Computer Lab 8: Model Choice and Multiple Regression

Complete all of the following questions, adding your inputs as code chunks (enclose within triple accent marks) within Rmarkdown.

The exercises are not marked and will not be factored into your course grade, but it is important to complete them to make sure you have the skills to answer assessment questions. You may consult any resource, including other students and the instructor. Please Knit this document to a PDF and upload your work via Canvas at the end of the session. Solutions will be posted for you to check your own answers.

Nested models

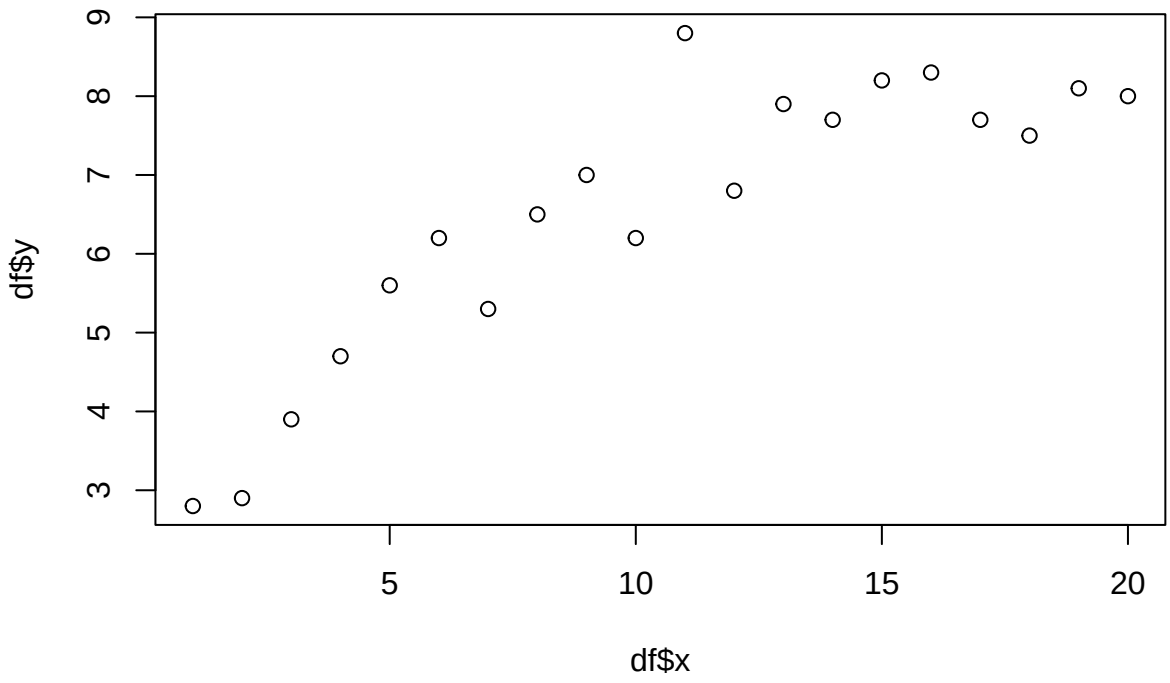
The file curve.csv contains some simple data (x versus y). You are trying to decide between two competing, nested non-linear models for these data:
Model 1: $y = ax^b$
Model 2: $y = ax^b e^{-(x/c)^2}$

1. Load the file and plot the data as a simple scatter plot.

```
df=read.csv('curve.csv')
df
```

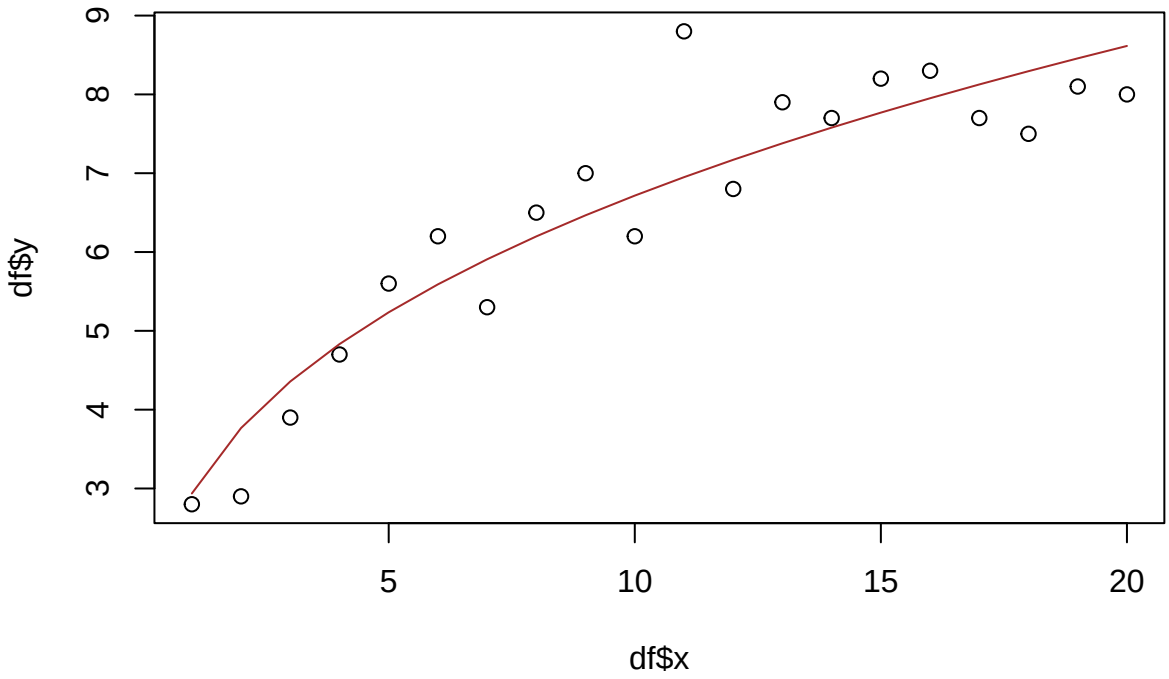
```
##     x   y
## 1   1 2.8
## 2   2 2.9
## 3   3 3.9
## 4   4 4.7
## 5   5 5.6
## 6   6 6.2
## 7   7 5.3
## 8   8 6.5
## 9   9 7.0
## 10 10 6.2
## 11 11 8.8
## 12 12 6.8
## 13 13 7.9
## 14 14 7.7
## 15 15 8.2
## 16 16 8.3
## 17 17 7.7
## 18 18 7.5
## 19 19 8.1
## 20 20 8.0
```

```r
plot(df$x,df$y)
```



2. Using nonlinear least squares (`nls` in R), fit Model 1 to the data (store the output model in variable model1). Overplot the model curve on the data (use `predict`).

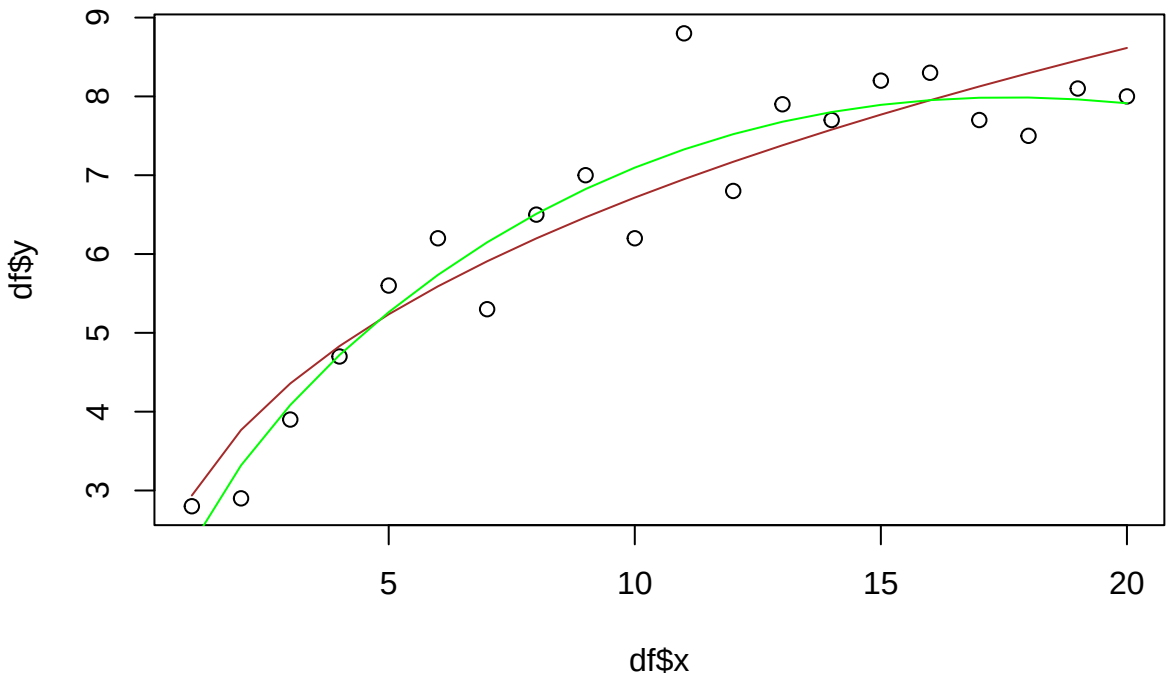
```r
model1 = nls(y~ a*x^b, data=df, start=list(a=1,b=0.5))
plot(df$x,df$y)
lines(df$x,predict(model1),col='brown')
```



3. Using nonlinear least squares, fit Model 2 to the data (store the output model in variable model2). Plot this curve and the curve from Model 1 on the data (as different colours). (Note: if you have trouble with the fit, try using your best-fit parameters for a and b from Model 1 as starting guesses.)

```
model2 = nls(y~a*x^b*exp(-(x/c)^2), data=df, start=list(a=3,b=0.35, c=10))

plot(df$x,df$y)
lines(df$x,predict(model1),col='brown')
lines(df$x,predict(model2),col='green')
```



4. Compute the SSE of model1 yourself using the data and the model curve. (Calculate the difference between the predictions and the data, then take the sum of squares.) Store this in the variable SSE1.

```
SSE1=sum((predict(model1)-df$y)^2)
SSE1
```

```
## [1] 7.986823
```

5. Compute the SSE of model2 yourself using the data and the model curve. Store this in the variable SSE2. Confirm that it is less than SSE1, since this is a more complex model nested inside Model 1.

```
SSE2=sum((predict(model2)-df$y)^2)
SSE2
```

```
## [1] 5.632781
```

6. Take the difference of these two values (SSE1-SSE2) to calculate the improvement in SSE provided by the more complicated model. Store this in the variable DSSE.

```
DSSE=SSE1-SSE2

DSSE
```

```
## [1] 2.354042
```

7. Calculate the degrees of freedom statistics and store them in variables:
    a. the degrees of freedom of Model 1 (store it in dofm1)
    b. the degrees of freedom of Model 2 (store it in dofm2)
    c. the difference between these two (store it in Ddof)
    d. the degrees of error freedom of Model 1 (store it in dofe1)
    e. the degrees of error freedom of Model 2 (store it in dofe2)

```
n = nrow(df)
dofm1 = 2
dofm2 = 3
Ddof= dofm2-dofm1
dofe1 = n - dofm1
dofe2 = n - dofm2
dofe1; dofe2; Ddof
```

```
## [1] 18
```

```
## [1] 17
```

```
## [1] 1
```

8. Calculate SSE1/dofe1 and SSE2/dofe2. Which model appears to be better (by this metric)?

```
SSE1/dofe1
```

```
## [1] 0.4437124
```

```
SSE2/dofe2
```

```
## [1] 0.3313401
```

9. Calculate the ANOVA F-ratio: F = (DSSE/Ddof)/(SSE2/dofe2).

```
F = (DSSE/Ddof)/(SSE2/dofe2)
F
```

```
## [1] 7.10461
```

10. Perform an F-test with the appropriate numbers of freedom to calculate a p-value: Ddof for the numerator, and dofe2 for the denominator. (This tells you the probability of obtaining a reduction in SSE as larger or larger than DSSE by chance if the simpler model is the correct one.) Remember that model-choice F-tests are almost always right-tailed.

```
1-pf(F, Ddof, dofe2)
```

```
## [1] 0.01630823
```

11. Check your results for #4 through #10 above using the shortcut function `anova()`.

```
anova(model1, model2)
```

```
## Analysis of Variance Table
##
## Model 1: y ~ a * x^b
## Model 2: y ~ a * x^b * exp(-(x/c)^2)
##   Res.Df Res.Sum Sq Df Sum Sq F value  Pr(>F)
## 1     18     7.9868
## 2     17     5.6328  1  2.354  7.1046 0.01631 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

12. Compare the Akaike information criteria of the two models using `AIC()`. Does this agree with the conclusion from the F-test (ANOVA) result?

```
AIC1 = AIC(model1)
AIC2 = AIC(model2)
AIC1
```

```
## [1] 44.39876
```

```
AIC2
```

```
## [1] 39.41496
```

13. Calculate the relative likelihood ratio, exp((AIC2-AIC1)/2). Based on this, about how likely is it that Model 1 is actually the correct one, assuming that one of the two models is right and given no additional prior knowledge? (Divide the relative likelihood of Model 1 by the sum of the relative likelihoods for both models.)

```
likelihood = exp((AIC2-AIC1)/2)
likelihood
```

```
## [1] 0.08275279
```

```
likelihood/(1+likelihood)
```

```
## [1] 0.07642815
```

14. Compare the Bayesian information criteria of the two models using `BIC()` and decide which model is better. Does this agree with the conclusions from AIC and from ANOVA?

```
bic1 = BIC(model1)

bic1
```

```
## [1] 47.38595
```

```
bic2 = BIC(model2)
bic2
```

```
## [1] 43.39789
```

ANCOVA on Covariant Data

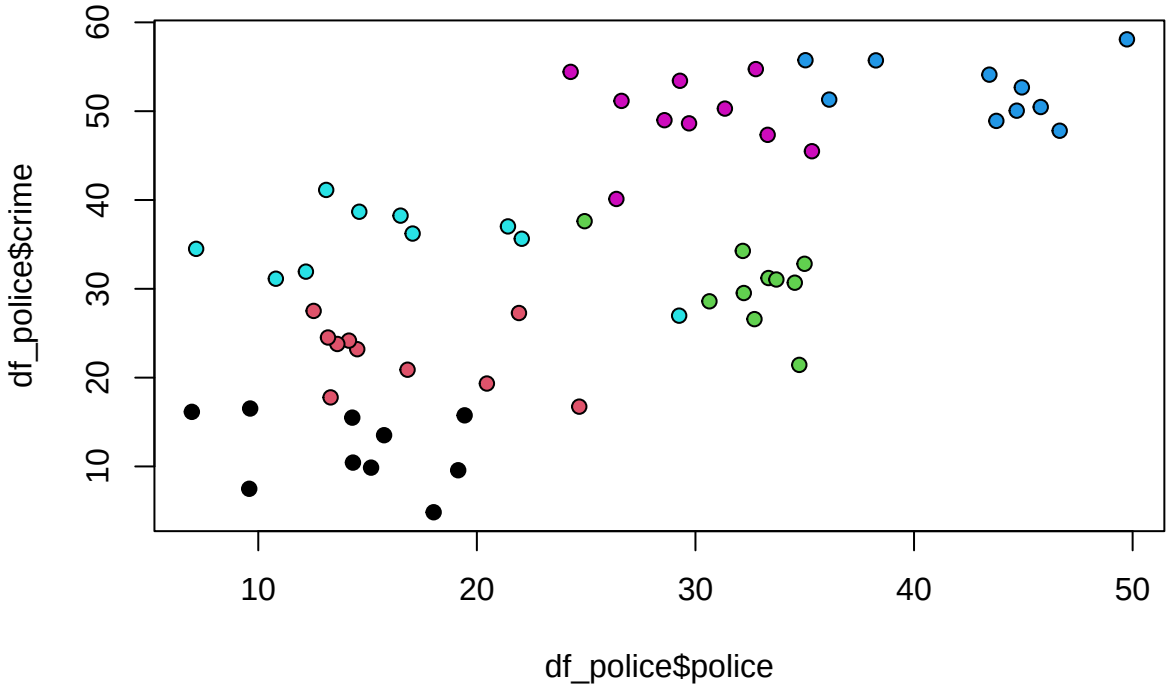
The data in police.csv are a (simulated) data set showing the number of crimes committed in individual neighbourhoods within several study zones in a city, versus the police presence in those same neighbourhoods.

15. Load the data in from disk and plot crime (y-axis) against police presence (x-axis), colour-coded by zone.

```
df_police=read.csv('police.csv', as.is=FALSE)
head(df_police)
```

```
##      police     crime zone
## 1 19.145494  9.571975    A
## 2 14.331187 10.432380    A
## 3 15.160330  9.865389    A
## 4  6.960505 16.146042    A
## 5 15.755502 13.518667    A
## 6 19.439775 15.753415    A
```

```
plot(df_police$police, df_police$crime, bg=df_police$zone, pch=21)
```



16. Fit a simple linear model for crime as a function of police (ignore zones). Examine the linear model summary. What would you conclude about the relation between police and crime based on only this information?

```
m1= lm(df_police$crime ~ df_police$police)
summary(m1)
```

```
##
## Call:
## lm(formula = df_police$crime ~ df_police$police)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -22.3597  -9.7687  -0.1187   9.8658  21.7103
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       11.3524     3.5046   3.239  0.00199 **
## df_police$police   0.8795     0.1262   6.968 3.31e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.05 on 58 degrees of freedom
## Multiple R-squared:  0.4557, Adjusted R-squared:  0.4463
## F-statistic: 48.56 on 1 and 58 DF,  p-value: 3.313e-09
```

17. Now fit a fully interacting model of crime as a function of police and zone. Use `summary.aov()` to

perform an ANOVA analysis and determine whether the interactions are really needed.

```
m2=lm(df_police$crime ~ df_police$police*df_police$zone)

summary.aov(m2)
```

```
##                                Df Sum Sq Mean Sq F value Pr(>F)
## df_police$police                1   5927    5927 364.871 <2e-16 ***
## df_police$zone                  5   6266    1253  77.150 <2e-16 ***
## df_police$police:df_police$zone 5     34       7   0.415  0.836
## Residuals                      48    780      16
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

18. Remove the interactions and fit an additive-only linear model. Examine the linear model summary. Now what would you conclude about the relation between police and crime?

```
m3=lm(df_police$crime ~ df_police$police+df_police$zone)
summary(m3)
```

```
##
## Call:
## lm(formula = df_police$crime ~ df_police$police + df_police$zone)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.2177  -2.3139   0.7978   3.0762   7.3832
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       15.6443     2.0906   7.483 7.53e-10 ***
## df_police$police  -0.2586     0.1183  -2.186   0.0333 *
## df_police$zoneB   11.1474     1.7728   6.288 6.28e-08 ***
## df_police$zoneC   23.1123     2.7728   8.335 3.27e-11 ***
## df_police$zoneD   47.9228     3.8117  12.572  < 2e-16 ***
## df_police$zoneE   23.7495     1.7709  13.411  < 2e-16 ***
## df_police$zoneF   41.5114     2.5387  16.352  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.918 on 53 degrees of freedom
## Multiple R-squared:  0.9375, Adjusted R-squared:  0.9304
## F-statistic: 132.4 on 6 and 53 DF,  p-value: < 2.2e-16
```

Multiple Regression: Artificial Data

A fake multi-parameter data set (similar to the demonstration set) is available in multireg2.csv.

