# Relationships

Computational Social Intelligence - Lecture 08

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This lecture is based on the following text (available on Moodle):

- D.C.Howell, "Statistical Methods for Psychology", Chapter 9, pp. 244-257 (included), pp. 273-274 (included), Cengage Learning, 2009.

# Outline

- Introduction
- Regression
- Correlation
- Conclusions

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# Relationships

"When we are concerned with relationships […] the experimenter is interested in showing that the dependent variable is some function of the independent variable."

D.C.Howell, "Statistical Methods for Psychology", Chapter 9, Cengage Learning, 2009.

# Scatterplot

"One of the most useful techniques for gaining insight into [a] relationship is a scatterplot (also called a scatter diagram or scattergram)."

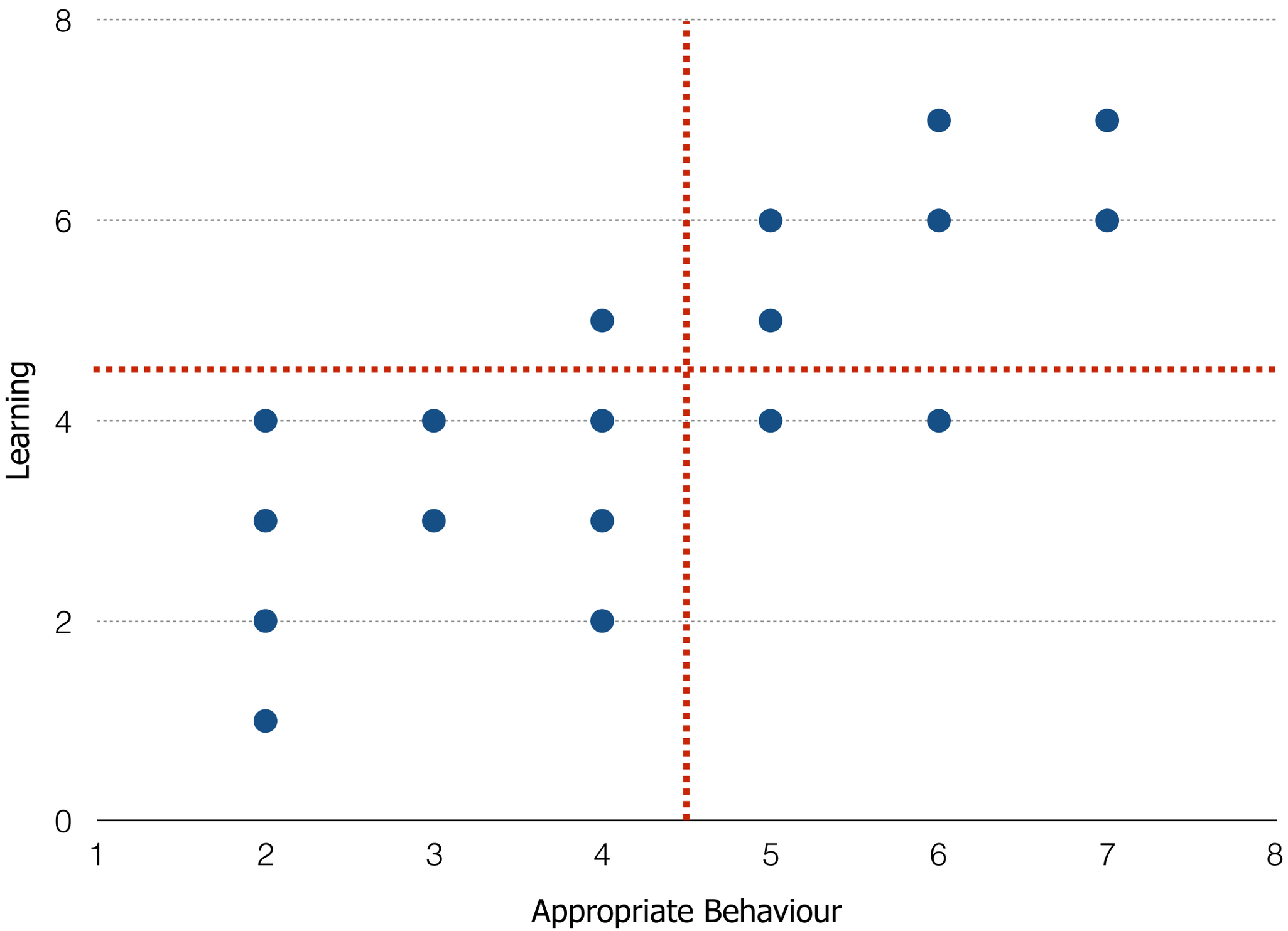
D.C.Howell, "Statistical Methods for Psychology", Chapter 9, Cengage Learning, 2009.

