

Learning-based Digital Backpropagation Scheme for Digital Subcarrier Multiplexing Optical Communication systems

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Abstract—In this paper, we propose a learning-based digital backpropagation (LDBP) scheme for digital subcarrier multiplexing (SCM) systems. Compared with the conventional digital backpropagation (DBP) technique, simulation results show that it can effectively improve the nonlinear mitigation capability and reduce computational complexity.

Keywords—Digital subcarrier multiplexing, learned digital backpropagation, fiber optic communication, nonlinear interference mitigation.

I. INTRODUCTION

Fiber nonlinearity and its interaction with dispersion are currently the main bottleneck in long-distance transmission over optical fibers. As a result, the study of techniques to reduce the effects of fiber nonlinearity has been one of the most intensively researched topics in recent years [1]. Several schemes have been proposed to compensate for digital nonlinearity, including digital backpropagation (DBP), perturbation based predistortion, and the Volterra series [2]–[4]. By numerically solving the nonlinear Schrödinger equation (NLSE) using the split-step Fourier method (SSFM), DBP compensates for both chromatic dispersion (CD) and nonlinear interference (NLI). The number of steps-per-span (StPS) used in the SSFM increases the performance of the DBP. The cost is the high computational complexity required to implement the Fast Fourier transform (FFT) and the inverse Fast Fourier transform (IFFT).

In recent years, machine learning for nonlinear interference compensation (NLC) has become increasingly popular for its adaptive properties and its potential for reducing computational complexity. Neural networks (NNs) are a popular choice for machine learning, with the ability to adapt to nearly all continuous functions. However, the dependence on NNs as an approximate of generic functions makes it difficult to integrate into existing domains or to explain the solutions obtained. Instead of relying on already existing generic NNs, it is better to start with an existing model or an algorithm based on a physical model and configure it. The technique used for the numerical solution of the NLSE is based on the Split-Step Method (SSM). The SSM is essentially a multi-stage NN, alternating between linear and nonlinear steps, where the linear steps correspond to linear propagation effects such as CD. Parameterizing each of these

steps as general linear functions, it is possible to obtain a parameterized multilayer model as follows [5].

In addition to direct fiber nonlinearity compensation by Digital signal processing (DSP) at the receiver side, another method to mitigate NLI by improving the tolerance of optical signals to fiber nonlinearity, digital subcarrier multiplexing (SCM) has been shown to be an effective fiber nonlinearity mitigation method [6], [7]. However, due to the effects of self-subcarrier nonlinearity (SSN) and cross-subcarrier nonlinearity (CSN), it can lead to distortion of SCM signals. To improve SCM system performance further, a DBP algorithm for SCM systems is proposed: the SCM-DBP, which is based on the cross-phase modulation (XPM) model and compensates for the SSN and CSN, respectively [8].

In this paper, in order to further reduce the SCM-DBP complexity in optical transmission systems, we propose two approaches. Firstly, a new simplified implementation structure is proposed without affecting the performance of SCM-DBP. We first perform SSN compensation for all subcarriers and then demultiplex them afterward to compensate each subcarrier CSN separately. Secondly, to address the high computational complexity in SCM-DBP, we propose a learning-based digital backpropagation (LDBP) scheme for SCM systems (SCM-LDBP). By parameterizing the linear steps as well as the nonlinear steps in SCM-DBP, the network parameters can be optimized through training, which leads to a significant performance improvement as well as a reduction in computational complexity. Simulation experiments using our proposed SCM-LDBP to transmit 32GBaud DP-16QAM SCM signals with 8 subcarriers over a 1200km single-mode fiber show that our proposed simplified 50StPS-SCM-DBP outperforms 50StPS-DBP by about 0.12dB. 6StPS-SCM-LDBP improves the performance compared to 50StPS-SCM-DBP and 50StPS-DBP by 0.35dB, and 0.47dB, respectively. When upgrading to 10StPS-SCM-LDBP, the performance improves by about 0.99dB, and 1.11dB, respectively. Compared to 100StPS-SCM-DBP, the gain is 0.82dB, which reduces the computational complexity due to the reduction of StPS.

II. THEORY AND METHOD

A. Simplified Digital Back Propagation for Subcarrier Multiplexing Systems

The SSN and CSN effects of the left subcarrier are illustrated in Fig.1, which shows the spectrum of an eight-

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subcarrier SCM signal. The SSN and CSN effects in SCM systems are similar to the self-phase modulation (SPM) and cross-phase modulation (XPM) effects in wavelength division multiplexing (WDM) systems. In SCM systems, we use traditional DBP technology for SSN compensation, while CSN compensation uses an XPM distortion analysis model based on WDM systems [8].

