Thermal noise suppression strategy of hybrid optomechanical gyroscope based on atomic ensemble

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***Abstract*—A ground-state cooling scheme based on hybrid optomechanical system is proposed. Atomic ensemble is introduced to suppress the thermal noise generated by mechanical resonator excited by external environment heat. The radiation pressure fluctuation spectrum of mechanical resonator can be changed by atom-cavity and cavity-cavity couplings. The peak value of the fluctuation spectrum can be leveraged to strengthen the cooling process, and its valley value can be utilized to suppress the heating process. The simulation results show that the optomechanical system effectively improves the cooling rate of the optomechanical gyroscope, cools the mechanical resonator to the ground state, and ultimately reduces the thermal noise of the optomechanical gyroscope.**

***Index Terms*—optomechanical system, gyroscope, ground-state cooling**

I. Introduction

Gyroscope is a core navigation and positioning device widely used in aircraft, submarines, autopilot and other fields. It can measure the angular velocity of the carrier relative to the inertial space, so as to sense and maintain the direction of the carrier [1]. With the continuous progress of science and technology, it is an important direction of gyro technology research to adopt new principles and technologies to develop new types of gyroscopes [2]. The gyroscope combined with the optomechanical system [3] is a new type of angular velocity measuring device. Compared with other types of gyroscopes, it has higher sensitivity and does not depend on the size of the optical path, so it is easier to realize in the micro structure [4].

The optomechanical system is based on the principle of radiation pressure [5]. It couples the optical cavity and mechanical resonator through radiation pressure, showing great advantages in detecting small mass, gravitational wave, displacement [6]–[8]. In recent years, based on the fact that the optomechanical cavity is very sensitive to the external force acting on it, the optomechanical system has been applied to the rotation detection. Sankar Davuluri [9] proposed that the optomechanical cavity can be designed as a very sensitive rotary detector because it can detect small changes in the degree of freedom of the mechanical resonator. M. Bhattacharya [10] theoretically discussed the optomechanical system coupling scheme of rotating nanoparticles and cavity mode with orbital angular momentum, and proved that the rotation frequency of nanoparticles can be detected through zero difference detection.

Mechanical resonator are easily excited by the external thermal environment to generate thermal noise, which is an important noise source of the optomechanical rotation detection model. Meanwhile, the display of the quantum properties of the optomechanical system must also suppress the uncontrollable thermal fluctuations. Ground-state cooling generally means that the mean phonon number of mechanical resonator is less than 1 [11], which is the best cooling effect that the mechanical mode can achieve. It is difficult to achieve ideal ground-state cooling effect for mechanical resonator only by means of low-temperature physical device and feedback measurement cooling method. Therefore, radiation pressure in optomechanical system is usually used to realize self cooling of mechanical resonator. In our previous work, we used the dual-cavity system model to suppress the thermal noise of the optomechanical gyroscope, and realized the ground-state cooling of the mechanical resonator [12].

In this paper, we propose a ground-state cooling strategy of mechanical resonator based on hybrid optomechanical system, introducing atomic ensemble [13] to improve the cooling efficiency of mechanical resonator and suppress the thermal noise of optomechanical gyroscope [4]. In the hybrid optomechanical gyroscope model, the two-level atomic ensemble is trapped in a Fabry Perot cavity [14] with a movable cavity mirror, and the optomechanical cavity is directly coupled with the auxiliary optical cavity. The radiation pressure generated by the optical field in the cavity makes the mechanical mode

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vibrate slightly, changing the length of the optical cavity. The change of the cavity length changes the number of photons in the cavity, which affects the radiation pressure acting on the mechanical mode. In our system model, the cooling of mechanical resonator comes from two ways, one is from the coupling of atoms and optical cavity, and the other is from the coupling of cavity and cavity. The combination of these two quantum interference effects not only enhances the cooling rate, but also reduces the mean phonon number of mechanical resonator, resulting in better cooling effect.

II. Hybrid optomechanical gyroscope

Considering the hybrid optomechanical gyroscope model as shown in Figure 1, the whole system is placed on a workbench with a rotation angular velocity of $\dot{\theta}$, so that the rotation axis passes through the center of the beam mirror, and the optical cavity is placed on a movable platform along the radius of the workbench. A beam of laser is emitted from the laser. The laser field l_0 is divided into $c_{in} = il_0/\sqrt{2}$ and $l_1 = l_0/\sqrt{2}$ through the 50:50 beam splitter (BS), where c_{in} is input into the hybrid optomechanical system, and the other beam l_1 of the beam splitter is directed to the reflector (FM). Finally, the light fields c_{out} and l_2 output by the hybrid optomechanical system and the mirror pass through the 50:50 beam splitter (BS) again, and recombine to generate two new beams $d_1 = (c_{out} + il_2)/\sqrt{2}$ and $d_2 = (l_2 + ic_{out})/\sqrt{2}$, which are measured by detectors D_1 and D_2.

