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Is Knowledge of Physical Reality Still Kantian? Some Remarks About the Transcendental Character of Loop Quantum Gravity

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Abstract In the following paper, the author will try to test the meaning of the transcendental approach in respect of the inner changes implied by the idea of quantum gravity. He will firstly describe the basic methodological Kant's aim, viz. the grounding of a meta-science of physics as the a priori corpus of physical knowledge. After that, he will take into account the problematic physical and philosophical relationship between the theory of relativity and the quantum mechanics; in showing how the elementary ontological and epistemological assumptions of experience result to be changed within them, he will also show the further modifications occurred in the development of the loop quantum gravity. He will particularly focus on the tough problem of the relationship space-matter, in order to settle the decisive question about the possibility of keeping a transcendental approach in the light of quantum gravity. He will positively answer by recalling Cassirer's theory of the invariants of experience, although he will also add some problematic issues arising from the new physical context.

Keywords Quantum gravity · Loop quantum gravity · Space–time · Ontological singularity

1 Introduction and Methodological Assumptions

It is well known that the relationship between Kant's philosophy and science is very remarkable, and it is not unusual to present Kantian philosophy as a wise interpretation of the actual structure of mathematical physics [17]. However, in the history of Kan-

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tianism we have also observed that the general architecture of reason and of the work of intellect upon nature cannot be held as it was in Kant: the modifications occurred in the theoretical grounds of physical knowledge have indeed implied a recalibration of the whole philosophical system.

There are two main topics about which the survival of Kantianism must be tested: (1) the definition of a metaphysical science of mathematical physics; and (2) the determination of space and time as a priori forms before of any empirical fulfilment of the two concepts.[1] I would like to point out that our inquiry should be limited, as far as possible, to the second point. Hence it will focus the possibility of holding a transcendental account of space and time as regards the very recent outcomes within the development of the idea of quantum gravity.

2 The Difficult Relationship Between General Relativity and Quantum Mechanics

In the history of physics, whenever there are two or more discordant theories in the description of reality, the inner tendency of physical thought is that of making an attempt to conciliate the different point of views into a unitary approach. This idea is still present in those authors who explicitly admit the variety of the different theories, for instance as it is in Heisenberg's *closed theories* [24, pp. 39–46]: if it is true that it is impossible to apply the same scientific principles to every specific field of reality, it is also recommendable to set the ideal goal of the pursuit of a unified field theory (UFT) summarizing the physical knowledge in a whole descriptive structure.

This is the exact situation at the beginning of the second half of the 20th century, when within physics we find two main theories which seemed to be incompatible: (1) on the one hand, GR as *continuous* geometrical description of the physical space in its interaction with matter, and (2) on the other, QM as theory of the elementary quanta composing the microphysical world. We can also say that, according to their standard form, GR and QM seem to be based on what they are supposed to go beyond: a stringent divergence between a theory of space–time and a theory of matter. Both the theories, indeed, do present a problematic element hidden between the very sincere success of their methods.

[1] As is clear, the two points are mutually connected. In *KrV*: B872/A 846 and ff., Kant distinguished between "*physica rationalis*" and "*physica generalis*" (the actual mathematical physics), in order to depict the possibility of a metaphysical science of physics which would be able to represent the transcendental *organon* for the critique of the physical concepts. The matter is that physicists do not simply attain empirical concepts: on the contrary, they start from some implicit metaphysical presumptions (see *loc. cit.*, footnote). *Physica rationalis* is thus needed to clarify the metaphysical heritage of physics, and at the same time it will become very useful to philosophy, insofar as it will provide meaningful empirical examples in the development of the critique of the intellect. This project will be recalled in *MAN*, which in turn seems to add a very important element to the *KrV*'s approach. Indeed, in respect of the old arrangement, a particular section of *physica rationalis*, pertaining to a more strict application of the transcendental move to the actual mathematical physics, is *ex novo* derived: the very matter is that upon this new part the weight of mathematics result to be strengthened, so that on the classical determination of the nature of the *transcendental method* mathematical physics seems to play a more decisive role than before [27, pp. 470–471, 17, pp. 136 and ff.].

GR seems in fact to overcome the dichotomy between space and matter in describing the language of a whole geometry capable to study the relationship between space and matter and the form of space according to the mass distribution; but it does not take into account that to some extent the theory will have to consider the quantum character of matter, the fact that we cannot part reality ad infinitum, since we encounter at some point the *minimum* of reality which is the actually ultimate physical substance, the *quantum of action.*[2] From the physical point of view, this problem appears to have no solution in the equations of GR when we approach space and time in the very microscopic scale of h: the gravitational field cannot be perfectly defined in so tiny a spot. This depends on the predictions of QM. As the uncertainty principle well explains, it is impossible to precisely determine both position and momentum of a particle at the same time: Heisenberg conceived this impossibility in the terms of the concept of trajectory becoming meaningless [20]. Therefore, if we wanted to describe the gravitational field in a very small space, we would have to consider the place occupied by a particle; but the tinier the place taken by the particle is, the bigger its momentum and then its own speed will be. It follows that if the position of the particle has to be so minutely determined as in the h scale, then its speed will become undefined. This also means that on the particle acts a high energy, which, according to GR, has to modify the space conformation. As a consequence, the space around the particle would appear to be curved on itself, until it may vanish into a black hole, and the particle with it [33, pp. 132–135].

As the above description may suggest, the problem of QM was instead exactly the opposite to that experienced by GR, since the abandoning of the concept of trajectory seemed to imply the impossibility of building a whole theory of space: there are only very individual physical events, and space as basic universal concept is replaced by a general statistical context. Indeed, there is no chance to link two different observations to one another, because every observation modifies the state of the things to be measured [20, pp. 181 and ff., 21, p. 48]. The space of trajectory, the realm of the old physical truth, has become a probability cloud.

In recalling the mentioned difficulties, we can sum them up as follows:

(1) GR cannot take into account the field quantization, which is also unavoidable for the gravitational field;
(2) Within QM, on the contrary, it seems impossible to start from the physical definition of particle to reach the definition of a whole spatial context, considering the effects of the gravitational field on the microscopic world.

