Partial Response Equalizer Design for Underwater OWC Systems Using Time-Reversal Waveforms

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***Abstract*—This paper proposes a partial response equalizer using the deterministic component of the channel response in the UOWC systems with time-reversal waveforms. Simulations illustrate the proposed equalizer is effective, albeit with an acceptable BER degradation.**

***Index Terms*—Time-reversal, Equalizer, Partial response, Underwater optical wireless communication, Channel model**

I. Introduction

Recently, there has been a growing interest in underwater optical wireless communications (UOWC) due to its potential for high-rate data transmission in both military and civilian applications, such as underwater navigation, sensor networks, ocean resource exploration, and oceanographic studies [1–3].

However, the scattering nature of the ocean water poses significant challenges to UOWC as it results in severe attenuation, multiple paths, and turbulence effects. The absorption and scattering of photons in the ocean water lead to severe intersymbol interference (ISI).

Time-reversal (TR) waveform design is a power-efficient and effective technique for mitigating the ISI in dispersive channels, which has been applied in underwater acoustic communication and radio frequency (RF) systems [4–7]. Sending the time-reversed channel response as the wavefrom can reduce the ISI at the receiver by utilizing the channel diversity and the auto-correlation characteristic of the channel. In UOWC systems, the scattered photons result in numerous independent and random multiple paths, making the TR waveform design naturally suitable for underwater optical transmissions. However, since only non-negative signals can be sent in the UOWC system, some residual ISI need to be further reduced even the TR waveform is transmitted. Designing a post-equalizer using the full channel response is difficult since the optical channels contains random and time-varying multiple paths, which is not easy to be estimated.

In this paper, we propose a partial response equalizer design method for UOWC systems when TR waveforms are used. Instead of using the full channel response for the equalizer design, the proposed technique only use the deterministic component of the channel, which is easier to be pre-estimated. From the simulation results, the partial response equalizer design only sacrifices the SNR less than 2 dB comparing with using the full channel response for the equalizer design when a similar BER is achieved.

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II. UOWC System Model using TR waveform

A block diagram of the UOWC system using the TR waveform for point-to-point transmission is depicted in Fig. 1. The transmitted signal $x[k] \in \{\pm\frac{M-1}{M},\ \pm\frac{M-3}{M},\ \cdots,\ \pm\frac{1}{M}\}$ is obtained by modulating a binary information signal with M-ary pulse magnitude modulation (M-PAM), where M represents the modulation constellation size. The modulated signal $x[k]$ is assumed to have a uniform probability distribution since the binary information is randomly distributed.

To mitigate ISI, the time-reversed channel impulse response (CIR) is used as the transmitted waveform, and the actual CIR can be pre-estimated. The TR waveform is obtained by

$$g(t) = \frac{h(T-t)}{\int_0^T h^2(t)dt}, \tag{1}$$

where $h(t)$ represents the CIR, and T is the duration of the channel. $g(t)$ is actually the normalized time-reversal $h(t)$. The transmitted signal $s(t)$ is modeled as a sum of the product of the modulated signal and the TR waveform as

$$s(t) = \sum_{k=0}^{\infty} x[k]g(t-kT_s), \tag{2}$$

where T_s is the transmitted symbol period assumed to be the same as the system sampling period. For ease of notation, the received signals and system performance are analyzed in the discrete-time domain. The kth sample of the received signal is given by

$$\begin{aligned} r[k] &= (s \otimes h)[k] + n[k] \\ &= \rho(x \otimes g \otimes h)[k] + n[k], \end{aligned} \tag{3}$$

where $\otimes$ is the time convolution operation. The responsivity of the light source, denoted by ρ, is assumed to be unity for simplicity. $n[k]$ is the kth sample of the added noise at the receiver, which is a combination of shot and thermal noises. Usually, we model $n[k]$ as an additive Gaussian noise with zero-mean and variance σ_n^2.

