Inversion of orbital-angular-momentum light field based on strongly scattering medium

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***Abstract*—Orbital Angular Momentum (OAM) beam is a vortex beam with helical phase term. The OAM multiplexing can make the beam carry more information. However, in practical communication, the scattering effect of the channel may lead to the distortion of the phase of the optical field. In this paper, an optimal spectral method based on the scaling is proposed for the inversion of the scattered light field, and the BFGS algorithm is used to iteratively optimize the initialized retrieved light field. Simulation results show that when the sampling rate is 4, the correlation of optical field inversion increases to more than 0.9999. The inversion error is reduced by 10^{-10} times compared with the method combining the speckle-correlation scattering matrix (SSM) and Gerchberg-Saxton (GS) algorithm, achieving a high-fidelity phase recovery.**

Keywords—OAM; BFGS; optimal spectral method; scattering medium; light field inversion

I. INTRODUCTION

Due to the orbital angular momentum (OAM) beams possessing an infinite number of mutually orthogonal eigenstates, the channel capacity of the communication system can be significantly improved via OAM mode multiplexing [1-4]. However, when the light propagates through the heterogeneous and strongly scattering medium such as the clouds and biological tissues, it is a challenge to recognize the original information from the received speckle light field [5]. Therefore, the study of optical field inversion based on the strongly scattering medium is of great significance to biomedicine, communication and other fields. The phase retrieval algorithm [6-9] is an important optical field inversion method, which obtains the phase information of the original signal according to the measured speckle field intensity information. In 2012, Bertolotti et al. proposed the non-invasive imaging through the scattering medium using the speckle-correlation method for the first time [10]. This method does not require the measurement of transmission matrix (TM), but is only applicable to weak scattering medium imaging. In 2016, KyeoReh Lee et al. proposed that the speckle-correlation scattering matrix (SSM) method and GS algorithm were combined to retrieve the original light field information from the speckle intensity pattern [11]. Due to the poor initialization effect of SSM, the reconstructed light field loses the edge and other detail information, so it is difficult to achieve a high-fidelity light field inversion. Therefore, it is necessary to propose a better initialization method and perform the iterative optimization on this basis.

The iterative phase retrieval algorithm is sensitive to the initial value, such that an appropriate initialization strategy [12] is an important means to avoid the local optimum problem. A good initial value needs to meet the following two requirements, i.e. ensuring that the algorithm can converge to a better solution and have a low time complexity, which is important to achieve a balance [13]. In 2015, Candes et al. proposed a spectral initialization method for the phase recovery in the diffraction imaging with the high complexity [14]. In 2016, the SSM method was used for the initial estimation of OAM light field in scattering imaging, and this method still required more iterations [11]. In 2017, Marco Mondelli et al. proposed an optimal spectral initialization algorithm [15] based on the spectral initialization, which could efficiently and accurately perform the phase recovery.

This paper proposes an accurate and efficient method for the OAM light field detection without the prior information, which is an optimal spectral method based on the idea of scaling. By constructing the matrix with TM and the received speckle intensity, most of the target information can be quickly recovered. Then, the Gerchberg-Saxton (GS) algorithm, hybrid input-output algorithm (HIOA) and other algorithms [16-18] are used to iteratively optimize the initial information. However, the alternate projection method has problems such as long iteration time and poor anti-noise performance. The BFGS algorithm [19] is a kind of gradient descent method, which can still converge quickly when the amount of data is large, and can effectively preserve the edge and other detail information during image reconstruction.

Firstly, this paper establishes a simulation model of light field inversion based on the strongly scattering medium, and then the phase initialization strategies including spectral method, optimal spectral method and SSM method are compared. Furthermore, the iterative optimization is carried out with different phase recovery algorithms. Finally, the optimal spectral initialization based on the idea of scaling and BFGS (optimal-BFGS) algorithm are proposed for optical field inversion.

II. Inversion System of OAM Light Field

A. System structure

The structure of an OAM optical field inversion system based on the scattering medium is shown in Fig. 1. This system consists of two modules, the scattering transmission module and optical field inversion module.