Hence, on the one hand, the definition of the space occupied by an elementary particle will become unapproachable, as if it were a being no standing in a place [19, p. 7]; on the other hand, we will conversely obtain many individual beings, although one of the prime assumptions of QM is the fact that each definition of a physical entity presupposes the relationship between measurement process and object [20].

[2] I am using the term "substance", but I am perfectly aware of the fact that atoms and particles of QM are not substances as the things and the bodies of our daily experience are [7, pp. 424–479, 9, pp. 131–184, 19, pp. 3–22, 81–11, 36, p. 58]. Afterwards, within Rovelli's QM, the "relational" or "structural" character of the identity of the objects is fully revealed [14, 29].

3 A Philosophical Interlude: On the Heno-ontological Equation in Aristotle and Its Relationship to QM

The whole quantum ontology can be genealogically explained in considering the evolution of the basic ontological postulates settled by Aristotle. To Aristotle, the basic premise is the mutual apprehension of one and being: every time we say that a thing actually *is* something, we are identifying it *as it were one*, viz. as *a thing* [1, 1003b22–34, 1053b9–1054a19]. However, the very crucial element which allows the identification is the localizability of the one-being. According to Aristotle, this possibility is based on the strict separation between things and space, which makes of space the *periechon*, the events-holder. The distinction space-matter was the key that allowed ancient physics to set the question of motion after its refutation within the Eleatic doctrine, according to which space and matter seem to be reunited in the concept of *pleon*. Parmenides conceived the fulfilment of space by matter as identical with space itself: this meant that continuous space (*synechēs*) is always full of matter, and that the perfection of being, whose expression is even continuous space, excludes motion, since once space is filled there will be no motion into it.[3] This also happens in respect of time, because *synechēs* is continuous both as space and in time [13, 28 B8]. To reintroduce motion in physics, it would have been obvious to distinguish space from matter, by claiming that the former is a sort of hollow basis through which matter moves itself. The hollowness of the basis works in turn as the condition of possibility to make of space a rational concept or at least an empirical concept rationally analyzable—for instance just as events-holder. Furthermore, against the monistic concept of being and space upheld by the Eleatic philosophers, it would have been plain to affirm the plurality of principles for physics.

Finally coming back to Aristotle, we must notice that he is perfectly aware that the limits of the holder might be assumed as the *peras* of the held thing [2, IV, 210b32 and ff.]. Considering this triadic definition (one, thing, localization) necessary to accomplish the very concept of the individual being, it is plain to understand why in QM the determination of an individuality as "being" has become problematic, if the localisation of the particle is only relative to statistical inferences (especially as regards the relationship between two measurements of a *same* particle). Therefore it should be asked: what happens if the representation of individuality directly concerns the intrinsic nature of a quantized space?

4 The Roots of QG and Its Ontological Structure

In this quote by Werner Heisenberg a first programmatic explanation of the idea of quantum gravity is contained:

> The theory of relativity is connected with a universal constant in nature, the velocity of light. This constant determines the relation between space and time

[3] However, the impossibility of motion is argued above all starting from mathematical reasons, as Zeno's paradoxes well show [35, pp. 147 and ff.]; furthermore, this implies that there was no precise distinction between geometrical and physical existence [37, p. 251].

> and is therefore implicitly contained in any natural law which must fulfill the requirements of Lorentz invariance. Our natural language and the concepts of classical physics can apply only to phenomena for which the velocity of light can be considered as practically infinite. When we in our experiments approach the velocity of light we must be prepared for results which cannot be interpreted in these concepts. Quantum theory is connected with another universal constant of nature, Planck's quantum of action. An objective description for events in space and time is possible only when we have to deal with objects or processes on a comparatively large scale, where Planck's constant can be regarded as infinitely small… There must exist a third universal constant in nature. This is obvious for purely dimensional reasons. The universal constants determine the scale of nature, the characteristic quantities that cannot be reduced to other quantities. One needs at least three fundamental units for a complete set of units. This is most easily seen from such conventions as the use of the c-g-s system… A unit of length, one of time, and one of mass is sufficient to form a complete set; but one must have at least three units. One could also replace them by units of length, velocity and mass; or by units of length, velocity and energy, etc.… Now, the velocity of light and Planck's constant of action provide only two of these units. There must be a third one, and only a theory which contains this third unit can possibly determine the masses and other properties of the elementary particles. Judging from our present knowledge of these particles the most appropriate way of introducing the third universal constant would be by the assumption of a universal length the value of which should be roughly 10^{-13} cm, that is, somewhat smaller than the radii of the light atomic nuclei. If we assume that the laws of nature do contain a third universal constant of the dimension of a length and of the order of 10^{-13} cm, then we would again expect our usual concepts to apply only to regions in space and time that are large as compared to the universal constant. We should again be prepared for phenomena of a qualitatively new character when we in our experiments approach regions in space and time smaller than the nuclear radii [21, pp. 64–166].

Nevertheless, the first steps toward the definition of a spatial constant of nature were made before Heisenberg by Matvei Bronštejn.[4] He understood the incompatibility of GR and QM with the idea of an indivisible *continuum*, and he reckoned a *minimum* for the divisibility of space (a): $L_p = \sqrt{\frac{\hbar G}{c^3}}$ [18, pp. 423–425]. The result of (a) represents the tiniest length a particle might have; besides, it evidently concerns gravity (G), SR (c) and obviously QM ($\hbar$). However, its basic premise is quite different from that of the classic representation of QM's unity, since we now encounter not only the idea there are very small entities floating within space, but we must assume that space itself is quantized. Despite all the modifications of the classic idea of nature occurred in QM,

[4] As far as I know, Heisenberg worked at the idea of a universal length at least since 1938, as proved by the publication of a memory about the question: *Über die in der Theorie der Elementarteilchen auftretende universelle Länge* [23, pp. 301–314]. However, he picks the *minimum* of nuclear radii as spatial constant, in considering Planck length to be not coherent with the physical implications of the theory of elementary particles [23, pp. 306 and ff.]. So the very point is that we are still talking of *matter* and not of *space*.