The independent variable (the predictor) is typically on the horizontal axis

The dependent variable (the criterion) is typically on the vertical axis

$$(x_i, y_i), i = 1, \ldots, N$$

The number of pairs at disposition


8
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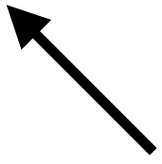
Learning
Appropriate Behaviour

Appropriate Behaviour



| | Above Mean | Below Mean |
|---|---|---|
| Above Mean | 7 | 1 |
| Below Mean | 2 | 10 |



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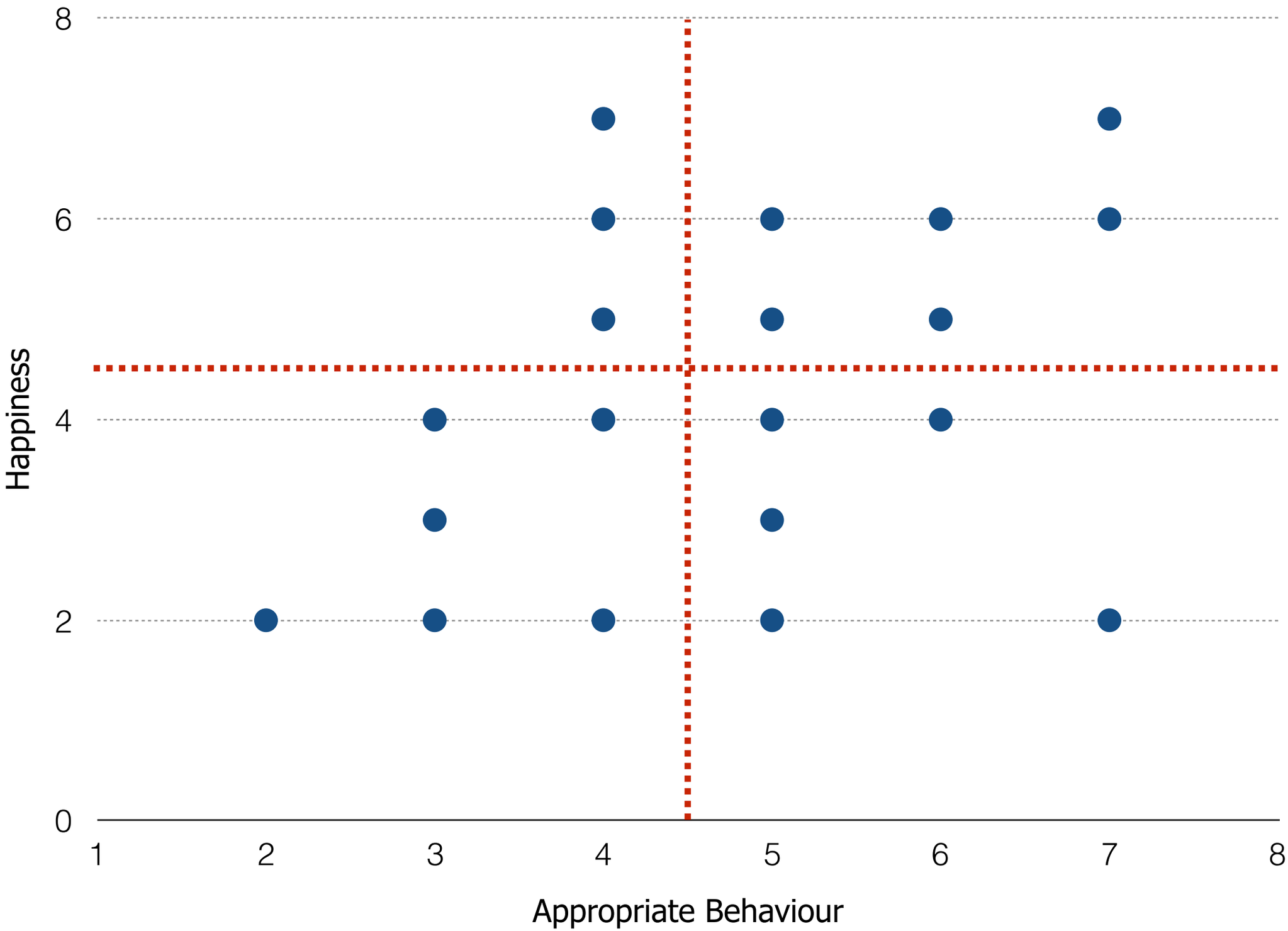
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Appropriate Behaviour

