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Problem Catalogue

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1 Integrals: Evil

1.1 Problem One

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Your final answer should be a single expression in the form $I_1 = [h(x)]_a^b$ where b and a are bounds of I_1 to be found

Conclude whether or not the below integral, I_1 :

$$I_1 = \int_{\int_x^{2x} (\ln x)^2}^{(3+x)^6} \left(\frac{1}{3e^x \cos x \sec^2 x} + \cot x \csc x \right) dx$$

can be written as a single expression, with all integration expressions evaluated.

Your conclusion must be supported, **either**, by an attempt to evaluate the integral or, a proof that I_1 has no real anti-derivative.

—12 Marks—

1.1.1 Worked Solution:

$$I_1 = \int_{\int_x^{2x}(lnx)^2}^{(3+x)^6} (\frac{1}{3e^x \cos x \sec^2 x} + \cot x \csc x)\ dx$$

$\Rightarrow\ I_1 = \int_b^a f(x)dx$
$\therefore\ let\ I_2 = b$

$$I_2 = \int_x^{2x} (\ln|x|)^2 = \int_{e^u}^{2e^u} u^2e^u du = [u^2e^u]_{e^u}^{2e^u} - \int_{e^u}^{2e^u} 2ue^ue^u du$$

$$= [u^2e^u]_{e^u}^{2e^u} - ([2ue^u]_{e^u}^{2e^u} - \int_{e^u}^{2e^u} 2e^u du = [u^2e^u]_{e^u}^{2e^u} - [2ue^u - 2e^u]_{e^u}^{2e^u} = [u^2e^u - 2ue^u - 2e^u]_{e^u}^{2e^u}$$

$$= [x(\ln|x|)^2 - 2x\ln|x| - 2x]_x^{2x} \because u = \ln|x| \Rightarrow e^u = x$$

$\therefore I_2 = x[2(\ln|2x|)^2 - (\ln|x|)^2 - \ln|16x^2|]$

This result can be obtained by evaluating the bounds

$$I_1 = \int_{I_2}^{(3+x)^6} (\frac{1}{3e^x \cos x \sec^2 x} + \cot x \csc x)\ dx = \int_a^b (\frac{1}{3e^x \cos x \sec^2 x)} + \int_a^b \cot x \csc x$$

$$let\ I_3 = \int_a^b (\frac{1}{3e^x \cos x \sec^2 x)} = \int_a^b (\frac{1}{3e^x \sec x)} dx = \frac{1}{3}\int_a^b (e^{-x}\cos x)dx$$

$$I_3 = 1/3(J)$$

$$J = [-e^{-x}\cos x]_a^b - \int_a^b -e^{-x}\sin x = [-e^{-x}\cos x]_a^b - ([-e^{-x}\sin x]_a^b - \int_a^b e^{-x}cosx$$

$$J = [-e^{-x}\cos x]_a^b - [e^{-x}\sin x]_a^b - J$$

$$\therefore\ J = \frac{1}{2}[e^{-x}\sin x - e^{-x}\cos x]_a^b$$

$$\therefore\ I_3 = \frac{1}{6}[e^{-x}\sin x - e^{-x}\cos x]_a^b$$

$$I_1 = I_3 + \int_a^b \cot x \csc x\ dx$$

$$let\ I_4 = \int_a^b \cot x \csc x\ dx$$

$$I_4 = \int_a^b (\frac{1}{\sin x})(\frac{\cos x}{\sin x})\ dx = \int_a^b \frac{\cos x}{1-\cos^2 x}\ dx$$

Integrate I_4 by parts:

$$I_4 = \int_a^b \frac{\cos x}{\sin^2 x}\ dx = \int_a^b \cos x(\frac{1}{\sin^2 x})\ dx = [\frac{1}{\sin x}]_a^b - \int_a^b \sin x(\frac{-2\sin x\cos x}{\sin^4 x})\ dx = [\frac{1}{\sin x}]_a^b - \int_a^b \frac{-2\cos x}{\sin x}\ dx$$

$$\therefore I_4 = [\frac{1}{\sin x}]_a^b + 2I_4$$

$$\Rightarrow I_4 = -\frac{1}{\sin x}$$

$$\therefore I_1 = [I_3 + I_4]_a^b$$

[1] $\therefore$ The integral I_1 can be written as a single expression within its bounds $\therefore$ Q.E.D

1.2 Problem 2

[1] **Warning:** Solution is unverified