it seemed indeed possible to sketch the presence of a particle in the terms of its own collocation into a place: for sure, it is not possible to depict a whole spatial evolution according to the idea of trajectory for the impulse would be uncontrollable, but the act of setting a particle into a given locus would still allow us to conceive that particle as it were *at least something localizable into a spot* [20, pp. 174–176]. In particular, there are no theoretical limits in conceiving place as if it were still taken from a continuous intellectual space: it is like, in this case, we may both uphold an imaginative continuous representation of space and a physically discontinuous conception of matter.[5]

Instead, if we assume that space is quantized, we should put aside the old assumption according to which the motion of a thing is made possible by its distinction from *space* (see Sect. 3). Therefore, the quantization of space is a clear argument in favour of the idea that reality is composed by elementary unities and significantly shows a tendency toward a discontinuous representation of nature. But from the physical point of view, such a representation would disable even that peculiar contrast, for it becomes in principle possible to show how *continuity* we should ascribe to space–time is the result of the composition of a strictly physical dynamics. The very progress is here gained in overcoming the old enigmatical Democritus' statement: *Mē mallon to den ē to mēden einai* [13, 68 B156], according to which space, as a material *nothing*, notwithstanding *is being* as the matter *is*. However, if we accept the idea that space is quantized, we may proceed in building reality through the settlement of an elementary composite dynamics which can correspond, ontologically speaking, to a monistic concept of *being*. Thus such an approach does not lead to the dangerous negation of motion [13, 28 B8, 2, I, 191a23 and ff], nor recalls the ontological paradox of the Atomists, but rather appears to be the realisation of *the physical concept of reality*, because space itself can be represented as the *minimum* element of a *whole dynamical context* [7, pp. 467 and ff.]. In a few words, one could say that in the premises of QG is expressed the idea of the accomplishment of the inner tendency of physical thought, aiming for the reunification of *becoming* and *being* in a sole concept [30, pp. 206–207].

As a consequence, the physical development of QG follows from the simple application of the quantum considerations made about matter to the problem of space: *continuum* itself is then justified as the macroscopic manifestation of a microscopic dynamics which is on the contrary discrete—an idea similar to that acting behind Boltzmann's equation for entropy. The mathematical model which allows us to describe the mechanism and the relationship between macroscopic and microscopic dimension is the Wheeler-DeWitt equation, within which *time* does not appear. This model lies at the basis of the *loop quantum gravity* (LQG)[6]: space is here quantized according to the idea that the spatial architecture can be derived from the interrelation of different rings, the so-called *loops*. Loops are the basic lines of the gravitational field, as happened for the lines of the electromagnetical field: the difference is naturally that these lines are now quantized. But in a more important acceptation, these lines are

[5] Such a distinction is also explicable starting from a genealogy of mathematical and geometrical definitions as interactions and progressive purifications of the intuitive manifold [7, pp. 403 and ff., 25, pp. 21–59, 103–135, 38, pp. 37 and ff., 65 and ff.].

[6] I will limit my exposition to LPQ, in considering the failure of the experiments concerning the supersymmetric particles and the difficulties thus arisen for the string theory [33, pp. 186–192].

not collocated into space, but *do build* space, for they are lines of the gravitational field [33, p. 141]. The *consistence* of space is then realised through the consideration that lines do not directly generate the *actual appearance of space*; this is rather due to the structure in its points of connections. The three-dimensionality of space is indeed obtained by considering the “graph”, viz. the cumulative result of the lines *linking* together the points of intersection (“nodes”) of the lines. The volume we perceive within our experience depends, then, on the nodes and on the graph.

Volume is quantized too: the nodes appear thus to be the elementary *quanta of space*. The space between two nodes is a small surface, which is also quantized: the values of a surface can be only discrete, and they are specifically multiples of a given quantity—(b): $A = 8\pi L_p^2 \sqrt{j(j+1)}$. As a consequence, from the physical point of view, space seems to be the sum of the different quanta composing the gravitational field [31, pp. 13 and ff., 33, p. 146].

Accordingly, the very physical implication appears to be the refutation of any merely theoretical *continuous* representation of space, since if it is true that the nodes of the graph are not quanta fitting space, they are however the actual structure whose combination originates space: this means that continuity is produced by discrete elements. As a consequence, the problem of their localization has to be seen under a different light, because we do not have a monad and a place to be tied together, but unities put together to *create* space—the monads of matter and the monads of space.[7] From this physical argumentation, it follows that the physical space has not only a material fulfilment, but rather *is* this fulfilment: in the differentiation between *having* and *being* a *content* took root the functional separation of the empirical definition of space from its logical use in a general theory of experience [6, pp. 409–429]. In this way, one could jointly justify the empirical meaning of the concept without rejecting its a priori nature as logical function of intellect. Indeed, when it was solely matter to be quantized, a continuous representation of space could more plainly survive at least as imaginative framework[8]; on the contrary, when space becomes quantized, the situation seems to change, because the theoretical emptiness has now to face an undeniable materialisation. In short, we could not say that space is not an “*empirischer Begriff*” [11, B37/A22 and ff.]; hence we must ask whether the representation of space as a priori concept is still possible.

However, it is also true that a substantial conception as such is not allowed at all; the reason is the same according to which the discovery of the elementary quantum of action does not represent the triumph of substance. The presentification of a particle depends on statistical predictions, which physically means that its definition is related to a specific interaction. There is no particle simply definable as a punctual and absolute one existing in space, because this oneness depends, for instance, on the rays of light we are using to determine the position of the particle itself. By the same token, spin networks (the whole structure of a graph endowed with a volume for each node and a j number for each line) are not *substances*, but they rather are the expression of how space *acts on* things; besides, if no interaction is taken into account, *space* is

7 On the contrary, monads were in the past conceived as unextended simple substances, for instance in Kant [17, pp. 25 and ff.].