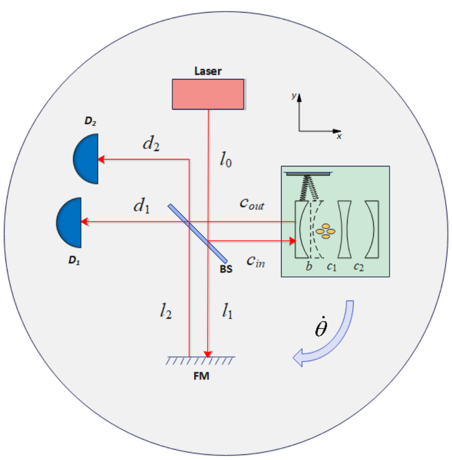

Fig. 1: Schematic diagram of hybrid optomechanical gyroscope

In the hybrid optomechanical system, the left optical cavity c_1 and the mechanical resonator are coupled by the radiation pressure, the right optical cavity c_2 and the left optical cavity c_1 are coupled by the exchange of photons (similar to the evanescent field coupling), and the two-level atomic ensemble is trapped in the optical cavity c_1, c_{in} and c_{out} are the driving and output light fields of the hybrid optomechanical system respectively. Mechanical resonator refers to a movable cavity mirror b suspended on a spring at one end. When it is subjected to the radiation pressure or Coriolis force of the light field in the cavity c_1, it will vibrate slightly and produce displacement changes. The direct measurement object of the hybrid optomechanical system is the displacement of the mechanical resonator caused by the Coriolis force in the rotating coordinate system. When the hybrid optomechanical gyroscope rotates at the angular speed $\dot{\theta}$, the Coriolis force acts on the mechanical resonator along the x-axis. Due to the vibration of the mechanical resonator, the output signal of the system will be proportional to the rotation angular velocity of the platform.

The Hamiltonian of the above-mentioned hybrid optomechanical system can be written as:

$$\begin{aligned} H &= \hbar\Delta_1 c_1^\dagger c_1 + \hbar\Delta_2 c_2^\dagger c_2 + \hbar\Delta_{\mathrm{a}} S_z + \hbar\omega_m b^\dagger b + i\hbar\varepsilon(c_1^\dagger - c_1) \\ &- \hbar g c_1^\dagger c_1(b + b^\dagger) + \hbar g_c(c_1^\dagger c_2 + c_2^\dagger c_1) + \hbar g_a(c_1^\dagger S_- + S_+ c_1) \end{aligned} \tag{1}$$

The first two items describe the light fields in the left and right optical cavity. c_i and ω_i are respectively the annihilation operator and frequency of optical cavity c_i, $\Delta_i = \omega_i - \omega_c$ and $\Delta_a = \omega_a - \omega_c$ are the frequency detuning of the laser field driven by the optical cavity and the atomic ensemble, and ω_c is the frequency of laser. The third and fourth iterms describe the energy of the atomic ensemble and mechanical resonator in the left cavity. b and ω_m are the annihilation operators and natural frequencies of the mechanical resonator. The fifth term describes the energy of the laser field driving the optical cavity system. $\varepsilon = \sqrt{2k_1}\bar{c}_{in}$ is the coefficient of the driving term and $\bar{c}_{in}$ is the amplitude of the driving laser field.

The last three items represent the coupling items of mechanical resonator and cavity, cavity and cavity, atomic ensemble and cavity. The optical force coupling coefficient is $g = g'\sqrt{\hbar/2m\omega_m}$, $g' = \omega_1/L_1$ is the optical force coupling constant, and L_i is the initial length of optical cavity c_i. g_c is the coupling constant of the optical cavity. N is the atomic number of the atomic ensemble. $g_a = \mu\sqrt{\omega_1/2\hbar\varepsilon_0 v}$ is the coupling strength between the cavity field and the atom, μ is the dipole moment of the atomic transition, ε_0 is the dielectric constant, and v is the volume of the cavity. The collective spin operators $S_\pm$, S_z of the atomic ensemble can be described by the Bose production and annihilation operators a^+ and a: $S_+ = \sqrt{N}a^+$, $S_- = \sqrt{N}a^-$, $S_z = a^+a - N/2$. Then the system Hamiltonian is:

$$\begin{aligned} H &= \hbar\Delta_1 c_1^\dagger c_1 + \hbar\Delta_2 c_2^\dagger c_2 + \hbar\omega_m b^\dagger b + \hbar\Delta_a a^+ a \\ &+ i\hbar\varepsilon(c_1^\dagger - c_1) - \frac{1}{2}\hbar\Delta_a N - \hbar g c_1^\dagger c_1(b + b^\dagger) \\ &+ \hbar g_c(c_1^\dagger c_2 + c_2^\dagger c_1) + \hbar\sqrt{N} g_a(c_1^\dagger a + a^+ c_1) \end{aligned} \tag{2}$$