19. Load this in from disk and use `summary()` and/or `head()` to briefly investigate the data set.

```
df_multipar=read.csv('multireg2.csv')
summary(df_multipar)
```

```
##        y                a                b                c
##  Min.   :0.8566   Min.   :-15.69   Min.   : 3.846   Min.   :0.0005935
##  1st Qu.:2.5842   1st Qu.: 27.17   1st Qu.:30.279   1st Qu.:0.2819484
##  Median :3.1089   Median : 50.50   Median :38.193   Median :0.5334972
##  Mean   :3.0827   Mean   : 51.11   Mean   :39.526   Mean   :0.5163659
##  3rd Qu.:3.5559   3rd Qu.: 76.09   3rd Qu.:48.237   3rd Qu.:0.7655886
##  Max.   :5.3289   Max.   :126.10   Max.   :84.484   Max.   :0.9995087
##
##        d                 e                 f                 g
##  Min.   :-92.680   Min.   :-79.509   Min.   :-24.193   Min.   :-2.191
##  1st Qu.:-17.954   1st Qu.:  5.326   1st Qu.:  2.495   1st Qu.: 5.617
##  Median :  3.165   Median : 30.500   Median : 10.465   Median : 9.163
##  Mean   :  1.822   Mean   : 33.324   Mean   : 10.422   Mean   :10.673
##  3rd Qu.: 22.848   3rd Qu.: 59.827   3rd Qu.: 18.191   3rd Qu.:14.161
##  Max.   : 83.056   Max.   :148.699   Max.   : 47.097   Max.   :42.704
##
##        h                  id                  j               k
##  Min.   :-6.652170   Length:600         Min.   : 5.341   Min.   :0
##  1st Qu.:-1.525722   Class :character   1st Qu.: 7.341   1st Qu.:0
##  Median : 0.092330   Mode  :character   Median : 9.341   Median :0
##  Mean   : 0.007944                      Mean   : 8.638   Mean   :0
##  3rd Qu.: 1.467511                      3rd Qu.:10.286   3rd Qu.:0
##  Max.   : 6.927176                      Max.   :11.231   Max.   :0
##                                         NA's   :597
```

```
head(df_multipar)
```

```
##          y        a        b          c          d         e         f
## 1 3.184311 51.42026 27.12527 0.68270165 -16.627974  56.10895  4.619684
## 2 4.078802 90.93125 43.26188 0.94534940   1.571907  47.07957 31.243675
## 3 3.286615 98.71964 57.81349 0.86691667  22.487176 100.08431 29.043086
## 4 3.582366 89.07057 34.81383 0.59954873  11.120274 -35.35449 -5.616698
## 5 3.182100 44.89312 38.22812 0.01036813  13.633426  72.30611  6.095976
## 6 2.464150 33.11191 46.04063 0.39760307 -48.057072  20.49485  6.499197
##           g          h      id      j k
## 1  3.589122 -0.7818687 ad00096     NA 0
## 2 13.066916 -4.6140785 fh00071     NA 0
## 3 20.370505  2.2817048 bc00034 11.231 0
## 4  5.218543 -0.9948896 eh00093     NA 0
## 5 11.635066  1.2807478 ax00045     NA 0
## 6 14.213027  0.4446310 fi00044     NA 0
```

20. Based on what you see in #19, edit the data frame by removing (dropping) any columns that are not likely to be useful for a multiple regression model. (Use `subset` or another method of your choice.)

```
df_multipar=subset(df_multipar, select = -c( j, k))

df_multipar
```

```
##            y           a        b           c            d           e
## 1  3.1843110  51.4202637 27.125270 0.6827016482 -16.62797430  56.10895150
## 2  4.0788021  90.9312531 43.261876 0.9453494020   1.57190720  47.07957245
## 3  3.2866154  98.7196415 57.813487 0.8669166700  22.48717625 100.08431156
## 4  3.5823658  89.0705696 34.813829 0.5995487254  11.12027388 -35.35448828
## 5  3.1820995  44.8931214 38.228120 0.0103681339  13.63342557  72.30610725
## 6  2.4641498  33.1119091 46.040632 0.3976030736 -48.05707193  20.49485347
## 7  2.2948768  76.1423394 72.523369 0.6293321918  -2.86673061  -7.24189499
## 8  3.1860679  32.7533381 21.918538 0.7277977758  50.39013154 -43.23933151
## 9  2.9209384  67.0009240 47.483481 0.4637902812  -9.37785018  38.20306367
## 10 2.8896518  88.0832643 66.111558 0.7791794979   7.46920915  -0.71210697
## 11 1.8720660  39.4291437 63.101919 0.5742710717 -24.98172695  17.26685829
## 12 3.5324869  55.0835689 33.179223 0.8836003654 -31.67810284  33.33733235
## 13 2.7784133  16.2111362 31.459398 0.9360883390  58.22242610 102.97541175
## 14 1.6188638   2.4584828 54.933629 0.8326913840 -77.56244958  28.60147868
## 15 3.4767303  38.8213052 24.189366 0.0653833176  -6.20971993 140.79440025
## 16 2.9581123  84.9821938 61.545459 0.9401420571 -18.44570169  62.27191062
## 17 2.7235389 -15.6851271 16.655304 0.2456351102 -11.44933799  11.22795577
## 18 3.6516780  77.1013850 38.824529 0.5901585429  -8.42542646   7.36218133
## 19 3.4334724  57.4529983 37.195523 0.9534708585   1.53844841  51.63316469
## 20 1.4359411   5.0809859 60.378352 0.4719502456  14.71117027 -25.67726063
## 21 2.4731786   8.2427250 33.235837 0.5208433906 -18.93646390 -13.60611637
## 22 1.5673268  12.1910799 59.740577 0.6768801401  29.41594342  16.78242216
## 23 4.3231488 101.6696348 33.152232 0.1044157897  42.56989063  77.31350697
## 24 2.5415922  15.7601455 38.473124 0.6000286059  51.42731377  33.05636090
## 25 2.3048742  40.3255631 54.441479 0.2885387065  59.64102837 -24.62610305
## 26 3.1163752  23.8094019 32.813576 0.8371764566  19.41474833   2.50273244
## 27 2.4577767  86.5288940 67.281564 0.6169900228 -58.97883414 -11.31003847
## 28 3.7194804  57.4136386 20.953902 0.2575552403  -2.19338133  17.88957795
## 29 4.3872106  97.2457363 30.796542 0.9592386771   8.03657338  62.30289638
## 30 4.1457685  58.8916531 21.492851 0.8974354735 -37.26344219 107.59451509
## 31 2.9427363  97.0777893 59.467963 0.3254023669   2.42562757  59.30045466
## 32 2.8189365  33.5190385 36.418454 0.8278977089 -33.88739631  18.13820063
## 33 3.6091717  38.3299313 12.013805 0.5243646759   1.48419267   2.53157911
## 34 4.5876230 107.6386332 22.671974 0.6641568963 -13.17448701  10.00246750
## 35 2.6392517  66.5152359 50.742385 0.3108219849  38.90377906  97.13655451
## 36 3.1251400  50.5338267 45.137707 0.0462148762 -17.39969397  79.29905820
## 37 2.6069449   1.1328789 28.826459 0.3384321467  15.88357396  31.69880461
## 38 2.4798622  66.1377769 62.142967 0.6569823525 -15.20805290 -28.44640899
## 39 2.4350042  39.8735263 51.446178 0.9242928405   7.99260100  54.87201907
## 40 2.6468152  12.1495639 38.684783 0.8858708488 -16.10420808  20.37541883
## 41 2.6534126  47.6514214 45.684023 0.5975779842 -24.59009892  -7.80718070
## 42 4.0032336  78.0727908 34.960817 0.0019021512  19.05838029  76.25224794
## 43 2.7127261  45.9263682 42.842516 0.6304926006 -16.13737463  -8.82867896
## 44 2.2010135  33.1979181 56.683841 0.8063030203   8.89771532  38.20613365
## 45 2.5481748  48.9703372 55.629653 0.7891689243 -16.26420162  15.97915463
## 46 3.3900884  49.7740451 36.880037 0.3266781811  36.62978704 -33.82435184
## 47 2.9709327  35.9191936 37.794933 0.8673742434  34.32815747  56.34722531
## 48 3.5026110  69.8588319 35.474152 0.3971823920  -7.92436488  31.73844046
## 49 3.6718242  89.8510074 37.393516 0.2826348518  -0.52011503  16.93485001
## 50 3.0628920  43.9488322 48.052504 0.1603990979 -73.36638462  79.48104018
## 51 3.1330889  27.8540222 21.308315 0.5946202911 -16.77225726 -47.51201577
## 52 2.3630320  12.1183238 39.530164 0.5538666388 -12.58167979 -24.23298504
## 53 3.3193319  86.0663377 48.258212 0.5008576228  45.10558657  -5.86934538
```

```
## 54  2.5677514  25.6176178 44.429460 0.1113350512  23.28669423  85.41261846
## 55  2.5724540  -1.5939407 29.407487 0.1873544671 -14.47177062  -7.83219749
## 56  2.2454494  12.0949973 49.275842 0.5708717974 -55.66790152   6.68019405
## 57  2.7342655  24.0290392 36.612668 0.7353378327 -23.16679808  61.61326899
## 58  2.8045659  85.7699127 51.122950 0.5422106297  13.36154211 -45.79267699
## 59  2.1050307   4.1800772 47.219891 0.0098789758 -43.63335203 122.42342561
## 60  3.2099505  53.8134131 38.794125 0.1331422119  11.54825530  49.58828220
## 61  3.5287262  40.7960992 24.013475 0.3608287747 -58.44117186  17.59097157
## 62  3.1022776  78.8048168 56.798450 0.2304072634  17.46181174  65.12067170
## 63  3.5820676  73.1175213 32.227489 0.3759240564 -72.09602171  11.53328762
## 64  0.9499290  27.3797127 83.667410 0.7708343745  -6.99597089  18.15337159
## 65  3.4794071  99.3772638 60.721207 0.6423414447 -24.13176077 -32.54578844
## 66  3.6892119  77.1495970 35.392038 0.4602417361  26.85041729  -7.90511744
## 67  3.3480197  59.9059625 36.404850 0.8792847528   5.59430376  24.65510244
## 68  4.3521603  76.6466672 12.273538 0.7663042983 -32.47294787  87.46614336
## 69  3.3650957  33.6988054 26.542809 0.3774947764  22.75653530  -5.66670049
## 70  2.1802548  29.9610378 45.442403 0.4167901003 -45.99663436  30.66762211
## 71  3.3259147  43.6852318 32.546538 0.3084158222  -9.56370707  17.05842236
## 72  3.3821596 101.8114320 56.762568 0.8143040454  56.92792479  25.19099252
## 73  3.6325943  63.8431281 32.278060 0.1525791639  -7.72283703  66.53061104
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## 486 3.3532224  61.7447368 34.946407 0.2178090652  19.44761349  66.78869246
## 487 2.3547306  18.9511824 34.672987 0.3613858509   9.55038942  23.25054069
## 488 1.3133204  25.0554444 71.999468 0.5870779934  -8.77315976  25.64948391
## 489 2.1737356  50.1877649 63.325220 0.6122901957  37.28594435   5.68536674
## 490 2.1880284  -2.1167089 44.374537 0.9995087443  46.37412631  44.59025448
## 491 2.5318257  79.8621438 60.164173 0.6979350538  19.63986454  -9.74790614
## 492 3.2007336  88.7277647 52.571598 0.3348187616 -16.71878948   6.49333917
## 493 3.2980815  95.5798965 44.730502 0.3788607842  80.62490020  -0.04765327
## 494 3.5903576  69.8204134 34.949398 0.0214240730  49.40900181  51.47309275
## 495 2.8846616  30.7331816 50.461354 0.9218001254  19.11941199  76.96188342
## 496 2.9468264  18.4824133 32.973551 0.7783087727 -15.25179186 -49.19554473
## 497 3.0243337  65.6175265 38.435256 0.4379650331 -43.50473203  11.58551664
## 498 3.7473707  54.4393284 24.306430 0.0441012450  36.24748367 106.55232237
## 499 2.3259765   4.5020157 39.223758 0.7395507167  14.74047720 -13.61639963
## 500 2.8861372  64.2422631 50.252121 0.4170037012  10.34996414   5.18552610
## 501 2.2043569  30.5480959 52.941394 0.5830936681  59.36481234 -30.69270686
## 502 2.6373918  28.6212450 43.523027 0.2021743748 -14.09058151  67.06114980
## 503 2.4957576 -13.3499329 26.950034 0.3067796512 -46.32532164 -11.53472529
## 504 2.1048420  17.7678183 40.885296 0.3840759953  46.95767727  23.37308566
## 505 3.3963848  32.5896971 27.678714 0.9905018259  33.96391376 148.69863478
## 506 4.6255162 105.3549352 24.291427 0.9530553373 -36.07603025  68.19960729
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## 508 4.0934605  72.7333095 27.777387 0.9600589240  19.03425643  24.25454701
## 509 2.6208869   7.4429416 31.881672 0.2856602450   4.82715821  26.52925909
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## 523 4.0015946  72.9155968 22.741755 0.8762523565  -4.15202856  33.23499807
## 524 2.2206220  18.4828056 45.061561 0.2577406778  23.26105361  -1.25647914
## 525 3.5700707  64.6114615 32.791089 0.3352748263 -35.32058792  46.05043829
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## 527 4.3099269  88.8118659 18.318878 0.8455094788  74.54229954 112.19129417
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## 529 2.0032671  61.5087695 66.919511 0.3532225804   1.69608269  11.26189824
## 530 3.2222646  84.5109494 51.158365 0.7750928442  -2.52583643  20.43354884
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## 535 4.2373086  97.7889666 34.652796 0.6052414258 -34.81685836 -36.19505913
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## 537 2.5899184  54.3068603 53.090015 0.5590687615  54.97452738  -5.89960407
## 538 2.9730356  46.3061538 33.517925 0.2368880583 -14.48121286  15.40122188
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## 540 3.1394447  26.0640705 25.605628 0.8164981513  57.89106993   9.22868929
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## 552 2.9571166  13.8662472 26.535547 0.9153133100 -30.70092445 114.90964456
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## 564 2.3603599   3.5170530 34.108128 0.5053108330  13.09785107  15.38535561
## 565 3.2075740  34.3116206 27.899592 0.7911266689  46.61927448  54.30345588
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## 585 3.2676105  64.7728064 39.509078 0.6924014008   7.64767811  37.90053513
## 586 3.4797201  52.0662896 33.948845 0.4135590412   2.88180620  48.46277034
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## 589 1.5560693  16.8812581 70.269077 0.4105966848  41.55333421  -3.72525566
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## 594 3.1529137  33.7354814 33.952352 0.0588084925 -18.09722619  71.67781037
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## 596 2.3555241  26.3298414 48.838226 0.1143663961  38.08314046  69.97171701
## 597 4.4267264  92.8622922 23.130280 0.8444770812  27.65659050  55.34904204
## 598 2.9234212  61.0345467 49.007494 0.7030357211 -14.38939917 -11.07418348
## 599 3.2229808  89.6317633 50.667386 0.6004191940  49.69463752  -6.30600101
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##                f           g            h      id
## 1     4.61968395  3.58912250 -0.781868672 ad00096
## 2    31.24367483 13.06691639 -4.614078453 fh00071
## 3    29.04308649 20.37050542  2.281704799 bc00034
## 4    -5.61669754  5.21854256 -0.994889565 eh00093
## 5     6.09597626 11.63506553  1.280747829 ax00045
## 6     6.49919701 14.21302660  0.444630993 fi00044
## 7    20.39702004 30.35440015  0.215779131 bg00087
## 8    13.34782634  2.98763441 -3.164797999 el00048
## 9    25.65127637 14.59229269  0.895767974 fv00068
## 10   17.45609933 21.85754668  2.103057126 di00079
## 11   29.38345430 23.66353778 -1.015274116 eb00041
## 12    8.82495727  5.14036477 -3.627283186 cq00086
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## 16   16.39808202 21.60849257 -0.933227024 fs00089
## 17   11.75585200  5.49391925  4.181652372 be00065
## 18    2.41561241 11.39738786 -1.985332060 fu00002
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## 22   15.33078667 22.14962577 -2.879039600 ad00092
## 23   -1.22170766  8.66037536 -1.395617138 af00038
## 24    4.77886755  8.55043429  2.582274434 eg00067
## 25    3.42499555 16.62319769  1.905121390 ap00006
## 26   16.77651376  7.05683939  0.801712678 ds00085
## 27   47.09706388 24.61352045  6.223320050 ag00013
## 28   -2.30183774  5.19913615 -2.703195915 bf00016
## 29   19.28714822  6.18855843 -0.354519881 ft00098
## 30    0.35626091 -2.19051268  3.887587968 fx00053
## 31   -2.10301340 23.00946094 -0.625291942 cq00036
## 32   11.11630180  9.41491496 -1.024867422 ar00020
## 33   -5.12626842  1.46377813  0.523980668 dy00043
## 34   -0.32182774  7.02898077  0.344431408 au00011
## 35   15.97156132 12.56409005 -3.063952318 cj00024
## 36  -11.63532196 14.14449163 -3.823492595 er00003
## 37   10.18220847  3.91405008  1.645363191 ei00084
## 38   23.90072367 25.40868339  4.574920624 ef00049
## 39   18.14195711 17.10103719  3.033011309 eh00081
## 40   22.66810173  8.82444134  0.908775844 et00027
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## 42  -23.14177478  6.19324623  2.533374833 dp00100
## 43   12.12999964 11.01456521  0.008172710 fh00047
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## 47   28.26427982 10.99560065 -2.674045175 dp00061
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## 50   13.05291499 13.62006382 -0.484302325 cf00017
## 51    0.29826954  4.21061632  0.092810296 bg00008
## 52   14.50532684  8.92511417  1.641519748 ds00056
## 53    5.11199731 15.63431954  0.151873696 fu00009
## 54    2.64037318 12.86157565  3.854472797 cw00088
## 55   -3.83061631  6.55995578 -0.233820937 ef00033
## 56    3.40797224 12.49948102  0.470731926 cs00063
## 57   17.01737697  7.76922239  0.076488580 da00052
## 58    8.49219924 15.78617313 -0.284475591 au00066
## 59   25.52558603 13.66913244 -1.599819679 dt00095
## 60    2.99942143 11.14649441  3.072620977 cm00075
## 61    0.07181456  5.17182072 -2.331462164 ad00096
## 62   14.01688102 17.02993740  4.512074367 fh00071
## 63   -5.49672550  8.15047056  0.435259479 bc00034
## 64   15.48498254 42.70391102 -1.443089126 eh00093
## 65   26.34464205 22.50472566  2.644143060 ax00045
## 66    9.17782115  4.69842922 -3.878158105 fi00044
## 67   23.59023284  7.64830531 -5.707616621 bg00087
## 68   21.87871020  6.54730878 -2.444368181 el00048
## 69    9.79157468  3.76108322  1.520037680 fv00068
## 70   19.95835125  9.01352105  1.381056510 di00079
## 71    3.00991145  5.45355962  1.341158556 eb00041
## 72   28.56247814 17.99085114  0.182991001 cq00086
## 73   15.08353938 11.74138369  0.617590068 ec00035
## 74   33.91968846 21.17824378 -3.203567993 dn00018
## 75   22.67085746 13.45299504 -1.457884333 ff00057
## 76    3.17531106  5.24122707  1.627919781 fs00089
## 77   24.85966570 16.49262657 -0.322335415 be00065
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## 79   22.32916531 10.56477514 -1.658342749 fa00028
## 80   17.56157785 12.50084688  2.521473584 bd00077
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## 82    8.99708111  2.78971504  3.222901386 ad00092
## 83    9.93842926 19.97613435  3.874279918 af00038
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## 86    4.56393314 13.77439589 -3.502508714 ds00085
## 87   11.82389900  9.92731258  0.675310850 ag00013
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## 89   -1.69652273 14.52038327  2.378775923 ft00098
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## 91   -4.63401627  9.59004643 -1.494020478 cq00036
## 92    2.64477592  9.99442363 -0.022211793 ar00020
## 93    5.79733120  9.42368515 -2.990414930 dy00043
## 94   23.42114349 16.93711489  1.148407494 au00011
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## 96   24.61419432 11.40432210 -1.005285581 er00003
## 97   14.34290237  6.08525939 -0.346090028 ei00084
## 98   23.09822632 19.61175947  0.638068457 ef00049
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## 100  -0.26878128 11.09449964 -0.227699727 et00027
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## 101   9.81984767  2.16559244 -3.442497960 fh00070
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## 103  12.55637514 17.94855525 -2.168214540 fh00047
## 104  19.64423882  3.09303975 -0.905780892 cc00004
## 105  21.39258759 21.03679693 -1.388287766 be00029
## 106   7.52387002 -0.81251225  1.403799433 ep00080
## 107   1.70991035 10.49909743 -0.843386903 dp00061
## 108   5.66577461  7.06981469 -1.827083524 bt00073
## 109  -4.36787466  1.03356269 -1.875997826 ce00015
## 110   8.99405213  6.01211771 -2.406224767 cf00017
## 111  -9.42360503  3.12555133  0.538193558 bg00008
## 112  -5.50881284  5.11346015  1.348912930 ds00056
## 113  13.69554107  5.62457876 -0.485832493 fu00009
## 114   7.07757730  6.15912614  3.005664745 cw00088
## 115  -7.35554225  5.33531491 -2.272664503 ef00033
## 116  21.21467954  6.07477554  0.444291862 cs00063
## 117  26.84876914  4.45757255  0.233557290 da00052
## 118  39.95583025  5.93679330  0.260304037 au00066
## 119  17.35704349 22.21203936  4.454135115 dt00095
## 120  17.36657319 22.58371417  1.753002051 cm00075
## 121   7.63100274 21.92952015  1.622923326 ad00096
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## 135 -14.68726671  2.19083326 -1.001494530 ff00057
## 136  -2.62674223  1.28247547 -4.065752480 fs00089
## 137  21.26541595 -0.36677832  0.263754754 be00065
## 138  -8.67958887  6.25090593  1.044134211 fu00002
## 139  -1.52100060  5.86215636 -3.681488554 fa00028
## 140  -9.57791376  2.93259753  5.238969446 bd00077
## 141 -11.62603925  7.30506286 -0.999691780 bu00042
## 142   8.85294545  9.63962179 -2.524609565 ad00092
## 143  10.32480824 20.11720733  4.620102835 af00038
## 144   4.80716691  3.30614963 -2.923966801 eg00067
## 145  20.06312290 32.02505753 -3.884026415 ap00006
## 146  17.41561174 -0.24668189  1.225476768 ds00085
## 147  17.97695864 18.25933141  2.854123090 ag00013
## 148  -3.11859802  7.00076515  1.721130130 bf00016
## 149  15.71748559  8.26944045 -2.142100609 ft00098
## 150  -2.52831010  2.50200005 -0.981138060 fx00053
## 151   9.44853124  9.56632543  1.477902613 cq00036
## 152 -14.60826524  2.35294174  1.464383359 ar00020
## 153   5.89238202 14.25069692 -3.759731638 dy00043
## 154   0.09547796 11.24901384  1.465785416 au00011
```