8 This is still possible in QM, since time is not fully determined as an operator [5, p. 62].

reduced to a physical *potential* [31, pp. 14 and ff.], as Heisenberg thought of matter [21, pp. 147–166].

Similar considerations may be made about time. Since SR, with Einstein's reinterpretation of Lorentz' equations, time cannot be conceived as an idle attribute of physical reality, but must be rather treated as an actual physical variable subject to a real computation and to an empirical and metrological definition. In considering that within the context of LQG granularity is also valid for time, the apparently paradoxical disappearance of time in Wheeler–DeWitt equation can be plainly explained by bearing in mind that this equation is the actual expression of the fact that, as for space, the definition of time can occur only after considering the inner dynamics of the basic structure of physical reality. At quantum scale, time is the result of the multiple interactions among microphysical events, and cannot be absolutely defined.

5 Space and Time Within LQG and the Concept of Physical Event

I would like now to consider the question under a more specific point of view, since GR has underlined the urgency of considering space and time not as isolated entities, but as the whole framework according to which physical events must be described [31, p. 15]. Hence the main question arising after our presentation, is the following: if space–time appear to be quantised, viz. if its discrete nature is revealed, how might a physical event be actually determined?

According to the general ideas of the theory of relativity and of QM, we may also think of a physical event as something happening in a place and within time, even if both theories apply some basic restrictions to the old *Weltanschauung*. If we consider Heisenberg's argumentation about the development of scientific world and language as a progressive implementation and purification of the natural language [21, pp. 167–186], this going far from ordinary experience seems to be perfectly clear: the starting point is represented by the first depuration of classical mechanics, which still considers physical events through mathematical functions describing the evolution of physical quantities according to time, whereas the approach of the theory of relativity and of QM is determined by the "observability principle"[9] and thus implies the urgency for treating time as a real physical magnitude. Both the theories recognise, in fact, a global metrification of physical reality: within them, a physically consistent description of a natural being becomes possible solely when its observation is made coherent with the theoretical formalism; in turn, the observations are purified from any other implication, and the empiric determination of a magnitude strictly depends only on what we *actually* observe. Therefore, both the theory of relativity and QM do limit experience according to some conditions of possibility[10]: the constancy of the speed of light and the uncertainty relations.

[9] The principle according to which the meaning of a physical concept exclusively depends on its empirical observability [22, p. 262].

[10] This sort of metrological a priori, divergent from the pure logical meaning of Neo-Kantianism, is clearly recognized by Cassirer [6, pp. 409 and ff.].

However, as I have already suggested, despite these relevant restrictions, there is still room for thinking of an event as though it were actually individuated in space and time. Cassirer has brilliantly showed us how to do that: if it is true that we cannot allow an intuitive representation of physical events as causal connection of substances in time anymore, and that the empirical determination of the content of a physical entity is prior to any intuitive definition, a functional primacy of the a priori can be recovered in considering that our approach to experience, even that of science, has always to be considered according to the work of some intellectual functions: space, time, number and function [6, pp. 266–270, 7, pp. 475 and ff.]. These elements represent the pivotal "invariants of experience", for their definition does not depend on their empirical fulfilment, but their content coincides with their function: viz. space is the concept showing the synthetic function of coordination of the "*Nebeneinandersein*", whereas time that of the "*Nacheinandersein*".[11] Accordingly, although it appears to be significantly modified, the transcendental approach seems to be once and for all preserved: indeed, in definitely conceiving the a priori in its independence from the empirical fulfilment, Cassirer's Neo-Kantianism creates a definitive form of transcendental science, which can be now thought as "*die radikal durchgeführte und ins Unendliche durchzuführende Wissenschaft vom Transzendentalen*"[12] and which can be considered prior to any empirical science.

The coherency of such an approach is guaranteed in the conceptual developments of the Copenhagen interpretation of QM, above all as far as the determination of an original level before that of actual scientific experience is concerned. Bohr and Heisenberg were fully aware that the physical determination of a quantum experience is connected to the basic separation between observer or observation context and quantum object [3, p. 3]: this means we cannot prescind from the classical world and its events in defining microphysical entities, because we would not obtain any specific determination without considering the passage from the observer world to the actual physical situation we are studying. This also entails that the classical world with its language is indispensable to the scientific definition of the quantum experience, both from a physical and an epistemological point of view: (1) on the one hand, the relational interpretation of QM is prepared by the argument of the interaction between the whole world and the microphysical quantum event [21, pp. 54–55][13]; (2) on the other hand, in order to express the premises and the results of an experiment, one

[11] We must notice that this functional definition was already clear in Kant [*KrV*: B42/A26-B43/A27, B48-49/A32]. By this point of view, the very problem of *Transcendental Aesthetics* is the representation of motion according to its dependence on the thing which is actually moving [11, B 57/A40, B58/A41]. The situation seems in any case to be partially changed in [27, pp. 476 and ff.].

[12] The statement is extrapolated from a letter Husserl sent to Cassirer on April 3, 1925 [10, pp. 84–86]. I only add that this constant application into which transcendental method consists, is interpreted by Cassirer in a quite orthodox version of a metaphysical meta-science of physics, whereas in Husserl it will lead to the idea of phenomenology and of the analysis of the genetic laws of objectivity, starting from the eidetic science of *Lebenswelt* [26, pp. 163 and ff.].

[13] I am convinced that Rovelli's refutation of the classic Copenhagen dichotomy between classical and quantum world is not decisive at all [14, pp. 12, 22]. I do clearly not discuss the physical foundation of this argument, which is related to the basic superposition principle [32, pp. 194 and ff., 34, pp. 113–114, 142); I only will try to show that from the *imaginative* point of view, and thus in considering space and time as intellectual functions, we might still need this division as pure methodological construction of experience.

must still use classical language, so that one does not only describe the supplies and the different tools, but also expresses the new picture of reality according to the only language we have. This state of things probably allows us to set the basic problem of a metaphysical theory of science conceived as *theory of original appearances*: if the empirical definition of a physical entity cannot lead us to an unproblematic definition of events happening in space and time, it is also true that the *a priority* of the functions of experience does also orient the construction of the concept of scientific experience: avoiding the urgency of the definition of a metaphysics of science would also mean risking the loss of the very meaning of scientific enterprise.

