Database Management System

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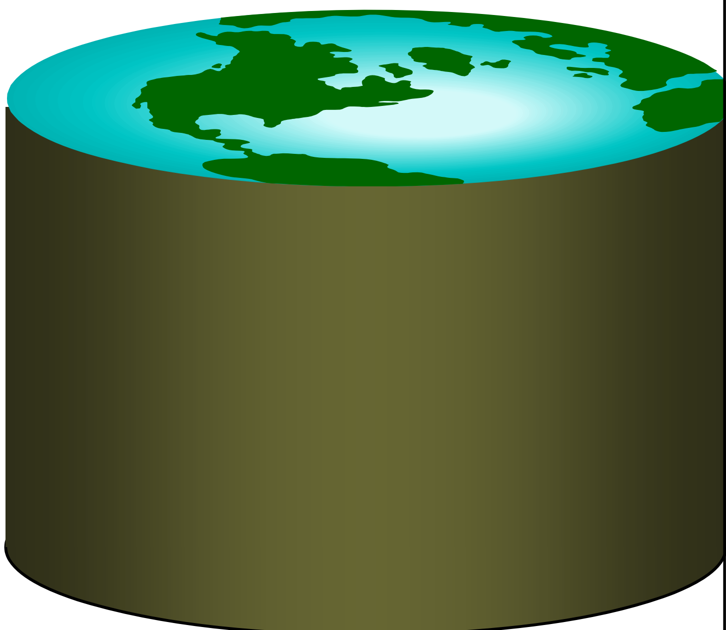
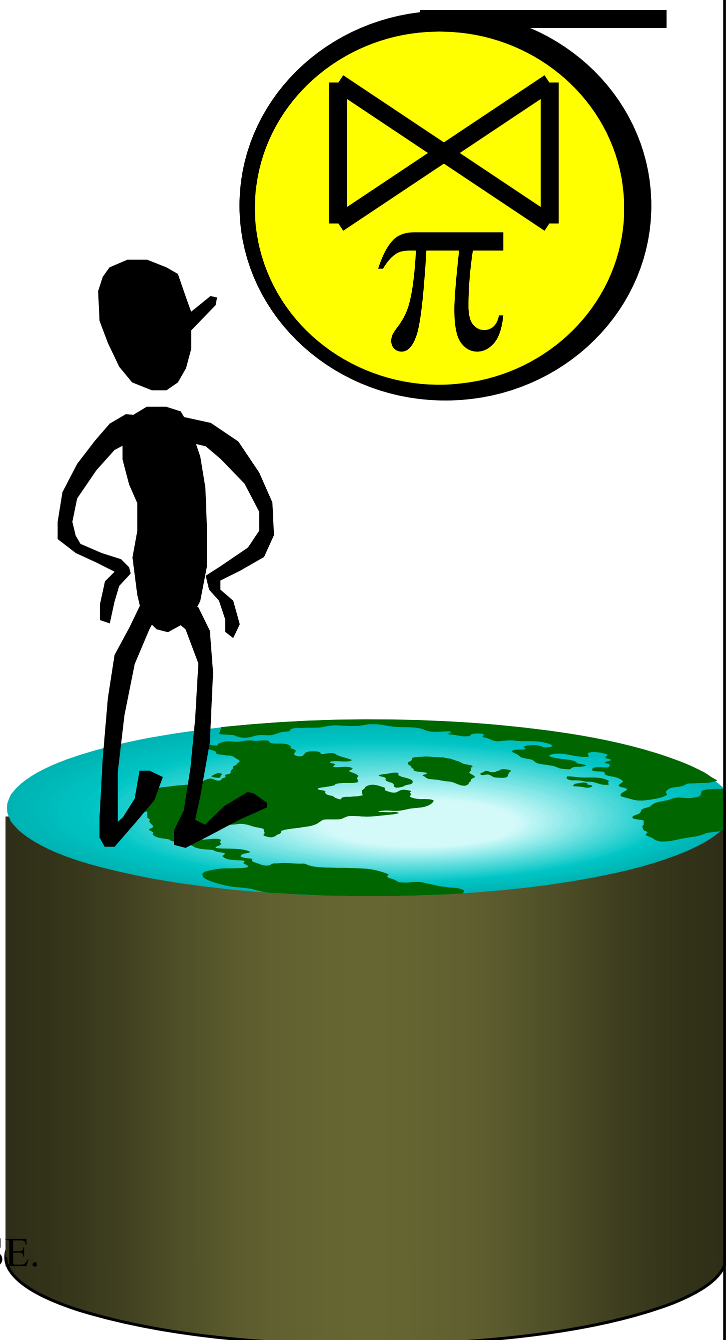


