DATE: 2-09-2024 **Day 01** DAY: Monday

MTH 262. Statistics and Probability Theory

Why this subject?	**Gratitude Attribute** → $\boxed{\text{NCEAC}}$
• Statistic Literacy	1. Knowledge
• Analytics	2. Problem solving.

شماریات
Statistics:- Provide insights of data.

1. Collection of (Raw) Data
2. Organization
3. Vizualization
4. Analysis
5. Interpretation.

$\boxed{\text{Data}}$

- $\boxed{\text{Qualitative . D}}$
  - → Attributes
  - → characteristics
  - $\boxed{\text{Ordinal}}$
    - → Rating in order
    - → opinions
  - $\boxed{\text{Nominal}}$
    - → Direct Questions
- $\boxed{\text{Quantitative . D}}$
  - → Numbers
  - $\boxed{\text{Discrete}}$
    - → Exact (count)
  - $\boxed{\text{Continuous}}$
    - → in decimals (Measurement)

×	×	✓	✓	✓	×
Name	Img	Age	Weight	Locality	Biography

Terms
NLP
LLM

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Phase-1 : Collection of Data

- → Direct Observation
- → Interview (1-1)
- → Survey (FB, Reviews)
  - ↳ (Broader Research Obs.)
- → Articles, Books (Numbers)
- → Census (مردم شماری)
  - ↳ [first data comes then insights]
- → Internet
- → Polls (opinions)
- → Newspaper
  - ↳ (pass, fail) (dead)
- → Forms (Adm. form)
  - ↳ [another type of survey but limited]
  - [Primary obj]

Phase-2 : Organization of Data (Representable form)

- How many siblings do you have?

Single value Grouping

No. of siblings	Freq.
0	1
1	3
2	6
3	4
4	7
5	0

Data type :-

→ Quantitative [Discrete]

Tabular form :-

→ Frequency Distribution

~~1~~, ~~0~~, ~~2~~, ~~1~~, ~~2~~, ~~3~~,
~~4~~, ~~2~~, ~~2~~, ~~3~~, ~~1~~, ~~3~~,
~~3~~, ~~2~~, ~~4~~, ~~4~~, ~~4~~, ~~2~~
~~4~~, ~~4~~, ~~4~~

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- Which Mobile Phone Brand you are currently used?

Mobile Phone Brand	Frequency
O	2
T	2
V	7
I	1
S	4
A	1
R	3
N	1

Categorial Group

Frequency Distribution of Qualitative / Categorial Data

Data type:-
→ Qualitative
Nominal

- How would you rate call Quality of your ViVo cell Phone? (survey)

*	Not satisfy	Bad
**		
***	Normal	
****		
*****	Satisfied	Good

Data type:-
→ Qualitative
Ordinal
→ order recommended

- Weight (Measurement) Data Type: Quantitative Continuous

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Data:- 21, 24, 19, 18, 31, 43, 47, 28, 16, 20,
22, 24, 19, 32, 38, 33, 36, 29, 41, 30

Note: Range ↑, that's why it's not single valued Grouping.

Step-1:- Find Range.

Range = Max value in Data − Min Value in Data

$= 47 - 16 = 31$

Step-2:- Find No. of Classes.

Note:

n↑ c↑
n↓ c↓

No. of Classes = $1 + (3.33 \log(n))$

∴ n = total no. of observations

So, $\boxed{n = 20}$

No. of classes = $1 + (3.33 \log(20))$

$= 5.33 \uparrow = 6$

Step-3:- Find Class Width

Class Width = Range / No. of Classes

$= 31 / 6$

Class Width $= 5.1 \uparrow = 6$

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$\boxed{22-16 = 6}$	Classes LCL-UCL	Freq. f	Class Boundary (LCL-0.5)↔(UCL+0.5)	Mid Point $\frac{LCL+UCL}{2}$	Cumulative freq. (CF)	Relative frequency $f/\Sigma f$	Percentage % (proportion data)
1	16-21	6	15.5-21.5	18.5	6 (1-6)	$\frac{6}{20}=0.3$	0.3 x 100 = 30%
2	22-27	3	21.5-27.5	24.5	9 (7-9)	0.45	45%
3	28-33	6	27.5-33.5	30.5	15 (10-15)	0.3	30%
4	34-39	2	33.5-39.5	36.5	17 (16-17)	0.85	85%
5	40-45	2	39.5-45.5	42.5	19 (18-19)	0.95	95%
6	46-51	1	45.5-51.5	48.5	$\boxed{20}$ = n	1	100%

$\boxed{\Sigma f = 20} = n$

- Class Boundary:- [Viz :- Histogram , Analysis: Median, Mode, etc]

- Mid Point:- [Viz : Freq. Polygon , OGive; Ana: Mean, variance, std.] [18.5 + classwidth (6) = 24.5]

- Cumulative Freq:- Provide order seq. [Viz:- OGive; Ana:- Mode, Median, etc]

  x_1 16, x_2 18, x_3 19, x_4 19, x_5 20, x_6 21 → C1

  x_7 22, x_8 24, x_9 24 → C2

- Relative Freq:- [PIE Chart : Viz]

Pg# 65 + Pg# 72 (2.28, 2.29, 2.30, 2.31

Pg# 75 (Table 2.6) → $\boxed{\text{Task}}$

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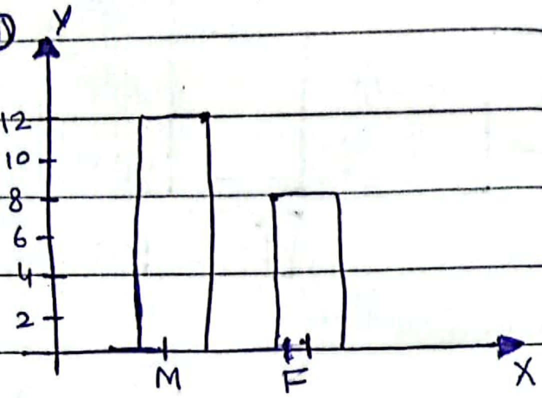
Phase-3 :- Vizualization of Data

- Qualitative Data :

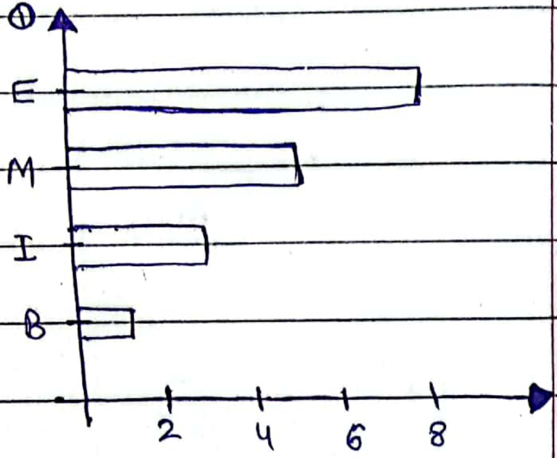
① → BAR Chart ② → PIE Chart

categ. ←

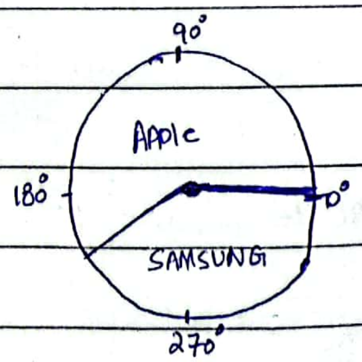
Gender	f
M	12
F	8

①


Int. Major	f
E	8
M	5
I	3
B	1

①


Mobile Phone	f	Rf
Apple	12	$12/20 \times 360° = 216°$
SAMSUNG	8	$8/20 \times 360° = 144°$

②


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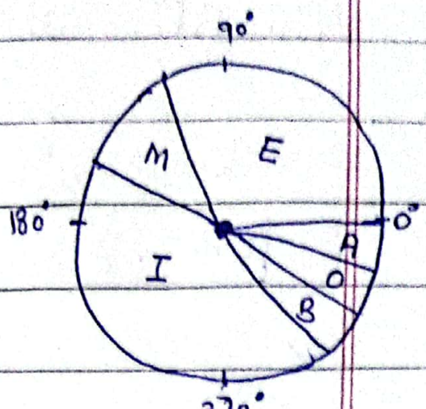
Int. Major	f	Rf	CF
E	8	$8/28 \times 360^\circ$ 102.8°	102.8°
M	5	64.3°	167.1°
I	10	128.5°	295.6°
B	3	38°	323.3°
O	1	12.8°	347°
A	1	12.8°	360°
	$\Sigma f = 28$		