Optomechanical systems are usually open systems, which will interact with the external environmental heat reservoir with multiple degrees of freedom. The effect of the heat reservoir on the system is to modify the motion equation of the system by adding noise terms and dissipation terms. When the optomechanical system rotates at the angular velocity $\dot{\theta}$, the Coriolis effect should be considered in the equation of motion, and the Coriolis force acts on the mechanical resonator along the x-axis. The optomechanical system is driven by the y-axis direction on the movable platform, and the instantaneous speed of the mechanical resonator along the y-axis is $\dot{y}$. After introducing the external noise operator, the quantum Langevin equation describing the system dynamics is:

$$\begin{cases} a = -i\Delta_a a - i\sqrt{N}g_a c_1 - \beta a + \sqrt{2\beta}R \\ \dot{c}_2 = -i\Delta_2 c_2 - k_2 c_2 - i g_c c_1 + \sqrt{2k_2}E_2 \\ \dot{c}_1 = -i\Delta_1 c_1 - i\sqrt{N}g_a a - k_1 c_1 + ig(b+b^\dagger)c_1 + \varepsilon \\ \quad - i g_c c_2 + \sqrt{2k_1}E_1 \\ b = -i\omega_m b + i g c_1^\dagger c_1 - \gamma b + \sqrt{2\gamma}W + \dfrac{2i}{\sqrt{2\hbar m\omega_m}} m\dot{y}\dot{\theta} \end{cases} \tag{3}$$

Where R, E_1, E_2, W are respectively the atomic ensemble, the noise operators of left and right optical cavities and mechanical resonator, β, k$_1$, k$_2$, γ are respectively the atomic ensemble, the attenuation rates of left and right optical cavities and mechanical resonator, and the last term of the fourth formula of formula (3) represents the classical Coriolis force. The correlation function of the noise operator is as follows [3], [15] :

$$\begin{cases} \left\langle E_i^\dagger(t)E_i(t')\right\rangle = 0, \left\langle E_i(t)E_j^\dagger(t')\right\rangle = \delta_{ij}\delta(t-t') \\ \left\langle R^\dagger(t)R(t')\right\rangle = 0, \left\langle R(t)R^\dagger(t')\right\rangle = \delta(t-t') \\ \left\langle W^\dagger(t)W(t')\right\rangle = \bar{n}_{th}\delta(t-t') \\ \left\langle W(t)W^\dagger(t')\right\rangle = (\bar{n}_{th}+1)\delta(t-t') \end{cases} \tag{4}$$

Where $\bar{n}_{th} = 1\big/(e^{\hbar\omega_m/k_B T} - 1)$ is the thermal phonon number, k_B is the Boltzmann constant, and T is the ambient temperature. After a long period of evolution, the system tends to be stable. The mean evolution equation can be obtained by taking the average of equation (3), and the Coriolis force and noise operator are treated as first-order fluctuation terms. The steady-state average value of the system can be obtained from the average value evolution equation:

$$\begin{cases} \bar{c}_1 = \dfrac{\sqrt{2k_1}\bar{c}_{in}}{i\Delta_1' + k_1 + \frac{g_c{}^2}{i\Delta_2 + k_2} + \frac{N g_a^2}{i\Delta_a + \beta}}, \bar{c}_2 = -\dfrac{i g_c \bar{c}_1}{i\Delta_2 + k_2} \\ \bar{b} = \dfrac{i g \bar{c}_1^2}{i\omega_m + \gamma}, \bar{a} = \dfrac{i\sqrt{N}g_a\bar{c}_1}{i\Delta_a + \beta} \end{cases} \tag{5}$$

Where $\Delta_1' = \Delta_1 - g(\bar{b}^\dagger + \bar{b})$. We write the fluctuation operator as the form of the difference between the operator and the steady-state average value: $\delta c_1 = c_1 - \bar{c}_1$, $\delta c_2 = c_2 - \bar{c}_2$, δb=$b - \bar{b}$, $\delta a = a - \bar{a}$, and substitute it into equation (3) for linearization and approximation, while ignoring the high-order fluctuation term to obtain the equation of the fluctuation operator:

$$\begin{cases} \delta\dot{a} = -i\Delta_a\delta a - i\sqrt{N}g_a\delta c_1 - \beta\delta a + \sqrt{2\beta}R \\ \delta\dot{c}_2 = -i\Delta_2\delta c_2 - k_2\delta c_2 - i g_c\delta c_1 + \sqrt{2k_2}E_2 \\ \delta\dot{c}_1 = -i\Delta_1'\delta c_1 - i\sqrt{N}g_a\delta a - k_1\delta c_1 - i g_c\delta c_2 + \sqrt{2k_1}E_1 \\ \quad + i g\bar{c}_1(\delta b + \delta b^\dagger) \\ \delta\dot{b} = -i\omega_m\delta b + ig(\bar{c}_1\delta c_1^\dagger + \bar{c}_1^\dagger\delta c_1) - \gamma\delta b + \sqrt{2\gamma}W \\ \quad + \dfrac{2i}{\sqrt{2\hbar m\omega_m}} m\dot{y}\dot{\theta} \end{cases} \tag{6}$$