Objectives

- **Relational Algebra**
- **Relational Calculus**

Relational Algebra



By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, *increases the mental power of the race.*

-- Alfred North Whitehead (1861 - 1947)

Formal Relational Query Languages

Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:

***Relational Algebra*: More operational, very useful for representing execution plans.**

***Relational Calculus*: Lets users describe what they want, rather than how to compute it. (even more *declarative*.)**

- *Understanding Algebra & Calculus is key to understanding SQL, query processing!*

Example: Joining Two Tables

Enrolled

cid	grade	sid
Carnatic101	C	53666
Reggae203	B	53666
Topology112	A	53650
History105	B	53666

Students

sid	name	login	age	gpa
53666	Jones	jones@cs	18	3.4
53688	Smith	smith@eecs	18	3.2
53650	Smith	smith@math	19	3.8

- **Pre-Relational:**
  write a program
- **Relational SQL**
  Select name, cid from students s, enrolled e where s.sid = e.sid
- **Relational Algebra**

$$\pi_{name,cid}(Students \bowtie_{sid} Enrolled)$$

- **Relational Calculus**
  {S | S∈Students ∧ ∃E(E∈Enrolled ∧ E.sid = S.sid)}

Relational Algebra: 5 Basic Operations

- *Selection* (σ) Selects a subset of ***rows*** from relation (horizontal).
- *Projection* (π) Retains only wanted ***columns*** from relation (vertical).
- *Cross-product* ($\times$) Allows us to combine two relations.
- *Set-difference* (—) Tuples in r1, but not in r2.
- *Union* ($\cup$) Tuples in r1 and/or in r2.

Since each operation returns a relation, operations can be *composed!* (Algebra is "closed".)

Example Instances

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Boats

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Projection

- Examples: $\pi_{age}(S2)$; $\pi_{sname,rating}(S2)$
- Retains only attributes that are in the "*projection list*".
- *Schema* of result:
  - exactly the fields in the projection list, with the same names that they had in the input relation.
- Projection operator has to *eliminate duplicates* (How do they arise? Why remove them?)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)

Projection

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$$\pi_{sname,rating}(S2)$$

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

age
35.0
55.5

$$\pi_{age}(S2)$$

Selection (σ)

- Selects rows that satisfy *selection condition*.
- Result is a relation.

  Schema of result is same as that of the input relation.
- Do we need to do duplicate elimination?

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$$\sigma_{rating>8}(S2)$$

sname	rating
yuppy	9
rusty	10

$$\pi_{sname,rating}(\sigma_{rating>8}(S2))$$

Union and Set-Difference

- All of these operations take two input relations, which must be ***union-compatible*:**
  - Same number of fields.
  - `Corresponding' fields have the same type.
- For which, if any, is duplicate elimination required?

Union

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$$S1 \cup S2$$

Set Difference

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
22	dustin	7	45.0

$S1 - S2$

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
44	guppy	5	35.0

$S2 - S1$

Cross-Product

- S1 × R1: Each row of S1 paired with each row of R1.
- Q: How many rows in the result?
- *Result schema* has one field per field of S1 and R1, with field names \`inherited' if possible.
  - *May have a naming conflict*: Both S1 and R1 have a field with the same name.
  - In this case, can use the *renaming operator*:

$$\rho\,(C(1 \to sid1, 5 \to sid2),\ S1 \times R1)$$

Cross Product Example

sid	bid	day
22	101	10/10/96
58	103	11/12/96

R1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

R1 X S1 =

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

Compound Operator: Intersection

- In addition to the 5 basic operators, there are several additional "Compound Operators"
  - These add no computational power to the language, but are useful shorthands.
  - Can be expressed solely with the basic ops.

- Intersection takes two input relations, which must be ***union-compatible*.**
- Q: How to express it using basic operators?

$$R \cap S = R - (R - S)$$

Intersection

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$$S1 \cap S2$$

Compound Operator: Join

- Joins are compound operators involving cross product, selection, and (sometimes) projection.
- Most common type of join is a "*natural join*" (often just called "join"). R$\bowtie$S conceptually is:
  - Compute R $\times$ S
  - Select rows where attributes that appear in both relations have equal values
  - Project all unique atttributes and one copy of each of the common ones.
- Note: Usually done much more efficiently than this.
- Useful for putting "normalized" relations back together.