Now we are finally ready to ask our question about the nature of physical event and the *apriority* of space and time more explicitly. The real difficulty, as I have already suggested, consists of the fact that the pure definition of space and time in terms of different intellectual functions of coordination seems to be more problematic than before in LQG. As far as the theory of relativity is concerned, it is easy enough to show that every inference which can be made by Einstein's observer is based on those functions, on the fact that in order to empirically define the concept of simultaneity the presumption of a judgment according to which simultaneity is the contemporary happening of two different events at any time is needed, as well as in GR. Despite the definition of the whole *continuum* of space–time, the orientation of an observer in a gravitation field still follows the basic rules of experience: the *Nebeneinandersein* and the *Nacheinandersein* of physical events [6, pp. 420 and ff.]. In respect of QM, I have instead pointed out that the theory of the primacy of language on the definition of the microphysical events is clearly expressed, and that from this point of view holding a Kantian determination of experience is even easier—the structures of our language are indeed built on those functions and needed them to allow significant representations of empirical concepts.

But in LQG a new situation arises from the puzzling determination of physical reality: the importance of their empirical fulfilment for the definition of space and time seems to cast growing shadow over the possibility of considering the a priori according to its pure *functionality*. Indeed, space–time is the result of the interaction between the quanta of space and matter, so that nature seems to coincide with an original whole, which can be indifferently described in terms of primitive matter or energy [21, pp. 61–62, 33, p. 167]. It appears, then, that the difference between space and matter can still be held only nominally, because in truth the elementary processes do only happen as results of the original interaction between the two—until the point that a clear separation solely appears to be an epistemological trick. To me, this seems to be especially evident when the abstract nature of the spin-network is emphasized [33, pp. 110 and ff.]. Accordingly, physical events seem to lose also that shape of individuality which is still possible to assign them in a standard quantum context or in a relativistic one.[14]

[14] As regards the question of individuality in QM, it must be pointed out that the experimental situation seeming to reject the possibility of a strict individuation, can be reconsidered in the light of the concept of "absolute becoming", according to which "an event *e* 'becomes' in an absolute sense (or 'comes into existence') at a certain time-place simply means that *e occurs* or *happens* at that-time place" [14, p. 24]. Indeed, if it is not possible to reconstruct the *whole space* of a particle, it is also true that the collapse of the wave function produced by measurement actualizes the particle and gives us the actual meaning of

The very matter is, then, that within LQG the description of space–time directly participates in physical events, so that the room for an independent and functional representation seems to be more exiguous. Within this framework, a *physical event* appears to be determined by the *motion* of a spin-network, and the intrinsic dynamics of the spin-network together with its relationship to the others determines the geometry of reality: physical events are conceivable as the *spinfoams* emerging from this basic movement [31, pp. 18–19 and 231–264, 33, pp. 161–165]. Of course, the computation about the spinfoams must be regulated by statistical rules and they cannot be conceived in a too intuitive way [34, pp. 138 and ff.], as foreseen by quantum principles.

In any case, it appears very clearly how the definition of a physical event lead us back to the verification of the ideas of the ancient Atomism [21, pp. 59–75, 33, pp. 209–210], according to which natural processes and events were the results of a general dynamics of monads geometrically determined. Therefore, physical events becomes that spinfoam originated by the transference of the spin-network and its relationship with other networks—into which it can be transformed too: space–time in particular appears to be the result of the motion of the whole spatial quantised framework. As a consequence, the very ancient aim of physical science to build a geometrical theory of motion is fully realised within LQG.

If one may ask Rovelli about the actual content of reality, he would "simply" answer: "Particles are quanta of quantum fields; light is made up by quanta of a field; space is also a quantum field; time arises from the processes of the same field. In other words, the world is entirely made up by quantum fields" [33, p. 166]. Hence, natural reality can be conceived as a superposition of different fields. The *reductio ad unum* is finally reached through the definition of the interactive context of covariant quantum fields.

Footnote 14 continued
quantum individuality. If we assume it is perfectly localised, we will not able to reckon particle's velocity, but this will solely mean we will not get any trajectory of the particle. Therefore, assuming the measurement perturbation, we may define individuality as it were more selective as regards its unity, as though the worth of this intensive definition loses its meaning as soon as we try to connect observations: in considering two observations, the concept of identity cannot be sustained anymore in its classical representation (as if the object were not changed by the physical interactions into which it is involved). As is well known, Heisenberg fixed the glitch concerning the problem of individuality recalling Aristotle's concept of potentiality [21, pp. 147–166]: hence he solved the question of the connection between the statistical considerations to be made as regards the evolution of particles, by recalling the concept of a unitary matter (energy). By the same token, the interpretation of Rovelli's QM seems to imply a "dispositionalist" or "propensitive" interpretation to justify the fact that "the only reality is given by events that are the outcome of interactions or mutual information between two different systems" [14, pp. 14, 10]: thus, at first glance, the locality of Rovelli's QM seems to be compatible with the meaning of individuality we upheld. However, I must add that a whole definition of individuality is not here accomplished, for I did not take into account one of the most important questions concerning quantum field theory: the problem of the identical particles, viz. of those particles which are in principle undistinguishable, for they have the same physical properties but they do not satisfy some basic assumptions of Aristotle's ontology and especially Leibniz's principle of indiscernibles. Finally, in settling the problem of individuality in quantum physics, it must be considered the basic distinction between the oneness of a particle and the oneness of a quantum, which do accomplish different requests, e.g. the circumstance that particles can still be considered at some extent, as said, in their concrete localizability, while the unity of quanta is not basically defined by localization [36, pp. 89–90]. Hence I would solely like to specify that my arguments do concern the attempt of giving a transcendental definition of the particle individuality as logical and general a priori pattern, the strict minimum necessary to address the main question we asked upon the nature of space and time.