Happiness

| | Above Mean | Below Mean |
|---|---|---|
| Above Mean | 6 | 3 |
| Below Mean | 5 | 6 |

Appropriate Behaviour

The covariance of X and Y

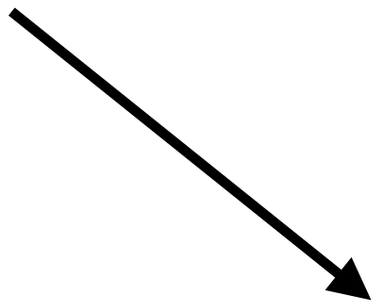
The average value of X

The average value of Y

$$\sigma_{xy} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y - \bar{y})}{N - 1}$$

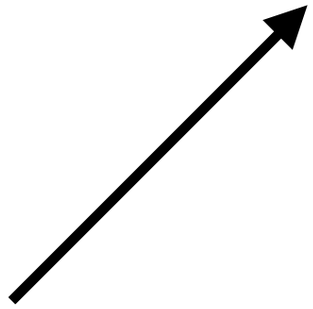
The total number of pairs (x,y)

Covariance of Appropriate Behaviour and Learning



$$\sigma_{xy} = 2.34$$

$$\sigma_{xy} = 0.96$$



Covariance of Appropriate Behaviour and Happiness

# Covariance

- The expression of the covariance is symmetric with respect to X and Y, its value does not change by switching predictor and criterion;
- It captures the relationship between variables as a systematic tendency to be on the same (or opposite) side of the mean;
- The value of the covariance is difficult to interpret.

# Recap

- Some random variables are expected to change according to one another;

- Conventionally, one of the two variables acts as predictor while the other acts as criterion;

- Switching between predictor and criterion does not change the results, the relationship has no direction.

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The estimated value of y when using a linear plot

The predictor (the independent variable)

$$\hat{y} = a + bx$$

The intercept

The slope

The prediction error

The estimated value of y when using a linear plot

$$E = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{N} (y_i - a - bx_i)^2$$

What is the "best" value for slope and intercept?

The derivative of the Error with respect to the intercept

The derivative of the Error with respect to the slope

$$\frac{\partial E}{\partial a} = 0 \qquad \frac{\partial E}{\partial b} = 0$$

The results of the equations are the values of intercept and slope that correspond to the minimum Error

The slope

The covariance of X and Y

$$b = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} = \frac{\sigma_{xy}}{\sigma_x^2}$$

