Purification-enabled Routing with Guaranteed Fidelity in Entanglement Distribution Networks

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***Abstract*—A purification-enabled routing algorithm was proposed to support remote entangled pairs sharing with limited quantum resources, upon satisfying the fidelity threshold for generated remote entanglements, and the proposed algorithm is further verified via simulation.**

Keywords—purification, fidelity, remote entanglement

I. Introduction

Entanglement-based network is a promising platform that can support ground-breaking quantum applications, including unconditional secure communication, distributed quantum computing, and quantum sensing. Most applications of quantum networks require sharing long-distance entangled particles between two remote end nodes. However, the success rate of establishing an entangled pair between two distant end nodes decays exponentially with the physical distance because of optical fiber lossy [1]. Fortunately, quantum repeater can serve as an intermediate "station" to be deployed, and quantum repeater capable of arbitrary quantum gates, with an optical interface for quantum bits (Qubits) storage, measurement and entanglement swapping. Accordingly, entanglement swapping [2] can be performed at each quantum repeater to facilitate end-to-end entanglement distribution, and qubits can be sent optically through standard optical fiber between source-destination nodes. Nevertheless, in this process, decoherence decays entanglement fidelity, and quantum repeaters sometimes might not generate entangled pairs with a certain desired fidelity. To overcome this obstacle, entanglement purification is an efficient technology to increase the fidelity of entangled pairs, at the expense of a reduced number of entanglement resources. By adopting entanglement purification, the fidelity of end-to-end entanglement distribution can be significantly affected. Simultaneously, considering the pre-established entanglement [3], i.e., the entanglement created in advance and accordingly stored in quantum memory for a limited time, to build long-distance entanglement, it's necessary to allocate the resources within a limited lifetime rather than waste them. Hence, due to the limited coherence time and limited memory cells on quantum memory, we propose a purification-enabled routing algorithm to effectively allocate limited quantum resources, upon satisfying the fidelity threshold for generated remote entanglements. Simulation results show that the proposed algorithm provides remote entanglement establishment and satisfy the fidelity threshold for the cases of different network scales and memory cells sizes.

II. Network Model

A general quantum network is described by a graph $G(V,E,B)$, where V is the set of quantum nodes (Q-Nodes) with V=$\{v_i\}_{i=1}^{N}$, E is the set of quantum channels (QChs) and classical channels (CChs) with $E = \{\ e_{i,j} \cup e_{EPS_i,i}, v_i, v_j \in V, b_{EPS_i} \in B\,\}$, B is the set of entangled pair sources (EPSs) with $B = \{b_{EPS_i}\}_{i=1}^{\mathfrak{N}}$. Note that quantum channels and classical channels are both optical fibers in physical deployment, and the EPS is responsible for producing entanglement between two nodes that share a direct physical connection. Each Q-Node in V holds the complete function of a quantum repeater, such as performing quantum measurements and storing qubits within a limited time. Arbitrary Q-Nodes equipped with quantum processors (e.g., quantum memory equipped with multiple memory cells) and quantum applications are deployed. Point-to-point entangled pairs can be generated in EPS and distributed to two adjacent Q-Nodes via QCh, accordingly stored in quantum memories in advance. In this process, each EPS has a valid distribution range to distribute entangle pairs, constituting a sub-network [4] (see Fig. 1 (a)). Simultaneously, the measurement messages (e.g., the messages heralding the success or failure of entanglement generation, swapping and purification) can be transmitted via CCh. Moreover, the quantum memory is the key technology to build long-distance entanglement because of its high multi-mode capability, long lifetime, and so on. In [5], the authors demonstrate a multiplexed quantum memory with 225 individually accessible memory cells. Meanwhile, each memory cell owns entangled particle with the same/different birth time and unified "cutoff" time T^{ch} (i.e., the storage time that keep entangled particles "alive"). Here, as shown in Fig. 1 (c), each entangled particle c_d^i ($C^i = \{c_d^i\}_{d=1}^{D}$) stored in advance has different waiting time τ_d^i ($\tau^i = \{\tau_d^i\}_{d=1}^{D}$), i.e., point-to-point pre-established entanglements have different residual lifetime to accomplish quantum operations.