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## 155  12.66372902 13.95893086  2.074209180 cj00024
## 156  13.41388064  9.40269138  0.251239951 er00003
## 157  27.03055384 10.55299655 -0.794696755 ei00084
## 158   0.01608637  3.39087635 -1.237832717 ef00049
## 159   6.94388014 11.39047719 -1.120642913 eh00081
## 160  13.49309205 14.29760055  1.545474103 et00027
## 161   0.91781371  9.17548015  0.348371815 fh00070
## 162  -4.23482838  5.15338316 -2.254117248 dp00100
## 163  19.09076393  3.89300805 -0.660514044 fh00047
## 164  21.60559779 21.18602119  3.583526925 cc00004
## 165   5.60066612 13.42699459  1.918318805 be00029
## 166  17.26771814  6.79231186 -1.747138941 ep00080
## 167  27.56199695  7.50174829  0.073247373 dp00061
## 168  18.23692768  8.66963695  0.475098851 bt00073
## 169  17.31747606  6.11915559 -4.207230179 ce00015
## 170   7.66508363  8.00750431 -1.554400948 cf00017
## 171  19.14159180 17.70299332 -0.765930659 bg00008
## 172  33.09041305 34.01937436 -0.448962624 ds00056
## 173  14.36524265 11.13469754  0.948405250 fu00009
## 174  12.48577507 20.67860246 -3.363613012 cw00088
## 175  17.32874332  9.14989245  2.043036802 ef00033
## 176  10.09908594 18.11191253 -2.636322619 cs00063
## 177   9.67369730  5.73473678 -4.939884301 da00052
## 178   3.96109284  5.95114942  0.496163204 au00066
## 179   1.44383860 19.45128762  6.927175751 dt00095
## 180  13.10664822  6.64324518  3.335752767 cm00075
## 181  -1.95001228  6.79750271 -0.080644198 ad00096
## 182   2.16120806  4.53305671 -1.189514936 fh00071
## 183   6.56388983  7.66557168  2.869101264 bc00034
## 184  -8.96693405 18.10419242 -1.581721747 eh00093
## 185   4.46654023 10.31686993 -1.362533539 ax00045
## 186  12.98183538 12.21892512  1.280904382 fi00044
## 187   0.39273579 11.93136518  2.028242295 bg00087
## 188  -8.60725554 10.81227293  0.785571357 el00048
## 189  21.93555273 19.37644940 -1.181258549 fv00068
## 190   6.93797471  7.15281860 -1.953721293 di00079
## 191  13.17706565  8.04255460  1.627351324 eb00041
## 192  31.12634898 18.30403993  0.091848905 cq00086
## 193  -8.65561758  5.26418107  2.263100481 ec00035
## 194   3.85910238 15.32848859  4.090932452 dn00018
## 195 -13.29611164  4.52577253  1.168143711 ff00057
## 196   0.47264910  4.93821470 -1.014725853 fs00089
## 197   4.05846526  4.90156729 -2.211950747 be00065
## 198  14.36331364  8.85510025  6.258366557 fu00002
## 199   7.82448152  4.43885359  0.026391354 fa00028
## 200  -0.55482552  2.27816326 -2.096345041 bd00077
## 201  24.92074538  7.99798077  0.013825441 bu00042
## 202   0.73086198  9.52163132 -0.670534437 ad00092
## 203  22.28538247  6.42539072  1.359142356 af00038
## 204  -5.68688378  6.76941858 -0.765102626 eg00067
## 205   8.42007835 11.09896787  2.213671275 ap00006
## 206  -1.88344359  2.11311837 -1.823131813 ds00085
## 207  15.28862510  5.64227418  2.193633452 ag00013
## 208 -13.29869756  0.18481871  1.277830361 bf00016
```

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## 209  10.17834672  7.73200593  1.452022184 ft00098
## 210   6.05063787 20.56162913  4.584274077 fx00053
## 211   5.32127943  7.66158048  1.032570739 cq00036
## 212 -15.83024691  0.65071936  0.528210312 ar00020
## 213  24.21671397  5.26555630 -0.613679328 dy00043
## 214  11.55682729  5.21132106 -1.546713287 au00011
## 215  20.96913897 19.37297201 -4.034483375 cj00024
## 216  25.00825570  6.85981576 -0.741304690 er00003
## 217  -5.42728787 10.20366038  0.432570947 ei00084
## 218  -5.65926951  4.13657534 -2.041718714 ef00049
## 219   4.24807216  8.68387889  1.853715762 eh00081
## 220  34.18561327 18.93053918 -2.613072639 et00027
## 221   9.96438469  3.77096676 -1.251719997 fh00070
## 222   9.28891780  7.95624439  0.530994329 dp00100
## 223  13.65703749 10.27043948  2.205021186 fh00047
## 224   6.74053252  5.05084507 -0.818953570 cc00004
## 225  -1.95902459  1.55887279  6.836935239 be00029
## 226  18.66897145 29.18569734  1.177234053 ep00080
## 227  -9.34299138  8.74856782  0.263597670 dp00061
## 228 -24.19336244  2.56882284  2.478616965 bt00073
## 229   5.43767695 13.52768748  1.104668567 ce00015
## 230  19.64663976  7.70271910  2.026567107 cf00017
## 231  10.01008281  8.46231507  2.857654921 bg00008
## 232  -5.10326401  1.26097210  0.753522604 ds00056
## 233  22.36714679 14.21032997  0.921137690 fu00009
## 234  30.72930634  6.68398442  2.637162882 cw00088
## 235  30.78096410 13.05287620  0.285338092 ef00033
## 236  19.33887291 12.16116020  4.459138047 cs00063
## 237  36.80417661 13.26941250  0.535360970 da00052
## 238  31.32464525 23.98607353 -2.296414373 au00066
## 239  -1.53061182 10.19486757  0.845751491 dt00095
## 240   8.16354665  4.61413500  1.266302927 cm00075
## 241  -1.76624646  8.20871551 -0.298823511 ad00096
## 242  26.71743895 16.59955500  1.797728717 fh00071
## 243  12.26217796  5.58100172  3.144344625 bc00034
## 244   2.46179877  2.18782524 -3.156703169 eh00093
## 245   1.47096086  4.47993349  1.901810855 ax00045
## 246  10.87769650 12.44818912  0.378481622 fi00044
## 247  16.48435183 13.52791704 -2.292736694 bg00087
## 248  12.84250551  3.67472686  4.649458656 el00048
## 249   7.68861225  4.72153516 -3.121316917 fv00068
## 250  25.59653849  5.91036145  0.714543761 di00079
## 251  20.87514288 13.69131386 -1.112631951 eb00041
## 252   3.66130656  5.10875651  0.122566239 cq00086
## 253  -8.72380551  7.08427833  4.004761893 ec00035
## 254  12.55485390  4.50620883 -0.275219868 dn00018
## 255   7.77709502  4.77108915 -3.290225224 ff00057
## 256  12.44258134  3.21787672  1.193812344 fs00089
## 257  33.09581699 14.93911191 -2.723166700 be00065
## 258  -5.77801265  3.97576852 -2.127910163 fu00002
## 259  -4.19825534 16.47694965 -3.681504318 fa00028
## 260   8.53487652  2.28986518  0.649423049 bd00077
## 261  10.88552810  5.35802738  3.699290521 bu00042
## 262   2.58624231  8.06314740 -4.565647204 ad00092
```

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## 263  15.97754635 23.76429977 -2.174879866 af00038
## 264   6.97699469  3.00144618 -0.251324862 eg00067
## 265   2.80248731  7.57605677 -5.653898745 ap00006
## 266  23.40866947  7.21803916  1.164832319 ds00085
## 267   3.48911198 25.40514082  1.927004605 ag00013
## 268  11.50954004 10.57787700  2.475334689 bf00016
## 269   6.81344103 25.39013221 -2.040768449 ft00098
## 270  11.56875000 10.17729226  2.848245032 fx00053
## 271  25.60242854  7.06941186 -0.168501998 cq00036
## 272  15.95388002 12.89557664  0.169108587 ar00020
## 273  20.69664853  4.08414647 -2.056417583 dy00043
## 274   8.82277884 20.07345359  1.208960282 au00011
## 275  19.26619814 12.04829038  1.156749655 cj00024
## 276   9.15226744 12.67300308 -1.521374808 er00003
## 277  20.58297489 30.06666368  2.316914098 ei00084
## 278   8.30831424  7.31373406 -4.501750054 ef00049
## 279   9.63645998  5.55843281 -0.367036744 eh00081
## 280  18.75557682 14.37065891  1.911320625 et00027
## 281   0.58952341  6.95212534  1.229097489 fh00070
## 282  22.89003732  7.64993237 -2.634012964 dp00100
## 283   4.28592269  5.40860085 -0.573995416 fh00047
## 284  10.88582958  8.89405708  3.612016491 cc00004
## 285  13.34108614  8.78426926 -2.072396479 be00029
## 286  -2.61727700  1.79555797 -1.737324570 ep00080
## 287 -12.94492852 11.72708677 -0.935542933 dp00061
## 288   5.18928421 19.67405705 -0.396326395 bt00073
## 289  15.77931441  7.22257073 -0.059311687 ce00015
## 290 -10.56441791  6.88300318 -0.606953418 cf00017
## 291  14.82013574 16.97398728 -1.602775026 bg00008
## 292 -23.25126850  2.39944292 -1.215999549 ds00056
## 293  -1.07789060  5.44112986 -3.370057613 fu00009
## 294  13.18121442  5.66822325 -3.042270439 cw00088
## 295  27.07276383  9.72821911  0.517425416 ef00033
## 296  12.08014091 13.48320344  1.015921800 cs00063
## 297   9.64221182  6.34232809  2.285821189 da00052
## 298   6.28891521  5.21655886 -3.164073911 au00066
## 299  10.07112386  9.75393278 -2.085358343 dt00095
## 300   3.41011557 10.17603865 -1.923458584 cm00075
## 301   3.41557475 15.15880427 -0.233863708 ad00096
## 302  14.93589809  5.99795839 -1.076675531 fh00071
## 303  26.17234246 11.89448320  4.012428984 bc00034
## 304  24.18599387 14.36948390 -0.749559178 eh00093
## 305  11.97213494 10.60735299  3.326474383 ax00045
## 306   4.46050212  7.87070709  2.412614070 fi00044
## 307  38.99173901 35.22406181 -1.202838029 bg00087
## 308  18.96167275  8.84832576 -2.654979138 el00048
## 309   8.49143643  9.23337811  1.021495964 fv00068
## 310  14.84189730  5.14571698 -1.387202985 di00079
## 311  19.03506942 10.28599142  3.045164936 eb00041
## 312  10.96218823  9.48771124 -0.736811456 cq00086
## 313  19.71123948 20.53781602 -4.662277619 ec00035
## 314  -0.77303272  7.35770135 -0.635094142 dn00018
## 315  15.25273038 19.09543335  0.113463534 ff00057
## 316  29.94054632  7.96241287 -1.463454930 fs00089
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## 317   8.55876866  6.76526015 -1.265338309 be00065
## 318  -5.22676978  8.31612158  2.756101982 fu00002
## 319  32.14798473 12.49439532  0.040499413 fa00028
## 320  38.30286529 29.96499928 -0.100923808 bd00077
## 321   9.21119523  5.72152597  0.008775520 bu00042
## 322   0.76604564  2.45877317  2.340388409 ad00092
## 323   9.97489740 15.45007499  2.825868280 af00038
## 324   6.50581432 12.32698582 -0.969288225 eg00067
## 325  16.99472233  5.10476336 -2.435548041 ap00006
## 326  -0.66902330  6.13472164 -0.927450139 ds00085
## 327   4.56570610 13.03171648  1.709657742 ag00013
## 328  -4.34940920  2.51902115 -0.463371822 bf00016
## 329  21.85212616 22.65740533  1.890764991 ft00098
## 330   9.21449330  6.08784396 -0.675926409 fx00053
## 331  17.51382886 14.62406561  1.863079470 cq00036
## 332  32.94226574 25.16060672  3.522686689 ar00020
## 333  13.62219064 26.09231546  1.197910780 dy00043
## 334   2.49037763  4.76270595 -0.270293335 au00011
## 335  -0.05611870  1.74924823  1.149155584 cj00024
## 336   7.15929316  9.03958608 -1.419371410 er00003
## 337   8.87726016 19.31513492 -0.464181539 ei00084
## 338  29.78670349 18.67629889 -2.018007426 ef00049
## 339 -13.53082729 13.93742779 -0.944648671 eh00081
## 340  12.36515225 12.10530124  3.069182874 et00027
## 341  -1.98530804 12.39108873 -1.553513333 fh00070
## 342   7.06414220 12.67528232  5.995014517 dp00100
## 343   7.46835262  4.75495746 -1.858655032 fh00047
## 344  15.08770749 13.39577709  0.552291355 cc00004
## 345  12.31035909 13.06825553  1.846772052 be00029
## 346   2.95896131  7.37880285  0.184378883 ep00080
## 347  18.99702001 13.71301203 -0.618764456 dp00061
## 348   8.32235663 13.03179241 -0.516416412 bt00073
## 349  10.36103331  5.09505516 -1.539324791 ce00015
## 350 -15.85935497 10.40174719  2.282718644 cf00017
## 351  16.07982325 18.81987568 -0.443533968 bg00008
## 352  31.18080354 19.83032313 -1.429872014 ds00056
## 353  14.32825903 11.06310026  1.472687262 fu00009
## 354  -1.51404429 10.56254325  0.561624251 cw00088
## 355  14.13627074 25.21112797 -1.343019900 ef00033
## 356  17.53039296  4.73951753  0.736655265 cs00063
## 357  -8.09371770  3.05688478  6.451118745 da00052
## 358  10.35659287 11.04819885 -0.390621012 au00066
## 359  -4.18129481 17.25742161 -1.098800435 dt00095
## 360  -0.77361562  4.20295281  1.221194057 cm00075
## 361   7.99335635  0.60233711 -1.676559935 ad00096
## 362  18.87010516 18.09161518  6.366650088 fh00071
## 363  26.38301510 23.03501741  3.044622439 bc00034
## 364  12.61083863  6.94544826 -1.918741514 eh00093
## 365  24.55315392 36.09767691  4.658228129 ax00045
## 366   3.75706200  1.13067177 -2.307048690 fi00044
## 367  -0.27224017  1.08701196 -2.656087773 bg00087
## 368  -1.13735229  4.65293391  1.656432258 el00048
## 369  12.99848760 10.11096021 -1.087247738 fv00068
## 370   5.93723806 11.60950311 -0.102087642 di00079
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## 371  15.47489423  4.66493016  0.409411796 eb00041
## 372 -16.40236318 -0.00256809  1.427763920 cq00086
## 373  12.27883293 10.87001888 -0.854018081 ec00035
## 374   8.62452549  9.70236819 -0.058039663 dn00018
## 375  29.69135262 21.18595266 -0.381245156 ff00057
## 376  12.72150600 10.64360364  0.464414801 fs00089
## 377 -11.68092076  8.08093198  0.440404291 be00065
## 378  -6.59610214  6.00862038  0.376710984 fu00002
## 379  -0.37311795  1.40457813 -2.900088253 fa00028
## 380   0.05298973  6.84339705  0.613360511 bd00077
## 381  -8.26772757  4.03699357  4.198808422 bu00042
## 382  11.22470612  6.44438617  0.713385435 ad00092
## 383  20.58846670  4.39023578 -0.324107247 af00038
## 384  10.91323767 18.91007275  1.089810433 eg00067
## 385  -0.52379337  6.88012995 -1.350210851 ap00006
## 386  29.70083136 11.26931804  0.282392882 ds00085
## 387  -5.11683387 14.72540329  1.613885738 ag00013
## 388   4.82674394 21.34977827 -0.234419565 bf00016
## 389  -3.64172794 11.98461644  0.134087769 ft00098
## 390  21.04664975 15.62860553  0.805633187 fx00053
## 391  15.51512558 -0.07190300  2.052875305 cq00036
## 392  21.79630858 16.44256270  1.034752112 ar00020
## 393   7.62954946  8.64535870 -0.696333361 dy00043
## 394  16.77543629  6.95052818  0.677904273 au00011
## 395  21.94058533 17.47289231 -3.421499955 cj00024
## 396  -0.07688646  7.42204427 -0.113836681 er00003
## 397  15.17348983 24.18342516  3.305694180 ei00084
## 398 -13.29329786  5.62454243  1.697888141 ef00049
## 399  25.87307332 40.25290500 -4.124115836 eh00081
## 400  19.19125107 10.39528699 -2.222977113 et00027
## 401  23.74168698  2.23605329  0.721079983 fh00070
## 402  25.69364806 14.05144222 -2.158431257 dp00100
## 403  32.95751238  6.02397794  1.016154896 fh00047
## 404  24.88588758  2.59249518 -1.538764502 cc00004
## 405  20.44457602 10.05370926  1.077269576 be00029
## 406  14.74294674  8.29638163 -4.102297000 ep00080
## 407  15.85950518 14.38820452 -0.796281112 dp00061
## 408  15.16104109 14.33747803  1.079386660 bt00073
## 409 -12.95284621 10.05371108 -4.855215319 ce00015
## 410  13.91079243  8.39945765 -1.578807352 cf00017
## 411   7.22094542 24.94520618 -0.926810171 bg00008
## 412  16.99768587 13.69456172 -0.655020378 ds00056
## 413   9.50093971 10.58786462  0.052911987 fu00009
## 414  15.61533418  8.21555473  0.284737546 cw00088
## 415   5.89160663  6.87320655  2.862677028 ef00033
## 416  -5.78295593  7.14015747  0.726517739 cs00063
## 417  19.14494131  9.85531698 -0.393346655 da00052
## 418  18.06377342  6.08484302 -3.219416488 au00066
## 419  15.66283034 12.39123351  3.536131058 dt00095
## 420  20.98183314 14.75096247  3.770499882 cm00075
## 421 -16.36820727 -1.00566386 -2.338138659 ad00096
## 422  13.74953402 13.09723009  2.936019906 fh00071
## 423   8.77324408 16.39100254 -3.175516088 bc00034
## 424   4.18548805  8.06737711 -5.606266892 eh00093
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## 425  20.94279117  9.78598724  1.371786522 ax00045
## 426  -1.79761346  3.85611851 -0.747936274 fi00044
## 427   2.63976157  1.95911170 -2.837842707 bg00087
## 428  15.12623548 11.91106571 -2.323498098 el00048
## 429  19.47351603  4.45000819  0.799207233 fv00068
## 430  28.23147358 13.23364173  1.928080215 di00079
## 431  15.42925821  6.70291954  2.845523095 eb00041
## 432  -2.17616839  1.71913064  3.082113864 cq00086
## 433  16.87315589  4.88169017  0.784130800 ec00035
## 434 -12.96592228  3.53524769 -2.553281263 dn00018
## 435  22.77221632  4.11181805 -0.868863460 ff00057
## 436  -0.30821262  5.99302877  1.721309388 fs00089
## 437   6.04885319 11.89318056 -2.132861430 be00065
## 438  28.27654974 11.30678128 -1.319398351 fu00002
## 439   2.46620911 25.41247304 -0.096530693 fa00028
## 440  21.37630232  5.12333057 -0.090839840 bd00077
## 441   7.09510964  7.43249661 -0.747352865 bu00042
## 442  16.60690563 19.44393033 -2.111541473 ad00092
## 443   4.49513433  7.66878226  0.097293178 af00038
## 444  13.12879993 10.98447029 -1.769707910 eg00067
## 445  16.09341012  2.14159636  2.077043526 ap00006
## 446  26.28102292  8.79534472  3.174017003 ds00085
## 447  17.29671841 13.17184880  2.735349193 ag00013
## 448  10.56891327 26.77716584  1.256264359 bf00016
## 449   2.45758661  4.54078426 -5.385127273 ft00098
## 450   0.73409083  4.45787246  1.490063647 fx00053
## 451  28.28181840  5.99346613 -4.835038775 cq00036
## 452  26.49961776 18.72574910 -0.519929307 ar00020
## 453   6.24284449  3.27410487  2.507983510 dy00043
## 454   7.38067505 10.70093677  0.835291785 au00011
## 455  10.33553469 29.29286929 -0.039508330 cj00024
## 456  11.41739145  2.41092806  1.938830028 er00003
## 457  21.96240371 13.93047741  0.142753902 ei00084
## 458 -19.42953416 -0.12897443  1.061019732 ef00049
## 459   5.91786457  3.26246262 -0.506519451 eh00081
## 460   7.64174931  3.95609916  0.719212905 et00027
## 461  15.73660372 21.34080470 -1.458170956 fh00070
## 462  -1.88883739  8.86062369 -0.719636428 dp00100
## 463  27.65881956 14.57739513 -4.302933112 fh00047
## 464  17.62130554  9.12684441 -2.115363622 cc00004
## 465   1.42863339 12.18396906  2.757772219 be00029
## 466  18.95294519  4.42521546  2.327759405 ep00080
## 467  22.45878736  7.05414903  2.341812207 dp00061
## 468   6.35755244  2.89275836  1.062872435 bt00073
## 469  19.61568058 11.65224237 -0.969712337 ce00015
## 470  29.97980422 16.82434029  1.754093137 cf00017
## 471  18.17518828 12.43875368  0.230795002 bg00008
## 472  26.93095045 13.77510209 -2.700324655 ds00056
## 473   8.66829886 17.62132491 -0.479485653 fu00009
## 474  16.20368358  3.80081242  0.309336681 cw00088
## 475  14.23611671 23.87588950 -1.961595012 ef00033
## 476  15.45190810 11.25367424  0.295357798 cs00063
## 477  13.26250977 16.90617088  1.487281740 da00052
## 478   1.98128699  7.70859639 -1.335783996 au00066
```