Natural Join Example

sid	bid	day
22	101	10/10/96
58	103	11/12/96

R1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

$$\mathbf{R1} \bowtie \mathbf{S1} =$$

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

Other Types of Joins

- ***Condition Join (or "theta-join")*:**

$$R \bowtie_{c} S = \sigma_{c}(R \times S)$$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

compare ids of both like cross product

- *Result schema* same as that of cross-product.
- May have fewer tuples than cross-product.
- ***Equi-Join*: Special case: condition *c* contains only conjunction of *equalities*.**

Compound Operator: Division

- Useful for expressing "for all" queries like: *Find sids of sailors who have reserved all boats*.
- For A/B attributes of B are subset of attrs of A.
  - May need to "project" to make this happen.
- E.g., let *A* have 2 fields, *x* and *y*; *B* have only field *y*:

$$A/B = \left\{ \langle x \rangle \middle| \forall \langle y \rangle \in B (\exists \langle x, y \rangle \in A) \right\}$$

***A/B* contains all tuples (x) such that for *every y* tuple in *B*, there is an *xy* tuple in *A*.**

Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

A

pno
p2

B1

pno
p2
p4

B2

pno
p1
p2
p4

B3

sno
s1
s2
s3
s4

A/B1

sno
s1
s4

A/B2

sno
s1

A/B3

Expressing A/B Using Basic Operators

- **Division is not essential op; just a useful shorthand.**
  - (Also true of joins, but joins are so common that systems implement joins specially.)
- ***Idea*: For *A/B*, compute all *x* values that are not `disqualified' by some *y* value in *B*.**
  - *x* value is *disqualified* if by attaching *y* value from *B*, we obtain an *xy* tuple that is not in *A*.

Disqualified *x* values: $\pi_{x}((\pi_{x}(A) \times B) - A)$

A/B: $\pi_{x}(A) -$ Disqualified *x* values

Examples

Reserves

sid	bid	day
22	101	10/10/96
58	103	11/12/96

Sailors

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Boats

bid	bname	color
101	Interlake	Blue
102	Interlake	Red
103	Clipper	Green
104	Marine	Red

E1: Find names of sailors who've reserved boat #103

- **Solution 1:** $\pi_{sname}((\sigma_{bid=103} \text{Reserves}) \bowtie Sailors)$
- **Solution 2:** $\pi_{sname}(\sigma_{bid=103}(\text{Reserves} \bowtie Sailors))$

E2: Find names of sailors who've reserved a red boat

- **Information about boat color only available in Boats; so need an extra join:**

$$\pi_{sname}((\sigma_{color='red'}\, Boats) \bowtie Reserves \bowtie Sailors)$$

- A more efficient solution:

$$\pi_{sname}(\pi_{sid}((\pi_{bid}\,\sigma_{color='red'}\, Boats) \bowtie Res) \bowtie Sailors)$$

- *A query optimizer can find this given the first solution!*

E3: Find sailors who've reserved a red or a green boat

- **Can identify all red or green boats, then find sailors who've reserved one of these boats:**

$$\rho\ (Tempboats, (\sigma_{color='red' \vee color='green'}\ Boats))$$

$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

E4: Find sailors who've reserved a red <u>and</u> a green boat

- **Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that *sid* is a key for Sailors):**

$$\rho\ (Tempred, \pi_{sid}((\sigma_{color='red'} Boats) \bowtie \text{Reserves}))$$

$$\rho\ (Tempgreen, \pi_{sid}((\sigma_{color='green'} Boats) \bowtie \text{Reserves}))$$

$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$

E5: Find the names of sailors who’ve reserved all boats

- **Uses division; schemas of the input relations to / must be carefully chosen:**

