Physics-Informed Machine Learning for Optical Fiber Communications: Opportunities and Challenges

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***Abstract*—A comprehensive evaluation and discussion of physics-informed machine learning (PIML) which revolutionized multiple fields is conducted from the perspective of optical fiber communications. Despite the challenges, PIML provides a promising solution with a fresh perspective.**

Keywords—nonlinear modeling, optical fiber communications, physics-informed machine learning, physics-informed neural network

I. INTRODUCTION

As the mathematical essence of physical phenomena, partial differential equations (PDEs) are of great significance for modeling and understanding the dynamics in various scientific and engineering fields, such as the Navier-Stokes equation in hydrodynamics, Burgers equation in aerodynamics, and heat transfer equation in thermodynamics. However, the analytical solutions are not always available in most nonlinear systems, and the numerical methods are usually required to approximate the exact solutions. Although multiple numerical methods have been widely applied, including the finite element method, finite difference method (FDM), finite volume method, and spectral method, they will suffer from increased complexity and extended computational time, especially for the systems with higher order nonlinearity, higher dimension, and larger scale. To address this problem, machine learning (ML) technology emerging in recent years has opened up new territories with respect to computational science. Generally, neural-network-based ML algorithms offer attractive approximation capabilities for any function by mapping the input features to the output targets in a data-driven manner. Compared with numerical methods, little specialized knowledge is required in data-driven algorithms, and the computational complexity and time can be reduced through parallel processing and matrix operating. However, such data-driven methods are usually uninterpretable and rely heavily on the data quality and quantity without considering the underlying mathematics and physics, which are critical for system modeling. Therefore, a new method that overcomes the limitations of both numerical and data-driven methods is required.

Recently, physics-informed machine learning (PIML) which combines the benefits of ML with physical prior knowledge has been proposed and widely applied in multiple fields [1]. Unlike purely data-driven ML algorithms, PIML takes the prior knowledge into full consideration, including the PDEs, physical laws, and other constraints. Owing to the nature of its composition, the prior knowledge stemming from our observational, empirical, physical, or mathematical understanding of the world can be leveraged by PIML, and this can yield more interpretable MLs that remain robust in the presence of imperfect data and provide accurate and physically consistent predictions even for extrapolatory or generalization tasks. Even earlier than this new proposal, PIML has been extensively investigated to fundamentally reassess a variety of physical systems and solve a large number of scientific and engineering problems, as illustrated in Fig. 1. In the field of optical fiber communications, the signal propagation dynamics are extremely complex and intractable. Nevertheless, PIML has been successfully applied for solving the nonlinear Schrodinger equation, Manakov equation and Helmholtz equation, and exhibits satisfactory results [2-5]. In fact, the potential of PIML in fiber optic communications has not been fully realized, and more challenges will occur with further exploration.

In this paper, the PIML for nonlinear modeling from the perspective of optical fiber communications has been introduced. A comprehensive comparison of numerical, approximate analytical, data-driven, and physics-informed methods has been done taking multiple factors into consideration, and an overview of applications, opportunities and challenges of PIML in the field of optical fiber communications in the future has also been presented.

II. NONLINEAR MODELING OF OPTICAL FIBER COMMUNICATION

The modeling and simulation of communication systems are essential for optimized system design, dynamic process prediction, transmission performance estimation, and numerical analysis. In optical communication systems, the modeling techniques aim to characterize the optical