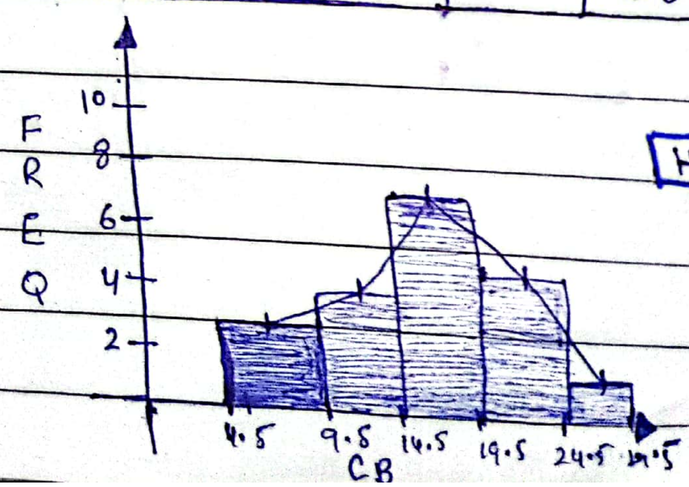


- Quantitative Data

① → Histogram ② → Freq. Polygon

③ → OGive ④ → PIE Chart

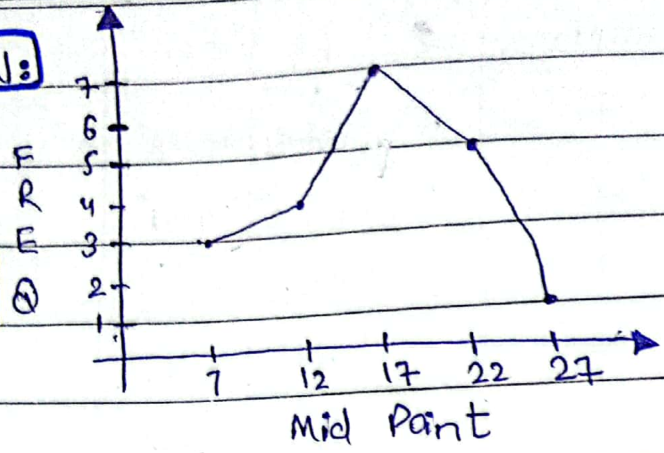
	Classes	f	CB	Mid.P	CF	Rf	CRF
1	5-9	3	4.5-9.5	7	3	$3/20 \times 360^\circ$ $= 54^\circ$	54°
2	10-14	4	9.5-14.5	12	7	72°	126°
3	15-19	7	14.5-19.5	17	14	126°	252°
4	20-24	5	19.5-24.5	22	19	90°	342°
5	25-29	1	24.5-29.5	27	20	18°	360°



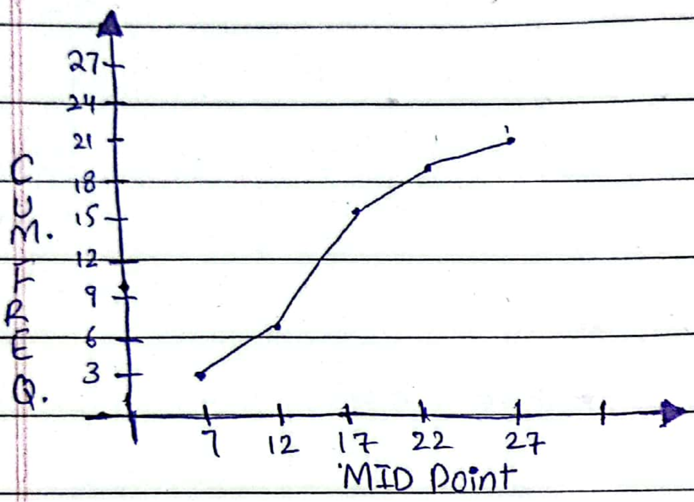
HISTOGRAM

FREQ. POLYGON:

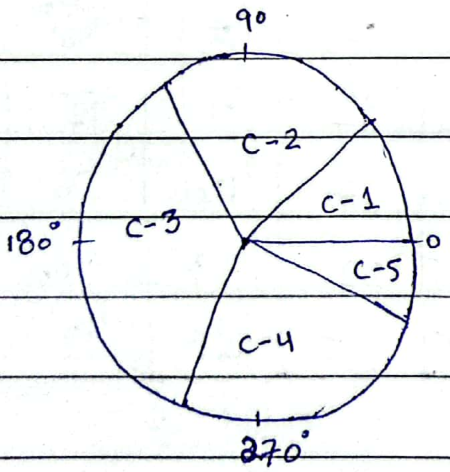
Also Known as Line Chart



OGIVE



PIE Chart



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PIV: ANALYSIS OF DATA

★ Measure of Central Tendency
1- Mean / Average [Tells origin]
2. Median [Tells Position wise center value]
3- Mode [Most Frequent value in data]

⇒ Unstructured / Ungrouped Data
⇒ Structured / Grouped Data

MOCT For Ungrouped Data:-

1. Mean / Average:

[Raw data present]

→ gives info about core value / origin
→ from where data originate.
→ it is a sensitive measure → [mean can change by adding data]
[60, 70, 51, 95] → Outliers (can effect the mean of data)
→ Very important observation to detect measure

Eg:- $\overset{x_1}{2}, \overset{x_2}{4}, \overset{x_3}{6}, \overset{x_4}{3}, \overset{x_5}{2}, \overset{x_6}{1}$ $\therefore n = 6$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$ $\sum x = 18$

$$\bar{x} = \frac{18}{6} = \boxed{3}$$

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2. Median:

- Tells location
- Divides data in two equal parts
- most important in Ranking.

Eg:- 2, 4, 3, 2, 1, 5, 6, 2, 9

Step1: No. of observations $n = 9$

Step2: Nature of n odd

Step3: Ordering (Ascending) 1, 2, 2, 2, 3, 4, 5, 6, 9

$x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9$

Formula: for (odd) Data

$$\left(\frac{n+1}{2}\right)^{th} \text{obs.} = \left(\frac{9+1}{2}\right)^{th} \text{obs.} = \left(\frac{10}{2}\right)^{th} \text{obs.}$$

$$= (5)^{th} \text{obs.} = \boxed{3}$$

Eg:- 15, 19, 21, 20, 13, 15, 12, 17

Step1: $n = 8$ No. of Observations.

Step2: Even — Nature of n

Step3: 12, 13, 15, 15, 17, 19, 20, 21 Ordering (Asc.)

Formula: for Even data

$$\frac{\left(\frac{n}{2}\right)^{th} \text{obs.} + \left(\frac{n}{2}+1\right)^{th} \text{obs.}}{2}$$

$$= \frac{\left(\frac{8}{2}\right)^{th} obs. + \left(\frac{8}{2}+1\right)^{th} obs.}{2} = \frac{(4)^{th} obs. + (5)^{th} obs.}{2}$$

$$= \frac{15+17}{2} = \frac{\cancel{32}^{16}}{\cancel{2}} = \boxed{16}$$

3: Mode: → Most Frequent value in Data

→ Provides $\boxed{\text{Peakness}}$ in graphs.

e.g: 2, 4, 3, 2, 1, 5, 6, 2, 9

Mode = 2 [Uni-Model]

e.g: 2, 3, 2, 2, 1, 3, 2, 0, 3

Mode = 2, 3 [Bi-Model]

e.g: 1, 1, 2, 1, 2, 3, 2, 3, 3, 4, 4, 5

Mode = 1, 2, 3 [Multi-Model]

منتشر/پھیلاؤ

* Measure of Dispersion:- (spread)

1- Range

2- Variance [Provide variation for complete data]

3- Standard Deviation

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$\therefore n = 5$

Data-1:- 2, 5, 3, 5, 6

Mean$_1$ = $\bar{X}_1 = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{21}{5} = 4.2$

Median$_1$ = $n = 5$ (odd) 2, 3, 5, 5, 6

$= \left(\frac{n+1}{2}\right)^{th}$ obs. $= \left(\frac{5+1}{2}\right)^{th}$ obs. $= \left(\frac{6}{2}\right)^{th}$ obs.