According to equation (6), we can get the effective linear Hamiltonian equation of the hybrid optomechanical system:

$$\begin{aligned} H' =& \hbar\Delta_1'\delta c_1^\dagger\delta c_1 + \hbar\Delta_2\delta c_2^\dagger\delta c_2 + \hbar\Delta_a\delta a^+\delta a \\ &+ \hbar\omega_m\delta b^\dagger\delta b - \hbar g(\delta c_1^\dagger + \delta c_1)(\delta b + \delta b^\dagger) \\ &+ \hbar g_c(\delta c_1^\dagger\delta c_2 + \delta c_2^\dagger\delta c_1) + \hbar\sqrt{N}g_a(\delta c_1^\dagger\delta a + \delta a^+\delta c_1) \end{aligned} \tag{7}$$

We choose the Fourier transform method to solve equations (6), according to the input-output relationship [11] $c_{out} = \sqrt{2k}c - c_{in}$, and adjusting the laser frequency to effectively detune $\Delta_1^{'} = 0$, we can obtain the frequency domain solution of the output field of the hybrid optomechanical system:

$$\bar{c}_{out} = \sqrt{2k_1}\bar{c}_1 - i\bar{l}_0\big/\sqrt{2} \tag{8}$$

$$\begin{aligned} &\delta c_{out}(\omega) = \Phi[(\Gamma(\omega)2k_1 - \frac{1}{\Phi})E_1(\omega) + \varphi_3^*(-\omega)2k_1E_1^\dagger(-\omega) \\ &+ \Gamma(\omega)\varphi_2^*(-\omega)2\sqrt{k_1k_2}E_2(\omega) + \varphi_3^*(-\omega)\varphi_2(\omega)2\sqrt{k_1k_2}E_2^\dagger(-\omega) \\ &+ \Gamma(\omega)\varphi_1^*(-\omega)2\sqrt{k_1\beta}R(\omega) + \varphi_3^*(-\omega)\varphi_1(\omega)2\sqrt{k_1\beta}R^\dagger(-\omega) \\ &+ \Gamma(\omega)\varphi_4^*(-\omega)2\sqrt{k_1\gamma}W(\omega) + \varphi_3^*(-\omega)\varphi_4(\omega)2\sqrt{k_1\gamma}W^\dagger(-\omega) \\ &- \sqrt{\frac{k_1}{\hbar m\omega_m}} g\bar{c}_1\eta_5(\omega)(\frac{1}{\eta_4^*(-\omega)} - \frac{1}{\eta_4(\omega)})F(2m\dot{y}\dot{\theta})] \end{aligned} \tag{9}$$

Where,

$$\begin{cases} \Phi = \dfrac{1}{(|\varphi_3(\omega)|^2 + \Gamma(\omega)\Gamma^*(-\omega)}, \varphi_1(\omega) = \dfrac{i g_a\sqrt{N}}{\eta_1(\omega)}, \varphi_2(\omega) = \dfrac{i g_c}{\eta_2(\omega)} \\ \varphi_3(\omega) = g^2|\bar{c}_1|^2(\dfrac{1}{\eta_4^\dagger(-\omega)} - \dfrac{1}{\eta_4(\omega)}), \eta_1(\omega) = -i\omega - i\Delta_a + \beta \\ \varphi_4(\omega) = -ig\bar{c}_1(\dfrac{1}{\eta_4^\dagger(-\omega)} + \dfrac{1}{\eta_4(\omega)}), \eta_2(\omega) = -i\omega - i\Delta_2 + k_2 \\ \eta_3(\omega) = -i\omega - i\Delta_1' + k_1, \eta_4(\omega) = -i\omega - i\omega_m + \gamma \\ \eta_5(\omega) = \eta_3(\omega) + \dfrac{g_a^2N}{\eta_1(\omega)} + \dfrac{g_c^2}{\eta_2(\omega)}, \Gamma(\omega) = \eta_5(\omega) - \varphi_3(-\omega) \end{cases}$$

$\bar{c}_{out}$ and $\delta c_{out}(\omega)$ are respectively the steady-state average value of the output operator of the system and the fluctuation operator in the frequency domain, $\bar{l}_0$ is the intensity of the laser field.$F(2m\dot{y}\dot{\theta}) = \sqrt{2\pi}m\dot{y}_0\dot{\theta}[\delta(\omega-\omega_m) + \delta(\omega+\omega_m)]$ is the Coriolis force after Fourier transform. We use the method of zero difference measurement to measure the output of the hybrid optomechanical system. Add a 90 degree phase to the light field output by the system, and the reference light field is l_2, then the difference between the detector D_1 and D_2 the measured light field intensity is:

$$I = I_1 - I_2 = l_2^\dagger c_{out} + c_{out}^\dagger l_2 \tag{10}$$

Where I_1=$d_1^+d_1$ and I_2=$d_2^+d_2$ are the light field intensities measured by detector D_1 and D_2 respectively. Linearize the operator in the above equation: δl_2=$l_2 - \bar{l}_2$, δc_{out}=$c_{out} - \bar{c}_{out}$, then equation (10) can be changed into:

$$\begin{aligned} I =& \bar{l}_2^*\bar{c}_{out} + \bar{c}_{out}^*\bar{l}_2 + \delta l_2^\dagger\bar{c}_{out} + \bar{l}_2^*\delta c_{out} \\ &+ \delta l_2^\dagger\delta c_{out} + \delta c_{out}^\dagger\bar{l}_2 + \bar{c}_{out}^*\delta l_2 + \delta c_{out}^\dagger\delta l_2 \end{aligned} \tag{11}$$