```
## 479  12.12521910 12.72415482  2.096196069 dt00095
## 480   0.39206661 14.28669156  3.160212035 cm00075
## 481  -0.43284865 14.30666742  5.086547177 ad00096
## 482  -7.54543828 11.47911873  2.520403787 fh00071
## 483   2.49592808 10.05157512  1.264965020 bc00034
## 484  22.23091459 19.90937700  0.786502502 eh00093
## 485  12.96160280  9.81203222  1.771505758 ax00045
## 486  20.82191313  7.32011405  1.309281800 fi00044
## 487  11.15619403  9.11658486  0.830583742 bg00087
## 488  17.29424087 29.14570373 -0.602210640 el00048
## 489  28.62908407 24.73550591  1.842755591 fv00068
## 490  16.73492299 13.33557913 -1.666556792 di00079
## 491  26.26664339 20.96873064  1.554209163 eb00041
## 492  22.27732131 15.76169940  1.213306234 cq00086
## 493  18.30666433 10.15882524 -1.969883526 ec00035
## 494   4.53138665 10.59826808 -3.808485331 dn00018
## 495  31.40960317  8.82385318 -0.618371633 ff00057
## 496  17.75970677  6.07613876  0.529243260 fs00089
## 497   9.53562146  6.13116182 -0.387812971 be00065
## 498 -18.05415632  3.96925210  2.328228950 fu00002
## 499  13.76646280 12.10649649  0.079256193 fa00028
## 500   7.03663282 15.75217151 -1.711263832 bd00077
## 501   3.18820696 16.37231000  1.737970716 bu00042
## 502  -6.44113714 10.38798463 -0.804344814 ad00092
## 503   5.50916075  6.00819174  2.132883428 af00038
## 504  -0.60657361  8.37115488 -2.659552048 eg00067
## 505  11.95854308  3.85494719  2.312011693 ap00006
## 506  17.33288366  3.53763626 -3.185869370 ds00085
## 507  28.49302689 10.55831461 -3.866211055 ag00013
## 508  18.68337728  2.95393420  0.658641875 bf00016
## 509 -15.06802492  9.06266267  1.596261802 ft00098
## 510  13.91873185 15.90001234 -5.019505210 fx00053
## 511  31.27859977  1.16700242 -2.103293523 cq00036
## 512   7.84759053  9.80261308  2.886760871 ar00020
## 513  38.62261501 22.51364194  0.522112564 dy00043
## 514  17.00863742  3.96301599 -3.237466670 au00011
## 515  11.49195400 12.98428740 -4.474196540 cj00024
## 516 -12.85388173 12.02483869 -3.290405011 er00003
## 517  18.66925795 11.54511100  1.060730813 ei00084
## 518  17.77339046  8.81014148 -3.990872025 ef00049
## 519  15.97225212  9.74891749 -2.652002141 eh00081
## 520 -14.79864864 16.70898655  0.153009755 et00027
## 521  -0.53241755  7.92641426 -2.202810717 fh00070
## 522  14.19102624  6.26965725  1.195019564 dp00100
## 523  10.77626926  4.41879147  1.152900625 fh00047
## 524  -1.76771085 13.34150220 -4.959893362 cc00004
## 525  16.90527242  5.08701798  0.086080749 be00029
## 526  -2.59081467 13.17050623  0.885492573 ep00080
## 527  -9.31798150  0.77379992 -6.652170379 dp00061
## 528  -5.64294453 10.80833638  0.127842132 bt00073
## 529  28.60262190 24.47486319 -3.369611692 ce00015
## 530  12.38009162 16.86537552  2.307888796 cf00017
## 531   7.72802127  5.26860445  1.317056902 bg00008
## 532 -16.09015284  5.59418148  3.566752668 ds00056
```

```
## 533  16.62958611  9.68386804 -1.093904873 fu00009
## 534  18.45132613  9.45725571 -1.206307780 cw00088
## 535  -7.01518706  6.24560096  1.843803072 ef00033
## 536   8.22338749  8.55979733 -0.518611408 cs00063
## 537  10.03920925 18.76571244 -0.859907475 da00052
## 538   4.61791090  4.91403322  3.471156832 au00066
## 539  28.57584944 15.82492194 -1.299504070 dt00095
## 540   8.09237805  3.57140968  0.493113258 cm00075
## 541  14.52054277 14.48768255  1.217891360 ad00096
## 542  11.26014102  8.69748413  1.267667611 fh00071
## 543   0.34495510  4.86122916 -0.135458052 bc00034
## 544  -6.72758195  3.30925959 -2.970312293 eh00093
## 545  -4.21153373 16.37445642  1.343850853 ax00045
## 546   6.42086966 17.28359758  0.524738303 fi00044
## 547   7.29367100  9.22114423  4.653128842 bg00087
## 548  15.72079750  6.67175617 -0.401319324 el00048
## 549   5.92646408 14.92824651 -0.189020161 fv00068
## 550 -20.03538856  4.57317462  0.661548550 di00079
## 551   8.31274187  8.24409871  1.424794178 eb00041
## 552   3.34638401  7.66615497 -1.491508732 cq00086
## 553  -5.28978370  6.67481121 -0.464529092 ec00035
## 554   8.02103841 12.19549213  4.715455380 dn00018
## 555  -9.94853985  6.17199732 -1.781012771 ff00057
## 556 -14.28178353  6.60074663 -1.447243097 fs00089
## 557 -14.98801130  9.77232402  1.116104021 be00065
## 558  32.43948269  4.67871164  0.006594457 fu00002
## 559  30.42973490 37.30183931 -2.766132693 fa00028
## 560  17.65968703 16.56895413 -1.660763773 bd00077
## 561   7.98170094  6.56706270  1.904413100 bu00042
## 562  21.09252999  8.24393397  0.291154856 ad00092
## 563  32.79713975 19.40670930  1.592079682 af00038
## 564  -2.69464666 11.79852917 -0.460819234 eg00067
## 565   7.92131366  7.31509482  1.658543332 ap00006
## 566  11.06301351 13.99007310  2.800245505 ds00085
## 567  21.48854086 15.11171636  1.094929618 ag00013
## 568  18.97848871 16.30536683 -3.281007578 bf00016
## 569  -9.21529241 16.48288394 -2.192602016 ft00098
## 570   3.27211380  7.14903556 -0.360053279 fx00053
## 571  21.55390506  4.01198996 -5.533163090 cq00036
## 572  12.39617506  6.95947366  0.622089494 ar00020
## 573  -3.90035839 11.09549345  0.103862871 dy00043
## 574 -18.53273826  2.24337445 -2.819632962 au00011
## 575  31.56227684  8.80054799  3.460961482 cj00024
## 576  33.84557686 31.46357604 -3.427737436 er00003
## 577  25.73246702  6.61389056  1.349218546 ei00084
## 578   4.37834299  4.65191695 -1.241860337 ef00049
## 579  22.03201250 16.25257965  2.129305512 eh00081
## 580  11.12929738 31.13976277  6.920675104 et00027
## 581  17.40899656 14.59262357 -1.661244678 fh00070
## 582   7.64638815  1.90045993 -0.021319363 dp00100
## 583  -7.46127375  3.64655449  2.027050312 fh00047
## 584  11.73122856  4.67977360 -5.511079853 cc00004
## 585  11.40890319  9.19999779  0.686423885 be00029
## 586  -3.95153778 13.30033149 -1.128599406 ep00080
```

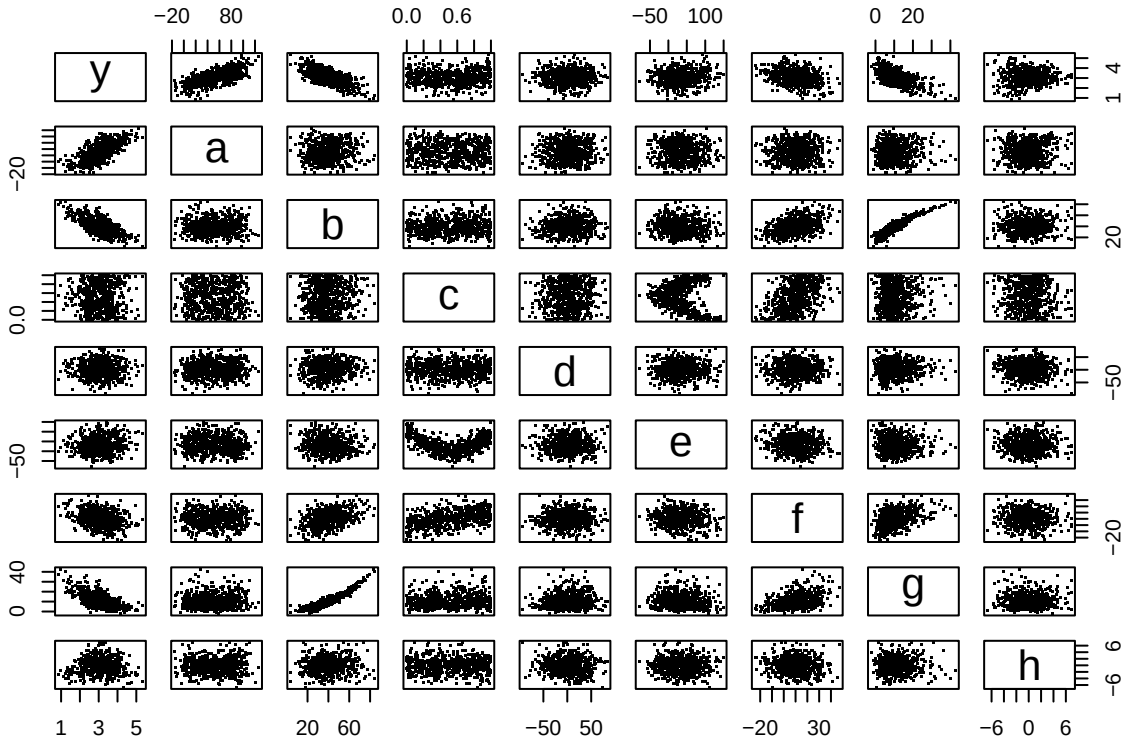
```
## 587   4.97844640 12.84601973  0.180497482 dp00061
## 588  17.34183361 17.63311590  0.493465217 bt00073
## 589  41.47278246 29.93573320 -4.955304133 ce00015
## 590  17.94721070  5.20722211 -0.401218554 cf00017
## 591   4.25894971 19.56585928  2.154336155 bg00008
## 592  -0.64943181  7.37731998  2.354506312 ds00056
## 593  -3.26333117  0.70590452  0.678259317 fu00009
## 594   8.19622036  6.70805327  1.461582533 cw00088
## 595   9.15063723  5.02791232  1.414652418 ef00033
## 596  -2.38092013 13.11538287 -1.948697690 cs00063
## 597   7.84780361  2.98680096 -0.962894256 da00052
## 598  45.22337066 14.87822261 -3.571198885 au00066
## 599  24.85389718 11.11882961 -0.100372090 dt00095
## 600  27.10710802 13.35597124  3.417154901 cm00075
```

```
df_multipar=within(df_multipar, rm(id))
head(df_multipar)
```

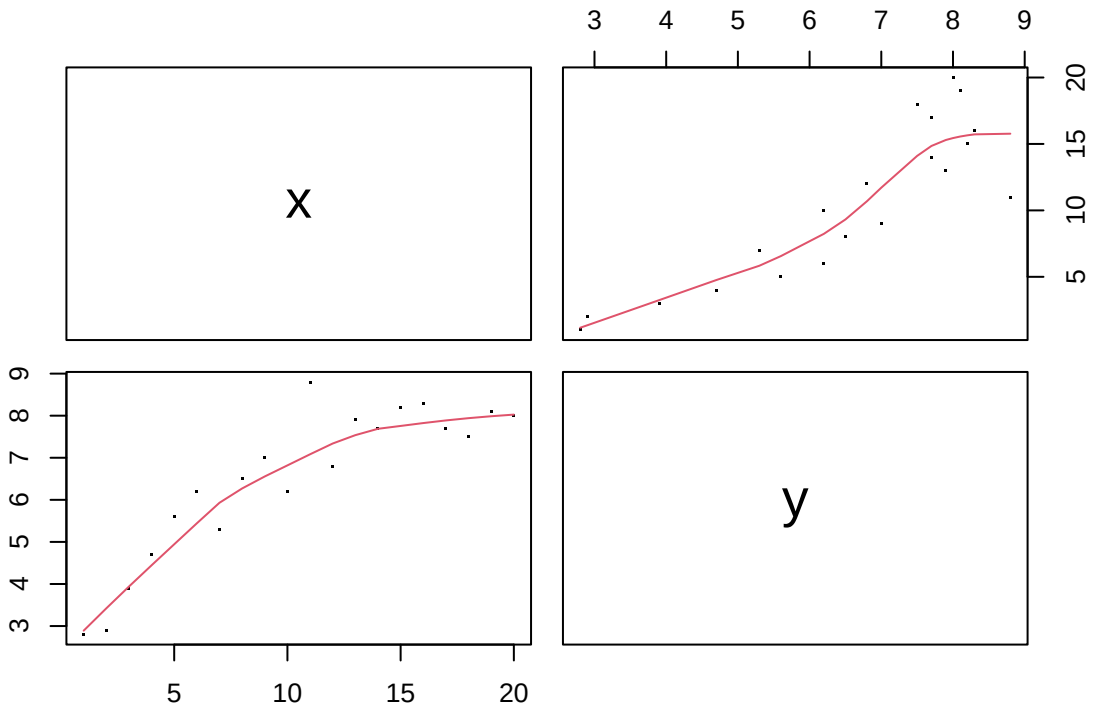
```
##          y        a        b          c          d         e         f
## 1 3.184311 51.42026 27.12527 0.68270165 -16.627974  56.10895  4.619684
## 2 4.078802 90.93125 43.26188 0.94534940   1.571907  47.07957 31.243675
## 3 3.286615 98.71964 57.81349 0.86691667  22.487176 100.08431 29.043086
## 4 3.582366 89.07057 34.81383 0.59954873  11.120274 -35.35449 -5.616698
## 5 3.182100 44.89312 38.22812 0.01036813  13.633426  72.30611  6.095976
## 6 2.464150 33.11191 46.04063 0.39760307 -48.057072  20.49485  6.499197
##           g          h
## 1  3.589122 -0.7818687
## 2 13.066916 -4.6140785
## 3 20.370505  2.2817048
## 4  5.218543 -0.9948896
## 5 11.635066  1.2807478
## 6 14.213027  0.4446310
```

21. Produce a `pairs()` plot of the remaining columns. Make sure you can interpret all the panels. (Note: you may need to change the plot margins to display the output efficiently.)

```
pairs(df_multipar,pch='.')
```




```
pairs(df,pch='.',panel=panel.smooth)
```




```
png('pairs.png', width=1500, height=1500)
pairs(df)
dev.off()
```

```
## pdf
##   2
```

22. Fit a multiple regression for y as a function of all the remaining parameters simultaneously, including interactions up to two-way. Examine its summary and check the values of RSE, R^2, adjusted R^2, and AIC.

```
m4=lm(y~.*., data=df_multipar )

summary(m4)
```

```
##
## Call:
## lm(formula = y ~ . * ., data = df_multipar)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.49720 -0.09626  0.00585  0.09859  0.47471
##
## Coefficients:
```

```
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.585e+00  9.083e-02  39.465  < 2e-16 ***
## a            1.775e-02  1.158e-03  15.339  < 2e-16 ***
## b           -3.497e-02  3.774e-03  -9.266  < 2e-16 ***
## c           -6.790e-02  1.293e-01  -0.525   0.5997
## d            5.698e-04  1.088e-03   0.524   0.6008
## e            1.023e-03  8.391e-04   1.219   0.2234
## f           -1.228e-03  3.109e-03  -0.395   0.6929
## g           -3.945e-03  8.499e-03  -0.464   0.6427
## h           -8.169e-03  1.547e-02  -0.528   0.5977
## a:b         -7.437e-05  4.705e-05  -1.581   0.1145
## a:c          9.581e-04  8.399e-04   1.141   0.2545
## a:d         -5.796e-06  6.990e-06  -0.829   0.4073
## a:e         -4.680e-06  5.429e-06  -0.862   0.3890
## a:f         -1.570e-05  2.159e-05  -0.727   0.4676
## a:g          1.808e-04  9.124e-05   1.982   0.0480 *
## a:h          1.918e-04  9.385e-05   2.044   0.0414 *
## b:c          3.908e-03  5.115e-03   0.764   0.4452
## b:d         -2.235e-05  4.328e-05  -0.516   0.6057
## b:e          2.791e-05  3.260e-05   0.856   0.3922
## b:f         -2.014e-04  1.220e-04  -1.651   0.0993 .
## b:g          1.483e-06  6.639e-05   0.022   0.9822
## b:h          5.547e-04  5.626e-04   0.986   0.3246
## c:d         -1.806e-05  8.368e-04  -0.022   0.9828
## c:e         -1.083e-03  6.550e-04  -1.653   0.0990 .
## c:f          1.231e-02  2.239e-03   5.499 5.81e-08 ***
## c:g         -8.723e-03  9.837e-03  -0.887   0.3756
## c:h          8.097e-03  1.124e-02   0.721   0.4715
## d:e         -4.501e-07  5.206e-06  -0.086   0.9311
## d:f          1.718e-05  2.197e-05   0.782   0.4346
## d:g          3.781e-05  8.767e-05   0.431   0.6664
## d:h          4.139e-05  8.848e-05   0.468   0.6401
## e:f          6.434e-06  1.761e-05   0.365   0.7150
## e:g         -3.008e-05  6.182e-05  -0.487   0.6267
## e:h         -9.577e-05  7.423e-05  -1.290   0.1975
## f:g          2.735e-04  2.478e-04   1.104   0.2701
## f:h         -6.745e-04  2.837e-04  -2.377   0.0178 *
## g:h         -1.127e-03  1.077e-03  -1.046   0.2958
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1534 on 563 degrees of freedom
## Multiple R-squared:  0.9574, Adjusted R-squared:  0.9546
## F-statistic: 351.3 on 36 and 563 DF,  p-value: < 2.2e-16
```

```
AIC(m4)
```

```
## [1] -509.2899
```

23. Perform a (single) stepwise regression simplification with `step()` to simplify the model. (You should probably specify `trace=0` to reduce output to the console.) Examine the`summary` of the simplified model and check the RSE, R^2, adjusted R^2, and AIC.