The variance of X

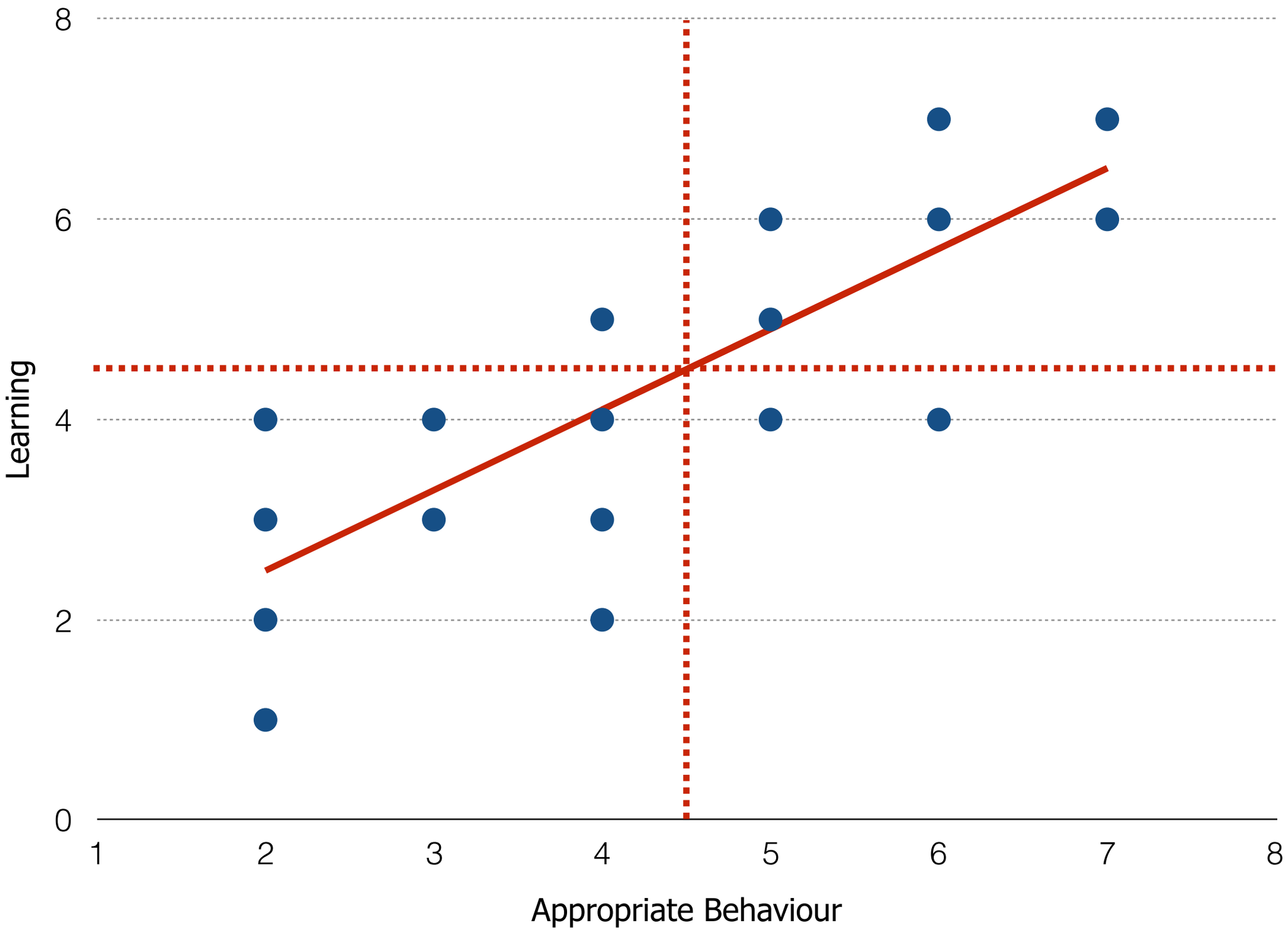
The averages of X and Y

The intercept

$$a = \frac{1}{N}\sum_{i=1}^{N} x_i - b\frac{1}{N}\sum_{i=1}^{N} y_i$$

The slope

The total number of pairs (x,y)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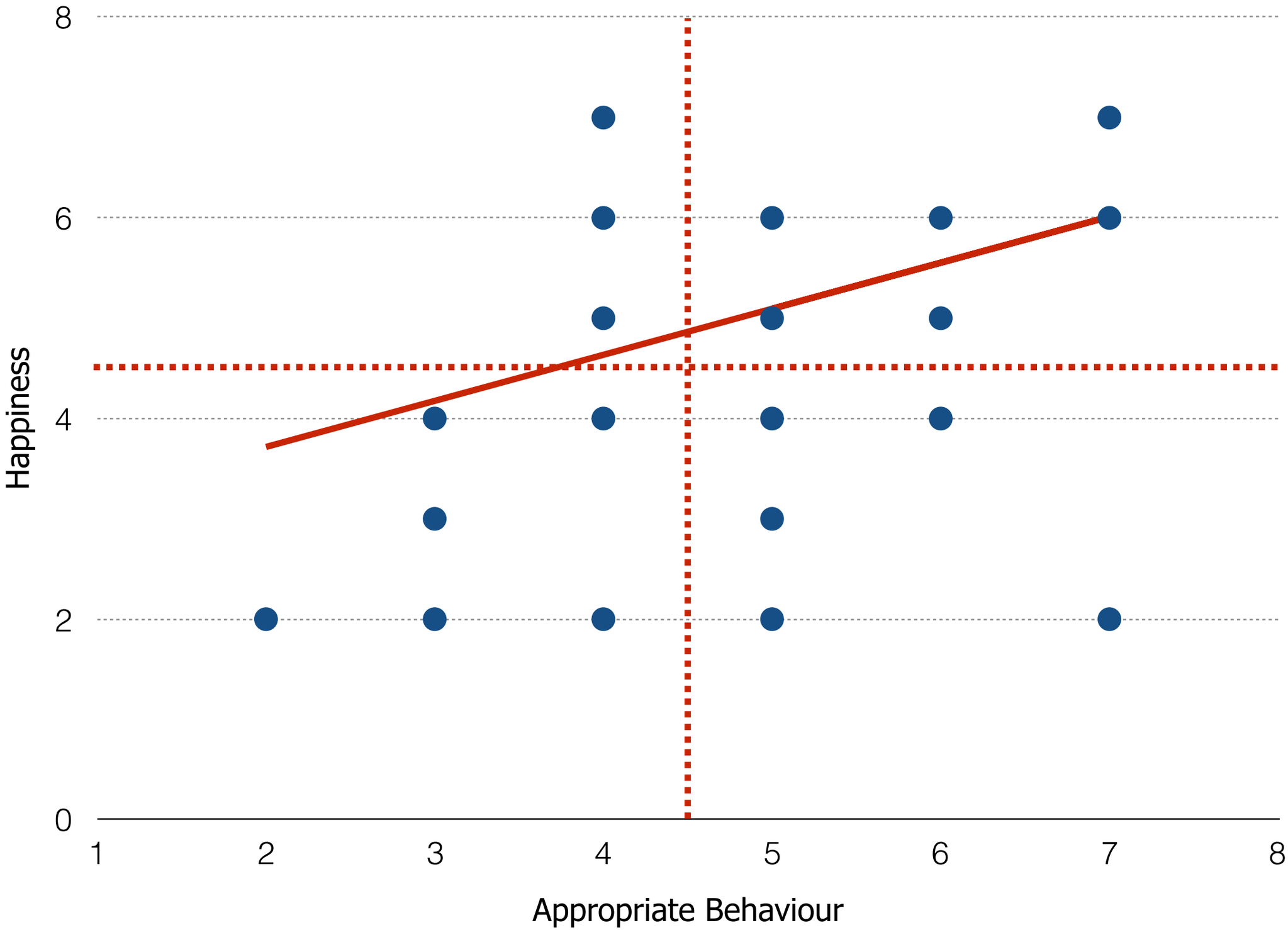
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Appropriate Behaviour