Hence the stepping back to the Presocratics is to be considered clearly enough as an accepted result of the philosophical enquiry about quantum gravity, both in considering the recalling of a strict concept for the unity of nature and for the idea that the relations among beings can be explained with the assumption that there are microscopic geometrical entities determining appearances.[15] Nevertheless, I am more interested in the definition of the guidelines of a theory of experience describing the grounding structures which lie at the basis of the physical concept of reality, rather than in knowing how this whole reality can be conceived as such.[16] We can start by pointing out that what changes between the old representation of the floating quantum as something moving into a place and *having* a spot and the new conception of the quantum of space, is that we do not have any possibility of arguing the motion of *something*, for we would need to allocate the particle in an undifferentiated *continuum* which does not seem to exist even in our imagination, since space itself is made up by those quanta of which we would like to grasp individuality. In the utter realisation of the idea of a physical description of reality without strictly distinguishing between space and matter, the perspective seems to be a return to Anaximander and to his idea of a primitive matter (*apeirōn*): physical events and beings are only the apparent result of the intrinsic modifications of this whole [15]. But to our crucial question no answer has yet been given: can it still make sense to talk about the primacy of space and time as logical and separated functions, if physical reality is only this quantized gravitational whole?

6 On the Possibility of Keeping a Kantian Point of View in the Description of Reality

To Rovelli, the quantisation of space would imply a patent rejection of Kantianism:

> The conceptual price to be paid—for QG—is the refutation of the idea of space and time as general structures within which world is collocated. Space and time are approximations rising up according to large scale. Kant was perhaps right in claiming that the subject and the object of knowledge are inseparable, but he was wrong in arguing that Newtonian space and time could be a priori forms of knowledge, the very basic elements of a crucial grammar according to which world can be comprehended. This grammar has evolved into something else and it evolves together with the growth of our knowledge [33, p. 168].

Excerpts like this one do misunderstand the actual spirit of transcendental approach, as well as the actual history of critical philosophy, which has made its milestone of the

[15] I think that the peculiar combination of ideas remounting both to the concept of *prōtē hylē* and to the concept of the existence of atoms allow a quite plain solution of the epistemological problems related to possibly upholding monism in Rovelli's QM (which summarized in one statement may be put as follows: "The problem with priority monism is that it concentrates exclusively on the dependence of the part from the whole, neglecting completely the converse type of dependence", [14, pp. 25–26]), for he refuses to consider LQG as UFT [31, pp. 9 and ff.].

[16] It is also clear that the analysis of the structures of knowledge does not mean to underestimate the influence that the structures themselves have within the ontological determination of reality [6, IV].

constant confrontation with the actual development of science [12, B6/C10, 8, p. 37]. The problem explained here by Rovelli results, indeed, to be already solved within Kantianism, thanks to Cassirer's work; thus, in order to be clear once and for all, I would like to summarize Cassirer's crucial arguments.

Cassirer was fully aware of the fact that the general metrification occurred in the history of physics with the theory of relativity would have implied the necessity of reconsidering the foundation of *Transcendental Aesthetics*. That system was indeed conceived on the basis of Euclidean geometry, the language, together with the algebra developed by Leibniz and Newton, of classical physics; it is then obvious enough that if the physical presumptions can be changed, a reconsideration of the philosophical question is needed. However, Cassirer did not modify Kant's specific point of view, but he rather stressed the functional attitude of the a priori forms which was already present in the text of *Kritik of reinen Vernunft*. In conceiving the *a priority* of forms according to a functional meaning, and by pointing out that this functional meaning is the content of the form insofar as we must conceive it a priori as a synthetical rule, he could also justify the empirical fulfilment of the form itself as a secondary moment. It is true that in this way the a priori forms of *Aesthetics* are more similar to categories than in Kant's own system, but it is also evident that this move allows Cassirer to keep the fundamental architecture of Kantianism safe against every specific development of physical science [6, pp. 409–429].

The main consequence of such an approach is the overcoming of the concept of *thing* or *substance*, and the settlement of the general aim for "a theory of the invariants",[17] able to wholly understand experience (*supra*, Sect. 5).

The remarks Cassirer makes about Einstein's theory can be of a particular interest. They are basically focused on the role of the observer and on his weight for the definition of the physical situations within the theory of relativity. The main thesis is that the definition within *judgement* has to be premised to any metrological determination of a physical concept.[18] In Cassirer's words:

> None of these concepts can be spared or be reduced to another so that, from the standpoint of the critique of cognition, each represents a specific and characteristic motive of thought; but on the other hand, each of them possesses an actual empirical use only along with the others and in systematic connection with them. The theory of relativity shows with especial distinctness how, in particular, the thought of function is effective as a necessary motive in each spatio-temporal determination. Thus physics knows its fundamental concepts never as logical "things in themselves", but only in their reciprocal combination; it must, however, be open to epistemology to analyze this product into its particular factors.

[17] Physically speaking, the refutation of the concept of substance completely agrees with the new ontological context of QM, according to which the primary matter and the very form of being must be conceived as a sort of "*omnipresence*" [9, p. 217].

[18] I should here remark that this argument does not imply a defence of subjectivism: observer, indeed, does not directly influence the definition of objectivity, which in this case would be meaningless as such, but he has to realize that every judgment upon scientific experience presupposes the work of some basic functions of coordination, which do not depend on any specific axiomatic system. Accordingly, the crucial problem becomes the grounding of the theoretical validity of those functions.

> It thus cannot admit the proposition that the meaning of a concept is identical with its concrete application, but it will conversely insist that this meaning must be already established before any application can be made. Accordingly, the thought of space and time in their meaning as connecting forms of order is not first created by measurement but is only more closely defined and given a definite content. We must have grasped the concept of the "event" as something spatio-temporal, we must have understood the meaning expressed in it, before we can ask as to the coincidence of events and seek to establish it by special methods of measurement [6, p. 420].

