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Problem Catalogue

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1 Integrals: Evil

1.1 Problem One

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Your final answer should be a single expression in the form $I_1 = [h(x)]_a^b$ where b and a are bounds of I_1 to be found

Conclude whether or not the below integral, I_1 :

$$I_1 = \int_{f_x^{2x}(\ln x)^2}^{(3+x)^6} \left(\frac{1}{3e^x \cos x \sec^2 x} + \cot x \csc x \right) dx$$

can be written as a single expression, with all integration expressions evaluated.

Your conclusion must be supported, **either**, by an attempt to evaluate the integral or, a proof that I_1 has no real anti-derivative.

—12 Marks—

1.1.1 Worked Solution:

$$I_1 = \int_{f_x^{2x}(\ln x)^2}^{(3+x)^6} \left(\frac{1}{3e^x \cos x \sec^2 x} + \cot x \csc x \right) dx$$

$$\Rightarrow I_1 = \int_b^a f(x) dx$$

\therefore let $I_2 = b$

$$I_2 = \int_x^{2x} (\ln |x|)^2 = \int_{e^u}^{2e^u} u^2 e^u du = [u^2 e^u]_{e^u}^{2e^u} - \int_{e^u}^{2e^u} 2ue^u e^u du$$

$$= [u^2 e^u]_{e^u}^{2e^u} - ([2ue^u]_{e^u}^{2e^u} - \int_{e^u}^{2e^u} 2e^u du) = [u^2 e^u]_{e^u}^{2e^u} - [2ue^u - 2e^u]_{e^u}^{2e^u} = [u^2 e^u - 2ue^u + 2e^u]_{e^u}^{2e^u}$$

$$= [x(\ln |x|)^2 - 2x \ln |x| + 2x]_{e^u}^{2e^u} \because u = \ln |x| \Rightarrow e^u = x$$

$$\therefore I_2 = x[2(\ln |2x|)^2 - (\ln |x|)^2 - \ln |16x^2|]$$

This result can be obtained by evaluating the bounds

$$I_1 = \int_{I_2}^{(3+x)^6} \left(\frac{1}{3e^x \cos x \sec^2 x} + \cot x \csc x \right) dx = \int_a^b \left(\frac{1}{3e^x \cos x \sec^2 x} \right) + \int_a^b \cot x \csc x$$

$$\text{let } I_3 = \int_a^b \left(\frac{1}{3e^x \cos x \sec^2 x} \right) = \int_a^b \left(\frac{1}{3e^x \sec x} \right) dx = \frac{1}{3} \int_a^b (e^{-x} \cos x) dx$$

$$I_3 = 1/3(J)$$

$$J = [-e^{-x} \cos x]_a^b - \int_a^b -e^{-x} \sin x = [-e^{-x} \cos x]_a^b - ([-e^{-x} \sin x]_a^b - \int_a^b e^{-x} \cos x)$$

$$J = [-e^{-x} \cos x]_a^b - [e^{-x} \sin x]_a^b - J$$

$$\therefore J = \frac{1}{2} [e^{-x} \sin x - e^{-x} \cos x]_a^b$$

$$\therefore I_3 = \frac{1}{6} [e^{-x} \sin x - e^{-x} \cos x]_a^b$$

$$I_1 = I_3 + \int_a^b \cot x \csc x dx$$

$$\text{let } I_4 = \int_a^b \cot x \csc x dx$$

$$I_4 = \int_a^b \left(\frac{1}{\sin x} \right) \left(\frac{\cos x}{\sin x} \right) dx = \int_a^b \frac{\cos x}{1 - \cos^2 x} dx$$

Integrate I_4 by parts:

$$I_4 = \int_a^b \frac{\cos x}{\sin^2 x} dx = \int_a^b \cos x \left(\frac{1}{\sin^2 x} \right) dx = \left[\frac{1}{\sin x} \right]_a^b - \int_a^b \sin x \left(\frac{-2 \sin x \cos x}{\sin^4 x} \right) dx = \left[\frac{1}{\sin x} \right]_a^b - \int_a^b \frac{-2 \cos x}{\sin x} dx$$

$$\therefore I_4 = \left[\frac{1}{\sin x}\right]_a^b + 2I_4$$

$$\Rightarrow I_4 = -\frac{1}{\sin x}$$

$$\therefore I_1 = [I_3 + I_4]_a^b$$

¹ \therefore The integral I_1 can be written as a single expression within its bounds \therefore Q.E.D

1.2 Problem 2

¹ **Warning:** Solution is unverified