At the input end of the scattering transmission module, the multiplexed OAM optical field passes through a 4f optical system and then passes through the strongly scattering medium. A charge coupled device (CCD) camera is placed at the output end of the scattering transmission module to receive the scattering speckle and send it into the optical field inversion module. Previously, the computer needs to store the TM of the scattering medium, which can be measured by the phase-shift interference method and phase recovery algorithm [20-22]. The speckle intensity pattern measured by the CCD camera and the TM are used as the input of OAM optical field inversion system to obtain an initial phase, and the target optical field is obtained by the iterative optimization.

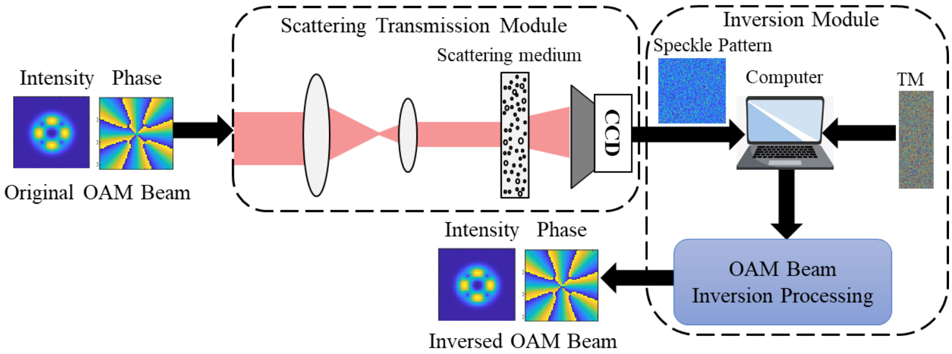

Fig. 1. System structure diagram of an OAM optical field retrieval based on the scattering medium.

B. Principle of optical transmission matrix based on the strongly scattering medium

The multiple scattering of light field through a strongly scattering medium can be viewed as a linear deterministic process [23,24], and the specific relationship is shown in Fig. 2.

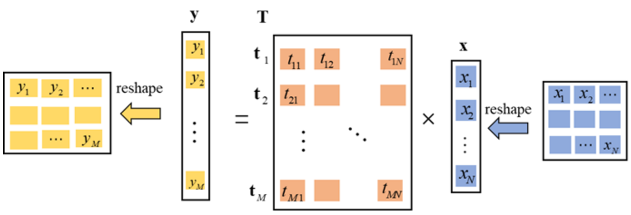

Fig. 2. The relationship among the output optical field **y**, the transmission matrix **T** and the input optical field **x.** T(TM, a $M \times N$ matrix)

In Fig. 2, the input optical field is a two-dimensional OAM beam, which is reshaped to an one-dimensional $N\times1$ column vector $\mathbf{x}$, where x_n represents the n-th element of $\mathbf{x}$. The output optical field is the speckle field of OAM, which is a $M\times1$ column vector $\mathbf{y}$, where y_m represents the m-th element of the output optical field. t_{mn} represents the element in the m-th row and n-th column of the optical transmission matrix $\mathbf{T}$. The relationship between the m-th output optical field element and the n input optical field element can be expressed by

$$y_m = \sum_{n=1}^{N} t_{mn} x_n \quad (n = 1,\ 2,\ \cdots,N,\ m = 1,\ 2,\ \cdots,M) \tag{1}$$

The sampling rate γ is the ratio of the number of sampling elements at the output to the number of input patterns, which can be expressed as [23]

$$\gamma = \frac{M}{N} \tag{2}$$

III. Principle and Method of OAM Beam Inversion

A. Optimal spectral initialization based on the scaling idea

Fig. 3 is an optimal spectral initialization flowchart.