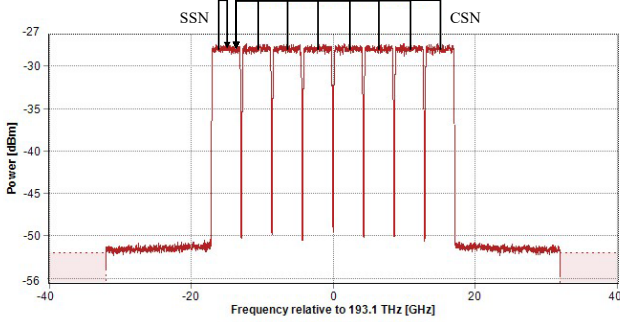


Fig. 1. The spectrum of SCM signal with eight subcarriers

The block diagram of an NLC step in a simplified version of SCM-DBP is shown in Fig. 2. First, the SSN compensation in the NLC step of SCM-DBP can be expressed as follows for an SCM system with N_{SCM} subcarriers, where the SSNs of all subcarriers are compensated at once.

$$S_{SSN,xy}(t) = S_{CDC,xy}(t) e^{-j\phi_{SSN,k}(t)} \quad (1)$$

where x/y denotes the x or y polarization, $S_{CDC,xy}(t)$ denotes the signal after chromatic dispersion compensation (CDC), and $\Psi_{SSN,k}(t)$ denotes the nonlinear phase offset caused by SSN, denoted as:

$$\Psi_{SSN,k}(t) = \xi \gamma L_{eff} \left(|S_{CDC,x}(t)|^2 + |S_{CDC,y}(t)|^2 \right) \quad (2)$$

Where L_{eff} is the effective length of the fiber, which is defined as follows:

$$L_{eff} = N(1 - \exp(-\alpha L)) / \alpha \quad (3)$$

Where L, α, N are the NLC fiber length per span, attenuation coefficient, and span step length, respectively. Subcarrier demultiplexing is then carried out, and for CSN compensation of the k -th probe subcarrier, the CSN compensation matrix is obtained as:

$$M'_k(t) = \begin{bmatrix} e^{j\xi\omega_{k,x}(t)} & -\xi\omega_{k,yx}(t)e^{j\xi\omega_{k,x}(t)} \\ \xi\omega_{k,yx}^*(t)e^{j\xi\omega_{k,y}(t)} & e^{j\xi\omega_{k,y}(t)} \end{bmatrix} \quad (4)$$

$$\overline{\Psi_k(t)} = \frac{1}{2} \times [\Psi_{k,x}(t) + \Psi_{k,y}(t)] \quad (5)$$

Where * , ξ , $\Psi_{k,x}(t)$, $\Psi_{k,y}(t)$, $\Psi_{k,yx}(t)$ denotes the conjugate operation, the compensation factor that needs to be optimized, the X-polarization CSN-induced nonlinear phase noise (NPN), the Y-polarization NPN, and the CSN-induced nonlinear polarization crosstalk (NPC), respectively. The frequency domain expressions for $\Psi_{k,x}(t)$ and $\omega_{k,yx}(t)$ are as follows:

$$\Psi_{k,x}(\omega) = \sum_{i \neq k}^{N_{SCM}} \left(F \left[2 \times |S_{SSN,i,x}(t)|^2 + |S_{SSN,i,y}(t)|^2 \right] \right) \times H_{i,N}(\omega) \quad (6)$$

$$W_{k,yx}(\omega) = \sum_{i \neq k}^{N_{SCM}} \left(F \left[S_{SSN,i,x}(t) S_{SSN,i,y}^*(t) \right] \right) \times jH_{i,N}(\omega) \quad (7)$$

where $i = 1, 2, \dots, N_{SCM}$ and F denote the FFT. $H_{i,N}(\omega)$ denotes the CSN transfer function of the i -th interference carrier associated with the k -th detection carrier, and each NLC step has N spans, represented by

$$H_{i,N}(\omega) = \frac{8\gamma}{9} \times \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}} \frac{1 - e^{-\alpha L + j\Delta\beta'_i \omega L}}{\alpha - j\Delta\beta'_i \omega} e^{-j\Delta\beta'_i \omega k L} \quad (8)$$