```r
m5 = step(m4)
```

```
## Start:  AIC=-2214.02
## y ~ (a + b + c + d + e + f + g + h) * (a + b + c + d + e + f +
##     g + h)
##
##        Df Sum of Sq    RSS     AIC
## - c:d   1   0.00001 13.244 -2216.0
## - b:g   1   0.00001 13.244 -2216.0
## - d:e   1   0.00018 13.245 -2216.0
## - e:f   1   0.00314 13.248 -2215.9
## - d:g   1   0.00438 13.249 -2215.8
## - d:h   1   0.00515 13.249 -2215.8
## - e:g   1   0.00557 13.250 -2215.8
## - b:d   1   0.00628 13.251 -2215.7
## - c:h   1   0.01221 13.257 -2215.5
## - a:f   1   0.01243 13.257 -2215.4
## - b:c   1   0.01373 13.258 -2215.4
## - d:f   1   0.01438 13.259 -2215.4
## - a:d   1   0.01618 13.261 -2215.3
## - b:e   1   0.01725 13.262 -2215.2
## - a:e   1   0.01748 13.262 -2215.2
## - c:g   1   0.01850 13.263 -2215.2
## - b:h   1   0.02287 13.267 -2215.0
## - g:h   1   0.02576 13.270 -2214.8
## - f:g   1   0.02867 13.273 -2214.7
## - a:c   1   0.03061 13.275 -2214.6
## - e:h   1   0.03916 13.284 -2214.2
## <none>              13.244 -2214.0
## - a:b   1   0.05877 13.303 -2213.4
## - b:f   1   0.06412 13.309 -2213.1
## - c:e   1   0.06425 13.309 -2213.1
## - a:g   1   0.09237 13.337 -2211.8
## - a:h   1   0.09826 13.343 -2211.6
## - f:h   1   0.13294 13.377 -2210.0
## - c:f   1   0.71126 13.956 -2184.6
##
## Step:  AIC=-2216.02
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:d + b:e + b:f + b:g + b:h + c:e + c:f +
##     c:g + c:h + d:e + d:f + d:g + d:h + e:f + e:g + e:h + f:g +
##     f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - b:g   1   0.00001 13.244 -2218.0
## - d:e   1   0.00017 13.245 -2218.0
## - e:f   1   0.00314 13.248 -2217.9
## - d:g   1   0.00437 13.249 -2217.8
## - d:h   1   0.00516 13.249 -2217.8
## - e:g   1   0.00556 13.250 -2217.8
## - b:d   1   0.00628 13.251 -2217.7
## - c:h   1   0.01228 13.257 -2217.5
## - a:f   1   0.01246 13.257 -2217.4
```

```
## - b:c   1   0.01376 13.258 -2217.4
## - a:d   1   0.01619 13.261 -2217.3
## - d:f   1   0.01702 13.261 -2217.2
## - b:e   1   0.01724 13.262 -2217.2
## - a:e   1   0.01750 13.262 -2217.2
## - c:g   1   0.01860 13.263 -2217.2
## - b:h   1   0.02287 13.267 -2217.0
## - g:h   1   0.02576 13.270 -2216.8
## - f:g   1   0.02868 13.273 -2216.7
## - a:c   1   0.03098 13.275 -2216.6
## - e:h   1   0.03923 13.284 -2216.2
## <none>              13.244 -2216.0
## - a:b   1   0.05876 13.303 -2215.4
## - b:f   1   0.06415 13.309 -2215.1
## - c:e   1   0.06427 13.309 -2215.1
## - a:g   1   0.09239 13.337 -2213.8
## - a:h   1   0.09841 13.343 -2213.6
## - f:h   1   0.13446 13.379 -2212.0
## - c:f   1   0.71364 13.958 -2186.5
##
## Step:  AIC=-2218.02
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:d + b:e + b:f + b:h + c:e + c:f + c:g +
##     c:h + d:e + d:f + d:g + d:h + e:f + e:g + e:h + f:g + f:h +
##     g:h
##
##        Df Sum of Sq    RSS     AIC
## - d:e   1   0.00017 13.245 -2220.0
## - e:f   1   0.00315 13.248 -2219.9
## - d:g   1   0.00439 13.249 -2219.8
## - d:h   1   0.00515 13.249 -2219.8
## - e:g   1   0.00563 13.250 -2219.8
## - b:d   1   0.00629 13.251 -2219.7
## - c:h   1   0.01227 13.257 -2219.5
## - a:f   1   0.01246 13.257 -2219.4
## - b:c   1   0.01377 13.258 -2219.4
## - a:d   1   0.01618 13.261 -2219.3
## - d:f   1   0.01703 13.261 -2219.2
## - b:e   1   0.01731 13.262 -2219.2
## - a:e   1   0.01749 13.262 -2219.2
## - c:g   1   0.01870 13.263 -2219.2
## - b:h   1   0.02293 13.267 -2219.0
## - g:h   1   0.02591 13.270 -2218.8
## - a:c   1   0.03104 13.275 -2218.6
## - f:g   1   0.03107 13.275 -2218.6
## - e:h   1   0.03944 13.284 -2218.2
## <none>              13.244 -2218.0
## - a:b   1   0.05876 13.303 -2217.4
## - c:e   1   0.06426 13.309 -2217.1
## - b:f   1   0.06457 13.309 -2217.1
## - a:g   1   0.09241 13.337 -2215.8
## - a:h   1   0.09846 13.343 -2215.6
## - f:h   1   0.13452 13.379 -2213.9
## - c:f   1   0.71365 13.958 -2188.5
```

```
##
## Step:  AIC=-2220.01
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:d + b:e + b:f + b:h + c:e + c:f + c:g +
##     c:h + d:f + d:g + d:h + e:f + e:g + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - e:f   1   0.00318 13.248 -2221.9
## - d:g   1   0.00439 13.249 -2221.8
## - d:h   1   0.00516 13.250 -2221.8
## - e:g   1   0.00565 13.250 -2221.8
## - b:d   1   0.00623 13.251 -2221.7
## - a:f   1   0.01242 13.257 -2221.4
## - c:h   1   0.01248 13.257 -2221.4
## - b:c   1   0.01368 13.258 -2221.4
## - a:d   1   0.01605 13.261 -2221.3
## - b:e   1   0.01717 13.262 -2221.2
## - d:f   1   0.01718 13.262 -2221.2
## - a:e   1   0.01773 13.262 -2221.2
## - c:g   1   0.01856 13.263 -2221.2
## - b:h   1   0.02288 13.267 -2221.0
## - g:h   1   0.02587 13.270 -2220.8
## - f:g   1   0.03090 13.275 -2220.6
## - a:c   1   0.03095 13.275 -2220.6
## - e:h   1   0.03927 13.284 -2220.2
## <none>              13.245 -2220.0
## - a:b   1   0.05859 13.303 -2219.4
## - b:f   1   0.06440 13.309 -2219.1
## - c:e   1   0.06501 13.309 -2219.1
## - a:g   1   0.09224 13.337 -2217.8
## - a:h   1   0.09896 13.344 -2217.5
## - f:h   1   0.13632 13.381 -2215.9
## - c:f   1   0.71349 13.958 -2190.5
##
## Step:  AIC=-2221.86
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:d + b:e + b:f + b:h + c:e + c:f + c:g +
##     c:h + d:f + d:g + d:h + e:g + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - d:g   1   0.00468 13.252 -2223.7
## - d:h   1   0.00593 13.254 -2223.6
## - e:g   1   0.00604 13.254 -2223.6
## - b:d   1   0.00648 13.254 -2223.6
## - a:f   1   0.01248 13.260 -2223.3
## - c:h   1   0.01323 13.261 -2223.3
## - b:c   1   0.01370 13.261 -2223.2
## - a:d   1   0.01619 13.264 -2223.1
## - d:f   1   0.01774 13.265 -2223.1
## - c:g   1   0.01806 13.266 -2223.1
## - a:e   1   0.01899 13.267 -2223.0
## - b:e   1   0.02256 13.270 -2222.8
## - b:h   1   0.02306 13.271 -2222.8
## - g:h   1   0.02548 13.273 -2222.7
```

```
## - f:g   1   0.03064 13.278 -2222.5
## - a:c   1   0.03163 13.279 -2222.4
## - e:h   1   0.04013 13.288 -2222.1
## <none>              13.248 -2221.9
## - a:b   1   0.05935 13.307 -2221.2
## - b:f   1   0.06541 13.313 -2220.9
## - c:e   1   0.06592 13.314 -2220.9
## - a:g   1   0.09314 13.341 -2219.7
## - a:h   1   0.09848 13.346 -2219.4
## - f:h   1   0.14011 13.388 -2217.6
## - c:f   1   0.71354 13.961 -2192.4
##
## Step:  AIC=-2223.65
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:d + b:e + b:f + b:h + c:e + c:f + c:g +
##     c:h + d:f + d:h + e:g + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - b:d   1   0.00196 13.254 -2225.6
## - d:h   1   0.00557 13.258 -2225.4
## - e:g   1   0.00586 13.258 -2225.4
## - a:f   1   0.01250 13.265 -2225.1
## - c:h   1   0.01332 13.266 -2225.1
## - b:c   1   0.01408 13.267 -2225.0
## - a:d   1   0.01623 13.269 -2224.9
## - a:e   1   0.01794 13.270 -2224.8
## - c:g   1   0.01845 13.271 -2224.8
## - d:f   1   0.01886 13.271 -2224.8
## - b:e   1   0.02247 13.275 -2224.6
## - b:h   1   0.02451 13.277 -2224.5
## - g:h   1   0.02782 13.280 -2224.4
## - a:c   1   0.03245 13.285 -2224.2
## - f:g   1   0.03329 13.286 -2224.2
## - e:h   1   0.04148 13.294 -2223.8
## <none>              13.252 -2223.7
## - a:b   1   0.06417 13.316 -2222.8
## - c:e   1   0.06592 13.318 -2222.7
## - b:f   1   0.06951 13.322 -2222.5
## - a:h   1   0.10116 13.354 -2221.1
## - a:g   1   0.10128 13.354 -2221.1
## - f:h   1   0.14083 13.393 -2219.3
## - c:f   1   0.71036 13.963 -2194.3
##
## Step:  AIC=-2225.56
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:e + b:f + b:h + c:e + c:f + c:g + c:h +
##     d:f + d:h + e:g + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - d:h   1   0.00569 13.260 -2227.3
## - e:g   1   0.00601 13.260 -2227.3
## - a:f   1   0.01202 13.266 -2227.0
## - c:h   1   0.01313 13.268 -2227.0
## - b:c   1   0.01428 13.269 -2226.9
```

```
## - d:f   1   0.01706 13.271 -2226.8
## - a:d   1   0.01722 13.272 -2226.8
## - a:e   1   0.01771 13.272 -2226.8
## - c:g   1   0.01839 13.273 -2226.7
## - b:e   1   0.02254 13.277 -2226.5
## - b:h   1   0.02424 13.279 -2226.5
## - g:h   1   0.02716 13.281 -2226.3
## - a:c   1   0.03214 13.287 -2226.1
## - f:g   1   0.03399 13.288 -2226.0
## - e:h   1   0.04207 13.296 -2225.7
## <none>              13.254 -2225.6
## - a:b   1   0.06507 13.319 -2224.6
## - c:e   1   0.06594 13.320 -2224.6
## - b:f   1   0.07112 13.325 -2224.3
## - a:g   1   0.10137 13.356 -2223.0
## - a:h   1   0.10255 13.357 -2222.9
## - f:h   1   0.14042 13.395 -2221.2
## - c:f   1   0.71141 13.966 -2196.2
##
## Step:  AIC=-2227.31
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:e + b:f + b:h + c:e + c:f + c:g + c:h +
##     d:f + e:g + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - e:g   1   0.00613 13.266 -2229.0
## - a:f   1   0.01140 13.271 -2228.8
## - c:h   1   0.01343 13.274 -2228.7
## - b:c   1   0.01444 13.274 -2228.7
## - d:f   1   0.01675 13.277 -2228.6
## - a:d   1   0.01812 13.278 -2228.5
## - c:g   1   0.01815 13.278 -2228.5
## - a:e   1   0.01939 13.279 -2228.4
## - b:e   1   0.02345 13.284 -2228.2
## - b:h   1   0.02497 13.285 -2228.2
## - g:h   1   0.02672 13.287 -2228.1
## - a:c   1   0.03112 13.291 -2227.9
## - f:g   1   0.03280 13.293 -2227.8
## - e:h   1   0.04049 13.300 -2227.5
## <none>              13.260 -2227.3
## - c:e   1   0.06686 13.327 -2226.3
## - a:b   1   0.06736 13.327 -2226.3
## - b:f   1   0.07046 13.331 -2226.1
## - a:g   1   0.10342 13.363 -2224.6
## - a:h   1   0.11153 13.371 -2224.3
## - f:h   1   0.14855 13.409 -2222.6
## - c:f   1   0.72273 13.983 -2197.5
##
## Step:  AIC=-2229.03
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:f +
##     a:g + a:h + b:c + b:e + b:f + b:h + c:e + c:f + c:g + c:h +
##     d:f + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
```

```
## - a:f   1   0.01182 13.278 -2230.5
## - b:c   1   0.01415 13.280 -2230.4
## - c:h   1   0.01419 13.280 -2230.4
## - d:f   1   0.01599 13.282 -2230.3
## - c:g   1   0.01774 13.284 -2230.2
## - a:d   1   0.01973 13.286 -2230.1
## - a:e   1   0.02038 13.287 -2230.1
## - b:h   1   0.02246 13.289 -2230.0
## - g:h   1   0.02390 13.290 -2229.9
## - a:c   1   0.03180 13.298 -2229.6
## - f:g   1   0.03256 13.299 -2229.6
## - e:h   1   0.03852 13.305 -2229.3
## <none>              13.266 -2229.0
## - b:e   1   0.04676 13.313 -2228.9
## - a:b   1   0.06421 13.330 -2228.1
## - c:e   1   0.06730 13.333 -2228.0
## - b:f   1   0.06952 13.336 -2227.9
## - a:g   1   0.10016 13.366 -2226.5
## - a:h   1   0.11143 13.378 -2226.0
## - f:h   1   0.15092 13.417 -2224.2
## - c:f   1   0.71907 13.985 -2199.4
##
## Step:  AIC=-2230.49
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:g +
##     a:h + b:c + b:e + b:f + b:h + c:e + c:f + c:g + c:h + d:f +
##     e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - d:f   1   0.01320 13.291 -2231.9
## - b:c   1   0.01547 13.293 -2231.8
## - c:h   1   0.01559 13.294 -2231.8
## - a:d   1   0.01890 13.297 -2231.6
## - c:g   1   0.01966 13.298 -2231.6
## - b:h   1   0.02036 13.298 -2231.6
## - a:e   1   0.02116 13.299 -2231.5
## - a:c   1   0.02139 13.299 -2231.5
## - g:h   1   0.02213 13.300 -2231.5
## - f:g   1   0.03732 13.315 -2230.8
## - e:h   1   0.03906 13.317 -2230.7
## <none>              13.278 -2230.5
## - b:e   1   0.04774 13.326 -2230.3
## - c:e   1   0.06758 13.346 -2229.4
## - b:f   1   0.07602 13.354 -2229.1
## - a:b   1   0.07894 13.357 -2228.9
## - a:g   1   0.10283 13.381 -2227.9
## - a:h   1   0.11136 13.389 -2227.5
## - f:h   1   0.15499 13.433 -2225.5
## - c:f   1   0.70817 13.986 -2201.3
##
## Step:  AIC=-2231.9
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:g +
##     a:h + b:c + b:e + b:f + b:h + c:e + c:f + c:g + c:h + e:h +
##     f:g + f:h + g:h
##
```

```
##        Df Sum of Sq    RSS     AIC
## - b:c   1   0.01568 13.307 -2233.2
## - a:d   1   0.01806 13.309 -2233.1
## - c:h   1   0.01885 13.310 -2233.1
## - c:g   1   0.01970 13.311 -2233.0
## - a:e   1   0.02161 13.313 -2232.9
## - b:h   1   0.02191 13.313 -2232.9
## - a:c   1   0.02243 13.314 -2232.9
## - g:h   1   0.02417 13.315 -2232.8
## - e:h   1   0.03549 13.327 -2232.3
## - f:g   1   0.03579 13.327 -2232.3
## <none>              13.291 -2231.9
## - b:e   1   0.04870 13.340 -2231.7
## - c:e   1   0.06534 13.357 -2231.0
## - b:f   1   0.07510 13.366 -2230.5
## - a:b   1   0.07707 13.368 -2230.4
## - a:g   1   0.10225 13.393 -2229.3
## - a:h   1   0.11446 13.406 -2228.8
## - f:h   1   0.16909 13.460 -2226.3
## - c:f   1   0.70271 13.994 -2203.0
##
## Step:  AIC=-2233.19
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:g +
##     a:h + b:e + b:f + b:h + c:e + c:f + c:g + c:h + e:h + f:g +
##     f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - c:g   1   0.00411 13.311 -2235.0
## - a:d   1   0.01895 13.326 -2234.3
## - a:c   1   0.01937 13.326 -2234.3
## - b:h   1   0.01948 13.326 -2234.3
## - a:e   1   0.02032 13.327 -2234.3
## - g:h   1   0.02162 13.329 -2234.2
## - f:g   1   0.02340 13.330 -2234.1
## - c:h   1   0.02525 13.332 -2234.1
## - e:h   1   0.03468 13.341 -2233.6
## <none>              13.307 -2233.2
## - b:e   1   0.04864 13.355 -2233.0
## - b:f   1   0.05955 13.366 -2232.5
## - c:e   1   0.06552 13.372 -2232.2
## - a:b   1   0.06959 13.376 -2232.1
## - a:g   1   0.09399 13.401 -2231.0
## - a:h   1   0.10842 13.415 -2230.3
## - f:h   1   0.17196 13.479 -2227.5
## - c:f   1   0.76664 14.073 -2201.6
##
## Step:  AIC=-2235
## y ~ a + b + c + d + e + f + g + h + a:b + a:c + a:d + a:e + a:g +
##     a:h + b:e + b:f + b:h + c:e + c:f + c:h + e:h + f:g + f:h +
##     g:h
##
##        Df Sum of Sq    RSS     AIC
## - a:c   1   0.01895 13.330 -2236.2
## - a:d   1   0.01978 13.331 -2236.1
```

```
## - a:e   1   0.01983 13.331 -2236.1
## - b:h   1   0.02012 13.331 -2236.1
## - f:g   1   0.02069 13.332 -2236.1
## - g:h   1   0.02212 13.333 -2236.0
## - c:h   1   0.02490 13.336 -2235.9
## - e:h   1   0.03350 13.345 -2235.5
## <none>              13.311 -2235.0
## - b:e   1   0.04807 13.359 -2234.8
## - b:f   1   0.05848 13.369 -2234.4
## - c:e   1   0.06352 13.374 -2234.2
## - a:b   1   0.06897 13.380 -2233.9
## - a:g   1   0.09463 13.406 -2232.8
## - a:h   1   0.11261 13.424 -2231.9
## - f:h   1   0.17662 13.488 -2229.1
## - c:f   1   0.82010 14.131 -2201.1
##
## Step:  AIC=-2236.15
## y ~ a + b + c + d + e + f + g + h + a:b + a:d + a:e + a:g + a:h +
##     b:e + b:f + b:h + c:e + c:f + c:h + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - a:d   1   0.01876 13.349 -2237.3
## - b:h   1   0.02052 13.350 -2237.2
## - f:g   1   0.02164 13.352 -2237.2
## - g:h   1   0.02259 13.352 -2237.1
## - a:e   1   0.02393 13.354 -2237.1
## - c:h   1   0.02754 13.357 -2236.9
## - e:h   1   0.03796 13.368 -2236.4
## <none>              13.330 -2236.2
## - b:e   1   0.04917 13.379 -2235.9
## - b:f   1   0.05879 13.389 -2235.5
## - c:e   1   0.06673 13.397 -2235.2
## - a:b   1   0.06750 13.397 -2235.1
## - a:g   1   0.09582 13.426 -2233.8
## - a:h   1   0.10324 13.433 -2233.5
## - f:h   1   0.18560 13.515 -2229.8
## - c:f   1   0.80581 14.136 -2202.9
##
## Step:  AIC=-2237.31
## y ~ a + b + c + d + e + f + g + h + a:b + a:e + a:g + a:h + b:e +
##     b:f + b:h + c:e + c:f + c:h + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - d     1   0.00268 13.351 -2239.2
## - b:h   1   0.02127 13.370 -2238.3
## - f:g   1   0.02162 13.370 -2238.3
## - g:h   1   0.02340 13.372 -2238.3
## - a:e   1   0.02667 13.375 -2238.1
## - c:h   1   0.03028 13.379 -2237.9
## - e:h   1   0.03462 13.383 -2237.8
## <none>              13.349 -2237.3
## - b:e   1   0.05071 13.399 -2237.0
## - b:f   1   0.05857 13.407 -2236.7
## - c:e   1   0.06677 13.415 -2236.3
```

```
## - a:b   1   0.07345 13.422 -2236.0
## - a:g   1   0.09985 13.448 -2234.8
## - a:h   1   0.10546 13.454 -2234.6
## - f:h   1   0.18943 13.538 -2230.8
## - c:f   1   0.81101 14.160 -2203.9
##
## Step:  AIC=-2239.19
## y ~ a + b + c + e + f + g + h + a:b + a:e + a:g + a:h + b:e +
##     b:f + b:h + c:e + c:f + c:h + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - b:h   1   0.02061 13.372 -2240.3
## - f:g   1   0.02116 13.373 -2240.2
## - g:h   1   0.02296 13.374 -2240.2
## - a:e   1   0.02715 13.379 -2240.0
## - c:h   1   0.02965 13.381 -2239.9
## - e:h   1   0.03522 13.387 -2239.6
## <none>              13.351 -2239.2
## - b:e   1   0.05174 13.403 -2238.9
## - b:f   1   0.05772 13.409 -2238.6
## - c:e   1   0.06686 13.418 -2238.2
## - a:b   1   0.07589 13.427 -2237.8
## - a:g   1   0.10242 13.454 -2236.6
## - a:h   1   0.10732 13.459 -2236.4
## - f:h   1   0.18794 13.539 -2232.8
## - c:f   1   0.81633 14.168 -2205.6
##
## Step:  AIC=-2240.26
## y ~ a + b + c + e + f + g + h + a:b + a:e + a:g + a:h + b:e +
##     b:f + c:e + c:f + c:h + e:h + f:g + f:h + g:h
##
##        Df Sum of Sq    RSS     AIC
## - g:h   1   0.00260 13.375 -2242.1
## - c:h   1   0.02134 13.393 -2241.3
## - f:g   1   0.02477 13.397 -2241.2
## - a:e   1   0.02494 13.397 -2241.1
## - e:h   1   0.03831 13.410 -2240.5
## <none>              13.372 -2240.3
## - b:e   1   0.04568 13.418 -2240.2
## - b:f   1   0.06349 13.435 -2239.4
## - c:e   1   0.06575 13.438 -2239.3
## - a:b   1   0.09618 13.468 -2238.0
## - a:g   1   0.11371 13.486 -2237.2
## - a:h   1   0.11472 13.487 -2237.1
## - f:h   1   0.16986 13.542 -2234.7
## - c:f   1   0.81317 14.185 -2206.8
##
## Step:  AIC=-2242.14
## y ~ a + b + c + e + f + g + h + a:b + a:e + a:g + a:h + b:e +
##     b:f + c:e + c:f + c:h + e:h + f:g + f:h
##
##        Df Sum of Sq    RSS     AIC
## - a:e   1   0.02488 13.399 -2243.0
## - f:g   1   0.02498 13.399 -2243.0
```

