Assignment 1

Due: 19 September, 2025 11:55 PM

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Instructions

- Ensure that all answers include complete and detailed workings for full credit.
- Collaboration between students is permitted, but copying answers is strictly prohibited.
- Any assistance received for the completion of this assignment must be clearly indicated.
- Assignments must be submitted on LMS prior to the specified deadline.
- Late submissions will not be accepted under any circumstances.
- Submit **both** the .pdf and the .tex file of your solution in a zipped folder named according to the convention `CS210_A1_RollNumber`.

Graded Problems

Question 1 [10 Marks]

Let p, q, r denote the following statements about a triangle ABC.

p : Triangle ABC is isosceles;
q : Triangle ABC is equilateral;
r : Triangle ABC is equiangular.

Translate each of the following into an English sentence. (2×5)

1. $q \to p$
2. $\neg p \to \neg q$
3. $q \leftrightarrow r$
4. $p \wedge \neg q$
5. $r \to p$

Question 2 [10 Marks]

(a) Let $p \oplus q$ denote exclusive-or. Decide whether the following is a tautology and prove your answer using a truth table: (4)

$$\big((p \oplus q) \leftrightarrow r\big) \;\leftrightarrow\; \big((p \leftrightarrow r) \oplus (q \leftrightarrow r)\big)$$

(b) Construct the complete truth table for

$$F(p, q) = (p \vee q) \;\wedge\; \big((p \to \neg q) \;\vee\; (q \to \neg p)\big)$$

showing all intermediate steps. (3)

(c) Simplify $F(p, q)$ using logical equivalence laws, clearly showing each step. (2)

(d) Identify the single logical operation that F is equivalent to. (1)

Question 3 [15 Marks]

Translate the following statements into logical form using the given predicates.

$P(x)$: "x submitted the project."
$Q(x)$: "x passed the final exam."
$R(x)$: "x is allowed to graduate."
$S(x)$: "x applied for an extension."

(a) If a student submitted the project and passed the final exam, then they are allowed to graduate. (1)

(b) If a student did not submit the project and did not apply for an extension, then they are not allowed to graduate. (2)

(c) If a student submitted the project and did not pass the final exam, then they are not allowed to graduate. (2)

(d) There exists a student who submitted the project and is not allowed to graduate. (2)

(e) Any student who applied for an extension and passed the final exam is allowed to graduate. (2)

(f) There exists a student who did not submit the project, did not pass the final exam, and is not allowed to graduate. (2)

(g) Every student who submitted the project has passed the final exam. (2)

(h) For every group, there exists a student who did not submit the project but is allowed to graduate. (2)

Question 4 [15 Marks]

Find the negations of the following two quantified predicates without "¬" in front of any quantifier. Show all your steps to receive full credit.

1. $\forall x \, \forall y \, [(x > y) \rightarrow (x - y > 0)]$
2. $\forall x \, \forall y \, [(x < y) \rightarrow \exists z (x < z < y)]$

Ungraded Problems

Question 1

Use a truth table to determine if the statements $[P \to (Q \vee R)] \equiv [\neg R \to (P \to Q)]$ is true.

Question 2

Use a truth table to establish whether the following statement forms a tautology or a contradiction or neither:

$$((Q \wedge R) \wedge (\neg P \wedge Q)) \wedge \neg Q$$

Question 3

Show that $((a \wedge b) \to c) \leftrightarrow ((a \to c) \vee (b \to c))$.

Question 4

Let p, q, r denote primitive statements. Use truth tables to prove the following logical equivalences.

1. $p \to (q \wedge r) \leftrightarrow (p \to q) \wedge (p \to r)$
2. $[(p \vee q) \to r] \leftrightarrow [(p \to r) \wedge (q \to r)]$

Question 5

Show that $(a \vee b \to c) \to (a \wedge b \to c)$ but the converse (i.e., $(a \wedge b \to c) \to (a \vee b \to c)$) is not true.

Question 6

If $p \to q$ is false, what is the truth value of $((\neg p) \wedge q) \leftrightarrow (p \vee q)$?

Question 7

Which of the following is a tautology?