where $\Delta\beta'_i = \beta_2(\omega_i - \omega_k)$ denotes the difference in group velocity between the k -th subcarrier and the i -th subcarrier, β_2 denote the second-order dispersion coefficient, ω_i denote the central angular frequency of the i -th subcarrier, ω_k denote the central angular frequency of the k -th subcarrier, respectively. Furthermore, $H_{i,N}(\omega)$ can be approximated as a low-pass filter, and the CSN compensation in one NLC step of the SCM-DBP for the k -th detected subcarrier is given by:

$$\begin{bmatrix} S_{CSN,k,x}(t) \\ S_{CSN,k,y}(t) \end{bmatrix} = M'_k(t) \times \begin{bmatrix} S_{SSN,k,x}(t) \\ S_{SSN,k,y}(t) \end{bmatrix} \quad (9)$$

For clarity, Fig. 3 depicts the block diagram for one of the NLC steps in the SCM-DBP, where no simplification is performed[8]. The Simplified SCM-DBP saves some computational complexity while reducing the SCM-DBP time delay.

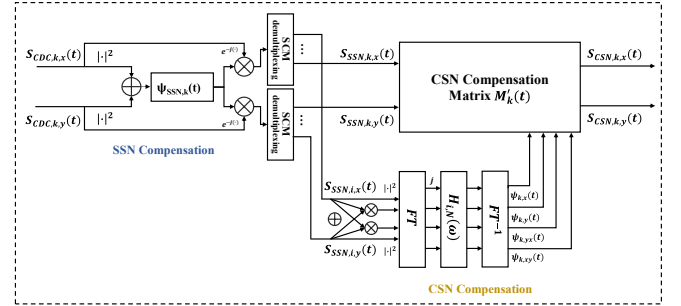


Fig. 2. Block diagram for simplifying one of the NLC steps in SCM-DBP

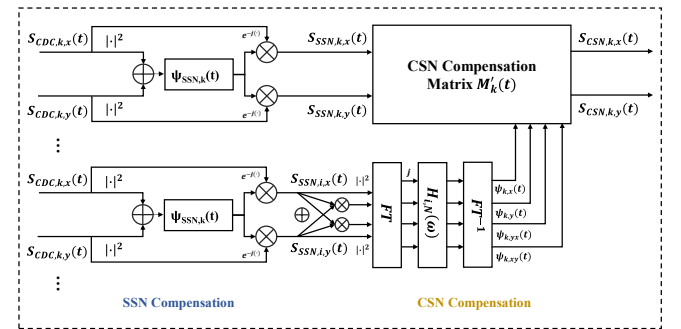


Fig. 3. Block diagram of one of the NLC steps in the un-simplified SCM-DBP

B. Learning-based digital backpropagation scheme for digital subcarrier multiplexing

To reduce the impact of nonlinearities on optical communication systems, we consider the DBP technique for SCM systems (SCM-DBP) for nonlinear compensation at the receiver side, focusing on efficient implementation with low complexity. In this paper, we propose an LDBP scheme for SCM systems (SCM-LDBP) from the perspective of machine learning, using its basic mathematical structure and establishing an optimization model applicable to SCM-DBP.

As shown in (1) and (2), we pay special attention to the parameters ξ , γ , which denote the compensation factor and the light nonlinear Kerr coefficient to be optimized, respectively. They regulate the phase rotation of each layer in SCM-DBP, which can be expressed as the rotation intensity.

In standard DBP implementations, ξ is often a certain fixed value, but the optimal compensation factor often depends on other factors such as the noise level and dispersion of the optical communication system, and the results are not the best if a specific value is used only empirically using a violent approach. Therefore, we combine ξ and γ into a single factor κ , and optimize it by training. After the later simulation experiments, it is shown that the optimized SCM-LDBP has better performance at low complexity compared with the traditional SCM-DBP. To optimize the parameters, we use a supervised learning approach, and the training will be done with Adam, an SGD variation, an algorithm that minimizes the cost function by adjusting the parameters of the grid.

The mean squared error (MSE) between the received and the predicted symbols is selected as the cost function, which is defined as follows: $\|\hat{s} - s\|^2 / N_{sym}$, and $\|s\|^2 \triangleq \sum_{i=1}^{N_{sym}} |s_i|^2$. Our proposed SCM-LDBP is blind to the polarization rotation state, frequency offset, and phase noise, since it is trained at the symbol level.