Cassirer's defence of transcendental approach is watertight: in reconsidering Kant's assumptions only according to a functional point of view, he can show the logical transparency and immutability of Kantianism for the building of a theory of experience; besides, he is also able to settle a real confrontation with the proper development of science, for science is set free from any particular definition of the a priori knowledge. The empirical content of the a priori is empty, for its own a priori content is only its *function* [16]: what we may expect to find in the a priori knowledge is a coordinative impulse in the building of experience, and not a particular content idealised into a concept.[19] In turn, this openness of the a priori allows us to solve any possible contrast between the definition of the *a priority* of the coordinative function and the empirical fulfilment due to measures.

As I have pointed out, this kind of interpretation is still more evident in QM, since the determination of the actual scientific experience is influenced by natural language and thus by our daily experience [21, pp. 144 and ff.]. This means I cannot approach quantum experience ignoring the coordinative functions acting through language: for instance, in describing the dynamics of a particle, even if I am perfectly aware of the pointlessness of the concept of trajectory, I am still forced to refer to the motion of the particles in the classical terms of *something* which is actually moving in a *continuous*, even though only imaginative, space at a given time. And this happens because I do not have a quantum language which is capable to perfectly hit the nail on the head as regards the intrinsic duplicity of quantum entities, which are jointly waves and particles.

It should now be finally asked if such an interpretation might also be held for LQG. As Dorato pointed out [14], the most important unexplained difficulty is to wholly understand the link between the quantized space and the macrophysics of our experience, if Rovelli is right in claiming that the latter is strictly determined by the former, and in considering that he denies a strict dichotomy between the two worlds (*supra:* footnote 13). This point is crucial, since the transcendental approach seems to imply the urgency of the preliminary definition of a *meta*- or *pre*-physical science of physics, which would be able to describe the articulation of the coordinative functions necessary to create every experience—thus it also seems to presuppose the priority of the context of our normal experience, from which it tries to derive a science of

[19] So the one and only preliminary assumption is the *Eindeutigkeit der Zuordung*, which means the "unity of nature", as Cassirer stated in a 1920s letter addressed to Schlick [10, pp. 50–55]; see also: [6, pp. 374 and ff., 414 and ff.].

the original appearances. Both these intuitions have been indeed developed within the history of transcendental philosophy: (1) on the one hand by Cassirer's theory of the logical invariants and (2) on the other hand by a *phenomenological* theory of nature inspired by Husserl's work.

The most important objection which can be upheld against the transcendental approach probably concerns the very complex question of the difference between space and matter, as a perfect clarification has hardly been made. For instance, in Rovelli there are many expressions which seem to go toward the direction of a perfect solution, above all when he recalls Heisenberg's idea of the whole physical reality as primitive energy defined in the terms of Anaximander's *apeirōn*; but the determination of the quantized nature of space and the definition of physical events according to the scheme of spinfoams still leaves room for the interaction of the quanta of space and matter [33, p. 159], viz. for a divergent representation of the nature of space and matter. And the inner contrast is paradoxically clear when he instead affirms: "The distinction between matter and spacetime (gravity) is not profound: it is largely *conventional*" [31, p. 206, italic mine]. In conclusion, from the imaginative point of view, I think there is an insufficient specification concerning the definition of the nature of the quantum of space and that of the quantum of action; in a few words, in which sense is the quantum of space *the basic oneness* as the quantum of action was? Which is the relationship between the *oneness* of space and the *oneness* of matter? Is it the same kind of *oneness*?

There would be a lot of reasons against the possibility of a transcendental metascience of physics, since physical reality would be determined according to the modifications of a primitive matter, within which space–time is conceived as the geometrical motion of a whole physical and interactive structure. However, I believe that the functionalist version of Kantianism can still be held. The definition of the quantized space does not indeed deny the urgency of keeping the basic coordinative functions of experience: "In fact, a spin network state is not *in* space: it *is* space. It is not localized with respect to something else: something else (matter, particles, other fields) might be localized with respect to it… It is 'where' with respect to which anything else can be localized" [31, p. 14]. There is still a relationship of something to a "where", and thus the possibility of arguing the logical study of this coordination emerges. Moreover, if it is true that a physical event is only the result of the spinfoam actualizations, we will still need the functions to establish a valid scheme of coordination in describing the physical situation [39]. Are we in truth able to represent the being next of the nodes of the spin-network without preliminary assuming the function of the *Nebeneinandersein*? Another quote by Rovelli:

> There is a crucial difference between the photons, quanta of the electromagnetical field, and the nodes of the graph, "quanta of space". Photons do live in space, whereas the quanta of space are space themselves. Photons are characterized by "where they are". On the contrary, the quanta of space have no place to be in, for they are "place". They carry out another crucial information, which characterizes them: the information about which are the other quanta of space next to each other – what is next to what. This information is expressed by the links of the graph. Two nodes tied by a link are two close quanta of space. They

> are two grains of space touching each other. It is this "touching" to build the structure of space. These quanta of gravity represented by nodes and lines, I repeat, are not *in* space, *they are space*… The localisation of the single quanta of space is not defined to something, but it is only determined by links, and only according to the relationship that the one has to the other… And then there is the other novelty of quantum mechanics: we must not think of things as "they are", but rather of as "they interact with each other". This means that we should not conceive spin-networks as physical entities, as though they were a grate upon which world is resting. We should think of them as they are the effect of space on things. Between two interactions, as well as an electron does not stand in a place, or it is diffused in a probability cloud over all places, space is not a specific spin-network, but a probability cloud over all possible spin-networks. At a very tiny scale, space is a floating pullulating of quanta of gravity acting on each other and all together they act on things, and they manifest themselves in these interactions as spin-networks, grains in relationship to each other. Physical space is the whole tissue deriving from the constant pullulating of these plots of relationships. Taken as they are, lines are nowhere nor at any place: they create, through their interactions, places. Space is made up by the interaction of the quanta of gravity [33, pp. 150–152].

As is clear, the question is considering to what extent the character of this proximity is purely logical, so to what extent the link tying two nodes is something physical or a mental connection. In order to do that, I start from some considerations about the definition of the transcendental argument.

