班级 Class________ 姓名 Name________ 学号 Student Number________ 装 订 线

专业：计算机[1191-1192]班，计算机[1193-1194]班
Major: 计算机[1191-1192]班,计算机[1193-1194]班

课程名称： Graph Theory And Modern Linear Algebra
Course Title: Graph Theory And Modern Linear Algebra

学分：
Credits： 2.5

试卷编号（A）
Paper Number (A)

课程编号： 1310700
Course Code： 1310700

考试方式： 报告
Test Method: Final Report(Open-Book)

考试时间：4 天
Exam Time 4 Days

拟卷人(签字)：
Designer Dr. Ammarah

拟卷日期：
Design Date : 05/06/2020

审核人(签字)：
Reviewer

题号 Item Number	I	II	III	IV	V	VI	VII	总分 Total
得分 Score	14	13	10	14	14	14	21	100

得分统计表：

Question I:

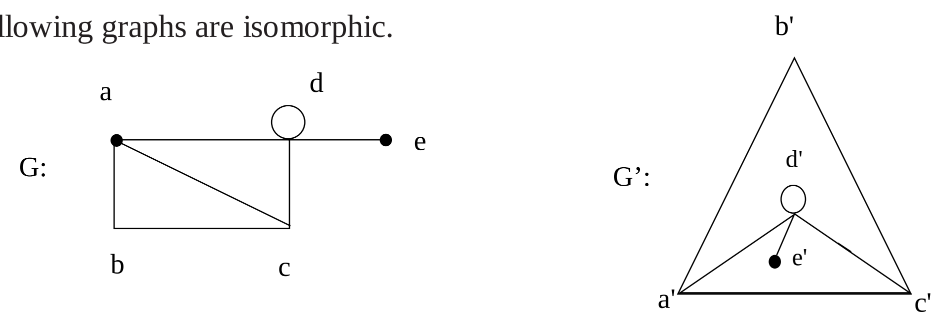
Part (a)

Prove that the set of residue classes modulo ***p*** is a commutative ring with respect to addition and multiplication of residue classes. Further show that the ring of residue classes modulo ***p*** is a field if and only if ***p*** is a prime.

(10)

Part (b)

Show that the following graphs are isomorphic.

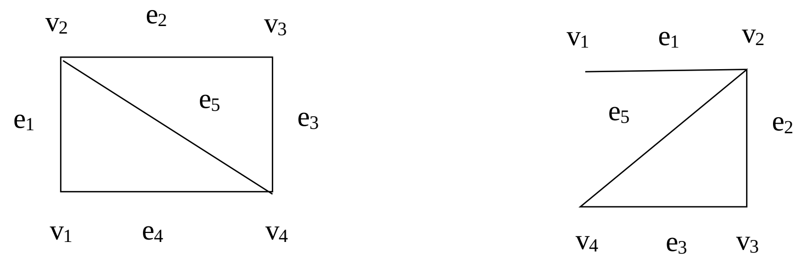


(04)

Question II:

Part (a)

Which of the graphs given in Figure below is Hamiltonian circuit. Give the circuits on the graphs that contain them.



(a) (b) (05)

Part(b)

Use Fleury's algorithm to construct a Euler circuit for the graph in Figure (1).

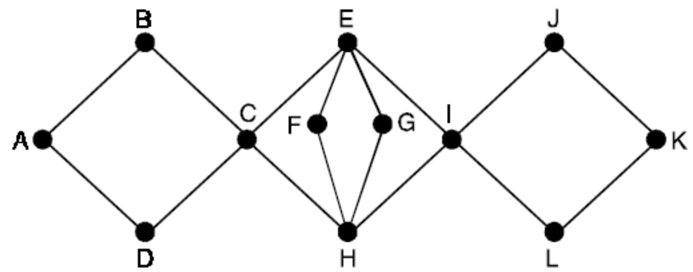


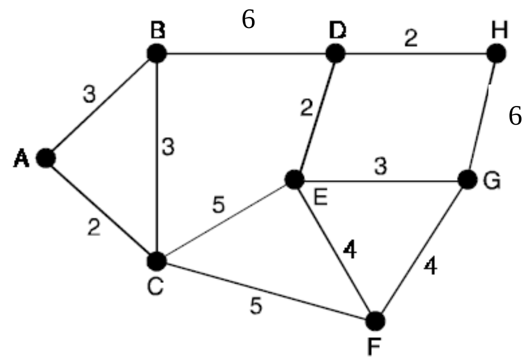
Fig. (1)

(08)

Question III:

Part (a)

Using Kruskal's algorithm, find the minimum spanning tree for the weighted graph of the Fig. given below.



(08)

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Part (b)
Does there exists a simple graph with seven vertices having degree (1,3,3,4,5,6,6)? Give reason.

(02)

Question IV:
Part (a)

Consider the groups $(G_1,*)$ and $(G_2,\oplus)$ with identity elements e_1 and e_2 respectively. If $f:G_1 \to G_2$ is a group homomorphism, then prove that

(i) If H_1 is a sub group of G_1 and $H_2 = f(H_1)$, then H_2 is a sub group of G_2.

(ii) If f is an isomorphism from G_1 onto G_2, then f^{-1} is an isomorphism from G_2 onto G_1.

(10)

Part (b)
Show that the set of all positive rational numbers forms an abelian group under the composition * defined by $a * \mathrm{b} = \frac{(ab)}{2}$.

(04)

Question V:
Part (a)

If R is the additive group of real numbers and R_+ be a group of all positive real numbers under multiplication. Show that the mapping $f:R \to R_+$ defined by $f(x) = e^x$ for all $x \epsilon R$ is an isomorphism of R onto R_+ .

(07)

Part (b)
Show that the mapping $f:C \to R$ such that $f(x+iy) = x$ is a homomorphism of the additive group of complex numbers onto additive group of real numbers and find the kernel of f.

(07)

Question VI:
Part(a)

Let C be the set of the ordered pairs (a,b) of real numbers. Define addition and multiplication in C by the equations

$$(\mathrm{a,b}) + (\mathrm{c,d}) = (\mathrm{a+c,b+d})$$
$$(\mathrm{a,b})(\mathrm{c,d}) = (\mathrm{ac-bd,bc+ad})$$

Prove that C is a field.

(10)

Part(b)
Prove that the set $G = \{1,2,3,4,5,6\}$ is a finite abelian group of order 6 with respect to multiplication modulo 7.

(04)

Question VII:
Part (a)

Let A be the set of all real valued functions on $(\infty,-\infty)$. Define

$$(\mathrm{f+g})(\mathrm{x}) = \mathrm{f(x)+g(x)}$$
$$(\mathrm{f.g})(\mathrm{x}) = (\mathrm{fg(x)})$$

For every x in $(\infty,-\infty)$. Is A a ring with respect to these two operations?

(08)

Part (b)
Construct a 2×2 matrix and prove that it is a skew field but not a field.

(09)

Part (c)
Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw two such graphs.

(04)