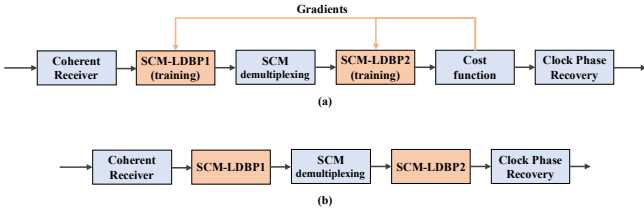


Fig. 4. The DSP procedures at (a) the training stage and (b) the implementation stage.

In the training stage, as shown in Fig. 4(a). We first receive the coherent optical signal coherently, pass the received signal sequentially into SCM-LDBP1(training), then perform subcarrier demultiplexing, and after demultiplexing, pass the signal sequentially into SCM-LDBP2(training), it propagates forward to the cost function and can pass the gradient back to SCM-LDBP. The cost function relative to the parameter The gradient is used for training, iteratively updating the nonlinear phase factor κ . Then after clock phase recovery (CPR), and other DSP modules for compensation of the associated losses. In the test stage, as shown in Fig. 4(b), SCM-DBP1 and SCM-DBP2 are replaced by trained SCM-LDBP1 and SCM-LDBP2, and then the signals are compensated by the correlation compensation modules in turn. In addition, for a fixed link setup, The optimal parameter depends on the optical input power P at the transmitter [9]. Therefore, each input power must be individually optimized for the best

performance, and the detailed results will be described in the next section.

III. SIMULATION SYSTEM SETTINGS AND RESULTS

A single-channel simulation for the 16QAM SCM signal was performed to verify the effectiveness of SCM-LDBP. The simulation setup is shown in Fig. 5. The transmitter-side DSP used to generate the SCM signal is the same as [6] with a symbol rate of 32-GBaud, and roll-raising-cosine (RRC) pulse shaping with a 0.05 roll-off factor is used to process the subcarriers. The SCM signal is generated in VPI and then transmitted in simulation. SCM has 8 subcarriers, which is approximately the optimal number of subcarriers [8]. At the transmitter side, it is I-Q modulated and polarization multiplexed and launched into the fiber, which consists of 16 spans. Each span contains a 5 dB noise inline EDFA and 75 km of standard single-mode fiber (SMF).

The fiber nonlinear coefficient is $1.3W^{-1} \cdot km^{-1}$, the dispersion coefficient is $-21ps^2/km$, and the fiber loss coefficient is $0.2dB/km$. The output signal of the link is filtered and the coherent receiver detects the coherent signal, and then the received signal is processed by offline DSP. The main algorithms are Clock Phase Recovery (CPR), Multiple-Input Multiple-Output Time Domain Equalizer(TDE-MIMO), Carrier Frequency Recovery (CFR), and Blind Phase Search (BPS). To compare the performance of our proposed SCM-LDBP, we use the formula: $Q^2 = 20 \times \log_{10}[2\text{erfc}^{-1}(2BER)]$ to calculate the Q factor, which is calculated from the bit error ratio (BER). For SCM systems, the average BER of all the sub-carriers is calculated to evaluate the performance of the system.

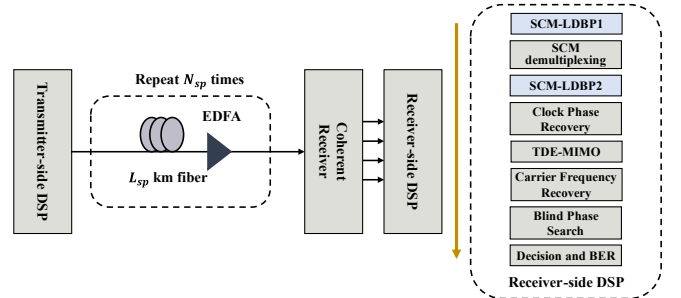


Fig. 5. Simulation Settings

Fig. 6 shows the performance of the 10StPS SCM-LDBP trained at -1, 0, 1, and 2 dBm transmit power, respectively, and from our analysis, if the test power is less than 2 dBm, we know that the best single training power for the 10StPS SCM-LDBP is 2 dBm. The optimal single training power of 10StPS SCM-LDBP is 1dBm when the test power is greater than 2dBm. From the above results, it can be concluded that the performance of the SCM-LDBP is dependent on the transmission power of the training data. In the following tests, we use the training data with the transmit power at 1 dBm, taking into account the above considerations.

