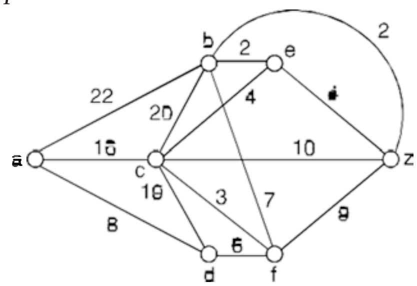
# ASSIGNMENT 2

**Problem 1.** *Determine a shortest path between the vertices a to z as shown below.*




**Solution.** The initial labelling is given by

| Vertex V | *a* | *b* | *c* | *d* | *e* | *f* | *z* |
|---|---|---|---|---|---|---|---|
| L($v$) | 0 | α | α | α | α | α | α |
| T | {*a*, | *b*, | *c*, | *d*, | *e*, | *f*, | *z*} |

**Iteration 1 :** $u = a$ has L($u$) = 0, T becomes T–{$a$}. There are three edges incident with $a$, *i.e.,* $ab$, $ac$, and $ad$ where $b, c, d \in$ T

L($b$) = min {old L($b$), L($a$) + $w(ab)$}
= min {α, 0 + 22} = 22

L($c$) = min {old L($c$), L($a$) + $w(ae)$}
= min {α, 0 + 16} = 16

L($d$) = min {old L($d$), L($a$) + $w(ad)$}
= min {α, 0 + 8} = 8.

Hence minimum label is L($d$) = 8.

| Vertex V | *a* | *b* | *c* | *d* | *e* | *f* | *z* |
|---|---|---|---|---|---|---|---|
| L($v$) | 0 | 22 | 16 | 8 | α | α | α |
| T | { | *b*, | *c*, | *d*, | *e*, | *f*, | *z*} |

**Iteration 2 :** $u = d$, the permanent label of $d$ is 8. T becomes T$-\{d\}$. There are two edges incident with $d$, *i.e.*, $dc$ and $df$ where $c, f \in$ T.

$$L(c) = \min \{\text{old } L(c), L(d) + w(dc)\}$$
$$= \min \{16, 8 + 10\} = 16$$
$$L(f) = \min \{\text{old } L(f), L(d) + w(df)\}$$
$$= \min \{\alpha, 8 + 6\} = 14.$$

Hence minimum label is $L(f) = 14$.

| Vertex V | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $z$ |
|---|---|---|---|---|---|---|---|
| L($v$) | 0 | 22 | 16 | 8 | α | 14 | α |
| T | { | $b$, | $c$, | | $e$, | $f$, | $z$} |

**Iteration 3 :** $u = f$, the permanent label of $f$ is 14. T becomes T$-\{f\}$. There are three edges incident with $f$, *i.e.*, $fc$, $fb$ and $fz$ where $a, b, z \in$ T.

$$L(c) = \min \{\text{old } L(c), L(f) + w(fc)\}$$
$$= \min \{16 + 14 + 3\} = 16 \{16, + 14 + 3\}$$
$$L(b) = \min \{\text{old } L(b), L(f) + w(fb)\}$$
$$= \min \{22, 14 + 7\} = 21$$
$$L(z) = \min \{\text{old } L(z), L(f) + w(fz)\}$$
$$= \min \{\alpha, 14 + 9\} = 23.$$

Hence minimum label is $L(c) = 16$.

| Vertex V | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $z$ |
|---|---|---|---|---|---|---|---|
| L($v$) | 0 | 21 | 16 | 8 | α | 14 | 23 |
| T | { | $b$, | $c$, | | $e$, | | $z$} |

**Iteration 4 :** $u = c$, the permanent label of $c$ is 16. T becomes T$-\{c\}$. These are three edges incident with $c$, *i.e.*, $cb$, $ce$ and $cz$, where $b, e, z \in$ T.

$$L(b) = \min \{\text{old } L(b), L(c) + w(cb)\}$$
$$= \min \{21, 16 + 20\} = 21$$
$$L(e) = \min \{\text{old } L(e), L(c) + w(ce)\}$$
$$= \min \{\alpha, 16 + 4\} = 20$$
$$L(z) = \min \{\text{old } L(z), L(c) + w(cz)\}$$
$$= \min \{23, 16 + 10\} = 23.$$

Hence minimum label is $L(e) = 20$

| Vertex V | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $z$ |
|---|---|---|---|---|---|---|---|
| L($v$) | 0 | 21 | 16 | 8 | 20 | 14 | 23 |
| T | | $b$, | | | $e$ | | $z$ |

**Iteration 5 :** $u = e$, the permanent label of $e$ is 20. T becomes T$-\{e\}$. There are two edges incident with $e$, *i.e.*, $eb$ and $ez$ where $b, z \in$ T.

$$L(b) = \min \{\text{old } L(b), L(e) + w(eb)\}$$
$$= \min \{21, 20 + 2\} = 21$$
$$L(z) = \min \{\text{old } L(z), L(e) + w(ez)\}$$
$$= \min \{23, 20 + 4\} = 23$$

Hence minimum label is $L(b) = 21$.

| Vertex V | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $z$ |
|---|---|---|---|---|---|---|---|
| L($v$) | 0 | 21 | 16 | 8 | 20 | 14 | 23 |
| T | | $b$, | | | | | $z$ |

**Iteration 6 :** $u = b$, the permanent label of $b$ is 21. T becomes T$-\{b\}$. There is one edge incident with $b$. *i.e.*, $bz$ where $z \in$ T.

$$L(z) = \min \{\text{old } L(z), L(b) + w(bz)\}$$
$$= \min \{23, 21 + 4\} = 23.$$

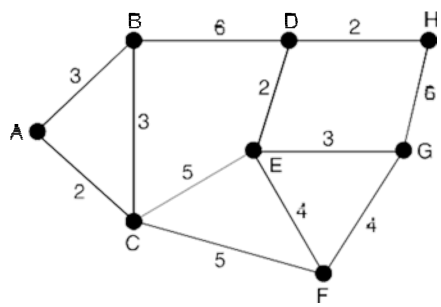
Hence minimum label is L($z$) = 23.

| Vertex V | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $z$ |
|---|---|---|---|---|---|---|---|
| L($v$) | 0 | 21 | 16 | 8 | 20 | 14 | 23 |
| T | { | | | | | | $z$} |

Since $u = z$, the only choice iteration stops.