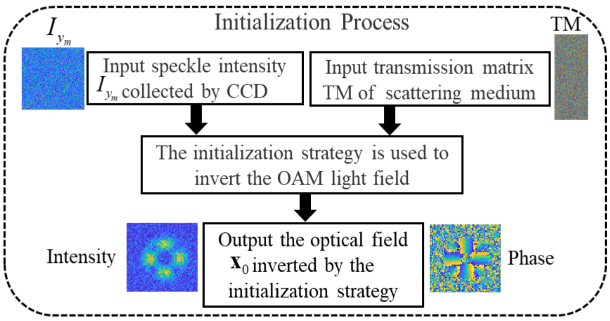

Fig. 3. Flowchart of the light field initialization based on the optimal spectral initialization.

The m-th element I_{y_m} of the output speckle intensity $\mathbf{I}$ is equal to the modular square of the product of the m-th row vector $\mathbf{t}_m$ of the $\mathbf{T}$ and the input column vector $\mathbf{x}$ [15].

$$I_{y_m} = \left|y_m\right|^2 = \left|\mathbf{t}_m \mathbf{x}\right|^2 = \sum_{a=1}^{N} t_{ma}^* x_a \sum_{b=1}^{N} t_{mb} x_b^* = \sum_{a=1}^{N}\sum_{b=1}^{N} t_{ma}^* t_{mb} x_a x_b^* \tag{3}$$

where * represents the conjugate operation. It can be seen from Eq. (3) that it is possible to reconstruct the input vector $\mathbf{x}$ from the received speckle field intensity I_{y_m} and TM. The semi-positive definite Hermitian matrix can be expressed as [14]

$$\mathbf{Y} = \frac{1}{M}\mathbf{T}^{\dagger}\mathrm{diag}\left(\mathbf{I}\right)\mathbf{T} = \frac{1}{M}\sum_{m=1}^{M} I_{y_m} \mathbf{t}_m^{\dagger} \mathbf{t}_m \tag{4}$$

where $\mathbf{t}_m^{\dagger}$ is the conjugate transpose of $\mathbf{t}_m$. The mathematical expectation of $\mathbf{Y}$ can be expressed as

$$E(\mathbf{Y}) = \|\mathbf{x}\|^2 \mathbf{I} + \mathbf{x}\mathbf{x}^\dagger \tag{5}$$

It can be seen that in the large sample rate, $\mathbf{Y}$ is equal to the matrix $\|\mathbf{x}\|^2 \mathbf{I} + \mathbf{x}\mathbf{x}^\dagger$, and the eigenvector corresponding to the principal eigenvalue of the matrix $\|\mathbf{x}\|^2 \mathbf{I} + \mathbf{x}\mathbf{x}^\dagger$ is of the form $\alpha\mathbf{x}$, where α is a scalar and $\alpha \in R$. Therefore, the eigenvector corresponding to the principal eigenvalue of $\mathbf{Y}$ is also close to $\alpha\mathbf{x}$. The estimated value $\alpha\mathbf{x}$ obtained is sufficiently related to the real solution $\mathbf{x}$, and its direction is theoretically close to that of the real solution, so $\mathbf{x}$ can be obtained by the spectral decomposition method described above. The sampling rate γ is crucial for the phase recovery. Although the spectral initialization into OAM optical field inversion without the prior information is possible, a good inversion effect can only be obtained at $\gamma \geq 9$. When γ is small, the spectral initialization and SSM methods cannot reverse the initial phase, while the optimal spectral initialization based on the scaling can reverse initial phase when γ=4 [15].

Since the eigenvector corresponding to the principal eigenvalue of the constructed matrix $\mathbf{Y}$ is of the form $\alpha\mathbf{x}$, there is a coefficient difference between the inversion signal $\mathbf{x}_{01} = \alpha\mathbf{x}$ and the real signal $\mathbf{x}$. The length of signal may affect the convergence of phase inversion. Therefore, after reversing the initial vector $\mathbf{x}_{01}$, the idea of scaling is used to scale $\mathbf{x}_{01}$ appropriately to eliminate the influence of coefficient on phase inversion. The solution is expressed as