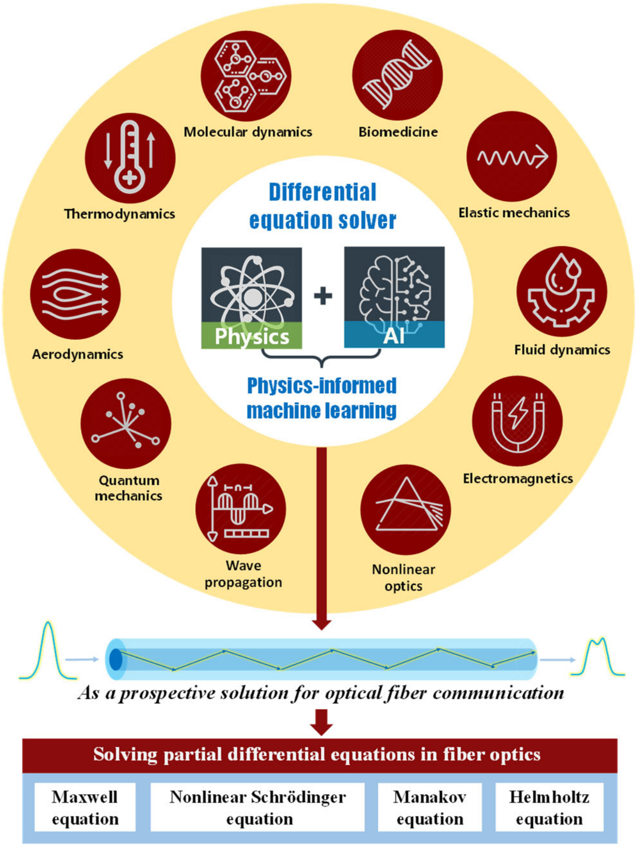

Fig. 1. Applications of physics-informed machine learning in a wide variety of physical systems. PIML is also a prospective solution for optical fiber communications by solving the partial differential equations in fiber optics.

propagation process from the transmitter through the fiber channel, optical amplifier to the receiver, where the nonlinear dynamics in fiber is the most intractable problem. A theoretical understanding of the nonlinear dynamics in fiber optics that is governed by certain fundamental PDEs is represented in Fig. 1. Similar to all electromagnetic phenomena, the propagation of optical fields in fibers is basically governed by Maxwell's equations. Derived from Maxwell's equations and based on the small-signal analysis theory, the wave equation in a nonlinear dispersive medium can be used to obtain a basic propagation equation—nonlinear Schrödinger equation (NLSE). Consequently, the Manakov equation can be derived from the coupled NLSEs to deal with both nonlinearity and varying birefringence in the fiber to describe the pulse evolution in both polarization-division multiplexed and space-division multiplexed systems. Furthermore, the Helmholtz equation can effectively characterize the spatial distribution of the complex electric field at any transverse plane for fiber and other waveguides with different geometries. However, these governing PDEs do not generally lend themselves to be solved analytically, and therefore, the following approximate solutions have been proposed to satisfy different modeling requirements and perform multiple simulation functions.

A. Numerical Methods

To solve PDEs numerically, integral or differential equations are converted into a system of linear algebraic equations that are mostly sparse block diagonal matrices. This process is known as the discretization of PDEs which is performed by taking only a finite number of grid points in the region of interest. A large number of numerical methods have been developed to solve the governing PDEs in fibers, and these can be roughly divided into the FDM and pseudo-spectral methods. Within the family of pseudo-spectral methods, split-step Fourier method (SSFM) provides an excellent methodology for solving the time-dependent NLSE and has been used extensively to simulate the optical pulse propagation over a fiber [6]. The SSFM obtains an approximate solution by assuming that the dispersive and nonlinear effects are executed separately within a short propagation distance. The linear operator can be evaluated in the Fourier domain to make the numerical calculation relatively fast. Thus, the SSFM is an effective and relatively accurate approach, but it has the disadvantages of complex calculation, repetitive iterations, and high time consumption: especially in systems with long distances, multiple channels, and strong nonlinearity.

B. Approximate Analytical Methods

To overcome the limitations of the numerical methods, an approximate analytical method named the Gaussian noise (GN) model was proposed and has become a common tool for the estimation of Kerr nonlinearity-induced interference (NLI) power that trades off accuracy degradation to some extent in an exchange for a reasonable computation time [7]. Based on the first-order regular perturbation, its main assumption is that the nonlinear noise of the system behaves like the GN which is generally satisfied in the long-haul and uncompensated transmissions. By considering signal launch power, fiber loss, amplifier gain, and linear noise together with nonlinear noise, the generalized signal to noise ratio (GSNR) in a given link that can be useful for quality of transmission estimation and network planning can be calculated. Several versions of the GN model in closed form have been proposed. A low-complexity model is the incoherent GN model, which assumes that the NLI is accumulated incoherently, whereas for the basic GN model the NLI is considered coherently. However, both versions underestimated the performance compared to the experimental results. To get more accurate results, the high-complexity enhanced GN model consisting of the GN model and a "correction" term that considers multiple nonlinear effects, such as self-phase modulation, cross-phase modulation, and four-wave mixing, has been proposed. The NLI power can be calculated quickly using the GN model even for wavelength division multiplexing (WDM) systems, but other information (e.g., bits, waveforms, amplitude, phase) cannot be obtained from it. Moreover, it is highly sensitive to the system parameters whose accurate values are generally unknown in practically deployed systems leading to inevitable estimation errors.

