Autoregression Assisted Extended Kalman Filtering for Laser Phase Noise Estimation with 1/f Frequency Noise

Kunjian Lian
School of Electronics and Information Technology
Sun Yat-sen University
Guangzhou, China
liankj@mail2.sysu.edu.cn

Meng Qiao
School of Electronics and Information Technology
Sun Yat-sen University
Guangzhou, China
qiaom@mail2.sysu.edu.cn

Jintao Wang
School of Electronics and Information Technology
Sun Yat-sen University
Guangzhou, China
wangjt37@mail2.sysu.edu.cn

Jie Liu
School of Electronics and Information Technology
Sun Yat-sen University
Guangzhou, China
liujie47@mail.sysu.edu.cn

Dawei Wang
School of Electronics and Information Technology
Sun Yat-sen University
Guangzhou, China
wangdw9@mail.sysu.edu.cn

***Abstract*—We develop an autoregression assisted extended Kalman filter for laser phase noise estimation. Simulation results confirm the performance improvement by using AR(1) and AR(4) against the existing methods under different SNR and 1/f noise levels.**

Keywords—Kalman filter, phase noise, 1/f noise

I. Introduction

The higher order modulation formats such as the m-array quadrature amplitude modulation (m-QAM) in optical fiber communication is sensitive to laser phase noise (PN). The digital signal processing (DSP) technology enables effective PN tracking and correction, and several DSP algorithms have been developed to mitigate PN in optical coherent receivers such as 4th-power, blind phase search (BPS), phase-lock loop (PLL) and Kalman filter [1-3]. Both plain extended Kalman filter (EKF), the state equation of which is described by (1), and the modified EKF [4] given by equation (2) have been proposed.

$$x_n = [\varphi_n] = [\varphi_{n-1}] + [q_{\varphi,n-1}] \tag{1}$$

$$x_n = \begin{bmatrix} \Omega_n \\ \varphi_n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \Omega_{n-1} \\ \varphi_{n-1} \end{bmatrix} + \begin{bmatrix} q_{\Omega,n-1} \\ q_{\varphi,n-1} \end{bmatrix} \tag{2}$$

The modified EKF uses a second state variable Ω_n for 1/f component of the laser frequency noise, in addition to the phase noise φ_n. The q_Ω and q_φ are the process noises. The 1/f noise is treated as an i.i.d random process, which however is never the case in a realistic situation.

II. Autoregression Assisted EKF

The state model incorporating a simple first-order AR process (AR(1)) is thus as follows.

$$x_n = \begin{bmatrix} \Omega_n \\ \varphi_n \end{bmatrix} = \begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \Omega_{n-1} \\ \varphi_{n-1} \end{bmatrix} + \begin{bmatrix} q_{\Omega,n-1} \\ q_{\varphi,n-1} \end{bmatrix} \tag{3}$$

The higher order AR process modeling the 1/f noise can be used for better accuracy. To this end, we add more state variables to the state model, and the process equation incorporating an AR(4) is now expressed as

$$x_n = \begin{bmatrix} \Omega_n \\ \Omega_{n-1} \\ \Omega_{n-2} \\ \Omega_{n-3} \\ \varphi_n \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Omega_{n-1} \\ \Omega_{n-2} \\ \Omega_{n-3} \\ \Omega_{n-4} \\ \varphi_{n-1} \end{bmatrix} + \begin{bmatrix} q_{\Omega,n-1} \\ q_{\Omega,n-2} \\ q_{\Omega,n-3} \\ q_{\Omega,n-4} \\ q_{\varphi,n-1} \end{bmatrix} \tag{4}$$

where $\{a_1, a_2, a_3, a_4\}$ and a in (3) are estimated by calibrating the laser frequency noise at the transmitter. The observation equation is the same for all EKF considered in this paper, which is expressed as usual

$$y_n = \begin{bmatrix} y_I \\ y_Q \end{bmatrix} = \hat{y} \begin{bmatrix} \cos(\varphi_n + n\Omega_n) \\ \sin(\varphi_n + n\Omega_n) \end{bmatrix} + \begin{bmatrix} q_{I,n} \\ q_{Q,n} \end{bmatrix} \tag{5}$$