Substitute the reference light field $\bar{l}_2 = -\bar{l}_0\big/\sqrt{2}$ into the above formula, and ignore the high-order fluctuation term. At this time, the measured signal in the frequency domain can be written as:

$$\bar{I}(\omega) = \sqrt{\frac{m\pi k_1}{\hbar\omega_m}}\bar{l}_0\dot{y}_0\dot{\theta}\Phi_1(\omega)g[\delta(\omega-\omega_m)+\delta(\omega+\omega_m)] - \bar{l}_0\sqrt{2\pi k_1}(\bar{c}_1+\bar{c}_1^*)\delta(\omega) \quad (12)$$

Where, $\Phi_1(\omega) = \Phi(\omega)(\eta_5^*(-\omega)\bar{c}_1^* - \eta_5(\omega)\bar{c}_1)(1/\eta_2^*(-\omega) - 1/\eta_2(\omega)\ )$. By performing inverse Fourier transform on equation (12), the solution of the signal in the time domain can be obtained:

$$\bar{I}(t) = \sqrt{\frac{mk_1}{2\hbar\omega_m}}\bar{l}_0\dot{\theta}\dot{y}_0\Phi_2 g\cos(\omega_m t+\phi) - \bar{l}_0\sqrt{k_1}(\bar{c}_1+\bar{c}_1^*) \quad (13)$$

Where, Phase ϕ meets:
$\tan(\phi) = (\Phi_1(-\omega_m) - \Phi_1(\omega_m))/(\Phi_1(\omega_m)+\Phi_1(-\omega_m))$
$\Phi_2 = \sqrt{(\Phi_1(\omega_m)+\Phi_1(-\omega_m))^2 + (\Phi_1(\omega_m)-\Phi_1(-\omega_m))^2}$.
we can find that signal $\bar{I}(\omega)$ is related to the angular velocity $\dot{\theta}$ of platform rotation. We can use the amplitude of system output signal to further deduce the angular velocity of system rotation. The noise part of the hybrid optomechanical gyroscope model is $\delta I(\omega) = I(\omega) - \bar{I}(\omega)$, then the quantum noise power spectrum is:

$$\begin{aligned} S(\omega) &= \int_{-\infty}^{\infty} \langle \delta I(t)\delta I(0)\rangle e^{i\omega t} dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} \iint d\omega' d\omega'' \delta I(\omega')\delta I(\omega'') e^{-i\omega' t} dt \\ &= S_1(\omega)+S_2(\omega)+S_3(\omega)+S_4(\omega) \end{aligned} \quad (14)$$

Where,$S_1(\omega) = \bar{l}_0^2/2\ (\Phi\eta_5 2k_1 - 1)(\Phi\eta_5^* 2k_1 - 1)$ represents the shot noise of optical cavity c_1 and the reaction noise to the mechanical resonator . $S_2(\omega) = 2\bar{l}_0^2 k_1 k_2 g_c^2 \Phi^2 |\eta_5|^2/|\eta_2|^2$ comes from the auxiliary optical cavity. The noise item $S_3(\omega) = 2\beta\bar{l}_0^2 k_1 g_a^2 N\Phi^2|\eta_5|^2/|\eta_1|^2$ comes from the atomic ensemble. $S_4(\omega) = 2k_1\gamma\bar{l}_0^2 g^2|\bar{c}_1|^2\Phi^2|\eta_5|^2(1/\eta_4\ + 1/\eta_4^*\ )^2(2n+1)$ represents thermal noise in mechanical mode.

The phonon number n of the mechanical resonator is proportional to the thermal noise $S_4(\omega)$. If we adopt a mechanical resonator ground-state cooling strategy to reduce the mechanical resonator phonon number, and use a hybrid optomechanical system to cool the mechanical resonator to phonon number less than 1, we can effectively suppress the thermal noise of the quantum gyroscope.