C. Data-driven Methods

In contrast to the aforementioned knowledge-driven methods, data-driven ML methods have played an important role in nonlinear modeling and have also been introduced to fiber channel modeling. As powerful feature extraction and nonlinear function approximating methods, several ML techniques were designed for different scenarios to address the corresponding issues. For example, bidirectional long short-term memory has been proven to model intensity modulation and direct detection systems for short-distance fiber transmission [8], in terms of learning the sequential correlation among adjacent symbols in the time domain. Moreover, a data-driven model featuring the multi-head attention mechanism is established to predict the transmission of 16-QAM 160Gbps signals [9]. For data-driven methods,

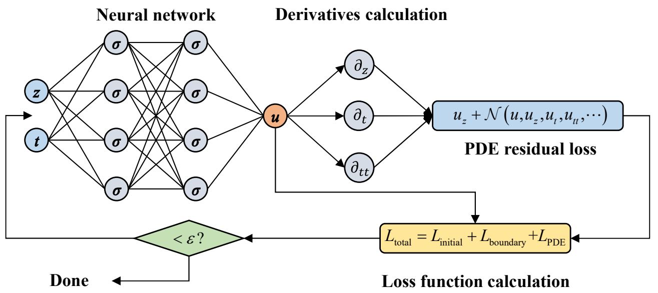

Fig. 2. Schematic of a physics-informed neural network.

only transmitted and received signals are needed to establish channel models without prior knowledge; this can dramatically reduce modeling difficulty and shorten running time. However, massive amounts of data must be prepared for training and processed in advance at a huge cost for training preparation and data collection. Furthermore, these methods ignore all physical principles owing to which the model performance is closely related to the distribution of training data; as a result, the limited training data impose restrictions on the generalization performance of these models.

III. Community of Physics-informed Machine Learning

Although numerical, approximate analytical, and data-driven methods have been widely studied for optical fiber communication modeling, these methods are still encumbered with their respective deficiencies. Nevertheless, PIML provides an alternative solution for optical fiber communications from a fresh perspective while taking full advantage of both knowledge-driven and data-driven methods.

A. *Physics-informed Neural Network*

One of the most well-known and widely used algorithms in the PIML fraternity is the physics-informed neural network (PINN) [10], which is a general framework developed recently for solving both the forward and inverse problems of various complex PDEs. Fig. 2 shows the schematic of a PINN, which consists of a neural network, derivatives calculation, and loss function calculation. All the derivatives in PDEs can be calculated with automatic differentiation, which enables PINN seamlessly integrate the information from the mathematical operators by embedding the governing equations into the loss function. This approach of enriching the neural network with prior knowledge establishes strong physical constraints on the basis of data thus avoiding the poor generalization performance and physical inconsistency or unreliability caused by extrapolation or data errors. In view of the matrix operation and parallel processing capabilities of neural networks, the efficiency and computational complexity of PINNs are far superior to those of conventional numerical methods and are of the same order as the data-driven methods. Both the forward problem of solving PDEs and inverse problem of inferring PDE parameters from observation data have been successfully solved in a variety of areas, including but not limited to the Navier-Stokes equation in turbulent fluid flows, Burgers equation in gas dynamics, heat conduction equation in thermodynamics, and Lame equation in elastic dynamics.

