

# Experimental Demonstration of the Widely Linear Sparse Volterra Equalizer Used in Probabilistic Shaping 1024-QAM Transmission with Spectral Efficiency of 16.57-bit/s/Hz

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**Abstract**—The widely linear sparse Volterra equalizer is proposed in this paper, and the performance is demonstrated in the polarization multiplexed probabilistic shaping 1024-QAM with the spectral efficiency of 16.57-bit/s/Hz.

**Keywords**—Complex-valued widely linear equalizer, high spectral efficiency, multi-input multi-output Volterra equalizer, ultra-high-order modulation.

## I. INTRODUCTION

With the development of information science and 5G networks, the demand for information acquisition and exchange is growing rapidly [1-2]. Ultra high-order quadrature amplitude modulation (QAM) is becoming the focus of the communication industry because of its high spectral efficiency [3-5]. Ultra high-order QAM signals are known to be challenging to generate and recover. Limitations from hardware such as quantization noise, isotropic quadrature modulator imbalance, bandwidth limitations of digital-to-analog/analog-to-digital converters, and laser linewidth [6-7].

In this paper, we focus on the polarization-multiplexed probabilistically shaped (PM-PS) 1024-QAM format and provide a feasible scheme for PM-PS-1024-QAM signals based on offline DSP, and utilize a  $2 \times 2$  MIMO widely linear complex-valued sparse Volterra (MIMO-WLCSV) equalizer based on the recursive least square (RLS) error, which consists of a 1<sup>st</sup>-order  $2 \times 2$  MIMO complex valued widely linear equalizer and a 3<sup>rd</sup>-order real-valued linear equalizer. The proposed MIMO-WLCSV can effectively compensate for IQ imbalances and nonlinear impairments induced by the device [8-11]. Sparseness is also utilized to reduce the complexity [12]. In this paper, the experimental results show spectral efficiency can reach 16.57 bit/s/Hz, and the BER can reach  $6.11 \times 10^{-4}$  using PM-PS-1024-QAM signals.

## II. PRINCIPLE OF THE ALGORITHM

The structure diagram of the general complex-valued  $2 \times 2$  MIMO equalizer is shown in Fig. 1 and can be presented mathematically by (1),

$$\begin{bmatrix} x_{out} \\ y_{out} \end{bmatrix} = \begin{bmatrix} w_{xx} & w_{xy} \\ w_{yx} & w_{yy} \end{bmatrix} \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} \quad (1)$$

where  $w$  denotes the weight of the equalizer taps,  $x_{in}$  and  $y_{in}$  are the inputs of the equalizer, and  $x_{out}$  and  $y_{out}$  are the outputs of the equalizer. Four  $N$ th-order real coefficient number filters typically need to implement the  $N$ th-order complex filter. For the  $2 \times 2$  MIMO complex-valued equalizer mentioned in Fig. 1, it is implemented by four  $2 \times 2$  real-valued equalizers.

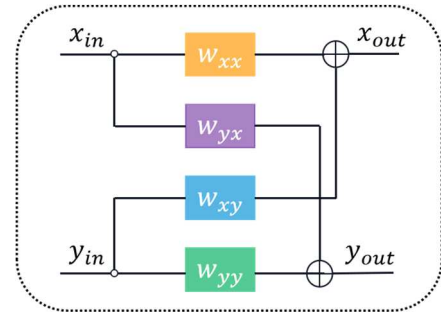


Fig. 1. Structure diagrams of  $2 \times 2$  MIMO complex-valued equalizer.

As the device characteristics of the two I/Q branches are not perfectly matched, for example, the I/Q features of the LO, the frequency response of the respective low-pass filters of the I/Q branches are not perfectly matched, causing the IQ imbalance. In addition, electrical amplifiers, AWG output RF lines, MZM modulators, and balanced detectors generally

cause nonlinear impairment, especially in ultra-high-order QAM. Considering that a complex-valued equalizer can overcome the IQ imbalance and a Volterra equalizer can simultaneously compensate for linear and nonlinear impairments. We utilized a 2×2 MIMO-WLCSV equalizer. The structure of MIMO-WLCSV is illustrated in Fig. 2, which can be represented as given below by (2):