III. Simulation

Numerical simulations are carried out to verify the proposed EKF algorithm under sampling rate 10GHz. Based on the conjecture in [5] that an continuous model for power law, or $1/f^\alpha$, noise can be derived as the limit of the fractional differencing (6) for small time, the laser phase noise is modeled as the integration of a colored frequency noise, and the pulse response values of the power law noises can be described using the recursive algorithm (7) and (8):

$$H_f(z) = \frac{1}{(1 - z^{-1})^{\alpha/2}}, \ \ z > 1 \tag{6}$$

$$h_0 = 1 \tag{7}$$

$$h_k = \left(\frac{\alpha}{2} + k - 1\right) \frac{h_{k-1}}{k} \tag{8}$$

The power spectrum of the colored frequency noise is shown in Fig. 1(a), which consists of both white noise and 1/f noise components. In the laser phase noise estimation part, we consider a pure laser carrier with the phase noise described above with no data modulation. In practice, this corresponds to the case when training sequence is used for stripping off the modulation before the PN estimation. The white noise component of laser frequency noise has a variance of 10^{-5}, corresponding to a laser linewidth of 15kHz. The variance of 1/f noise is varied to investigate the performance of various algorithms. An independent white noise is added to the laser field $\exp(i\varphi)$ to simulate a finite SNR. No other system impairments are considered in this work.

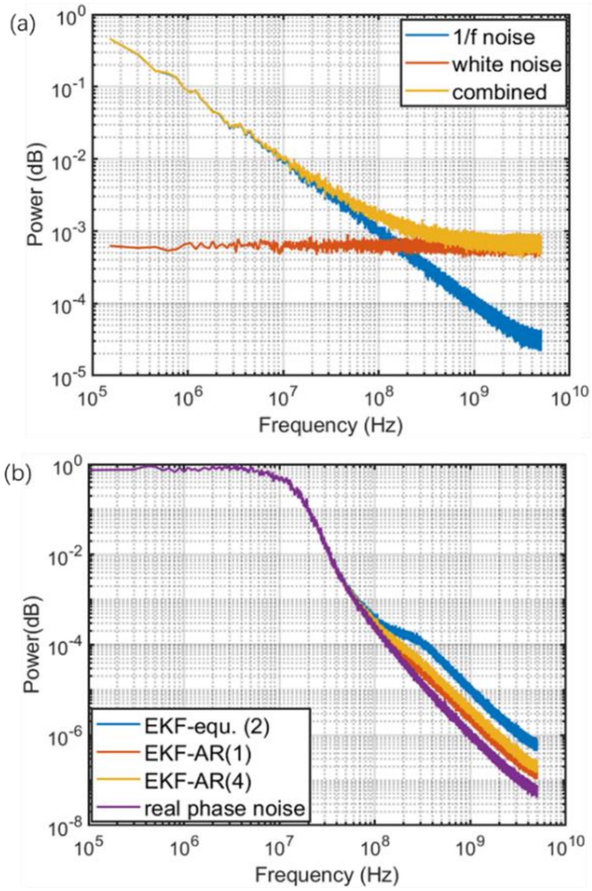

Fig. 1. Power spectrum of (a) simulated phase noise and (b) phase noise factor $e^{i\varphi}$ estimated via different EKF models.

Fig. 1 (b) shows the power spectrum of estimated phase term $\exp(i\varphi)$ via different EKF models. It can be seen that, the results of incorporating the AR model are more consistent with the real phase noise compared with the method in [4], while AR(1) is slightly better than AR(4).