III. Ground-state cooling

In this section, we use the method in [11], [16] to obtain the light pressure fluctuation spectrum and the mean phonon number based on the effective Hamiltonian of the hybrid optomechanical system, and analyze the ground-state cooling of the mechanical resonator. The radiation pressure on the mechanical resonator under the optical cavity is obtained by the interaction term between the optical cavity and the mechanical resonator in the effective Hamiltonian:F=$c_1 + c_1^\dagger$ (For simplification, all δ in the effective Hamiltonian is omitted below). At $g \ll \omega_m$, we can ignore the reaction of mechanical resonator to optical cavity [11], effective Hamiltonian H''=$\hbar\Delta_1' c_1^\dagger c_1 + \hbar\Delta_2 c_2^\dagger c_2 + \hbar\Delta_a a^\dagger a + \hbar \mathrm{g_c}(c_1^\dagger c_2 + c_2^\dagger c_1) + \sqrt{N}g_a(c_1^\dagger a + a^\dagger c_1)$, and only consider the optical cavity and atomic ensemble in the effective Hamiltonian quantity, then the radiation pressure fluctuation spectrum of the mechanical resonator [3](the light pressure fluctuation spectrum):

$$\begin{aligned} S_{FF}(\omega) &= \int_{-\infty}^{\infty} e^{i\omega t}\langle F(t)F(0)\rangle dt \\ &= \frac{1}{|Y(\omega)|^2}(2k_1 + \frac{2g_c^2 k_2}{|-\mathrm{i}\omega + \mathrm{i}\Delta_2 + k_2|^2} + \frac{2\beta N g_a^2}{|-\mathrm{i}\omega + \mathrm{i}\Delta_a + \beta|^2}) \end{aligned} \quad (15)$$

Among them,
$Y(\omega)$= $-\mathrm{i}\omega + \mathrm{i}\Delta_1' + k_1 + \frac{Ng_a^2}{-\mathrm{i}\omega+\mathrm{i}\Delta_a+\beta} + \frac{g_c^2}{-\mathrm{i}\omega+\mathrm{i}\Delta_2+k_2}$

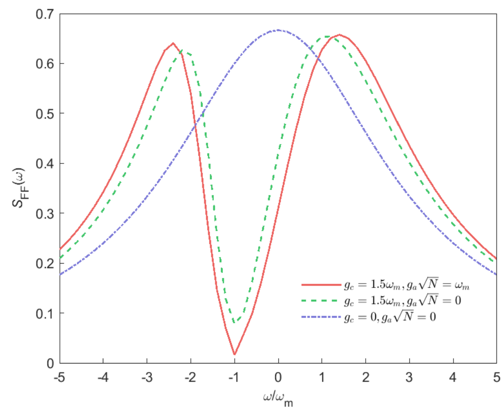

Fig. 2: Radiation pressure fluctuation spectra $S_{FF}(\omega)$ under different coupling coefficients g_c and $g_a\sqrt{N}$. Effective mistuning of optical cavity and atomic ensemble: $\Delta_1' = 0$, $\Delta_2 = -\omega_m$, $\Delta_a = -\omega_m$, decay rate of optical cavity and atomic ensemble: $k_1 = 3\omega_m$, $k_2 = 0.1\omega_m$, $\beta = 0.01\omega_m$

Figure 2 compares the radiation pressure fluctuation spectra $S_{FF}(\omega)$ of the three models. When the coupling coefficient $g_c = 0$ between optical cavities and the coupling coefficient $g_a\sqrt{N} = 0$ between atomic ensemble and optical cavities, the hybrid optomechanical model degenerates to the standard optomechanical system form [4], that is, there is no auxiliary cavity and atomic ensemble, and the fluctuation spectrum is a single peak Lorentz spectrum. After introducing atomic ensemble or auxiliary optical cavity into the optomechanical system, the fluctuation spectrum is split from a single Lorentz peak into two narrower peaks and a lower valley. When $g_c = 1.5\omega_m$, $g_a\sqrt{N} = 0$, the hybrid optomechanical model degenerates into a dual-cavity model [12] consisting of a standard optomechanical system and an auxiliary optical cavity, that is, the atomic ensemble of the left optical cavity does not exist. When $g_c = 1.5\omega_m$, $g_a\sqrt{N} = \omega_m$, that is, the hybrid optomechanical system model, it can be seen that the fluctuation spectrum of the hybrid optomechanical system has a lower valley value than the other two models.

During the evolution of the system, the radiation pressure F will cause the mechanical resonator phonon state to transition between energy levels. According to Fermi's golden rule [3], the transition rate of the phonon of the mechanical resonator between different phonon states is related to the effective rise and fall force of the cavity light field on the phonon. The transition rates of phonon states $|n\rangle$ to $|n-1\rangle$ and $|n+1\rangle$ are

$$\begin{cases}\Gamma_{n\to n-1} = g^2 n S_{FF}(\omega_m) \\ \Gamma_{n\to n+1} = g^2 (n+1) S_{FF}(-\omega_m)\end{cases} \tag{16}$$