Thus the length of the shortest path is 23 and the shortest path is ($a$, $d$, $f$, $z$).

**Problem 2.** *Using Kruskal's algorithm, find the minimum spanning tree for the weighted graph of the Fig. given below.*




**Solution.** Let S = ($V_s$, $E_s$) be the spanning tree to be found from G.

Initialize, there are eight nodes so the spanning tree will have seven arcs.

The iterations of algorithm applied on the graph are given below and it runs at the most seven times.

The number indicates iteration number.

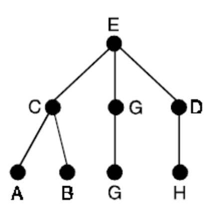
1. Since arcs AC, ED, and DH have minimum weight 2. Since they do not form a cycle, we select all of them and $E_s$ = {(A, C), (E, D), (D, H)} and E = E – ({A, C), (E, D), (D, H)}.
2. Next arcs with minimum weights 3 are AB, BC, and EG. We can select only one of the AB and BC. Also we can select EG.

   Therefore, $E_s$ = ({A, C), (E, D), (D, H), (A, B), (E, G)} and E = E – {(A, B), (E, G)}
3. Next arcs with minimum weights 4 are EF and FG. We can select only one of them.

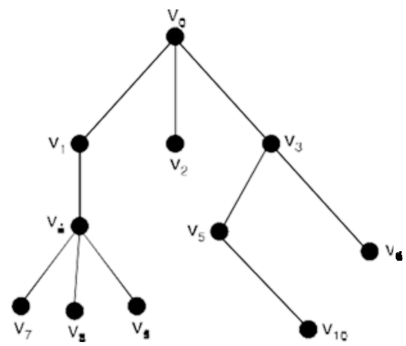
   Therefore, $E_s$ = {(A, C), (E, D), (D, H), (A, B), (E, G), (F, G)} and E = E – {(F, G)}.
4. Next arcs with minimum weights 5 are CE and CF. We can select only one of them.

   Therefore, $E_s$ = ({A, C), (E, D), (D, H), (A, B), (E, G), (F, G), (C, E)} and E = E – {(C, E)}.

   Since number of edges in $E_s$ is seven process terminates here. The spanning tree so obtained is shown in the Fig.



**Problem 3.** Consider the tree with root $v_0$ shown in Figure.

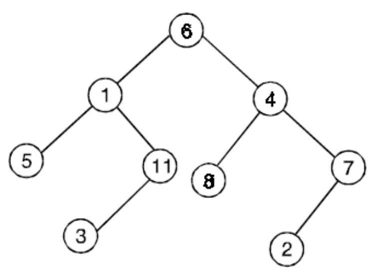


($a$) what are the levels of $v_0$ and $v_4$ ?
($b$) what are the children of $v_3$ ?
($c$) what is the height of this rooted tree ?
($d$) what is the parent of $v_5$ ?
($e$) what are the siblings of $v_7$ ?
($f$) what are the descendants of $v_3$ ?

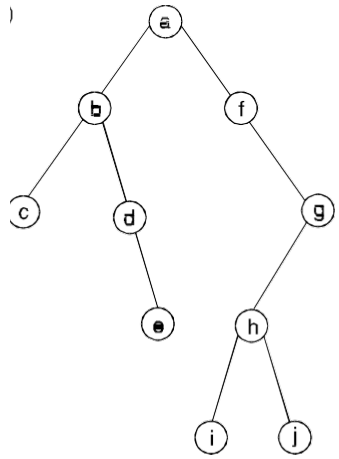
($a$) 0, 2 ($b$) $v_5$, $v_6$ ($c$) 3 ($d$) $v_3$ ($e$) $v_8$, $v_9$ ($f$) $v_5$, $v_6$, $v_{10}$.

**Problem 4.** Construct a binary tree whose in-order and pre-order traversal is given below
($a$) ($i$) In-order : 5, 1, 3, 11, 6, 8, 2, 4, 7
($ii$) Pre-order : 6, 1, 5, 11, 3, 4, 8, 7, 2

**Solution:**




**Problem 5.** Determine the order in which the vertices of the binary tree given below will be visited under ($i$) In-order ($ii$) pre-order ($iii$) post-order.



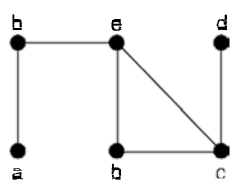

**Solution:**

i) $c\ b\ d\ e\ a\ f\ i\ g\ h\ j\ g$

ii) $a\ b\ c\ d\ e\ f\ g\ h\ i\ j$

iii) $c\ e\ d\ b\ i\ j\ h\ g\ f\ a.$

**Problem 6.** *Find all spanning trees for the graph G shown in Figure given below*



**Solution.** The graph G has one cycle *cbec* and removal of any edge of the cycle gives a tree. There are three trees which contain all the vertices of G and hence spanning trees.