H_∞ Filter-Based Joint RSOP and Phase Noise Tracking in Coherent PDM Systems

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***Abstract*—In this paper, an H_∞ filter-based approach is proposed for jointly tracking the rotation of state of polarization (RSOP) and laser phase noise (LPN). The H_∞ filter dynamically and robustly estimates the high-dimensional states by limiting the worst-case estimation error, which exhibits excellent tolerance to the noise disturbances. Simulation results verify that the proposed H_∞ filter outperforms the conventional extended Kalman filter (EKF) with respect to the tracking accuracy and the tolerance under ultra-fast RSOP.**

***Index Terms*—H_∞ filter, rotation of state of polarization, laser phase noise**

I. Introduction

With the increasing demand of network traffic data, the coherent optical polarization division multiplexing (PDM) system has become a promising technique in improving system capacity. The PDM system exhibits capacity in doubling the spectral efficiency by allowing the transmission of information signals over two independent and orthogonal polarization states of the same optical carrier wave. However, the PDM signals are sensitive to the rotation of state of polarization (RSOP) caused by random fluctuations in fiber birefringence and the time-varying laser phase noise (LPN) caused by the non-zero laser linewidth, which can severely degrade the system performance. Therefore, a joint polarization-phase recovery scheme, with dynamic and robust tracking capability under high disturbance scenario, is highly demanded for the next generation of coherent optical communications.

Various methods are utilized for tracking the impairments in polarization and phase noise. A conventional solution is to apply constant modulus algorithm (CMA) in compensating the RSOP [1] followed by a phase estimate algorithm, such as Viterbi-Viterbi phase estimation (VVPE) in mitigating the effect of LPN [2]. However, the CMA+VVPE method has the disadvantage of low convergence speed and the estimation performance will degrade dramatically at a ultra-high of RSOP [3].

The Kalman filter (KF) is an optimum adaptive filter algorithm, which can jointly track and compensate several optical transmission impairments. In [3], an unscented Kalman filter (UKF) was applied for the joint mitigation of polarization and phase distortions, and the tracking performance is verified via simulations and experiments, which outperforms the conventional CMA+VVPE approach. However, in [3], the parameters are restricted to be real-valued in order to avoid the singularity issue. In [4], Pakala and Schmauss proposed an adaptive cascaded Kalman filter, with a series of extended Kalman filter (EKF) and linear Kalman filter (LKF), for the tracking LPN and RSOP, with the process noise covariance Q being adaptively updated. In [5], EKF with diagonalized state error covariance matrix P was proposed to jointly tracking the RSOP, polarization model dispersion (PMD), and residual chromatic dispersion (RCD) in time-frequency domain with a reduction in computational complexity. However, the aforementioned KF-based compensation scheme requires the noise distribution to be Gaussian. Furthermore, to ensure the estimation performance, accurate knowledge of noise statistics should be obtained, which is usually unknown in real optical transmission systems.

In this paper, we propose an H_∞ filtering approach to attain a robust, dynamic, and joint tracking of RSOP and LPN. With the objective of minimizing the worst-case estimation error, the H_∞ filter can achieve the robust compensation for RSOP and LPN recursively [6]. Simulation results justify the tracking performance and robustness of the proposed H_∞ filter by comparing with the conventional EKF method.

II. H_∞-Based Joint RSOP and LPN Tracking

A. Signal Model

Assuming that the fiber nonlinearities, dispersions, timing and frequency offsets have been compensated by existing methods, the k-th received time-domain signal $\boldsymbol{r_k}$ in the presence of RSOP and LPN can be expressed as:

$$\mathbf{r}_k = J_k \mathbf{s}_k e^{j\theta_k} + \mathbf{v}_k \tag{1}$$

where $\mathbf{r}_k = [r_x^{(k)}, r_y^{(k)}]^T$ and $\mathbf{s}_k = [s_x^{(k)}, s_y^{(k)}]^T$ are the vector of the k-th received and transmitted signals in PDM system, with subscripts representing the polarization x and y, respectively. $\mathbf{v}_k = [v_x^{(k)}, v_x^{(k)}]^T$ is the complex additive white Gaussian noise (AWGN) vector with the covariance matrix $R = diag\{\sigma_v^2, \sigma_v^2\}$. θ_k is the LPN which is commonly modelled as a Wiener process, i.e.,

$$\theta_k = \theta_{k-1} + \nu_k \tag{2}$$

where $\nu_k \sim N(0, \sigma_p^2)$ and $\sigma_p^2 = 2\pi\Delta\nu T_s$, $\Delta\nu$ is the combined laser linewidth (CLW) and T_s is the sample time interval.

