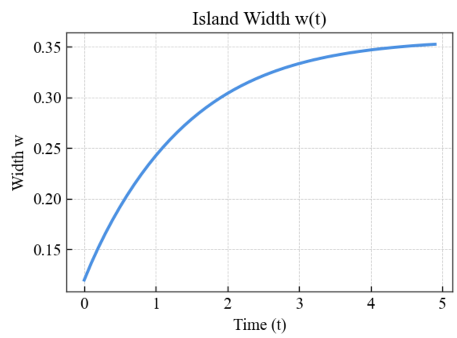

Figure 4: Island width w(t) over time. Bootstrap drive: 0.2, initial width = 0.01, delta scale = 10, saturation = 1.0

Figure 4 shows the exponential growth of the magnetic island over time. The island grows from 0.01 to 0.35 units, a significant increase in growth over time. This increase is primarily caused by bootstrap current term in the MRE. The island growth plateaus due to the saturation term, due to the non-linear damping effect included via the Dw$\hat{2}$ term. Physically, this widening of the island represents a progressive degradation of the magnetic confinement.

Figure 5 represents the exponential de-

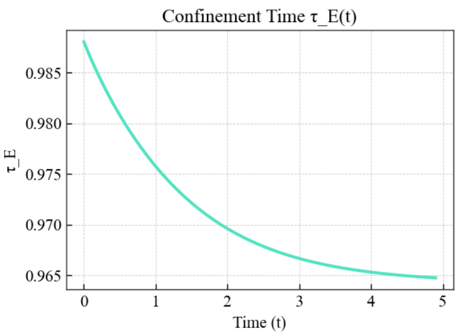

Figure 5: Energy confinement time $\tau_E(t)$ as island width increases

cline in energy confinement time ($\tau_E(t)$) as the island widens over time. This occurs due to wider islands form conduits for heat to leak out of the core, "breaking" the magnetic cage which isolates the plasma from reactor walls. This shows that even small increases in magnetic island width can significantly degrade energy retention, emphasising controlling island growth in fusion reactor operations.

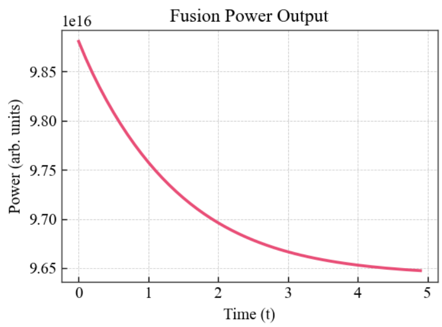

Figure 6: Fusion power output over time, calculated via the triple product $nT\tau_E$

Figure 6 tracks the decline in fusion power output, having a similar relationship as confinement time. Since fusion output scales with confinement time through the triple product, any loss in confinement directly reduces the effective yield of the plasma. Despite temperature and density being constant within the simulation, the loss in confinement time alone leads to a major drop in output power, highlighting the sensitivity of confinement degradation caused by magnetic islands.

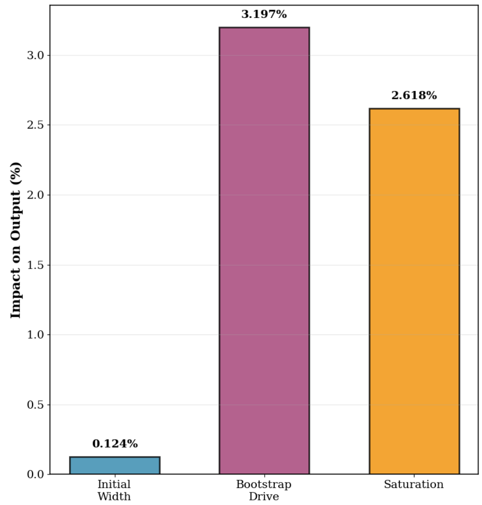

Figure 7: Percentage impact of island parameters on final fusion power output, showing bootstrap drive and saturation as dominant factors.

Understanding how each input parameter effects the final fusion output is crucial for optimising confinement strategies and designing control systems in future Tokamak. Figure 7 quantifies the influence of each parameter on fusion output, based on percentage change. It can be seen that bootstrap drive exerts the largest influence at 3.197% followed by saturation at 2.618%, while island width has a minor impact of just 0.124% [5,6]. The physical reason as to bootstrap drive impacting fusion output the most is due to its self-sustaining nature. Since bootstrap current is generated within the island due to pressure gradients, it acts as a positive feedback mechanism: as the island grows, pressure gradients increase, causing a further rise in bootstrap current. This makes bootstrap drive the primary destabilising force in the MRE, therefore the main contributor to confinement degradation. In contrast, saturation introduces a non-linear damping effect, limiting runaway island growth at large widths. In summation when assessing the effectiveness of fusion systems, fusion output is the key performance metric, as it ultimately determines whether a plasma configuration can achieve ignition and sustain a net-positive energy gain. This simulation has proved that controlling the bootstrap drive, whether through profile shaping, external current drive or localised heating, is vital for improving reactor performance [7]. These findings reinforce the value of predictive simulations in fusion research and underscore the importance of magnetic island management in the roadmap toward commercially viable fusion energy.

References:

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