```
## - c:h   1   0.02560 13.400 -2243.0
## - e:h   1   0.03643 13.411 -2242.5
## <none>              13.375 -2242.1
## - b:e   1   0.04766 13.422 -2242.0
## - b:f   1   0.06324 13.438 -2241.3
## - c:e   1   0.06407 13.439 -2241.3
## - a:b   1   0.09387 13.468 -2239.9
## - a:h   1   0.11238 13.487 -2239.1
## - a:g   1   0.11519 13.490 -2239.0
## - f:h   1   0.23090 13.605 -2233.9
## - c:f   1   0.81177 14.186 -2208.8
##
## Step:  AIC=-2243.03
## y ~ a + b + c + e + f + g + h + a:b + a:g + a:h + b:e + b:f +
##     c:e + c:f + c:h + e:h + f:g + f:h
##
##        Df Sum of Sq    RSS     AIC
## - f:g   1   0.02416 13.424 -2243.9
## - c:h   1   0.02905 13.428 -2243.7
## - e:h   1   0.03676 13.436 -2243.4
## - b:e   1   0.04175 13.441 -2243.2
## <none>              13.399 -2243.0
## - b:f   1   0.06225 13.462 -2242.2
## - c:e   1   0.06417 13.464 -2242.2
## - a:b   1   0.08662 13.486 -2241.2
## - a:g   1   0.10763 13.507 -2240.2
## - a:h   1   0.12130 13.521 -2239.6
## - f:h   1   0.23118 13.631 -2234.8
## - c:f   1   0.80223 14.202 -2210.1
##
## Step:  AIC=-2243.95
## y ~ a + b + c + e + f + g + h + a:b + a:g + a:h + b:e + b:f +
##     c:e + c:f + c:h + e:h + f:h
##
##        Df Sum of Sq    RSS     AIC
## - c:h   1   0.02898 13.453 -2244.7
## - e:h   1   0.03363 13.457 -2244.4
## - b:e   1   0.03792 13.461 -2244.3
## <none>              13.424 -2243.9
## - c:e   1   0.06266 13.486 -2243.2
## - a:b   1   0.08417 13.508 -2242.2
## - b:f   1   0.09053 13.514 -2241.9
## - a:g   1   0.10238 13.526 -2241.4
## - a:h   1   0.11844 13.542 -2240.7
## - f:h   1   0.24563 13.669 -2235.1
## - c:f   1   0.79899 14.223 -2211.3
##
## Step:  AIC=-2244.65
## y ~ a + b + c + e + f + g + h + a:b + a:g + a:h + b:e + b:f +
##     c:e + c:f + e:h + f:h
##
##        Df Sum of Sq    RSS     AIC
## - e:h   1   0.03183 13.484 -2245.2
## - b:e   1   0.03548 13.488 -2245.1
```

```
## <none>              13.453 -2244.7
## - c:e   1   0.06592 13.518 -2243.7
## - b:f   1   0.08311 13.536 -2243.0
## - a:b   1   0.08589 13.539 -2242.8
## - a:g   1   0.10173 13.554 -2242.1
## - a:h   1   0.12251 13.575 -2241.2
## - f:h   1   0.21699 13.670 -2237.1
## - c:f   1   0.77961 14.232 -2212.8
##
## Step:  AIC=-2245.24
## y ~ a + b + c + e + f + g + h + a:b + a:g + a:h + b:e + b:f +
##     c:e + c:f + f:h
##
##        Df Sum of Sq    RSS     AIC
## - b:e   1   0.02676 13.511 -2246.1
## <none>              13.484 -2245.2
## - c:e   1   0.06501 13.549 -2244.3
## - b:f   1   0.07967 13.564 -2243.7
## - a:b   1   0.08652 13.571 -2243.4
## - a:g   1   0.10397 13.588 -2242.6
## - a:h   1   0.12397 13.608 -2241.8
## - f:h   1   0.19333 13.678 -2238.7
## - c:f   1   0.77491 14.259 -2213.7
##
## Step:  AIC=-2246.05
## y ~ a + b + c + e + f + g + h + a:b + a:g + a:h + b:f + c:e +
##     c:f + f:h
##
##        Df Sum of Sq    RSS     AIC
## <none>              13.511 -2246.1
## - c:e   1   0.06684 13.578 -2245.1
## - a:b   1   0.07705 13.588 -2244.6
## - b:f   1   0.08931 13.601 -2244.1
## - a:g   1   0.09463 13.606 -2243.9
## - a:h   1   0.12057 13.632 -2242.7
## - f:h   1   0.20297 13.714 -2239.1
## - c:f   1   0.77215 14.283 -2214.7
```

```
m5 = step(m4, trace=0)
summary(m5)
```

```
##
## Call:
## lm(formula = y ~ a + b + c + e + f + g + h + a:b + a:g + a:h +
##     b:f + c:e + c:f + f:h, data = df_multipar)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.49316 -0.09907  0.00321  0.10281  0.48921
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.520e+00  6.268e-02  56.163  < 2e-16 ***
## a            1.816e-02  9.447e-04  19.221  < 2e-16 ***
```

```
## b           -3.334e-02  2.654e-03 -12.565  < 2e-16 ***
## c            4.674e-02  4.786e-02   0.977  0.32918
## e            1.586e-03  3.242e-04   4.892 1.29e-06 ***
## f           -3.558e-03  1.674e-03  -2.126  0.03396 *
## g           -5.086e-03  5.099e-03  -0.997  0.31899
## h            1.356e-03  6.168e-03   0.220  0.82608
## a:b         -7.662e-05  4.195e-05  -1.826  0.06829 .
## a:g          1.654e-04  8.169e-05   2.024  0.04341 *
## a:h          2.036e-04  8.911e-05   2.285  0.02268 *
## b:f         -7.218e-05  3.670e-05  -1.966  0.04972 *
## c:e         -9.772e-04  5.744e-04  -1.701  0.08944 .
## c:f          1.158e-02  2.002e-03   5.782 1.20e-08 ***
## f:h         -6.419e-04  2.165e-04  -2.964  0.00316 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.152 on 585 degrees of freedom
## Multiple R-squared:  0.9565, Adjusted R-squared:  0.9555
## F-statistic: 919.2 on 14 and 585 DF,  p-value: < 2.2e-16
```

```
AIC(m5)
```

```
## [1] -541.3211
```

24. Simplify the model further by removing some additional interactions of low significance out of the linear model using `update()`. Give final estimates of RSE, R^2, adjusted R^2, and AIC.

```
m6 = m5
m6 = update(m6,~.-c:g)

summary(m6)
```

```
##
## Call:
## lm(formula = y ~ a + b + c + e + f + g + h + a:b + a:g + a:h +
##     b:f + c:e + c:f + f:h, data = df_multipar)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.49316 -0.09907  0.00321  0.10281  0.48921
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.520e+00  6.268e-02  56.163  < 2e-16 ***
## a            1.816e-02  9.447e-04  19.221  < 2e-16 ***
## b           -3.334e-02  2.654e-03 -12.565  < 2e-16 ***
## c            4.674e-02  4.786e-02   0.977  0.32918
## e            1.586e-03  3.242e-04   4.892 1.29e-06 ***
## f           -3.558e-03  1.674e-03  -2.126  0.03396 *
## g           -5.086e-03  5.099e-03  -0.997  0.31899
## h            1.356e-03  6.168e-03   0.220  0.82608
## a:b         -7.662e-05  4.195e-05  -1.826  0.06829 .
```

```
## a:g          1.654e-04  8.169e-05   2.024  0.04341 *
## a:h          2.036e-04  8.911e-05   2.285  0.02268 *
## b:f         -7.218e-05  3.670e-05  -1.966  0.04972 *
## c:e         -9.772e-04  5.744e-04  -1.701  0.08944 .
## c:f          1.158e-02  2.002e-03   5.782 1.20e-08 ***
## f:h         -6.419e-04  2.165e-04  -2.964  0.00316 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.152 on 585 degrees of freedom
## Multiple R-squared:  0.9565, Adjusted R-squared:  0.9555
## F-statistic: 919.2 on 14 and 585 DF,  p-value: < 2.2e-16
```

```
AIC(m6)
```

```
## [1] -541.3211
```

Multiple Regression: Real Data

Next consider some real data on driving fatalities in the United States during the mid-1980's. This is available in fatality.csv. The columns are:

state: state ID code
year: year
mrall: traffic fatality rate (deaths per 10000)
beertax: tax on case of beer
mlda: minimum legal drinking age
jaild: mandatory jail sentence (yes/no)
comserd: mandatory community service (yes/no)
vmiles: average miles per driver
unrate: unemployment rate
perinc: per capita personal income

This data set has some pseudoreplication features (repeat measurements in time as well as spatial/geographical effects), so significance statements should be treated with great care, but it still represents a useful attempt to put linear models in to practice.

25. Load the data from disk. Convert the "state" numeric code to a factor.

```
df_fat=read.csv('fatality.csv')
df_fat
```

```
##     state year   mrall    beertax  mlda jaild comserd    vmiles unrate
## 1       1 1982 2.12836 1.53937948 19.00    no      no  7.233887   14.4
## 2       1 1983 2.34848 1.78899074 19.00    no      no  7.836348   13.7
## 3       1 1984 2.33643 1.71428561 19.00    no      no  8.262990   11.1
## 4       1 1985 2.19348 1.65254235 19.67    no      no  8.726917    8.9
## 5       1 1986 2.66914 1.60990703 21.00    no      no  8.952854    9.8
## 6       1 1987 2.71859 1.55999994 21.00    no      no  9.166302    7.8
## 7       1 1988 2.49391 1.50144362 21.00    no      no  9.674323    7.2
```

```
## 8       4 1982 2.49914 0.21479714 19.00   yes     yes  6.810157    9.9
## 9       4 1983 2.26738 0.20642203 19.00   yes     yes  6.587495    9.1
## 10      4 1984 2.82878 0.29670331 19.00   yes     yes  6.709970    5.0
## 11      4 1985 2.80201 0.38135594 21.00   yes     yes  6.771263    6.5
## 12      4 1986 3.07106 0.37151703 21.00   yes     yes  8.129008    6.9
## 13      4 1987 2.76728 0.36000001 21.00   yes     yes  9.370654    6.2
## 14      4 1988 2.70565 0.34648702 21.00   yes     yes  9.815721    6.3
## 15      5 1982 2.38405 0.65035802 21.00    no      no  7.208500    9.8
## 16      5 1983 2.39570 0.67545873 21.00    no      no  7.175917   10.1
## 17      5 1984 2.23785 0.59890109 21.00    no      no  7.084820    8.9
## 18      5 1985 2.26367 0.57733053 21.00    no      no  7.253918    8.7
## 19      5 1986 2.54323 0.56243551 21.00    no      no  7.468999    8.7
## 20      5 1987 2.67588 0.54500002 21.00    no      no  7.665831    8.1
## 21      5 1988 2.54697 0.52454287 21.00    no      no  8.024625    7.7
## 22      6 1982 1.86194 0.10739857 21.00    no      no  6.858677    9.9
## 23      6 1983 1.80672 0.10321102 21.00    no      no  7.216292    9.7
## 24      6 1984 1.94611 0.09890110 21.00    no      no  7.619176    7.8
## 25      6 1985 1.88128 0.09533899 21.00    no      no  7.874067    7.2
## 26      6 1986 1.94548 0.09287926 21.00    no      no  8.034910    6.7
## 27      6 1987 1.98966 0.09000000 21.00    no      no  8.180633    5.8
## 28      6 1988 1.90365 0.08662175 21.00    no      no  8.531990    5.3
## 29      8 1982 2.17448 0.21479714 21.00    no     yes  7.742842    7.7
## 30      8 1983 2.05144 0.20642203 21.00    no     yes  7.656063    6.6
## 31      8 1984 1.90596 0.19780220 21.00    no     yes  7.707853    5.6
## 32      8 1985 1.79201 0.19067797 21.00    no     yes  8.092209    5.9
## 33      8 1986 1.84630 0.18575852 21.00    no     yes  8.131375    7.4
## 34      8 1987 1.79308 0.18000001 21.00    no     yes  8.182028    7.7
## 35      8 1988 1.50560 0.17324351 21.00    no     yes  8.380769    6.4
## 36      9 1982 1.64695 0.22434367 18.50    no      no  6.440054    6.9
## 37      9 1983 1.39490 0.23356308 19.25    no      no  6.570043    6.0
## 38      9 1984 1.48653 0.24801099 20.00    no      no  6.680193    4.6
## 39      9 1985 1.41147 0.23907840 20.33   yes     yes  6.979215    4.9
## 40      9 1986 1.40933 0.23291023 21.00   yes     yes  7.661745    3.8
## 41      9 1987 1.39832 0.22569001 21.00   yes     yes  8.338534    3.3
## 42      9 1988 1.49706 0.21721849 21.00   yes     yes  8.061235    3.0
## 43     10 1982 2.03333 0.17303102 20.00    no      no  7.651654    8.5
## 44     10 1983 1.81518 0.16628440 20.00    no      no  8.062700    8.1
## 45     10 1984 2.11726 0.15934065 21.00    no      no  8.368063    6.2
## 46     10 1985 1.67203 0.15360169 21.00    no      no  8.625425    5.3
## 47     10 1986 2.14850 0.14963880 21.00    no      no  9.045817    4.3
## 48     10 1987 2.26708 0.14500000 21.00    no      no  9.450308    3.2
## 49     10 1988 2.42424 0.13955726 21.00    no      no  9.703021    3.2
## 50     12 1982 2.53197 1.07398570 19.00    no     yes  7.587130    8.2
## 51     12 1983 2.49768 1.17041278 19.00    no     yes  7.604254    8.6
## 52     12 1984 2.54661 1.18681324 19.00    no     yes  7.735305    6.3
## 53     12 1985 2.49164 1.14406788 20.00    no     yes  7.747311    6.0
## 54     12 1986 2.42004 1.11455119 21.00    no     yes  7.768756    5.7
## 55     12 1987 2.36131 1.08000004 21.00    no     yes  7.788331    5.3
## 56     12 1988 2.49534 1.03946102 21.00    no     yes  8.538230    5.0
## 57     13 1982 2.17484 2.72076368 19.00    no      no  8.623444    7.8
## 58     13 1983 2.26060 2.61467886 19.00    no      no  8.518590    7.5
## 59     13 1984 2.41356 2.50549436 19.00    no      no  8.641914    6.0
## 60     13 1985 2.27744 2.41525412 19.25    no      no  8.988107    6.5
## 61     13 1986 2.50820 2.35294104 20.25    no      no  9.344768    5.9
```

```
## 62     13 1987 2.56991 2.27999997 21.00    no      no  9.690281    5.5
## 63     13 1988 2.60643 2.19441772 21.00    no      no  9.817396    5.8
## 64     16 1982 2.61759 0.40274465 19.00    no      no  8.033752    9.8
## 65     16 1983 2.66194 0.38704130 19.00    no      no  8.387642    9.8
## 66     16 1984 2.42242 0.37087911 19.00    no      no  7.775768    7.2
## 67     16 1985 2.53731 0.35752121 19.00    no      no  7.671632    7.9
## 68     16 1986 2.57485 0.34829721 19.00    no      no  7.899201    8.7
## 69     16 1987 2.62525 0.33750001 20.50    no      no  8.135269    8.0
## 70     16 1988 2.56231 0.32483158 21.00    no      no  8.102682    5.8
## 71     17 1982 1.43840 0.18854415 21.00    no      no  5.696535   11.3
## 72     17 1983 1.32800 0.18119267 21.00    no      no  5.862868   11.4
## 73     17 1984 1.34265 0.17362638 21.00    no      no  6.067528    9.1
## 74     17 1985 1.32987 0.16737288 21.00    no      no  6.141676    9.0
## 75     17 1986 1.38170 0.16305470 21.00    no      no  6.345777    8.1
## 76     17 1987 1.43326 0.15800001 21.00    no      no  6.540846    7.4
## 77     17 1988 1.58171 0.15206930 21.00    no      no  6.757613    6.8
## 78     18 1982 1.75269 0.30913246 21.00    no      no  7.149917   11.9
## 79     18 1983 1.85605 0.29707912 21.00    no      no  7.277506   11.1
## 80     18 1984 1.68427 0.28467363 21.00    no      no  7.478887    8.6
## 81     18 1985 1.77123 0.27442053 21.00    no      no  7.416253    7.9
## 82     18 1986 1.88624 0.26734054 21.00    no      no  7.714322    6.7
## 83     18 1987 1.90743 0.25905299 21.00    no      no  7.977216    6.4
## 84     18 1988 1.98164 0.24932915 21.00    no      no  9.201576    5.3
## 85     19 1982 1.65119 0.37589499 19.00    no      no  6.653263    8.5
## 86     19 1983 1.76997 0.36123854 19.00    no      no  6.770308    8.1
## 87     19 1984 1.44678 0.34615386 19.00    no      no  7.060630    7.0
## 88     19 1985 1.64355 0.33368644 19.00    no      no  7.001038    8.0
## 89     19 1986 1.54737 0.38312694 20.00    no      no  7.192990    7.0
## 90     19 1987 1.73253 0.42750001 21.00    no      no  7.342257    5.5
## 91     19 1988 1.96542 0.48492688 21.00    no      no  7.730063    4.5
## 92     20 1982 2.06811 0.48400956 21.00   yes     yes  7.333069    6.3
## 93     20 1983 1.69345 0.46513763 21.00   yes     yes  7.479611    6.1
## 94     20 1984 2.09016 0.44571429 21.00   yes     yes  7.670888    5.2
## 95     20 1985 1.98367 0.42966104 21.00   yes     yes  7.867333    5.0
## 96     20 1986 2.03335 0.41857585 21.00   yes     yes  8.100053    5.4
## 97     20 1987 1.98304 0.40560001 21.00   yes     yes  8.304131    4.9
## 98     20 1988 1.93587 0.39037538 21.00   yes     yes  8.481355    4.8
## 99     21 1982 2.22523 0.21599045 21.00    no      no  6.937466   10.6
## 100    21 1983 2.09478 0.20756879 21.00    no      no  7.194143   11.7
## 101    21 1984 2.02688 0.19890109 21.00    no      no  7.513703    9.3
## 102    21 1985 1.91090 0.19173728 21.00    no      no  7.654335    9.5
## 103    21 1986 2.16049 0.18679050 21.00    no      no  7.859621    9.3
## 104    21 1987 2.26456 0.18099999 21.00    no      no  8.135244    8.8
## 105    21 1988 2.24846 0.17420596 21.00    no      no  8.482436    7.9
## 106    22 1982 2.48916 0.86634845 18.00   yes     yes  6.137799   10.3
## 107    22 1983 2.10088 0.83256882 18.00   yes     yes  6.208742   11.8
## 108    22 1984 2.15423 0.79780221 18.00   yes     yes  7.080938   10.0
## 109    22 1985 2.07766 0.76906782 18.00   yes     yes  7.445878   11.5
## 110    22 1986 2.07157 0.74922603 18.00   yes     yes  7.108686   13.1
## 111    22 1987 1.85384 0.72600001 20.50   yes     yes  6.859208   12.0
## 112    22 1988 2.09846 0.69874883 21.00   yes     yes  7.867977   10.9
## 113    23 1982 1.46127 0.80548930 20.00   yes      no  6.733286    8.6
## 114    23 1983 1.95633 0.77408260 20.00   yes      no  6.920517    9.0
## 115    23 1984 2.00692 0.74175823 20.00   yes      no  8.083908    6.1
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## 116    23 1985 1.76976 0.71504241 20.50   yes      no  7.969934    5.4
## 117    23 1986 1.82594 0.74980700 21.00   yes      no  8.550348    5.3
## 118    23 1987 1.95451 0.78750002 21.00   yes      no  9.069937    4.4
## 119    23 1988 2.11618 0.75794035 21.00   yes      no  9.461399    3.8
## 120    24 1982 1.49778 0.24105012 21.00    no      no  6.768094    8.4
## 121    24 1983 1.52523 0.23165138 21.00    no      no  7.118825    6.9
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## 123    24 1985 1.65984 0.21398306 21.00    no      no  7.590410    4.6
## 124    24 1986 1.75745 0.20846234 21.00    no      no  7.826705    4.5
## 125    24 1987 1.79493 0.20200001 21.00    no      no  8.046975    4.2
## 126    24 1988 1.69191 0.19441772 21.00    no      no  8.112946    4.5
## 127    25 1982 1.14669 0.28639618 20.00    no      no  6.380051    7.9
## 128    25 1983 1.12903 0.27522936 20.00    no      no  6.510740    6.9
## 129    25 1984 1.14867 0.26373625 20.00    no      no  6.646591    4.8
## 130    25 1985 1.27448 0.25423729 20.50    no      no  6.818296    3.9
## 131    25 1986 1.28900 0.24767801 21.00    no      no  7.027964    3.8
## 132    25 1987 1.17677 0.23999999 21.00    no      no  7.225436    3.2
## 133    25 1988 1.23111 0.23099133 21.00    no      no  7.358473    3.3
## 134    26 1982 1.52682 0.54565394 21.00    no      no  6.712743   15.5
## 135    26 1983 1.45129 0.52437842 21.00    no      no  6.721328   14.2
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## 140    26 1988 1.84416 0.44009432 21.00    no      no  8.430646    7.6
## 141    27 1982 1.38156 0.34606203 19.00    no      no  7.059264    7.8
## 142    27 1983 1.33896 0.33256879 19.00    no      no  7.494075    8.2
## 143    27 1984 1.39803 0.31868130 19.00    no      no  7.644966    6.3
## 144    27 1985 1.45004 0.30720338 19.00    no      no  7.795873    6.0
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## 162    30 1982 3.15528 0.34644747 19.00   yes      no  8.284474    8.6
## 163    30 1983 3.50490 0.33293921 19.00   yes      no  8.800240    8.8
## 164    30 1984 2.89186 0.31903625 19.00   yes      no  8.974486    7.4
## 165    30 1985 2.69976 0.31907839 19.00   yes      no  9.167078    7.7
## 166    30 1986 2.71726 0.32208154 19.00   yes      no  9.575281    8.1
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## 244    41 1987 2.27606 0.18871000 21.00   yes     yes  8.565328    6.2
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## 246    42 1982 1.53127 0.28639618 21.00    no      no  6.003269   10.9
## 247    42 1983 1.44731 0.27522936 21.00    no      no  6.080384   11.8
## 248    42 1984 1.45285 0.26373625 21.00    no      no  6.250283    9.1
## 249    42 1985 1.49414 0.25423729 21.00    no      no  6.363636    8.0
## 250    42 1986 1.59240 0.24767801 21.00    no      no  6.476124    6.8
## 251    42 1987 1.66471 0.23999999 21.00    no      no  6.587292    5.7
## 252    42 1988 1.60903 0.23099133 21.00    no      no  6.769258    5.1
## 253    44 1982 1.10063 0.17422435 20.00    no      no  6.192878   10.2
## 254    44 1983 1.04603 0.16743119 20.00    no      no  6.290825    8.3
## 255    44 1984 0.82121 0.16043955 20.50    no      no  5.509383    5.3
## 256    44 1985 1.12603 0.15466101 21.00    no      no  6.015479    4.9
## 257    44 1986 1.27179 0.15067080 21.00    no      no  6.064592    4.0
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## 276    47 1984 2.31697 0.31134394 19.67   yes      no  7.728100    8.6
## 277    47 1985 2.31205 0.30013028 21.00   yes      no  7.614016    8.0
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## 278    47 1986 2.56250 0.29238698 21.00   yes      no  8.165000    8.0
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## 284    48 1985 2.24679 0.46144068 19.00    no      no  8.751546    7.0
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## 298    50 1985 2.14953 0.63162076 18.00    no      no  8.762605    4.8
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## 302    51 1982 1.60503 0.75894988 21.00    no      no  7.547831    7.7
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## 313    53 1986 1.57517 0.20837875 21.00   yes      no  8.166685    8.2
## 314    53 1987 1.71882 0.20191900 21.00   yes      no  8.488326    7.6
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## 325    55 1984 1.72617 0.15934065 18.50    no      no  7.426941    7.3
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## 73  15390.110
## 74  15602.754
## 75  15988.648
## 76  16417.000
## 77  16915.303
## 78  12282.816
## 79  12364.679
## 80  13008.791
## 81  13161.017
## 82  13582.043
## 83  13937.000
## 84  14363.812
## 85  12968.974
## 86  12573.395
## 87  13203.297
## 88  13351.695
## 89  13812.178
## 90  14284.000
## 91  14111.646
## 92  14094.272
## 93  13917.432
## 94  14308.791
## 95  14631.355
## 96  14977.296
## 97  15152.000
## 98  15167.469
## 99  11071.599
## 100 10913.991
## 101 11441.758
## 102 11405.721
```