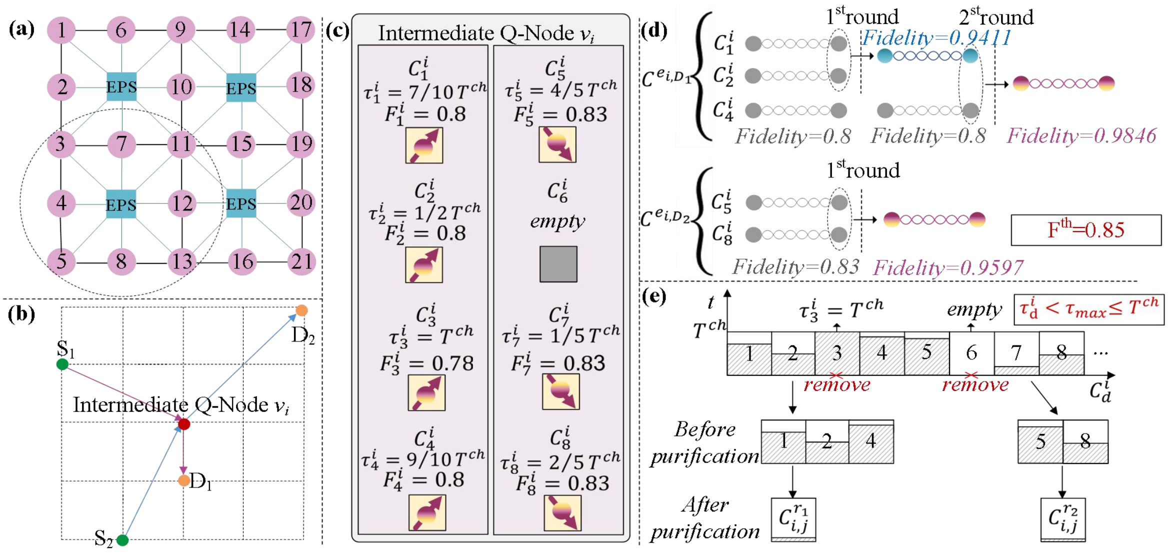


Fig. 1. (a) Network topology; (b) a sample routing scenario; (c) examples of storage entangled particle's state in v_i; (d) the process of purification operations; (e) examples of resource allocation for two requests

Simultaneously, we assume all entangled pairs with the memory output fidelities following a normal distribution $N(F_{mean}, F_{std})$. The following sets are then created as shown in Table 1.

III. Purification-Enabled Routing with Guaranteed Fidelity

The proposed algorithm provides efficient solutions to the remote entanglement fidelity problem, with advanced design features (pre-established entanglement, multi-memory cells, multi-waiting time and multi-fidelities). Given a routing request from source-destination Q-Nodes, and a quantum network with topology as $G(V, E, B)$, finding routing solution, to share the remote entangled pairs between source-destination Q-Nodes with satisfying (Rule. 1) and (Rule. 3) (see Table 1).

An illustration of the local entanglement purification ((Rule. 2)) in an intermediate Q-Node v_i for two routing requests (r_1: $S_1 \leftarrow D_1$; r_2: $S_2 \leftarrow D_2$) is depicted in Figs. 1 (d)-1(e). Local entanglement purification is carried out on each entangled particle having fidelity below the threshold F^{th}. Note that the value of F^{th} may vary in practice for different network functionalities. What's more, by considering bit flip errors, [6] calculated the fidelity resulting after entanglement purification as $f(x_1, x_2) = \frac{x_1 x_2}{x_1 x_2 + (1-x_1)(1-x_2)}$, where x_1, x_2 is the fidelity of two Bell pairs in the purification operation. For example, the initial fidelities on e_{i,D_1} are 0.8 below the $F^{th} = 0.85$, accordingly all lower-fidelity entangled pairs on e_{i,D_1} after 2st rounds entanglement purification are turned to one higher-fidelity (i.e., 0.9846) entangled pair. Consequently, this higher-fidelity entangled pair is used for r_1 to establish point-to-point entanglement. Besides, due to decoherence and limited memory cells on quantum memory, the waiting time of entangled particles is another factor to be considered for a given routing solution within "cutoff" time T^{ch}, able to ensure below-threshold waiting time τ_{max}. For instance, the waiting time τ_3^i of c_3^i isn't satisfied with the (Rule. 3). Hence, given routing requests, entangled particle c_3^i is removed.