$= (3)^{th}$ obs. $= \boxed{5}$

Mode$_1$ = 5 (uni-model)

Range$_1$ = $6 - 2 = 4$ (Max. value - Min)

Variance$_1$ = $S_1^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

x	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
2	$2 - 4.2 = -2.2$	4.84
5	$5 - 4.2 = 0.8$	0.64
3	$3 - 4.2 = -1.2$	1.44
5	$5 - 4.2 = 0.8$	0.64
6	$6 - 4.2 = 1.8$	3.24
		$\sum (x_i - \bar{x})^2 = 10.8$

Why Range is not compatible measure for Dispersion? Bcz the Range depends on 2-values.

$S_1^2 = \frac{10.8}{5-1} = \boxed{2.7}$

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Data-2: 0, 5, 10, 5, 1 $\therefore n = 5$ (odd)

$$\text{Mean}_2 = \bar{x}_2 = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{21}{5} = 4.2$$

$$\text{Mode}_2 = \left(\frac{5+1}{2}\right)^{th} \text{obs.} = \left(\frac{6}{2}\right)^{th} \text{obs.} = (3)^{th} \text{obs.}$$

$\text{Mode}_2 = 5$ (uni-model) 0, ~~0~~1, 5, 5, 10

$\text{Range}_2 = 10 - 0 = 10$

$$\text{Variance}_2 = S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

x	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
0	$0 - 4.2 = -4.2$	17.64
5	$5 - 4.2 = 0.8$	0.64
10	$10 - 4.2 = 5.8$	33.64
5	$5 - 4.2 = 0.8$	0.64
1	$1 - 4.2 = -3.2$	10.24
		$\sum (x_i - \bar{x})^2 = 62.8$

$$S_2^2 = \frac{62.8}{5-1} = \frac{62.8}{4}$$

$$\boxed{S_2^2 = 15.7}$$

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⋆ Measure of Dispersion:

Data1: 2,3,5,3,6 $S_1^2 = 2.7$

Data2: 0,5,10,5,1 $S_2^2 = 15.7$

3. Standard Deviation:-

$S_1 = \sqrt{S_1^2} = \sqrt{2.7} = 1.69$ Reliable Data

$S_2 = \sqrt{S_2^2} = \sqrt{15.7} = 3.96$ Disp. ↓

⋆ MOCT for Grouped Data:

∴ L.C.B = Lower Class Boundary

$$\text{Average} = \bar{x}_f = \frac{\sum f(x_i)}{\sum f}$$ $\to \boxed{x_i = \text{Mid Point}}$

$$\text{Median}_f = l + \left(\frac{n/2 - CF_b}{f_m}\right) \times h$$

$$\text{Mode}_f = l + \left(\frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)}\right) \times h$$

l = L.C.B of Medianth class	l = L.C.B of Modal class
CF_b = Cumulative Freq. before " "	f_m = Freq. of " "
f_m = Freq. of " "	f_1 = Freq. before " "
h = class width of " "	f_2 = Freq. after " "
	h = class width of " "

Note: If medianth class is 1st class then $\boxed{CF_b = 0}$

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	Classes	Freq (f_i)	Mid.P (x_i)	Class Boundary	Cum. Freq.	$f_i \cdot x_i$
1	50-54	3	52	49.5-54.5	(1-3) 3	3x52 = 156
2	55-59	8	57	54.5-59.5	(4-11) $\boxed{11}$ CF_b	8x57 = 456
3 ✓	60-64	f_m $\boxed{11}$	62	l $\boxed{59.5}$-64.5	(12-22) 22	11x62 = 682
4	65-69	6	67	64.5-69.5	(13-28) 28	6x67 = 402
5	70-74	2	72	69.5-74.5	(29-30) 30	2x72 = 144
		$\Sigma f = 30$				$\Sigma fx = 1840$

$$\bar{X}_f = \frac{\Sigma f_x}{\Sigma f} = \frac{1840}{30} = 61.37$$

★ For Medianth class $= \left(\frac{n}{2}\right)^{th}$ obs. $n = f = 30$

$= \left(\frac{30}{2}\right)^{th}$ obs. $= (15)^{th}$ obs.

Since, 15th obs. lies in 3rd class

so class-3 is medianth class.

$$\text{Median}_f = 59.5 + \left(\frac{\left(\frac{30}{2}\right)_{15} \text{obs.} - 11}{11}\right) \times 5$$

$h = 55-50$

$\boxed{h = 5}$

$$= 59.5 + \left(\frac{15-11}{11}\right) \times 5$$

$\boxed{\text{Median}_f = 61.3}$

Note: answer must be in range of medianth class (60-64)

Imp Note: If Model class is 1st Class then $\boxed{f_1 = 0}$.
If Model class is last Class then $\boxed{f_2 = 0}$

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$$\text{Mode}_f = l + \left(\frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \right) \times h$$

Modal Class:- The Class which have high frequency (max. no. of observations).

$$\text{Mode}_f = 59.5 + \left(\frac{11-8}{(11-8)+(11-6)} \right) \times 5$$

$$= 59.5 + \left(\frac{3}{3+5} \right) \times 5$$

$$= 59.5 + \left(\frac{3}{8} \right) \times 5$$

$$\boxed{\text{Mode}_f = 61.37}$$

[mode must also be in range of model class]

* MOD for Grouped Data:-

$$\text{Variance} = S_f^2 = \frac{\Sigma \left(f_i \left(x_i - \overline{x}_f \right)^2 \right)}{n-1}$$

$$\text{Standard Deviation} = S_f = \sqrt{\frac{\Sigma \left(f_i \left(x_i - \overline{x}_f \right)^2 \right)}{n-1}}$$

$$\boxed{\overline{x}_f = 61.3}$$

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x_i	$(x_i - \bar{x}_f)^2$	f_i	$f_i(x_i - \bar{x}_f)^2$
52	$(52-61.3)^2 = 86.49$	3	$3 \times 86.49 = 259.47$
57	$(57-61.3)^2 = 18.49$	8	$8 \times 18.49 = 147.92$
62	$(62-61.3)^2 = 0.49$	11	$11 \times 0.49 = 5.39$
67	$(67-61.3)^2 = 32.49$	6	$6 \times 32.49 = 194.94$
72	$(72-61.3)^2 = 114.49$	2	$2 \times 114.49 = 228.98$
			$\Sigma f_i(x_i - \bar{x}_f)^2 = 836.7$

$$S_f^2 = \frac{836.7}{30-1} = \frac{836.7}{29}$$

$$\boxed{S_f^2 = 28.85}$$

$$S_f = \sqrt{28.85} = 5.37$$

$$\boxed{S_f = 5.37}$$

Quiz#1 (3-Phases)

Pg#139 ⇒ 3.95

Part (b) Grouped Data

(Solve)

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3.95 : Part (b)

333	106	19	74	189	39	146	242
216	85	181	83	224	63	147	155
193	127	207	22	54	94	34	112
34	87	92	172	43	76	92	114
28	44	139	65	75	116	172	98

[MOCT For Grouped Data]

Step1: Find Range:-

Range = 333 - 19 = 314

Step2: No. of Classes = 1 + (3.33 log(n)) 	∴ n = 40

= 1 + (3.33 log(40))

No. of Classes = 6.33 = 7

Step3: Class width = Range / No. of Classes.

= 314 / 7

Class Width = 44.8 = 45

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	Classes	(f_i) Freq.	Class Bound.	Cum. Freq.	Mid. Point (x_i)	$f_i x_i$	$(x_i-\bar{x}_f)^2$	$f_i(x_i-\bar{x}_f)^2$
1	19-63	13	18.5-63.5	(1-13) 13 cf_b	41	533	4112.01	53456.13
2	64-108	f_m 12	l 63.5-108.5	(14-25) 25	86	1032	365.76	4389.12
3	109-153	6	108.5-153.5	(26-31) 31	131	786	669.51	4017.06
4	154-198	5	153.5-198.5	(32-36) 36	176	880	5023.26	2516.3
5	199-243	3	198.5-243.5	(37-39) 39	221	663	13427.01	40281.03
6	244-288	0	243.5-288.5	39	266	0	25880.76	0
7	289-333	1	288.5-333.5	40	311	311	42384.51	42384.51
		$\Sigma f=40$				$\Sigma f_i x_i = 4205$		$\Sigma f_i(x_i-\bar{x})^2 = 169643.88$

① Average $= \bar{X}_f = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{4205}{40} = 105.125$

② For Medianth class $= \left(\dfrac{n}{2}\right)^{th}$ obs. $= \left(\dfrac{\cancel{40}^{20}}{\cancel{2}}\right)^{th}$ obs.

