Impact of Non-Gaussian Noise Distribution by Artificial Neural Network-based Equalizers

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***Abstract*—The non-Gaussian noise distribution by an artificial neural network (ANN) equalizer causes degradation to LDPC performance. Accurate estimation of LLR based on non-Gaussian noise distribution can improve receiver sensitivity by about 0.7 dB.**

Keywords—Artificial Neural Network; LDPC; Noise Distribution; LLR; GMI

I. Introduction

There has been steady demand for higher capacity in datacenter networks [1] driven by the popularity of many AI-centric applications such as ChatGPT. In the meantime, direct-detection (DD) systems are preferred over coherent detection systems for short-reach communication in datacenters because of their lower cost and simple arrangement. However, the square-law detection of DD converts the linear response of the optical channel into a nonlinear one that is difficult to compensate by a linear equalizer. Research in machine learning has become a popular topic in recent years in the optical communication community, and many machine learning techniques such as ANN have also been explored as nonlinear equalizers to compensate for nonlinear channel penalties[2].

Low-Density Parity Check Code (LDPC) has been adopted to improve noise margin by a large number of communication systems and international standards because of its excellent performance [3] and can be only 0.0045 dB away from the Shannon limit in a Gaussian noise channel [4]. However, the ideal performance of LDPC is often based on the assumption of Gaussian noise distribution, while the nonlinear equalization of DSP may modify the Gaussian noise distribution. For example, in DFE, the output noise distribution is altered because of the feedback structure [5]. The different distribution of noise affects the mutual information of the channel, and the value of the mutual information under the same communication system is a good metric of the LDPC performance [6]. Thus it is anticipated that LPDC performance will be influenced by non-linear Gaussian noise in ANN-equalized systems.

This paper investigates the effect of ANN on channel noise distribution and the impact of non-Gaussian noise distribution on LDPC performance in a 20-km 50-Gb/s pulse amplitude modulation4 (PAM4) DD system. For a fixed post-corrected BER, the required GMI for the signal after nonlinear equalization is higher than that for a Gaussian distribution. The reason is that nonlinear equalization reshapes the original Gaussian noise into a non-Gaussian distribution which subsequently reduces the LDPC coding gain. For the same ANN-equalized signal, soft-decoding by computing the LLR using the true distribution brings about a 0.7 dB improvement in the receiver sensitivity after LDPC decoding.

II. Principle

A. Schematic of Feedforward (A)NN (F-NN)

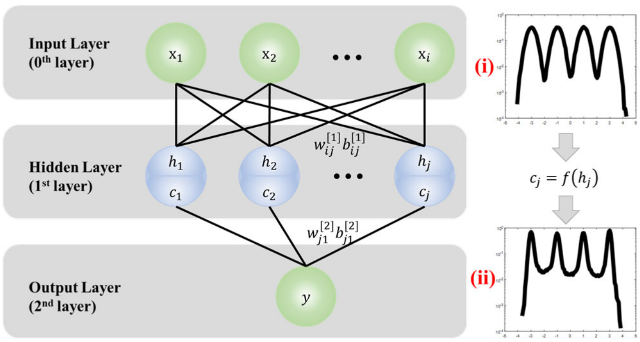

Fig. 1. Schematic of a 2-layer FNN [7]. The insets (i) and (ii) are the signal distribution at the input (Gaussian) and output of FNN (non-Gaussian).

A schematic diagram of the two-layer feedforward NNs (F-NN) equalization is shown in Fig. 1. This paper uses a simple NN with a single hidden layer, which is sufficient to equalize the penalty of the PAM4 signal in the DD system. A hidden layer ANN with enough number of neurons capable of estimating many continuous functions has been demonstrated.

The sequence $[x_1, x_2, \cdots, x_i]$ is the detected signal and is the input into the FNN. The output layer is a single ideal signal y, rather than multiple levels of categorical output [7]. the relationship between the output and input layers of the FNN can be expressed as:

$$y = \sum_j \sum_i w_{j1}^{[2]} f\left(w_{ij}^{[1]} x_i + b_{ij}^{[1]}\right) + b_{j1}^{[2]} \tag{1}$$

where $w_{ij}^{[1]}$ $(b_{ij}^{[1]})$, $w_{j1}^{[2]}$($b_{j1}^{[2]}$) are respectively the weights (biases) of the connections between the i-th neuron in the 0th layer and the j-th neuron in the 1st layer, and those between the j-th neuron in the 1st layer and the 1st neuron in the 2nd layer. $f(*)$ is the nonlinear activation function, which is the key element of the equalizer that can compensate for the nonlinear distortion. Commonly used nonlinear activation functions include hyperbolic tangent (Tanh) and rectified linear unit (Relu), etc.