B. *Deep Operator Network*

Another promising PIML is the deep operator network (DeepONet) which belongs to the group of learning operators. Unlike the PINN, which solves PDEs by embedding prior knowledge of physical systems into the loss function, the

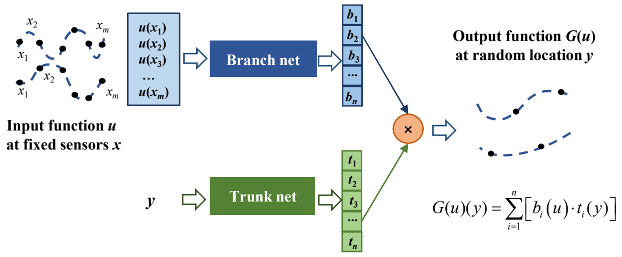

Fig. 3. Schematic of a deep operator network.

DeepONets learn nonlinear operators in a supervised learning manner [11]. Inspired by the universal approximation theorem, a neural network with a single hidden layer can approximate almost any nonlinear continuous operator. As shown in Fig. 3, the DeepONet learns complex operators or system equations from discrete data through a branch network and a trunk network. Specifically, the input of the branch network is the exact value of the input function at multiple fixed locations (sensor locations), and the input of the trunk network is the location to be evaluated. To learn complex operators (e.g., integral operator), the input function can be trigonometric or exponential functions, and for learning system equations (e.g., differential equation for heat transfer), the input function can be the initial conditions, boundary conditions, or forcing terms of the system. Generally, DeepONets generate multiple input functions to enhance the generalization ability. To date, the DeepONets have been successfully verified in multiple problems, including Legendre transform, diffusion-reaction, advection-diffusion, and even stochastic differential equations and statistical operators.

C. *Physics-informed Neural Operator*

While techniques such as PINN and DeepONet have demonstrated powerful performance in various scenarios related to the solution of PDEs, there remain important problems to be addressed. For instance, the computational cost is markedly amplified when multiple evaluations of PDEs corresponding to different initial/boundary conditions need to be conducted through PINN, while DeepONet requires a significant amount of training data to operate effectively. To tackle such challenges, the physics-informed neural operator (PINO) has been proposed [12], and the schematic of PINO is shown in Fig. 4. The main idea behind the PINO is to leverage the advantages of both PINN and DeepONet, while avoiding their respective limitations. By incorporating physical constraints into the training process of DeepONet, PINO enables the neural network to simultaneously learn from limited or even no data and satisfy the underlying physical laws. This is done by formulating a physics-informed loss function that considers the physical constraints imposed by the

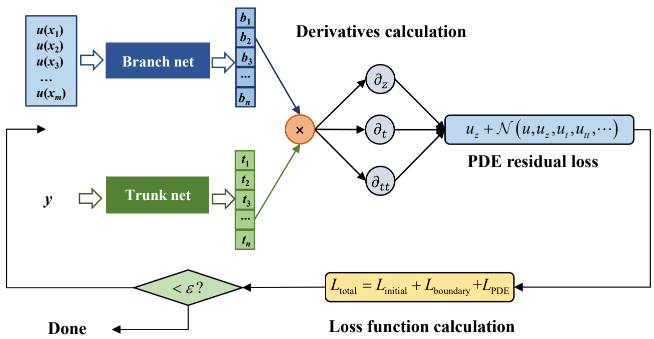

Fig. 4. Schematic of a physics-informed neural operator.

TABLE I. COMPARISON AMONG SSFM, GN MODEL, DATA-DRIVEN MODEL, AND PHYSICS-INFORMED MODEL (MAINLY REFERRING TO PINN)

	SSFM-based model	GN-based model	Data-driven model	Physics-informed model
Modeling basis	Mathematics & physics	Mathematics & physics	Training data & machine learning	Physics & machine learning
Modeling theory	Numerical approximation	Perturbation & Gaussian assumptions	Data statistics analysis	Automatic differentiation
Computing complexity	High	Low	Medium	Low
Running time	Long	Short	Short	Short
Training time	—	—	Long	Long
Model accuracy	High (at small step)	Relatively high (w/o assumptions)	Medium (depend on data)	High (high resolution)
Interpretability	High	High	Low	High
Adaptivity	High	High	Low	Medium
Scalability	High	Medium	Low	Medium
Input features	Transmitted complex signal	System parameters	Transmitted complex signal	Governing equations, initial & boundary conditions
Output targets	Received complex signal	GSNR value	Received complex signal	Solutions of governing equation
Information richness	Rich (bit-level)	Poor (power-level)	Dependent on data	Rich (bit-level)
Functions	Single	Single	Single	Multiple
Knowledge dependence	Strong	Strong	Weak	Medium
Data dependence	—	—	High	Low
Parameter dependence	High	Extremely high	Low	Medium

PDEs. It should be noted that the proposed physics-informed loss function can be combined with other neural operator techniques, such as Fourier neural operator (FNO) [13]. PINO has shown promising results in a wide range of applications, including fluid dynamics, materials science, and quantum mechanics, among others.