$= 20^{th}$ obs.

Thus, Class-2 is Medianth Class because 20th obs. lies in class-2.

$$\text{Median}_f = l + \left(\frac{n/2 - CF_b}{f_m}\right) \times h$$

$$= 63.5 + \left(\frac{40/2 - 13}{12}\right) \times 45$$

$$= 63.5 + (7/12) \times 45 = 63.5 + 26.25$$

$\text{Median}_f = 89.75$

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③ For Model class, since, highest freq. (no. of observations) lie in 1st-class so class-1 is our Model class.

$$Mode_f = l + \left(\frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \right) \times h$$

$$= 18.5 + \left(\frac{13 - 0}{(13-0)+(13-12)} \right) \times 45$$

$$= 18.5 + (13/14) \times 45$$

$$= 18.5 + 41.78$$

$$Mode_f = 60.28$$

[MOD for Grouped Data]

① $$\text{Variance}(S^2) = \frac{\Sigma \left(f \cdot (x - \bar{x}_i)^2 \right)}{n-1}$$

$$= \frac{169.643.88}{45-1} = \frac{169643.88}{44}$$

$$\text{Variance}(S^2) = 3855.54$$

② $$\text{Standard Deviation}(S) = \sqrt{\frac{\Sigma (f \cdot (x - \bar{x}_i)^2}{n-1}}$$

$$= \sqrt{3855.54}$$

$$\text{Standard Deviation}(S) = 62.09$$

DATE: Day 07 DAY:

→ Quartiles [seasonal variation/impact] data enhance.
→ Outlier Direction [Data can impact by outlier]
→ Box-Plot

Quartiles for Ungrouped Data:-

→ Data divides into four parts
→ For Seasonal measures

4, 5, 6, 3, 2, 1, 0, 6, 3, 2, 1

Step 1: Find Median $\therefore n = 11$ (odd)

Step 2: $\left(\frac{n+1}{2}\right)^{th}$ obs.

Step 3: $\underbrace{\overset{\boxed{Part 1}}{0, 1, 1}}_{Q_1}, \overset{\boxed{Part 2}}{2, 2}, \underbrace{\overset{Med}{3}}_{Q_2}, \overset{\boxed{Part 3}}{3, 4}, \underbrace{5}_{Q_3}, \overset{\boxed{Part 4}}{6, 6}$

Median $= \left(\frac{11+1}{2}\right)^{th}$ obs. $= 6^{th}$ obs.

$Median_{right} = \left(\frac{5+1}{2}\right)^{th}$ obs. $= 3^{th}$ obs.

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for Even = $\dfrac{\left(\frac{n}{2}\right)^{th} \text{obs.} + \left(\frac{n}{2}+1\right)^{th} \text{obs.}}{2}$

$= \dfrac{\left(\frac{12}{2}\right)^{th} \text{obs.} + \left(\frac{12}{2}+1\right)^{th} \text{obs.}}{2}$

$= \dfrac{6^{th} \text{obs.} + 7^{th} \text{obs.}}{2}$

for Even = $\dfrac{3+3}{2} = \dfrac{9}{2} = 3$

P-1	P-2	P-3	P-4
0, 1, 1,	2, 2, 3,	3, 4, 5	6, 6, 7

Q_1 Q_2 Q_3

$\text{Median}_f = \dfrac{\left(\frac{6}{2}\right)^{th} \text{obs.} + \left(\frac{6}{2}+1\right)^{th} \text{obs.}}{2}$

$= \dfrac{(3)^{th} \text{obs.} + (4)^{th} \text{obs.}}{2}$

$= \dfrac{1+2}{2}$ | $= \dfrac{5+6}{2}$

$= \dfrac{3}{2}$ | $= \dfrac{11}{2}$

$\text{Median}_f = 1.5$ | $= 5.5$

DATE: ____________ DAY: ____________

Quartlies for Grouped Data:-

$$\text{Median}_{f} = l_{Q_i} + \left(\frac{i n/4 - CF_{bQ_i}}{f_{mQ_i}} \right) \times h$$

To find Medianth class (3 - Medianth class)

$$Q_i = \left(\frac{in}{4} \right)^{th} \text{obs.} \qquad \therefore i = 1, 2, 3, 4, \ldots$$

$$Q_1 = \left(\frac{n}{4} \right)^{th} \text{obs.} = \left(\frac{28}{4} \right)^{th} \text{obs.} = 7\text{th obs.}$$

$$Q_2 = \left(\frac{n}{2} \right)^{th} \text{obs.} = \left(\frac{28}{2} \right)^{th} \text{obs.} = 14^{th} \text{obs.}$$

$$Q_3 = \left(\frac{3n}{4} \right)^{th} \text{obs.} = \left(\frac{3(28)}{4} \right)^{th} \text{obs.} = 21^{th} \text{obs.}$$

	Classes	f	C.B	CF
	0-4	2	-0.5-4.5	(1-2) 02
Q_1	5-9	05	4.5-9.5	(3-7) 07
Q_2	10-14	12	9.5-14.5	(8-19) 19
	15-19	06	14.5-19.5	(20-25) 25
	20-24	03	19.5-24.5	(26-28) 28
		$\Sigma f = 28$		

DATE:

$$Q_1 = 4.5 + \left(\frac{28/4 - 02}{05} \right) \times 5$$

$Q_1 =$

$$Q_2 = 9.5 + \left(\frac{14 - 07}{12} \right) \times 5$$

$$Q_2 = 9.5 + 2.915 = 12.415$$

$$Q_3 = 14.5 + \left(\frac{21 - 19}{6} \right) \times 5$$

$$Q_3 = 14.5 + 10 = 24.5$$

Detection of Outlier

* **Inter Quartile Range (IQR).**

$$IQR = Q_3 - Q_1$$

* **Limits for Detection:-**

Lower Limit $= Q_1 - (1.5 \times IQR)$.

Upper Limit $= Q_3 + (1.5 \times IQR)$

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Ex#:- Find the values of Q_1, Q & Q_3 for following data.

15, 13, 12, 14, 16, 17, 19, 18, 0, 20, 16, 39, 12, 15

0, 12, 12, 13, 14, 15, 15 | 16, 16, 17, 18, 19, 20, 39

Q_2

$n = 14$ (Even)

$$\text{Median}_f = \frac{\left(\frac{n}{2}\right)^{th} \text{obs.} + \left(\frac{n}{2}+1\right)^{th} \text{obs.}}{2}$$

$$= \frac{\left(\frac{14}{2}\right)^{th} \text{obs.} + \left(\frac{14}{2}+1\right)^{th} \text{obs.}}{2}$$

$$= \frac{(7)^{th} \text{obs.} + (8)^{th} \text{obs.}}{2} = \frac{15+16}{2}$$

$$\text{Median}_f^{(Q_2)} = 15.5$$

$$Q_1 = \left(\frac{n+1}{2}\right)^{th} \text{obs.} \qquad \because n = 7 \text{ (odd)}$$

$$= \left(\frac{7+1}{2}\right)^{th} \text{obs.} = \left(\frac{8}{2}\right)^{th} \text{obs.}$$

$$Q_1 = 4^{th} \text{obs.} = 13$$

$$Q_3 = 18$$

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$IQR = Q_3 - Q = 18 - 13 = 5$

Lower limit $= 13 - (1.5 \times 5) = 5.5$

Upper limit $= 18 + (1.5 \times 5) = 25.5$

→ Remove Outlier

→ important outliers keep

→ Replace outlier with mean of data.

