An investigation of the OSNR penalty for overcoming the impairments of distributed PDL*

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***Abstract*—Using SVD, we derived OSNR penalty is weakly correlated with PDL distribution. The relationship between OSNR penalty and average PDL value is quadratic function for the both polarization tributaries and linear function for the worst.**

Keywords—polarization dependent loss, OSNR penalty, Polynomial fitting.

I. INTRODUCTION

Polarization Dependent Loss (PDL) is considered to have a strong impact on the performance of next-generation optical fiber transmission systems. The PDL of the elements in fiber link such as optical amplifiers and reconfigurable optical add-drop multiplexers induces polarization dependent optical power gain and attenuation, with the result of an inequality of received signal powers between two orthogonal polarization tributaries and imbalance of optical signal-to-noise ratio (OSNR) [1,2]. Up to now, the practical and economical solution for a commercial system to overcome PDL impairments is by the addition of SNR margin, or equivalent OSNR penalty. However, underestimation of OSNR penalty results in an excessive outage probability, while overestimation leads to extra-power consumption. Therefore, it is most important for us to estimate an exact OSNR penalty for a distributed PDL fiber link although it is a troublesome and difficult issue.

In this paper, based on Singular Value Decomposition (SVD), which means the dual-polarization system is perfectly demultiplexed, we obtain an accurate OSNR penalty value for a PDL distributed system. On this basis, we show that OSNR penalty is weakly correlated to the PDL distribution. The relationship between OSNR penalty and average PDL value is linear for the worst polarization tributary and is quadratic for the when considering the mean OSNR for both polarization tributaries. Besides, we show that OSNR penalty of the worst polarization tributary is independent of baseline OSNR, which is the OSNR without PDL in the fiber channel.

II. PRINCIPLE

The model for distributed PDL with distributed ASE noise is shown in Fig. 1. The whole fiber link is considered having several spans, each of them contains a PDL matrix H_i and an additive white Gaussian noise Z_i. The PDL matrix consists of a PDL block and two blocks representing the rotation of state of polarization (RSOP). The expressions of the PDL matrix is expressed as [3]

$$\mathbf{H}_i = \mathbf{R}_{eig,i}\Lambda_i\mathbf{R}_{eig,i}^{-1} \tag{1}$$

Matrix of PDL blocks can be expressed as

$$\Lambda_i = \begin{bmatrix} \sqrt{1+\gamma_i} & 0 \\ 0 & \sqrt{1-\gamma_i} \end{bmatrix} \tag{2}$$

where $\gamma_i \in [0,1]$, $\Gamma_{dB}=10\log_{10}\left[\sqrt{(1+\gamma_i)/(1-\gamma_i)}\right]$ is the PDL attenuation value. The RSOP matrix is expressed using a three-parameter model.

$$R_{eig,i} = \begin{bmatrix} \cos(\theta_i)e^{j\varphi_i} & -\sin(\theta_i)e^{j\beta_i} \\ \sin(\theta_i)e^{-j\beta_i} & \cos(\theta_i)e^{-j\varphi_i} \end{bmatrix} \tag{3}$$

where θ_i is the azimuth rotation angle, and φ_i, β_i are phase rotation angles. The two columns of $\mathbf{R}_{eig,i}$ denote the low/high loss eigenmodes of the PDL matrix in between. The transmitted signal $\mathbf{E_{in}}$, ASE noise $\mathbf{Z_i}$, received signal $\mathbf{E_{out}}$ are all 2 × 1 vectors. $\mathbf{H}$ is the channel impairment matrix experienced by the signal. According to the model in Fig.1, the received signal can be expressed as

$$\begin{aligned}\mathbf{E}_{out} &= \mathbf{H}_n \ldots \mathbf{H}_1\mathbf{E}_{in} + \mathbf{H}_n \ldots \mathbf{H}_2\mathbf{Z}_1 + \ldots + \mathbf{H}_n\mathbf{Z}_{n-1} + \mathbf{Z}_n \\ &= \left[\prod_{i=n}^{1}\mathbf{H}_i\right]\mathbf{E}_{in} + \left[\sum_{i=1}^{n}\left(\prod_{j=n+1}^{i+1}\mathbf{H}_j\right)\mathbf{Z}_i\right] \\ &= \mathbf{H}\mathbf{E}_{in} + \mathbf{Z}\end{aligned} \tag{4}$$

