

Clock Recovery and Equalization Algorithms for Coherent Optical Communication Systems with Non-Integer Oversampling

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Abstract—With the increase of symbol rate, the sampling rate of the signals is lower than the requirement of Nyquist sampling rate. However, non-integer oversampling causes spectrum overlap and seriously affect the transmission performance. In order to solve this problem, this paper proposes a modified clock recovery and equalization algorithm corresponding to the non-integer oversampling. The experimental results of 20-Gbaud SP-16-QAM transmission demonstrate the performance of the clock recovery and equalization algorithms, which can compensate the inherent distortions without additional performance penalty.

Keywords—DSP, non-integer oversampling, clock recovery, TDE, FDE

I. INTRODUCTION

As the ever increasing Internet traffic, the requirement of optical fiber transmission capacity has pushed the development of new technologies and spectral efficient schemes to maximize the channel transmission capacity[1]. With the further increase of symbol rate of transmitted signal, the limited sampling rate of digital converters (ADC) or digital to analog converters (DAC) cannot meet the upgrade requirement of Nyquist sampling rate [2]. In order to relax the limited sampling rate from ADCs/DACs, non-integer oversampling has been proposed to applied in optical fiber communication transmissions. Non-integer oversampling (sub-Nyquist sampling) in coherent optical communication systems can increase the symbol rate when the sampling rate of ADCs/DACs is limited, so as to realize high-speed information transmission [3].

Clock recovery and adaptive equalization algorithms are two key techniques to guarantee the transmission performance with non-integer oversampling [4]. Because transmitters and receivers usually use different clock sources, the sampling points on the receiver often cannot aligned to the input symbol, which causes the phase shift of the sampling points and greatly reduce the transmission performance. The performance degradation caused by this clock offset is particularly significant in systems with applying non-integer oversampling. The suitable timing recovery algorithm can estimate the phase error to deal with this problem [5, 6].

Moreover, non-integer oversampling can also cause spectrum overlap. The traditional 2×2 MIMO equalizer applied to the integer multiple sampling system lead to performance degradation. Therefore, it is necessary to modify the MIMO equalizer to adapt to the system with non-integer oversampling.

In this paper, we propose a modified clock recovery and equalization algorithm in coherent optical transmissions with non-integer oversampling. The clock recovery for non-integer oversampling is processed in parallel. We use time-domain and frequency-domain fractional equalizations to mitigate the distortions from the spectrum alias, respectively. We conduct DP-16QAM simulations to study the influence from different oversampling factors and clock offsets. We also conduct a 20-Gbaud SP-16-QAM coherent transmission experiment to verify the feasibility of the algorithm. The simulation and experimental results show that the modified clock recovery and equalization can be used for $1.25 \times$ oversampling signal without additional penalties.

II. CLOCK RECOVERY ALGORITHM

The traditional Godard algorithm is a technique to estimate the signal timing error based on the frequency domain coefficient. It needs the oversampling factor more than 2. The estimation function of phase error can be expressed as [7],

$$\hat{\tau} = \sum_{k=0}^{\frac{N}{2}-1} \text{Im}\{\tilde{r}_k \tilde{r}_{k+N/2}^*\} \quad (1)$$

where \tilde{r}_k is the discrete Fourier transform (DFT) of the received time domain signal r_n , N is the DFT size, and $(.)^*$ represents the complex conjugate.

The estimation of τ with Godard algorithm is like a sine curve. If the estimated value is positive, the sampling time is too late. While the sampling time is too early if the estimated value is negative. Godard algorithms cannot estimate the timing error when the sampling rate is less than two times of the symbol rate. In order to be feasible for non-integer oversampling, the roll-down factor is introduced as[8],

$$\hat{\tau} = \sum_{k=\frac{1-\beta}{2\eta}N}^{\frac{1+\beta}{2\eta}N-1} \text{Im}\{\tilde{r}_k \tilde{r}_{k+(1-1/\eta)N}^*\}$$

with $\eta \geq \beta + 1$ and $0 < \beta \leq 1$ (2)

The values of η and N should produce an integer on the lower and upper bounds of the sum. Meanwhile, this algorithm also effectively reduces the complexity of Godard algorithm because some terms that contribute less to τ estimation are removed.