$$\rho\ (Tempsids, (\pi_{sid,bid} Reserves) \ / \ (\pi_{bid} Boats))$$

$$\pi_{sname} (Tempsids \bowtie Sailors)$$

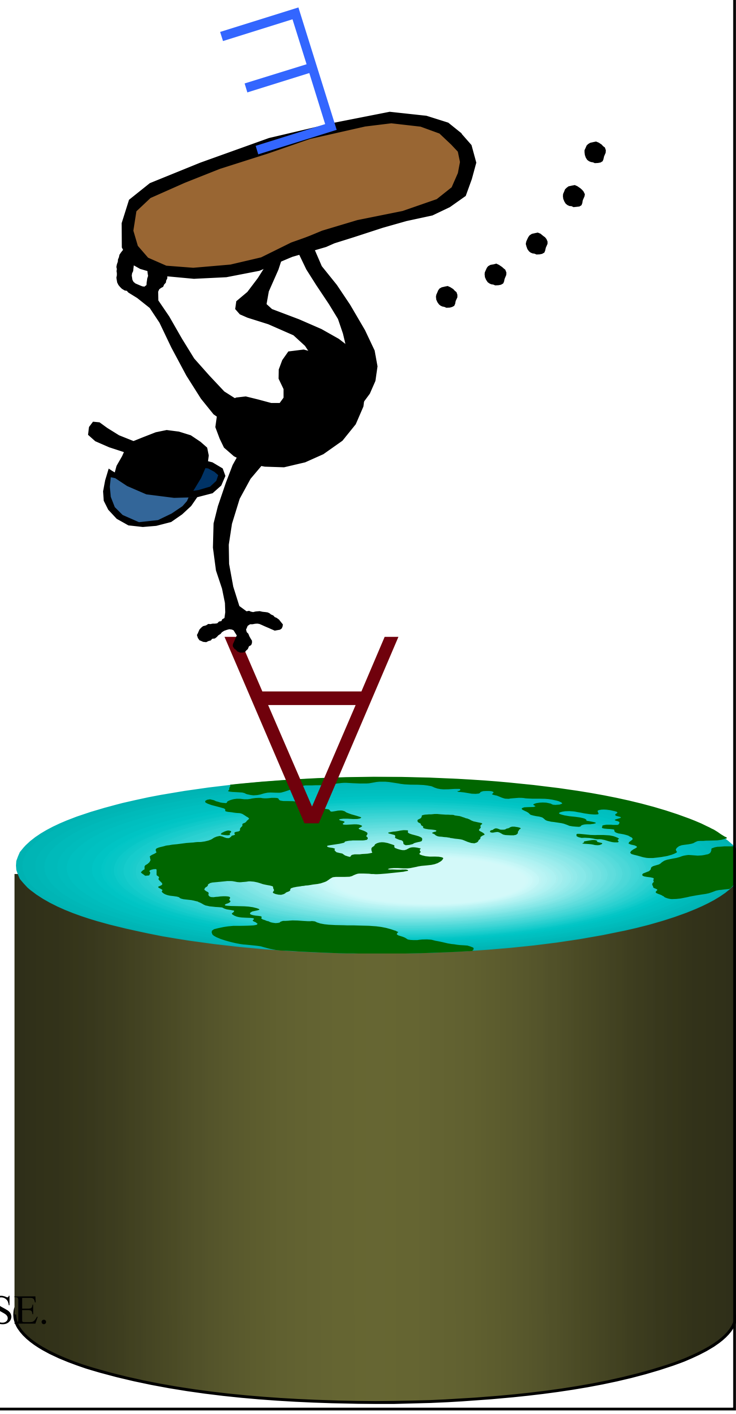
- To find sailors who’ve reserved all ‘Interlake’ boats:

$$..... \quad / \ \pi_{bid} (\sigma_{bname='Interlake'} Boats)$$

Summary

- **Relational Algebra: a small set of operators mapping relations to relations**
  - Operational, in the sense that you specify the explicit order of operations
  - A *closed* set of operators! Can mix and match.
- **Basic ops include:** σ, π, $\times$, $\cup$, **—**
- **Important compound ops:** $\cap$, $\bowtie$, **/**

Relational Calculus



We will occasionally use this arrow notation unless there is danger of no confusion.

-- Ronald Graham (Elements of Ramsey Theory)

Relational Calculus

- **High-level, first-order logic description**

- **English:**
  Find all sailors with a rating above 7

- **More logical English:**
  From the universe of all things, find me the set of things that happen to be tuples in the Sailors relation and whose rating field is a number greater than 7.

- **Relational Calculus (TRC)**
  $\{S \mid S \in Sailors \wedge S.rating > 7\}$

So what is Relational Calculus?

- **A formal, logical description, of what you want from the database.**

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Relational Calculus

- **Comes in two flavors:**
  - *Tuple relational calculus* (TRC)
  - *Domain relational calculus* (DRC)

- **English Example: Find all sailors with a rating above 7**
  - Tuple R.C.: From the universe of all things, find me the set of things that happen to be tuples in the Sailors relation and whose rating field is a number greater than 7.