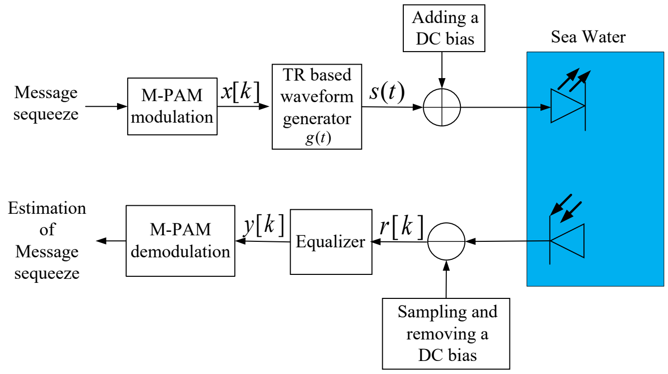


Fig. 1. The schematic diagram of TR based UOWC system with equalizer.

Finally, the received signals are equalized and demodulated to recover the binary data. The sampling period is assumed to be T_s, which is equivalent to applying a matched filter and sampling the data at the peak values of the correlation signals at the receiver.

III. Partial Response Equalizer Design

In Sec. II, we derive the received signal model of the UOWC system with the TR waveform, which suggests that the transmitting signal $x[k]$ undergoes the equivalent channel $(g \otimes h)[k]$, i.e., similar to the auto-correlation of channel $h[k]$. Although transmitting the TR waveforms can eliminate partial ISI caused by the random scattering, the non-negative channel and imperfect auto-correlation still result in residual ISI. In this section, we analyze the channel of UOWC systems and derive the principles of the proposed partial response equalizer design.

Considering the complex propagation of photons in an underwater environment, the random scattering of photons causes variations in propagation delay and transmission path, leading to a time-varying and dispersive channel response. As the IM/DD scheme is used in UOWC, the CIR is real-valued and non-negative. Thus, we model the discrete-time version of the CIR as

$$h[k] = h_\ell[k] + c[k], \quad k = 0, 1, \cdots, L-1, \tag{4}$$

where $h[k]$ represents the kth sample of the CIR. Since $h[k]$ is greater than or equal to zero, $c[k]$ is the bias that is the deterministic component of the channel. In (4), the truncated length of the channel is denoted by L. Due to the random scattering of the photons, $h_\ell[k]$ is modeled as a Gaussian distributed random process with zero mean as

$$h_\ell[k] \sim \mathcal{N}(0, \sigma_k^2). \tag{5}$$

σ_k^2 denotes the variance of $h_\ell[k]$. In (4), since the variance of $h[k]$ decays, the bias term $c[k]$ can be modeled as an exponential function, which represents the optical power attenuation over the time, as shown below

$$c[k] = a_0 e^{-b_0 k T_s}, \quad k = 0,\ 1, \cdots, L-1, \tag{6}$$

where a_0 and b_0 depend on the transmission distance between the light source and the receiver as well as the quality of the seawater. The values of a_0 and b_0 can be obtained by fitting the experimental CIR data. In Sec. IV, we present Monte-Carlo simulations of the CIR that is consistent with our proposed channel model.

Based on the previous analysis presented, it is observed that the equivalent channel consists of two parts: the first part comes from the auto-correlation of $c[k]$, which is the main source of ISIs; the second part comes from the auto-correlation of the random part $h_\ell[k]$, which leads to the some time-varying interferences. Thus, a post-equalizer is necessary for UOWC systems with TR waveforms. However, designing an equalizer using the full equivalent channel is challenging since the random part, $h_\ell[k]$, is time-varying, which is not easy to be estimated.

