Impact of Super-Gaussian Distribution on System Gain of Probabilistic Shaping 64QAM

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***Abstract*—We investigated the gain provided by optimal-super-Gaussian in the DP-PCS-64QAM system. Results indicate that the maximum 0.24/0.18/0.15-dB Q^2 factor gain is achieved compared with Maxwell-Bolzmann distribution in the 96-GBaud 500/1000/1500-km transmission system, respectively.**

***Index Terms*—Super-Gaussian, Maxwell-Bolzmann, PCS**

I. Introduction

Upcoming applications and innovations such as the internet of thing (IoT), live streaming online, and cloud computing services are driving optical networks to meet the ever-increasing traffic demands [1], [2]. Doubling the transmission capacity compared with the intensity modulation/direct detection (IM/DD), coherent detection scheme has been widely deployed in transponders from short to long-haul applications scenarios [3]–[5]. In general, state-of-the-art digital signal processing (DSP) algorithms could efficiently mitigate various linear impairments, such as chromatic dispersion (CD) and polarization mode dispersion (PMD), which also enables the widespread application of higher-order quadrature amplitude modulation (QAM) format. Nonlinear impairment imposed by the fiber Kerr effect, however, has been always regarded as the essential constraint which puts a cap on the achievable information capacity of optical transmission systems [6], [7], especially in modern long-haul high-speed scenarios [8], [9].

On the other hand, the probabilistic constellation shaping (PCS) technique has been vastly explored in coherent optical fiber communication systems, bringing the system capacity closer to the Shannon limit together with a finely adjustable net system spectral efficiency (SE) over a wide range [10]. In addition, PCS is usually realized by the probabilistic amplitude shaping (PAS) architecture which maps the signals with Maxwell-Boltzmann (MB) distribution and possesses the advantage that the underlying forward error correction (FEC) and DSP algorithms remain the same, making it share the same transceiver as the conventional modulation format. Nevertheless, partly because the Kerr effect breaks the assumption that the signal is conveyed into additive white Gaussian noise (AWGN) channel in which the MB distribution is optimal [11], it has been found that the accumulated nonlinear impairment is proved to be aggravated by PCS and the shaping gain can be significantly offset [12]–[14]. Consequently, extensive research on the interplay between shaping and nonlinear effects has been reported [13], [15], [16], and solutions to retrieve the shaping gain are urgently required.

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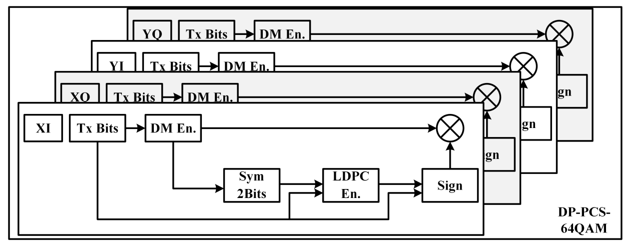

Fig. 1. The architecture of the PAS scheme. DM En.: distribution matcher encoder; Sym2Bits: Symbols-to-bits conversion; LDPC En.: LDPC encoder.

In this work, we evaluate the performance of the optimal super-Gaussian distribution (O-Super-Gaussian, throughout the paper, the term "O-Super-Gaussian" refers to super-Gaussian with the optimal Gaussian order unless otherwise specified), which enhances the nonlinear impairment tolerance from the perspective of modifying the constellation point of MB distribution [11]. The gain of the O-Super-Gaussian is verified in a 96-GBaud DP-PCS-64QAM transmission system, where the entropy in this work ranges from 5.0 bits/symbol to 6.0 bits/symbol with the step of 0.1 bits/symbol. We find that, compared with the MB distribution, the maximum shaping gain provided by the O-Super-Gaussian distribution is related to the entropy, and the maximum gain is achieved at the 5.3 bits/symbol entropy for three transmission scenarios. Specifically, compared with the MB distribution, the O-Super-Gaussian distribution provides 0.24-dB, 0.18-dB, and 0.15-dB Q^2 factor improvement over 500-km, 1000-km, and 1500-km fiber transmission systems, respectively.