1. $(a \leftrightarrow b) \to (a \wedge b)$,
2. $(a \leftrightarrow b) \leftrightarrow (a \wedge b) \vee (\neg a \wedge \neg b)$

Question 8

Show that the propositions $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

Question 9

Consider a standard 9×9 Sudoku puzzle. In this puzzle, each cell in the grid can contain a number from 1 to 9 , and the puzzle must satisfy the following constraints:

Let $p(i,j,n)$ be a predicate such that $p(i,j,n)$ is true if and only if the number n is in the cell located at row i and column j, where i,j,n are integers in the range 1 to 9 .

Formalize the Constraints for a 9×9 Sudoku Puzzle

1. Row Constraint: Write a logical expression using predicates and quantifiers that asserts that each number from 1 to 9 appears exactly once in each row.
2. Column Constraint: Write a logical expression using predicates and quantifiers that asserts that each number from 1 to 9 appears exactly once in each column.
3. Block Constraint: Write a logical expression using predicates and quantifiers that asserts that each number from 1 to 9 appears exactly once in each 3×3 block.
4. Cell Constraint: Write a logical expression using predicates and quantifiers that asserts that each cell contains at most one number.

Question 10

1. Suppose a software system has 9 components $\{A,B,C,D,E,F,G,H,I\}$. Each component has either exactly one of the two types of bugs (Bug 1 and Bug 2) or has no bug (is clean). We want to identify which components have Bug 1 or Bug 2 or are clean. Findings are summarized as follows.

   (a) Let $P(x)$ be the predicate that component x has Bug 1, let $Q(x)$ be the predicate that component x has Bug 2, and let $R(x)$ be the predicate that component x is clean. Translate each of the below findings in terms of the predicates $P(x), Q(x)$ and $R(x)$.

   (i) E and H do not have the same bug.
   (ii) If G has Bug 1 then all components have Bug 1.
   (iii) If E has Bug 1 then H has Bug 1 too.
   (iv) If C has Bug 1 then D and F do not have Bug 1.
   (v) If either E or H has Bug 1 then I does not have Bug 2.
   (vi) At least 4 components have Bug 1.
   (vii) If A has either bug then all components have Bug 2.
   (viii) A and F are not in the same category.
   (ix) B has Bug 2.
   (x) At least one of C and G have the same bug as B.
   (xi) Exactly 2 components have Bug 2.
   (xii) If I has Bug 2 then at least one of D, F and A have Bug 2 too.
   (xiii) If E or G have Bug 2 then all components have either Bug 1 or Bug 2.

   (b) Determine, using the above findings, which components have Bug 1, which ones have Bug 2, and which are clean.

Question 11

Original Statement: "If an element x is in set $A \cap B$, then $x \in A$."

(a) Write the converse, inverse, and contrapositive.

(b) Determine truth values for each.

(c) Provide a counterexample for any false statements.

Question 12

Original Statement ($P \to Q$): "If an integer n is prime, then $n^2 + 2$ is odd."

(a) **Converse, Inverse, and Contrapositive**

  i. Write the converse, inverse, and contrapositive of the original statement.

  ii. Clearly indicate which symbols represent P and Q in each form.

(b) **Truth Analysis** For the original statement and each of its forms, determine whether it is true or false. If false, provide a counterexample.

Question 13

Simplify the following formula step by step using laws of logical equivalence. Determine whether it is a **tautology** or a **contradiction**.

$$F_1 = \big((p \wedge q \wedge r) \to (t \vee (u \wedge v))\big) \;\vee\; \big((p \wedge q \wedge r) \wedge (\neg u \vee (v \wedge \neg t)) \vee (s \vee \neg s)\big).$$

Question 14

Using logical equivalence only, simplify the expression below. Clearly state whether it is a **tautology** or a **contradiction**.

$$F_2 = ((p \wedge q \wedge r \wedge s) \wedge (\neg(p \wedge q \wedge r \wedge s) \vee (t \vee (u \wedge (v \vee \neg w))))) \wedge \neg(t \vee (u \wedge (v \vee w)))).$$

Question 15

Determine whether the following formulas are logically equivalent:

$$\big(P(a) \to (Q(a) \wedge R(b))\big) \;\vee\; (\neg R(b) \vee \neg Q(a))$$

and

$$\neg P(a) \;\vee\; Q(a) \;\vee\; \neg R(b).$$