In this paper, we propose a partial response equalizer design technique, which only employs a partial information of the channel response instead of using the full channel. Since $c[k]$ in (4) is easier estimated than $h_\ell[k]$, we use $c[k]$ only for the equalizer design. Thus, the equivalent partial response channel can be described as

$$h_p[k] = \epsilon[k] + \sigma^2 \delta(k - \frac{2L-1}{2}), \quad k = 0, 1, \cdots, 2L-1, \tag{7}$$

where $\sigma^2 = \sum_{k=1}^{L} \sigma_k^2$, and $\epsilon[k]$ represents the auto-correlation of $c[k]$, which is calculated as

$$\begin{aligned} \epsilon[k] &= \overleftarrow{c} \otimes c \\ &= a_0^2 e^{-b_0 k T_s} \otimes e^{-b_0 (L-k) T_s}, \end{aligned} \tag{8}$$

where $\overleftarrow{(\cdot)}$ denotes the time-reversal operation. In this paper, we employ the Zero-Forcing (ZF) design method as an example by using the partial response of the channel. Other equalizer design methods, such as minimum mean squared error (MMSE) can also be used. The ZF equalizer with $2N+1$ taps can be designed as

$$\boldsymbol{e} = \mathbf{H}_p^{\dagger} \mathbf{d}, \tag{9}$$

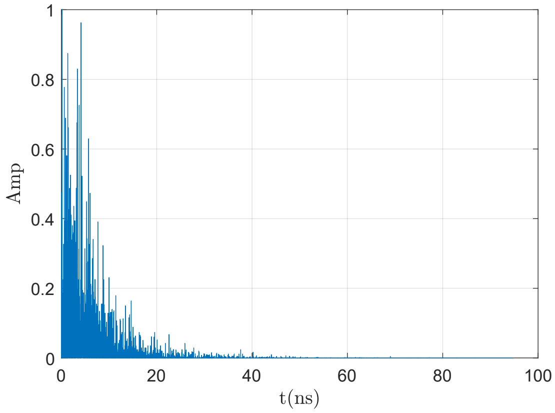
where $(\cdot)^\dagger$ is pseudo-inverse operation. $\mathbf{H}_p$ is a symmetric matrix of the element from (7), shown as

$$\mathbf{H}_p = \begin{bmatrix} \epsilon[k_0] + \sigma^2 & \epsilon[k_0+1] & \cdots & \epsilon[k_0+2N] \\ \epsilon[k_0+1] & \epsilon[k_0] + \sigma^2 & \cdots & \epsilon[k_0+2N-1] \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon[k_0+2N] & \epsilon[k_0+2N-1] & \cdots & \epsilon[k_0] + \sigma^2 \end{bmatrix}, \tag{10}$$

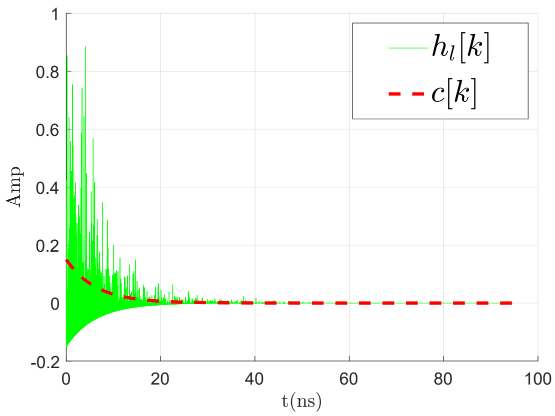
In (10), $k_0 = (2L-1)/2$ and

$$\mathbf{d} = [0 \ \cdots \ 0 \quad 1 \quad 0 \ \cdots \ 0]^T, \tag{11}$$

which is a $(2N+1)$ vector, where $(\cdot)^T$ represents the transpose operation.

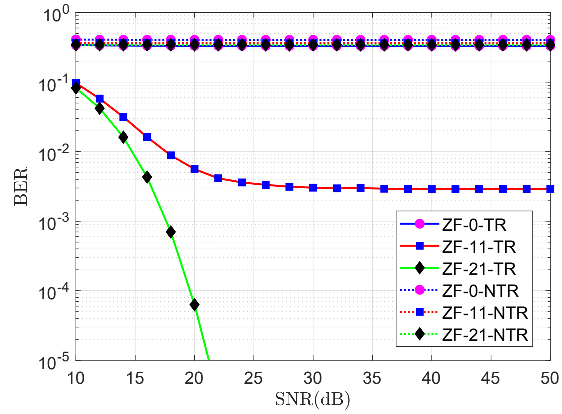


(a)