II. Principles of Operation

PCS is practically enabled by the PAS architecture shown in Fig. 1, which elegantly resolves the long-standing problem

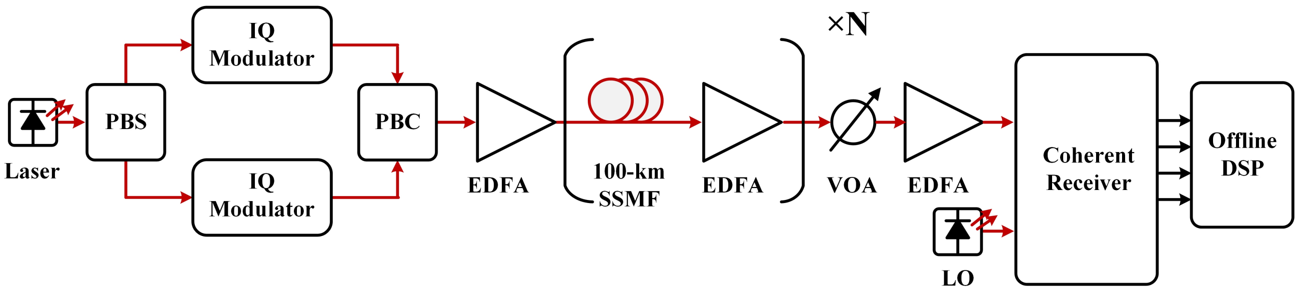

Fig. 2. Block diagram of the DP-PCS-64QAM system. PBS: polarization beam splitter; PBC: polarization beam combiner; EDFA: erbium-doped fiber amplifier; SSMF: standard single-mode fiber; VOA: variable optical attenuator; LO: local oscillator.

of PCS in terms of combining shaping and coding. To begin with, the offline DSP algorithm divides the binary information bits into two parts. One part is sent to the constant composition distribution matcher (DM) encoder to generate symbols with the desired probability [17]. Then, the symbols are converted into bits based on symbol-to-bit mapping rules. Next, the converted bits, together with the remaining uniform bits, are sent to the low-density parity-check (LDPC) encoder to generate redundant bits. These redundant bits are combined with the remaining uniform equiprobable binary information bits to create the sign bits. Finally, the DP-PCS-64QAM is composed of four-tributary PCS-PAM symbols.

Generally, utilizing the DM encoder in PAS architecture, the pseudo-random binary sequences (PRBS) are mapped to the signals characterized by MB distribution. The probability mass function (PMF) of MB distribution is given by

$$P_x = \frac{exp(-\lambda(|X_m|)^2)}{\sum_{k=1}^{M} exp(-\lambda(|X_k|)^2)} \tag{1}$$

where λ is the shaping factor, which ranges from 0 to 1, the choice of which should be cautious as it is closely related to information entropy and other system key parameters.

Super-Gaussian distribution is achieved by adjusting the index value "2" to a variable θ, shown as

$$P_x = \frac{exp(-\lambda(|X_m|)^\theta)}{\sum_{k=1}^{M} exp(-\lambda(|X_k|)^\theta)} \tag{2}$$

Eq. (2), and extending Eq. (1) to the family of MB distribution. Note that the probability of the occurrence of the constellation points varies with θ and changes the 4-th and 6-th moment [11], [13], which contributes to nonlinear distortion. As a result, the super-Gaussian distribution supplies an optional way to decrease the nonlinear impairment and achieves a better nonlinear impairment tolerance and an overall improvement in system performance from the perspective of optimizing the constellation points in the PCS scenarios.

III. Simulation setup

Figure 2 depicts the simulation system setup for a 96-GBaud DP-PCS-64QAM coherent optical communication system. The target entropy is adjusted from 5.0 bits/symbol to 6.0 bits/symbol with an incremental step of 0.1 bits/symbol. Notably, when the entropy is 6 bits/symbol, both the MB and super-Gaussian distributions are uniform distribution. The Gaussian order is varied from 1 to 5 with a step of 0.2 to implement the super-Gaussian distribution and regulate the probability distribution of the constellation points. It should be noted that the Gaussian order P is the primary variable that needs to be determined at first in the given range above. Once the Gaussian order is fixed, the shaping factor λ can be determined for the target entropy according to the family of MB distribution in Eq. (2).