The DD system is dominated by electronic thermal noise which is commonly considered as Gaussian. However, while compensating the signal nonlinear distortion, the nonlinear activation function also reshapes the Gaussian noise of the signal at the ANN input to non-Gaussian one at the output, as shown in the insets of Fig. 1. In a nutshell, in general, the noise at the output of an ANN equalizer is non-Gaussian, which will subsequently influence the performance of LDPC.

B. LLR in LDPC decoding

The Log likelihood ratio (LLR) needs to be calculated for the output signal of the equalizer for LDPC soft-decoding. The LLR is given by [8]:

$$LLR = \log\left(\frac{\Pr\left(b=0 \middle| r=(x,y)\right)}{\Pr\left(b=1 \middle| r=(x,y)\right)}\right) \tag{2}$$

where $\Pr\,(b = 0|r = (x, y))$ is the (a posteriori) probability that the bit 0 is being sent when the received symbol is (x, y). Typically, all symbols are assumed to be independent and identically Gaussian distribution for simplicity of calculation, namely

$$LLR = \log\left(\frac{\sum_{s \epsilon S_0} e^{-\frac{1}{\sigma^2}((x-s_x)^2+(y-s_y)^2)}}{\sum_{s \epsilon S_1} e^{-\frac{1}{\sigma^2}((x-s_x)^2+(y-s_y)^2)}}\right) \tag{3}$$

However, this simplified formula can only accurately estimate the LLR for the Gaussian noise channel. For non-Gaussian distributed noise, the formula (3) inevitably inaccurately computes the LLR, which leads to a lower decoding performance of LDPC than optimum.

To obtain the true LLR, the noise distribution of the known signal (pilot signals or training signals) is first fitted with a statistical probability density distribution. Namely, the 'true' probability density distribution of the output noise corresponding to each input symbol is acquired from the curve-fitting results. After that, for each output signal, $\Pr\left(b = 0 \middle| r = (x, y)\right)$ and $\Pr\left(b = 1 \middle| r = (x, y)\right)$ are calculated based on the calculated probability density to obtain the true LLR. For non-Gaussian distributed noise, such method will obtain a different value of LLR than the conventional Gaussian noise based estimation, enabling a better decision with a lower error probability for soft LDPC decoding.

III. System Setup and Results

A 50-Gb/s PAM4 DD optical communication system simulated with VPI Photonics 11.3 is shown in Fig. 2. The PAM4 electrical signal at the transmitter after the digital-to-analog converter (DAC) is modulated onto a 1550 nm CW laser with a Mach-Zehnder modulator (MZM). The PAM4 signal is quadruple up-sampled in the Tx DSP and then pulse shaped with a root-raised cosine filter (RRC) with a 0.1 roll-off factor. Two groups of simulation are performed for comparison: (i) One group is 10-GHz bandwidth-limited signal passing through 20-km fiber with a dispersion coefficient of 1.7 $ps/nm/km$, and (ii) the other group is that the signal bypasses the fiber (or back-to-back). At the receiver side, the signal is input into a variable optical attenuator (VOA) to adjust the received optical power (ROP) for the two groups. The signals of both groups are added with the same level of Gaussian noise at the detection of Photodetector (PD). The digitalized signal is subsequently fed into an NN-based nonlinear equalizer. The input layer of NN has 21 neurons, the hidden layer has 30 neurons, and the nonlinear activation functions are Relu or Tanh. The equalizers are trained with high SNR. The code rate of LDPC is 5/6, and maximum decoding iterations is 20.

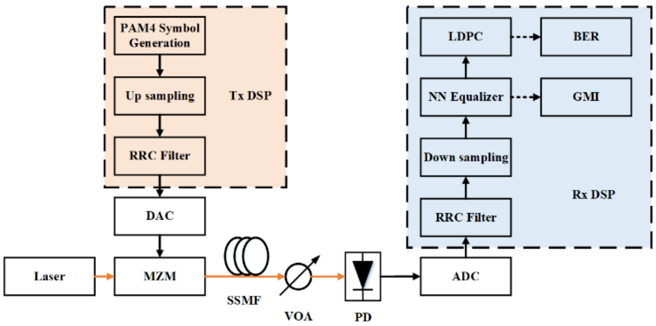

Fig. 2. Setup of a 50-Gb/s PAM4 DD optical communication system.