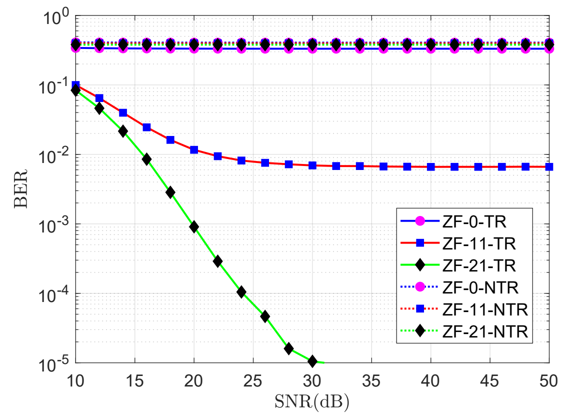


(b)

Fig. 2. Underwater optical CIR and its equivalent channel. (a) is the original CIR, and (b) is the decomposition of (a) as shown in (4).



(a)



(b)

Fig. 3. BER performance comparison under 4-PAM modulation with ZF equalizer, bit rate is 4 Gbps. (a) Equalizer aiming for the overall channel, (b) Partial response equalizer.

IV. Simulation Results

In this section, the simulation results are presented. We use Monte-Carlo photon tracing algorithm to obtain the UOWC channel impulse response. In this paper, a 15 m propagation distance in the harbor seawater environment is assumed. The beam width of the light source is assumed to be $0.75 \times 10^{-6}\pi$, and the laser diode is used. The bandwidth of the overall channel is not limited by the light source and the photodetector. Other parameters, such as, absorption and scattering coefficient can be found in [8].

Fig. 2 (a) illustrates the normalized channel impulse responses using the parameters discussed. In this result, the response contains time-varying random multiple paths. Using the model shown in (4), the relatively fixed component,$c[k]$, which is used for partial response equalizer design can be estimated, as shown in Fig. 2 (b).

The comparison of the BER performances using the equalizer with the whole channel response and the partial response equalizer are illustrated in Fig. 3. The cases with and without an equalizer are also tested. In this work, a ZF equalizer with different numbers of taps is employed and denoted as "ZF-K" in the figure's legend, where K is the number of taps used for the ZF equalizer design. Particularly, "ZF-0" is the case without using ZF. In addition, "TR" and "NTR" denoted in the figure respectively represent the system using and without using the TR waveforms. Fig. 3 (a) presents the BER performance with an equalizer for the entire channel, Fig. 3 (b) shows the BER with the proposed partial response equalizer.

From the figure, the ISI is dominated, and the system cannot work properly. Comparing the results in Figs. 3 (a) and (b), using equalizer with a larger number of taps improves the BER performances for both using the full channel and the partial channel response. When the number of taps for the ZF equalizer is 11, the BER improves firstly as the SNR increases. Then, the performance does not improve for a further increasing of the SNR due to the ISI. The performances of using the full channel and the partial channel response are similar. When a larger number of taps is used for the equalizer, the performance of using the partial channel response is worse than that using the full channel response. To achieve a similar BER, using the partial channel response only has a 2 dB penalty for the SNR, which is acceptable.

In summary, the results in Figs. 3 (a) and (b) demonstrate the BER performances using the full channel equalizer and the partial response equalizer, respectively. From the results, the proposed partial response equalizer is effective, albeit exhibiting an acceptable degradation in BER performance.

V. Conclusion

Due to the rich-scattering channel for underwater optical wireless communication systems, using TR waveforms cannot fully eliminate the ISI. To reduce the residual ISI, we propose a partial response equalizer design method instead of using the full channel response. After analysing the properties of the underwater optical channel response, a new channel model consists of the random and the deterministic components is derived. We only use the deterministic channel component for the equalizer design. From the simulation results shown in this paper, the proposed partial response equalizer design method is effective, albeit exhibiting an acceptable degradation in BER performance.

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