In terms of (Rule. 4), based on the high-fidelity entangled pairs generated by purification operations, this algorithm attempts to find "the optimal one" with minimum $\tau_{max} - \tau_{i,j}^r$ via an extended Dijkstra algorithm, simultaneously, remove the entangled pairs do not meet the waiting time constraint. In this process, the entangled pairs are about to expire will be used to establish point-to-point entanglement or end-to-end entanglement, instead of being wasted. Moreover, by adopting local entanglement purification, the expected fidelity for routing request r can be calculated as $F(s,d) = \prod_{i,j \in P(s,d)} F_{i,j}^r$, where $P(s,d)$ donates the routing path for request r.

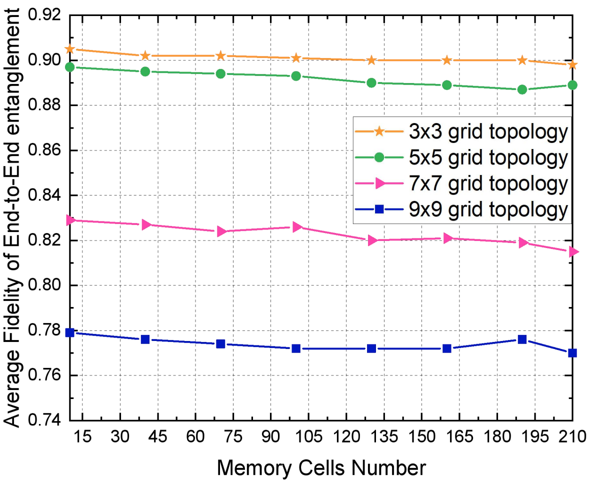
TABLE I. Definations of Sets, Integers and Rules

Sets	
$C^{e_{i,j}}$	The set of entangled particles between Q-Nodes v_i and v_j within T^{ch}
C^i	The set of entangled particles in Q-Node v_i within T^{ch}
$C_{i,j}^r$	The set of entangled particles between Q-Nodes v_i and v_j for request r within T^{ch}
$\tau^{e_{i,j}}$	The set of waiting time of $C^{e_{i,j}}$ within T^{ch}
τ^i	The set of waiting time of C^i within T^{ch}
$\tau_{i,j}^r$	The set of waiting time of $C^{e_{i,j}}$ for request r within T^{ch}
$F^{e_{i,j}}$	The set of fidelity of $C^{e_{i,j}}$ within T^{ch}
F^i	The set of fidelity of C^i within T^{ch}
$F_{i,j}^r$	The set of fidelity of $C^{e_{i,j}}$ for request r within T^{ch}
Integers	
F^{th}	Fidelity threshold
T^{ch}	Maximum storage time in a quantum memory
τ_{max}	Waiting time threshold
Rules	
(Rule. 1)	$F^{e_{i,j}}$ Lower/Upper bounds of $C^{e_{i,j}}$: $F^{th} \le F_d^{e_{i,j}} \le 1$, $F^{e_{i,j}} = \{F_d^{e_{i,j}}\}_{d=\mathcal{D}_1}^{\mathcal{D}_2}$
(Rule. 2)	*Local entanglement purification:* $C^{e_{i,j}} \leftarrow$ new $C^{e_{i,j}}$, $C^i \leftarrow$ new C^i, $\tau^{e_{i,j}} \leftarrow$ new $\tau^{e_{i,j}}$, $\tau^i \leftarrow$ new τ^i, $F^{e_{i,j}} \leftarrow$ new $F^{e_{i,j}}$, $F^i \leftarrow$ new F^i
(Rule. 3)	$\tau^{e_{i,j}}$ *upper bounds of* $C^{e_{i,j}}$: $\tau_d^{e_{i,j}} < \tau_{max} \le T^{ch}$, $\tau^{e_{i,j}} = \{\tau_d^{e_{i,j}}\}_{d=\mathcal{D}_1}^{\mathcal{D}_2}$
(Rule. 4)	*Path search:* $P(s,d) \leftarrow$ *Using extended Dijkstra algorithm to search the path with minimum* $\tau_{max} - \tau_{i,j}^r$, *remove the entangled pairs with* $\tau_{max} - \tau_{i,j}^r = 0$