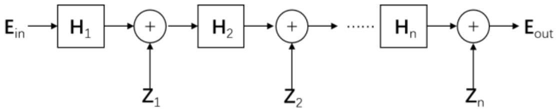

Fig.1 Schematic of the distributed PDL and noise model for the fiber link

Since the aggregated PDL appears stochastic, with the result that the BER performance of the optical communication system also appears stochastic [4]. For example, outage probability (OP) and OSNR penalty. In this paper, we use outage probability (OP) to evaluate the failure rate of the performance of the communication system, which is defined as $\mathrm{OP}=\mathrm{Prob}\{\mathrm{BER}_{\mathrm{PDL}}>\mathrm{BER}_0\}$, where BER_0 is the threshold BER corresponding to the baseline OSNR measured without PDL effect, $\mathrm{BER}_{\mathrm{PDL}}$ is BER values under the influence of PDL. The OSNR penalty for a certain 99% confidence is defined as the least OSNR margin required for which only 1% outage probability occurs in one year, and 99.9% confidence corresponds to 0.1% OP occurs. In general, we focus on the value of the mean OSNR penalty obtained in both polarization tributaries. However, for communication performance guarantee, we need to also consider the OSNR penalty for the case of the high loss (worst) polarization tributary.

In order to accurate estimation of OSNR penalty, we need to calculate the OSNR penalty under the condition of complete polarization demultiplexing, for example using SVD method. By using SVD, we rewrite the channel matrix **H** and obtain (5)

$$\mathrm{SVD}(\mathbf{H})=\mathbf{U}\begin{bmatrix}\sqrt{1+\gamma_{\mathrm{SVD}}} & 0\\ 0 & \sqrt{1-\gamma_{\mathrm{SVD}}}\end{bmatrix}\mathbf{V}^{-1} \tag{5}$$

where the two columns of **V** represent two input orthogonal eigenmodes and two columns of **U** are two output orthogonal eigenmodes of channel **H**. These eigenmodes refer to aggregated PDL respectively. $\sqrt{1-\gamma_{\mathrm{SVD}}}$ and $\sqrt{1+\gamma_{\mathrm{SVD}}}$ are the eigenvalues of **H**. Given each combination of RSOP $(\theta_i,\varphi_i,\beta_i)$ which are all distributed in $(0,2\pi)$, we obtain a aggregated PDL value of $\Gamma_{\mathrm{dB},i}$ (following Maxwellian distribution as Fig. 2), and displayed $\mathrm{OSNR}_{\mathrm{best},i}$ and $\mathrm{OSNR}_{\mathrm{worst},i}$, which correspond to low/high loss polarization tributaries. Through SVD we can find the exact input and output polarization eigenmodes, and perfectly align the receiver's two polarization tributaries with these eigenmodes, achieving complete polarization demultiplexing.

