Nanyang Technological University
SPMS/Division of Mathematical Sciences

2022/23 Semester 1 MH1810 Math 1 Take Home Test

Version A

Name:

Matric Number: **Tutorial Group:**

Question 1 carries 1 mark and the rest 2 marks each. Total 9 marks. Answer ALL questions.

1. If f is a differentiation function such that $f'(x) < 0$ for all $x \in (a, b)$. Show that f is decreasing on (a, b).

2. A rain gutter is to be constructed from a metal sheet of width 30 cm by bending one-third of the sheet on each side through an angle θ. How should θ be chosen so that the gutter will carry maximum amount of water? Express the answer in terms of π.

3. (a) Find the derivative of the function $F(x) = \displaystyle\int_1^{x^2} \frac{t}{\sqrt{t^3+1}} dt$.

   (b) Find the limit
   $$\lim_{x\to 0} \frac{\displaystyle\int_0^x \frac{t}{\sqrt{t^3+1}} dt}{x^2}.$$

4. Express the following as a definite integral $\int_0^1 f(x)\ dx$ and find its **exact** value. (Express your answer in the form $\frac{\ln a}{b} + \frac{\sqrt{3}}{c}\pi$).
   $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{i^2 + in + n^2}$$

5. Show that

   (a) $\displaystyle\int_{-\frac{1}{4}}^{1} \frac{x+3}{\sqrt{4-3x-x^2}}dx = a\pi + b\sqrt{3}$, where the numbers a, b are to be determined.

   (b) A wedge is cut out of a circular cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The other plane intersects the first plane at an angle 60° along a diameter of the cylinder. Find the exact volume of the wedge.