```
## 103 11602.684
## 104 12008.000
## 105 12340.712
## 106 12213.604
## 107 11994.266
## 108 12017.582
## 109 11972.458
## 110 11602.684
## 111 11515.000
## 112 11830.606
## 113 11442.721
## 114 11795.871
## 115 12271.429
## 116 12609.110
## 117 13292.054
## 118 13984.000
## 119 14538.979
## 120 15198.091
## 121 15644.495
## 122 16313.187
## 123 16921.609
## 124 17475.748
## 125 18167.000
## 126 18755.533
## 127 15215.990
## 128 15801.605
## 129 16735.164
## 130 17271.186
## 131 18145.512
## 132 19050.000
## 133 20034.648
## 134 13247.017
## 135 13606.651
## 136 14317.582
## 137 14830.509
## 138 15278.638
## 139 15418.000
## 140 15930.702
## 141 13781.623
## 142 13840.597
## 143 14734.066
## 144 14983.051
## 145 15464.396
## 146 15910.000
## 147 16048.123
## 148  9553.699
## 149  9513.762
## 150  9792.308
## 151  9797.670
## 152  9996.904
## 153 10303.000
## 154 10698.749
## 155 12968.974
## 156 13186.927
```

```
## 157 13727.473
## 158 14033.898
## 159 14368.421
## 160 14648.000
## 161 14871.992
## 162 12033.413
## 163 11954.129
## 164 11906.594
## 165 11669.491
## 166 12076.367
## 167 12291.000
## 168 12383.061
## 169 13192.124
## 170 12919.725
## 171 13540.659
## 172 13735.170
## 173 13971.104
## 174 14300.000
## 175 14219.441
## 176 14914.081
## 177 14863.532
## 178 15214.286
## 179 15564.618
## 180 15976.265
## 181 16412.000
## 182 16853.705
## 183 13834.129
## 184 14662.844
## 185 15451.648
## 186 16280.721
## 187 17132.096
## 188 17906.000
## 189 18704.523
## 190 16665.871
## 191 17275.229
## 192 18065.934
## 193 18662.076
## 194 19421.053
## 195 20313.000
## 196 21168.432
## 197 11347.255
## 198 11288.991
## 199 11539.561
## 200 11861.229
## 201 11825.594
## 202 11898.000
## 203 12019.249
## 204 15158.711
## 205 15573.395
## 206 16335.165
## 207 16708.686
## 208 17326.109
## 209 18005.000
## 210 18580.365
```

```
## 211 11078.759
## 212 11455.275
## 213 12089.011
## 214 12353.813
## 215 12839.010
## 216 13325.000
## 217 13767.084
## 218 12553.699
## 219 12389.908
## 220 12690.110
## 221 12661.017
## 222 12817.338
## 223 12971.000
## 224 12351.300
## 225 13039.380
## 226 13236.238
## 227 13784.615
## 228 13992.585
## 229 14279.670
## 230 14598.000
## 231 14952.839
## 232 13552.506
## 233 12784.403
## 234 12881.318
## 235 12904.661
## 236 12656.347
## 237 12607.000
## 238 12822.906
## 239 12626.491
## 240 12925.459
## 241 13246.154
## 242 13376.060
## 243 13649.123
## 244 14019.000
## 245 14326.275
## 246 13651.552
## 247 13706.422
## 248 13987.912
## 249 14356.991
## 250 14713.106
## 251 15200.000
## 252 15623.677
## 253 13326.969
## 254 13759.174
## 255 14312.088
## 256 14595.339
## 257 15109.392
## 258 15633.000
## 259 16257.940
## 260 10393.795
## 261 10693.808
## 262 11160.439
## 263 11369.703
## 264 11674.923
```

```
## 265 12027.000
## 266 12440.809
## 267 11323.389
## 268 11091.743
## 269 11661.538
## 270 11684.322
## 271 12175.438
## 272 12545.000
## 273 12276.228
## 274 10988.066
## 275 11183.486
## 276 11704.396
## 277 11919.491
## 278 12371.517
## 279 12876.000
## 280 13352.262
## 281 13942.721
## 282 13692.660
## 283 14039.561
## 284 14270.127
## 285 13950.465
## 286 13889.000
## 287 14038.499
## 288 10788.783
## 289 10779.816
## 290 11120.879
## 291 11284.958
## 292 11339.525
## 293 11389.000
## 294 11735.322
## 295 12064.439
## 296 12186.927
## 297 12680.220
## 298 13112.288
## 299 13740.970
## 300 14325.000
## 301 14727.623
## 302 13878.281
## 303 14299.312
## 304 14906.594
## 305 15323.093
## 306 15915.377
## 307 16486.000
## 308 17011.549
## 309 14342.482
## 310 14534.403
## 311 14758.242
## 312 14909.958
## 313 15375.645
## 314 15630.000
## 315 15854.668
## 316 10748.210
## 317 10451.835
## 318 10641.758
```

```
## 319 10669.491
## 320 10888.545
## 321 10992.000
## 322 11294.514
## 323 13213.604
## 324 13291.284
## 325 13818.682
## 326 13952.330
## 327 14351.909
## 328 14720.000
## 329 14941.290
## 330 14600.238
## 331 13574.541
## 332 13456.044
## 333 13595.339
## 334 13126.935
## 335 12719.000
## 336 13098.171
```

```
state=as.factor(df_fat$state)
```

26. Make a pairs plot of all the continuous response and explanatory variables: mrall, beertax, mlda, vmiles, unrate, perinc. (Define a new dataframe containing only these variables, or use numerical subscripts on the columns of the original dataframe.) Can you spot the outlier in this data set?

```
df_fatnew=df_fat[c('mrall','beertax', 'mlda', 'vmiles', 'unrate', 'perinc')]
df_fatnew
```

```
##       mrall    beertax  mlda    vmiles unrate    perinc
## 1   2.12836 1.53937948 19.00  7.233887   14.4 10544.152
## 2   2.34848 1.78899074 19.00  7.836348   13.7 10732.798
## 3   2.33643 1.71428561 19.00  8.262990   11.1 11108.791
## 4   2.19348 1.65254235 19.67  8.726917    8.9 11332.627
## 5   2.66914 1.60990703 21.00  8.952854    9.8 11661.507
## 6   2.71859 1.55999994 21.00  9.166302    7.8 11944.000
## 7   2.49391 1.50144362 21.00  9.674323    7.2 12368.624
## 8   2.49914 0.21479714 19.00  6.810157    9.9 12309.069
## 9   2.26738 0.20642203 19.00  6.587495    9.1 12693.808
## 10  2.82878 0.29670331 19.00  6.709970    5.0 13265.934
## 11  2.80201 0.38135594 21.00  6.771263    6.5 13726.695
## 12  3.07106 0.37151703 21.00  8.129008    6.9 14107.327
## 13  2.76728 0.36000001 21.00  9.370654    6.2 14241.000
## 14  2.70565 0.34648702 21.00  9.815721    6.3 14408.085
## 15  2.38405 0.65035802 21.00  7.208500    9.8 10267.303
## 16  2.39570 0.67545873 21.00  7.175917   10.1 10433.486
## 17  2.23785 0.59890109 21.00  7.084820    8.9 10916.483
## 18  2.26367 0.57733053 21.00  7.253918    8.7 11149.364
## 19  2.54323 0.56243551 21.00  7.468999    8.7 11399.381
## 20  2.67588 0.54500002 21.00  7.665831    8.1 11537.000
## 21  2.54697 0.52454287 21.00  8.024625    7.7 11760.347
## 22  1.86194 0.10739857 21.00  6.858677    9.9 15797.136
## 23  1.80672 0.10321102 21.00  7.216292    9.7 15970.184
## 24  1.94611 0.09890110 21.00  7.619176    7.8 16590.109
```

```
## 25  1.88128 0.09533899 21.00  7.874067    7.2 16985.170
## 26  1.94548 0.09287926 21.00  8.034910    6.7 17356.037
## 27  1.98966 0.09000000 21.00  8.180633    5.8 17846.000
## 28  1.90365 0.08662175 21.00  8.531990    5.3 18049.086
## 29  2.17448 0.21479714 21.00  7.742842    7.7 15082.339
## 30  2.05144 0.20642203 21.00  7.656063    6.6 15131.881
## 31  1.90596 0.19780220 21.00  7.707853    5.6 15486.813
## 32  1.79201 0.19067797 21.00  8.092209    5.9 15569.915
## 33  1.84630 0.18575852 21.00  8.131375    7.4 15616.099
## 34  1.79308 0.18000001 21.00  8.182028    7.7 15605.000
## 35  1.50560 0.17324351 21.00  8.380769    6.4 15845.043
## 36  1.64695 0.22434367 18.50  6.440054    6.9 17255.369
## 37  1.39490 0.23356308 19.25  6.570043    6.0 17744.266
## 38  1.48653 0.24801099 20.00  6.680193    4.6 18760.439
## 39  1.41147 0.23907840 20.33  6.979215    4.9 19312.500
## 40  1.40933 0.23291023 21.00  7.661745    3.8 20152.734
## 41  1.39832 0.22569001 21.00  8.338534    3.3 21192.000
## 42  1.49706 0.21721849 21.00  8.061235    3.0 22193.455
## 43  2.03333 0.17303102 20.00  7.651654    8.5 14263.724
## 44  1.81518 0.16628440 20.00  8.062700    8.1 14500.000
## 45  2.11726 0.15934065 21.00  8.368063    6.2 14925.274
## 46  1.67203 0.15360169 21.00  8.625425    5.3 15408.898
## 47  2.14850 0.14963880 21.00  9.045817    4.3 15822.497
## 48  2.26708 0.14500000 21.00  9.450308    3.2 16407.000
## 49  2.42424 0.13955726 21.00  9.703021    3.2 16998.074
## 50  2.53197 1.07398570 19.00  7.587130    8.2 13502.387
## 51  2.49768 1.17041278 19.00  7.604254    8.6 13924.312
## 52  2.54661 1.18681324 19.00  7.735305    6.3 14307.692
## 53  2.49164 1.14406788 20.00  7.747311    6.0 14760.593
## 54  2.42004 1.11455119 21.00  7.768756    5.7 15102.167
## 55  2.36131 1.08000004 21.00  7.788331    5.3 15584.000
## 56  2.49534 1.03946102 21.00  8.538230    5.0 15979.788
## 57  2.17484 2.72076368 19.00  8.623444    7.8 11774.463
## 58  2.26060 2.61467886 19.00  8.518590    7.5 12237.386
## 59  2.41356 2.50549436 19.00  8.641914    6.0 12957.143
## 60  2.27744 2.41525412 19.25  8.988107    6.5 13364.407
## 61  2.50820 2.35294104 20.25  9.344768    5.9 13891.641
## 62  2.56991 2.27999997 21.00  9.690281    5.5 14306.000
## 63  2.60643 2.19441772 21.00  9.817396    5.8 14687.199
## 64  2.61759 0.40274465 19.00  8.033752    9.8 11078.759
## 65  2.66194 0.38704130 19.00  8.387642    9.8 11346.330
## 66  2.42242 0.37087911 19.00  7.775768    7.2 11386.813
## 67  2.53731 0.35752121 19.00  7.671632    7.9 11459.746
## 68  2.57485 0.34829721 19.00  7.899201    8.7 11541.796
## 69  2.62525 0.33750001 20.50  8.135269    8.0 11859.000
## 70  2.56231 0.32483158 21.00  8.102682    5.8 12189.605
## 71  1.43840 0.18854415 21.00  5.696535   11.3 14743.437
## 72  1.32800 0.18119267 21.00  5.862868   11.4 14745.413
## 73  1.34265 0.17362638 21.00  6.067528    9.1 15390.110
## 74  1.32987 0.16737288 21.00  6.141676    9.0 15602.754
## 75  1.38170 0.16305470 21.00  6.345777    8.1 15988.648
## 76  1.43326 0.15800001 21.00  6.540846    7.4 16417.000
## 77  1.58171 0.15206930 21.00  6.757613    6.8 16915.303
## 78  1.75269 0.30913246 21.00  7.149917   11.9 12282.816
```

```
## 79  1.85605 0.29707912 21.00  7.277506   11.1 12364.679
## 80  1.68427 0.28467363 21.00  7.478887    8.6 13008.791
## 81  1.77123 0.27442053 21.00  7.416253    7.9 13161.017
## 82  1.88624 0.26734054 21.00  7.714322    6.7 13582.043
## 83  1.90743 0.25905299 21.00  7.977216    6.4 13937.000
## 84  1.98164 0.24932915 21.00  9.201576    5.3 14363.812
## 85  1.65119 0.37589499 19.00  6.653263    8.5 12968.974
## 86  1.76997 0.36123854 19.00  6.770308    8.1 12573.395
## 87  1.44678 0.34615386 19.00  7.060630    7.0 13203.297
## 88  1.64355 0.33368644 19.00  7.001038    8.0 13351.695
## 89  1.54737 0.38312694 20.00  7.192990    7.0 13812.178
## 90  1.73253 0.42750001 21.00  7.342257    5.5 14284.000
## 91  1.96542 0.48492688 21.00  7.730063    4.5 14111.646
## 92  2.06811 0.48400956 21.00  7.333069    6.3 14094.272
## 93  1.69345 0.46513763 21.00  7.479611    6.1 13917.432
## 94  2.09016 0.44571429 21.00  7.670888    5.2 14308.791
## 95  1.98367 0.42966104 21.00  7.867333    5.0 14631.355
## 96  2.03335 0.41857585 21.00  8.100053    5.4 14977.296
## 97  1.98304 0.40560001 21.00  8.304131    4.9 15152.000
## 98  1.93587 0.39037538 21.00  8.481355    4.8 15167.469
## 99  2.22523 0.21599045 21.00  6.937466   10.6 11071.599
## 100 2.09478 0.20756879 21.00  7.194143   11.7 10913.991
## 101 2.02688 0.19890109 21.00  7.513703    9.3 11441.758
## 102 1.91090 0.19173728 21.00  7.654335    9.5 11405.721
## 103 2.16049 0.18679050 21.00  7.859621    9.3 11602.684
## 104 2.26456 0.18099999 21.00  8.135244    8.8 12008.000
## 105 2.24846 0.17420596 21.00  8.482436    7.9 12340.712
## 106 2.48916 0.86634845 18.00  6.137799   10.3 12213.604
## 107 2.10088 0.83256882 18.00  6.208742   11.8 11994.266
## 108 2.15423 0.79780221 18.00  7.080938   10.0 12017.582
## 109 2.07766 0.76906782 18.00  7.445878   11.5 11972.458
## 110 2.07157 0.74922603 18.00  7.108686   13.1 11602.684
## 111 1.85384 0.72600001 20.50  6.859208   12.0 11515.000
## 112 2.09846 0.69874883 21.00  7.867977   10.9 11830.606
## 113 1.46127 0.80548930 20.00  6.733286    8.6 11442.721
## 114 1.95633 0.77408260 20.00  6.920517    9.0 11795.871
## 115 2.00692 0.74175823 20.00  8.083908    6.1 12271.429
## 116 1.76976 0.71504241 20.50  7.969934    5.4 12609.110
## 117 1.82594 0.74980700 21.00  8.550348    5.3 13292.054
## 118 1.95451 0.78750002 21.00  9.069937    4.4 13984.000
## 119 2.11618 0.75794035 21.00  9.461399    3.8 14538.979
## 120 1.49778 0.24105012 21.00  6.768094    8.4 15198.091
## 121 1.52523 0.23165138 21.00  7.118825    6.9 15644.495
## 122 1.47850 0.22197802 21.00  7.289488    5.4 16313.187
## 123 1.65984 0.21398306 21.00  7.590410    4.6 16921.609
## 124 1.75745 0.20846234 21.00  7.826705    4.5 17475.748
## 125 1.79493 0.20200001 21.00  8.046975    4.2 18167.000
## 126 1.69191 0.19441772 21.00  8.112946    4.5 18755.533
## 127 1.14669 0.28639618 20.00  6.380051    7.9 15215.990
## 128 1.12903 0.27522936 20.00  6.510740    6.9 15801.605
## 129 1.14867 0.26373625 20.00  6.646591    4.8 16735.164
## 130 1.27448 0.25423729 20.50  6.818296    3.9 17271.186
## 131 1.28900 0.24767801 21.00  7.027964    3.8 18145.512
## 132 1.17677 0.23999999 21.00  7.225436    3.2 19050.000
```

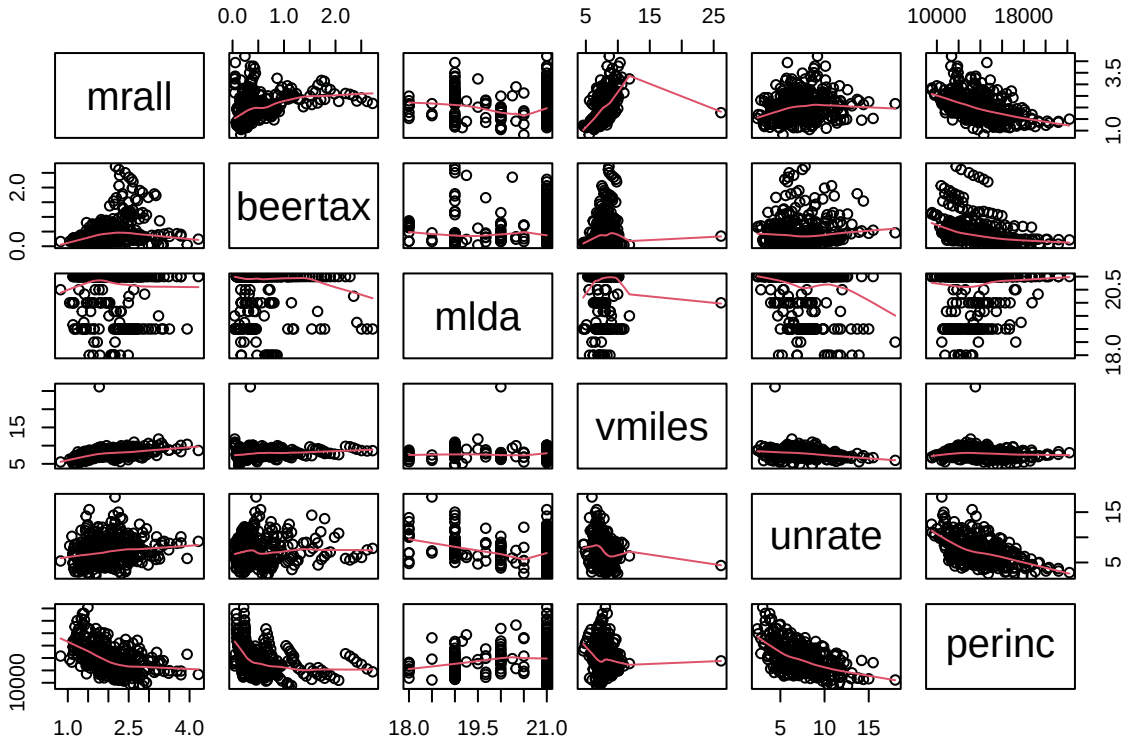
```
## 133 1.23111 0.23099133 21.00  7.358473    3.3 20034.648
## 134 1.52682 0.54565394 21.00  6.712743   15.5 13247.017
## 135 1.45129 0.52437842 21.00  6.721328   14.2 13606.651
## 136 1.69022 0.50248128 21.00  7.007071   11.2 14317.582
## 137 1.70004 0.48438346 21.00  7.416576    9.9 14830.509
## 138 1.75621 0.47188646 21.00  7.829523    8.8 15278.638
## 139 1.73587 0.45725799 21.00  8.228915    8.2 15418.000
## 140 1.84416 0.44009432 21.00  8.430646    7.6 15930.702
## 141 1.38156 0.34606203 19.00  7.059264    7.8 13781.623
## 142 1.33896 0.33256879 19.00  7.494075    8.2 13840.597
## 143 1.39803 0.31868130 19.00  7.644966    6.3 14734.066
## 144 1.45004 0.30720338 19.00  7.795873    6.0 14983.051
## 145 1.35533 0.29927760 19.67  8.053176    5.3 15464.396
## 146 1.24823 0.31625000 21.00  8.282359    5.4 15910.000
## 147 1.42094 0.32098171 21.00  8.462255    4.0 16048.123
## 148 2.84379 1.14594281 21.00  6.679401   11.0  9553.699
## 149 2.76810 1.10126150 21.00  6.891988   12.6  9513.762
## 150 2.61355 1.05527472 21.00  7.098540   10.8  9792.308
## 151 2.53349 1.07663143 21.00  7.321097   10.3  9797.670
## 152 2.93826 1.06538701 21.00  7.489324   11.7  9996.904
## 153 2.88000 0.96030003 21.00  7.684952   10.2 10303.000
## 154 2.75573 0.92425412 21.00  8.413373    8.4 10698.749
## 155 1.80089 0.34661219 21.00  7.082759    9.2 12968.974
## 156 1.83558 0.33309749 21.00  7.363074    9.9 13186.927
## 157 1.93361 0.31918791 21.00  7.705446    7.2 13727.473
## 158 1.85126 0.30769175 21.00  7.811482    6.4 14033.898
## 159 2.22946 0.29975337 21.00  8.161917    6.1 14368.421
## 160 2.04586 0.29046100 21.00  8.500705    6.3 14648.000
## 161 2.14550 0.27955824 21.00  8.864047    5.7 14871.992
## 162 3.15528 0.34644747 19.00  8.284474    8.6 12033.413
## 163 3.50490 0.33293921 19.00  8.800240    8.8 11954.129
## 164 2.89186 0.31903625 19.00  8.974486    7.4 11906.594
## 165 2.69976 0.31907839 19.00  9.167078    7.7 11669.491
## 166 2.71726 0.32208154 19.00  9.575281    8.1 12076.367
## 167 2.89246 0.32380000 20.33  9.980224    7.4 12291.000
## 168 2.45963 0.32291049 21.00 10.109327    6.8 12383.061
## 169 1.64151 0.37589499 20.00  7.191827    6.1 13192.124
## 170 1.59774 0.36123854 20.00  7.226798    5.7 12919.725
## 171 1.77570 0.34615386 20.00 26.148271    4.4 13540.659
## 172 1.47572 0.36943856 21.00  7.505625    5.5 13735.170
## 173 1.81477 0.46439627 21.00  7.867966    5.0 13971.104
## 174 1.86324 0.46687499 21.00  8.212686    4.9 14300.000
## 175 1.62921 0.49807507 21.00  8.368896    3.6 14219.441
## 176 3.18907 0.16109785 21.00  7.304109   10.1 14914.081
## 177 2.82051 0.19997133 21.00  7.661085    9.8 14863.532
## 178 2.71538 0.22252747 21.00  7.995649    7.8 15214.286
## 179 2.76709 0.21451271 21.00  8.083322    8.0 15564.618
## 180 2.40951 0.20897833 21.00  8.253348    6.0 15976.265
## 181 2.60179 0.20250000 21.00  8.337645    6.3 16412.000
## 182 2.71347 0.19489895 21.00  8.528455    5.2 16853.705
## 183 1.82489 0.48329356 20.00  7.353357    7.4 13834.129
## 184 1.99166 0.56766057 20.00  7.488016    5.4 14662.844
## 185 1.96319 0.74175823 20.00  7.458077    4.3 15451.648
## 186 1.91383 0.71504241 20.00  7.553116    3.9 16280.721
```