$$\min_{\beta} \left\| \beta |\mathbf{T}\mathbf{x}_{01}| - \sqrt{\mathbf{I}} \right\| \tag{6}$$

where $\mathbf{T}$ and $\mathbf{I}$ can be measured, and $\mathbf{x}_{01}$ is initialized by inversion, so u and l can be obtained

$$u = \mathbf{T}\mathbf{x}_{01} \cdot * \sqrt{\mathbf{I}} \tag{7}$$

$$l = \mathbf{T}\mathbf{x}_{01} \cdot * \mathbf{T}\mathbf{x}_{01} \tag{8}$$

The ratio of u normalized to l normalized is β

$$\beta = \frac{\mathrm{norm}(u(:))}{\mathrm{norm}(l(:))} \tag{9}$$

where β is actually $1/\alpha$, and finally the initialization $\mathbf{x}_{02}$ is the real signal $\mathbf{x}$.

$$\mathbf{x}_{02} = \beta\mathbf{x}_{01} = \frac{1}{\alpha}\alpha\mathbf{x} = \mathbf{x} \tag{10}$$

B. *Iterative optimization based on the BFGS algorithm*

The BFGS algorithm is used to iteratively optimize the initialized light field $\mathbf{x}_0$ obtained in Fig. 4, and when the inversion error of the $\tau+1$ and τ light field lower than the threshold value of 10^{-15}, the iteration is terminated and the final retrieved light field $\mathbf{x}_{\tau+1}$ is the output.

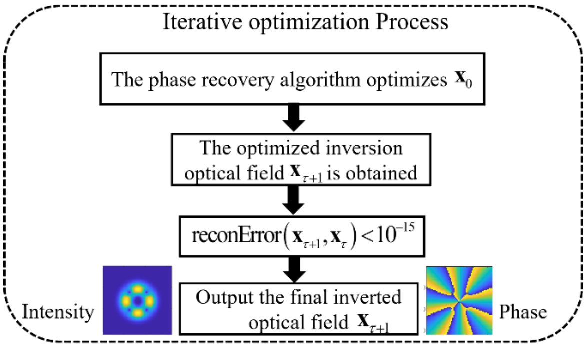

Fig. 4. Flowchart of the light field iterative optimization based on the BFGS algorithm.

In order to measure the light field inversion effect of this system, the inversion error (reconError) and inversion correlation (CorrelationRate) of the original OAM light field and the inverted OAM light field are used

$$\mathrm{reconError} = \frac{\mathrm{norm}(\mathbf{x}_{\mathrm{true}} - \mathbf{x}_{\mathrm{estimate}})}{\mathrm{norm}(\mathbf{x}_{\mathrm{true}})} \tag{11}$$

$$\mathrm{CorrelationRate} = \left| \frac{\mathbf{x}_{\mathrm{true}} \times \mathbf{x}_{\mathrm{estimate}}^\dagger}{\mathrm{norm}(\mathbf{x}_{\mathrm{true}}) \times \mathrm{norm}(\mathbf{x}_{\mathrm{estimate}}^\dagger)} \right| \tag{12}$$

where $\mathbf{x}_{\mathrm{true}}$ is the original OAM light field, † is conjugate transpose, $\mathbf{x}_{\mathrm{estimate}}$ is the inversion of light field, $|\cdot|$ is modulus operation. The sampling rate (γ), signal-to-noise ratio (SNR), iteration number (iteration) and running time (time) are used as the indicators to compare the inversion effect of different methods.

IV. Simulation and Discussion

The transmission matrix model is established according to the principle of circular Gaussian distribution matrix [24,25] to simulate the strongly scattering medium. The incident OAM beam has the wavelength $\lambda = 532\mathrm{nm}$, the radial index $p = 0$, the incident aperture $D_R = 0.1\mathrm{m}$, and the beam waist $\omega_0 = 0.01\mathrm{m}$.