Where $S_{FF}(\omega_m)$ and $S_{FF}(-\omega_m)$ are the values of $\omega = \omega_m$ on the positive frequency side and $\omega = -\omega_m$ on the negative frequency side of the fluctuation spectrum, respectively. The rate equation usually describes the dynamic equation of an open system, so we need to take into account the external thermal reservoir environment in addition to the force of the optical field in the cavity on the mechanical resonator. The phonon state of the mechanical resonator only transits between adjacent states [17]. We can write the complete form of the rate equation of the mechanical resonator:

$$\begin{aligned}&\dot{P}_n = \Gamma_{n-1\to n} P_{n-1} + \Gamma_{n+1\to n} P_{n+1} - \Gamma_{n\to n-1} P_n \\ &- \Gamma_{n\to \mathrm{n}+1} P_n + \gamma \bar{n}_{th} n P_{n-1} + \gamma(\bar{n}_{th}+1)(n+1) P_{n+1} \\ &- \gamma(\bar{n}_{th}+1) n P_n - \gamma \bar{n}_{th}(n+1) P_n\end{aligned} \tag{17}$$

Where P_n is the probability that the mechanical resonator is in the phonon number eigenstate $|n\rangle$, $\Gamma_{n'\to n}$ is the transition rate from $|n'\rangle$ state to $|n\rangle$ state, $\bar{n}_{th}$ is the thermal average phonon number, and γ is the dissipation coefficient of the mechanical resonator. The above formula describes the increment of probability of mechanical resonator in phonon state $|n\rangle$. The first four terms and the last four terms are rate equations under the influence of radiation pressure and heat reservoir respectively. Formula (17) can be simplified and rewritten as follows:

$$\begin{aligned}\dot{P}_n =& \gamma_f n_f n P_{n-1} + \gamma_f (n_f+1)(n+1) P_{n+1} \\ &- \gamma_f (n_f+1) n P_n - \gamma_f n_f (n+1) P_n\end{aligned} \tag{18}$$

In the above formula, $\gamma_f = \gamma + \gamma_c$ is the rate at which the average number of phonons tends to the steady-state value, and n_f is the mean phonon number of the mechanical resonator.

$$n_f = \frac{\gamma \bar{n}_{th} + \gamma_c n_c}{\gamma + \gamma_c} \tag{19}$$

Where, n_c represents the cooling limit and γ_c represents the cooling rate.

$$n_c = \frac{S_{FF}(-\omega_m)}{S_{FF}(\omega_m) - S_{FF}(-\omega_m)} \tag{20}$$

$$\gamma_c = g^2 [S_{FF}(\omega_m) - S_{FF}(-\omega_m)] \tag{21}$$

From equation (19), we can see that part of the cooling process of the mechanical resonator is due to its own attenuation, and the other part is to transfer energy to the optical cavity through coupling to reduce the number of phonons. Our cooling scheme utilizes two quantum interference effects, atom-cavity coupling and cavity-cavity coupling, to increase the cooling rate to ground-state cooling. In the ground-state cooling theory, $\gamma_c \gg \gamma$ is usually required. At this time, the mean phonon number of the mechanical resonator can be written as:

$$n_f = \frac{\gamma \bar{n}_{th} + \gamma_c n_c}{\gamma + \gamma_c} \approx \frac{\gamma \bar{n}_{th}}{\gamma_c} + n_c \tag{22}$$

When the decay rate of the mechanical resonator is $\gamma \to 0$, the mean phonon number tends to a non-zero cooling limit n_c. In order to make the mechanical resonator reach ground-state cooling ($n_f < 1$), we need to make the cooling rate γ_c as large as possible and the cooling limit n_c as small as possible. It can be seen from Formula (20) that the larger the positive frequency side value $S_{FF}(\omega_m)$ of the fluctuation spectrum, and the smaller the negative frequency side value $S_{FF}(-\omega_m)$, the smaller the mean phonon number of the mechanical resonator.

Combined with equation (16), we find that the positive frequency side value is related to the downward transition of phonon state and determines the cooling process; The negative frequency side value is related to the upward transition of phonon state, which determines the heating process. Therefore, we need to use the radiation pressure fluctuation spectrum to enhance its positive frequency side value $S_{FF}(\omega_m)$, suppress the negative frequency side value $S_{FF}(-\omega_m)$, so as to make the mechanical resonator cool down in the ground state and reduce the thermal noise of the system. It can be seen from Figure 2 that the difference between the positive frequency side value $S_{FF}(\omega_m)$ and the dual cavity model is not large for the hybrid optomechanical system, while the negative frequency side value $S_{FF}(-\omega_m)$ at the trough of the fluctuation spectrum is lower, so the cooling effect is better, and the mechanical resonator is easier to cool to the ground state. In order to verify the ground-state cooling effect of our scheme, we take a set of experimentally feasible parameters [18]: mechanical resonator frequency $\omega_m = 2\pi \times 20\mathrm{MHZ}$, thermal reservoir temperature $T = 300\mathrm{mk}$, thermal average phonon number $\bar{n}_{th} = 312$, mechanical resonator quality factor $Q = \omega_m/\gamma = 8 \times 10^4$.