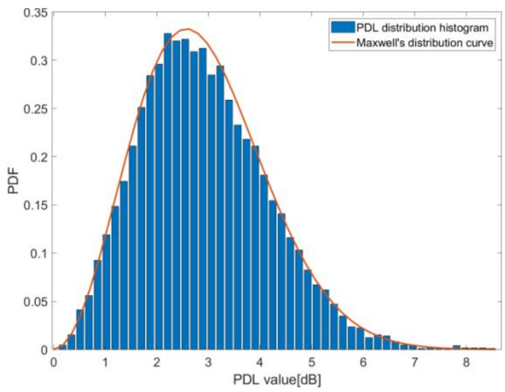

Fig.2 PDL values obey Maxwell distribution

III. Simulation and Analysis

To study the PDL induced OSNR penalty, we built a simulation platform as shown in Fig.3, where 20 PDL elements and ASE noise are uniformly distributed along the fiber link, and the orientations of PDL devices are random, which means the combination of $(\theta_i,\varphi_i,\beta_i)$ are all distributed in $U(0,2\pi)$. We use QPSK system, each fiber amplification span is 80km, and fiber attenuation coefficient is 0.24dB/km. BER threshold = 4e-2. Since CD and PMD can be compensated by the receiver DSP, they are not considered in this paper. Ignoring the effect of BER floor, the SNR margin and OSNR penalty are the same values. 100000 Monte Carlo experiments are conducted in each case to calculate OSNR penalty for a certain degree of confidence. We calculate the average PDL for a n PDL elements through $\mathrm{E[PDL]}=0.92*\sqrt{\mathrm{PDL}_1^2+...+\mathrm{PDL}_n^2}$.

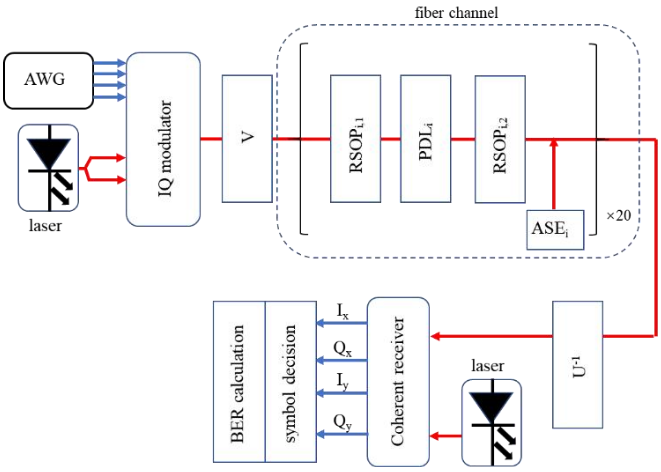

Fig.3 Simulation platform model

For research simplicity, we hope that this estimated OSNR penalty is only related to the average PDL value of the link, and is independent of a specific PDL distribution [5,6]. We model the first modelling with per-span PDL is 0.7dB and the average PDL value is 2.88dB. The others modelling with per-span PDL being uniformly distributed on the interval [0.55dB， 0.85dB] subject to a constraint on an average PDL value of 2.88dB. The OSNR penalties of all model are presented in Table I. It can be seen that the three models' penalty are very close at either confidence level, and the difference is within 0.1dB. From this, it can be seen that OSNR penalty is weak dependence of the PDL distributions. For the sake of simplicity of the ensuing discussion, we will keep a fixed PDL value for per-span PDL.

TABLE I. Relationship between OSNR penalty and PDL distribution in the link

Confidence	90%	99%	99.9%
First model	0.54dB	0.98dB	1.41dB
Second model	0.53dB	0.97dB	1.39dB
Third model	0.56dB	1.02dB	1.47dB

Consider the $\frac{1}{\mathrm{SNR}}$-$\frac{1}{\text{baseline OSNR}}$ curve. Generally speaking the curve is expressed as a linear with the corresponding expression as (6)

$$\frac{1}{\mathrm{SNR}}=\mu\frac{1}{\text{baseline OSNR}}+b \tag{6}$$

The $\frac{1}{\mathrm{SNR}}$-$\frac{1}{\text{baseline OSNR}}$ curve of the worst polarization tributary as shown in Fig.4, it can be seen that the effect of

PDL on the $\frac{1}{\text{SNR}}$-$\frac{1}{\text{baseline OSNR}}$ curve for the worst polarization tributary is only a change in slope. That is, changing the value of μ in (6).