$$\begin{bmatrix} x_{out} \\ y_{out} \end{bmatrix} = \begin{bmatrix} w_{xx} & w_{xy} \\ w_{yx} & w_{yy} \end{bmatrix} \begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} + \begin{bmatrix} w_{xx}^* & w_{xy}^* \\ w_{yx}^* & w_{yy}^* \end{bmatrix} \begin{bmatrix} x_{in}^* \\ y_{in}^* \end{bmatrix} \\ + \begin{bmatrix} w_{xr2} & w_{xi2} \\ w_{yr2} & w_{yi2} \end{bmatrix} \begin{bmatrix} x_{in2} \\ y_{in2} \end{bmatrix} + \begin{bmatrix} w_{xr3} & w_{xi3} \\ w_{yr3} & w_{yi3} \end{bmatrix} \begin{bmatrix} x_{in3} \\ y_{in3} \end{bmatrix} \quad (2)$$

where  $[\cdot]^*$  means complex conjugate,  $w_{xi}$  is the weight of  $i$ th-order equalizer taps.  $x_{in2}$  and  $x_{in3}$  mean inputs of the 2<sup>nd</sup>-order and 3<sup>rd</sup>-order kernel.

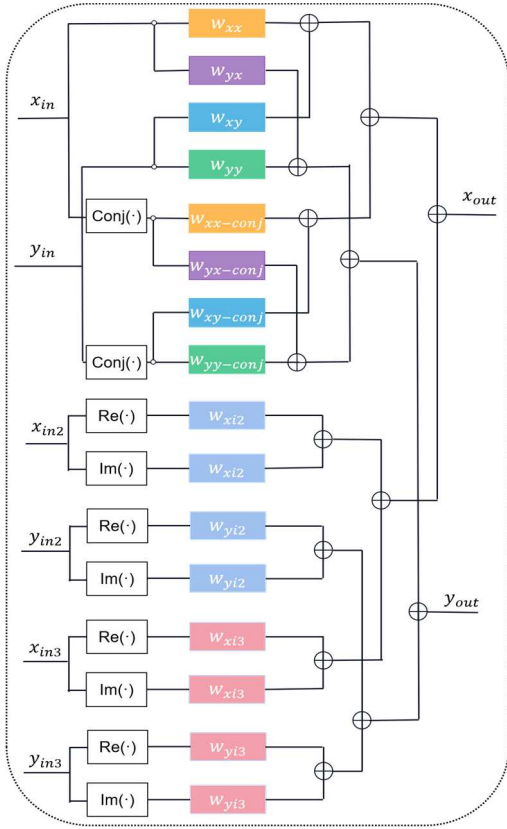


Fig. 2. Structure diagrams of the proposed MIMO-WLCSV.

In addition to the Volterra series, we know the complexity mainly comes from the 2<sup>nd</sup>-order and 3<sup>rd</sup>-order kernels. We utilize a sparse method based on the  $l_0$  norm to reduce the complexity. Defined  $k$ th-order normalized kernel weights are the ratio of the  $k$ th-order weights to the  $k$ th-order maximum weights, which means the smaller normalized weights are less critical and have less impact on the system equilibrium performance. By selecting the minimum threshold of normalized weight and deleting all taps below the threshold point, the updated equation for RLS weights can be given as:

$$w(k+1) = w(k) + e(k)z(k) - S(k) \quad (3)$$

$$S(k) = \gamma(1 - \lambda)P(k)\nabla^s f(w(k-1)) \quad (4)$$

where  $e(k)$  are the error at the moment  $k$ ,  $S(k)$  are terms for sparse,  $\gamma$  is the regularization parameter,  $\lambda$  is the forgetting factors,  $z(k)$  and  $P(k)$  are  $k$ th-order iteration value,  $f(w(k))$  is the  $l_0$  norm of  $w(k)$ ,  $\nabla^s f(w(k))$  is the subdifferential of  $f(w(k))$ .

We compare the commonly used real-valued 2×2 MIMO equalizer, based on the RLS and least mean square (LMS) error, respectively. The experimental results verified the proposed MIMO-WLCSV shown below. Furthermore, the number of 1<sup>st</sup>-order, 2<sup>nd</sup>-order, and 3<sup>rd</sup>-order memory lengths are set to 121, 41, and 5 concerning performance and complexity. In the experiments, we mainly focused on 2<sup>nd</sup>-order kernels. Fig. 3(a) shows the modulus of weights for the first 200 2<sup>nd</sup>-order kernels after training. Deleting all taps below the threshold, which are marked in red in Fig. 3(b) can reduce the complexity of Volterra equalizer.