Average Prediction Error (Appropriate Behaviour vs Learning)

$$\frac{E}{N} = 1.07$$

$$\frac{E}{N} = 3.00$$

Average Prediction Error (Appropriate Behaviour vs Happiness)

# Recap

- The regression allows one to express the criterion as a function of the predictor;
- The use of a line is a constraint that is not necessarily respected by the data, it is an approximation;
- It is difficult to quantify how well the data fits the relationship.

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# Correlation

"The degree to which the points cluster around the regression line (in other words, the degree to which the actual values of Y agree with the predicted values) is related to the correlation between X and Y."

D.C.Howell, "Statistical Methods for Psychology", Chapter 9, Cengage Learning, 2009.

The Pearson correlation between X and Y

The covariance of X and Y

$$r = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

The standard deviation of X

The standard deviation of Y

The covariance of X and Y (multiplied by N-1)

The Pearson correlation coefficient between X and Y

$$\frac{\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i-\bar{x})^2\sum_{i=1}^{N}(y_i-\bar{y})^2}}$$

The variance of X (multiplied by N-1)

The variance of Y (multiplied by N-1)

The correlation can be adjusted when N (the number of pairs) is small

The non-adjusted value of the correlation

$$r_{adj} = \sqrt{1 - \frac{(1 - r^2)(N - 1)}{N - 2}}$$

The total number of pairs (x,y)

The Student's t variable can show whether r is statistically significant

The total number of pairs (x,y)

$$t = \frac{r\sqrt{N-1}}{\sqrt{1-r^2}}$$

The value of the correlation

# Research Hypothesis

- Research Hypothesis: The correlation is higher (or lower for the negative values) than what is expected by chance;
- Null Hypothesis: The correlation is lower (or higher for the negative values) than what is expected by chance.

The Null Hypothesis can be rejected with confidence level 0.01 (two-tailed)

Correlation between Appropriate Behaviour and Learning

$r = 0.79$

$r = 0.38$

The Null Hypothesis fails to be rejected

Correlation between Appropriate Behaviour and Happiness

# Recap

- The correlation measures the fraction of common variance with respect to the variance of the individual variables;
- The values of the correlation are comparable and easy to interpret;
- It is possible to test whether the correlation is statistically significant, i.e., higher than what is expected by chance.

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# Conclusions

- It is possible to measure the relationship between two variables, i.e., their tendency to change according to each other;
- The relationship has no direction (it is not possible to say whether one variable influences the other or vice versa);
- However, the analysis of the relationships can provide insight about the phenomena under exam.

# Thank You!