```
## 187 1.67478 0.69659442 20.50  8.133395    2.8 17132.096
## 188 1.69347 0.67500001 21.00  8.672647    2.5 17906.000
## 189 1.52995 0.64966315 21.00  8.762188    2.4 18704.523
## 190 1.42799 0.08949881 19.00  6.971983    9.0 16665.871
## 191 1.24799 0.08600917 21.00  6.992092    7.8 17275.229
## 192 1.22655 0.08241758 21.00  6.959141    6.2 18065.934
## 193 1.27480 0.07944915 21.00  7.023037    5.7 18662.076
## 194 1.36262 0.07739938 21.00  7.224902    5.0 19421.053
## 195 1.33342 0.07500000 21.00  7.438867    4.0 20313.000
## 196 1.36122 0.07218479 21.00  7.598872    3.8 21168.432
## 197 4.21784 0.24164678 21.00  8.662289    9.2 11347.255
## 198 3.78745 0.34833717 21.00  8.329537   10.1 11288.991
## 199 3.48527 0.44505495 21.00  8.718084    7.5 11539.561
## 200 3.68966 0.42902541 21.00  9.151046    8.8 11861.229
## 201 3.37390 0.41795665 21.00  9.596345    9.2 11825.594
## 202 3.78667 0.40500000 21.00 10.077343    8.9 11898.000
## 203 3.23159 0.38979790 21.00 10.141354    7.8 12019.249
## 204 1.22932 0.11933175 19.00  4.576346    8.6 15158.711
## 205 1.17444 0.13285550 19.00  4.737511    8.6 15573.395
## 206 1.16082 0.13599999 19.00  4.917594    7.2 16335.165
## 207 1.12804 0.13110170 19.16  5.090125    6.5 16708.686
## 208 1.19247 0.12771930 21.00  5.296995    6.3 17326.109
## 209 1.30884 0.12376000 21.00  5.498026    4.9 18005.000
## 210 1.25914 0.11911453 21.00  5.789922    4.2 18580.365
## 211 2.16589 1.43198097 21.00  7.164226    9.0 11078.759
## 212 2.03061 1.37614679 21.00  7.411233    8.9 11455.275
## 213 2.35161 1.31868136 21.00  7.814157    6.7 12089.011
## 214 2.36930 1.27118647 21.00  7.981280    5.4 12353.813
## 215 2.60148 1.23839009 21.00  8.254921    5.3 12839.010
## 216 2.46998 1.20000005 21.00  8.513946    4.5 13325.000
## 217 2.42410 1.15495670 21.00  8.929410    3.6 13767.084
## 218 2.20238 0.42959428 21.00  7.815473    5.9 12553.699
## 219 1.70338 0.41284406 21.00  7.875196    5.6 12389.908
## 220 1.45560 0.39560440 21.00  7.826761    5.1 12690.110
## 221 1.31387 0.38135594 21.00  7.864242    5.9 12661.017
## 222 1.47275 0.37151703 21.00  8.150198    6.3 12817.338
## 223 1.50298 0.36000001 21.00  8.453891    5.2 12971.000
## 224 1.55922 0.34648702 21.00  8.643176    4.8 12351.300
## 225 1.49155 0.42959428 21.00  6.659627   12.5 13039.380
## 226 1.47327 0.41284406 21.00  6.818204   12.2 13236.238
## 227 1.53259 0.39560440 21.00  6.973471    9.4 13784.615
## 228 1.53202 0.38135594 21.00  7.031749    8.9 13992.585
## 229 1.55657 0.37151703 21.00  7.196974    8.1 14279.670
## 230 1.64318 0.36000001 21.00  7.340248    7.0 14598.000
## 231 1.62414 0.34648702 21.00  7.553218    6.0 14952.839
## 232 3.26215 0.86634845 21.00  9.288461    5.7 13552.506
## 233 2.56116 0.83256882 21.00  8.929328    9.0 12784.403
## 234 2.40785 0.94739008 21.00  9.359799    7.0 12881.318
## 235 2.25386 0.96133476 21.00  9.445915    7.1 12904.661
## 236 2.11434 0.93653256 21.00  9.496078    8.2 12656.347
## 237 1.82457 0.90750003 21.00  9.659524    7.4 12607.000
## 238 1.95558 0.87343603 21.00  9.990114    6.7 12822.906
## 239 1.94080 0.22519094 21.00  7.262638   11.5 12626.491
## 240 2.06767 0.21641056 21.00  7.728198   10.8 12925.459
```

```
## 241 2.13752 0.20737363 21.00  7.826238    9.4 13246.154
## 242 2.08039 0.19990467 21.00  7.985869    8.8 13376.060
## 243 2.29090 0.19474716 21.00  8.288321    8.5 13649.123
## 244 2.27606 0.18871000 21.00  8.565328    6.2 14019.000
## 245 2.44669 0.18162657 21.00  9.108771    5.8 14326.275
## 246 1.53127 0.28639618 21.00  6.003269   10.9 13651.552
## 247 1.44731 0.27522936 21.00  6.080384   11.8 13706.422
## 248 1.45285 0.26373625 21.00  6.250283    9.1 13987.912
## 249 1.49414 0.25423729 21.00  6.363636    8.0 14356.991
## 250 1.59240 0.24767801 21.00  6.476124    6.8 14713.106
## 251 1.66471 0.23999999 21.00  6.587292    5.7 15200.000
## 252 1.60903 0.23099133 21.00  6.769258    5.1 15623.677
## 253 1.10063 0.17422435 20.00  6.192878   10.2 13326.969
## 254 1.04603 0.16743119 20.00  6.290825    8.3 13759.174
## 255 0.82121 0.16043955 20.50  5.509383    5.3 14312.088
## 256 1.12603 0.15466101 21.00  6.015479    4.9 14595.339
## 257 1.27179 0.15067080 21.00  6.064592    4.0 15109.392
## 258 1.14604 0.14600000 21.00  6.088211    3.8 15633.000
## 259 1.25881 0.14051972 21.00  5.894251    3.1 16257.940
## 260 2.26286 2.06205249 21.00  7.508355   10.8 10393.795
## 261 2.59055 1.98165143 21.00  7.666370   10.0 10693.808
## 262 2.77408 1.89890110 21.00  7.865243    7.1 11160.439
## 263 2.84135 1.83050847 21.00  7.970419    6.8 11369.703
## 264 3.13221 1.78328180 21.00  8.414968    6.2 11674.923
## 265 3.17080 1.72800004 21.00  8.824518    5.6 12027.000
## 266 2.97983 1.66313767 21.00  9.152459    4.5 12440.809
## 267 2.13256 0.71837711 21.00  9.165686    5.5 11323.389
## 268 2.50358 0.69036698 21.00  9.037208    5.4 11091.743
## 269 2.02837 0.66153848 21.00  9.079438    4.3 11661.538
## 270 1.83616 0.63771188 21.00  8.865828    5.1 11684.322
## 271 1.89266 0.62125903 21.00  8.817817    4.7 12175.438
## 272 1.88999 0.60925299 21.00  8.757423    4.2 12545.000
## 273 2.06171 0.59336478 21.00  9.304343    3.9 12276.228
## 274 2.26152 0.33809426 19.00  7.458300   11.8 10988.066
## 275 2.21156 0.32491168 19.00  7.733209   11.5 11183.486
## 276 2.31697 0.31134394 19.67  7.728100    8.6 11704.396
## 277 2.31205 0.30013028 21.00  7.614016    8.0 11919.491
## 278 2.56250 0.29238698 21.00  8.165000    8.0 12371.517
## 279 2.57055 0.28332299 21.00  8.676843    6.6 12876.000
## 280 2.58631 0.27268815 21.00  9.028183    5.8 13352.262
## 281 2.74034 0.43317422 19.00  8.144788    6.9 13942.721
## 282 2.41717 0.41628441 19.00  8.338572    8.0 13692.660
## 283 2.43238 0.41884616 19.00  8.564129    5.9 14039.561
## 284 2.24679 0.46144068 19.00  8.751546    7.0 14270.127
## 285 2.13734 0.44953561 19.67  8.821696    8.9 13950.465
## 286 1.94234 0.43560001 21.00  9.005048    8.4 13889.000
## 287 2.01473 0.41924930 21.00  9.290322    7.3 14038.499
## 288 1.89345 0.35684010 21.00  7.012183    7.8 10788.783
## 289 1.77429 0.62925458 21.00  7.035091    9.2 10779.816
## 290 1.94085 0.87734836 21.00  7.184842    6.5 11120.879
## 291 1.84195 0.84574890 21.00  7.317344    5.9 11284.958
## 292 1.88101 0.82392877 21.00  7.426684    6.0 11339.525
## 293 1.76190 0.79838699 21.00  7.547003    6.4 11389.000
## 294 1.75740 0.76841867 21.00  7.847945    4.9 11735.322
```

```
## 295 2.05769 0.71151549 18.00  7.678837    6.9 12064.439
## 296 1.79048 0.68377292 18.00  7.906684    6.9 12186.927
## 297 2.15094 0.65521979 18.00  8.307534    5.2 12680.220
## 298 2.14953 0.63162076 18.00  8.762605    4.8 13112.288
## 299 2.01479 0.61532509 19.50  8.990770    4.7 13740.970
## 300 2.17153 0.59625000 21.00  9.195244    3.6 14325.000
## 301 2.31598 0.57386911 21.00  9.969486    2.8 14727.623
## 302 1.60503 0.75894988 21.00  7.547831    7.7 13878.281
## 303 1.62080 0.72935778 21.00  7.609125    6.1 14299.312
## 304 1.79737 0.69890106 21.00  7.900444    5.0 14906.594
## 305 1.71048 0.67372876 21.00  8.399579    5.6 15323.093
## 306 1.94305 0.65634674 21.00  8.866417    5.0 15915.377
## 307 1.72934 0.63599998 21.00  9.287623    4.2 16486.000
## 308 1.78055 0.61212701 21.00  9.551628    3.9 17011.549
## 309 1.74848 0.23175895 21.00  7.306683   12.1 14342.482
## 310 1.62137 0.23155849 21.00  8.395816   11.2 14534.403
## 311 1.71534 0.22188902 21.00  7.874928    9.5 14758.242
## 312 1.68746 0.21389724 21.00  7.796564    8.1 14909.958
## 313 1.57517 0.20837875 21.00  8.166685    8.2 15375.645
## 314 1.71882 0.20191900 21.00  8.488326    7.6 15630.000
## 315 1.67384 0.19433975 21.00  8.995922    6.2 15854.668
## 316 2.29475 0.47636396 18.00  5.574713   13.9 10748.210
## 317 2.16505 0.45779014 18.50  5.958217   18.0 10451.835
## 318 2.24500 0.43867362 19.00  6.494611   15.0 10641.758
## 319 2.16942 0.42287391 19.00  6.541318   13.0 10669.491
## 320 2.29525 0.41196388 19.67  6.887315   11.8 10888.545
## 321 2.48287 0.39919299 21.00  7.244076   10.8 10992.000
## 322 2.45203 0.38420886 21.00  7.400866    9.9 11294.514
## 323 1.62242 0.17303102 18.00  6.909823   10.7 13213.604
## 324 1.52728 0.16628440 18.00  7.184746   10.4 13291.284
## 325 1.72617 0.15934065 18.50  7.426941    7.3 13818.682
## 326 1.55812 0.15360169 19.00  7.681489    7.2 13952.330
## 327 1.56178 0.14963880 19.67  8.036372    7.0 14351.909
## 328 1.65800 0.14500000 21.00  8.361979    6.1 14720.000
## 329 1.66220 0.13955726 21.00  8.745190    4.3 14941.290
## 330 3.94118 0.05369928 19.00 10.354911    5.8 14600.238
## 331 3.35271 0.05160551 19.00  9.804255    8.4 13574.541
## 332 3.06043 0.04945055 19.00  9.994155    6.3 13456.044
## 333 2.98625 0.04766949 19.00 10.611011    7.1 13595.339
## 334 3.31361 0.04643963 19.00 10.619331    9.0 13126.935
## 335 2.63265 0.04500000 19.00 10.953050    8.6 12719.000
## 336 3.23591 0.04331088 19.50 11.812115    6.3 13098.171
```

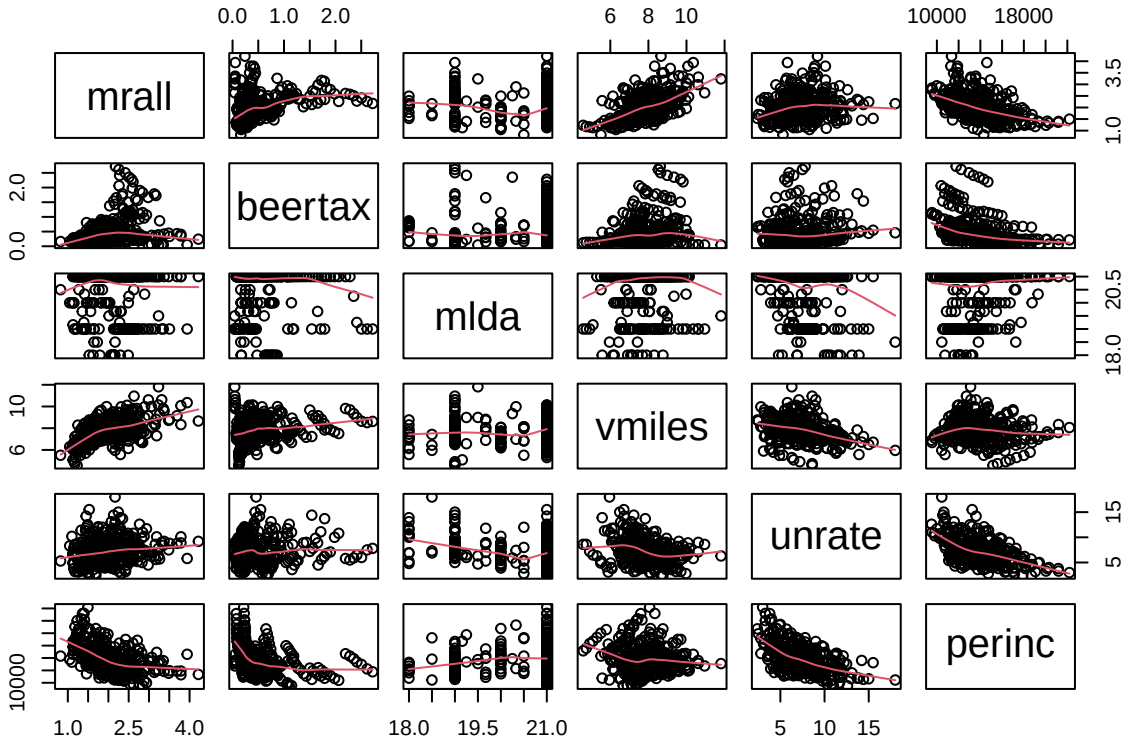
```
pairs(df_fatnew, panel=panel.smooth)
```



27. Identify the outlier row and delete it from the data frame using a logical operator, or alternatively a negative element subscript. (You may wish to save the modified data frame to a new variable, in case you make a mistake.) Make another pairs plot. Use the `panel=panel.smooth` option to visualize possible correlations.

```
df_fatnew1=df_fatnew[df_fatnew$vmiles<20,]

pairs(df_fatnew1[,c('mrall','beertax', 'mlda', 'vmiles', 'unrate', 'perinc')], panel=panel.smooth)
```



28. Examine the top row of the pairs plot. Considering individual variables in isolation, how do each of the following variables seem to relate to traffic mortality (increase, decrease, no obvious effect?)

    a. Tax rate on beer

    b. Drinking age

    c. Vehicle miles driven

    d. Unemployment rate

    e. Per capita income

29. Fit a linear model on the (outlier-cleaned) dataset. Do not consider interactions (which would be very difficult to interpret in this case). Do consider state and year (while this drastically reduces the degrees of freedom, it removes the pseudoreplication by only considering changes in mortality in response to changes in state policies over this period, with the national trend removed).

```
#dmodel = lm(mrall~state+year+beertax+mlda+jaild+comserd+vmiles+unrate+perinc, data=df_fatnew1)
#summary.aov(dmodel)
# I tried this but I did not figure why it is not working for me so that is why I left th 31 and 32 que
```

30. Simplify the model by successive updates or stepwise regression.

31. Look at the final linear model summary, and examine the signs and t-significances of the remaining terms. How do the following seem to correlate with traffic mortality?

a. Tax rate on beer

b. Drinking age

c. Vehicle miles driven

d. Unemployment rate

e. Per capita income

f. Mandatory jail sentence (=yes)

g. Mandatory community service (=yes)

32. Compare the conclusions of #31(a-e) to #28. Are there any differences? Can you explain why this is?