In this paper, we change the algorithm to parallel structure to further improve resource utilization and save computing time. Specifically, instead of taking the DFT length N of the whole signal, the signal is divided into several small segments, and the phase error of each small segment is calculated and compensated. Suppose that the original signal is divided into L data blocks, each of which is N/L in length, and the phase error of the m -th data block is shown in Eq.(3).

$$\hat{\tau}_m = \sum_{m=\frac{1-\beta N}{2\eta L}}^{\frac{1+\beta N}{2\eta L}-1} \text{Im}\{\tilde{r}_m \tilde{r}_{m+(1-1/\eta)N/L}^*\} \quad (3)$$

III. FRACTIONALLY SPACED ADAPTIVE EQUALIZATION ALGORITHMS

The commonly equalization algorithms can be divided into blind equalization and training sequence based equalization. The latter realizes the update of filter coefficient by comparing the training sequence with the received signal. It has the advantages of fast convergence speed and better real-time performance. Commonly convergence algorithms include least mean square algorithm (LMS) and recursive least squares algorithm (RLS). At present, fractionally spaced equalizers are mainly using T/2 and T/4 symbol space, which are used for integer oversampling [9-12]. However, for non-integer oversampling, it needs to be modified to adapt different oversampling factors. The following is a fractionally spaced adaptive equalization algorithm based on LMS, which can be processed in the time domain and frequency domain, respectively.

A. Time Domain Equalization (TDE)

Taking 1.25 \times oversampling as an example, the structure of the corresponding 4T/5 fractional symbol space time-domain equalizer is shown in the Fig. 1.

Here, the 1.25 \times oversampling means that five sampling points will be generated for every four symbol points. Correspondingly, the four symbols are recovered by five sampling symbols at the receiver. In the case of dual polarization multiplexing, the signal received by the receiver is the transmission signal on two polarization states: x_{Rx} and y_{Rx} . The x_{Rx} and y_{Rx} take a set of data every five sampling points. This set of data needs to be convolved with the butterfly filter architecture to obtain four outputs of each polarization states: $x_{Rx_out}(i)$ and $y_{Rx_out}(i)$.

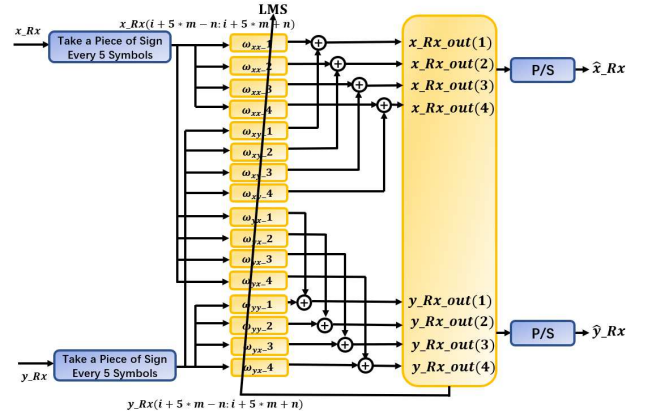


Fig. 1. 1.25 \times oversampling time domain equalization.

$$x_{Rx_out}(i) = x_{Rx} * \omega_{xx_i} + y_{Rx} * \omega_{xy_i}, (i = 1, 2, 3, 4) \quad (4)$$

$$y_{Rx_out}(i) = x_{Rx} * \omega_{yx_i} + y_{Rx} * \omega_{yy_i}, (i = 1, 2, 3, 4) \quad (5)$$

The error function is shown as follows,

$$e_x(i) = x_{Tx}(i) - x_{Rx_out}(i) \quad (6)$$

$$e_y(i) = y_{Tx}(i) - y_{Rx_out}(i) \quad (7)$$

The iterative process is,

$$\omega_{xx_i} = \omega_{xx_i} + \mu * e_x(i) * \text{conj}(x_{Rx}) \quad (8)$$

$$\omega_{xy_i} = \omega_{xy_i} + \mu * e_x(i) * \text{conj}(y_{Rx}) \quad (9)$$

$$\omega_{yx_i} = \omega_{yx_i} + \mu * e_y(i) * \text{conj}(x_{Rx}) \quad (10)$$

$$\omega_{yy_i} = \omega_{yy_i} + \mu * e_y(i) * \text{conj}(y_{Rx}) \quad (11)$$

where μ is the step size. Convergence during iteration can be observed in terms of the magnitude of the error value.