Fig. 7 shows the performance of the proposed SCM-LDBP, SCM-DBP and DBP at different StPS and different transmit powers. It can be seen that our proposed simplified SCM-DBP, by compensating the SSN of all subcarriers at once, with a performance gain of approximately 0.12 dB. In addition, it can be observed that when 50StPS-SCM-DBP is upgraded to 100StPS-SCM-DBP, the performance improvement is about 0.17dB, and when the upgrade is continued to 500StPS, the

performance improvement is about 0.05dB, we believe that 100StPS-SCM-DBP is the best SCM-DBP performance benchmark, and when the benchmark is exceeded, the performance improvement is not significant. The performance improvement of 6StPS-SCM-LDBP is 0.35 dB and 0.47 dB compared to 50StPS-SCM-DBP and 50StPS-DBP, respectively, and the performance improvement is about 0.99 dB and 1.11 dB when upgrading to 10StPS-SCM-LDBP, respectively. There is a gain of 0.82 dB compared to the 100StPS-SCM-LDBP.

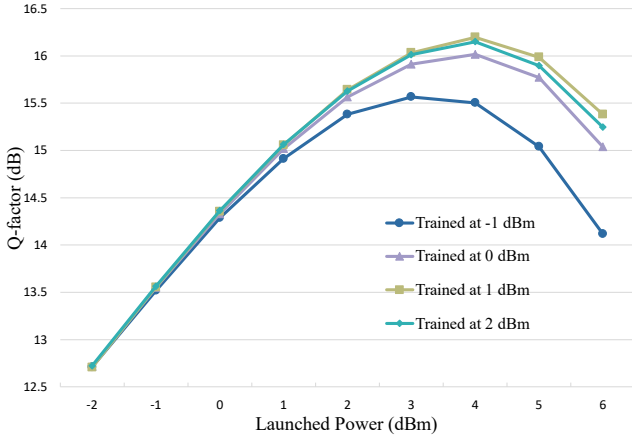


Fig. 6. Performance of the 10StPS-SCM-LDBP scan with all single training powers

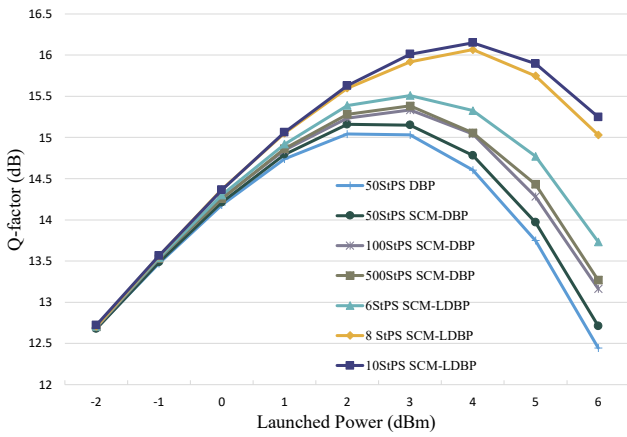


Fig. 7. Q-factor of SCM-DBP and SCM-LDBP versus signal transmit power

It is important to note that the performance improvement in SCM-LDBP is a result of parameter optimization in SCM-DBP and does not add any additional computational complexity. 10StPS-SCM-LDBP and 10StPS-SCM-DBP have the same complexity but are 5 and 10 times simpler than

50StPS-SCM-DBP and 100StPS-SCM-DBP, respectively. In addition, since training is typically done offline, this comparison does not take into account the complexity of training SCM-LDBP. It is important to note that the optimal transmit power of SCM-LDBP has a 1 dBm shift compared to the optimal transmit power of SCM-DBP, indicating that the extension of transmission distance is possible.

IV. CONCLUSION

The proposed simplified SCM-DBP and SCM-LDBP are used for a 1200 km simulated transmission. Firstly, our proposed simplified structure, which does not affect its initial performance, has a gain of about 0.12 dB compared to the DBP. Secondly, since it is trained at the symbol level, it allows SCM-LDBP to compensate for fiber nonlinearity in the presence of polarization rotation states, frequency offset, and phase noise. The simulation shows that the gain of 6StPS-SCM-DBP is 0.35dB compared to 50StPS-SCM-DBP and 0.47dB compared to 50StPS-DBP. When upgrading to 10StPS-SCM-LDBP, the performance improvement is about 0.99 dB, and 1.11 dB, respectively. Gain of 0.82dB compared with 100StPS-SCM-DBP. Furthermore, the performance gain of our proposed SCM-LDBP, SCM-DBP requires more than 10 times the number of StPS to achieve the same performance gain.

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