Important Outliers:-

Monthly pay

5 → 4000

5 → 5000

5 → 4500

5 → 5500

5 → $\boxed{16000}$ ✓

Day 08 → Quiz-1

DATE: DAY:

Day 09

L.L = 5.5
U.L = 25.5

→ Box-Plot :-

Q2 (between 15 and 16)

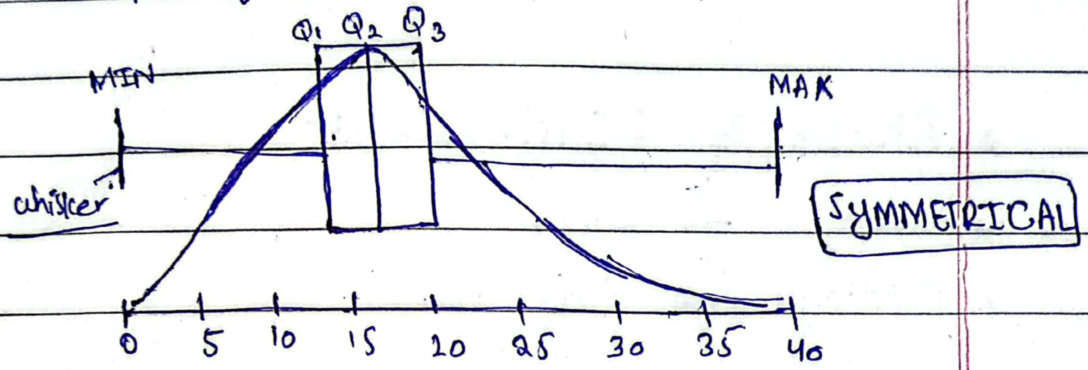
6, 12, 12, 13, 14, 15, 15, | 16, 16, 17, 18, 19, 20, 39

- 6: Min → Outlier
- 12: Min
- 13: Q_1
- 18: Q_3
- 20: Max
- 39: Max → Outlier

• Five-Number Summary:-

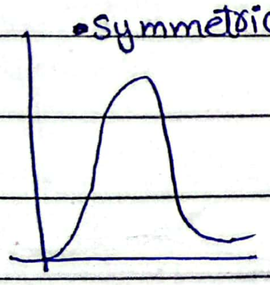
→ Min → Max → Q_1 → Q_2 → Q_3

→ Shape of data distribution provides.

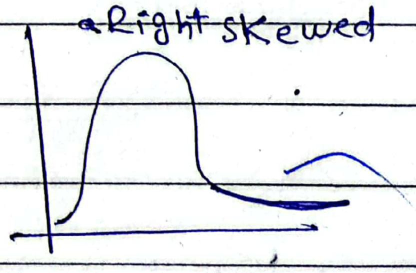


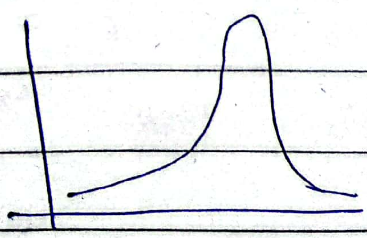
→ Box contains outliers (Q_1, Q_2, Q_3)
→ Whiskers.

• Symmetrical



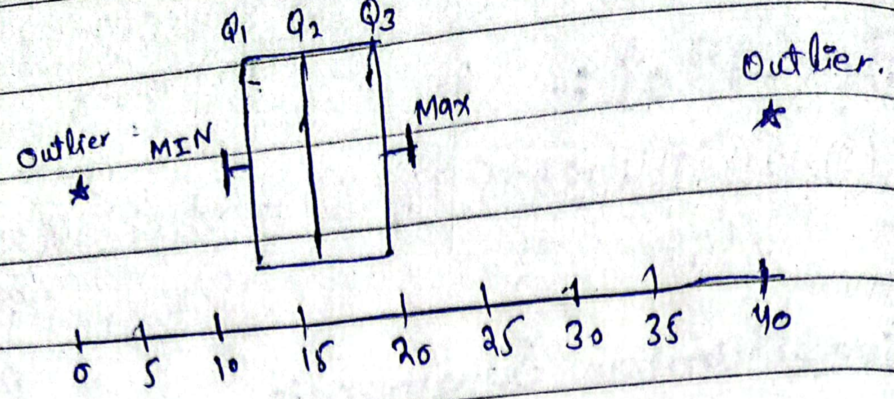
• Right skewed





• Left-skewed.

DATE: DAY:

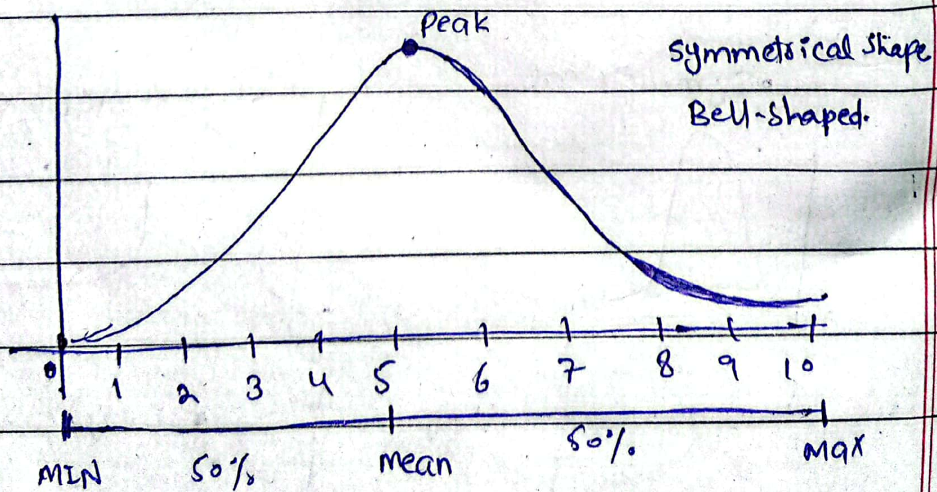


→ Mainly used for detection of outlier.

★ Shape of Distribution:-

★ Mean → Origin
★ Median → Center Value
★ Mode → Most Freq. Value.

e.g. Mean = 5 , Median = 5 , Mode = 5



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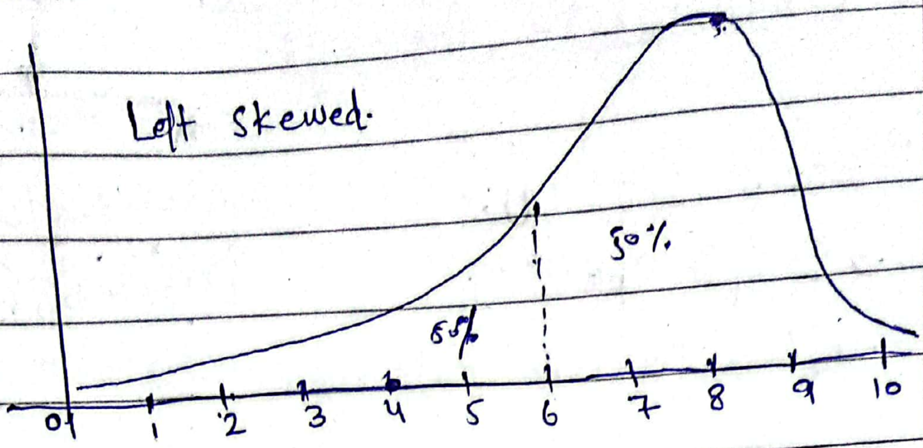
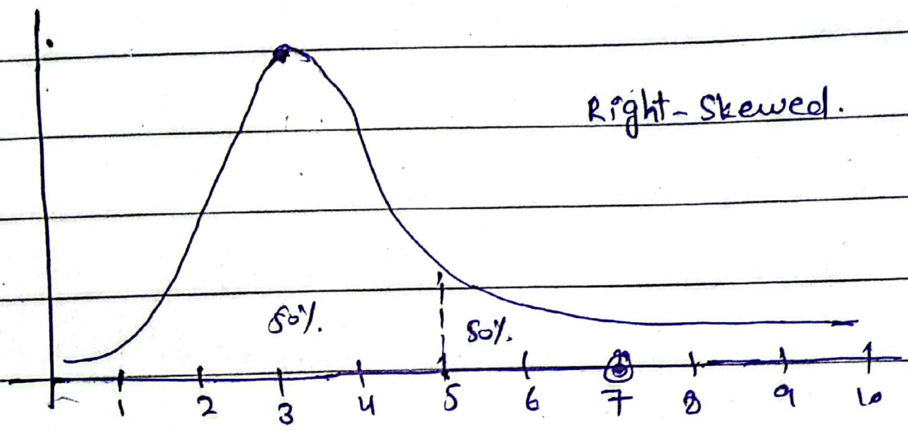
e.g. Mean = 4 , Median = 6, Mode = 8



e.g. Mean = 7, Median = 5, Mode = 3



* Mean = Median = Mode (Symmetrical)
* Mean > Median > Mode (Right-Skewed)
* Mean < Median < Mode (Left-Skewed)

Q: Find the mean, median & mode of data and based upon measures what will be the shape of data?

