# CS 210 Homework 2

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## Problem 1

1. $\exists o\ W(o)$
2. $\exists i\ \exists c\ U(i,c) \wedge P(i)$
3. $\exists c\ M(c) \wedge \neg F(c)$
4. $\forall c\ B(c) \wedge W(c)$
5. $\exists s\ \forall t\ G(s,t)$
6. $\forall s\ \exists c\ A(s,c) \wedge S(c) \wedge O(c)$
7. $\forall c\ \exists i\ F(i,c)$
8. $\forall d\ \exists i\ H(i,d) \wedge G(i)$
9. $\forall l\ (l \neq \text{Daniyal})\ \exists v\ P(l,v)$
10. $\forall q\ \exists t\ C(q,t) \wedge P(q)$
11. $\forall d\ \exists c\ \exists j\ T(d) \wedge B(j,c,d)$
12. $\forall c\ \forall s\ E(c,s) \rightarrow \neg L(s, \text{Starset}) \wedge \neg L(s, \text{Spiritbox})$
13. $\forall h\ \exists s\ \exists q\ H(h) \wedge A(s,q,h)$
14. $\forall i\ \exists p\ \forall s\ M(i) \rightarrow O(i,p) \wedge \neg R(s,p)$
15. $\exists c\ \forall f\ (f \neq \text{Tottenham Hotspur})\ C(c) \wedge F(f)$
16. $\forall s\ \exists c\ U(s,c) \rightarrow P(s)$

## Problem 2

1. False
   $\emptyset$ is not in $\{0\}$
2. True
   $A \subset B \equiv\ \forall x\ x \in A \rightarrow x \in B$
   Since the Empty Set has no elements to check for, it is a subset of all sets by definition.
3. False
   if $0 \in \mathbb{Z}\ 0^2 \not> 0$
4. False
   There is not integer whose square is 2
5. False
   Cartesian Product is not associative
   (Assuming that's what it meant, if it's intersection then it is true since that's associative)

# Problem 3

1.

$$
\begin{aligned}
&(P \cap Q \cap R \cap Y) \cup (P \cap R) \cup (Q \cap R) \cup (R \cap Y) \\
&= R \cap [(P \cap Q \cap Y) \cup P \cup Q \cup Y] \quad \text{(Factor out } R \text{ from each term)} \\
&= R \cap (P \cup Q \cup Y) \quad \text{(Since } (P \cap Q \cap Y) \subseteq P \cup Q \cup Y\text{)} \\
&= R \quad \text{(Because } R \cap (P \cup Q \cup Y) = R \text{ when } R \subseteq P \cup Q \cup Y\text{)}
\end{aligned}
$$

# Problem 4

1.

$$
\begin{aligned}
(X \setminus Y) \setminus Z &= (X \cap \overline{Y}) \cap \overline{Z} \quad \text{(since } A \setminus B \equiv A \cap \overline{B}\text{)} \\
&= X \cap \overline{Y} \cap \overline{Z} \\
&= X \cap \overline{(Y \cup Z)} \quad \text{(De Morgan's law)} \\
&= X \setminus (Y \cup Z)
\end{aligned}
$$

| $x \in X$ | $x \in Y$ | $x \in Z$ | $x \in (X \setminus Y) \setminus Z$ | $x \in X \setminus (Y \cup Z)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |

2.

$$
\begin{aligned}
\overline{(\overline{X \cap Y}) \cup (X \cap \overline{Z})} &= \overline{\overline{X \cap Y} \cup (X \cap \overline{Z})} \\
&= \overline{\overline{X \cap Y}} \cap \overline{X \cap \overline{Z}} \quad \text{(De Morgan)} \\
&= (X \cap Y) \cap \overline{X \cap \overline{Z}} \quad \text{(Double Negation)} \\
&= (X \cap Y) \cap (\overline{X} \cup Z) \quad \text{(De Morgan)} \\
&= (X \cap Y \cap \overline{X}) \cup (X \cap Y \cap Z) \quad \text{(Distributive Law)} \\
&= \emptyset \cup (X \cap Y \cap Z) \quad \text{(Since } X \cap \overline{X} = \emptyset\text{)} \\
&= X \cap Y \cap Z
\end{aligned}
$$

| $x \in X$ | $x \in Y$ | $x \in Z$ | $\overline{X \cap Y}$ | $X \cap \overline{Z}$ | LHS | RHS |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 |

## Promblem 5

1.

$$\begin{aligned} D \times E \times F = \{\, &(p,7,r),(p,7,s),(p,7,t),\\ &(p,8,r),(p,8,s),(p,8,t),\\ &(q,7,r),(q,7,s),(q,7,t),\\ &(q,8,r),(q,8,s),(q,8,t)\,\} \end{aligned}$$

2.

$$\begin{aligned} E \times E \times D = \{\, &(7,7,p),(7,7,q),\\ &(7,8,p),(7,8,q),\\ &(8,7,p),(8,7,q),\\ &(8,8,p),(8,8,q)\,\} \end{aligned}$$

3.

$$\begin{aligned} D \times D \times D = \{\, &(p,p,p),(p,p,q),\\ &(p,q,p),(p,q,q),\\ &(q,p,p),(q,p,q),\\ &(q,q,p),(q,q,q)\,\} \end{aligned}$$

## Problem 6

1.

$$\begin{aligned} \emptyset &= \text{the empty set}\\ \mathcal{P}(\emptyset) &= \{\emptyset\}\\ A &= \mathcal{P}(\mathcal{P}(\emptyset)) = \mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\} \end{aligned}$$

Now compute:

$$\begin{aligned} B &= \mathcal{P}(A) = \mathcal{P}(\{\emptyset, \{\emptyset\}\})\\ &= \left\{ \begin{array}{l} \emptyset,\\ \{\emptyset\},\\ \{\{\emptyset\}\},\\ \{\emptyset, \{\emptyset\}\} \end{array} \right\} \end{aligned}$$

Define:

$$\begin{aligned} b_1 &= \emptyset\\ b_2 &= \{\emptyset\}\\ b_3 &= \{\{\emptyset\}\}\\ b_4 &= \mathcal{P}(A) = \{\emptyset, \{\emptyset\}\} \end{aligned}$$

So we can write:

$$B = \{b_1, b_2, b_3, b_4\}$$

Now compute:

$$C = \mathcal{P}(B) = \left\{ \begin{array}{l} \emptyset, \\ \{b_1\}, \{b_2\}, \{b_3\}, \{b_4\}, \\ \{b_1, b_2\}, \{b_1, b_3\}, \{b_1, b_4\}, \{b_2, b_3\}, \{b_2, b_4\}, \{b_3, b_4\}, \\ \{b_1, b_2, b_3\}, \{b_1, b_2, b_4\}, \{b_1, b_3, b_4\}, \{b_2, b_3, b_4\}, \\ \{b_1, b_2, b_3, b_4\} \end{array} \right\}$$