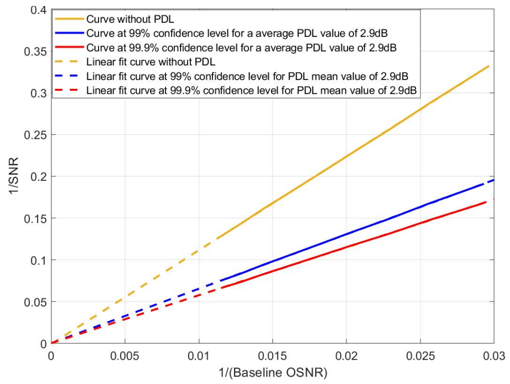

Fig.4 The 1/SNR-1/baseline OSNR curve for the worst polarization tributary.

For Fig.4, OSNR penalty can be defined as the SNR difference value under the same baseline OSNR, corresponding to SNR of the corresponding confidence level under the influence of PDL minus the baseline SNR. The slope of the 1/SNR-1/baseline OSNR curve without PDL is μ_1 while the slope of the curve at corresponding confidence level under the influence of PDL is μ_2. So, the penalty of the worst polarization tributary can be obtained as

$$\begin{aligned} \text{OSNR penatly} &= 10\log_{10}\left(\frac{\text{SNR}_{\text{PDL}}}{\text{SNR}_{\text{noPDL}}}\right) \\ &= 10\log_{10}\left(\frac{\mu_1 \text{*baseline OSNR}}{\mu_2 \text{*baseline OSNR}}\right) \\ &= 10\log_{10}\left(\frac{\mu_1}{\mu_2}\right) \end{aligned} \tag{7}$$

Equation (7) shows that the SNR penalty of the worst polarization tributary is only related to the slope ratio of the curve, but not to the baseline OSNR. That is, the SNR penalty is consistent for the worst tributary under different baseline OSNR.

Fig. 5 shows OSNR penalty for the worst polarization tributary with respect to baseline OSNR changes from 16.5dB to 21dB. We can see that OSNR penalty remains almost unchanged under different baseline OSNR, which means the estimated penalty in the worst polarization tributary is independent of the baseline OSNR, that can be easily used to overcome PDL impairment for a degree of confidence.

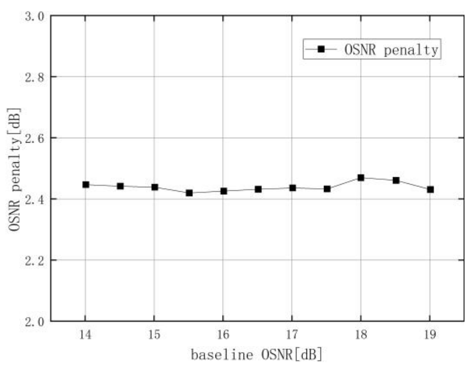

Fig.5 OSNR penalty with regard to baseline OSNR for the worst polarization tributary

Next, we try to establish the relationship between <PDL> and μ_1/μ_2 to get the relationship between penalty and <PDL>. However distributed PDL not only polarizes the signal, but also polarizes the noise, which makes the theoretical analysis more difficult. We fit the curve of <PDL> and μ_1/μ_2 of 90% confidence by simulation, and the obtained simulation curve is shown in Fig. 6. The fitting function used is $\frac{\mu_1}{\mu_2}=10^{0.062<\text{PDL}>-0.012}$. So, we can get the relationship between penalty and <PDL> as penalty=0.62<PDL>-0.12, which means the penalty for the worst polarization tributary is linearly related to <PDL>.