A. *Optimal spectral initialization compared to other initialization algorithms*

Fig. 5 (a) and Fig. 5 (b) show the results of light field inversion of OAM beam with the preset number of input elements $N = 1024$ and the topological charge number $l = 8$ under different γ and SNR respectively. When γ is as low as 4, the CorrelationRate based on the optimal spectral initialization reaches 0.74. When SNR is as low as 1 dB, the CorrelationRate reaches 0.86. The SSM initialization and spectral initialization connot clearly invert the contours of the original light field. It can be seen that the optimal spectral initialization based on the idea of scaling proposed in this paper has a better performance under small γ and SNR.

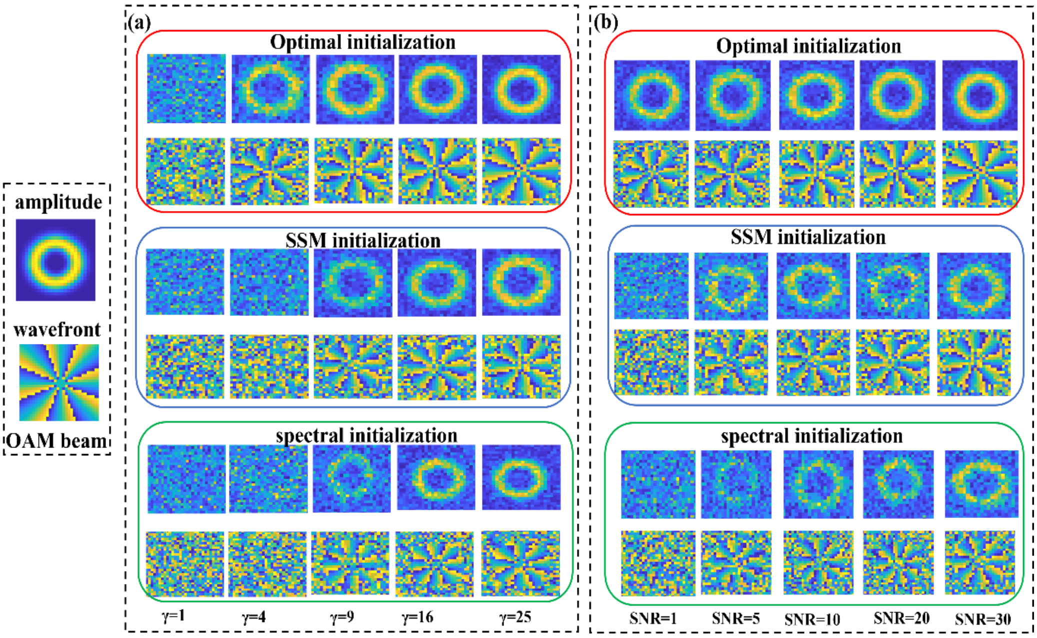

Fig. 5. Comparison plots of OAM beam retrieval under different initialization strategies. (a) Comparison of OAM beam retrieval under different γ. (γ=[1,4,9,16,25], SNR=30dB) (b) Comparison of OAM beam retrieval at different SNR. (SNR=[1,5,10,20,30]dB, γ =9)

The fidelity of the inversion still needs to be improved. The effect of light field inversion is directly affected by γ. With the increase of γ, the amount of data of the TM required to generate the **Y** will also increase exponentially, which makes the computer spend more time on the inversion calculation, and the requirements for the CPU, memory and other configuration of the computer are also higher. Therefore, the solution of initialization method inversion needs to be further optimized.

B. Optimal spectral initialization compared to the BFGS optimization

Fig. 6 shows the simulation result of the OAM multiplexed beam with the order $O=\{4,8\}$ after passing through the scattering medium using the optimal-BFGS algorithm. The number of sampling elements is $M=16384$, and the number of preset input elements is $N=4096$, that is γ=4. Fig. 6(a) shows the light field intensity and phase distribution of the incident OAM beam, and Fig. 6(b) shows the light field intensity and phase distribution obtained by using the optimal spectral initialization. At this time, the OAM light field reconError is 0.4, and the CorrelationRate is 0.52. Fig. 6 (c), (d) and (e) show the intensity and phase distribution of the OAM light field obtained by the optimal-BFGS algorithm, and the reconError are 10^{-5}, 10^{-10} and 10^{-15}, respectively. Finally, almost all errors are eliminated, and the original OAM light field is restored with a high fidelity.