IV. COMPARISON AMONG DIFFERENT MODELING TECHNIQUES

In Table I, a comprehensive comparison of SSFM, GN model, data-driven ML, and PIML (mainly referring to PINN) is summarized considering multiple factors, and it can be observed that these four modeling techniques present different advantages depending on the factors under consideration. The main strengths of SSFM are high adaptivity, high accuracy at small step sizes, and rich information (signal waveforms with complete bit information); the main weaknesses are high computing complexity and long running time. The main strengths of the GN model are low computing complexity and short running time, whereas the main weaknesses are poor information (only NLI power) and extremely high parameter dependence. For data-driven ML, the main strengths are low knowledge and parameter dependence, while the weaknesses are high data dependence, low interpretability, and low adaptivity. PIML merges the merits of both physics and data into account thus achieving a good tradeoff between these factors. PIML is superior in computing complexity, running time, data dependency, and interpretability; however, it needs to be improved further in terms of adaptivity and scalability.

V. APPLICATIONS, OPPORTUNITIES AND CHALLENGES IN OPTICAL FBER COMMUNICATIONS

As described above, we can conclude that PIML is an advanced technique for scientific computing and intelligent modeling that has attracted the attention of researchers in multiple fields. Benefitting from its powerful strength to solve PDEs, PIML provides exciting opportunities for optical communications to develop some prospective applications. Even then, when compared to conventional methods, PIML is a new-born technique and still faces some challenges that need to be further studied in the future.

A. Applications

Before PIML was proposed, the idea of machine learning combined with physics has been gradually practiced and applied in the field of optical fiber communications. As a typical representative, inspired by the similar functional forms of SSFM and neural network, a NN-based learning digit backpropagation algorithm was proposed to alleviate NLI, where the NN structure was designed by unrolling SSFM [14]. A Phynet that combined with SSFM was proposed to solve the nonlinear pulse propagation without data [15]. By continuously optimizing the neural network that cascaded with SSFM, the input of SSFM can be successfully predicted by the neural network. Moreover, a physics-guided neural network that sets the known physical law as a physical regularization based on a semi-supervised learning was proposed for nonlinear noise modeling [16]. Furthermore, a physics-informed Gaussian process regression method was also proposed to facilitates Bayesian inference of the signal variation with fewer measurements of the SNR [17].

Although PIML is newly introduced into the field of optical fiber communications [5], it rapidly promotes the multiple applications in this field, as shown in Fig. 5. As the most fundamental application, PINN was first adopted to model the forward propagation of the optical pulses in fiber optics [2,18]. Next, for further applications in more complex problems, variants of PINN are proposed and used to characterize the nonlinear dynamics of complex pulses in fiber [3,4], model multiple modes in few-mode fiber [5], and predict the power spectrum evolution in wideband WDM system in presence of stimulated Raman scattering (SRS) [19]. Moreover, PINN has also been used in inverse problems such as fiber parameter estimation [20], and photonic material designing [21]. In addition to PINN, the neural operators, including PINO and FNO, have also been adopted