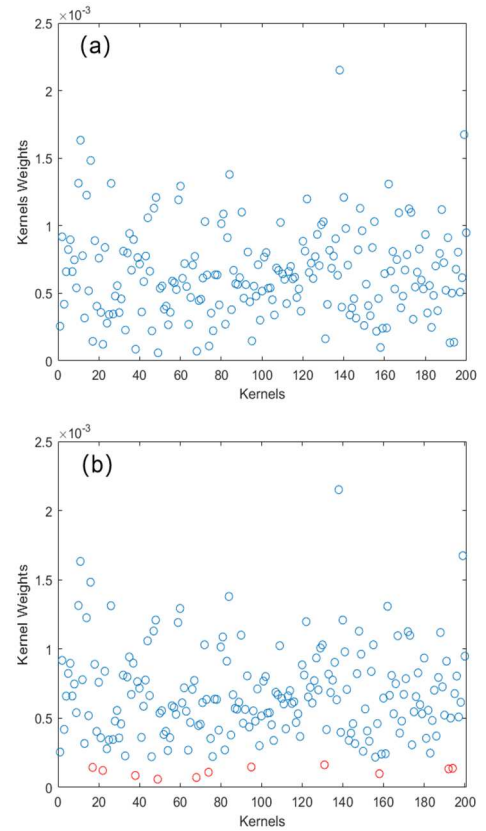


Fig. 3. Principle of sparse Volterra.

The complexity of the system is calculated and can be represented as:

$$C_1 = M_1 + 2M_2 + 3M_3 \quad (5)$$

$$C_2 = M_1 + 2(M_2 - N_2) + 3(M_3 - N_3) \quad (6)$$

where  $M_1$ ,  $M_2$  and  $M_3$ , mean the number of taps of 1<sup>st</sup>-order, 2<sup>nd</sup>-order and 3<sup>rd</sup>-order,  $N_1$  and  $N_2$  mean the number of deleted taps of 2<sup>nd</sup>-order and 3<sup>rd</sup>-order.  $C_1$  and  $C_2$  mean the full Volterra complexity and the sparse Volterra complexity. The complexity of the system reduction can be represented as:

$$P = 1 - \frac{C_2}{C_1} \quad (7)$$

### III. EXPERIMENTAL SETUP

The setup is illustrated in Fig. 4. The PS-1024-QAM signals are generated at the transmitter by a high-precision arbitrary waveform generator (AWG) and passed through an IQ modulator, which mixes the data with an optical carrier. The laser generates light with a linewidth of 1 kHz and a wavelength of 1550 nm, then divided into two paths by a  $2 \times 2$  coupler. The two light paths are fed to the IQ modulator as the optical carrier and the receiver as the local oscillation source, respectively. Then the modulated light is implemented by the delay line for polarization multiplexing. An EDFA and VOA control the signal-to-noise ratio of the signal. After 10 km of standard single-mode fiber, the signals are received by a homodyne integrated coherent receiver (ICR) with a bandwidth of 20 GHz. Then collected through a digital storage oscilloscope with a 3-dB bandwidth of 20 GHz and a sampling rate of 100-GSa/s and processed by offline DSP.

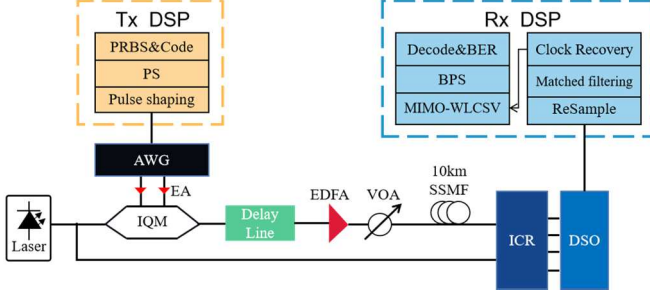


Fig. 4. Experimental setup for PM-PS-1024-QAM system. AWG: arbitrary waveform generator, EA: electric amplifier, IQM: IQ modulator, SSMF: standard single mode fiber, EDFA: Erbium-doped fiber amplifier, VOA: variable low pass filter, ICR: intradyne coherent receiver, DSO: digital storage oscilloscope, MIMO-WLCSV: MIMO widely linear complex-valued sparse Volterra.

The offline DSP algorithm at the transmitter and receiver sides of the system is illustrated in Fig. 4. At the transmitter side, the pseudo-random binary sequence is encoded with PS-1024-QAM pulse shaping, and the roll-off factor is 0.05. The DSP algorithm includes resampling, matched filtering, clock recovery, MIMO-WLCSV, BPS, and decoding algorithm on the receiver side.