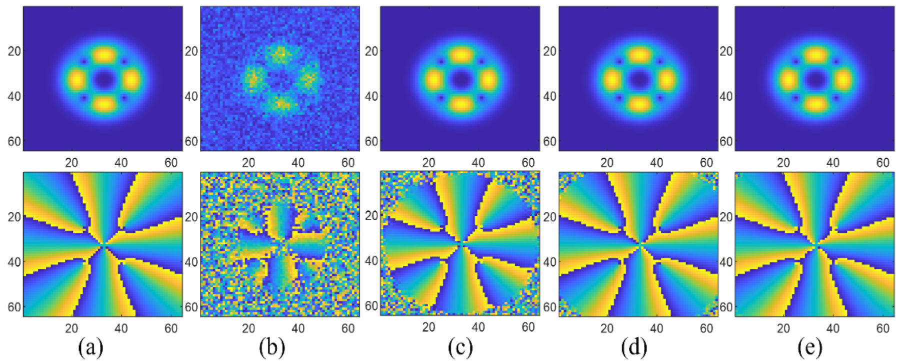

Fig. 6. (a) Light field of the incident OAM beam. (b) Inversion of OAM light field by optimal spectral initialization. (c-e) OAM light field after iterative optimization of BFGS. (OAM state $O=\{4,8\}$)

In order to display the OAM optical field inversion effect more accurately, the correlation diagram between the original phase and the inverted phase shown in Fig. 7. Fig. 7 (a) is the correlation graph of the original signal itself, which can be seen to be completely positively correlated. Fig. 7(b) corresponds to Fig. 6(b), where the relation graph is disorganized and does not present a positive correlation. Fig. 7(c-e) corresponds to Fig. 6(c-e), respectively, and as the recovery error decreases, the original OAM phase and the retrieved OAM phase are finally realized to be almost completely positively correlated.

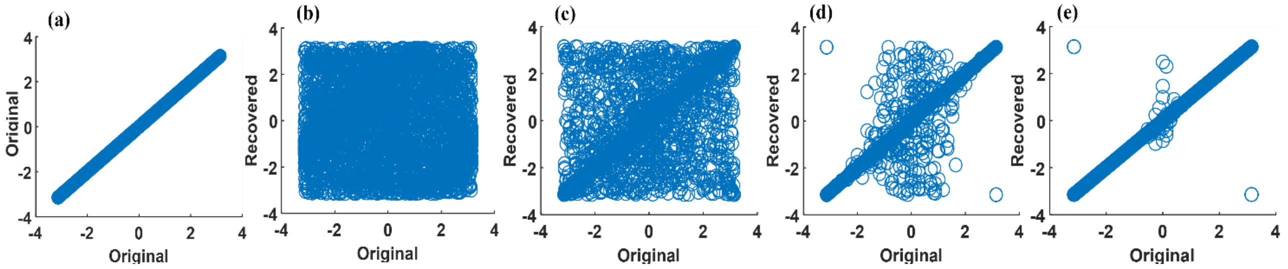

Fig. 7. Plot of correlation between the original phase and the retrieved phase. (a) The correlation of the original signal itself. (b) Optimal spectral initialization method inversion. (c-e) Correlation plots with the inversion errors of 10^{-5}, 10^{-10} and 10^{-15}.