B. Frequency Domain Equalization (FDE)

However, the computational complexity of FIR filter increases with the number of delay taps. Therefore, in application specific integrated circuits (ASIC) or field programmable gate arrays (FPGA), it is difficult to implement FIR filters with a large number of delay taps due to large power consumption and high gate density. Frequency domain equalization can reduce the computational cost through block-by-block signal processing and effective implementation of DFT, so processing the equalization algorithm in frequency domain can reduce the complexity and save the cost [13].

Taking 1.25 \times oversampling as an example, the structure of the corresponding 4T/5 fractional space frequency domain equalizer proposed in this paper is shown in Fig. 2.

Different from the time domain equalization, frequency domain equalization performs FFT and IFFT at the beginning and the end of the input signal, respectively. The input signal is converted from series to parallel, and each polarization state is divided into five branches. The final output of $X_{Rx_out}(k)$ and $Y_{Rx_out}(k)$ are written as ($k = 1, 2, 3, 4$),

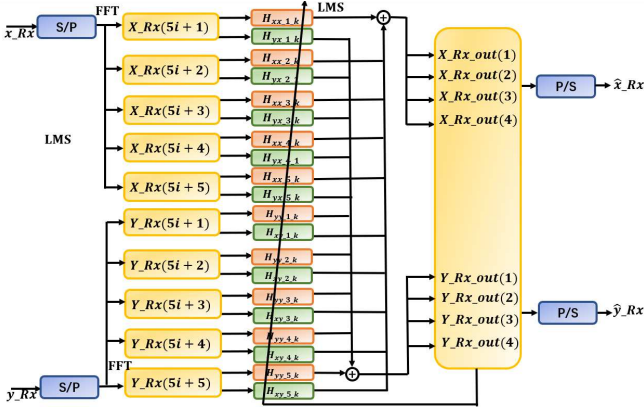


Fig. 2. 1.25 \times oversampling frequency domain equalization.

$$X_{Rx_out}(k) = \sum_{m=1}^5 [X_{Rx}(5i+m) \cdot H_{xx_m_k} + Y_{Rx}(5i+m) \cdot H_{yx_m_k}] \quad (12)$$

$$Y_{Rx_out}(k) = \sum_{m=1}^5 [Y_{Rx}(5i+m) \cdot H_{yy_m_k} + X_{Rx}(5i+m) \cdot H_{yx_m_k}] \quad (13)$$

where i represents the i -th Fourier transform block, m and k represent the m -th wight value correspond to k -th output symbol. The error functions $E_x(i)$ and $E_y(i)$ are the FFT transformation of time domain error function. In this algorithm, a parallel architecture is adopted to divide the original signal into multiple FFT blocks for processing, which improves the computational efficiency and reduces the complexity.

$$H_{xx_m_k} = H_{xx_m_k} + u \cdot \text{conj}(X_{Rx}(5i+m)) \cdot E_x(k) \quad (14)$$

IV. SIMULATION

We conduct the dual polarization transmission of coherent 16-QAM modulation at the symbol rate of 20 Gbaud with VPItransmissionMaker 9.1. The oversampling factor η is set to 1.25 or 1.125, respectively. The fiber length is 100 km. The optical signal noise ratio (OSNR) is 25 dB. Fig. 3 shows the bit error rate (BER) curve versus the received optical power (ROP) for 1.25 \times or 1.125 \times oversampling signal. We can find that the non-integer oversampling equalization effectively improves the performance compared with traditional 2×2 MIMO equalizer. The performances of TDE and FDE are the same.

Different sampling clock offsets are added during the oversampling process to simulate the penalty caused by clock instability. Fig. 4 depicts the BER versus ROP with different clock offsets. The performance after adding the clock recovery function is the same as that without the clock offset. It indicates that the clock recovery algorithm is suitable for non-integer oversampling signal.