Let J_k denote the RSOP effect at time k. Since the polarization state can be one-to-one mapped onto the Jones space, the equalization of RSOP impairment can be modelled by [7]

$$J_k^{-1} = \begin{bmatrix} e^{j\alpha_k}\cos(\gamma_k) & -e^{j\beta_k}\sin(\gamma_k) \\ e^{-j\beta_k}\sin(\gamma_k) & e^{-j\alpha_k}\cos(\gamma_k) \end{bmatrix} \tag{3}$$

where α_k, β_k are phase rotation angles, and γ_k is azimuth rotation angle. Since J_k^{-1} is a unitary matrix, J_k is also a unitary matrix, with the Cayley-Klein form:

$$J_k = \begin{bmatrix} J_{xx}^{(k)} & J_{xy}^{(k)} \\ J_{yx}^{(k)} & J_{yy}^{(k)} \end{bmatrix} = \begin{bmatrix} a_k + jb_k & c_k + jd_k \\ -c_k + jd_k & a_k - jb_k \end{bmatrix} \tag{4}$$

where $J_{yy}^{(k)} = {J_{xx}^{(k)}}^*$, $J_{yx} = -{J_{xy}^{(k)}}^*$, and we have $a_k^2 + b_k^2 + c_k^2 + d_k^2 = 1$.

B. State Space Model

Let $\mathbf{z}_k = [a_k, b_k, c_k, d_k, \theta_k]^T$ be the state vector to estimate, then the state space model can be expressed as:

$$\begin{aligned} \mathbf{z}_k &= F_{k-1}\mathbf{z}_{k-1} + \mathbf{w}_{k-1} \\ \mathbf{r}_k &= H_k \mathbf{z}_k + \mathbf{v}_k \end{aligned} \tag{5}$$

where $F_k = I_{5\times5}$, and $\mathbf{w}_k \in \mathbb{R}^5$ is the process noise vector, with $\mathbf{w}_k = [0, 0, 0, 0, \nu_k]^T$. The second equation in (5) is the first-order Taylor expansion of the signal model in (1), and we have

$$\begin{aligned} H_k &= \nabla_{\mathbf{z}_k}^T h(\mathbf{z}_k) = \nabla_{\mathbf{z}_k^T}(J_k \mathbf{s}_k e^{j\theta_k}) \\ &= je^{j\theta_k}\begin{bmatrix} s_x^{(k)}, & js_x^{(k)}, & s_y^{(k)}, & js_y^{(k)}, & j(J_{xx}^{(k)}s_x^{(k)} + J_{xy}^{(k)}s_y^{(k)}) \\ s_x^{(k)}, & js_x^{(k)}, & s_y^{(k)}, & js_y^{(k)}, & j(J_{yx}^{(k)}s_x^{(k)} + J_{yy}^{(k)}s_y^{(k)}) \end{bmatrix}. \end{aligned}$$

Algorithm 1: H_∞ filter-based RSOP and LPN tracking

Input: λ, received signal $\mathbf{r}$;
Output: Estimated state vectors $\hat{\mathbf{z}}_1, \hat{\mathbf{z}}_2, ..., \hat{\mathbf{z}}_{N-1}$
Initialize $\hat{\mathbf{z}}_\mathbf{0}, P_0, Q_0, R_0$;
for *k = 1,2, ... , N-1* **do**
  Update the H_∞ gain K_k:
    $K_k = P_k[I - \lambda P_k + H_k^H R_k^{-1} H_k P_k]^{-1} H_k^{-1} R_k^{-1}$;
  Update the posterior state estimate:
    $\hat{\mathbf{z}}_{k+1} = F_k \hat{\mathbf{z}}_k + F_k K_k [\mathbf{r}_k - H_k \hat{\mathbf{z}}_k]$;
  Update the posterior estimation error covariance:
    $P_{k+1} = F_k P_k [I - \lambda P_k + H_k^H R_k^{-1} H_k P_k]^{-1} F_k^H + Q_k$.
end