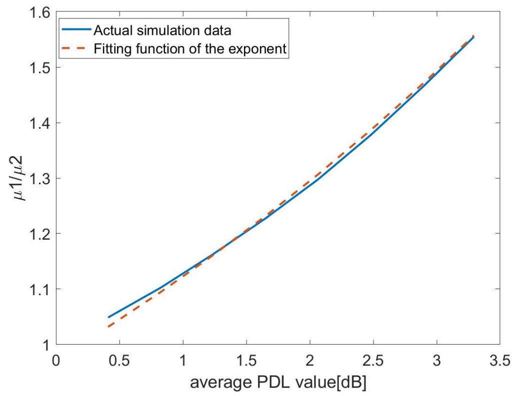

Fig.6 The μ_1/μ_2-average PDL value curve

The estimated OSNR penalty as the function of average PDL value corresponding to all models are shown in Fig.7 for the worst polarization tributary. According to the previous theory, we use a linear function to fit the curve. It can be seen that the fit is good. So, the relationship between penalty for the worst polarization tributary and <PDL> is proved to be linear. In addition, the fitting curve of OSNR penalty at 90% confidence is penalty=0.619<PDL>-0.1189, which is consistent with the previous theoretical analysis. In addition, with the increase of confidence, the slope of the curve increases, which means higher OSNR penalty.

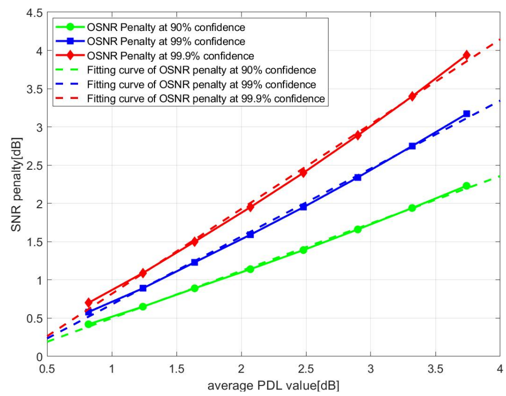

Fig.7 OSNR penalty with regard to average PDL value for the worst polarization tributary

The penalty of the worst polarization tributary has been analyzed above. However, for industrialists, the penalty considering the mean OSNR for both polarization tributaries is also important. So, we use the same method to analyze the penalty for the both polarization tributaries again. As shown in Fig.8, the effect of PDL on the $\frac{1}{SNR}$-$\frac{1}{\text{baseline OSNR}}$ curve for the mean OSNR for both polarization tributaries is not only a change in slope, but also a change in intercept distance. That is, changing both the value of μ and the value of b in (6). This means that penalty for both polarization tributaries is related to baseline OSNR and is more complex to analyze. Therefore, we will use curve fitting to get the relationship between penalty and <PDL> under the same baseline OSNR.

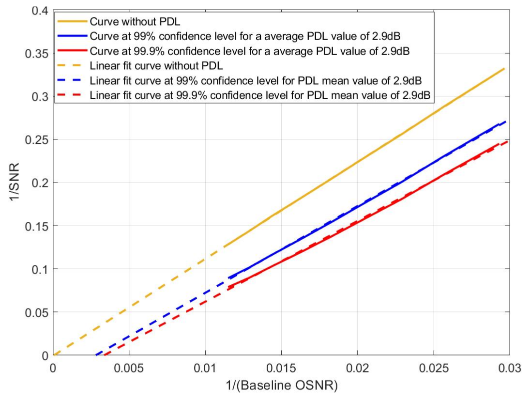

Fig.8 The 1/SNR-1/baseline OSNR curve for the both polarization tributaries

The estimated OSNR penalty for the both polarization tributaries as the function of average PDL value corresponding to all models are shown in Fig.9. Besides, we performed a polynomial fit to the curve. Since the curve fit is good, the relationship between OSNR penalty for the both polarization tributaries and average PDL value is quadratic function.

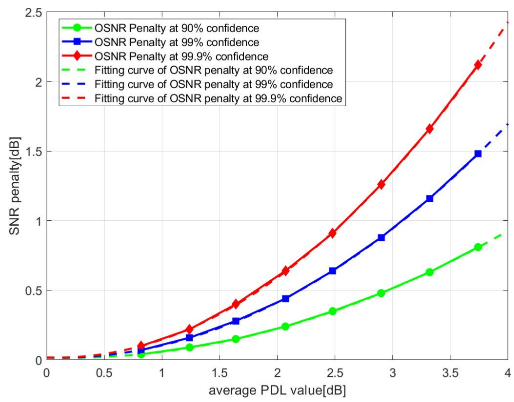

Fig.9 OSNR penalty for the both polarization tributary with regard to average PDL value.