DATE: ______ **Day 10** DAY:

uncertainity ⟷ Experiment → Set Theory → Universal set

Quantify ← **Probability Theory:-**

- Universal Set (U.S) — Dice Rolling / coin / Toss
- in (P.T) • Sample Space (S) — $\{1,2,3,4,5,6\}$ $\{H,T\}$
  - → All possible outcomes.

SUBSET

- **Event:** That information in which you show interest.

$S.S = \{H,T\} = n(S) = 2$

$S.S = \{1,2,3,4,5,6\} = n(S) = 6$

$S.S = \{W,L,D\} = n(S) = 3$

$A = C/T = \{H\} = n(A) = 1$

$B = E.D = \{6\} = n(B) = 1$

$C = M.P = \{W,D\} = n(C) = 2$

$$P(A) = \frac{\text{\# of element in event A}}{\text{Total no. of " " sample space}}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2} = 0.5 = 50\%$$

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$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{6} = 0.166 = 16.67\%$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{3} = 0.75 = 75\%$$

[can't change in any circumstances]

- **Limitations:-** (Axioms of Prob.)

① → Prob. will always lies between 0 and 1, inclusively. 0-100%

→ ② $P(S) = 1$ Sum of probability of all the elements in a sample space.

e.g.
→ $S = \{H, T\} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

e.g. $S = \{1, 2, 3, 4, 5, 6\} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$

$S = \{W, L, D\} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$

③ → $n(S) \neq 0$ e.g. $\frac{1}{0}$ = undefined.

Ex#: What is the probability of selecting a $\boxed{\text{Vowel}}$ (event) in an $\boxed{\text{Eng. alphabet}}$? (sample space)

⇒ $S = n(S) = 26$

⇒ $A = \{a, e, i, o, u\} = n(A) = 5$

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$\Rightarrow P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5}{26}$ $\Rightarrow$ Answer.

$\Rightarrow P(S) = \dfrac{1}{26} + \dfrac{1}{26} + \dfrac{1}{26} + \dots + \dfrac{1}{26} = \dfrac{26}{26} = 1$

Ex#2: What is the prob. of getting an even number on throwing a dice?

$S = \{1, 2, 3, 4, 5, 6\}$

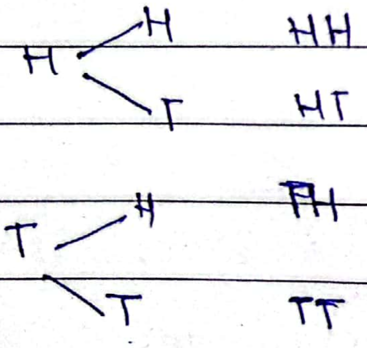
$A = \{2, 4, 6\}$

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

$\boxed{\text{2-Times}}$

$\boxed{\text{C.T}}$ Tree-Diagram.

Toss-1



$S = \{H, H, H, T, TH, TT\}$

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1 → 1, 2, 3, 4, 5, 6 ...... 6 → 1, 2, 3, 4, 5, 6

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) ...... (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

Formula:- n^{r} (r → replace, n → no. of outcomes) $= 2^2 = 4$

$= 6^2 = 36$

$2\times2\times2$

$2\times2\times2$

Distribution of Cards:-

N-P cards

Heart ♡	A	2	3	4	5	6	7	8	9	10	J	Q	K	13
SPADE ♤					5									13
Club ♧														13
Diamond ◇														13

P-cards

$4+4+4+4+4+4+4+4+4+4+4+4+4 = \boxed{52}$

Redcards = 26 Black cards = 26

$P(H) = \frac{n(H)}{n(S)} = \frac{13}{52}$

$P(7) = \frac{4}{52}$

Picture-cards $= \frac{12}{52}$

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if one data has 2- connected sources then we use $\cap$ & $\cup$
AND OR

Day 11

$\cap \rightarrow$ common information A "AND" B

$\cup \rightarrow$ All information A "OR" B

A = King of Hearts $\{K^{H}\}$

B = King $\{K^{H}, K^{S}, K^{C}, K^{D}\}$

C = Heart Card $\{A^{H}, 2^{H}, 3^{H}, \ldots, J^{H}, Q^{H}, K^{H}\}$

D = Face Card (Picture Cards)

$\{J^{H}, Q^{H}, K^{H}, J^{S}, Q^{S}, K^{S}, J^{C}, Q^{C}, K^{C}, J^{D}, Q^{D}, K^{D}\}$

(A) $P(\text{not } D) = P(D')$

$\Rightarrow P(D) = \frac{n(D)}{n(S)} = \frac{12}{52}$

$\Rightarrow P(D') = 1 - P(D) = 1 - \frac{12}{52}$

$\Rightarrow \boxed{P(D') = \frac{40}{52}}$

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★ Law of complement event:-

$P(A) + P(A') = 1$

$\downarrow$ occurrence $\quad \downarrow$ non-occurrence

Exam . Result = {Pass, Fail}

$P(P) + P(F) = 1$

$0.60 + ? = 1$

$P(F) = 1 - 0.60$

$\boxed{P(F) = 0.40}$

(B) $P(B \& C) = P(B \cap C) = \frac{n(B \cap C)}{n(S)}$

$\Rightarrow B \cap C = \{K^{(H)}\}$

$\Rightarrow n(B \cap C) = 1 \quad \Rightarrow \boxed{P(B \cap C) = \frac{1}{52}}$

(C) $P(B \text{ or } C) = P(B \cup C)$

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$

$P(C) = \frac{n(C)}{n(S)} = \frac{13}{52}$

$P(B \cup C) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

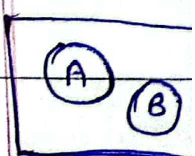
$\boxed{P(B \cup C) = \frac{16}{52}}$

★ For "OR" Case

[law of Addition]

• Mutually Exclusive Events

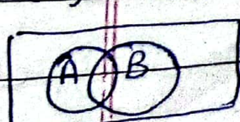
$P(A \cup B) = P(A) + P(B)$



$\therefore A \cap B = \phi$

• Non-Mutually E.E:-

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$\therefore A \cap B \neq \phi$

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(D) $P(C \& D) = P(C \cap D) = \dfrac{n(C \cap D)}{n(S)}$

$C \cap D = \{ \overset{(H)}{J}, \overset{(H)}{Q}, \overset{(H)}{K} \}$

$$\boxed{P(C \cap D) = \frac{3}{52}}$$

$P(B \text{ or } E) = P(B \cup E)$ [~~Non~~-Mutually]

$P(E) = \dfrac{n(E)}{n(S)} = \dfrac{40}{52}$

$P(B \cup E) = P(B) + P(E)$

$P(B \cup E) = \dfrac{04}{52} + \dfrac{40}{52} = \dfrac{44}{52}$

$$\boxed{P(B \cup E) = \frac{43}{52}}$$

$P(B \cup E) =$ [Non-Mutually Exclusive]

E = non-face cards

$\{ \overbrace{A, 2, 3, \dots, 10}^{(H)}, \overbrace{A, 2, 3, \dots, 10}^{(S)}, \overbrace{A, 2, 3, \dots, 10}^{(C)}, \overbrace{A, 2, 3, \dots, 10}^{(D)} \}$

$B = \{ \overbrace{A, 2, 3, \dots, J, Q, K}^{(H)} \}$

$B \cap E = \{ \overbrace{A, 2, 3, \dots, 10}^{(H)} \} \Rightarrow n(C \cap E) = 10$

$P(B \cup E) = P(B) + P(E) - P(C \cap E)$

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$$P(B \cup E) = \frac{13}{52} + \frac{40}{52} - \frac{10}{52}$$

$$\boxed{P(C \cup E) = \frac{17}{52}}$$

Ex # 4.7:-

Age(yr)	freq.
17	1
18	1
19	9
20	7
21	7
22	5
23	3
24	4
26	1
35	1
36	1

$\Sigma f = 40$

$A = 1+1+9+7 = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{40}$$

$B = 1+1 = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{40}$$

$C = 7+7+5+3+4+1 = 27$

$$P(C) = \frac{n(C)}{n(S)} = \frac{27}{40}$$

$D = 9+7+7+5+3+4+1+1+1 = 38$

$$P(D) = \frac{n(D)}{n(S)} = \frac{38}{40}$$

(a) $P(\text{not } D) = P(D') = 1 - P(D)$

$$P(D') = 1 - \frac{38}{40} = \frac{40-38}{40} = \frac{2}{40}$$

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b) $P(A \& D) = P(A \cap D)$

