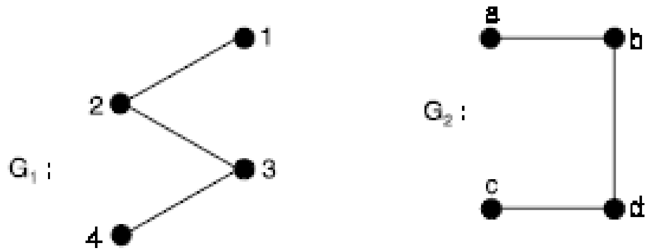
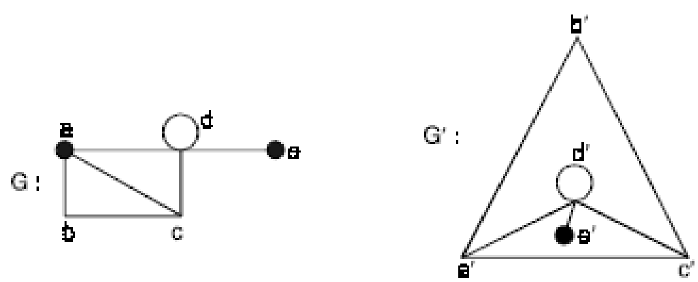
Assignment 1:

Problem 1. *Show that the two graphs shown in Figure are isomorphic.*



Solution. Here, $V(G_1) = \{1, 2, 3, 4\}$, $V(G_2) = \{a, b, c, d\}$
$E(G_1) = \{\{1, 2\}, \{2, 3\}, \{3, 4\}\}$ and $E(G_2) = \{\{a, b\}, \{b, d\}, \{d, c\}\}$
Define a function $f : V(G_1) \rightarrow V(G_2)$ as
$f(1) = a$, $f(2) = b$, $f(3) = d$, and $f(4) = c$
f is clearly one-one and onto, hence an isomorphism.
Further, $\{1, 2\} \in E(G_1)$ and $\{f(1), f(2)\} = \{a, b\} \in E(G_2)$
$\{2, 3\} \in E(G_1)$ and $\{f(2), f(3)\} = \{b, d\} \in E(G_2)$
$\{3, 4\} \in E(G_1)$ and $\{f(3), f(4)\} = \{d, c\} \in E(G_2)$
and $\{1, 3\} \notin E(G_1)$ and $\{f(1), f(3)\} = \{a, d\} \notin E(G_2)$
$\{1, 4\} \notin E(G_1)$ and $\{f(1), f(4)\} = \{a, c\} \notin E(G_2)$
$\{2, 4\} \notin E(G_1)$ and $\{f(2), f(4)\} = \{b, c\} \notin E(G_2)$.
Hence f preserves adjacency as well as non-adjacency of the vertices.
Therefore, G_1 and G_2 are isomorphic.

Problem 2. *Show that the following graphs are isomorphic.*



Solution. Let $f : G \rightarrow G'$ be any function defined between two graphs degrees of the graphs G and G′ are as follows :

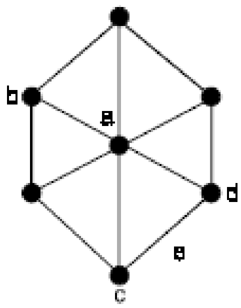
deg (G) deg (G′)
deg $(a) = 3$ deg $(a') = 3$
deg $(b) = 2$ deg $(b') = 2$
deg $(c) = 3$ deg $(c') = 3$
deg $(d) = 5$ deg $(d') = 5$
deg $(e) = 1$ deg $(e') = 1$
Each has 5-vertices, 6-edges and 1-circuit.
$\deg(a) = \deg(a') = 3$
$\deg(b) = \deg(b') = 2$
$\deg(c) = \deg(c') = 3$

deg(d) = deg(d') = 5
deg(e) = deg(e') = 1
Hence the correspondence is $a - a'$, $b - b'$, ..., $e - e'$.
Therefore, the given two graphs G and G′ are isomorphic.

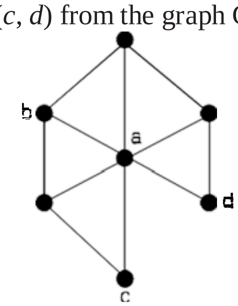
Problem 3. *For the graph G shown below, draw the subgraphs*
(i)G – e

(*ii*) *G – a*

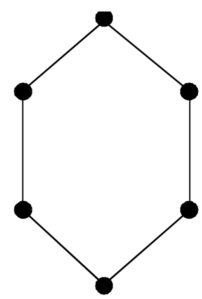
(*iii*) *G – b*.



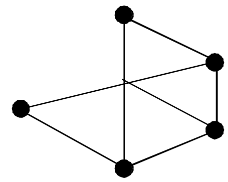
Solution. (*i*) After deleting the edge $e = (c, d)$ from the graph G, we get a subgraph G – e as shown below



(*ii*) After deleting the vertex a form the graph G, and all edges incident on this vertex, we set the subgraph G – a as shown below:



(*iii*) The subgraph is obtained after deleting the vertex b.



Problem 4. *Consider the following directed graph G :*
V(G) = {a, b, c, d, e, f, g}
E(G) = {(a, a), (b, e), (a, e), (e, b), (g, c), (a, e), (d, f), (d, b), (g, g)}.
(*i*) *Identify any loops or parallel edges.*
(*ii*) *Are there any sources in G ?*
(*iii*) *Are there any sinks in G ?*
(*iv*) *Find the subgraph H of G determined by the vertex set V = {a, b, c, d}.*

Solution.
(*i*) (*a*, *a*) and (*g*, *g*) are loops
(*a*, e) and (*a*, *e*) are parallel edges.
(*ii*) d is source
(*iii*) f and c are sinks
(*iv*) V = {*a*, *b*, *c*, *d*}
E = {(*a*, *a*), (*d*, *b*)}
H = H(V, E).

Problem 5. *Does a 3-regular graph on 14 vertices exist? What can you say on 17 vertices?*

Solution. We have q = (P x r) / 2
given r = 3, P = 14
Now q = (14 x 3)/2= 21, is a positive integer.
Hence 3-regular graphs on 14 vertices exist.
Further, if P = 17, then q = (17 x 3)/2 =51/ 2 is not a positive integer.
Hence no 3-regular graphs on 17 vertices exist.