The digital DP-PCS-64QAM signal is first generated at the transmitter side by offline DSP, following the PAS architecture illustrated in Fig. 1. Subsequently, a square root-raise-cosine (RRC) filter with a roll-off factor of 0.1 is applied to perform Nyquist pulse shaping in the time domain. Next, an external laser with a central wavelength of 1550 nm is used to modulate the signal. The optical DP-PCS-64QAM signal is then transmitted through the standard single-mode fiber (SSMF). Within the transmission link, we consider 100-km SSMF (α = 0.2 dB/km, $D = 17.0$ $ps/nm/km$, γ = 1.4 $W^{-1}km^{-1}$) as a span. Three transmission distances are considered, i.e., 5 × 100-km (500 km), 10 × 100-km (1000 km), and 15 × 100-km (1500 km), respectively, where each span is equipped with an Erbium-doped fiber amplifier (EDFA) with a 5-dB noise figure to compensate for signal power attenuation. Finally, the signal is detected by the coherent receiver. It should be noted that the offline DSP mainly involves downsampling, chromatic dispersion compensation (CDC), matched filtering, and hard decision for Q^2 factor calculation.

IV. Results and discussion

Figure 3 (a) depicts the Q^2 factor gain achieved by the O-super-Gaussian distribution in comparison to the MB distribution, as a function of the entropy. It should be mentioned that the results are based on the condition that only linear CDC is conducted as the nonlinear impairment tolerance is focused on in this work. In addition, the Q^2 factor gain is obtained at the optimal launch power. It can be found that the Q^2 factor gain is related to entropy, that is, the shaping gain varies by the signal entropy. Moreover, with the fiber length increasing, the maximum Q^2 factor gain decreases. Meanwhile, for 500-km, 1000-km, and 1500-km SSMF transmission scenarios, the

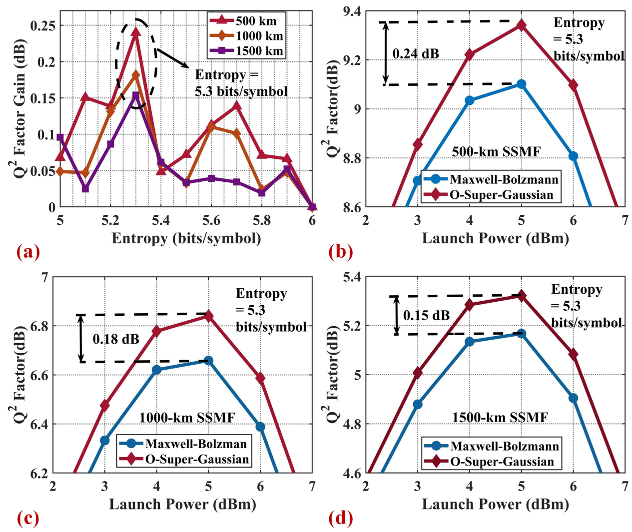

Fig. 3. Simulation results in the 96-GBaud DP-PCS-64QAM simulation transmission. (a) The Q^2 factor gain achieved by the O-super-Gaussian distribution in comparison to the MB distribution, as a function of the entropy; (b) The Q^2 factor versus launch power in 500-km SSMF transmission system; (c) The Q^2 factor versus launch power in 1000-km SSMF transmission system; (b) The Q^2 factor versus launch power in 1500-km SSMF transmission system.

maximum shaping gain is obtained at the same entropy of 5.3 bits/symbol, which has been marked with the black dotted line.

To make a clear performance comparison between the MB-distribution and O-Super-Gaussian distribution, for the case of entropy setting to 5.3 bits/symbol, Fig. 3 (b), Fig. 3 (c), and Fig. 3 (d) present the relation of launch power versus the Q^2 factor. The results indicate that the O-super-Gaussian distribution reaches a maximum of 0.24-dB, 0.18-dB, and 0.15-dB Q^2 factor improvement compared with the MB distribution in the optimal launch power over 500-km, 1000-km, and 1500-km SSMF transmission systems, respectively. Moreover, it can be seen that from the linear region to the nonlinear region, the O-super-Gaussian distribution outperforms the MB distribution and achieves an overall improvement in system performance.

V. Conclusion

Super-Gaussian distribution supplies an optional way to enhance the nonlinear impairment tolerance from the perspective of constellation points in the PCS scenarios. In this work, we evaluate the performance of O super-Gaussian distribution in a 96-GBaud DP-PCS-64QAM simulation transmission. Our results indicate that the maximum shaping gain of the O-super-Gaussian distribution over the MB distribution is related to the entropy. Specifically, compared with MB, the O-super-Gaussian distribution provides 0.24-dB, 0.19-dB, and 0.15-dB Q^2 factor improvement over 500-km, 1000-km, and 1500-km SSMF transmission systems, respectively, when the entropy is 5.3 bits/symbol.

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