$n(A \cap D) = 9 + 7 = 16$

$P(A \cap D) = \frac{n(A \cap D)}{n(S)} = \frac{16}{40}$

c) $P(A \text{ or } D) = P(A \cup D)$

$P(A \cup D) = P(A) + P(D) - P(A \cap D)$

$P(A \cup D) = \frac{18}{40} + \frac{38}{40} - P(A \cap D)$

$P(A \cap D) = \frac{n(A \cap D)}{n(S)} = \frac{16}{40}$

$P(A \cup D) = \frac{18}{40} + \frac{38}{40} - \frac{16}{40} = \frac{40}{40} = 1$

d) $P(B \text{ or } C) = P(B \cup C)$

$P(B \cup C) = P(B) + P(C)$

$P(B \cup C) = \frac{2}{40} + \frac{27}{40} = \frac{29}{40}$

Ex# 4.13:- M = event the person obtained is Male.

$M \cup E$ = Male or under 18 years of age

E = under 18

$P(M \cup E) = P(M) + P(E) - P(M \cap E)$

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$P(M \cup E) = 0.739 + 0.120 - 0.085$

$P(M \cup E) = 0.774 = 77.4\%$

Note: Bi-characteristics of data of table is called contingency.

Task given: 4.89 + 4.99 Pg # 201

4.89:

a) the person is engaged in household work.

$P(A) = \frac{n(A)}{n(S)} = \frac{48.0}{100} = 0.48$

b) The person is either old or disabled

$P(C \cup D) = P(C) + P(D) = \frac{20.7}{100} + \frac{6.1}{100} = \frac{26.8}{100} = 0.268$

c) The person is engaged in studies

$P(B) = \frac{n(B)}{n(S)} = \frac{20.5}{100} = 0.205$

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4.99: P = event of students attend public schools
C = event of students attend college
$P \cap C$ = event of students attend public colleges

$P(P \cup C) = P(P) + P(C) - P(P \cap C)$
$= 85.3 + 27.9 - 20.1$
$P(P \cup C) = 93.1\%$

Day 12

Contingency: Joint Prob & Marginal Prob

Q: 4.111

Years of experience

Weight (lb)	Rookie Y_1	1–5 Y_2	6–10 Y_3	Over 10 Y_4	Total
Under 200 W_1	$W_1 \cap Y_1$ 3	$W_1 \cap Y_2$ 5	$W_1 \cap Y_3$ 0	$W_1 \cap Y_4$ 0	W_1 8
200–300 W_2	$W_2 \cap Y_1$ 11	$W_2 \cap Y_2$ 21	$W_2 \cap Y_3$ 7	$W_2 \cap Y_4$ 2	W_2 41
Over 300 W_3	$W_3 \cap Y_1$ 4	$W_3 \cap Y_2$ 4	$W_3 \cap Y_3$ 5	$W_3 \cap Y_4$ 0	W_3 13
Total	Y_1 18	Y_2 30	Y_3 12	Y_4 2	$n(S)$ 62

DATE: DAY:

$$P(W_1 \cap Y_1) = \frac{n(W_1 \cap Y_1)}{n(S)} = \frac{3}{62}$$ → Joint Prob. ↓ 2 features at a time

$$P(W_1) = \frac{n(W_1)}{n(S)} = \frac{8}{62}$$ → Marginal Prob.

$$P(W_1 \cap Y_2) = \frac{n(W_1 \cap Y_2)}{n(S)} = \frac{5}{62}$$

⋮

4.113:-

Pay Grade

Marital Status	Enlisted G_1	officer G_2	warrant G_3	Total
Single W/O children M_1	118,116	13,915	75	132,106
Single with children M_2	14,784	1482	99	16,365
Joint Service marriage M_3	14,722	2,681	67	17,470
Civilian marriage M_4	124,976	32,031	1,423	158,430
Total	272,598	50,109	1,664	324,371

$$P(M_1 \cap G_1) = \frac{n(M_1 \cap G_1)}{n(S)} = \frac{118,116}{324,371}$$

⋮

DATE: Bayesian Theorem foundation. DAY:

★ Conditional Probability:- A & B

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \begin{array}{l} \rightarrow \text{Joint. Prob.} \\ \rightarrow \text{Marginal Prob.} \end{array}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$ [The probability of A given B.]

We call B the "given that".

4.111. $$P\left(\frac{W_1}{Y_4}\right) = \frac{P(W_1 \cap Y_4)}{P(Y_4)} = \frac{0}{62} = 0$$

$$P\left(\frac{W_2}{Y_3}\right) = \frac{P(W_2 \cap Y_3)}{P(Y_3)} = \frac{7/62}{12/62} = \frac{7}{12}$$

4.146:- B = event for awarding Bachelor's degree
W = event for awarding bachelor's degree to women

dependent event

$$P\left(\frac{W}{B}\right) = \frac{P(W \cap B)}{P(B)} = \frac{0.378}{0.752}$$

DAY. ______

DATE: **Note imp:-**

First event occur independent then another event occur from the first one [dependent event].

Task given:- 4.143 (imp) [4.148 + 4.149]

4.143:-

Education	Owner: Yes S_1	Owner: No S_2	$P(E_i)$
Not HS grad E_1	0.046	0.083	0.129
HS grad E_2	0.144	0.168	0.312
Some college E_3	0.101	0.067	0.168
College Grad E_4	0.274	0.117	0.391
$P(S_j)$	0.564	0.436	1.000

a). owns a smartphone

$$P(S_1) = \frac{n(S_1)}{n(S)} = \frac{0.564}{1.000} = 0.564$$

b) is a college grad

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{0.391}{1.000} = 0.391$$

c) owns a smartphone, and is a college grad.

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$$P(E_4 \cap S_1) = \frac{n(E_4 \cap S_1)}{n(S)} = \frac{0.274}{1.000} = 0.274$$

d) owns a smartphone, given that the person is a college grad.

$$P(S_1 | E_4) = \frac{P(S_1 \cap E_4)}{P(E_4)} = \frac{0.274}{0.391} = 0.701$$

e) is a college grad, given that the person owns a smartphone.

$$P(E_4 | S_1) = \frac{P(E_4 \cap S_1)}{P(S_1)} = \frac{0.274}{0.564} = 0.486$$

f) Interpret your answers in parts (a)-(e) in terms of percentages:-

a) 56.4%

b) 39.1%

c) 27.4%

d) 70.1%

e) 48.6%

DATE: DAY:

4.148:

A = event for members in Army National Guard.

$A \cap B$ = event of member are Black & in Army National Guard.

$$P\left(B \middle| A\right) = \frac{P(A \cap B)}{P(A)} = \frac{10.2\%}{77.0\%} = 0.1325$$

$= 13.25\%$

So, the percentage of members in the Army National Guard who are Black is 13.25%.

4.149:

I = event of active U.S physicians specialize in I.M.

$I \cap U$ = event of active U.S physicians are under age 55 and specialize in I.M.

$$P\left(U/_{I}\right) = \frac{P(I \cap U)}{P(I)} = \frac{8.67}{13.6} = 0.637$$

$= 63.7\%$

So, the %age of active U.S. physicians who specialize in internal medicine and are under age 55 is 63.7%.

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Quiz # 02.

Day 14

★ **Multiplication Law:-**

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P\left(\frac{B}{A}\right) \cdot P(A)$$

Ex# 4.21:-

$P(M) = \frac{17}{40}$, $P(F) = \frac{23}{40}$

condition

$$P(F \cap M) = P(F) \cdot P\left(\frac{M}{F}\right)$$

$$= \frac{23}{40} \cdot \frac{17}{39}$$

$$P(F \cap M \cap F \cap F \cap M) = \frac{23}{40} \cdot \frac{17}{39} \cdot \frac{22}{38} \cdot \frac{21}{37} \cdot \frac{16}{36}$$

4.184 & 4.183 :- (Task) $\boxed{\text{imp.}}$

$$\therefore \boxed{P(B|A) = P(B)}$$

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* Independent Events Prob:-

$P(A\cap B) \Rightarrow P(A) \cdot P(B|A)$ $\quad \because B$ is dependent on A.