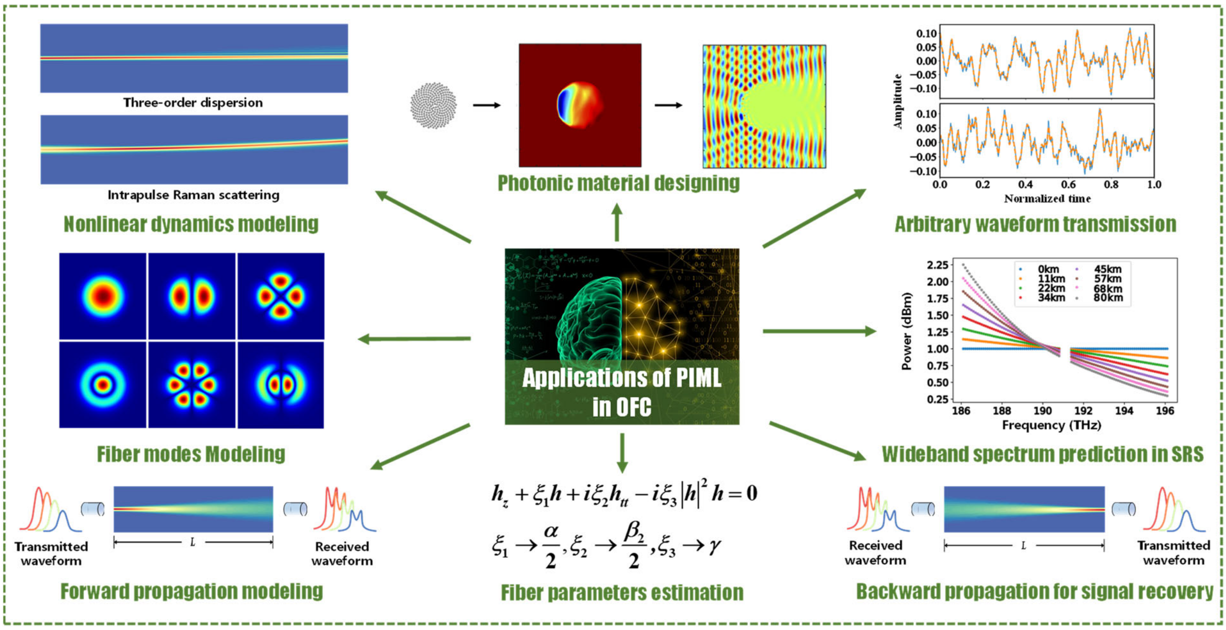

Fig. 5. Applications of PIML in optical fiber communications [2-5,18-24].

for the received signals recovery through backward propagation in digital signal processing module [22], and arbitrary waveforms transmission in a multi-span link [23,24].

B. Opportunities

First, PIML can realize multi-physics and multi-dimensional modeling by solving nonlinear dynamics in fiber optics to simulate waveform propagation, field distribution, and spectrum evolution. Second, with the ability to derive the initial condition, PIML can perform a back-propagation process for signal recovery by compensating both linear and nonlinear impairments from the transmission link as demonstrated previously. Third, in addition to common optical communication modeling other special cases based on nonlinear effects in the fiber can also be solved by PIML. Moreover, femtosecond laser modeling and optical signal processing where higher order nonlinearity are confronted with severe difficulties owing to the much more complex dynamics and extremely high complexity can also be solved by PIML at a relatively low cost.

As another important problem solved by PIML, the identification of PDE parameters will provide a fresh perspective in multiple applications. First, system parameters such as fiber loss, chromatic dispersion, and nonlinear coefficients can be estimated accurately by solving the inverse problem of NLSE. Moreover, PIML also enables the predictive design of artificial optical materials by formulating an inverse design as a boundary condition matching problem, and PIML-based inverse design could also play an important role in optical amplifier setting, system planning, fiber fabrication, and optimal input signal pre-calculation.

C. Challenges

It would not be an exaggeration to state that both opportunities and challenges are intertwined in the future of PIML in optical fiber communications. Research is still in its initial stages and there are certain challenges that need to be addressed. First, to execute optical propagation simulation flexibility, PIML should be adaptive to various simulation conditions for optical signals, viz., various symbol lengths, waveforms, and modulation formats should be satisfied by PIML. However, according to the principle of PINN, all the information of input optical signals are set as initial conditions in the loss function thus implying that retraining is necessary for different simulation conditions; the longer symbol lengths will lead to a larger space-time domain, greater searching region, and more calculation samples. Further, PIML should be scalable to other complex scenarios where multi-channel, multi-mode, and multi-span cases have not yet been well solved by SSFM. In [4], PINN was successfully proved in a simple case with limited pulse lengths and typical pulse shapes at a single channel over fixed fiber. As a surrogate model of SSFM, it would be of great significance for PIML to implement these complex simulations adaptively and flexibly in a more efficient manner.