    $\{S \mid S \in \textit{Sailors} \wedge S.\textit{rating} > 7\}$

  - Domain R.C.: From the universe of all things, find me S, N, R, and A, where S is an integer, N is a string, R is an integer greater than 7, and A is a floating point number, and <S, N, R, A> is a tuple in the Sailors relation.

    sid,sname,rating,age

    $\{\langle S,N,R,A\rangle \mid \langle S,N,R,A\rangle \in \textit{Sailors} \wedge R > 7\}$

Relational Calculus (cont.)

- **Calculus has**
  - *variables*
    - *TRC*: Variables range over (i.e., get bound to) *tuples*. Like SQL.
    - *DRC*: Variables range over *domain elements* (= field values). Like Query-By-Example (QBE)
  - *constants,* e.g.7, “Foo”, 3.14159, etc.
  - *comparison ops,* e.g. =, <>, <, >, etc.
  - *logical connectives*

    $\neg$ - not

    $\wedge$ – and

    $\vee$ - or

    $\Rightarrow$- implies

    $\in$ - is a member of
  - *quantifiers*

    $\forall X(p(X))$ – $p(X)$ must be true for every X

    $\exists X(p(X))$ – $p(X)$ is true for some X

- **Both TRC and DRC are simple subsets of first-order logic. We’ll focus on TRC here**

Tuple Relational Calculus

- ***Query*** **has the form: {*T* | *p*(*T*)}**
  - ***p(T)*** denotes a formula in which tuple variable ***T*** appears.

- ***Answer*** **is the set of all tuples *T* for which the *formula p*(*T*) evaluates to *true*.**

- ***Formula*** **is recursively defined:**
  - start with simple *atomic formulas* (get tuples from relations or make comparisons of values)
  - build bigger and better formulas using the *logical connectives*.

TRC Formulas

- ***An Atomic formula is one of the following:***

  R $\in$ *Rel*

  R.a op S.b

  R.a op constant

  op is one of $<, >, =, \leq, \geq, \neq$

- **A *formula* can be:**
  - an atomic formula
  - $\neg p, p \wedge q, p \vee q$ where p and q are formulas
  - $\exists R(p(R))$ where variable *R* is a tuple variable
  - $\forall R(p(R))$ where variable *R* is a tuple variable

Free and Bound Variables

- **The use of quantifiers $\forall$X and $\exists$X in a formula is said to *bind* X in the formula.**
  - A variable that is not bound is free.

- **Let us revisit the definition of a query:**
  - $\{T \mid p(T)\}$

- **There is an important restriction**
  - the variable T that appears to the left of \`|' must be the *only* free variable in the formula $p(T)$.
  - in other words, all other tuple variables must be bound using a quantifier.

Use of $\forall$

- $\forall x\ (P(x))$ - is only true if P(x) is true for *every* x in the universe
- Usually:

  $\forall \mathbf{x}\ ((\mathbf{x} \in \textbf{Boats}) \Rightarrow (\mathbf{x.color} = \textbf{“Red”}))$

- $\Rightarrow$ logical implication,

  a $\Rightarrow$ **b** means that if a is true, b must be true

  a $\Rightarrow$ **b** is the same as $\neg a \vee b$

$a \Rightarrow b$ is the same as $\neg a \lor b$

		b: T	b: F
a	T	T	F
a	F	T	T

- **If a is true, b must be true!**
  - If a is true and b is false, the implication evaluates to false.
- **If a is not true, we don't care about b**
  - The expression is always true.

Quantifier Shortcuts

$$\forall x\ ((x \in \text{Boats}) \Rightarrow (x.\text{color} = \text{“Red”}))$$

can also be written as:

$$\forall x \in \text{Boats}(x.\text{color} = \text{“Red”})$$

$$\exists x\ ((x \in \text{Boats}) \wedge (x.\text{color} = \text{“Red”}))$$

can also be written as:

$$\exists x \in \text{Boats}(x.\text{color} = \text{“Red”})$$

Selection and Projection

- **Find all sailors with rating above 8**

$$\{S \mid S \in Sailors \ \wedge S.rating > 8\}$$

- **Find names and ages of sailors with rating above 8.**

$$\{S \mid \exists S1 \in Sailors(S1.rating > 8 \\ \wedge S.sname = S1.sname \\ \wedge S.age = S1.age)\}$$

  – Note: *S* is a tuple variable of 2 fields (i.e. {S} is a projection of S*ailors)*
    - only 2 fields are ever mentioned and *S* is never used to range over any relations in the query.