Kant's approach worked until we conceive physical reality as given [28, remark III: pp. 44 and ff.]: according to such an assumption, Kantianism is good for conventionalism as well as for realism. In fact, on the one hand we may consider the circumstance that we can reduce knowledge to some logical premises in grasping external reality, and, on the other hand, the assumption that reality is given saves the basic argument of realistic theories. Critical philosophy solely works as long as we presuppose external reality, even though we shall conclude that it does not exist as such, but only in respect of knowledge, whatever could be its specific form. In a few words, we always risk to be Kantian, if we perceive ourselves either as rationalist or empiricist. But how things stand in LQG? The question is to understand the nature of the relationship between the different quanta of space. According to Rovelli, space as dimensionality is produced by links, but we must still think of the relationship between quanta: hence it seems that whilst the mere being of the different quanta precedes the connection and then space, it is actually the link to create space. Therefore, if place[20] exists only as a result of the interaction of specific quanta, it might be considered as the synthetic product of

[20] It could be perhaps desirable to specify the terms of the question by highlighting the difference between space and place, recalling Aristotle's definition of space as "extension of magnitude" [2, 209b5 and ff.]. In its general meaning, *space* is the general opening of dimensionality, originated by the run and the different history of the quanta of space; here we still find ourselves in a world of pure potentiality, within which nothing *has* its position yet. *Place* is on the contrary the single spot as a result of the interaction among quanta, so that we may say that between space and place runs the same relation valid between potency and act.

a network of relations. It follows that in this case the relational character is assumed *before* the existence of quanta, and we solely have to decide if the relation is a logical attribute.

Hence there are basically two questions: (1) how to conceive proximity and (2) how to depict the nature of the concept of space. We must begin from the second point. Kantian's revolution started from a simple but decisive idea, namely that of considering space as a non-empirical concept: this allowed Kant to treat it as "intuition", as a peculiar form of knowledge amidst intellect and sensitivity. On the contrary, in LQG space is composed by quanta, and the synthesis, which in Kant still remained an intellectual function, appears to become physical. Thus the proximity function, into which space appears to consist, loses its merely theoretical character, and becomes a physical information owned by the quanta of space. The synthesis of experience is then realized from inside. But the fact is that the link appears to be more a geometrical creation than a spontaneous process of nature. Accordingly, the link seems to stay once again as the exigency of claiming the logical function of the *Nebeneinandersein*. In fact, *in order to think that quanta are next to each other, we should presuppose the logical relationship of their proximity*, and we express that through links. Proximity seems then to subsume the same existence of quanta and to present itself as somehow a priori.

As regards time, instead, it is sufficient to say that the motion of the spin-network would still confirm an orientation, despite the motion of the spinfoam cannot be envisaged according to the concept of *something* univocally moving in space and time, and then the reference to the function of the *Nacheinandersein* is still required. This means we may empirically define time as the product of the motion of a basic space–time structure, according to the specific rules of computation, but we would never fill the logical function of the *Nacheinander* with an empirical content: we only use the function to establish the general schema of the existence of a *path*, whose content only after can be actually added.[21] Therefore, we shall presuppose the distinction of the two a priori logical functions to reconstruct the image of the inner dynamics of the new physical whole in the terms of *experience*.

7 Conclusion

I did not discuss some important issues about QG, above all the tendency of expressing the theory according to the concept of "information" and the relationship of the theory with cosmology and thermodynamics. As far as the first point is concerned, the concept of "information" implies the full acknowledgment of the statistical charac-

[21] Zinkernagel applies this argumentation to a field into which LQG has proved its fruitfulness, viz. the Big Bang theory. He points out that if the model foresees that it is impossible to speak of the very early stage of the Universe in respect of time, it is also true that the concept of a chronological coordination must be premised, otherwise the same attempt to individuate the very beginning of the Universe would not make sense. Indeed, no time would mean also the impossibility of arguing the concept of beginning [39, pp. 13–15]. I also mean that such an interpretation can be endorsed in considering that the definition of the quanta of space does not allow us to consider physical reality as if it were actually built by ultimate substances and thus empirically filled: the quanta appear rather to be the consequent manifestation of "a quantum superposition of states whose geometry has discrete features, not a collection of elementary discrete objects" [32, p. 110].

ter of physical knowledge, and clearly expresses the interactive spirit of the quantum approach, according to which no isolated physical event actually exist: every physical system carries a piece of “information” upon another system with which it has interacted, and the physical explanation, in order to be exhaustive, must take it into account [14, pp. 4–5, 33, pp. 207–223]. As regards the second point, the most important developments which seem to prove the success of the theory are related to it. From the cosmological point of view, QG appears to be very helpful in explaining the Big Bang theory and especially the foreseen very concentration of mass and energy, which is subsumed into a cyclical model explaining our actual universe as the result of the implosion of a precedent system which collapsed standing at h scale [4]. From this point of view, the Big Bang would cease to be a sort of metaphysical beginning without starting point (see footnote 21). Furthermore, a very important step forward seems to have been made for what concerns the question of the heath of the black holes and in the explanation of the nature of time as derived from the behaviour of heath too.

However, I did not treat these problems considering that the theory has not yet reached a full physical definition and especially because for the discussion about the validity of an a priori representation of space and time it is enough to present its basic architecture. I have here proposed some argumentations in defence of a transcendental meta-science of physics, and I suggested that its more appropriate form could be conceived in the terms of a theory of logical invariants, following Cassirer’s methodology. In any case, I finally point out the problems related to the definition of such a meta-theory: (1) firstly, the relationship between LQG’s world and the macroscopic one in which we live,[22] if the latter is somehow mechanically derived from the dynamics of the former and no strict distinction is allowed; (2) secondly, the hidden tension acting behind the logical invariants, which seems to imply the urgency of a theory of original appearances historically connected to the phenomenological primacy of the *Lebenswelt*; (3) thirdly, the possibility that the primacy of the proximity between the quanta of space confuse the level of representation with that of physical reality.

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[22] As far as I can see, this relationship seems to be more clear in the string theory, above all for the intrinsic tendency of the theory to give a unified account of natural forces [5, pp. 71 and ff].

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