Although PIML is independent of labeled training data, it strongly relies on prior knowledge, including specific governing equations, accurate physical parameters, as well as initial and boundary conditions. However, it is difficult to obtain accurate prior information in the case of practical optical communication systems, and therefore despite other advantages the physics-informed nature of PIML can be a disadvantage. Although the governing equations in optical fiber are determinate, in optical communication systems the random noise will induce some uncertainties, including amplified spontaneous emission noise from an erbium-doped fiber amplifier, nonlinear noise from the fiber, and thermal noise from the transceiver. As a result, the typical PINNs may have to be modified as stochastic or Bayesian PINNs to address these uncertain situations and the resulting solutions of PDEs may not be determinate in such cases [25]. Besides, PIML and most other modeling techniques still face certain common problems in scientific computing, such as multiscale problems, multi-physics problems, and high-dimensional problems.

VI. Conclusions

In this study, we introduced the techniques of PIML from scientific computing to optical fiber communication,

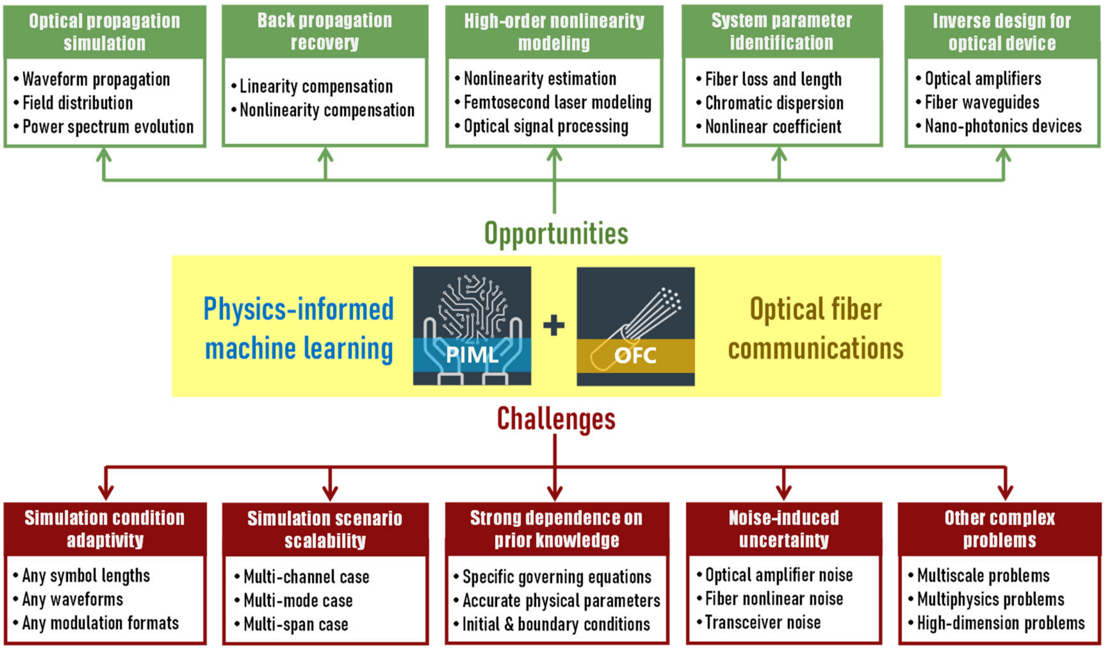

Fig. 6. The summarization of potential opportunities and possible challenges of physics-informed machine learning in optical fiber communications.

investigated its comparative advantages, and discussed its applications, opportunities and challenges. Compared to conventional methods, PIML yielded lower complexity, shorter running time, less data dependence, and stronger interpretability. As a promising technology, PIML has been applied in the field of optical fiber communications, but its capabilities have not yet been fully exploited. While it is a fact that both opportunities and challenges coexist in the future of PIML in optical communications, it can be envisaged with optimism that PIML as a prospective PDE solution would fundamentally revolutionize a wide range of scientific and engineering fields.

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