بسم الله الرحمن الرحيم

CS-323 Artificial Intelligence

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Lecture 14

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Probabilistic Reasoning

Chapter 13

(Continued - - -)

Total Probability Theorem

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

The result is extendable to any number of events.

$$\boxed{P(B) = \sum_k P(B|A_k)P(A_k)}$$

Joint Probability Distribution

- Captures probabilities of all possible combinations of a set of random variables.
- Also known as *probabilistic model*.
- E.g. $\mathbf{P}(MyGradeInAI, MyGradeInCA)$

$MyGradeInAI$	$MyGradeInCA$	$\mathbf{P}(MyGradeInAI, MyGradeInCA)$
a	a	0.10
a	b	0.05
$\vdots$	$\vdots$	$\vdots$
d	d	0.00

Example

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Toothache	*Cavity*	*Catch*	**P**(*Toothache*, *Cavity*, *Catch*)
yes	*yes*	*yes*	**0.108**
yes	*yes*	*no*	**0.012**
yes	*no*	*yes*	**0.016**
yes	*no*	*no*	**0.064**
no	*yes*	*yes*	**0.072**
no	*yes*	*no*	**0.008**
no	*no*	*yes*	**0.144**
no	*no*	*no*	**0.576**

Example

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

$P(toothache \lor catch) =$ 0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144 = 0.416

$P(toothache \land cavity) =$ 0.108 + 0.012 = 0.120

$P(toothache) =$ 0.108 + 0.012 + 0.016 + 0.064 = 0.200

Marginal Probability

Marginalization $\boxed{\mathbf{P}(\mathbf{Y}) = \sum_{\mathbf{z} \in \mathbf{Z}} \mathbf{P}(\mathbf{Y}, \mathbf{z})}$

Conditioning $\boxed{\mathbf{P}(\mathbf{Y}) = \sum_{\mathbf{z}} \mathbf{P}(\mathbf{Y} \mid \mathbf{z}) P(\mathbf{z})}$

Example

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$\mathrm{P}(cavity \mid toothache) = \frac{P(cavity, toothache)}{P(toothache)} = \frac{0.108 + 0.012}{0.200} = 0.60$$

$$\mathrm{P}(\neg cavity \mid toothache) = \frac{P(\neg cavity, toothache)}{P(toothache)} = \frac{0.016 + 0.064}{0.200} = 0.40$$

$$\mathbf{P}(Cavity \mid toothache) = \langle 0.60, 0.40 \rangle$$

$$\mathbf{P}(Cavity \mid toothache) = \boxed{\frac{1}{P(toothache)}} \langle 0.120, 0.080 \rangle = \langle 0.60, 0.40 \rangle$$

α: Normalization Constant

General Inference Procedure

- Let,
  - X: query variable
  - E: set of evidence variables
  - e: observed values for E
  - Y: set of unobserved variables (also known as hidden variables)
  - query: $\mathbf{P}$(X | e)

$$\boldsymbol{P}(X|e) = \alpha \boldsymbol{P}(X, e) = \alpha \sum_{y} \boldsymbol{P}(X, e, y)$$

Table size for a full joint probability distribution of *n* variables each having *k* values? $\boxed{k^n}$

Reducing the Size of Domain Representation

- Let's add another random variable *Weather* to our dentistry example.
- Suppose *Weather*'s domain is {*sunny*, *rainy*, *fog*, *cloudy*}.
- Size of full joint probability distribution table?
  - ✓2 * 2 * 2 * 4 = 32

- $\mathbf{P}(Weather \mid Toothache, Cavity, Catch) = \mathbf{P}(Weather)$
- $\mathbf{P}(Weather, Toothache, Cavity, Catch)$

$$= \mathbf{P}(Weather \mid Toothache, Cavity, Catch)\ \mathbf{P}(Toothache, Cavity, Catch)$$

$$= \mathbf{P}(Weather)\ \mathbf{P}(Toothache, Cavity, Catch)$$

Reducing the Size of Domain Representation