The standard deviation (STD) of the estimated phase noise φ is calculated, and the results versus system SNR are shown in Fig. 2(a). Clearly, an AR(1) is sufficient to achieve a performance improvement in the high SNR regime but for lower SNR, an AR(4) is more suitable, at the expenses of a higher complexity. It is worth nothing that the model without 1/f noise is even better than the method in [4] at high SNR. Compared to the model without considering 1/f noise, AR(4) shows a gain of over 5dB in SNR, whereas the model in [4] has only about 2 dB improvement when the STD is 0.08 rad. Next, the variance of seed white noise (Qvar) used for generating the 1/f noise is varied with a system SNR of 15 dB. The STD of estimated phase noise φ is compared in Fig. 2(b) versus various Qvar values, corresponding to different levels of laser linewidth broadening due to 1/f noise. When Qvar is lower than 1e-5, the scheme in [4] degrades the estimated performance, which can be attributed to the imprecision of the model. It confirms again that the AR model assisted EKF is effective. Note that for a large 1/f contribution, AR(1) is sufficient to obtain a PN estimation improvement, and thus the additional complexity is minimum.

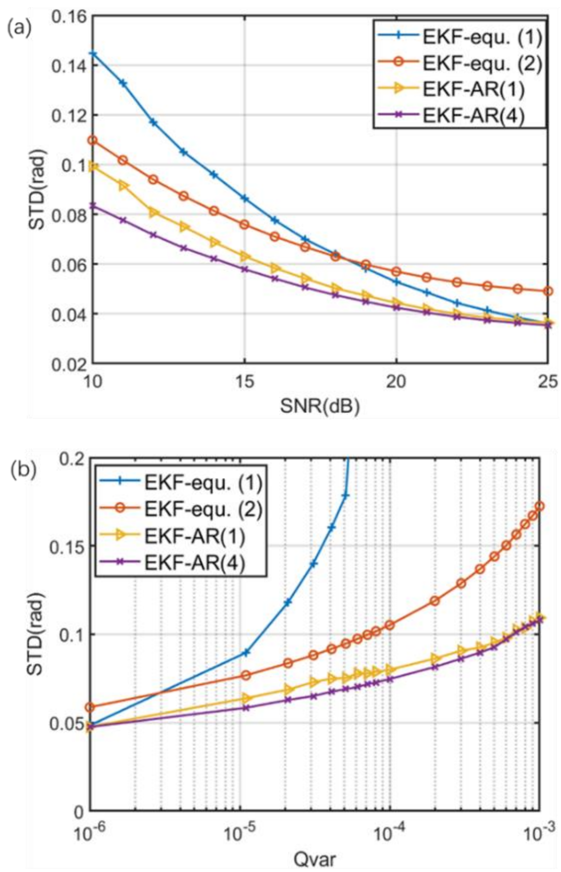

Fig. 2. Standard deviation (STD) of phase noise φ. versus (a) SNR and (b) Qvar with different EKF models.

IV. CONCLUSION

We use AR models to facilitate a more effective state equation of the extended Kalman filter for laser phase noise estimation when the 1/f frequency noise is no longer negligible. Besides, we use simulations to demonstrate the performance improvement of the proposed algorithm in comparison with other EKF methods.

REFERENCES

[1] Fatadin, Irshaad, David Ives, and Seb J. Savory. "Laser linewidth tolerance for 16-QAM coherent optical systems using QPSK partitioning." IEEE Photonics Technology Letters 22.9 (2010): 631-633.

[2] Jain, Ankita, and Pradeep Kumar Krishnamurthy. "Phase noise tracking and compensation in coherent optical systems using Kalman filter." IEEE Communications Letters 20.6 (2016): 1072-1075.

[3] Pakala, Lalitha, and Bernhard Schmauss. "Extended Kalman filtering for joint mitigation of phase and amplitude noise in coherent QAM systems." Optics Express 24.6 (2016): 6391-6401.

[4] Kleis, Sebastian, and Christian G. Schaeffer. "Improving the secret key rate of coherent quantum key distribution with Bayesian inference." Journal of Lightwave Technology 37.3 (2018): 722-728.

[5] Kasdin, N. Jeremy. "Discrete simulation of colored noise and stochastic processes and $1/f^{\alpha}$ power law noise generation." Proceedings of the IEEE 83.5 (1995): 802-827