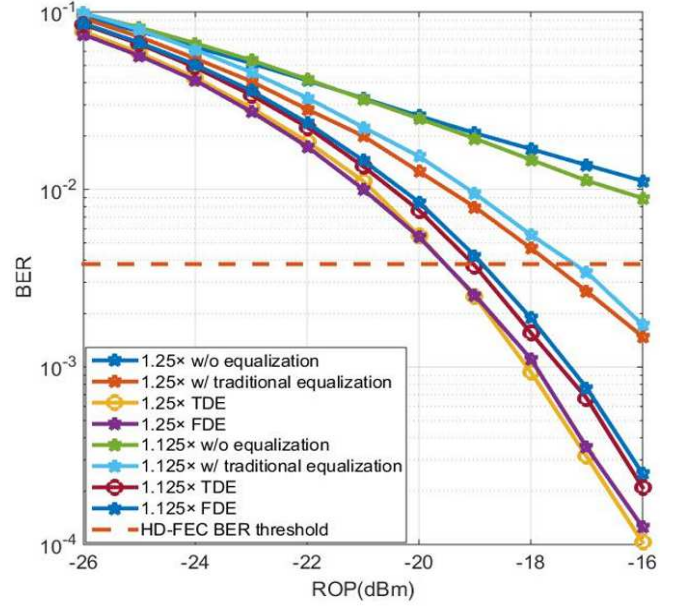


Fig. 3. The influence of equalization algorithm on the performance of 1.25 \times and 1.125 \times oversampling signal.

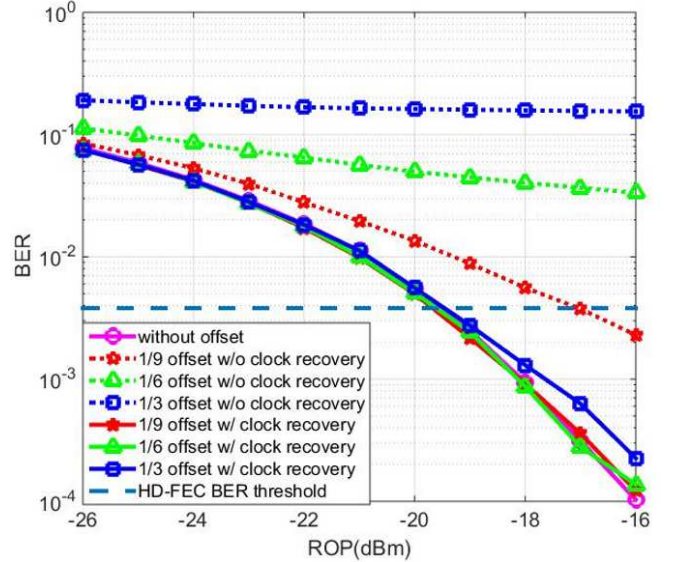


Fig. 4. The influence of clock recovery algorithm on the performance of 1.25 \times oversampling signal.

V. EXPERIMENTAL VERIFICATION

Fig. 5 shows the experimental setup of SP-16-QAM coherent transmission. At the transmitter side, a pseudo random binary sequence with 2^{17} bits is used generate the 16-QAM signal. The AWG works at 92 GS/s with 1.25-times oversampling rate, and the roll-off factor of the Nyquist-pulse-shaping filter is 0.1. The electrical signal is used to modulate the carrier of 1550 nm by an optical I/Q modulator. Then, the modulated optical signal becomes a single -polarization signal. After 100-km fiber transmission, the received optical signal is detected by the coherent receiver and digitized by a digital processing oscilloscope (DPO) operating at 25 GS/s. After the coherent detection, DSP is operated offline, e.g. chromatic dispersion compensation (CDC), frequency offset estimation (FOE), clock recovery, fractionally spaced equalization, phase compensation and demapping.

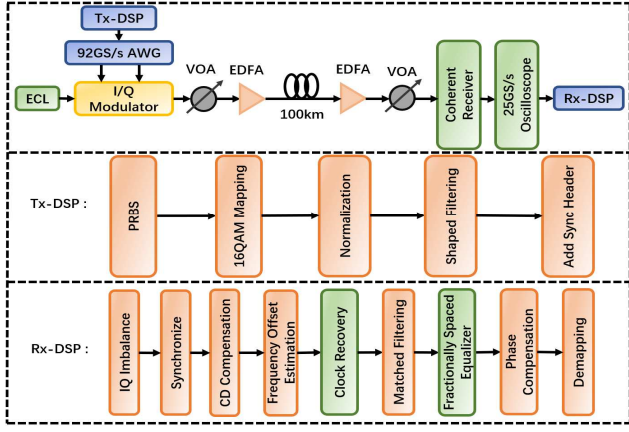


Fig. 5. Experimental system setup for SP 16-QAM coherent transmission.