C. H_∞ Filtering

The H_∞ filter is designed to estimate the state vector $\mathbf{z}_k$ by minimizing the worst-case estimation error. According to the H_∞ filtering theory [8], the cost function J_{cost} measuring estimation error is defined as,

$$J_{cost} = \frac{\sum_{k=0}^{N-1} ||\mathbf{z}_k - \hat{\mathbf{z}}_k||^2}{||\mathbf{z}_k - \hat{\mathbf{z}}_k||_{P_0^{-1}}^2 + \sum_{k=0}^{N-1}(||\mathbf{w}_k||_{Q^{-1}}^2 + ||\mathbf{v}_k||_{R^{-1}}^2)} \tag{6}$$

where $\hat{\mathbf{z}}_k$ is the estimate of state $\mathbf{z}_k$. Define $||x||_A^2 = x^T A x$ and let P_k, Q_k and R_k be the covariance matrices of state estimation error, phase noise and measurement noise respectively. The optimal estimate of $\mathbf{z}_k$ is the one which can minimize the H_∞ norm of J_{cost}. Therefore, the optimization problem is formed as:

$$\begin{aligned} &\min_{\mathbf{z}_k} \max_{v_k, w_k, \mathbf{z}_0} J_{cost} \\ &s.t. \quad J_{cost} < 1/\lambda \end{aligned} \tag{7}$$

where λ is the predefined level of disturbance attenuation.

By solving the min-max optimization problem with game theory approach, the updated $(k+1)$-th state estimate is given by

$$\hat{\mathbf{z}}_{k+1} = F_k \hat{\mathbf{z}}_k + F_k K_k (\mathbf{r}_k - H_k \hat{\mathbf{z}}_k) \tag{8}$$

where K_k is the H_∞ gain, which can be expressed as

$$K_k = P_k[I - \lambda P_k + H_k^H R_k^{-1} H_k P_k]^{-1} H_k^H R_k^{-1} \tag{9}$$

where $(\cdot)^H$ is conjugate transpose operation. P_k is the covariance of posterior estimation error, which can be updated as

$$P_{k+1} = F_k P_k [I - \lambda P_k + H_k^H R_k^{-1} H_k P_k]^{-1} F_k^H + Q_k. \tag{10}$$

According to H_∞ filtering theory, to guarantee that the state estimate $\hat{\mathbf{z}}_k$ minimizes $J_{cost}^{(k)}$, the matrix $(I + H_k^H R_k^{-1} H_k P_k - \lambda P_k)$ is required to be symmetric and positive definite. Therefore, when designing the H_∞ filter, properly selecting the user-specified performance bound λ is required for the positive definite condition to be hold. The algorithm of H_∞ filter-based RSOP and LPN tracking is summarized in **Algorithm** 1.

After obtaining the state estimates of the pilot symbols, we can then apply the decision-feedback scheme to iteratively recover the unknown latter signals and compensate for the

RSOP and phase noise. For instance, for $k = N, N+1, \cdots$, we can utilize $\hat{\mathbf{z}}_k$ to compensate for the J_k and θ_k on $\mathbf{r}_{k+1}$, then make decision on the compensated signal to obtain the estimate of $\hat{\mathbf{s}}_{k+1}$ and $\hat{\mathbf{z}}_{k+1}$.

III. Simulation Results and Discussion

Simulations are carried out through Matlab. Two independent pseudo-random input data sequence (PRBS) of length $2^{15}-1$ is mapped onto QPSK. The sample rate of arbitrary waveform generator (AWG) is 25 GSa/s. To illustrate the superior tracking performance, we utilize 512 pilot symbols in total. Assume the value of initial phase is uniformly distributed in $[-\pi, \pi)$, and the initial estimate $\hat{\theta}_0$ is set to be its mean value 0, with initial estimation variance being $(2\pi)^2/12$. Let initial values for estimation error covariance $P_0 = diag[\frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{(2\pi)^2}{12}]$. The CLW is set to be 200 kHz. The initial parameters in Jones matrix J_0 is selected to be $a_0 = 0.5, b_0 = 0.5, c_0 = 0.1, d_0 = 0.7$. In addition, we select the covariance matrix $Q = diag\{q, q, q, q, \sigma_p^2\}$, where q is selected to be 0.1. The polarization rotation speed is set to be 100 Mrad/s.

Firstly, we investigate the tracking performance of the proposed H_∞ filter compared with conventional EKF at 20 dB SNR in Fig. 1. It can be observed that at a ultra-high rotation speed of state of polarization, the transmission matrix or the Jones matrix varies fast with respect to time. In this case, the H_∞ estimate of a, b, c and d can always accurately track the actual state at each time index. It can be observed that the EKF estimate of a, b, c, d seems to have a larger variance than that of the H_∞ filter, which illustrates that the accumulated mean squared error (MSE) of EKF will be much larger than H_∞ filter. Moreover, the phase tracking performance of EKF is severely degraded under this ultra-high speed polarization rotation scenario, but the phase tracking performance of H_∞ filter is still robust and accurate.