$P(A\cap B) \Rightarrow P(A) \cdot P(B)$ $\quad \because B$ not dependent on A.

Ex# 4.23 :- • equally likely. (no condition).

$$P(B) = \frac{18}{38}, \quad P(R) = 18/38, \quad P(G) = 2/38.$$

$$P(G\cap B\cap B) = \frac{2}{38} \cdot \frac{18}{38} \cdot \frac{18}{38}$$

4.188:- (b) $(G_1 \& F_1)$ independence.

$$P(G_1|F_1) = P(G_1) \rightarrow \text{independent events.}$$

$$P(G_1|F_1) = \frac{P(G_1\cap F_1)}{P(F_1)} = P(G_1)$$

$$P(G_1|F_1) = \frac{0.300}{0.582} \neq 0.419$$

So, prove that G_1 and F_1 are dependent events.

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4.195:-

(a) $P(A) = 0.255 \times 0.238 \times 0.393$

(b) $P(B) = 0.255 \times 0.255 \times 0.114$

Task: (4.202) $\underset{\text{passing on 1st try}}{P_1 = 0.60}$, $\underset{\text{passing on 2nd try}}{P_2 = 0.54}$, $\underset{\text{3rd try}}{P_3 = 0.4}$

(a) Students fail in first try $= 1 - P_1$

$= 1 - 0.60$

$= 0.40$

Out of them, that passed on 2nd try $= P_2 = 0.54$

$= 0.40 \times 0.54$

$\boxed{\text{Passes on 2nd try} = 0.216 = 21.6\%}$

(b) Student passes on third try.

Probability of failing on 1st try $= 0.40$

Probability of failing on 2nd try $= 1 - 0.54$

$= 0.46$

Passed on third try $= 0.46 \times 0.40 \times 0.48$

$= 0.08832$

$= 8.832\%$

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(c) **Percentage of students passed:**

P (total pass) = P_1 + P (pass on 2nd try) + P (pass on third try)

= 0.60 + 0.216 + 0.08832

= 0.90432

= 90.43%

Day 15

→ Law of Total Probability.

$P\left(B/A\right) = \frac{P(A \cap B)}{P(A)}$ · A & B are dependent events

C-1

$P(A \cap B) = P(A) \cdot P(B|A)$

C-2

$P(A \cap B) = P(A) \cdot P(B)$ ← A & B are independent events

C

↑

CS

ComΛC	RΛC	SΛC
↓	↓	
P(ComΛC) = P(Com) · P(C/Com)	P(RΛC) = P(R) · P(C/R)	P(SΛC) = P(S) · P(C/S)
COMSATS (COM)	RIPHA (R)	UOS (S)

Com = 0.37 × 50% ≈ 0.18%

Ripah = 0.17 × 60% ≈ 10

UOS = 0.46 × 0.25 = 0.12

$\boxed{0.40}$

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$$P(C) = P(COM \cap C) + P(R \cap C) + P(S \cap C)$$
$$= P(COM) \cdot P(C|_{COM}) + P(R) \cdot P(C|_R)$$
$$+ P(S) \cdot P(C|_S)$$

Ex# 4.24:-

$R_1 \cap S$	$R_2 \cap S$	$R_3 \cap S$	$R_4 \cap S$
R_1	R_2	R_3	R_4

$$P(R_1 \cap S) = P(R_1) \cdot P(S|_{R_1})$$
$$= 0.179 \times 0.141$$
$$=$$

$$P(R_2 \cap S) = P(R_2) \times P(S|_{R_2})$$
$$=$$
$$=$$

$$P(R_3 \cap S) =$$
$$=$$
$$=$$

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$P(R_4 \cap S) =$

$=$

$=$

$$P(S) = P(R_1) \cdot P(S|R_1) + P(R_2) \cdot P(S|R_2) + P(R_3) \cdot P(S|R_3) + P(R_4) \cdot P(S|R_4)$$

4.2208-

(a)

$$P(S) = P(E) \cdot P(S|E) + P(St) \cdot P(S|St)$$

(b) trick (which asked before than it is given that)

$P(S|E) =$

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*4.221 imp. Question.

Ex #: Baye's Theorem

4.25

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$P(A \cap B) \rightarrow$ c-1, c-2

$P(A) \rightarrow$ Law of total Prob.

$$P(R_1|S) = \frac{P(R_1 \cap S)}{P(S)} = \frac{P(R_1) \cdot P(S/R_1)}{\sum_{i=0} P(R_i) \cdot P(S/R_i)}$$

4.221

$A_1 = 18\text{-}34 \qquad P(A_1) = 0.425$

$A_2 = 35\text{-}49 \qquad P(A_2) = 0.285$

$A_3 = 50\text{-}64 \qquad P(A_3) = 0.164$

$A_4 = 65$ over $\qquad P(A_4) = 0.126$

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$P(O|A_1) = 0.411$ $\quad$ $P(O|A_2) = 0.579$

(a) $P(O)$ $\quad$ (b) $P(O|A_2)$

(c) $$P(A_2|O) = \frac{P(A_2 \cap O)}{P(O)} = \frac{P(A_2) \cdot P(O|A_2)}{\sum_{i=1}^{4} P(A_i) \cdot P(O|A_i)}$$

Pg #227 :-

Ex #:- 4.26 imp

L_1 = A person has lung disease.

$P(L_1) = 0.07$.

$P(S|L_1) = 0.90$

L_2 = A person does not having lung. d.

$P(S|L_2) = 0.253$

$P(L_2) = 1 - P(L_1) = 1 - 0.07 = 0.930.$

$$P(L_1|S) = \frac{P(L_1 \cap S)}{P(S)} = \frac{P(L_1) \cdot P(S|L_1)}{P(L_1) \cdot P(S|L_1) + P(L_2) \cdot P(S|L_2)}$$

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Counting of Sample Space:-

$\boxed{\text{Sample Space}}$ → collection of all possible outcomes in the result of experiment.

Coin Toss = $\frac{n_1}{2} \cdot \frac{n_2}{2} \cdot \frac{n_3}{2} = 8$

Dice Roll = $\frac{n_1}{6} \cdot \frac{n_2}{6} \times \frac{n_3}{6} = 216$

→ **Multiplication Rule:** [how many times you experiment].

Position ← **Permutation:** $^nP_r = \frac{n!}{(n-r)!}$
(Order) [without replacement]

Random ← **Combination:** $^nC_r = \frac{n!}{r!(n-r)!}$
Randomness.

e.g. $n = 10 \quad r = 3$

$$^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}}$$

$$= 10 \cdot 9 \cdot 8 = 720$$

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$10 \to 3$ (Randomly)

$$^{10}C_3 = \frac{10!}{3!(7!)} = \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{3!\cdot \cancel{7!}}$$

$$^{10}C_3 = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = \frac{720}{6} = 120$$

Ex# 4.36:-

100

Defec 6 | 94 Non.D

$n(S) = {}^{100}C_5$

P (exactly 2 of 5 TVs are Defective) =

$$\frac{^{6}C_2 \cdot {}^{94}C_3}{^{100}C_5}$$

$$\frac{^{6}C_3 \cdot {}^{94}C_2}{^{100}C_5}$$

at least & at most

$$P(\text{at most 2 defective}) = \frac{^{6}C_0 \cdot {}^{94}C_5}{^{100}C_5} + \frac{^{6}C_1 \cdot {}^{94}C_4}{^{100}C_5} + \frac{^{6}C_2 \cdot {}^{94}C_3}{^{100}C_5}$$

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P(at least 3 defectives)

$$\frac{{}^{6}C_{3}\cdot{}^{94}C_{2}}{{}^{100}C_{5}}+\frac{{}^{6}C_{4}\cdot{}^{94}C_{1}}{{}^{100}C_{5}}+\frac{{}^{6}C_{5}\cdot{}^{94}C_{0}}{{}^{100}C_{5}}$$

P(All 5 defective) :-

$$\frac{{}^{6}C_{5}\cdot{}^{94}C_{0}}{{}^{100}C_{5}}$$

P(6 defective): we cannot find probability because selected TVs are 5.