Table I shows the comparison of transmission performance with or without clock recovery under the same channel conditions. It can be found that the BER is unstable due to the jitter of the sampling point when the clock recovery is not carried out. After clock recovery, the performance of the five groups of data is improved and the final performance is similar, indicating that the clock recovery algorithm is feasible.

TABLE I. CLOCK RECOVERY PERFORMANCE

Parameter	LOP=-6dBm, ROP=-25dBm, Length=100km, $\eta=1.25$			
	with clock recovery		without clock recovery	
	SNR/dB	BER	SNR/dB	BER
1	15.3556	0.0036214	13.1301	0.017029
2	15.2904	0.0040291	11.3822	0.0392
3	15.354	0.0034739	10.4215	0.051878
4	15.1646	0.0045018	13.1343	0.017767
5	15.5009	0.0031737	12.9765	0.02001

Then, we demonstrate the feasibility of the equalization algorithm. Fig. 6 shows the BER performance with different equalizations.

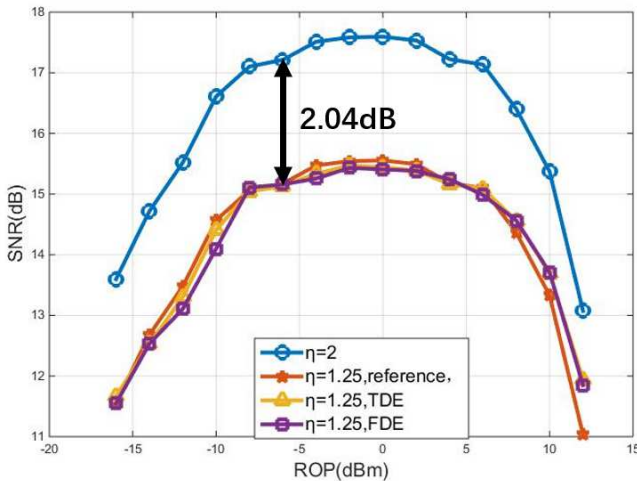


Fig. 6. The blue line represents $2\times$ oversampling, and then subtract 2.04 dB to get the red line as the reference curve of $1.25\times$ oversampling. The yellow line represents $1.25\times$ oversampling time domain equalization, and the purple line represents $1.25\times$ oversampling frequency domain equalization.

TABLE II. COMPLEXITY COMPARISON

Equalization	Complexity	$a=5, b=4, n=100$
TDE	$12bL/a$	$9.6L$
FDE	$12abL/n$	$2.4L$

As we all know, the relationship between the SNR and the sampling rate is as follows,

$$SNR = 6.02N + 1.76 + 10 * \log(f_s/2B) \quad (15)$$

where N is the quantization bit number, the sampling rate is f_s , and B is the bandwidth. When the bandwidth is fixed, f_s increases and the effect is equivalent to extending the quantization noise over a wider frequency range, thus increasing the SNR requirement. Substituting into Eq.(15), it can be obtained that there is a theoretical difference between the $2\times$ and $1.25\times$ oversampling, which is 2.04 dB. This value corresponds exactly to the SNR difference between the twice sampled signal and the 1.25 times oversampled signal in Fig. 6. It indicates that the equalization method can be used for non-integer oversampling without additional performance degradation.

In addition, the performance of time domain equalization and frequency domain equalization are similar. The frequency domain equalization algorithm complexity is lower, which is benefit to cost saving. If the sampling factor η is a fraction, it can be written as a/b (e.g. 1.25 can be written as $5/4$). In the TDE, it is assumed that the length of the training sequence is L , a set of data is taken every a sampling point, L/a times of training is needed. The algorithm requires $12bL/a$ multipliers. In the FDE, it is assumed that the length of the training sequence is L , and a group of data is taken every n points to make Fourier changes for FFT block processing, which requires L/n times of training. The algorithm requires $12abL/n$ multipliers. After calculation, the complexity of TDE and FDE is compared as shown in the Table II.

VI. CONCLUSIONS

We propose a modified clock recovery and equalization algorithm corresponding to the non-integer oversampling. The simulation and experimental results of 20-Gbaud SP-16-QAM transmission show that the modified algorithm is feasible for clock recovery and adaptive equalization with 1.25-times oversampling.

VII. ACKNOWLEDGMENT

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