### IV. RESULTS AND DISCUSSION

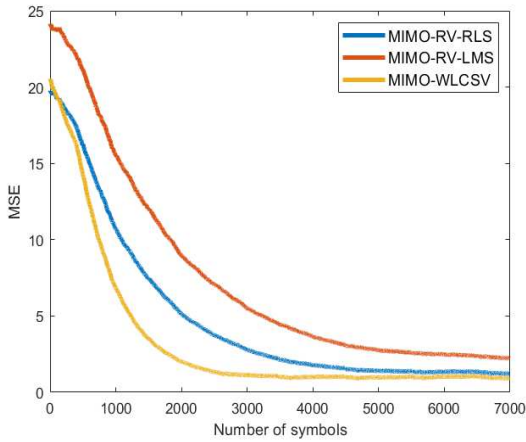


Fig. 5. X polarization MSE versus symbol number for MIMO-RV-RLS, MIMO-RV-LMS, and MIMO-WLCSV.

Fig. 5 illustrates the MSE and symbol number relationship. It can be seen MIMO-WLCSV converges faster and better than the MIMO-RV-LMS and MIMO-RV-RLS

equalizer. The experimental results prove that MIMO-WLCSV can effectively deal with the ultra-high-order modulation format's IQ imbalance and nonlinear impairment.

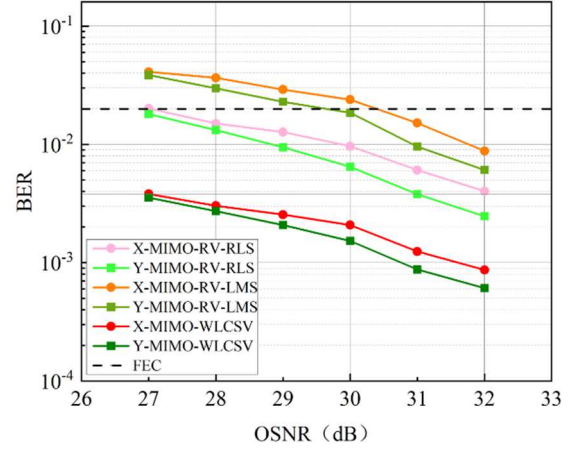


Fig. 6. The relationship of SNR versus BER for PM-PS-1024-QAM.

Fig. 6 illustrates the OSNR versus BER relationship of 1G Baud PS-1024-QAM after 10km fiber. The shaping factor is 0.02 and the spectral efficiency is 16.57 bit/s/Hz. It can be seen that the BER can meet FEC threshold ( $2 \times 10^{-2}$ ) and the system performance is significantly improved by using MIMO-WLCSV with the same SNR. When the OSNR is 32dB, the system BER reaches  $6.11 \times 10^{-4}$ .

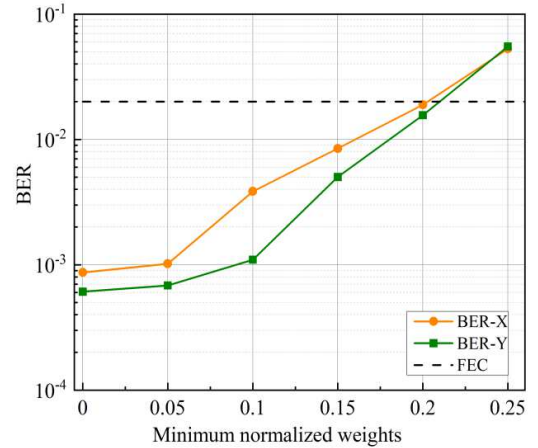


Fig. 7. BER versus minimum normalized weights of 2<sup>nd</sup>-order Volterra kernels for PM-PS-1024-QAM at 32dB.

Fig. 7 illustrates BER results versus minimum normalized weights of 2<sup>nd</sup>-order Volterra kernels for PM-PS-1024-QAM at 32dB. The results show that when 2<sup>nd</sup>-order threshold is 0.2, the BER still meets  $2 \times 10^{-2}$ , at which point the reduced complexity is 8.64%.

### V. CONCLUSION

This paper provides a feasible scheme for PM-PS-1024QAM based on offline DSP. The spectral efficiency achieves 16.57 bit/s/Hz. We utilize a MIMO-WLCSV equalizer compensating the IQ imbalances and nonlinear impairments of the PM-PS-1024-QAM signals. In addition, we introduce sparse based on  $l_0$  norm regularization to reduce the complexity of the Volterra equalizer, which can reduce the algorithm's complexity with less performance loss. The experimental results show that the BER still satisfies the FEC threshold.

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