Tutorial Problem 1: Question and Answer

On a new game show, the contestant who reaches the final stage of the game is confronted with a challenge: they must correctly guess the answer to a True/False question. If they answer correctly, they must also answer another question of the same nature, and then a third question after that. If all three questions are answered correctly, they win the grand prize.

1. Suppose that the questions are so obscure that the contestant can only guess the answers randomly. Under this assumption, what is the probability that a contestant guesses the correct answer to a given question?

2. Under the assumption above, what is the probability that the finalist wins the grand prize (on a single episode)?

After watching the first season of the show (45 episodes), you noticed that only one contestant won (on episode #34), and become suspicious. You decide to investigate using statistical inference, acknowledging that because you formed your suspicions after seeing the result, this will be an *a posteriori* calculation and thus not following all the rules of statistical inference.

3. Come up with a *test statistic*, an easy-to-measure single number that reflects how unusual (or not) the outcome of the whole "experiment" (season) was. Describe it in one sentence, and state its value given the results above.

4. Framing the situation in a similar way as the coin flip example from class, come up with a *null hypothesis*: a specific mathematical statement expressed in words or equations about the nature of the system under the least interesting possibility that might be true.

5. Also describe the *alternative hypothesis*, a mathematical statement about what scenarios exclusive of the null hypothesis would be considered interesting to you.

6. Calculate the probability that a result (test statistic) "as or more extreme" than the actual result would result from the scenario under the null hypothesis.

7. Based on your result from Part 6, draw a conclusion about the null hypothesis, and about the alternative hypothesis.

8. The game show is renewed for a second season. This season will be another 45 episodes in duration. What possible outcomes for Season 2 would confirm your conclusion from Question 7 above? If this happens, would you still be concerned that you are not strictly following the rules of statistical inference?

Tutorial Problem 2: Probability Distributions

1. Plot each of the following probability density functions. Use an x-axis that spans at least from the minimum value to the maximum value, with an extra margin on either side.

PDF1(x) =	1	(if 0 ≤ x ≤ 1)
	0	(otherwise)
PDF2(x) =	e^{-x}	(if x > 0)
	0	(otherwise)
PDF3(x) =	0.1	(if 0 ≤ x ≤ 2, or if 4 ≤ x ≤ 7, or if 10 ≤ x ≤ 13)
	0	(otherwise)
PDF4(x) =	$2x^{-3}$	(if x > 1)
	0	(otherwise)

2. What is the value of the integral from negative to positive infinity of each of these four functions? *Hint: It should be possible to answer this question without performing any calculations.*

3. Analytically determine the equation(s) of the cumulative distribution function (CDF) for each of the four PDFs above. Make sure that your results are defined for every possible value of x.

4. Plot each of the four CDFs you derived in Part 3. Confirm graphically that they obey the "rules" for CDFs: specifically, the range is zero to one, and the function does not decrease.

5. For each of the above PDFs, what is the probability of drawing a value between 0.5 and 1.5?

6. Which of the following statements are **true**, and which are **false**?
   (a) The mean of PDF1 is 0.5.
   (b) The median of PDF2 is ln(2).
   (c) The first quartile of PDF3 is 4.5.
   (d) The mode of PDF4 is 2.

Tutorial Problem 3: Standardized Test

A particular standardized test taken by hundreds of thousands of primary school pupils has an average mark of 50 and a standard deviation of 10. The distribution of marks can be treated as normal.

1. What proportion of pupils scored better (more) than 75?

2. What proportion of pupils scored between 52 and 58?

3. Among the pupils who scored better than 75, what proportion of them scored better than 80?

4. What mark value did exactly 1% of the pupils exceed?

5. A group of 12 randomly-selected pupils was put through an experimental after-school enrichment programme before taking the test. For this group, the average mark was 56. Assuming that the programme produces no change in variance, is the improvement significant? Give both a yes or no answer, and a p-value.

The academic standards board is developing a new version of the test intended to replace the old one. The distribution of marks for this version is unknown, but is expected to be normal in shape. A group of 25 randomly selected students are chosen to trial the test, and they receive an average mark of 46 with a standard deviation of 7.

6. Provide a 95% confidence interval on the population average for the new test.

7. Can you rule out the possibility that the average mark is the same as it is on the old test (50)? Quote a p-value.

8. You would like a more precise estimate, such that the difference between the upper and lower limits of the 95% confidence interval is (about) 2. How many more students do you need to take the test to expect to achieve this?