Joins

Find sailors rated > 8 who've reserved boat #103

$$\{S \mid S \in Sailors \wedge S.rating > 8 \wedge \\ \exists R(R \in Reserves \wedge R.sid = S.sid \\ \wedge R.bid = 103)\}$$

Note the use of $\exists$ to find a tuple in Reserves that `joins with' the Sailors tuple under consideration.

Joins (continued)

$$\{S \mid S \in Sailors \wedge S.rating > 8 \wedge \exists R(R \in Reserves \wedge R.sid = S.sid \wedge \exists B(B \in Boats \wedge B.bid = R.bid \wedge B.color = \text{'red'}))\}$$

Find sailors rated > 8 who've reserved a red boat

- **Observe how the parentheses control the scope of each quantifier's binding.**
- **This may look cumbersome, but it's not so different from SQL!**

Division (makes more sense here???)

Find sailors who've reserved all boats
(hint, use $\forall$)

$$\{S \mid S \in Sailors \wedge \forall B \in Boats\ (\exists R \in Reserves\ (S.sid = R.sid \wedge B.bid = R.bid))\}$$

- **Find all sailors *S* such that for all tuples *B* in Boats there is a tuple in Reserves showing that sailor *S* has reserved *B*.**

Exercises

- Find the names of sailors who've reserved boat #103

$$\{N \mid \exists S \in Sailors\ (S.name = N.name \wedge \exists R \in Reserves(S.sid = R.sid \wedge R.bid = 103))\}$$

Exercises

- Find the names of sailors who've reserved any red boat

$$\{N \mid \exists S \in Sailors\ (S.name = N.name \wedge \exists R \in Reserves\ (S.sid = R.sid \wedge \exists B \in Boats(B.color = \text{"Red"} \wedge B.bid = R.bid)))\}$$

Exercises

- Find sailors who've reserved a red boat or a green boat

$$\{S \mid S \in Sailors \wedge$$
$$(\exists R \in Reserves$$
$$(S.sid = R.sid \wedge$$
$$\exists B \in Boats(B.bid = R.bid \wedge$$
$$(B.color = \text{“Red”} \vee$$
$$B.color = \text{“Green”}))))\}$$

Exercises

- Find sailors who've reserved a red boat <u>and</u> a green boat

$$
\begin{aligned}
\{S \mid\ & S \in Sailors \wedge \\
& ((\exists R \in Reserves \\
& \quad (S.sid = R.sid \wedge \\
& \quad \exists B \in Boats(B.color = \text{“Red”} \wedge B.bid = R.bid))) \\
& \wedge \\
& (\exists R \in Reserves \\
& \quad (S.sid = R.sid \wedge \\
& \quad \exists B \in Boats(B.color = \text{“Green”} \wedge B.bid = R.bid))))\}
\end{aligned}
$$

Exercises

- Find sailors who've reserved all red boats

$$\{S \mid S \in Sailors \wedge \forall B \in Boats(B.color = \text{'red'} \Rightarrow \exists R(R \in Reserves \wedge S.sid = R.sid \wedge B.bid = R.bid))\}$$

$$\{S \mid S \in Sailors \wedge \forall B \in Boats\ (B.color \neq \text{'red'} \vee \exists R(R \in Reserves \wedge S.sid = R.sid \wedge B.bid = R.bid))\}$$

Unsafe Queries, Expressive Power

- **∃ syntactically correct calculus queries that have an infinite number of answers!** ***Unsafe*** **queries.**
  - e.g., $\{S \mid \neg(S \in Sailors)\}$
  - Solution???? Don't do that!

- **Expressive Power (Theorem due to Codd):**
  - every query that can be expressed in relational algebra can be expressed as a *safe* query in DRC / TRC; the converse is also true.

- ***Relational Completeness*:** Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus. (actually, SQL is more powerful, as we will see…)

Summary

- **The relational model has rigorously defined query languages — simple and powerful.**
- **Relational algebra is more operational**
  - useful as internal representation for query evaluation plans.
- **Relational calculus is non-operational**
  - users define queries in terms of what they want, not in terms of how to compute it. (*Declarative*)
- **Several ways of expressing a given query**
  - a *query optimizer* should choose the most efficient version.
- **Algebra and safe calculus have same *expressive power***
  - leads to the notion of *relational completeness*.