ASSIGNMNET 4

Problem 1: Do the following sets form integral domains with respect to ordinary addition and multiplication? If so state if they are fields.

i) The set of numbers of the form b√2 with b rational.
ii) The set of even integers
iii) The set of positive integers

Problem 2: Let C be the set of the ordered pairs (a, b) of real numbers. Define addition and multiplication in C by the equations

$$(a, b) + (c, d) = (a + c, b + d)$$

$$(a, b)(c, d) = (ac - bd, bc + ad)$$

Prove that C is a field.

Problem 3: How many generators are there of the cyclic group G of order 8?

Problem 4: Taking a group {e,a,b,c} of order 4, construct two composition tables which are not isomorphic.

Problem 5. If H⊆K are two subgroups of a finite group G, then show that [G:H]=[G:K][K:H].

1. i) No

ii) Yes No

iii) Yes Yes

2. Associativly of addition

$[(a,b)+(c,d)]+(e,f) = (a+c, b+d)+(e,f)$

$= ([a+c]+e, [b+d]+f) = (a+[c+e], b+[d+f])$

$= (a,b)+(c+e, d+f) = (a,b)+[(c,d)+(e,f)]$

commutativity of addition

$(a,b)+(c,d)=(a+c,b+d)=(c+a,d+b)=(c,d)+(a,b)$

$\because (0,0)\in C \therefore (0,0)+(a,b)=(0+a,0+b)=(a,b)$

$\therefore (0,0)$ is the additive identity

If $(a,b)\in C$, than $(-a,-b)\in C$

$(-a,-b)+(a,b)=(-a+a,-b+b)=(0,0)$

$\therefore (-a,-b)$ is the additive inverse of (a,b)

$\therefore$ Associativity of multiplication

$\therefore [(a,b)(c,d)](e,f)=(ac-bd,bc+ad)(e,f)$

$=([ac-bd]e-[bc+ad]f,[bc+ad]e+[ac-bd]f)$

$=(a[ce-df]-b[de+cf],b[ce-df]+a[de+cf])$

$=(a,b)(ce-df,de+cf)=(a,b)[(c,d)(e,f)]$

$(1,0)\in C$ if $(a,b)\in C$, then $(a,b)(1,0)=(a1-b0,b1+a0)=(a,b)=(1,0)(a,b)$

if (c,d) is the multiplicative inverse of (a,b)

then: $(a,b)(c,d)=(1,0)$ or $(ac-bd,bc+ad)=(1,0)$

$ac-bd=$ and $bc+ad=0$ $\quad c=\dfrac{a}{a^2+b^2}$ $\quad d=\dfrac{-b}{a^2+b^2}$

now $a\neq 0$ or $b\neq 0 \Rightarrow a^2+b^2\neq 0$

either c or d both are non-zero real numbers

Hence C is a field

3. just count the number of integers which are than equal to 8 and relatively prime to 8, which are

1, 3, 5, 7

So They are 4

4. if $a^2=e$

$\cdot$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

or

$\cdot$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

if $a^2=b$

$\cdot$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

5. Proof since K is a subgroup of G, Hence $|G|=|G,K||K|$
G is a subgroup of K, $|K||K:H|$. Thus $|G|=|G:K||K:H||H|$
H is a subgroup of G, $|G|=|G:H||H|$ Hence $|G:K||K:H||H|$
$=|G:K||H|$ and

$|G:K||K:H|=|G:H|$