In order to verified the validity of the given OSNR penalty, We tested the outage probability value of the system regarding the BER threshold after increasing the 99% confidence OSNR penalty by Monte Carlo simulation. The Fig.10 corresponds to the OP case of the worst polarization tributary and the Fig.11 corresponds to the OP case of the both polarization tributaries. It can be seen that these OPs are really 1% both for the worst polarization tributary and the both polarization tributaries, which proves that the estimated OSNR penalty is accurate enough.

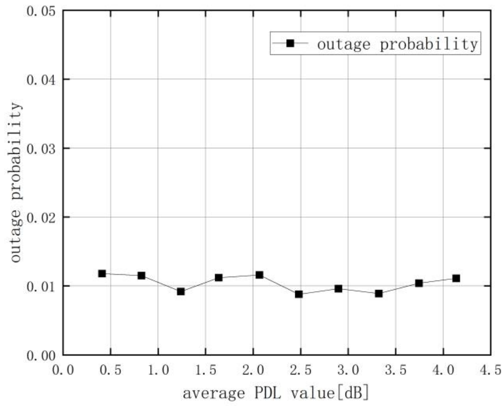

Fig.10 OP with regard to average PDL value for the worst polarization tributary

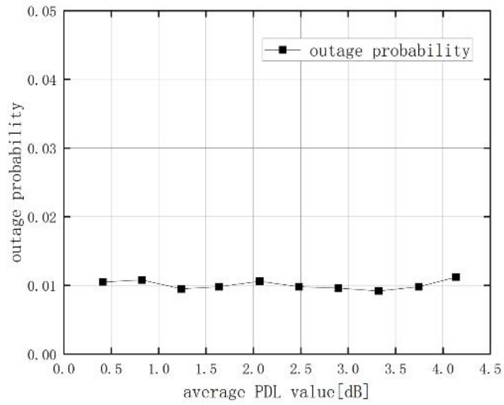

Fig.11 OP with regard to average PDL value for the both polarization tributaryies

IV. CONCLUSION

In this paper, based on SVD, we give a scheme to estimate the exact OSNR penalty induced by distributed PDL link. It is verified that the OSNR penalty is weakly correlated with PDL distribution both for the both polarization tributaries and the worst polarization tributary. Besides, we find the OSNR penalty of the worst tributary is independent of the OSNR value of the system. There is a linear relationship between the OSNR penalty and the average PDL value for the worst polarization tributary and a quadratic function for the both polarization tributaries.

REFERENCES

[1] Jay N. Damask, "Polarization Optics in Telecommunications," Springer, 2005.

[2] A. Dumenil, "Polarization dependent loss in next-generation optical networks: challenges and solutions," Institute Polytechnique de Paris, 2020.

[3] P. Serena, C. Lasagni, A. Bononi, "The enhanced Gaussian noise model extended to polarization-dependent loss," Journal of Lightwave Technology, vol. 38, pp. 5685-5694, 2020.

[4] C. D. Poole, R. E. Wagner, "Phenomenological approach to polarization dispersion in long single-mode fibers," Electronics Letters, vol. 19, pp. 1029-1030, 1986.

[5] J. Cartledge, A. I. Abd El-Rahman, "Estimating the outage probability due to polarization dependent loss using threshold exceedances," Journal of Lightwave Technology, vol. 39, pp. 136-145, 2020.

[6] J. C. Cartledge, A. I. Abd El-Rahman, "Impact of transceiver noise on the outage probability due to polarization dependent loss," IEEE Photonics Technology Letters, vol. 33, pp. 59-62, 2020.