Hierarchical Coordinate of a Family Dynamic in a Family Tree by an Inherited Family Name

Let $G = (V, E)$ be a directed graph representing a family tree, where each vertex $v \in V$ corresponds to a **single-sided Family Dynamic** encoded as a JSON object with the following fixed schema:

- **Id**: A unique identifier for the vertex (Family Dynamic node).
- **Inherited Family Names** v_F: A set of family names inherited by the member.
- **MemberId**: A reference to the person collection in the DocumentDB.
- **In-law Id** (optional): A reference to the in-law in the person collection.
- **FamilyDynamicId** (optional): A reference to the family dynamic record in the DocumentDB.

Each directed edge $(v, u) \in E$ represents a **parent-child** relationship, indicating that u is a child of v.

Subgraph Extraction by Family Name

Given a specific inherited family name f, define a subgraph $G' = (V', E')$ such that:

$$V' = \{v \in V \mid f \in v_F\}$$

and E' retains all edges from E that connect nodes within V'. To ensure a tree structure, we introduce a synthetic root vertex r, connected to each vertex in V' that has no parent in V'. This yields a rooted tree:

$$T = (V' \cup \{r\}, E' \cup E_r)$$

where E_r is the set of edges from r to root-level Family Dynamics in the subgraph.

Hierarchical Coordinate Assignment

Let $w \in T$ be a target Family Dynamic. To compute its **hierarchical coordinate**:

1. Find the unique path $P = (r, v_1, v_2, \ldots, w)$ from the root r to w in the tree T.
2. At each step along this path, enumerate the children of the current parent vertex *from left to right*, assigning labels $1, 2, \ldots$ in the order they appear among that parent's children.

3. For each vertex $v_i \in P$, let $L(v_i)$ denote its **position among its siblings** (children of its parent).

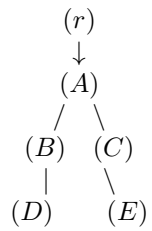
4. Collect the labels into an ordered list:

$$\mathcal{H}(w) = [L(v_1), L(v_2), \ldots, L(w)]$$

This list uniquely encodes the **hierarchical position of** w within the tree rooted at r, as filtered by the inherited family name f.

Example

Consider the following tree structure:



- A is the first child of r: $L(A) = 1$
- B, C are children of A: $L(B) = 1$, $L(C) = 2$
- D is the first child of B: $L(D) = 1$
- E is the first child of C: $L(E) = 1$

Now consider the path $r \to A \to C \to E$. The hierarchical coordinate of E is:

$$\mathcal{H}(E) = [1, 2, 1]$$

Note that although D and E are at the same depth, their sibling groups differ due to different parent vertices, and each is the first among its siblings.