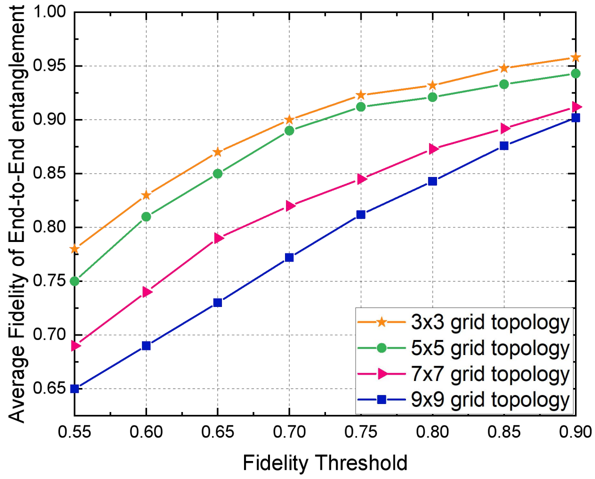
IV. Results Analysis

We evaluate the performance of purification-enabled routing algorithm in different grid network topologies, i.e., 3×3, 5×5, 7×7 and 9×9 grid networks. For example, 5×5 grid network topology consist of 21 Q-Nodes and 4 EPSs is shown in Fig. 1 (a). A bidirectional link (i.e., CCh) with the black line is deployed between two adjacent Q-Nodes, and a QCh with the blue line is deployed to distribute point-to-point entangled particles from EPS to Q-Node. The simulation is repeated 100 times to narrow the error, and we provide the error bar of the 95% confidence intervals for all cases in Fig. 2. Here, the original fidelity of entangled pairs follows $N(0.8, 0.1)$, the typical qubit lifetime is 1.46s [7], the other simulation parameters in the procedure are set according to [7].

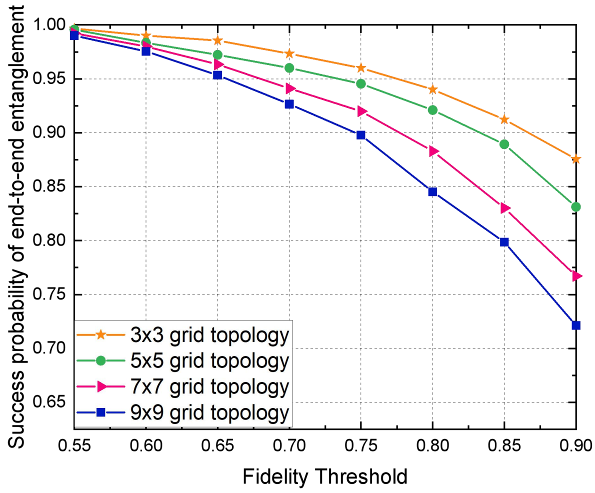
For the case of different memory cells set sizes (see Fig. 2 (a)), the smaller network scale can obtain higher fidelity, and all end-to-end fidelities are keeping relatively stable with increase the memory cells sizes in a quantum memory. For example, when we vary the memory cells size in 3×3 grid network, the fidelities obtained at 10 and 210 are 0.905 and 0.898 respectively. These results observed in Fig. 2 (a) imply that there is a weak correlation between end-to-end fidelity and memory cells size, and a strong correlation between network scale and average fidelity. In Fig. 2 (b), when we set the fidelity threshold to different values, we can observe that all scales of network topologies performance better as fidelity threshold increases, i.e., fidelity threshold has a positive effect on the end-to-end fidelity. Simultaneously, purification-enabled routing algorithm achieves a performance that can satisfy the fidelity constraint even with a higher fidelity threshold (i.e., 0.90). In particular, for the case of 9×9 grid network, when the size of the fidelity threshold fixes to 0.9, the average fidelity is 0.902 which is above the threshold. This phenomenon show that the local entanglement purification and path selection obtained from this algorithm can provide end-to-end fidelity guarantee. Fig. 2 (c) shows the success probability of end-to-end entanglement in different network sizes, where we fix the memory capacity set size to 130 and vary the fidelity threshold. It can be clearly seen that the larger the fidelity threshold value, the lower the success probability of end-to-end entanglement establishment in different topology sizes. What's more, these results observed in Fig. 2 (c) imply that the large-scale topology (i.e., 9×9 grid topology) shows better performance.



(a)



(b)



(c)

Fig. 2. (a) Fidelity versus Memory Cells Size (fidelity threshold=0.7); (b) Fidelity versus Fidelity Threshold (memory cells size=130); (c) success probability versus Fidelity Threshold (memory cells size=130)

V. Conclusion

We propose a purification-enabled routing algorithm to support end-to-end entanglement establishment with fidelity guarantee, considering pre-established entanglement, multi-memory cells, multi-waiting time and multi-fidelities. Simulation results show that the proposed algorithm performance well (i.e., the end-to-end fidelity above fidelity threshold) for the cases of different network scales and memory cells sizes..

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