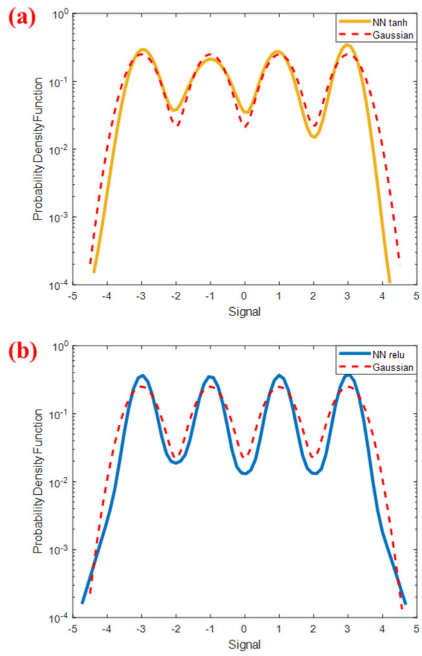

Fig. 3. Noise distribution after ANN with (a) Tanh and (b)Relu versus Gausssion distribution

The noise distribution of the ANN equalization output using different activation functions is shown in Fig. 3. The red dashed line in both figure is the Gaussian distribution. It can be seen that both equalizers reshape the noise into a non-Gaussian distribution. Further, the ANN equalizer output noise with Relu seems to have more probability density distortion than that with Thanh.

Fig. 4 shows decoded BER as a function of GMI or ROP at Back-to-Back (BtB) and 20-km transmission with ANN equalizers. The dashed line in the figure is the estimated LLR based on the Gaussian distribution (GN-LLR), and the solid line is the estimated true LLR (True-LLR). The green line in Fig. 4b shows that the signal pass through 20-km fiber and is unequalized. The GMI is calculated based on the true LLR by fitting the signal distribution as discussed in last section.

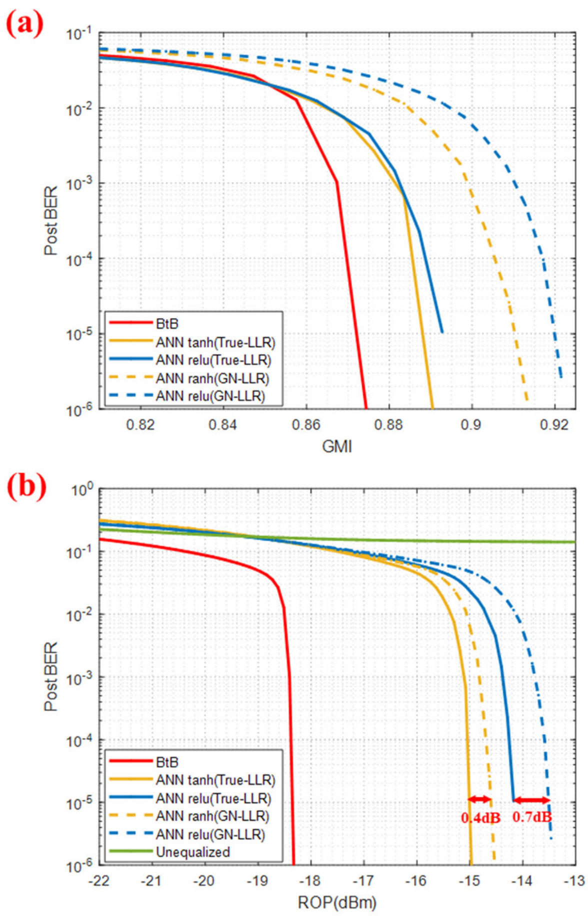

Fig. 4. BER performance versus (a) GMI and (b)ROP of the LDPC code. True-LLR: LLR is estimated by true noise distribution; GN-LLR: LLR is estimated by Gaussion distribution.

As shown in Fig.4a, compared to the BtB, ANN-equalized signal requires a higher GMI for a BER at $1x10^{-5}$ after LDPC decoding. It is already known that before LDPC, the GMI at the same pre-corrected BER could decrease because of non-Gaussian noise distribution and error propagation[5]. Judging from Fig 3, the GMI also decreases for the same ROP of the signal after ANN equalization caused by both the nonlinear distortion of channel and the non-Gaussian distribution noise. While for the same GMI, the non-Gaussian distributed noise in the ANN-equalized signal still degrades the post-corrected BER. The reason for the LDPC performance penalty could be that LDPC is designed for a Gaussian channel, and is not optimal for non-Gaussian channel.

For signals with the same noise distribution, accurate estimation of LLR can improve the performance of LDPC decoding. As shown in Fig. 4b, when LLR is estimated accurately, there is a gain of 0.4 dB for ANN with Tanh and 0.7 dB for ANN with Relu, with almost no change in BtB. This result may vary depending on the conditions of neural network training. Therefore, when using the ANN equalizer, the noise distribution of the results needs to be estimated in advance with the training data to improve the LDPC performance. Also, the penalty caused by incorrect estimation in ANN with Relu is larger.

IV. Conclusion

The effect of ANN on channel noise distribution and the impact of non-Gaussian noise distribution on LDPC performance is demonstrated in a 50-Gb/s PAM4 DD optical communication system. Compared to a Gaussian channel, the ANN-equalized signals would require a higher GMI to have the same post-corrected BER. For the ANN equalizer, accurate estimation of LLR can improve the receiver sensitivity by about 0.7 dB.

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