Fig. 2 shows the MSE performance of the proposed H_∞ filter and conventional EKF with respect to the sample index, in order to investigate their ability for real-time dynamic tracking. It can be observed that at a polarization state rotation of 100 Mrad/s, the EKF converges to the steady-state at a very low speed, while that of H_∞ filter always shows superior estimation accuracy of a, b, c, d and θ. This is mainly because we utilize the fixed state covariance matrix Q for updating the state error covariance matrix as in (10), which may not tolerable to such a fast speed of RSOP. However, H_∞ filter estimates the states under the worst cases, which leads to its better estimating performance compared with EKF.

In Fig. 3, we investigate the robustness of the proposed H_∞ filter by considering various polarization rotation angular frequencies. It can be seen that both EKF and H_∞ filter show robustness to ultra-fast RSOP, which can be as high as 1000 Mrad/s. However, the MSE of the proposed H_∞ filter is always the smallest, especially the phase estimation accuracy, which is 1.5 times order smaller than that of EKF. Such a phase estimation error in EKF can lead to severe signal detection errors, thus degrade the system performance.

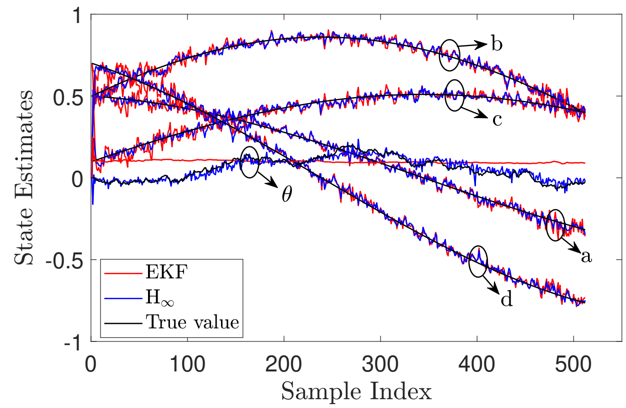

Fig. 1. Tracking performance at 20dB SNR

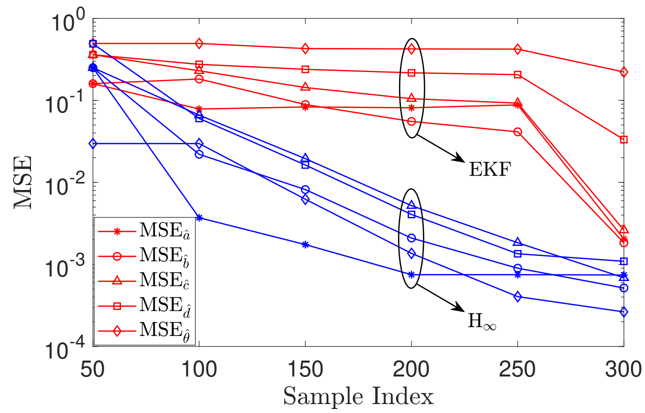

Fig. 2. MSE performance comparison of proposed H_∞ filter and EKF with SNR = 20 dB.

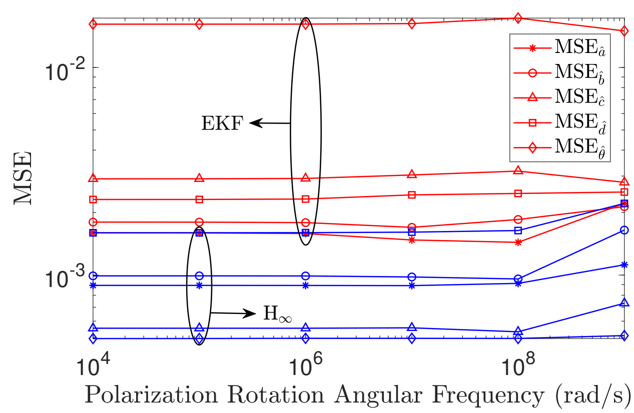

Fig. 3. MSE vs. polarization rotation angular frequency for BTB configuration with SNR = 20 dB.

IV. Conclusion

This paper proposed an H_∞ filter-based algorithm for the joint and dynamic tracking of RSOP and LPN for coherent PDM systems. Simulation results verify the superior estimation accuracy and robustness of the proposed algorithm under ultra-fast RSOP scenario compared with conventional EKF. The RSOP tracking speed can up to 1000 Mrad/s and the phase estimation accuracy is more than one order better than that of EKF, which indicates that H_∞ filter is a promising candidate for real-time state estimation in high-speed optical communications.

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