Fig. 8 shows the comparison curves of BFGS algorithm based on different initialization strategies. At this time, the number of preset input elements $N = 1024$. Fig. 8(a) is the graph of the relationship between the correlation degree of OAM optical field inversion and γ, when γ is 1, 4, 9, 16 and 25, respectively. Fig. 8(b) is the graph of the relationship between the inversion error of the OAM light field and γ under the same conditions. Fig. 8(c) is the graph of the relationship between the inversion error of the OAM light field and the SNR. Even if the SNR is as low as 5dB, the optimal-BFGS algorithm can reconstruct the light field with a high fidelity. Fig. 8(d) shows the graph of the relationship between the inversion error and the iteration time. The optimal-BFGS algorithm is faster than other algorithms, and the speckle field can be recovered almost without error within 10 seconds. For each algorithm, the average of three experiments is taken as the result of the algorithm, so the results are universal. From the results, it can be seen that the optimal-BFGS algorithm has the best inversion effect.

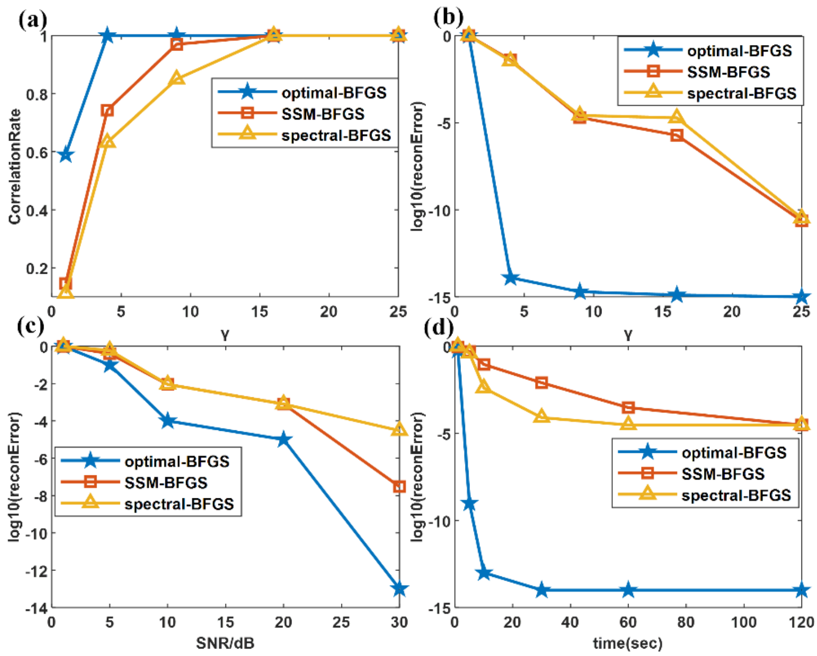

Fig. 8. (a) The graph of the relationship between CorrelationRate and γ. (γ=[1,4,9,16,25]， SNR=30dB). (b) The graph of the relationship between the inversion error and γ. (γ=[1,4,9,16,25]， SNR=30dB) (c) The graph of the relationship between the inversion error and SNR. (SNR=[1,5,10,20,30]dB, γ =9). (d) The graph of the relationship between the inversion error and iteration time. (γ =9, SNR=30dB, time=[1,5,10,30,60,120]second).

V. Conclusion

In this paper, the light field inversion simulation model based on the strongly scattering medium is established, and the optimal spectral initialization strategy based on the scaling idea is proposed, and the BFGS optimization algorithm is used to achieve a high fidelity recovery of the light field.

(1) When the SNR of the system is 30 dB, optimal-BFGS algorithm is used to reduce the inversion error to 10^{-15}, which is 10^{-10} times lower than the previous algorithms (GS based on SSM, HIOA based on SSM).

(2) The optimal-BFGS algorithm can invert the edge and other detail information of the target with the low error and high correlation at lower sampling rate γ, weaker signal-to-noise ratio, less running time and iteration times.

(3) In the light field reconstruction technology based on the optical TM, the optimal-BFGS algorithm can preserve a lot of detail information of the target, and can effectively suppress the influence of noise, which has a certain effect on the cases where the measurement of optical TM is inaccurate or the detection signal is weak.

The method and results of this paper are of great significance for OAM beam pattern detection, imaging and focusing through the strongly scattering medium, and lesion observation in biomedicine.

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