

Concavity

In calculus, concavity is a function's rate of change in relation to its derivative. It is determined by the function's first and second derivatives:

Concave up

The function's second derivative is greater than zero ($f''(x) > 0$) for all x in an interval. The graph of the function is cup-shaped and lies above all of its tangent lines.

Concave down

The function's second derivative is less than zero ($f''(x) < 0$) for all x in an interval. The graph of the function is cap-shaped and lies below all of its tangent lines.

A function's concavity can change at a point of inflection, which is where the concavity switches. A point of inflection occurs when $f''(x) = 0$ or $f''(x)$ is undefined. At a point of inflection, the graph of the function "wiggles".

To determine the intervals where a function is concave up or concave down, you can:

1. Find the function's second derivative, $f''(x)$
2. Find where $f''(x) = 0$
3. Divide the domain of the function into smaller intervals
4. Determine the sign of f'' over each of the smaller intervals