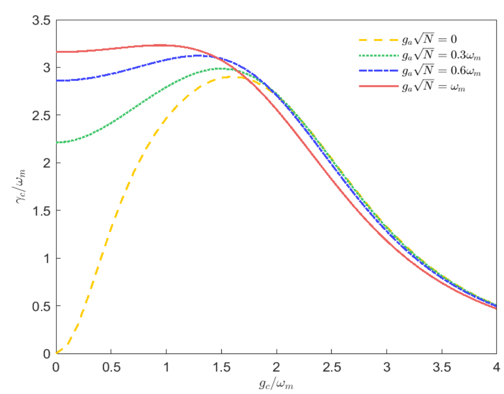

Fig. 3: The cooling rate γ_c at different coupling coefficients $g_a\sqrt{N}$. Effective mistuning of optical cavity and atomic ensemble: $\Delta_1^{'} = 0$, $\Delta_2 = -\omega_m$, $\Delta_a = -\omega_m$, decay rate of optical cavity and atomic ensemble: $k_1 = 3\omega_m$, $k_2 = 0.1\omega_m$, $\beta = 0.01\omega_m$

The yellow dotted line in Figure 3 is consistent with the blue dotted line in Figure 2. When the coupling coefficient $g_c = 0$, $g_a\sqrt{N} = 0$, the positive frequency side $S_{FF}(\omega_m)$ and the negative frequency side $S_{FF}(-\omega_m)$ are equal, that is, the cooling rate γ_c of the original model is 0, and the optomechanical system in the original model has no cooling effect. $g_c \neq 0$, the yellow dotted line in Figure 3 describes the change of the cooling rate of the dual-cavity model with the coupling strength g_c between optical cavities. The other three lines describe the change of the cooling rate with the coupling strength $g_a\sqrt{N}$ between optical cavities when the atomic ensemble and optical cavity coupling coefficient g_c are different. The larger the optical cavity coupling coefficient $g_a\sqrt{N}$, the greater the maximum cooling rate. Through comparison, it can be seen that the cooling rate of the hybrid optomechanical system model is higher than that of the dual-cavity system model and the original model, and the thermal noise suppression effect is better.

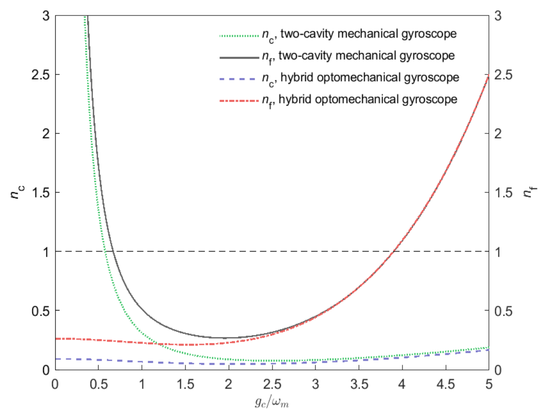

Fig. 4: The variation of cooling limit n_c and the mean phonon number of mechanical resonator n_f with coupling strength of optical cavity g_c. Effective mistuning of optical cavity and atomic ensemble: $\Delta_1^{'}=0$, $\Delta_2=-\omega_m$, $\Delta_a=-\omega_m$, decay rate of optical cavity and atomic ensemble: $k_1=3\omega_m$, $k_2=0.1\omega_m$, $\beta=0.01\omega_m$

Figure 4 describes the change of the cooling rate and cooling limit of the dual-cavity system model and the hybrid optomechanical system model with the coupling strength g_c between optical cavities. It can be seen from Figure 4 that the optomechanical system with the introduction of atomic ensemble can reduce the mean phonon number to below 1 under appropriate parameters, that is, achieve ground-state cooling. The cooling limit and the mean phonon number of the hybrid optomechanical system model are smaller than those of the dual-cavity system model, and the ground-state cooling of the mechanical resonator occurs in a larger parameter range of the optical cavity coupling strength g_c.

In hybrid optomechanical gyroscope model, atomic ensemble and auxiliary optical cavity are partially introduced into the hybrid optomechanical system, so that the valley value $S_{FF}(-\omega_m)$ of fluctuation spectrum is lower, and higher cooling rate γ_c and lower mean phonon number n_f are achieved. When proper parameters are adopted, the mechanical resonator can achieve ground-state cooling, thus reducing the thermal noise of the gyroscope model.

IV. Conclusion

A hybrid optomechanical gyroscope model is established, and a ground-state cooling strategy of mechanical resonator for suppressing the thermal noise of optomechanical gyroscope is proposed. the mean phonon number of the mechanical resonator is cooled to less than 1 by using a hybrid optomechanical system. Through the combination of atom cavity coupling and cavity cavity coupling quantum interference effects, the peak and valley values of the radiation pressure fluctuation spectrum of the hybrid optomechanical system are affected. The peak value is used to enhance the cooling process and the valley value is used to suppress the heating process. Compared with the original model and the dual-cavity model, the scheme of this paper can effectively enhance the cooling rate, reduce the mean phonon number of the mechanical resonator, and make it easier to achieve the ground-state cooling of the mechanical resonator, thus suppressing the thermal noise of the gyroscope. The research in this paper is helpful to design a more sensitive gyroscope.

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